

July 2006

Colloids

- What's a colloid?
- Colloidal Interactions
- Playing with Colloidal Crystals
- Hard Spheres and thermodynamics
- The Crystal – Liquid Transition
- Packings and Candies
- Soft Spheres and electrostatics
- Controlling Colloids with field gradients
- What's new?

Colloid

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It has been suggested that *[Colloidal system](#)* be [merged](#) into this article or section. ([Discuss](#))

In general, a **colloid** or **colloidal dispersion** is a substance with components of one or two [phases](#), a type of [mixture](#) intermediate between *[homogeneous solution](#)* and *[heterogeneous mixture](#)* with properties also intermediate between a solution and a mixture. Typical membranes restrict the passage of dispersed colloidal particles more than they restrict the passage of dissolved ions or molecules; i.e. ions or molecules may diffuse through a membrane through which dispersed colloidal particles will not diffuse. The dispersed phase particles are largely affected by the [surface chemistry](#) existent in the colloid.

Many familiar substances, including [butter](#), milk, cream, aerosols (fog, smog, smoke), [asphalt](#), inks, paints, [glues](#), and sea foam are colloids. This field of study was introduced in 1861 by [Scottish](#) scientist [Thomas Graham](#).

The size of dispersed phase particles in a colloid range from one nanometer to one micrometer. Dispersions where the particle size is in this range are referred to as **colloidal aerosols**, **colloidal emulsions**, **colloidal foams**, or **colloidal suspensions or dispersions**. Colloids may be colored or translucent because of the [Tyndall effect](#), which is the scattering of light by particles in the colloid.

Colloids?

COLLOIDS. In a paper entitled "Liquid Diffusion applied to Analysis" published in 1861 in the *Philosophical Transactions*, Thomas Graham (*q.v.*) described the results of an investigation carried out with very simple means. Aqueous solutions were placed in a cylindrical vessel the bottom of which was formed by a piece of animal membrane, like pig's bladder, or by the recently invented parchment paper, and the membrane immersed in water. The amount of dissolved substance which diffused into the outer water was determined from time to time. Graham found that the numerous substances examined fell into two classes: those which diffused in appreciable amounts and those which hardly passed through the membrane in perceptible quantities. The former were without exception substances known to crystallize from their solutions, like various salts or sugar, while the latter, among which were albumin, gum arabic or gelatin, had never been known as crystals. Graham accordingly called the first class *crystalloids* and the second class *colloids* (from *colla*, glue).

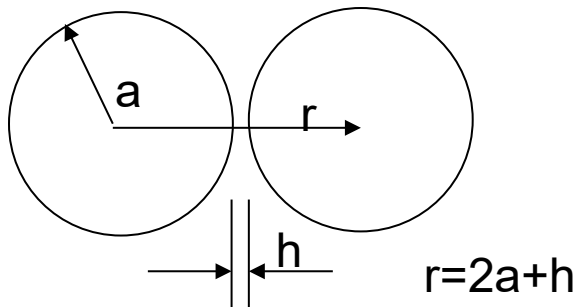
van der Waals attraction

Induced dipole-dipole

$$E \propto -\frac{\alpha_1 \alpha_2}{r^6}$$

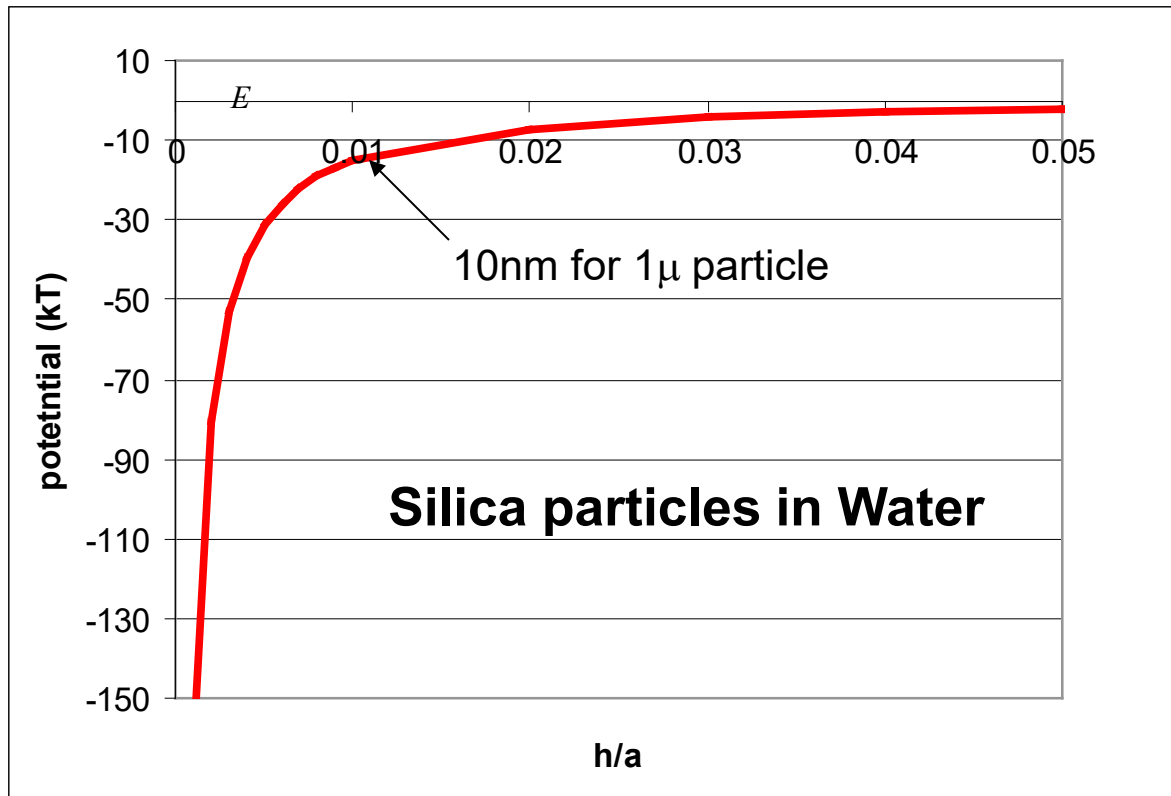
points

$$E = \frac{-A(0)}{6} \left[\frac{2a^2}{r^2 - 4a^2} + \frac{2a^2}{r^2} + \ln \left(\frac{r^2 - 4a^2}{r^2} \right) \right]$$



$$E \propto -\frac{1}{r^6}, r \gg a$$

$$E \propto -\frac{1}{h}, h \ll a$$



	$A_{\text{eff}}(0)/10^{-20} \text{ J}$	
	Vacuum Water	
polystyrene	7.9	1.3
hexadecane	5.4	—
gold	40	30
silver	50	40
copper	40	30
water	4.0	—
pentane	3.8	0.34
decane	4.8	0.46
hexadecane	5.2	0.54
water	3.7	—
quartz		
fused	6.5	0.83
crystalline	8.8	1.70
fused silica	6.6	0.85
calcite	10.1	2.23
calcium fluoride	7.2	1.04
sapphire	15.6	5.32
poly(methyl methacrylate)	7.1	1.05
poly(vinyl chloride)	7.8	1.30
polyisoprene	6.0	0.74
poly(tetrafluoroethylene)	3.8	0.33

Parsegian & Weiss (1981)

Hough & White (1980)

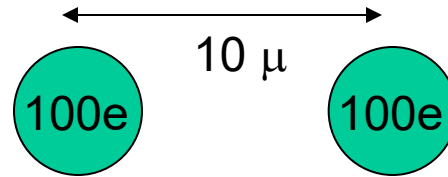
Electrostatic interactions

Bare

$$\frac{e^2}{\epsilon_0 r} = .69 k_B T$$

Single electron charge,
r=1 micron, room temp

$$\frac{Z^2 e^2}{\epsilon r} = 230 k_B T$$



$\epsilon=30$

(water $\epsilon=80$, decalin $\epsilon=3$)

Screened by conducting ions/electrons

$$\frac{Z^2 e^2}{\epsilon r} e^{-r/\lambda}$$

Yukawa Potential

Colloids

Debye Length

$$\lambda^{-2} = \frac{4\pi n_{ions} e^2}{\epsilon k_B T}$$

Metals

Thomas-Fermi length

$$\lambda^{-2} = \frac{4\pi n_{els} e^2}{\epsilon_0 k_B T_F}$$

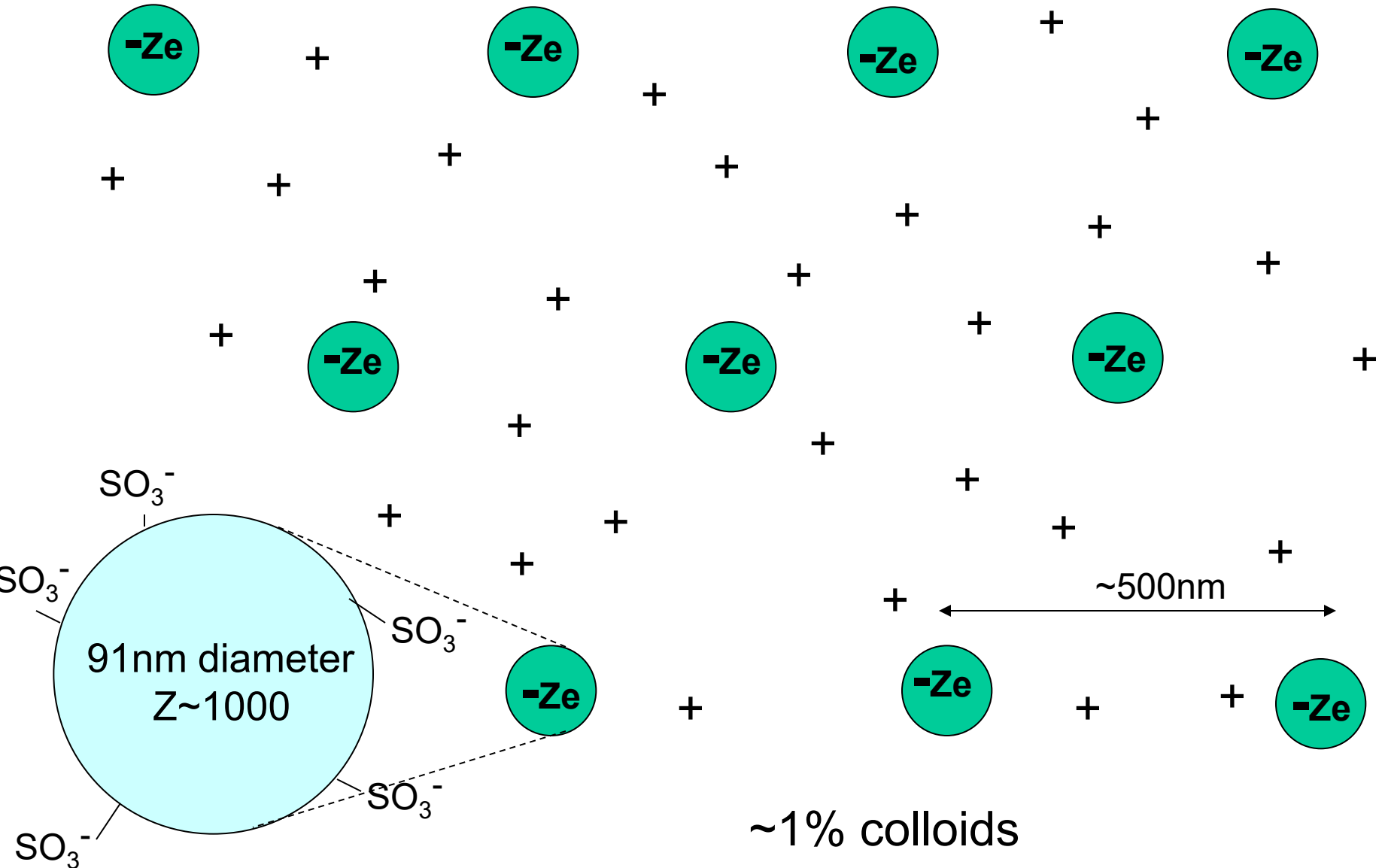
Typical screening length: 1 milliMole NaCl in water $\lambda=10\text{nm}$

Colloidal Interactions

		Range	Strength
Van der Waals	Attractive	0.3-10nm	0-100$k_B T$
Electrostatic	Attractive Repulsive	0.3-1000nm	0-1000 $k_B T$
Depletion	Attractive	10-300nm	0-20 $k_B T$
Brushes	Hard repulsion Soft repulsion Soft-repulsion Reversible attraction	5-50nm	0-50 $k_B T$
DNA	Repulsion Specific attraction Reversible	10-5000nm	0-100 $k_B T$

Colloidal Crystals - Polyballs

Polystyrene Spheres in Water

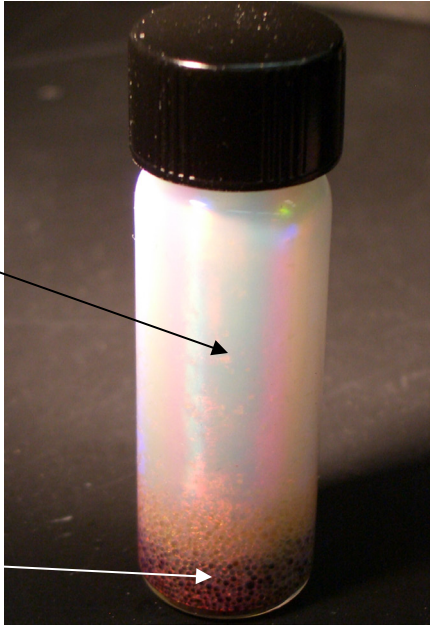


Playing with Polyballs

Crystal → long range order, rigidity

Colloidal
Crystal
Bragg
Scattering

Ion
Exchange
Resin



Vigorously twist
sample
Shear melts
To liquid

Bragg gone



Shake sample

Resin doesn't
fall
→ finite shear
modulus



Twist slowly
and stop
Shear aligns
layers

Brilliant Bragg
Colors

Shear Modulus → weak interactions?

Colloidal Crystals ~ 10 dynes/cm²

Typical metals $\sim 10^{12}$ dynes/cm²

$G \propto Vn$

modulus ↑ ↓

metals $V \sim \frac{e^2}{1A^0} \sim 10eV \gg k_B T$ That's why they're both solid at room temp.

colloids $V \sim \frac{Z^2 e^2}{\epsilon(1\mu m)} e^{-\kappa r} \sim \frac{10^6 e^2}{(100)(1000A^0)} e^{-3} \sim 10eV \gg k_B T$

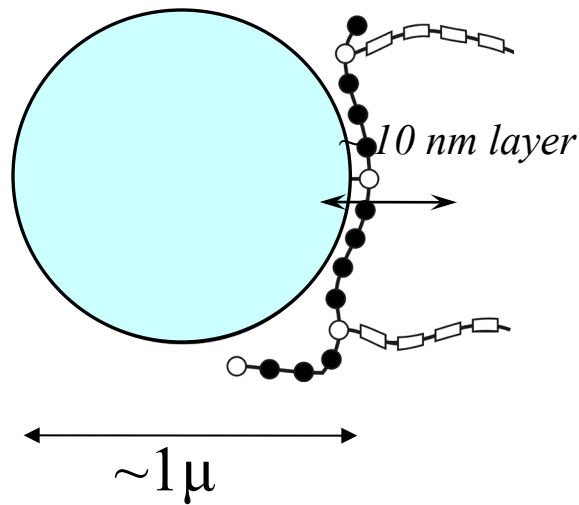
But $n_{\text{metal}} \sim 10^{24}/\text{cm}^3$

$n_{\text{colloid}} \sim 10^{13}/\text{cm}^3$

Difference is just density
- same for all “soft” condensed matter

Polymer **Hard Spheres** - Colloids in Oil

PMMA-PHSA

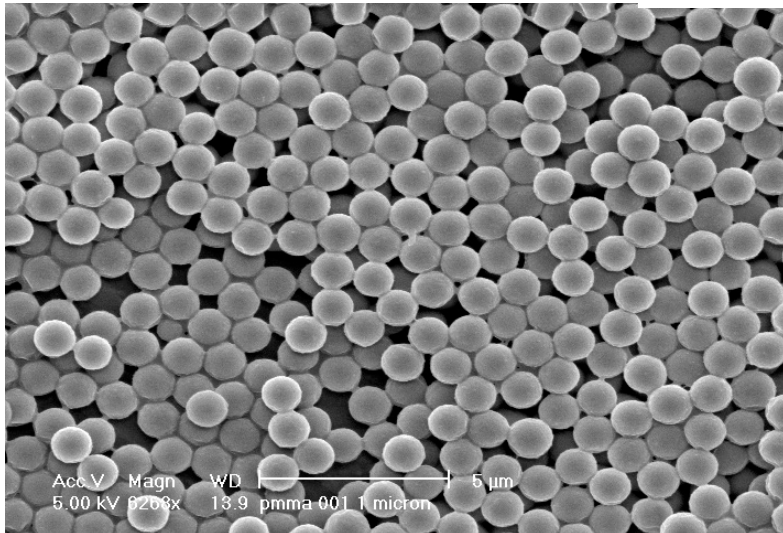
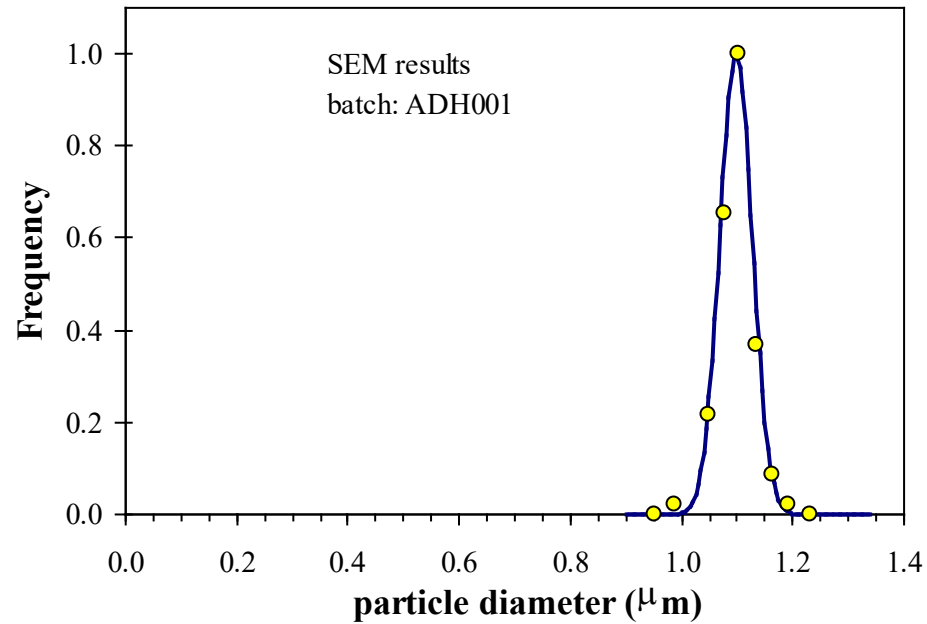


Steric stabilization

methyl methacrylate (•); methacrylic acid (◻); glycidyl
methacrylate (◊); 12-hydroxystearic acid (◻)

Originally Ron Ottewill - Bristol
ours were from Andy Schofield - Edinburgh
now home grown by **Andy Hollingsworth**

Poly(methyl methacrylate)
 latices stabilized by
poly(12-hydroxy-steric acid)
 in nonaqueous media

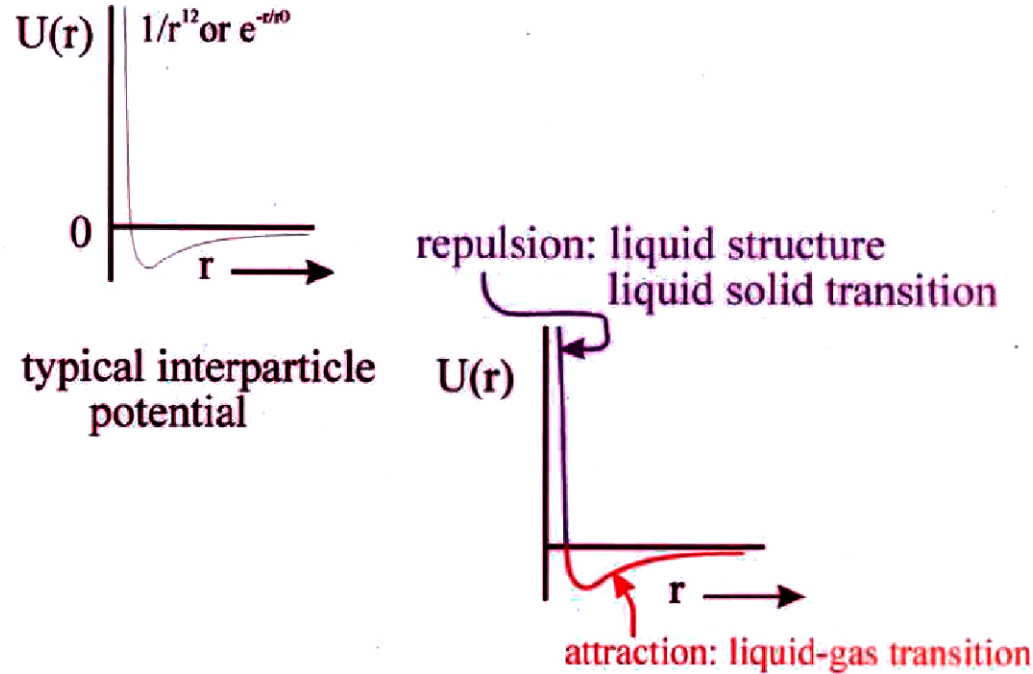


$$\langle D_p \rangle = 1.098 \mu m$$

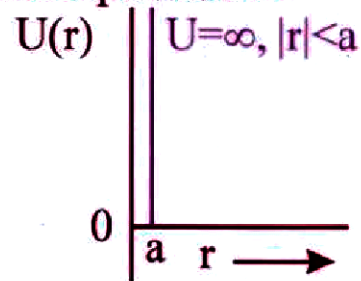
$$\sigma^2 = 0.029$$

$$PDI \equiv \sigma^2 / \langle D_p \rangle = 0.026$$

Why Hard Particles?



