Exotic Quantum Phases (II) [Senthil]

- Last time, we consider trial wavefunction for Mott state:

$$\Psi_F = \Psi_B^{solid} \cdot \Psi_{Slater}$$

▲ In the extreme limit, we can freeze the charge degree of freedom, by forbidding double occupation everywhere

$$\Longrightarrow \Psi_B^{solid}\, \Psi_{Slater} \sim \underbrace{P_G}_{\text{Gutzwiller projector}} \Psi_{Slater}$$

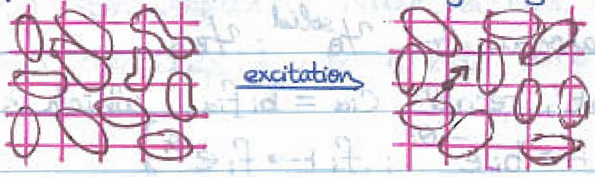
▲ Now consider $\Psi_F = \Psi_B^{solid} \cdot \Psi_{BCS}$

$\Psi_{BCS} \Longrightarrow$ spins are paired to singlet

$\Psi_B^{solid} \Longrightarrow$ no phase coherence

This results in "resonating valence bond"

i.e. RVB = spin correlated state + charge degree freezed out



The excitation of the system is a spin at site i that's NOT part of the pairing $\Longrightarrow$ "spinon" (which carries spin-1/2)

NOTE that sound wave mode in Ψ_{BCS} is killed upon Ψ_B^{solid} x, since charge degree is freezed

But vortex excitation can survive. $P_G|\text{vortex w/ } hc/2e \text{ flux}\rangle = |\text{"vison"}\rangle$

▲ To understand vison, first consider BCS wavefunction:

$$|BCS\rangle \propto e^{\sum_k g_k c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger}|0\rangle$$

$$\Longrightarrow |BCS\rangle_{2N} = (\textstyle\sum_k g_k c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger)^N|0\rangle = (\sum_{ij} g\, c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger)^N|0\rangle$$

Then $|hc/e \text{ vortex}\rangle \propto e^{i\sum_i \theta_i \hat{n}_i}|BCS\rangle_{2N}$ $\quad \theta_i = \frac{2\pi x}{L_x}$

To create $|\frac{hc}{2e} \text{ vortex}\rangle$, circumvent multivaluedness by using anti-periodic boundary condition. i.e. $|\frac{hc}{2e}\text{ vortex}\rangle \propto \underbrace{e^{i\sum_i \frac{\theta_i \hat{n}_i}{2}} \overbrace{|BCS^{(AP)}\rangle_{2N}}^{\text{antiperiodic ...}}}_{\text{the full object is periodic}}$

Upon projection, $|\frac{hc}{e}\text{ vortex}\rangle$ does nothing ($\because$ phase unimportant)

But $|\frac{hc}{2e}\text{ vortex}\rangle$ is non-trivial. spinon winds around it induce a phase of π.

NOTE: spinon has antiperiodic BC while e^- has periodic BC.

NOTE: Since phase coherence is destroyed by projection, the broken symmetry in BCS state is restored.

▲ Note that vison & spinon has non-local "statistical" interaction. By analogy with Abaranov-Bohm effect, we can introduce gauge fields.

In the case of spinons & visons, we need only $\mathbb{Z}_2$-gauge field.

To formalize, recall in last time we wrote $\mathcal{H}_{MF} = \mathcal{H}_b + \mathcal{H}_f$

$$\mathcal{H}_b = -t_c \sum_{ij} (b_i^\dagger b_j + h.c.) + U_b \sum_i \frac{n_i(n_i-1)}{2}$$

$$\mathcal{H}_f = -\sum_{ij} t_{ij}^s (f_i^\dagger f_j + h.c.) + \sum_{ij} (\Delta_{ij} f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger + h.c.)$$

If $U_b \gg t_c$, then boson form insulator, and we obtain a state analogous to $\Psi_B^{solid} \cdot \Psi_{BCS}$.

