Exotic Quantum Phases (III) [Senthil]

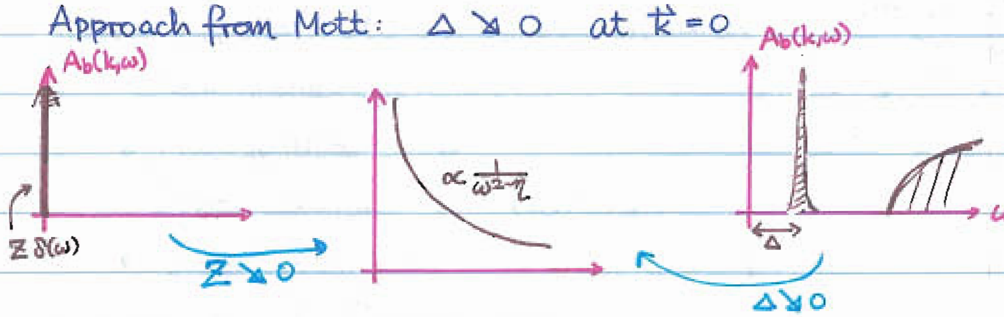
- Mott transition — metal / superfluid / superconductor & insulator

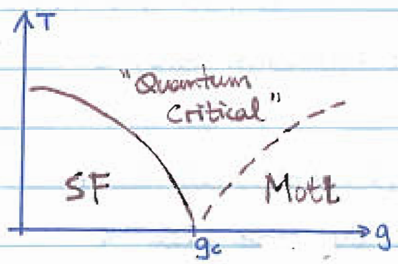
▲ EXAMPLE: Bose–Hubbard $-t\sum(b_i^\dagger b_j + h.c.) + U\sum_i \frac{n_i(n_i-1)}{2}$

$t \gg U \implies$ superfluid , $U \gg t \implies$ Mott insulator

Approach from SF : $Z, \rho_s \searrow 0$:

Approach from Mott : $\Delta \searrow 0$ at $\vec{k}=0$



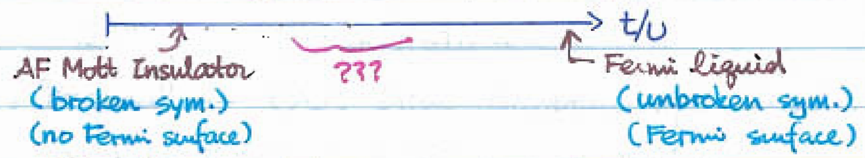


$$S = \int d\tau\, d^2x \left(|\partial_\tau \psi|^2 + |\nabla\psi|^2 + r|\psi|^2 + u|\psi|^4 \right)$$

Can approach by ϵ-expansion, $1/N$-expansion, Monte Carlo, etc.

▲ Example: one-band Hubbard on non-bipartite lattice

$$\mathcal{H} = -t\sum(c_{i\alpha}^\dagger c_{j\alpha} + h.c.) + U\sum \frac{n_i(n_i-1)}{2}$$

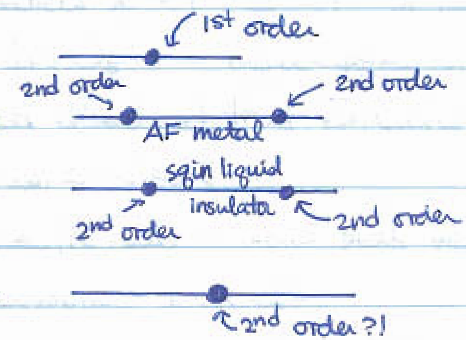


case (1) :

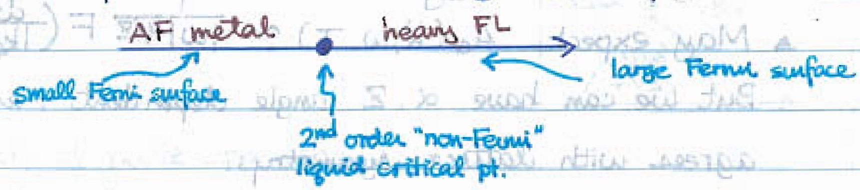
case (2) :

case (3) :

case (4) :



▲ A similar problem occur in heavy electron problem:



($\approx$ one band Mott transition + conduction bands)
of f-electron — of d-electron

- Issues : (1) Can Fermi surface disappear at same time when magnetic order appears?