Most interesting things depend on repulsive part
simplest repulsion is:



The essence of the problem -- **Hard Spheres**

Packing Densities for Spheres

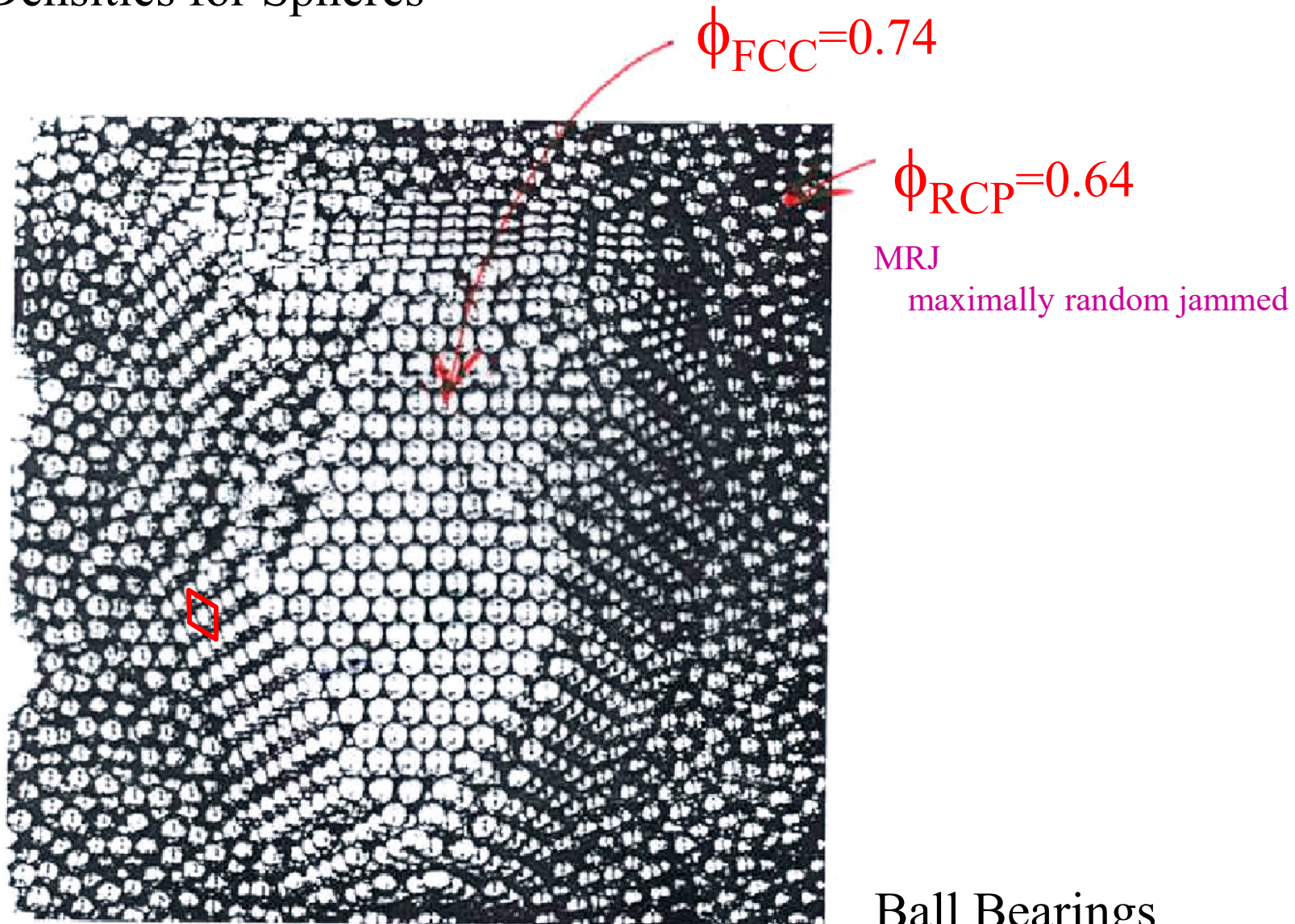
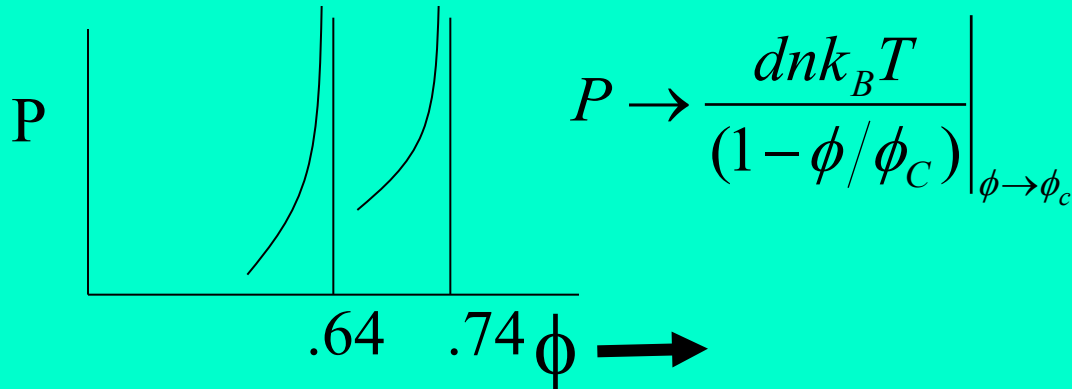


FIGURE 14. Face-centred cubic 'crystal' surrounded by 'liquid' caused by shearing ball-bearing mass. 111 face is shown at the top surface.

van der Waals and excluded volume

$$S = Nk_B \ln(V - Nb) = Nk_B \ln(V(1 - \phi/\phi_C)) \quad P = \frac{Nk_B T}{V - Nb} = \frac{Nk_B T}{V(1 - \phi/\phi_C)} \quad \text{Exact in 1D}$$

Exact asymptotic form in any dimension



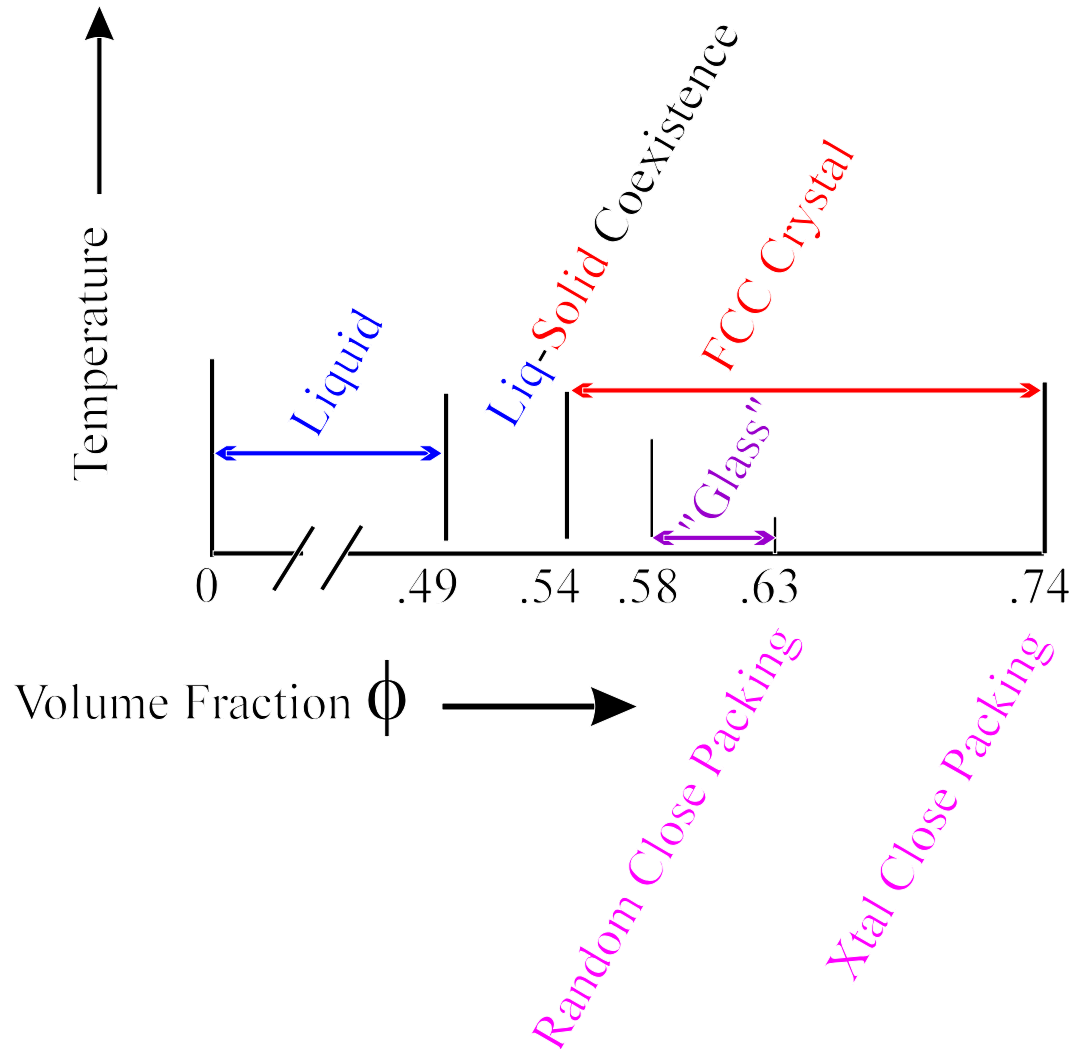
$\Rightarrow$ *Entropy drives liquid to crystal*

$$S_{\text{liquid}} \rightarrow 0 \text{ as } \phi \rightarrow .64$$

$$S_{\text{crystal}} \rightarrow 0 \text{ as } \phi \rightarrow .74$$

Highest Packing Fraction determines Stable High Density Phase

Hard Sphere Equilibrium Phase Diagram



Phase Diagram by 1g Experiment

325 nm PMMA/decalin/CS₂



Pusey & van Megen, *Nature*, 320 (1986) 340

PHaSE

The Physics
of Hard Spheres
Experiment



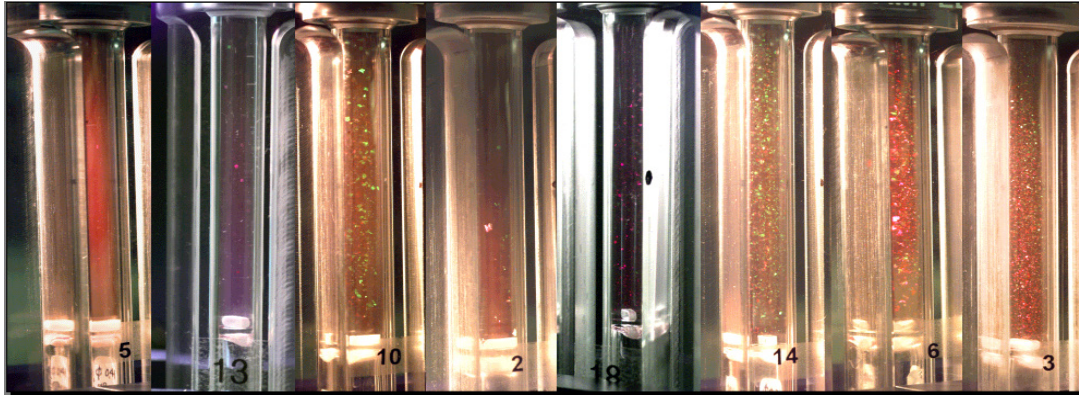
NASA

nyma

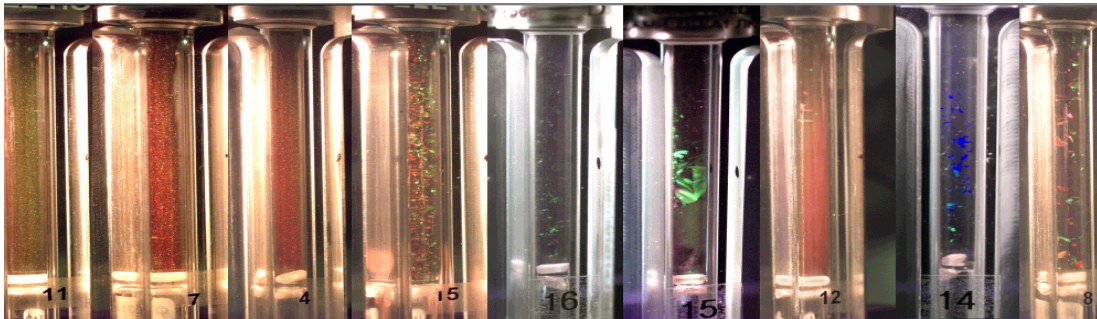
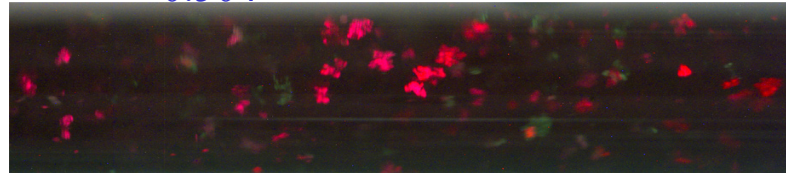
ADF

Princeton University

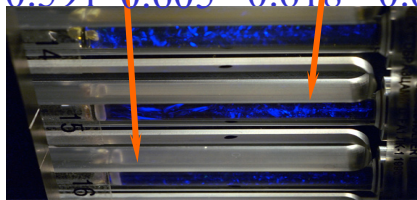
CDOT 1998



0.50 0.502 0.504 0.512 0.516 0.518 0.524 0.537



0.547 0.549 0.561 0.591 0.605 0.618 0.619 0.633 0.634



Bewegung v. in ruhenden Flüssigkeiten suspendierten Teilchen. 551

$$p = \frac{RT}{V^*} \frac{n}{N} = \frac{RT}{N} \cdot \nu,$$

wobei N die Anzahl der in einem Gramm-Molekül enthaltenen wirklichen Moleküle bedeutet. Im nächsten Paragraph soll gezeigt werden, daß die molekularkinetische Theorie der Wärme wirklich zu dieser erweiterten Auffassung des osmotischen Druckes führt.

§ 2. Der osmotische Druck vom Standpunkte der molekularkinetischen Theorie der Wärme.¹⁾

Sind $p_1 p_2 \dots p_l$ Zustandsvariable eines physikalischen Systems, welche den momentanen Zustand desselben vollkommen bestimmen (z. B. die Koordinaten und Geschwindigkeitskomponenten aller Atome des Systems) und ist das vollständige System der Veränderungsgleichungen dieser Zustandsvariablen von der Form

$$\frac{\partial p_\nu}{\partial t} = \varphi_\nu(p_1 \dots p_l) (\nu = 1, 2 \dots l)$$

gegeben, wobei $\sum \frac{\partial \varphi_\nu}{\partial p_\nu} = 0$, so ist die Entropie des Systems durch den Ausdruck gegeben:

$$S = \frac{\bar{E}}{T} + 2\pi \lg \int e^{-\frac{E}{2\pi T}} dp_1 \dots dp_l.$$

Hierbei bedeutet T die absolute Temperatur, $\bar{E}$ die Energie des Systems, E die Energie als Funktion der p_ν . Das Integral ist über alle mit den Bedingungen des Problems vereinbaren Wertekombinationen der p_ν zu erstrecken. π ist mit der oben erwähnten Konstanten N durch die Relation $2\pi N = R$ verbunden. Für die freie Energie F erhalten wir daher:

$$F = -\frac{R}{N} T \lg \int e^{-\frac{E}{RT}} dp_1 \dots dp_l = -\frac{RT}{N} \lg B.$$

1) In diesem Paragraph sind die Arbeiten des Verfassers über die Grundlagen der Thermodynamik als bekannt vorausgesetzt (vgl. Ann. d. Phys. 9. p. 417. 1902; 11. p. 170. 1903). Für das Verständnis der Resultate der vorliegenden Arbeit ist die Kenntnis jener Arbeiten sowie dieses Paragraphen der vorliegenden Arbeit entbehrlich.

§ 3. Theorie der Diffusion kleiner suspendierter Kugeln.

In einer Flüssigkeit seien suspendierte Teilchen regellos verteilt. Wir wollen den dynamischen Gleichgewichtszustand derselben untersuchen unter der Voraussetzung, daß auf die einzelnen Teilchen eine Kraft K wirkt, welche vom Orte, nicht aber von der Zeit abhängt. Der Einfachheit halber werde angenommen, daß die Kraft überall die Richtung der X -Achse habe.

Es sei ν die Anzahl der suspendierten Teilchen pro Volumeneinheit, so ist im Falle des thermodynamischen Gleichgewichtes ν eine solche Funktion von x , daß für eine beliebige virtuelle Verrückung δx der suspendierten Substanz die Variation der freien Energie verschwindet. Man hat also:

$$\delta F = \delta E - T \delta S = 0.$$

Es werde angenommen, daß die Flüssigkeit senkrecht zur X -Achse den Querschnitt 1 habe und durch die Ebenen $x=0$ und $x=l$ begrenzt sei. Man hat dann:

$$\delta E = - \int_0^l K \nu \delta x dx$$

und

$$\delta S = \int_0^l R \frac{\nu}{N} \frac{\partial \nu}{\partial x} dx = - \frac{R}{N} \int_0^l \frac{\partial \nu}{\partial x} \nu dx.$$

Die gesuchte Gleichgewichtsbedingung ist also:

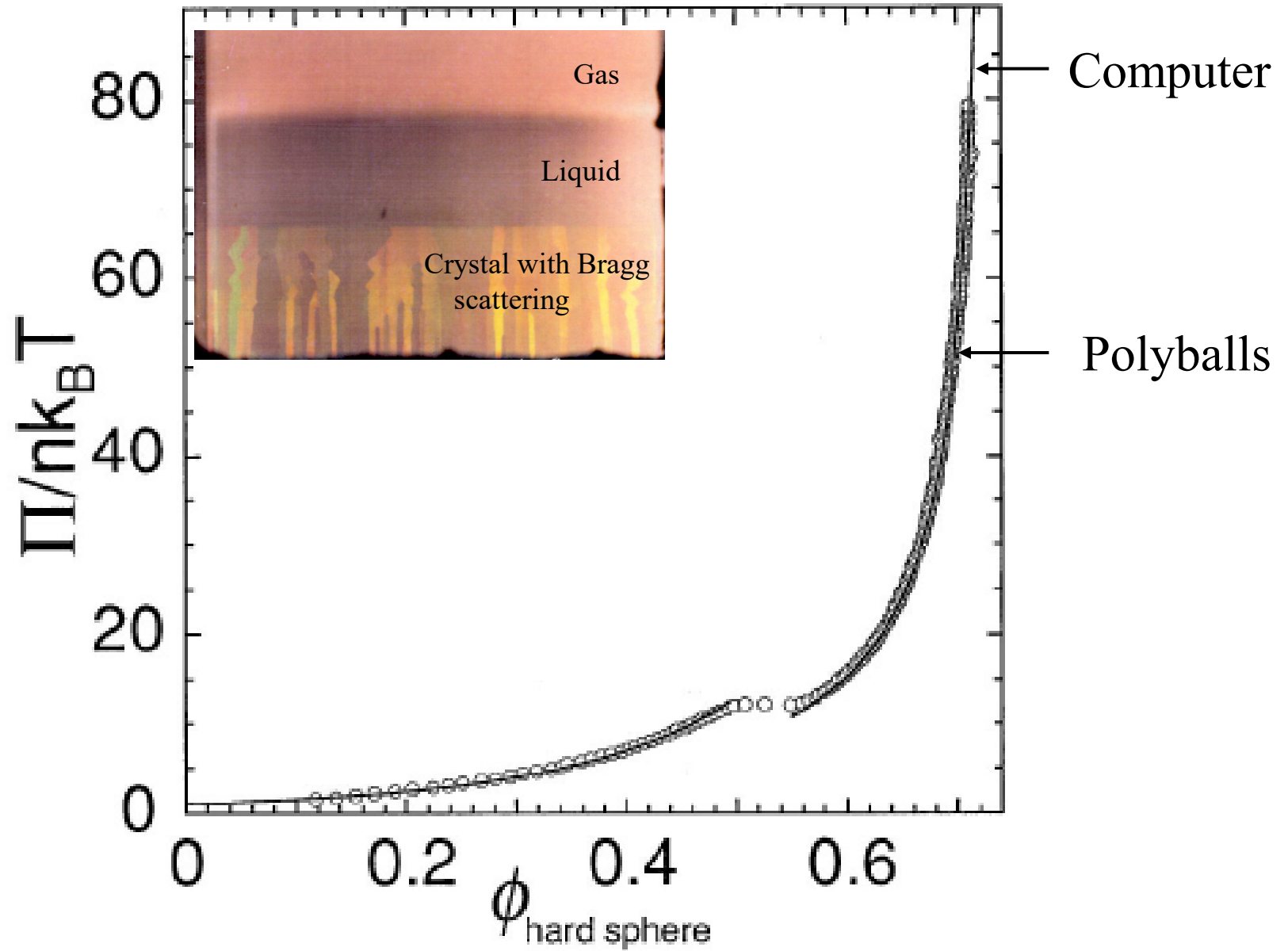
$$(1) \quad -K\nu + \frac{RT}{N} \frac{\partial \nu}{\partial x} = 0$$

oder

$$K\nu - \frac{\partial p}{\partial x} = 0.$$

Die letzte Gleichung sagt aus, daß der Kraft K durch osmotische Druckkräfte das Gleichgewicht geleistet wird.

Die Gleichung (1) benutzen wir, um den Diffusionskoeffizienten der suspendierten Substanz zu ermitteln. Wir können den eben betrachteten dynamischen Gleichgewichtszustand als



Colloids are just like Granular Materials

But:

- Gravity irrelevant
- Inertia irrelevant
- Friction irrelevant
- Nonequilibria irrelevant
- Set “Effective Temperature” = Temperature

What’s left?