Next, recall $c_{i\alpha} = b_i f_{i\alpha}$, which is invariant under $\{b_i \mapsto b_i e^{-i\theta_i} ; f_i \mapsto f_i e^{i\theta_i}\}$

But once ff paired, the invariance is reduced down to the $\mathbb{Z}_2$ subgroup $\left\{ \begin{matrix} b_i \longmapsto -b_i \\ f_{i\alpha} \longmapsto -f_{i\alpha} \end{matrix} \right\}$

If instead we consider $\Psi_F = \Psi_b^{solid} \cdot \Psi_{slater}$. In the corresponding theory the $U(1)$ gauge symmetry is unbroken.

▲ Stability of spin liquids

The lattice gauge theory of slave-boson theory has several phases:

(1) "confined phase": gauge fluctuation strong; f & b disappears from physical spectrum

(2) "deconfined": f, b becomes useful degree of freedom.

(3) "Higgs": $\langle b \rangle \neq 0$

For slave-boson theory, stability depends on:

(1) gauge group

(2) spatial dimension

(3) spectrum of (internal) gauge charged excitations.

▲ Confinement vs. deconfinement

consider two spinons seperated by distance L. If there is tension between them (e.g. Energy $\sim L$), then spinons are confined.

Linear potential ⟷ E-field not fluctuate ⟷ B-field strongly fluctuatin

$\Longrightarrow$ confinement.

Instead, B-field not fluctuate ⟷ E-field fluctuate strongly

$\Longrightarrow$ deconfinement.

▲ Examples for spin liquid

(1) $\mathbb{Z}_2$ spin liquid, stable for $d \geq 2$

(2) gapped $U(1)$ spin liquid, stable for $d \geq 3$.

(3) Gapless $U(1)$ spin liquid, potentially stable at $d = 2$.

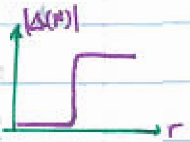
▲ Gapped vs Gapless Spin liquid

(1) gapped spin liquid — best understood theoretically

(2) gapless spin liquid — non-trivial thermal conductivity, even though an insulator. Spinons may form Fermi surface / line / point.

Appendix:

Start with $\mathcal{H}_{BCS} = \sum \epsilon_k c_k^\dagger c_k + \sum \left(\Delta(\vec{r})\, c_\uparrow^\dagger(\vec{r})\, c_\downarrow^\dagger(\vec{r}) + h.c.\right)$



Transform $d_\uparrow(\vec{r}) = c_\uparrow(\vec{r})$, $d_\downarrow(\vec{r}) = c_\downarrow^\dagger(\vec{r})$; $d = \begin{bmatrix} d_\uparrow \\ d_\downarrow \end{bmatrix}$

$$\Longrightarrow \mathcal{H}_{BCS} \sim \int d^2x\; d^\dagger \left[\left(-\frac{\nabla^2}{2m} - \mu\right)\tau^3 + |\Delta(\vec{r})| \left(e^{i\theta}\tau^+ + e^{-i\theta}\tau^-\right)\right] d$$

Need solve $\left[\left(-\frac{\nabla^2}{2m} - \mu\right)\tau^3 + |\Delta(\vec{r})| \left(e^{i\theta}\tau^+ + e^{-i\theta}\tau^-\right)\right] \begin{bmatrix} u(r,\theta) \\ v(r,\theta) \end{bmatrix} = E \begin{bmatrix} u(r,\theta) \\ v(r,\theta) \end{bmatrix}$

Solutions:
So
$$\begin{cases} u(r,\theta) = \tilde{u}(r,\theta)\, e^{i\theta/2} \\ v(r,\theta) = \tilde{v}(r,\theta)\, e^{-i\theta/2} \end{cases}$$

$[\tilde{u}, \tilde{v} \text{ real}]$