  (2) How to understand QCP where whole Fermi surface disappears?

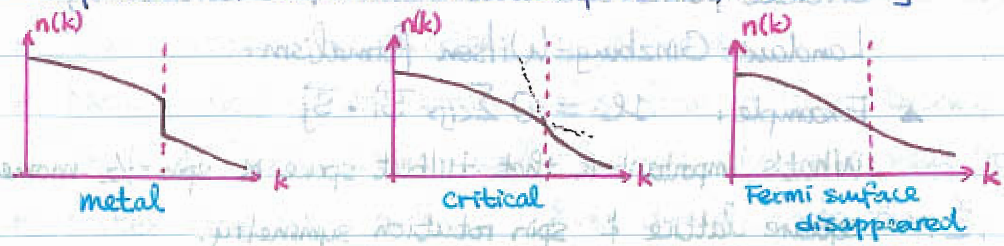
- Possible Directions: (1) theory of "deconfined criticality".

  (2) concept of "critical Fermi surface".

- Fermi Suface Disappearence :

  ▲ Quasiparticle weight may vanishes continuously and everywhere on Fermi surface. (Brinckman — Rice 1973)

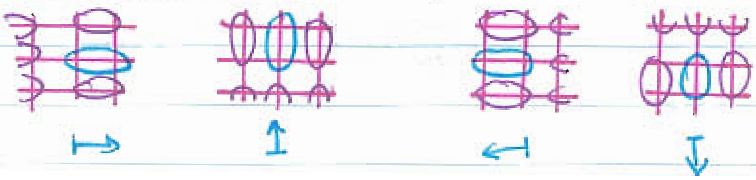
  ▲ Claim: at critical point, Fermi surface is still sharply defined ("critical Fermi surface"), even though $Z = 0$.



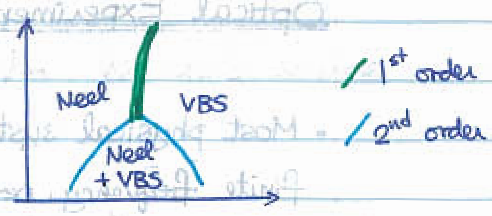
▲ In case of heavy fermion, both large/small Fermi surface must disappear.

- Scaling Phenomenology near critical Fermi surface
  - ▲ May expect $A_c(\vec{k},\omega,T) \sim \frac{1}{|\omega|^{\alpha/z}} F\left(\frac{\omega}{|k_\parallel|^z}, \frac{\omega}{T}\right)$
    But we can have α, z angle dependent, as long as it agrees with lattice symmetry.
  - ▲ Expect scale invariant spectrum cutoff at $k_\parallel \sim \xi^{-1}$, $\omega \sim \xi^{-z}$, where $\xi \sim |g-g_c|^{-\nu}$ with ν depends on angle.
  - ▲ Prediction : $C_V \sim T\int_{FS} \frac{1}{V_F} \sim T\int_{FS} |\delta g|^{-\nu(1-z)}$
    For ν, z depends on θ, result dominated by portion with maximum $\nu(1-z)$
  - ▲ Different portions of Fermi surface may emerge out of criticality at different energy scale
    $\Longrightarrow$ richer finite-T crossover than usual.
  - ▲ NOTE: we still assume only ONE critical point g_c instead of multiple critical point

- Simultaneous disappearence of Fermi surface & appearence of magnetic order.
  - ▲ Start with simpler situations where both side of critical points are well known & describe by Landau-Ginzburg-Wilson formalism.
  - ▲ Example: $\mathcal{H} = J\sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j$
    What's important is that Hilbert space is spin-1/2 moments with square lattice & spin rotation symmetry.
    Consider $\underset{O(3)}{\underline{\text{Neel}}} \longleftrightarrow \underset{\mathbb{Z}_4}{\underline{\text{VBS}}}$ state
    - ▶ VBS state has $\mathbb{Z}_4$ order parameter



▸ Ginzburg–Landau prediction:



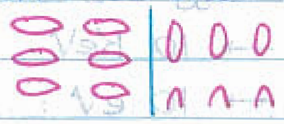
▸ Failure of naive approach:

Neel $\neq$ usual $O(3)$ transition in $d=3$

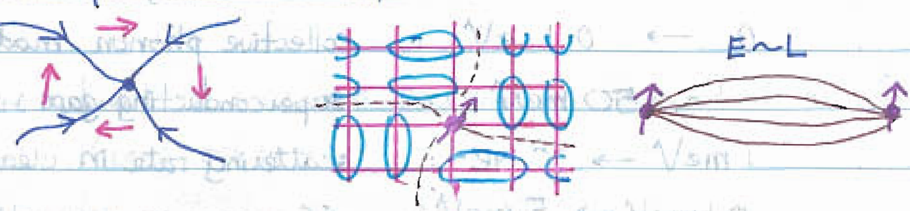
VBS $\neq$ usual $\mathbb{Z}_4$ transition in $d=3$

Reason: topological defects carry non-trivial quantum numbers

▸ Consider approaching from VBS side. Topological defects are domain walls



▸ These defects meet at $\mathbb{Z}_4$ vortex:



▸ This lead too $\mathbb{Z}_4$ transition, but with vortices carrying spin-1/2 quantum number.

When these vortices condense we obtain Neel state

▸ We want to transform such that vortices become primary variable

$\Longrightarrow$ duality transformation.

▸ The vortices are described by XY-order.

▸ Near critical points domain walls from vortices become "thick" & "soft". Domain wall thickness divergences near transition

▸ Vortex core size $\sim \xi$; ξ_{VBS} diverges faster than ξ.

▸ Best numerical evidence from J–Q model:

$$\mathcal{H} = J\sum_{\langle ij\rangle} \vec{S}_i\cdot\vec{S}_j - Q\sum_{\langle\langle\langle ij\rangle\langle kl\rangle\rangle\rangle} (\vec{S}_i\cdot\vec{S}_j - 1/4)(\vec{S}_k\cdot\vec{S}_l - 1/4)$$

$\begin{cases} Q/J \text{ small} \Rightarrow \text{Neel} \\ Q/J \text{ large} \Rightarrow \text{VBS} \end{cases}$

Appendix of Senthil's talk

Consider XY-model in (2+1)-dimensions

Recall $E(\ast) \sim \ln R$, so it can be interpreted as charges

Formally, $Z = \int \prod_i d\theta_i \, e^{-K\sum\cos(\theta_i-\theta_j)}$

$$\approx \int \prod_i d\theta_i \sum_{\{\vec{J}\}} e^{-\sum_{\text{links}} u\vec{J}^2 + i\vec{J}\cdot\vec{\Delta}\theta}$$

$i \xrightarrow{\Delta\theta = \theta_j - \theta_i} j$

Performing θ-integral $\Longrightarrow$ $\vec{\Delta}\cdot\vec{J} = 0$

$$Z \approx \sum_{\{\vec{J}\}} \delta(\vec{\Delta}\cdot\vec{J}) \, e^{-u\sum_{\text{links}}\vec{J}^2}$$

Solve the constraint via $\vec{J} = \vec{\Delta}\times\vec{A}$ ($\vec{A}$ lives on dual lattice), with $\vec{A}$ an integer

$$Z = \sum_{\{\vec{A}\}} e^{-u\sum_{\text{plaq.}}(\vec{\Delta}\times\vec{A})^2}$$

$$Z \mapsto \int \mathcal{D}\vec{A} \sum_{\{\vec{J}_v\}} e^{-u\sum_{\text{plaq.}}(\vec{\nabla}\times\vec{A})^2 + 2\pi i\vec{J}_v\cdot(\vec{A} + \overbrace{e_c\vec{J}_v}^{\text{core energy}} - \overbrace{\vec{\Delta}\phi}^{\vec{A}\text{ define up to phase}})}$$

$$\mapsto \int \mathcal{D}\vec{A}\,\mathcal{D}\phi \; e^{-\int u(\vec{\nabla}\times\vec{A})^2 + t_v\cos[2\pi(\vec{\nabla}\phi - \vec{A})]}$$