--*Geometry*

Packing of Hard Particles

Important for:

- Grain*
- Structure of Liquids, Solids and Glasses
- Freezing Transition
- Flow and Plasticity of Sand
- Optimization of data storage and transmission in the presence of Noise
- Opto-electronic Materials,
Light localization and Photonic Bandgaps

In closing we must not forget the commentary on Random Packing which Saint Luke attributes to Jesus, “Give and it shall be given unto you; good measure, pressed down, and shaken together, and running over, shall men give into your bosom. For with the same measure that ye mete withal shall be measured to you again.” -Bernal

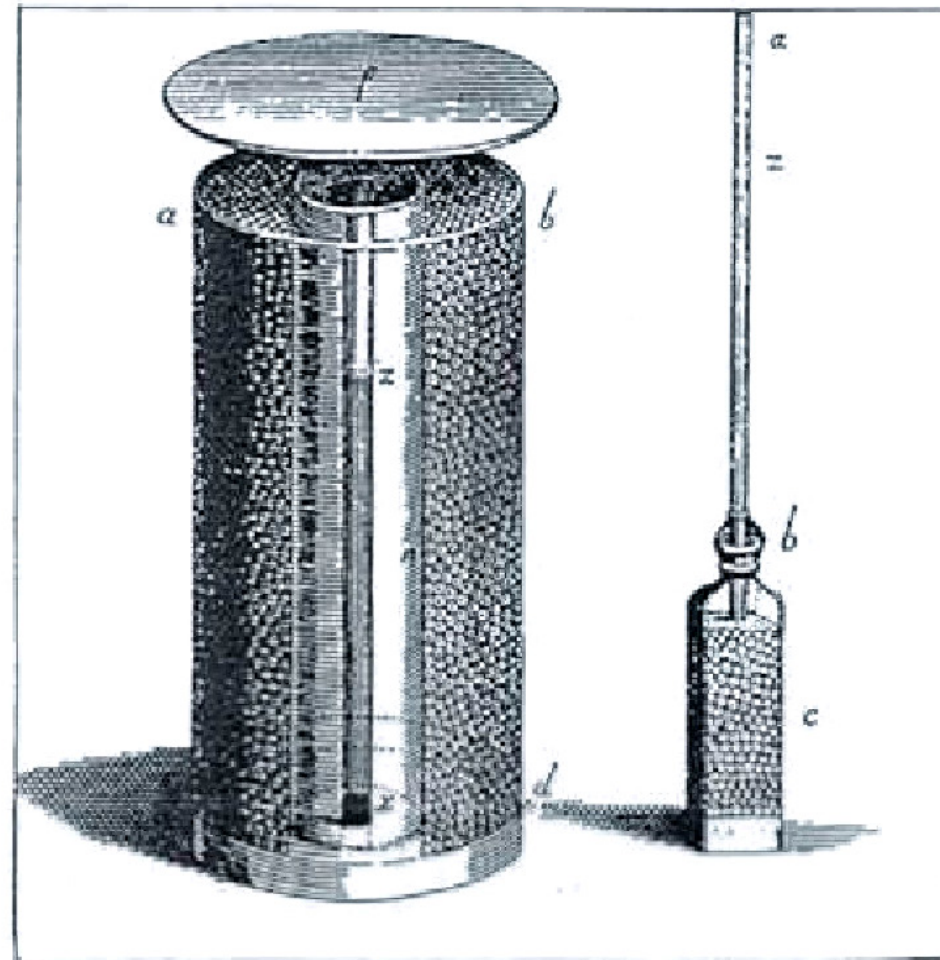


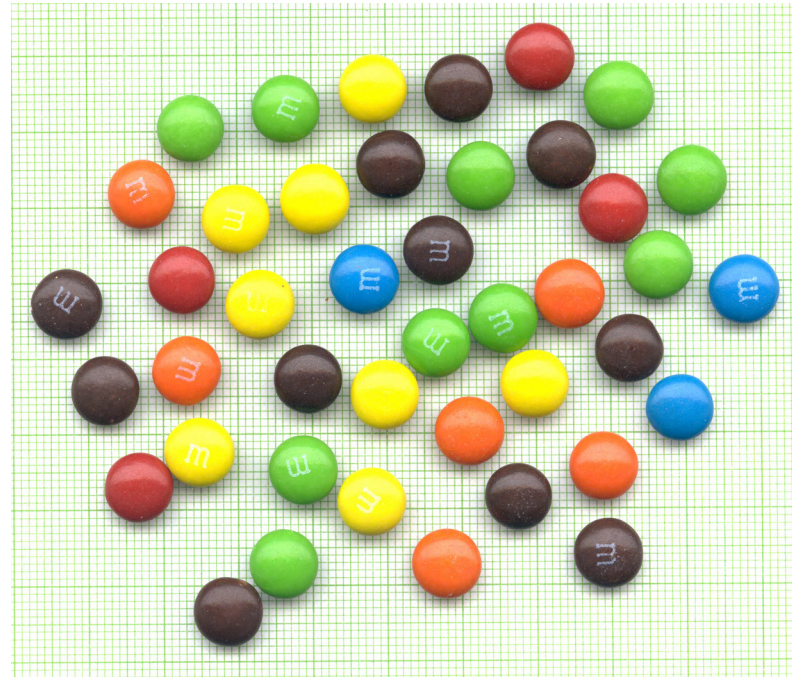
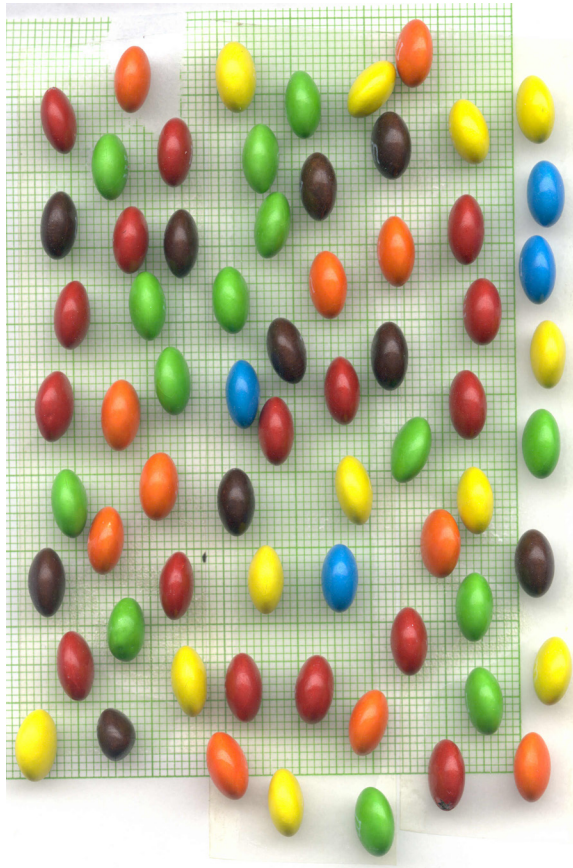
Figure 2.9 Stephen Hales (1727) was a highly gifted biologist whose interests included the uptake of water by plants. The diagram on the left shows the apparatus he used to demonstrate the substantial force exerted by dilating peas. However, when the lid (*l*) was covered with a weight great enough to prevent its lifting, the dilated peas deformed into the Wigner-Seitz cells of the rcp structure. The peas in the bottle at right, used by Hales in a related experiment, would not have served so well to illustrate random close packing because of the presence of the crystallinity-inducing planar walls.

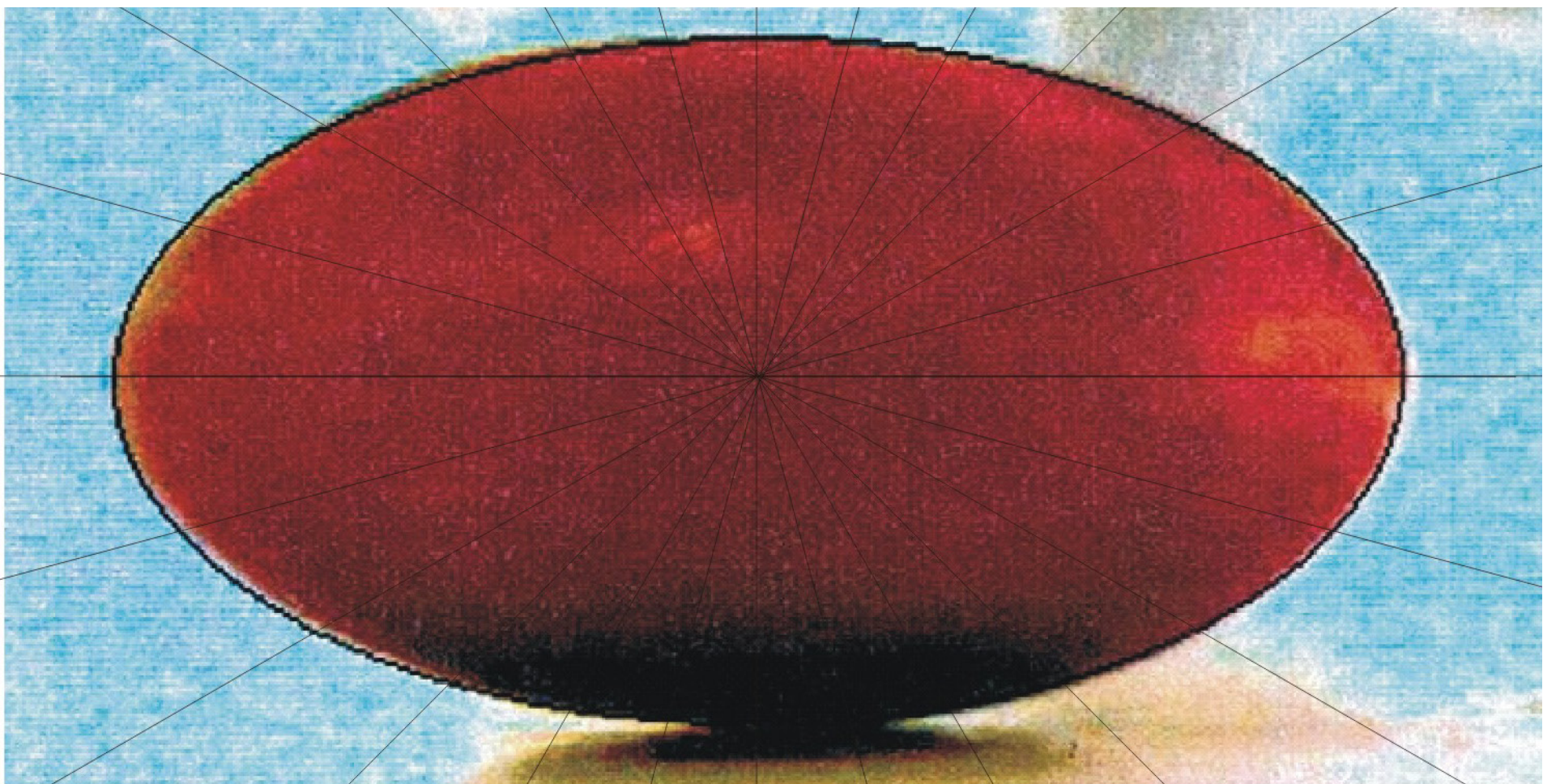




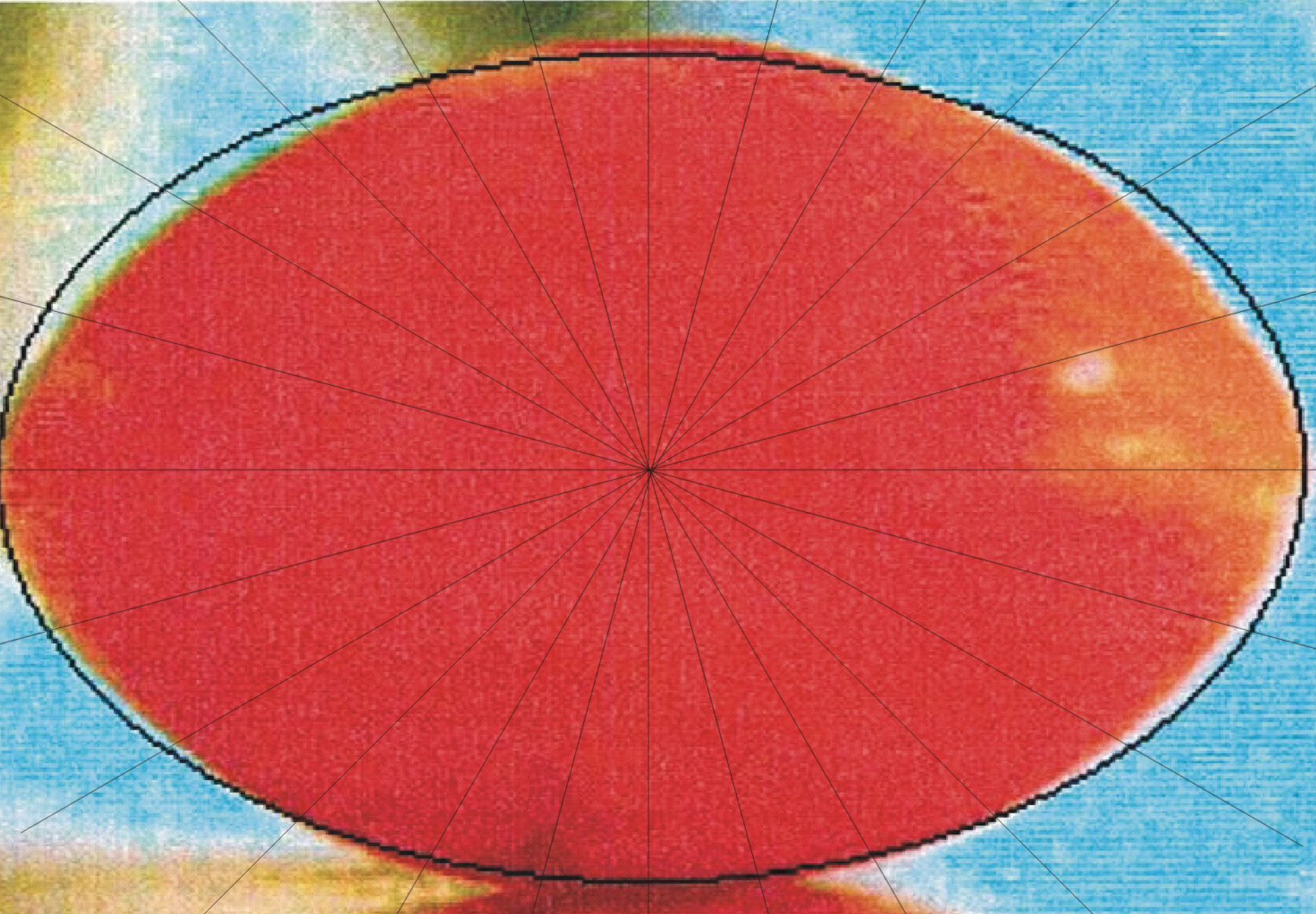


David Sachs





$$\text{M\&M}^{\text{®}} (\delta r^2 / \langle r \rangle^2)^{1/2} < .01$$

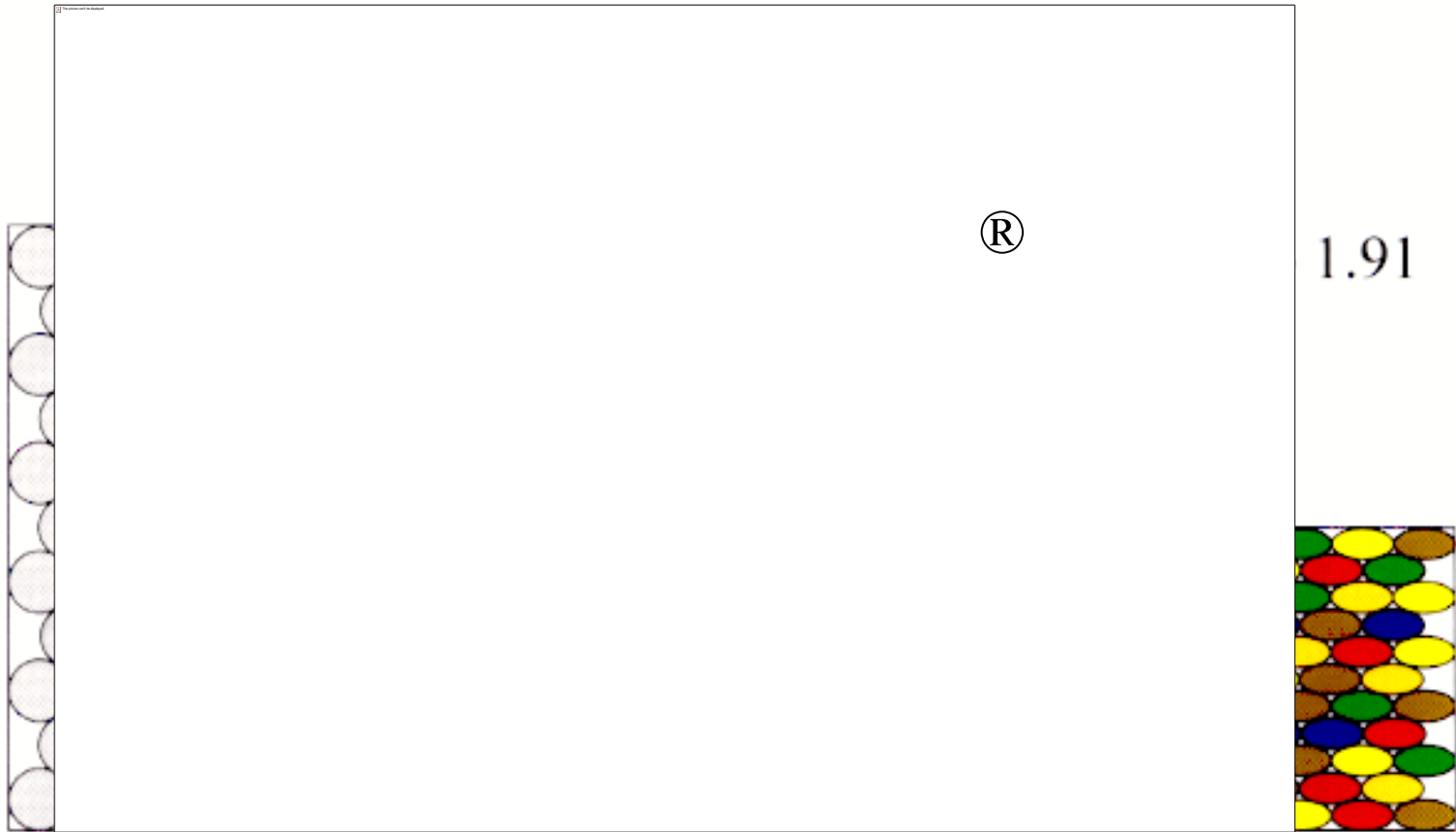


Skittle[®] $(\delta r^2 / \langle r^2 \rangle)^{1/2} \sim .03$



~28,000 m&m's[®] minis

Crystal Packing of Spheres and Ellipsoids

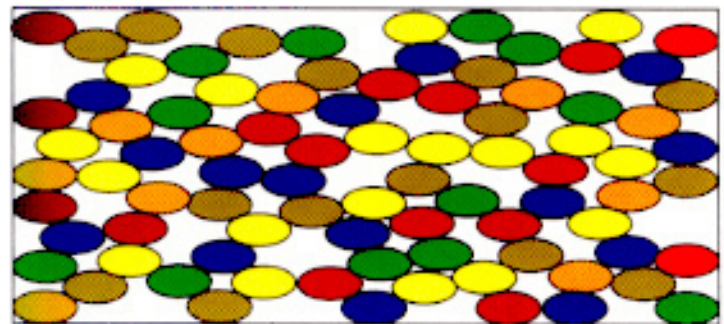
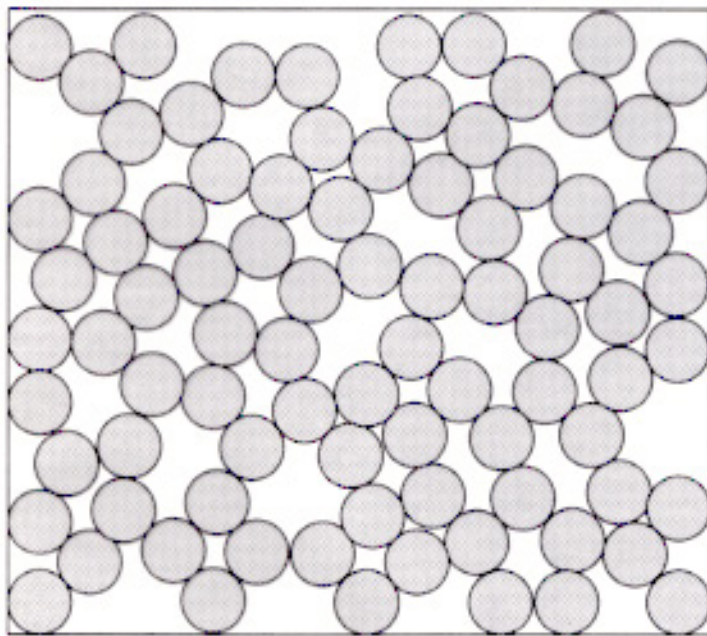


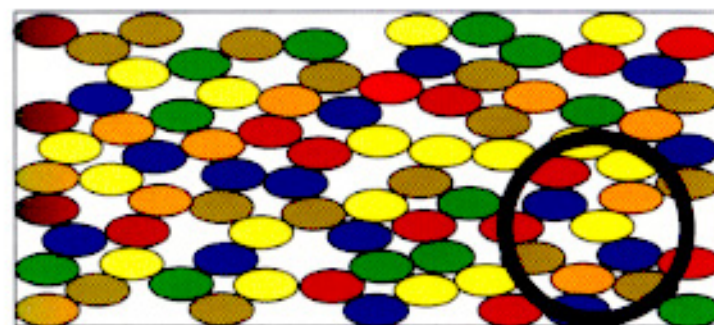
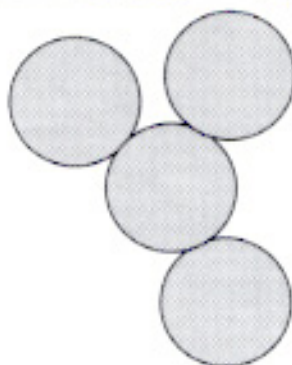
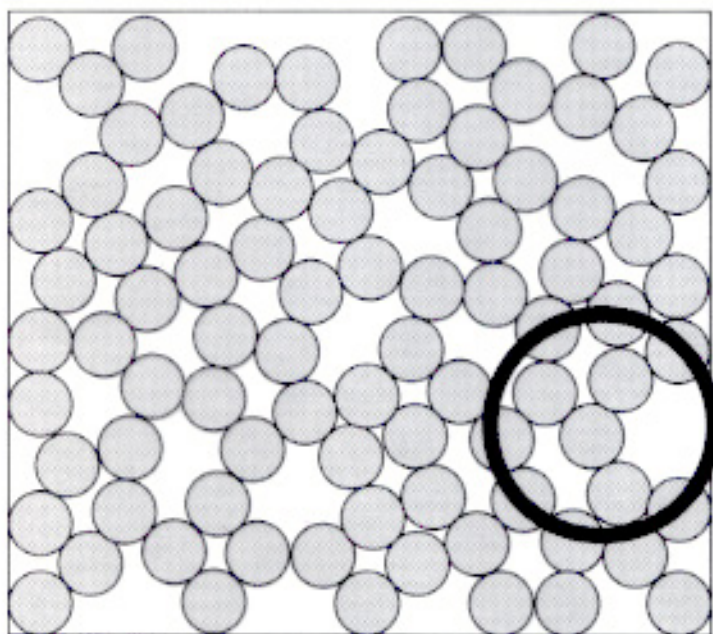
$$\varphi = \frac{N \frac{4}{3} \pi r^3}{L^3}$$

$$\varphi = \frac{N \frac{4}{3} \pi r^2 (r / 1.91)}{L^2 (L / 1.91)}$$

Volume Fraction 0.7404

Random Packing of Spheres and Ellipsoids





Do ellipsoids pack denser than spheres?

