Boulder Summer School, 2024: Das Lectures' Problem Set.

1. Consider an equilibrium fiber network where it only costs energy to stretch fibers, i.e. this is a network made purely of springs, each of which has a spring constant α. You dilute this network by randomly removing springs with a uniform probability until only a given fraction, say $0 < p < 1$, of springs remain.

   (a) Use the Maxwell constraint counting (number of constraints equal number of degrees of freedom) to obtain the rigidity percolation threshold p_{rp} for such a network with a coordination number z in d dimensions. What is p_{rp} for 2D disordered networks based on a triangular lattice and a square lattice?

   (b) Following the discussion in class, do an effective medium theory (EMT) calculation to find the rigidity percolation threshold for a disordered 2d triangular lattice-based network made only of springs. It should agree with your result from the constraint counting argument.

   (c) The EMT gives you the effective medium spring constant α_{em}/α as a function of the occupation probability of springs. Plot α_{em}/α from (b) as a function of p.

2. Now consider two variants of this problem.

   (a) The springs in one of the three directions of the triangular-lattice-based network are 50 times stiffer than the other two directions. Do an EMT calculation and calculate the effective medium spring constant as a function occupation probability of springs. Plot α_{em}/α as a function of p. Comment on the rigidity percolation threshold.

   (b) The stiffer springs above (say in the x-direction) occur with a larger probability (say p') than the softer springs (other two directions) (say p). Once again do an EMT. How do your results change?