	Aspect ratio	5 litre	1 litre	0.5 litre
Regular M&M's [®]	1.89+/- .005	.685	.676	.674
M&M minis [®]	1.91+/- .005	.689 ±.01	.677	.676
Ball bearings	1+/- .001			.636

YES!

The *isostatic* conjecture – Shlomo Alexander, Sam Edwards

Coordination number $Z=2d$

Random packings are just barely constrained.

d degrees of freedom per particle

→ Nd constraints to define system

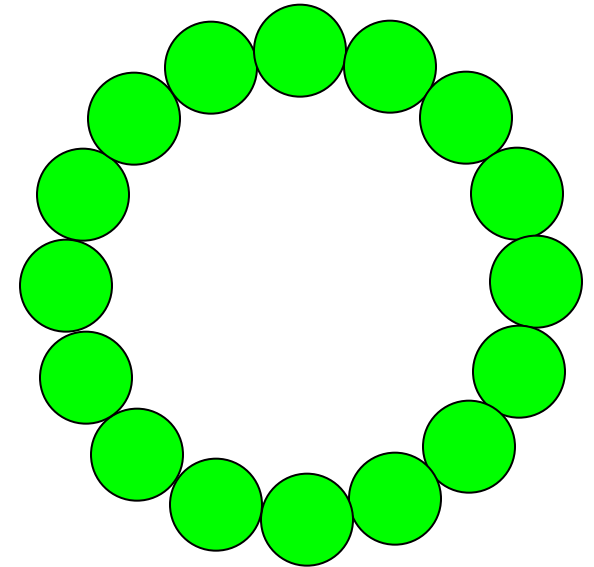
→ Nd contacts → $2Nd$ touching neighbors

Spheres 1D, $d=1$, $Z=2$

2D, $d=2$, $Z=4$

3D, $d=3$, $Z=6$

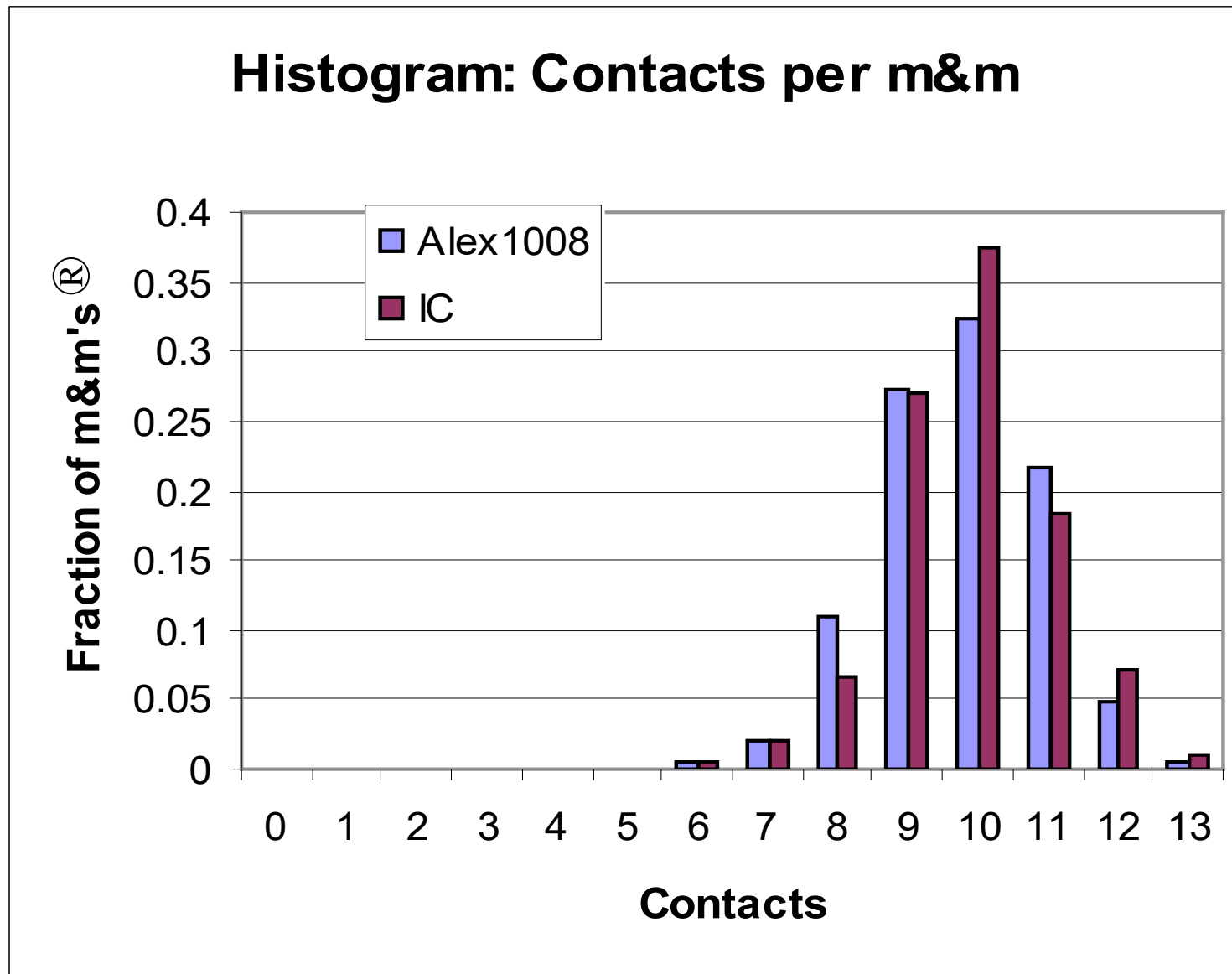
RCP 3D, $6+\epsilon$ neighbors



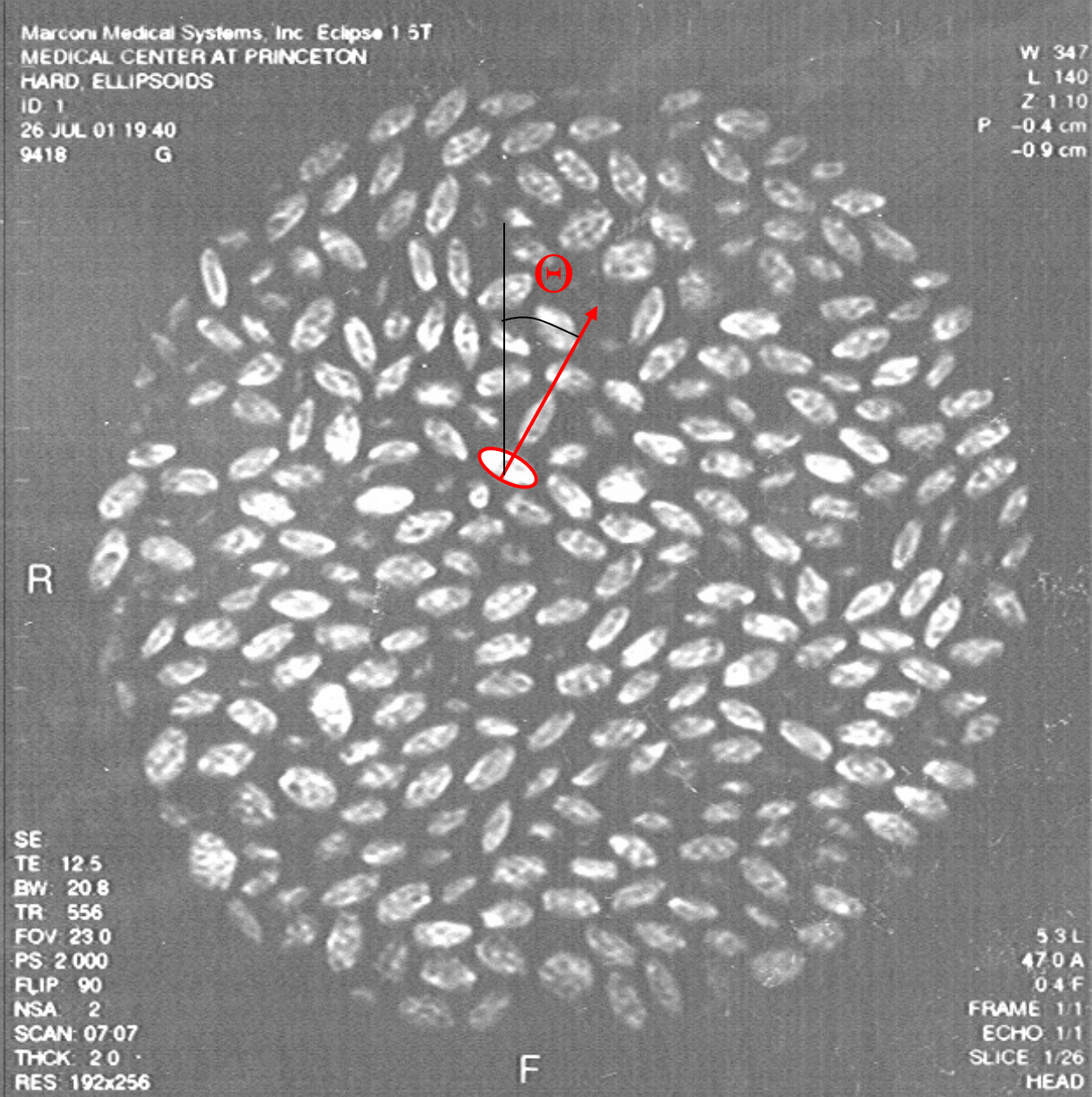
m&m's[®] 3D, $d=3\text{trans}+2\text{rotate}=5$, $Z=10$

To get 10 neighbors needs to pack denser





Average # contacts/m&m = 9.87



No orientational order

Nematic order parameter

$$\eta = 2 \cos^2(\Theta) - 1 = .04$$

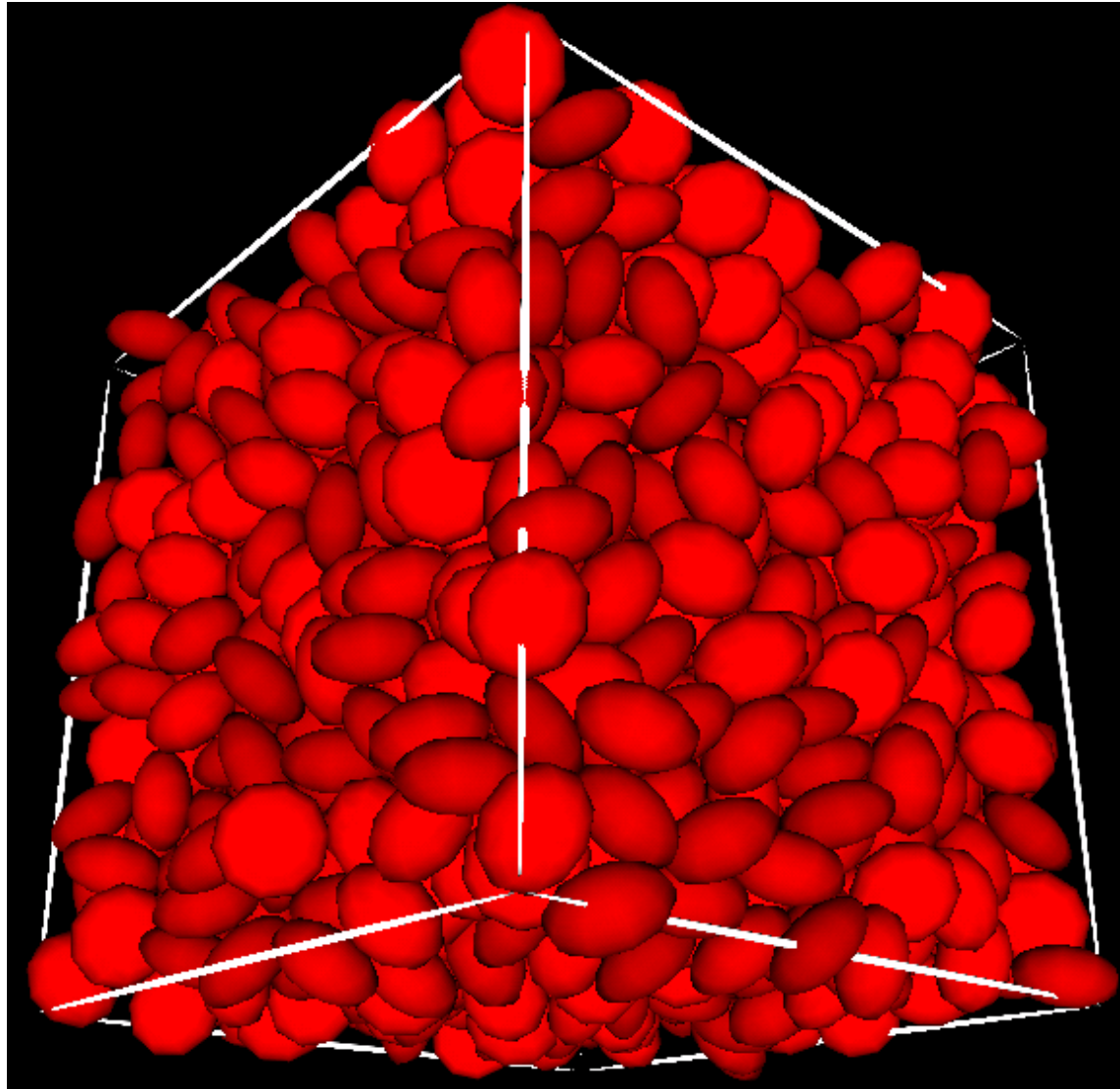
if random

$$\eta = 0 \pm .06$$

Full Head Scan MRI – Princeton Hospital

Torquato, Chaikin

Stillinger-Lubachevsky-Donev algorithm - MD & particles expand

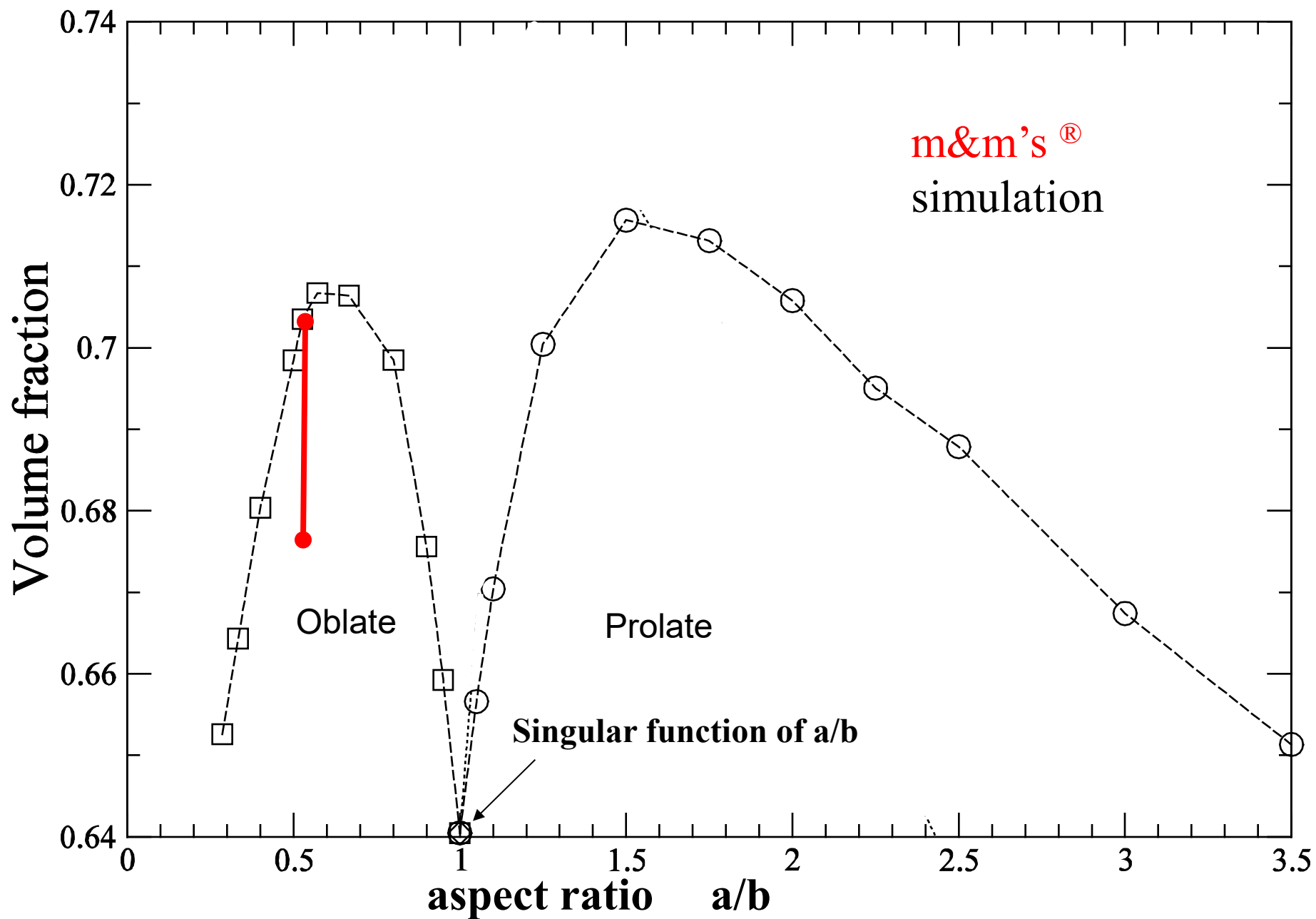


1000
1.91:1
Ellipsoids

$$\eta = (3 \cos^2(\Theta) - 1) / 2 = .01$$

$$\bar{\eta} = \frac{1}{2} \sqrt{\frac{3}{N}}$$

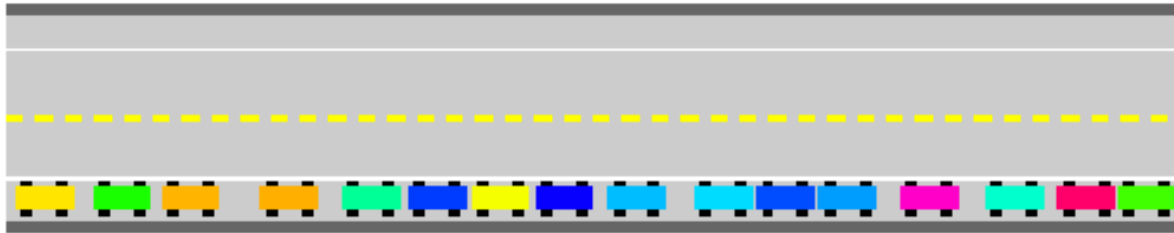
Ellipsoids of revolution



“The Parking Problem”

Random Sequential Addition – a random tiling

Rényi's Parking Constants



Given the closed interval $[0, x]$ with $x > 1$, let one-dimensional "cars" of unit length be parked randomly on the interval. The mean number $M(x)$ of cars which can fit (without overlapping!) satisfies

$$M(x) = \begin{cases} 0 & \text{for } 0 \leq x < 1 \\ 1 + \frac{2}{x-1} \int_0^{x-1} M(y) dy & \text{for } x \geq 1. \end{cases} \quad (1)$$

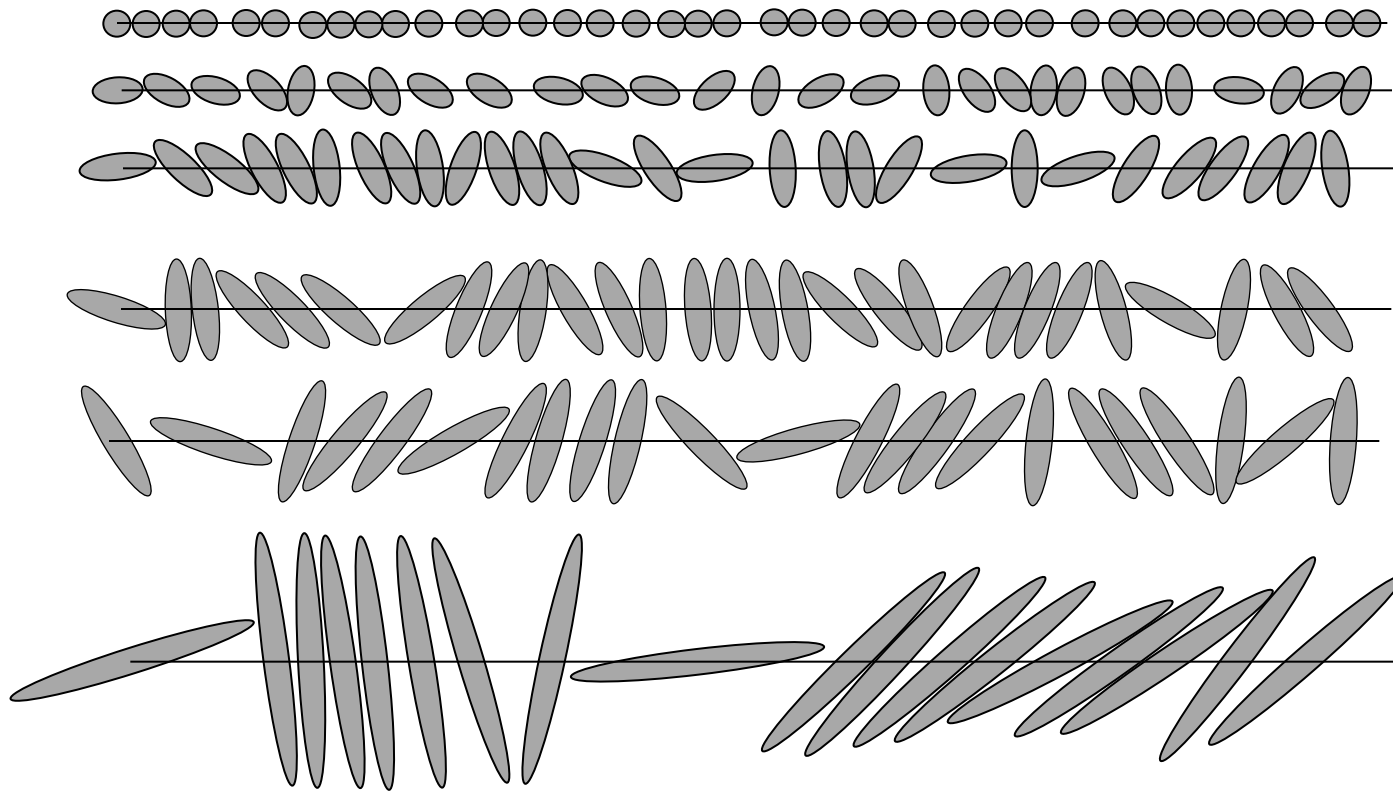
The mean density of the cars for large x is

$$m \equiv \lim_{x \rightarrow \infty} \frac{M(x)}{x} \quad (2)$$

$$= \int_0^\infty \exp \left(-2 \int_0^x \frac{1 - e^{-y}}{y} dy \right) dx \quad (3)$$

$$= \text{[Redacted]} \quad (4)$$

“Paris” Parking Problem - Ellipses



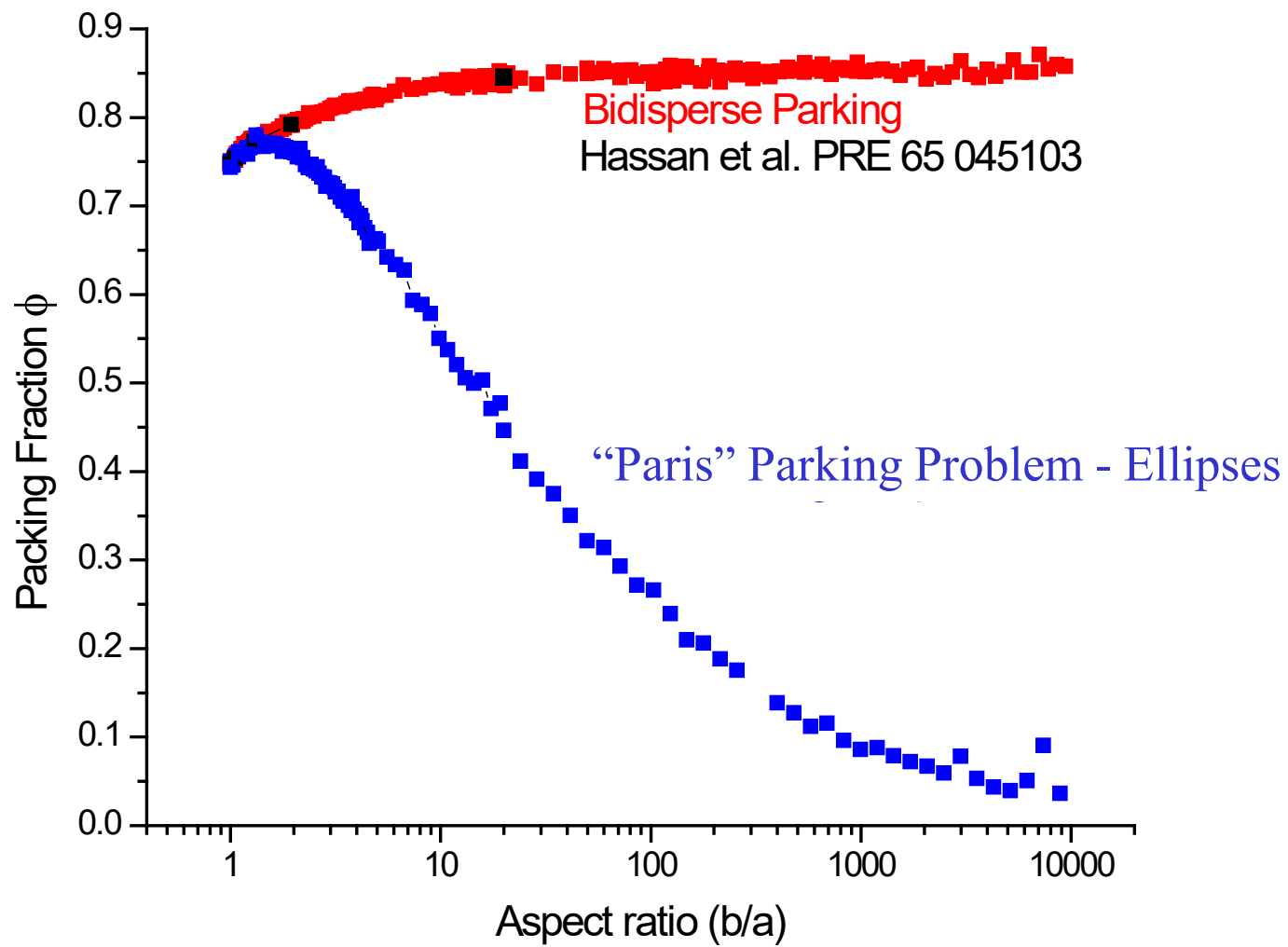
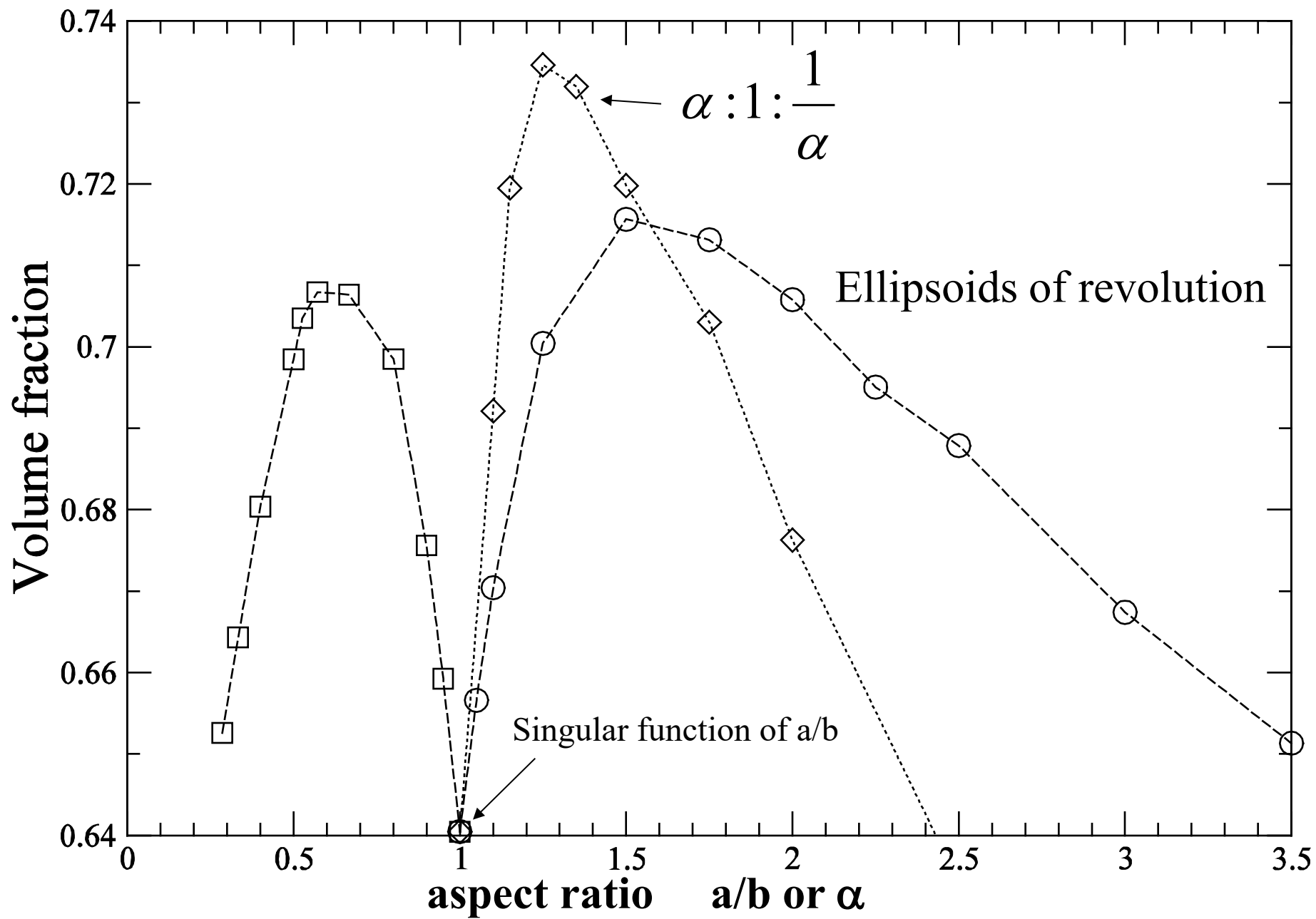
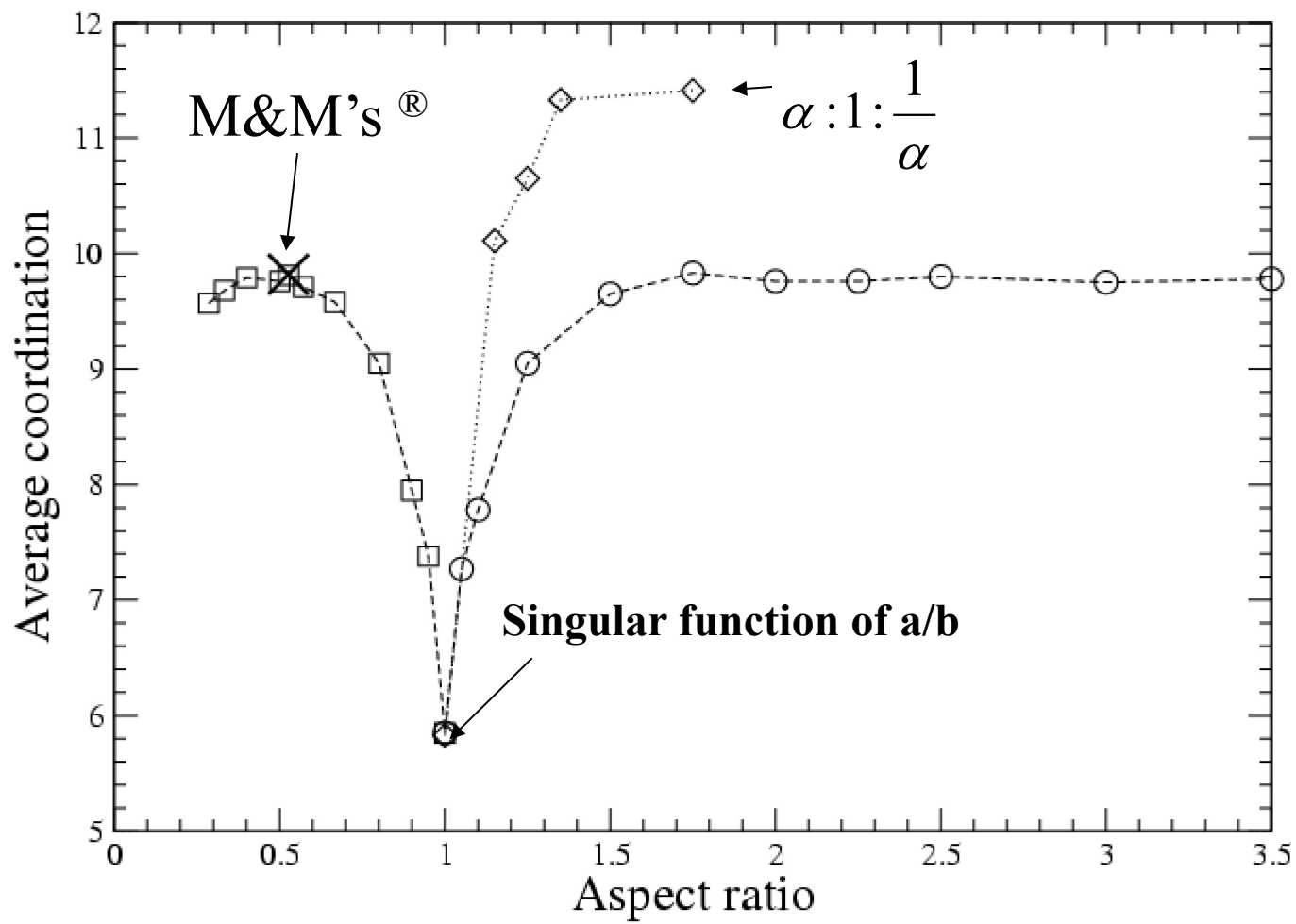


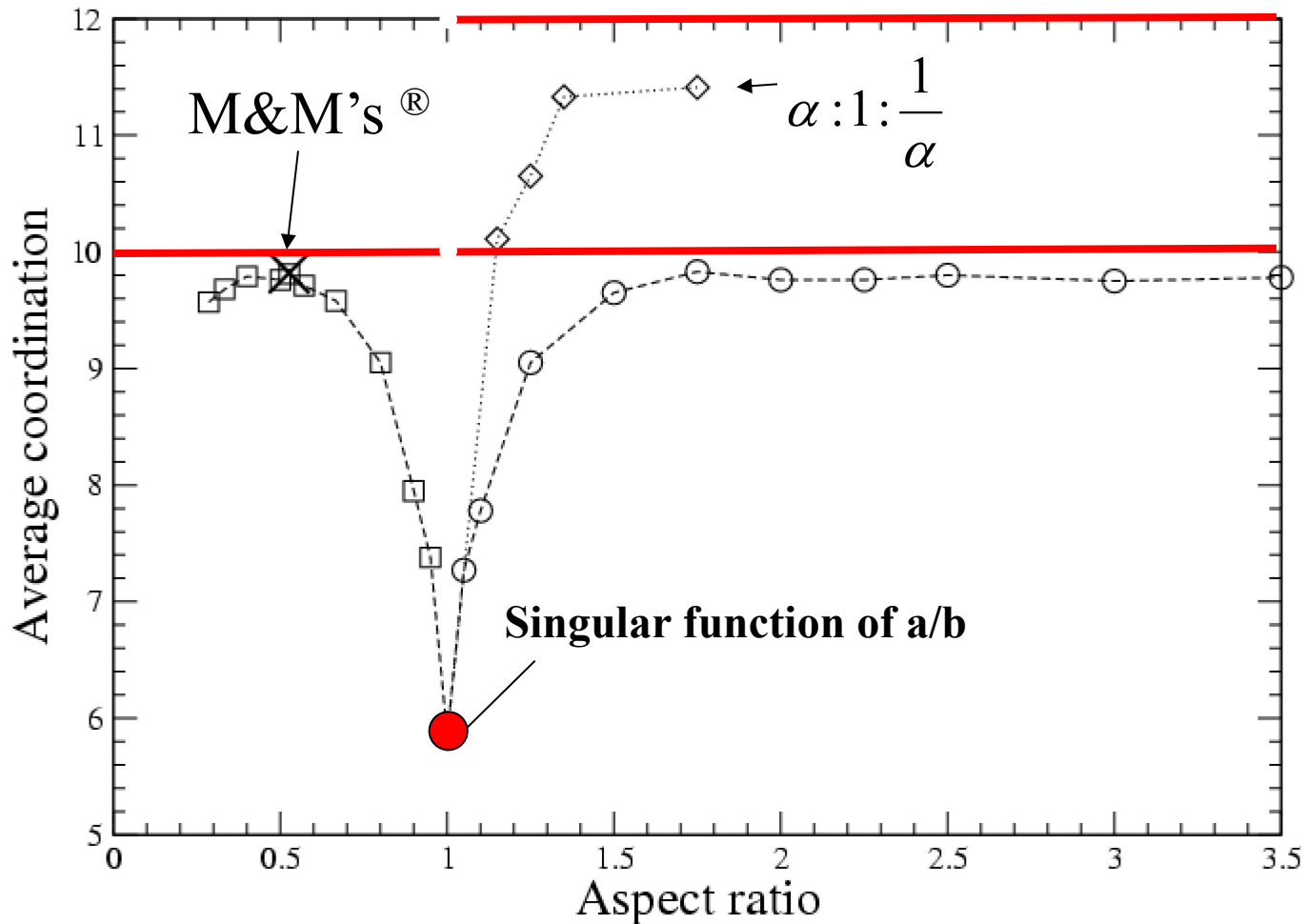
Fig. 5





Why is isostatic result only approximately right?

Jammed states don't maximize configurations/entropy?



Isostatic Conjecture

Equations

For each particle

$$\sum_{\text{contacts}} F_i = 0 \quad d \text{ force eqs.}$$

$$\sum_{\text{contacts}} (r \times F)_i = 0 \quad n \text{ torque eqs*}.$$

2D $n=1$
3D $n=3$

Degrees of freedom $f = d + n$

Unknowns

F_i 's at the N_c contacts $= dN_c$

Frictional Case: $dN_c = fN$ $Z = 2f/d$

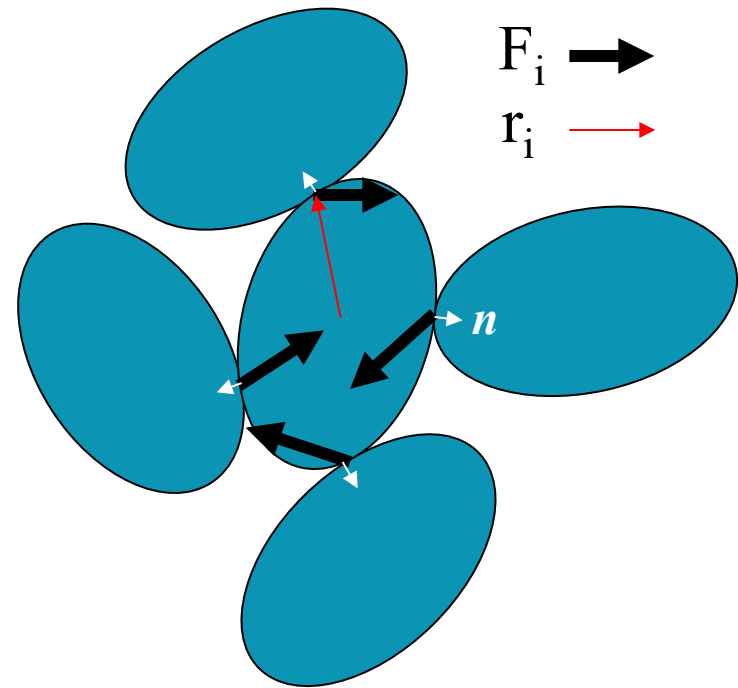
1D $d=1$ $n=0$ $f=1$ $f/d=1$ $Z=2$

2D $d=2$ $n=1$ $f=3$ $f/d=3/2$ $Z=3$

3D $d=3$ $n=3$ $f=6$ $f/d=2$ $Z=4$

$$Z = d + 1$$

$$*n_{\max} = d(d-1)/2$$



coordination number $Z = 2 N_c / N$

Frictionless Case $(n \times F)_i = 0$

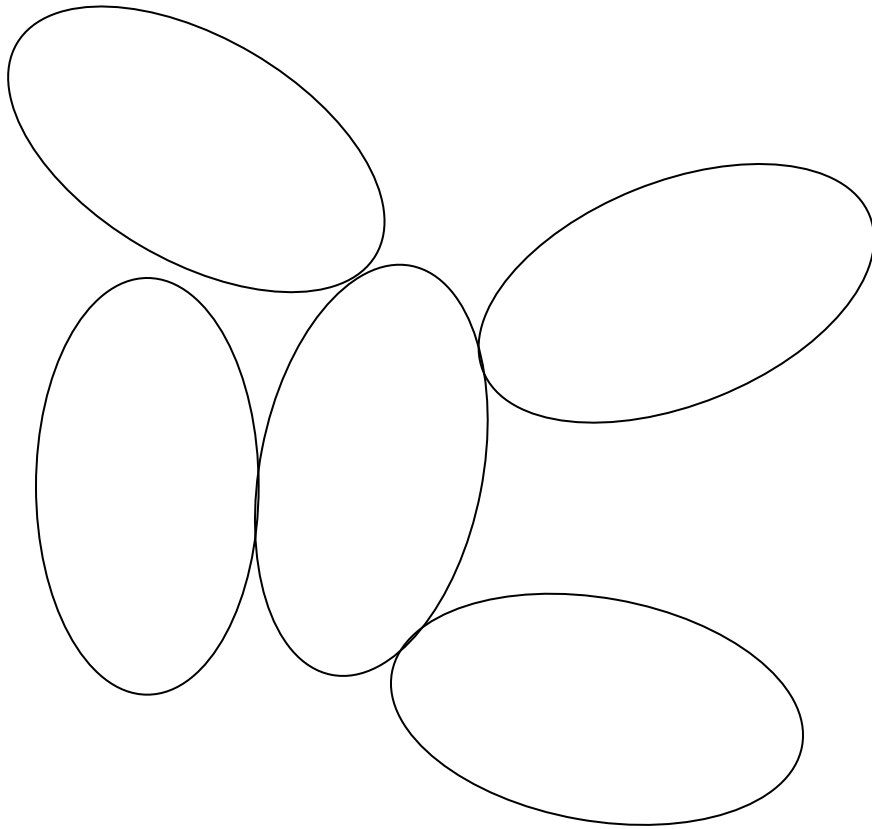
Additional $(d-1)N_c$ equations

$$dN_c = (d-1)N_c + fN : N_c = fN$$

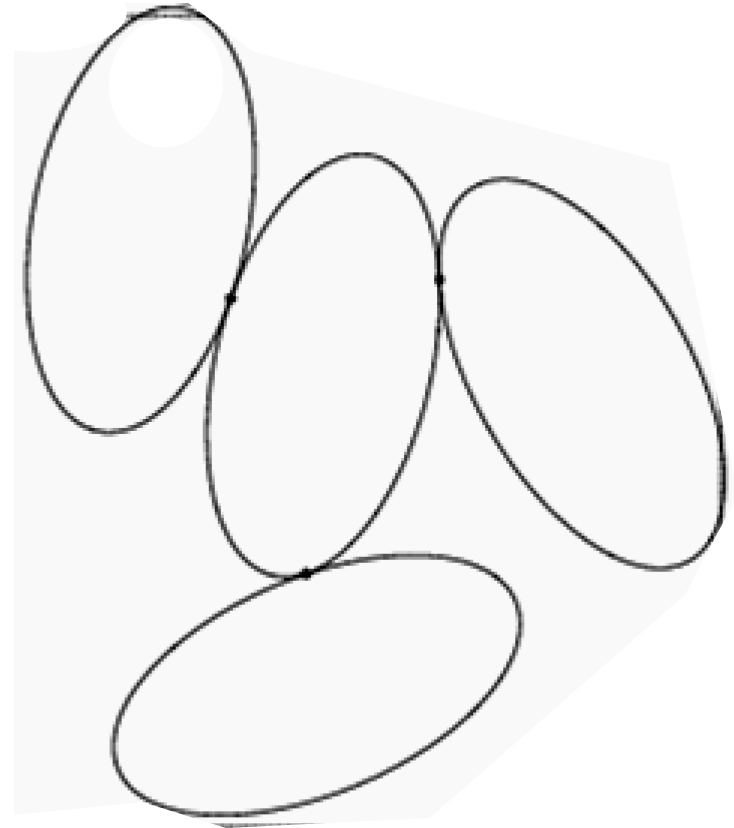
$$Z = 2f$$

**If contacts are random
i.e. most likely case?**

Local Jamming (hold neighbors fixed)



4 neighbors always works

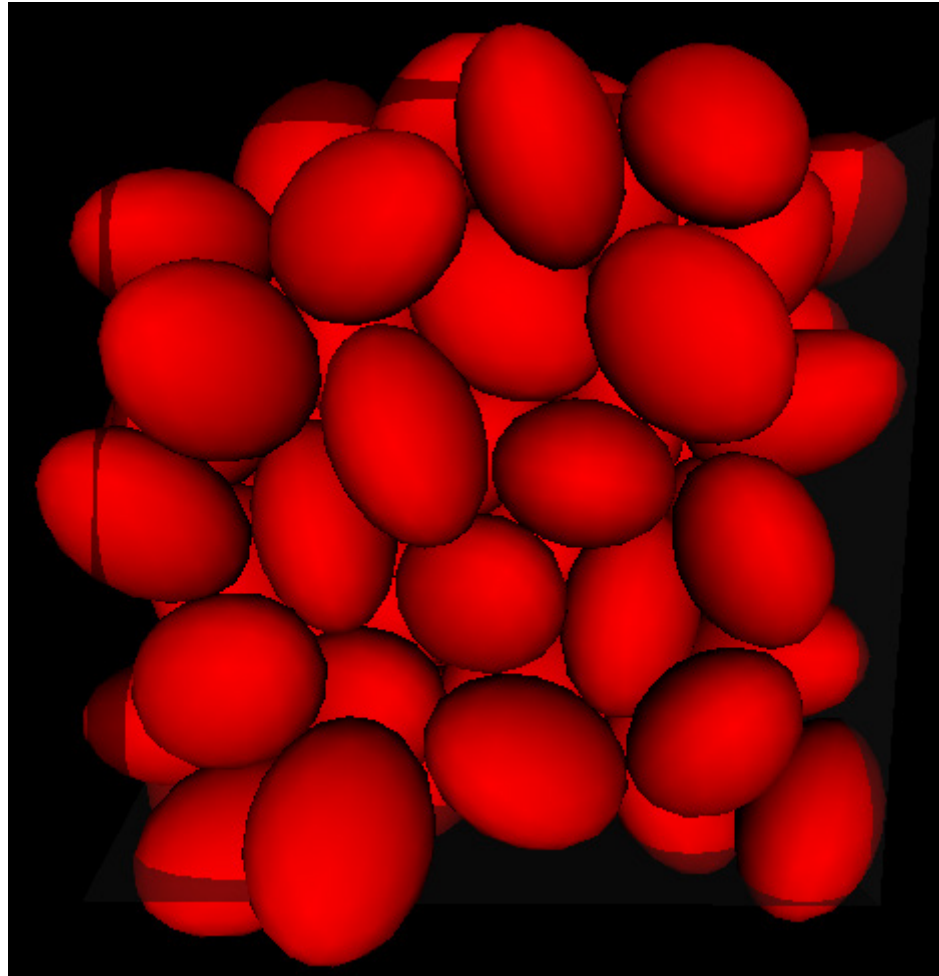


3 neighbors sometimes works
- But less “entropy”

Translationally trap, expand,
-always find Jammed state

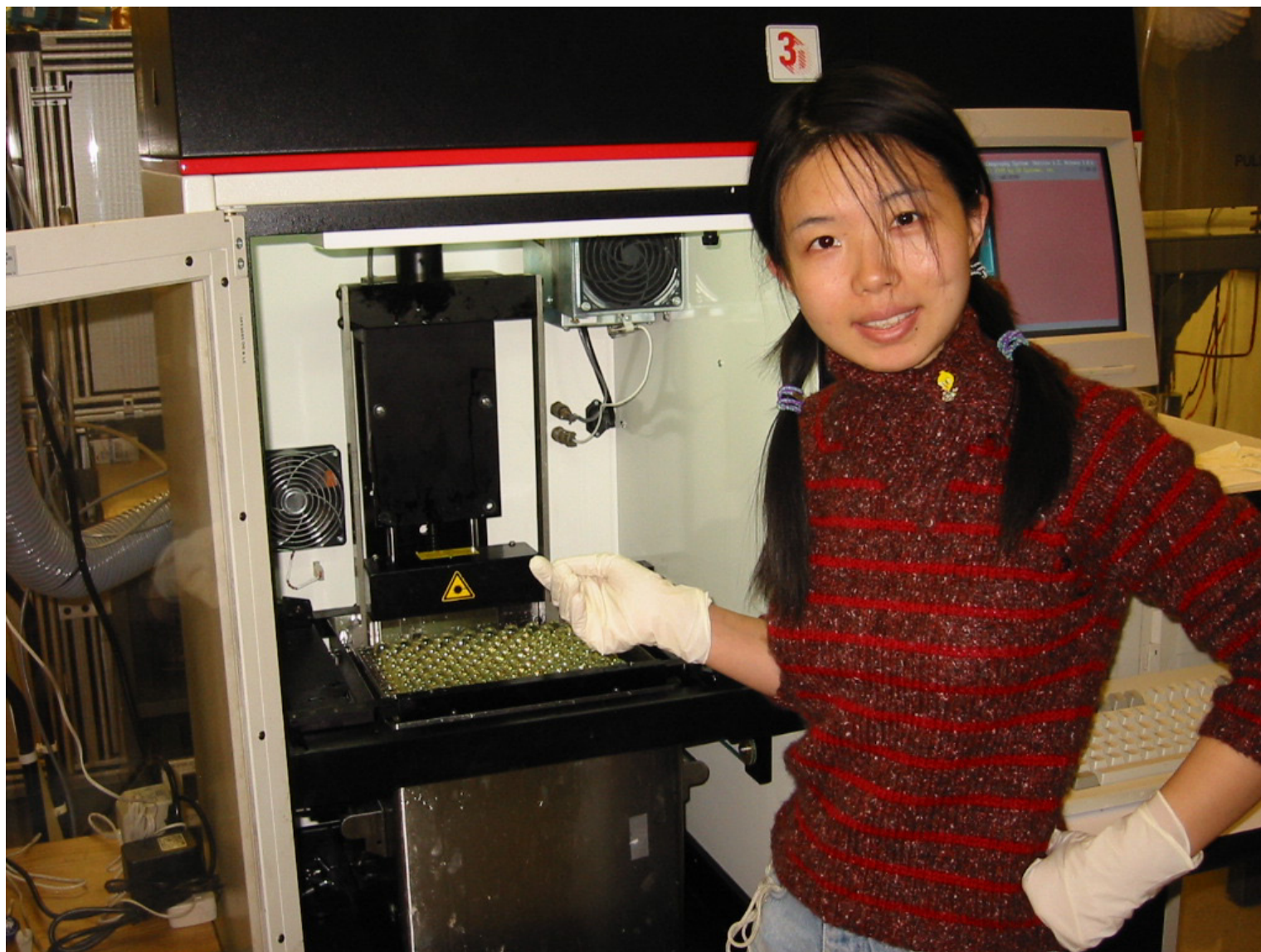
Random Ellipsoid Packing Exceeds FCC Spheres

$$\Phi = .747$$



$$a:b:c = 1.25:1:0.8$$

32 ellipsoids
Periodic boundaries



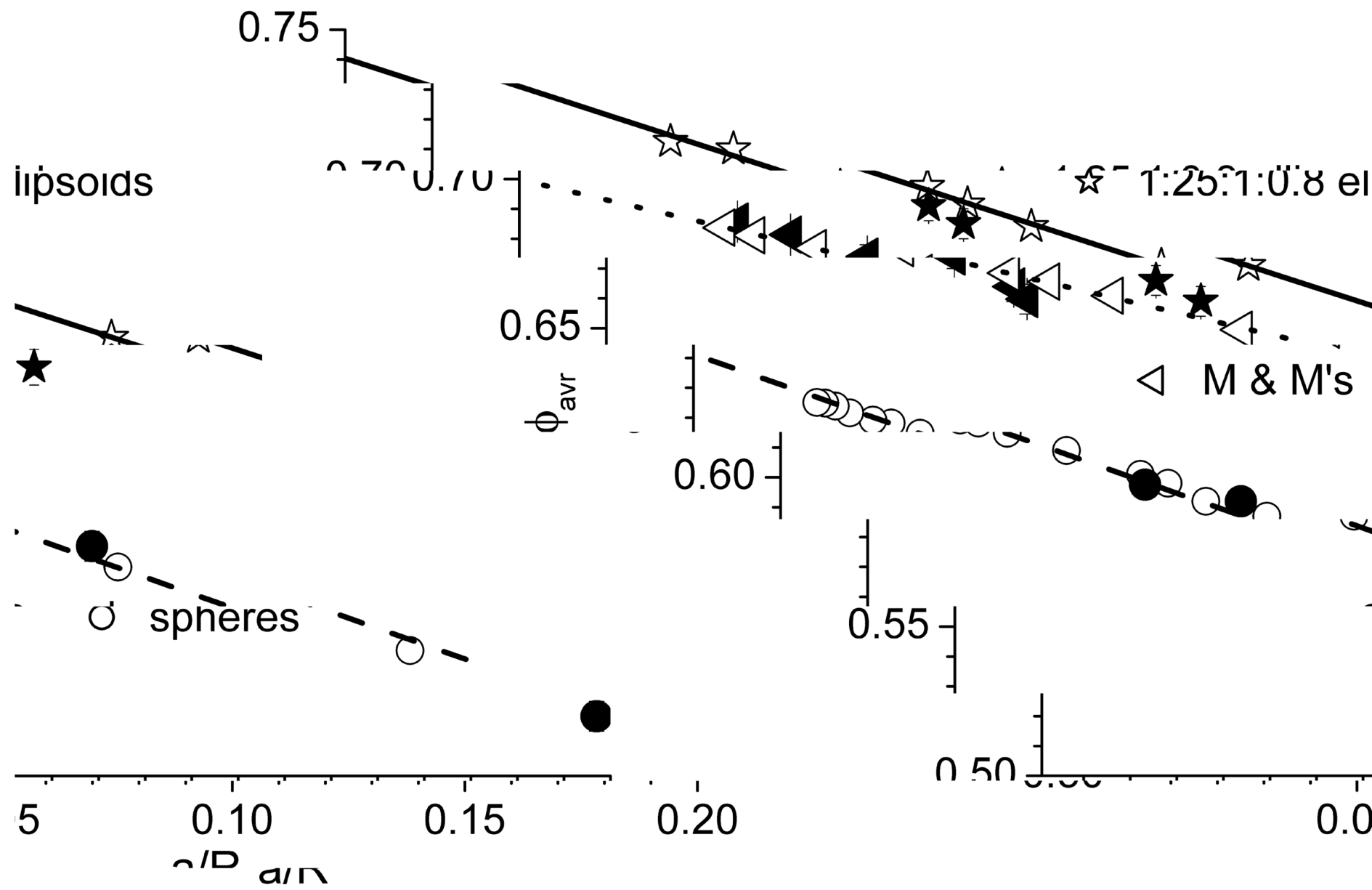
Weining Man

Stereolithography for 1.25:1:0.8 ellipsoids



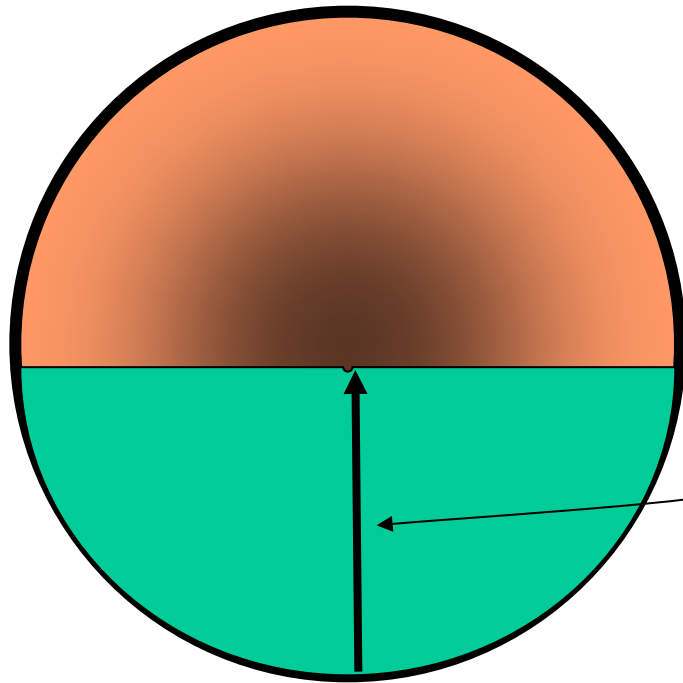
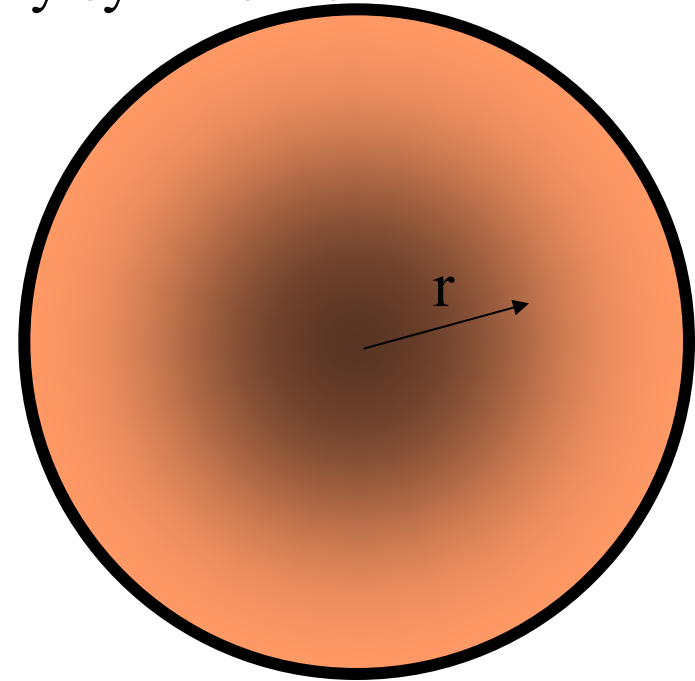
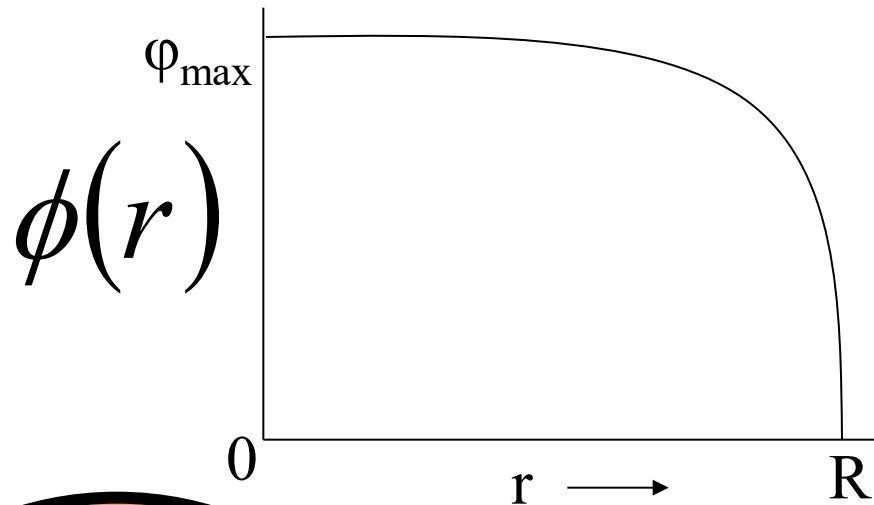


Average Packing in Spherical Container



Packing fraction from finite spherical container

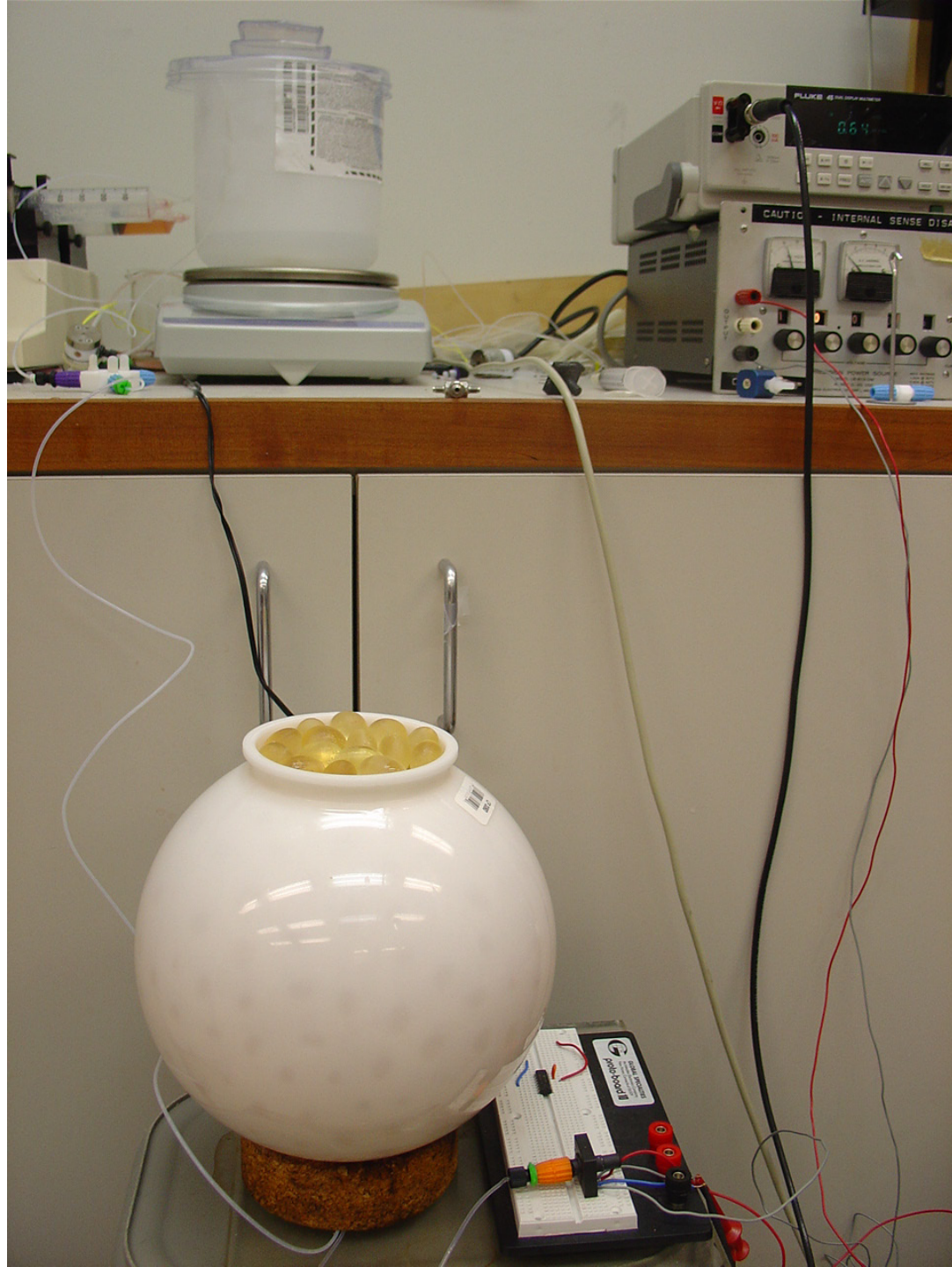
Expect packing less dense at walls, spherically symmetric

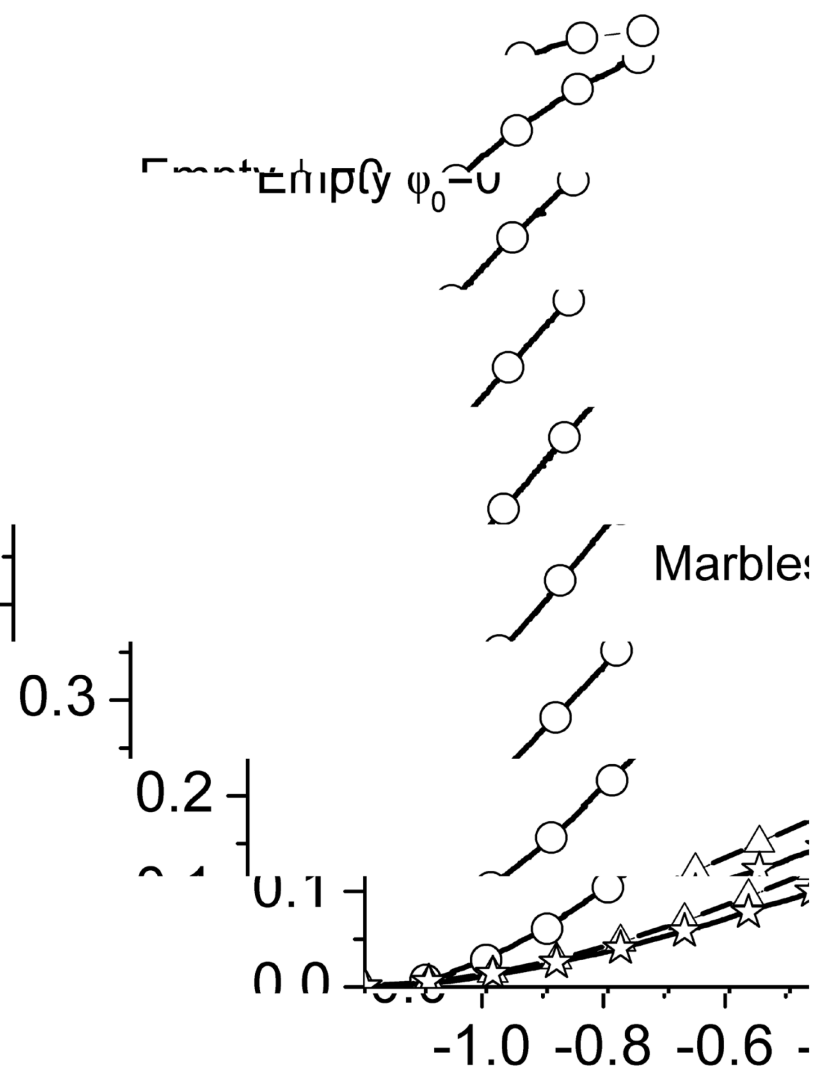
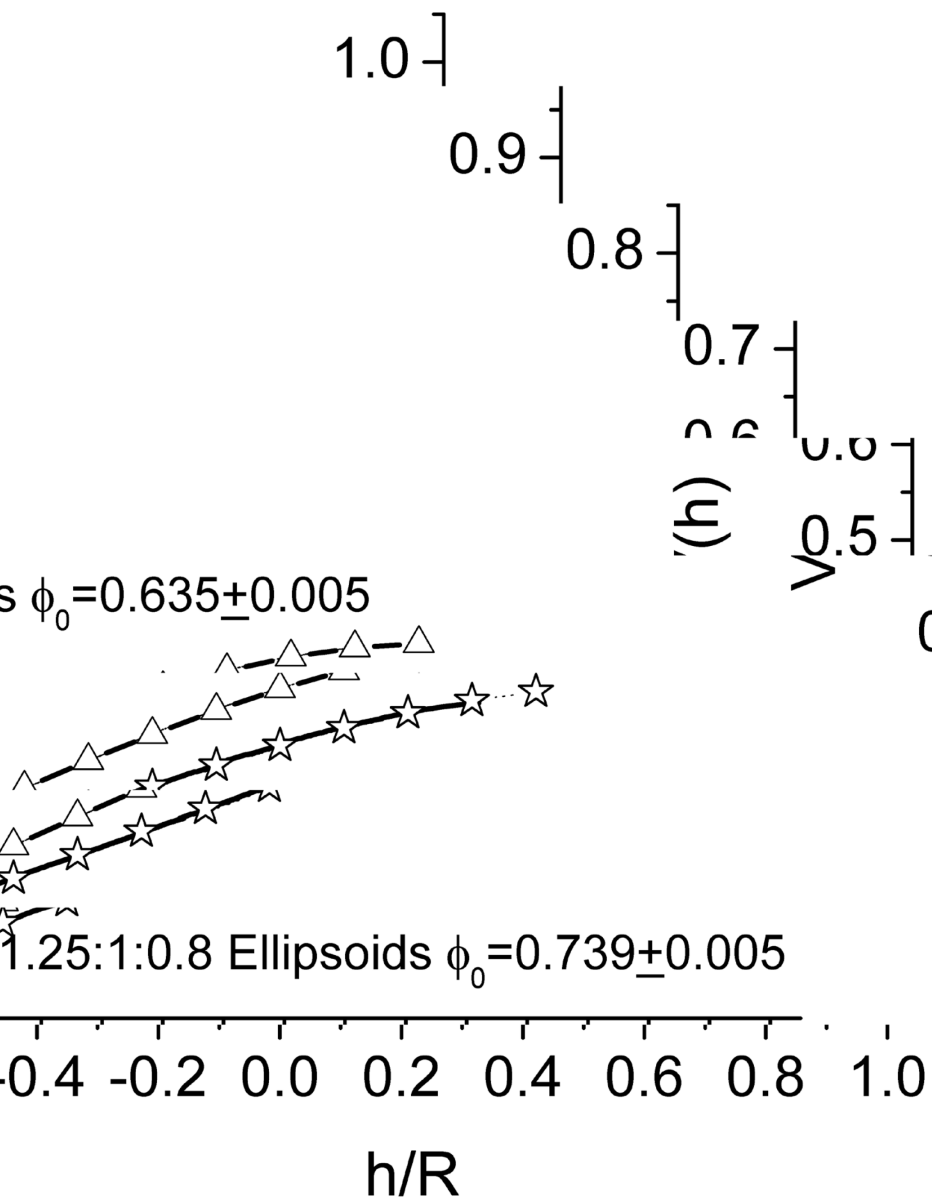


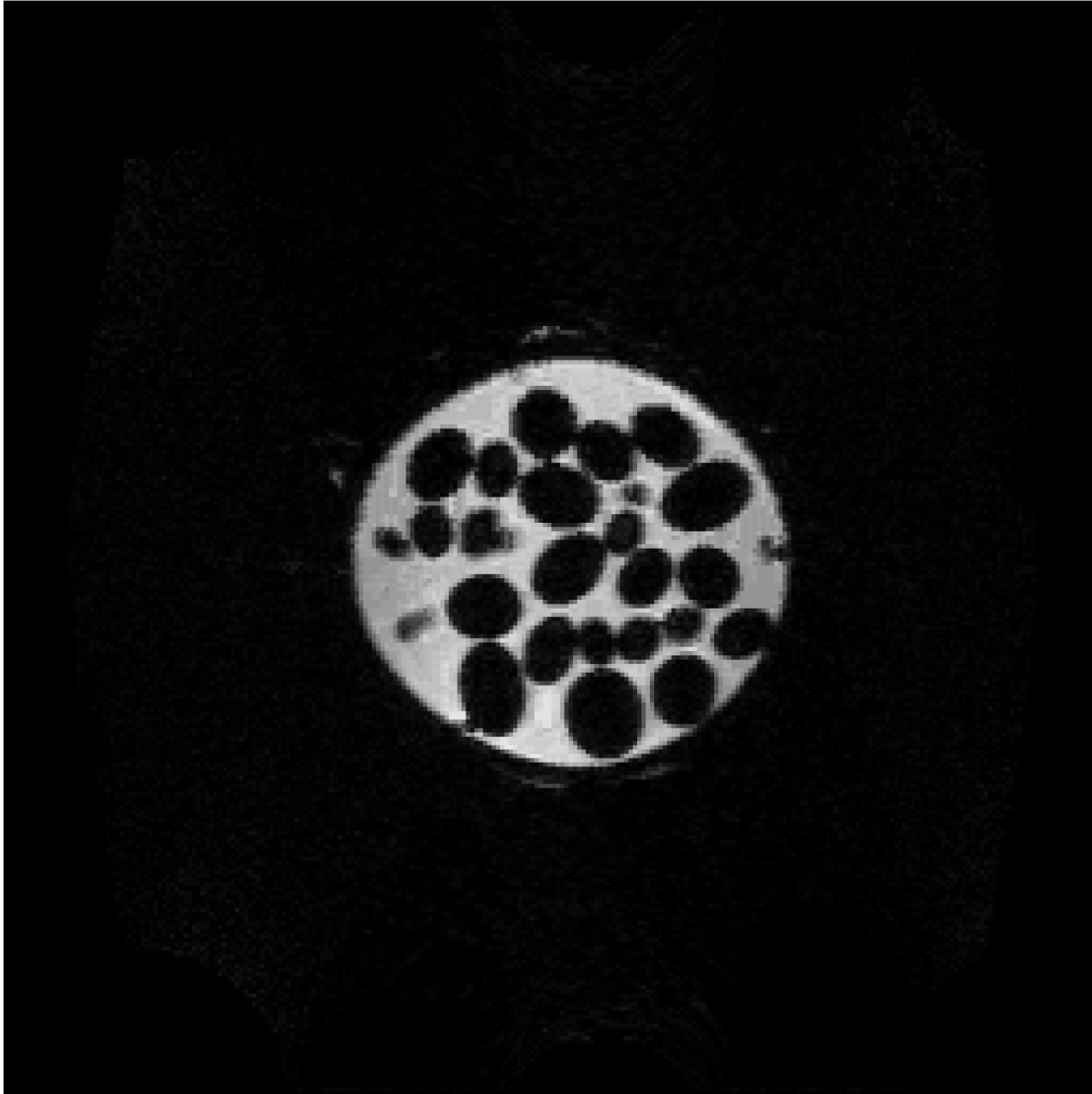
Measure height as a function of added fluid

$$V(h) = \int_h^R (1 - \phi(r)) \left(1 - \frac{h}{r}\right) 2\pi r^2 dr$$

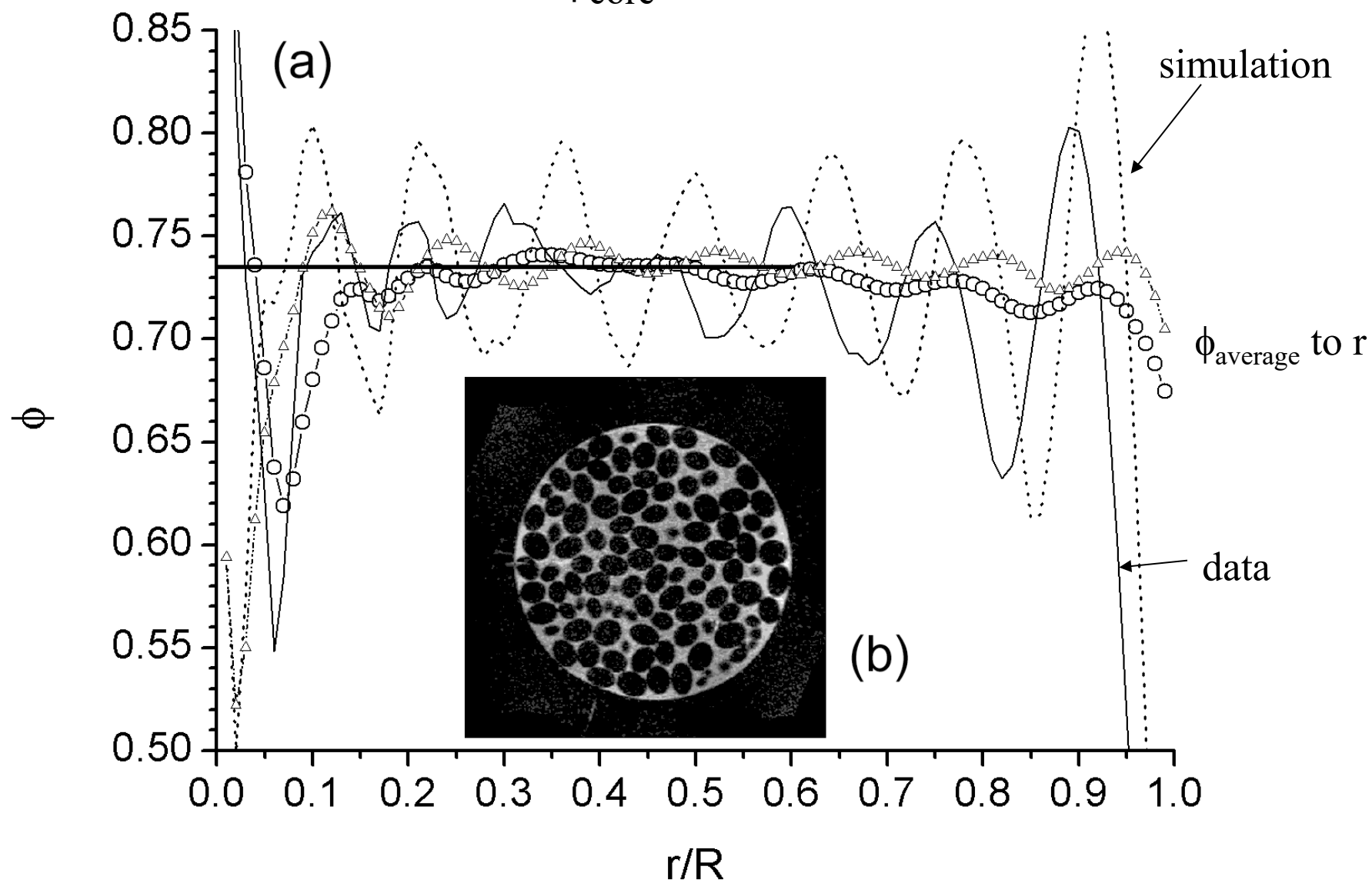
Invert to get
$$\phi(r) = 1 - \frac{1}{2\pi} \left. \frac{\partial^2 V(h)}{\partial h^2} \right|_{h=r}$$







$$\phi_{\text{core}} = .735 \pm .005$$



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SCIENCE & SPACE

M&M's obsession leads to physics discovery

Monday, February 16, 2004 Posted: 1939 GMT (3:39 AM HKT)

WASHINGTON (Reuters) -- Princeton physicist Paul Chaikin's passion for M&M's candies was so well known that his students played a sweet practical joke on him by leaving a 55-gallon drum of the candies in his office.

Little did they know that their prank would lead to a physics breakthrough.

The barrel full of the oblate little candies made Chaikin think about how well they packed in. A series of studies have shown they pack more tightly than perfect spheres -- something that surprises many physicists and Chaikin himself.

"It is a startling and wonderful result," said Sidney Nagel, a physicist at the University of Chicago. "One doesn't normally stop to think about this. If you did, you might have guessed what would happen, but you'd have guessed wrongly."

The issue of how particles pack together has intrigued scientists for centuries and has implications for fields such as the design of high-density ceramic materials



Professors Paul Chaikin, left, and Salvatore Torquato used M&M candies to reveal fundamental principles governing the random packing of particles.

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Princeton profs Paul
Chaikin, left, and
Salvatore Torquato



The physics of M&M's Who knew candy could solve an ancient physics problem? It all began as a prank on Princeton physics professor Paul Chaikin, whose students left a 55-gallon drum of M&M's in his office to sweeten him up. Instead, it inspired an "Aha!" moment as Chaikin pondered a centuries-old physics mystery: how objects (in this case, "oblate spheroids" — science-speak for the M&M shape) settle. He and Princeton chemist Salvatore Torquato supervised experiments like pouring paint on a container of the candies to see how it covered them. Their scientific conclusion: Unlike sphere shapes, oblate spheroids act like tiny levers, rotating until they find the most stable packing position, making for better packaging and shipping. In short, Chaikin says: "You can pack more M&M's than gum balls [of similar size] into the same space."

Public opinions as to why M&M's[®] pack denser:

They're flatter and tend to orient and order

- Nope: If they oriented it's an affine deformation of spheres, packing is 63.6%. No evidence of order.

If you make them flat they're like pennies and stack better

If you make them long they're like cylinders and pack better.

- Nope: Cylinders and pennies pack to 91%, but an ellipsoid only fills a cylinder to $2/3 \Rightarrow 67\%$



$.67 \times .91 = .604$ and that's crystal packing,
random pennies and sticks pack much less dense.

They can rotate to fit into open spaces

- Yep, that's it.

Matt Sullivan's life savings



**Packing Fraction 0.574 ± 0.005
Weining Man**

Comparison of Packings

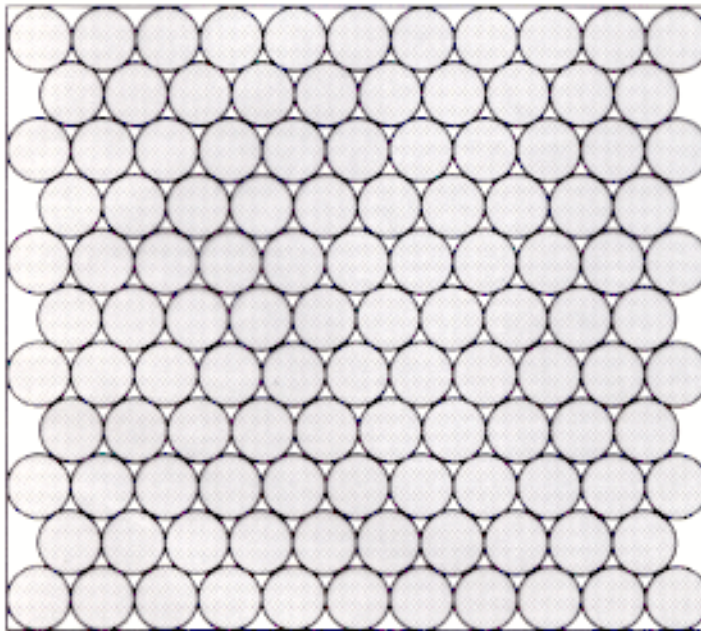
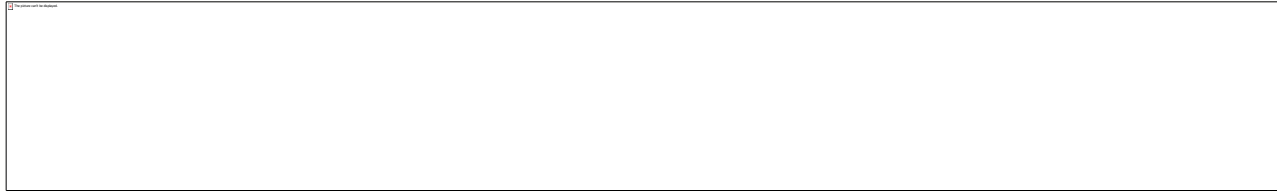
	Degrees of Freedom	Touching Neighbors	Volume Fraction
FCC Crystal	3	12	.7404
MRJ Spheres	3	6	.636
MRJ Ellipsoid of Revolution	5	10	.68-.70
MRJ $a \neq b \neq c$ Ellipsoid	6	>11.4	>.747

We have found a random packing $\geq .7405$
higher than crystal packing of spheres

Do we have a higher random packing
than crystal packing??????

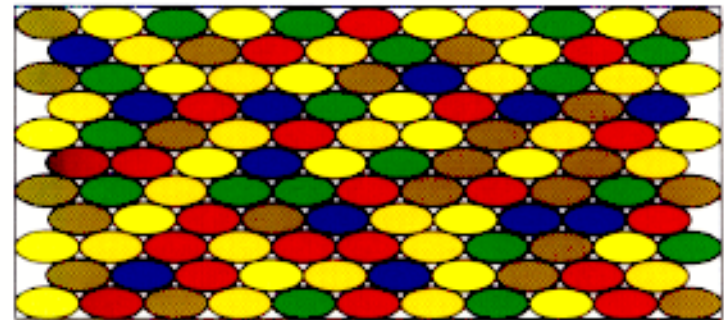
What is the densest packing of ellipsoids?

Crystal Packing of Spheres and Ellipsoids



$$\varphi = \frac{N \frac{4}{3} \pi r^3}{L^3}$$

M&M[®]s aspect ratio 1.91



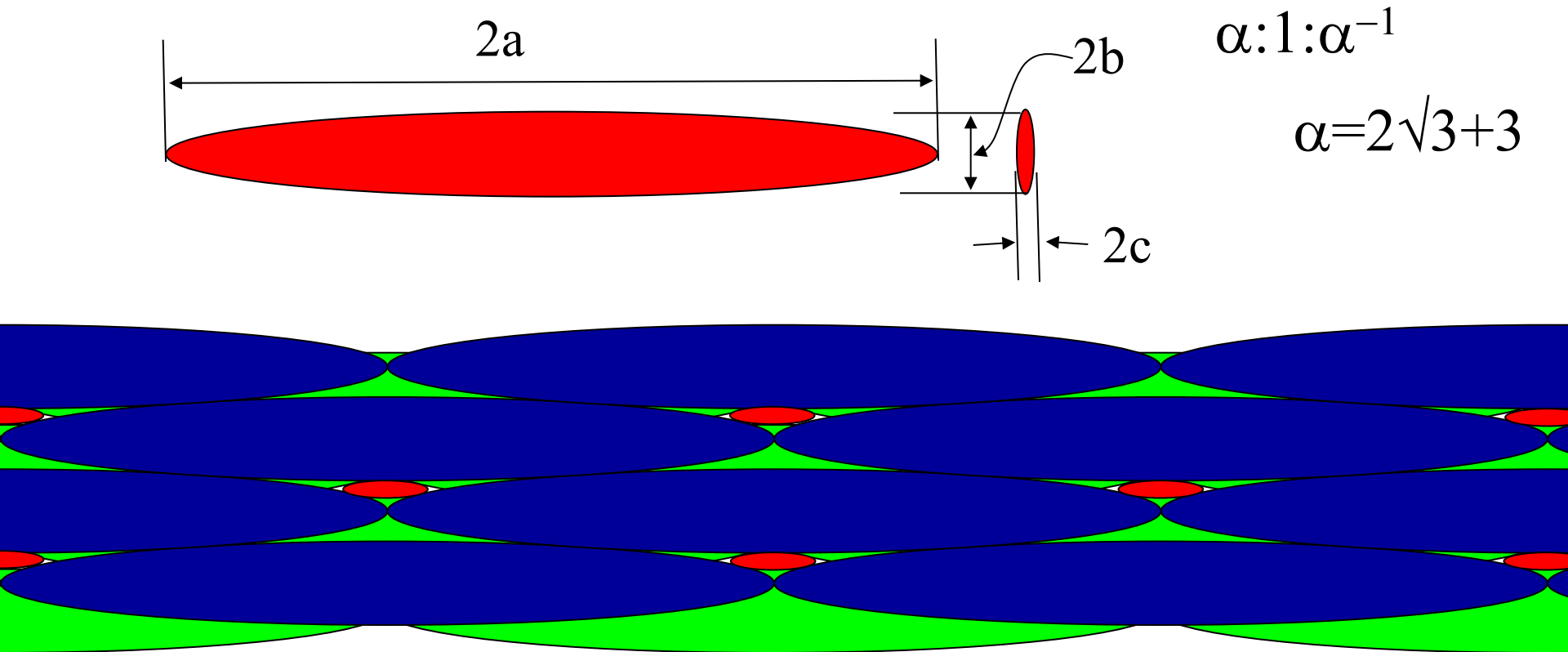
$$\varphi = \frac{N \frac{4}{3} \pi r^2 (r / 1.91)}{L^2 (L / 1.91)}$$

Volume Fraction 0.7404

Densest previous ellipsoid packing .7585

J. M. Wills, *Mathematika*, 38:318-320 '91

For Needle/knife blade like ellipsoids can feed blades into interstices of HCP Lattice



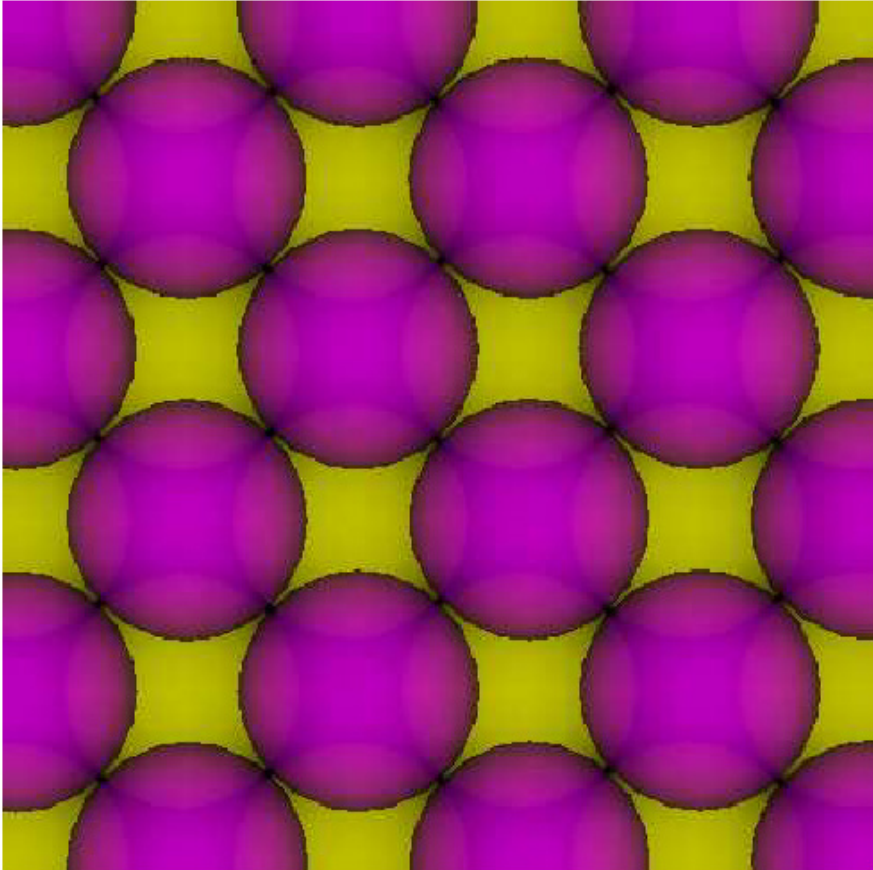
Notes: Incommensurate – a/c irrational

Not 3D crystal – 1D lattice in slots not in registry

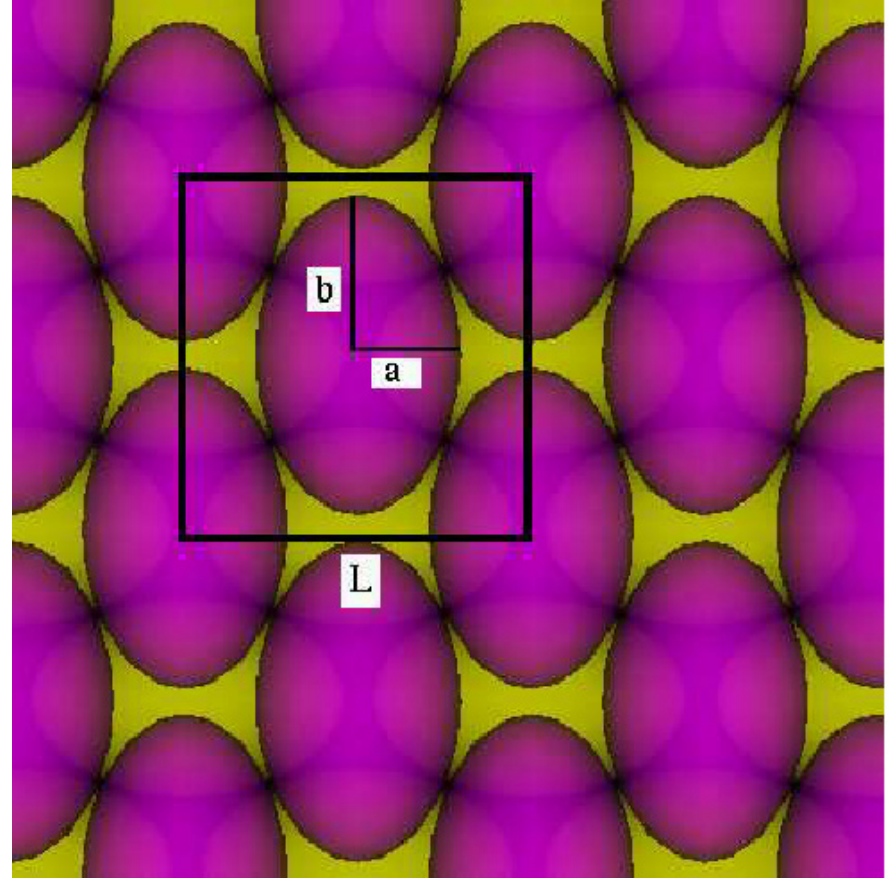
Our Conjectured Optimal Ellipsoidal Packing Close to Spheres

Donev Close Packed

Square cell

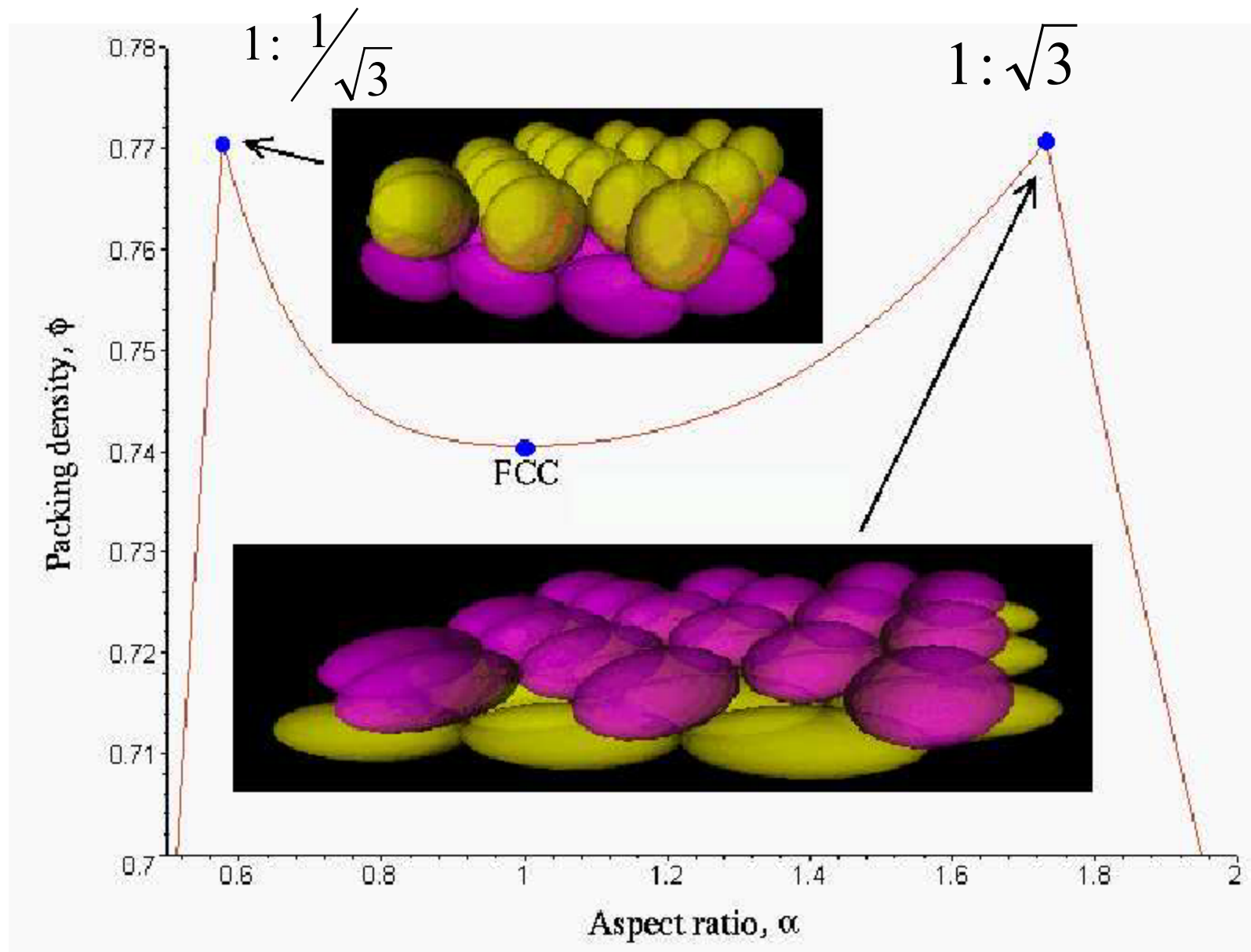


FCC 001 face

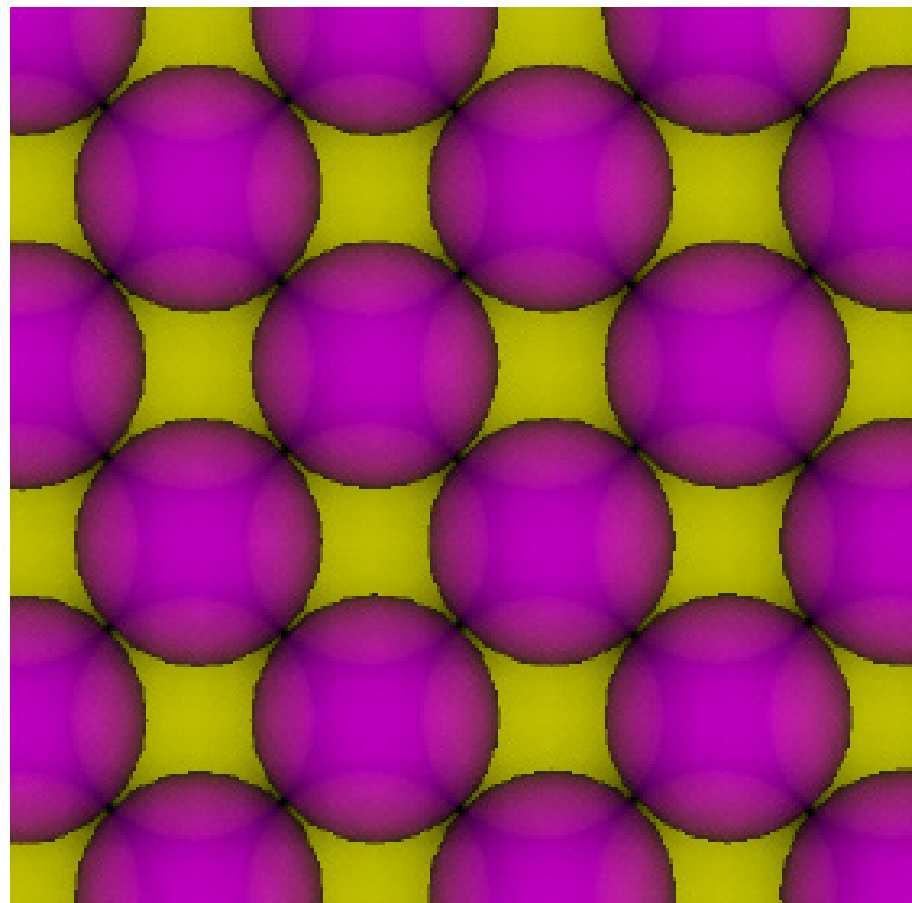


Alternate elongation direction
in adjacent layers

Donev Close Packed

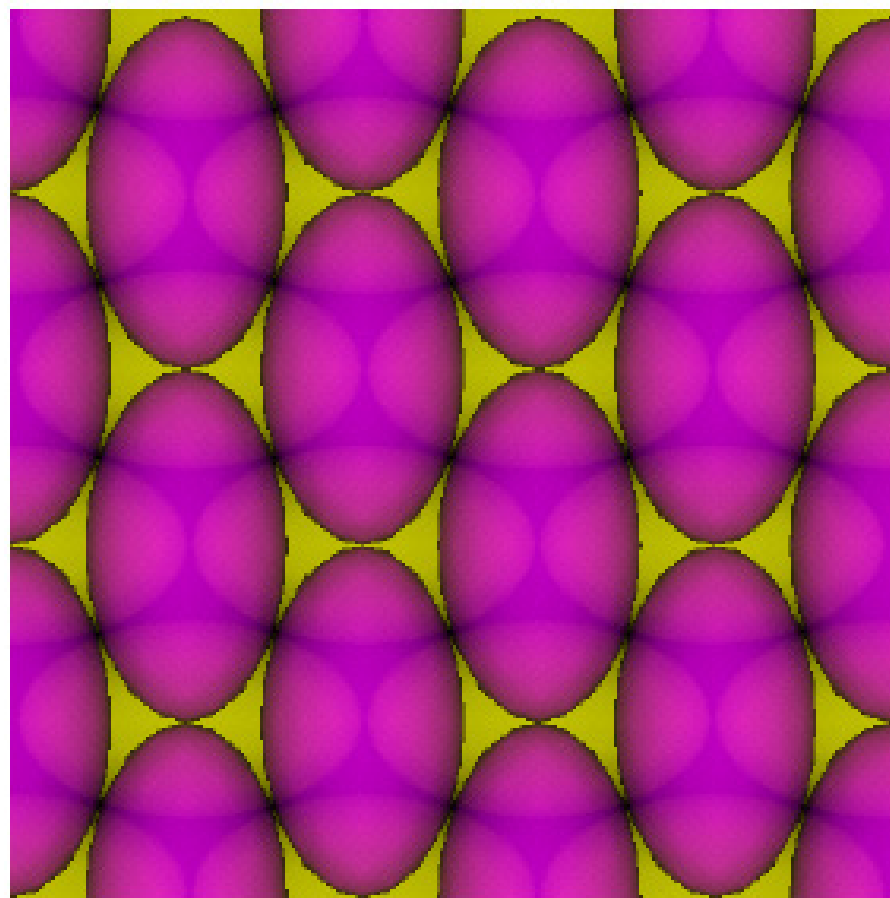


12 nearest neighbors



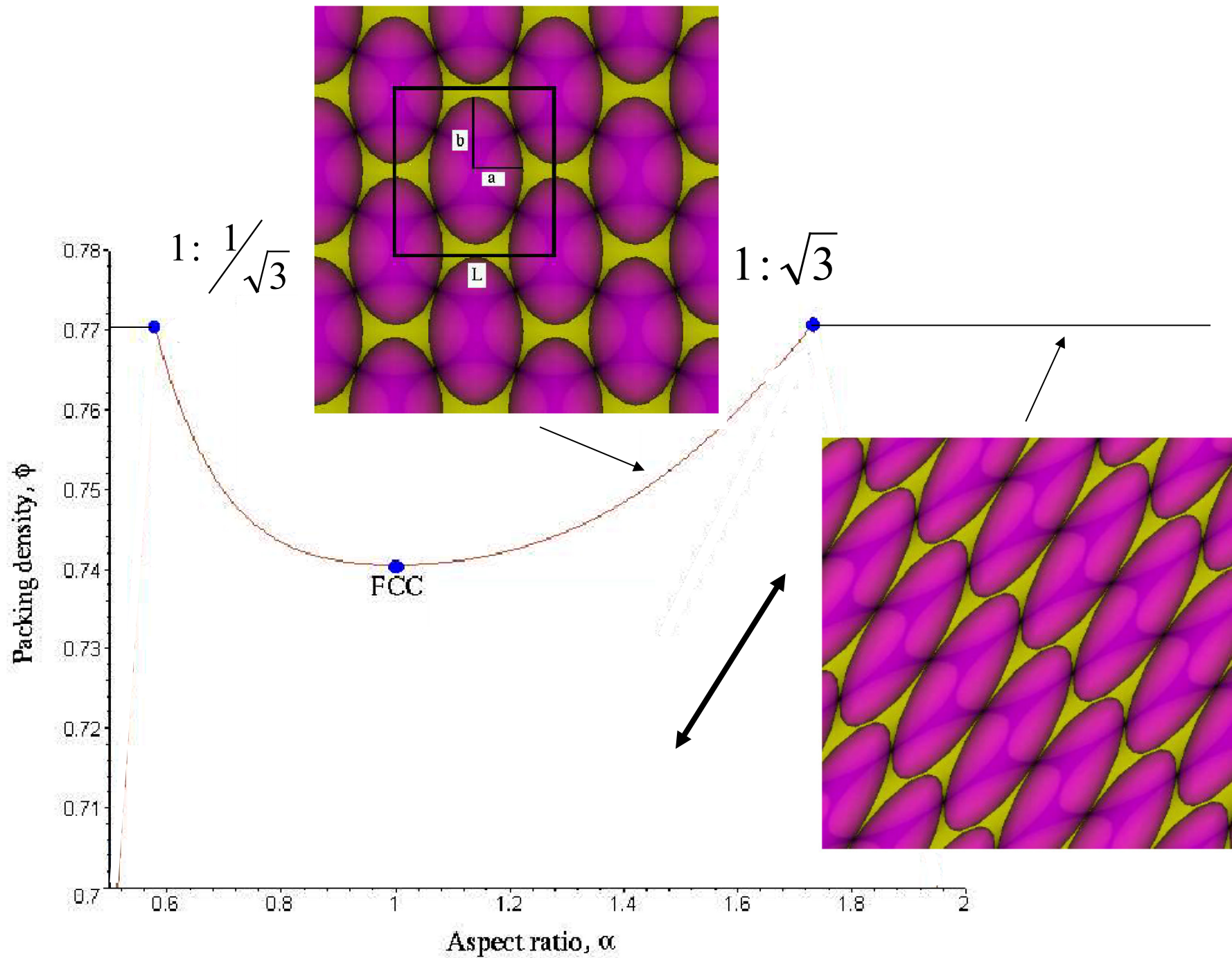
.7405

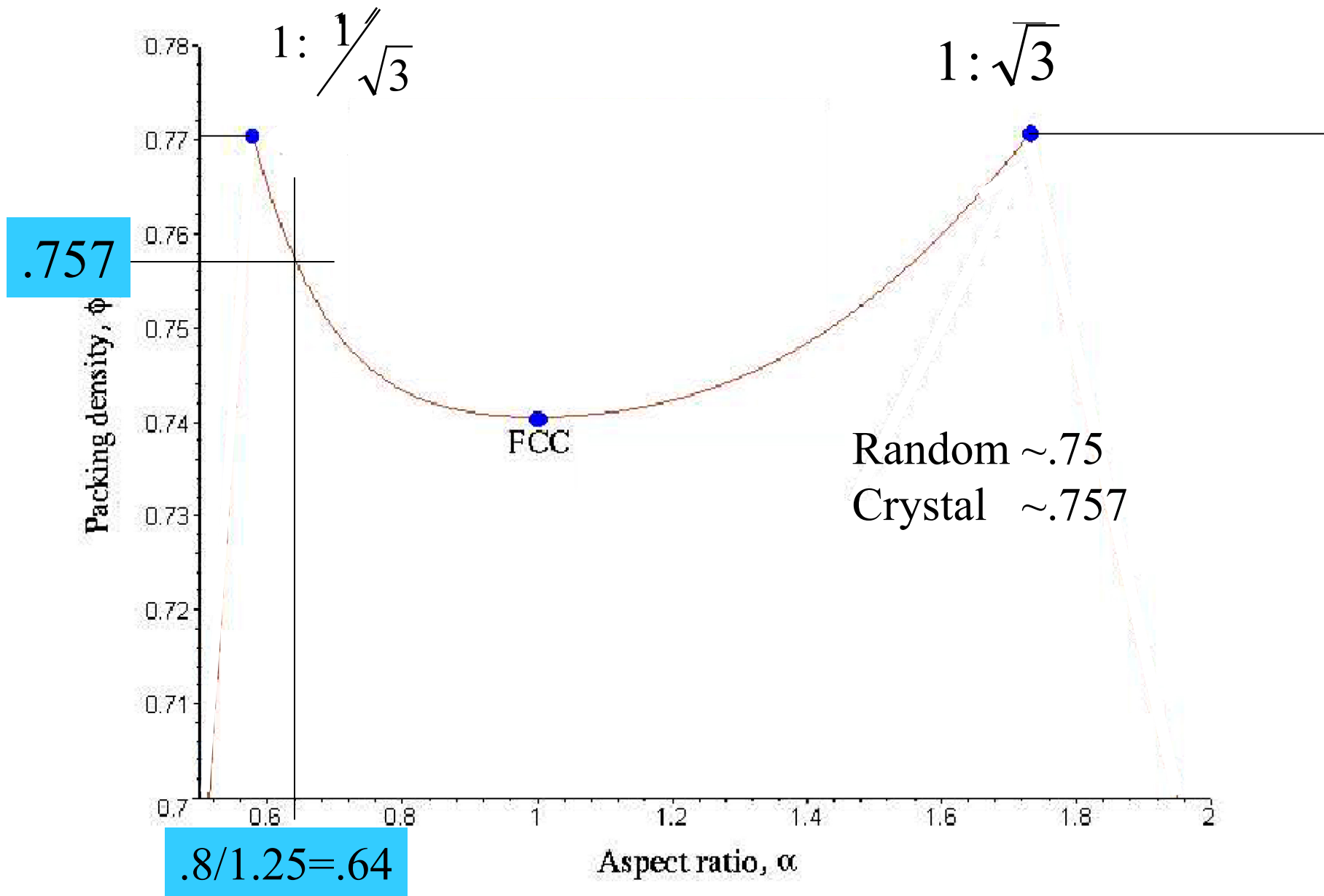
14 nearest neighbors



.7707

Highest Ellipsoid Packing yet





Random packing of .8:1:1.25 ~.75



Tetrahedra?

Who's to Blame

Mirjam Leunissen, Alfons van Blaaderen
Debye Institute, Univ. Utrecht

Bill Russel

Princeton - Chem Eng.

Matt Sullivan

Bob Austin

Weining Man

Kun Zhao

Princeton - Physics

Princeton - Chemistry

Sal Torquato

Frank Stillinger

Aleksandar Donev

Bob Connelly (Cornell)

Andrew Hollingsworth

PC

Center for Soft Condensed Matter Research

NYU - Physics.



Kun
Zhao

Matt
Sullivan

Bill
Russel

Marie-Pierre
Valignant

Weining
Man

Mirjam
Leunissen

Andy
Hollingsworth

Summary

Random Ellipsoid packings :

- Are singular at $a/b=1$
- Can be denser than .74 with 12 neighbors*
- Tend toward the isostatic limit: $Z=2d$
- Are comparable to (and may yet surpass) crystal packings

Crystal ellipsoidal packing:

Conjecture - Donev Close Packed (DCP) is densest

$\phi=.7707$ for $1:\sqrt{3}$, 14 touching neighbors

Basis of 2, Square face

Does Random ever beat crystal?

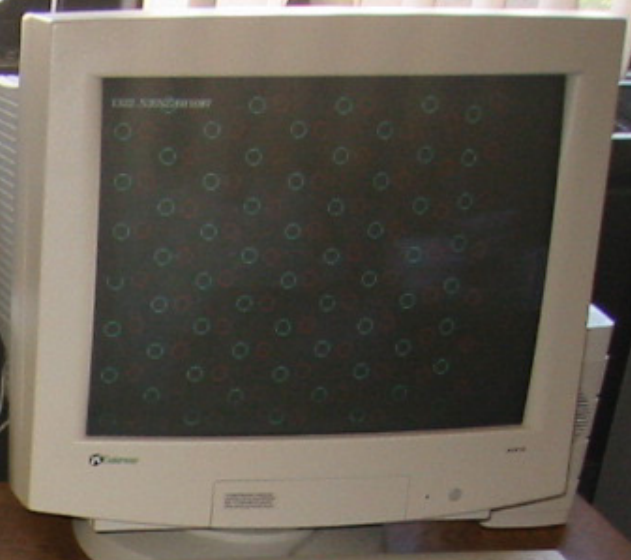
For $a:b:c=1.25:1:0.8$, $\phi_{\text{random}} > .743$ but , $\phi_{\text{DCP}} = .757$

Not yet but.....

and Ellipsoids are much harder to crystallize than spheres

m & m's

CHAINLAX





125 lbs of Almond M&M's
Polydispersity $\sim 17\%$



2 liters packs to $\phi \sim .67$

