Exotic Quantum Phases (I) [Senthil]

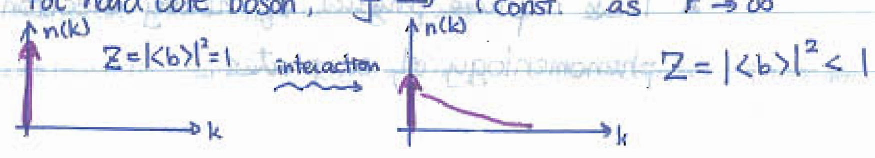
- Central Ideas in "conventional" condensed matter:
  (1) e^- retains its integraty as quasiparticles
  (2) Notion of order parameter / spontaneous symmetry breaking
  ▲ These ideas have been challenged by experimental findings such as fractional quantum Hall and high-Tc superconductivity

- Some conceptual questions
  ▲ Does every quantum phase has elementary excitation?
  ▲ Can interacting bosons have metallic (i.e. ρ finite) phase?
  ▲ Is order in phase necessarily described by Landau order parameter?
  ▲ Does e^- have to survive as quasipaticle in phases?
  ▲ Does clean metal always have sharp Fermi surface?
  ▲ Can solid with odd e^- per cell have non-symmetry breaking ground state?

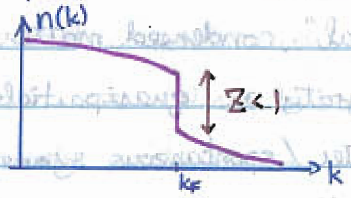
- In general, interacting system has Hamiltonian of form:
  $$\mathcal{H} = T + V$$
  e.g. Hubbard model: $\mathcal{H} = -t\sum(c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U\sum n_{i\uparrow}n_{i\downarrow}$
  ▲ The two extreme limits ($\underbrace{T \gg V}_{e^-\text{ wavelike}}$ or $\underbrace{V \gg T}_{e^-\text{ particle like}}$) is easy.

  ▲ Interesting physics occurs in intermediate regime $T \sim V$.

- Consider delocalized limit
  ▲ Bose fluid: $\psi \propto \prod_{i<j} f(\vec{r}_i - \vec{r}_j) = e^{-\frac{1}{2}\sum_{i<j} u(\vec{r}_i - \vec{r}_j)}$
  For ideal gas, $f = \text{const.}$;
  For hard core boson, $f \to \begin{cases} 0 & \vec{r} \to 0 \\ \text{const.} & \text{as } \vec{r} \to \infty \end{cases}$



▲ Fermi fluid: $\psi \propto e^{-\frac{1}{2}\sum_{i<j} u(\vec{r}_i - \vec{r}_j)} \psi_{\text{Slater}}(\{\vec{r}_i, \sigma_i\})$



▲ Special case: Gutzwiller wavefunction

$$\psi_{\text{Gutz}} = \left[\prod_i (1-(1-g)n_{i\uparrow}n_{i\downarrow})\right]\psi_{\text{Slater}} = g^{\sum_i n_{i\uparrow}n_{i\downarrow}}\,\psi_{\text{Slater}}$$

▲ Note that in general we can write down a Fermion wavefunction by: $\psi_F = \psi_B \cdot \psi_{\text{Slater}}$

▲ Thus, we can think ψ_B as a spinless "slave" of the fermion. This motivates "slave boson" mean-field theory.

• Slave Boson Mean-field Theory

▲ Write $c_{i\alpha} = b_i f_{i\alpha}$. Replace microscopic $\mathcal{H}$ by approximate $\mathcal{H}_{MF}$ in which holon & spinon are non-interacting (but which $\mathcal{H}_{MF}$ is determined self-consistently)

$$\mathcal{H}_{MF} = \mathcal{H}[b] + \mathcal{H}[f]$$

$$\mathcal{H}[b] = \sum_{ij} t^c_{ij}(b_i^\dagger b_j + h.c.) + V_{int}[b^\dagger b]$$

$$\mathcal{H}[f] = -\sum_{ij} t^s_{ij}(f^\dagger_{i\alpha} f_{j\alpha} + h.c.)$$

▲ The metalic phase given by condensing b:

$$c_{i\alpha} = \langle b \rangle f_{i\alpha}$$

$$\Longrightarrow \langle c\bar{c}\rangle = |\langle b\rangle|^2 \langle f\bar{f}\rangle$$

$\Longrightarrow$ quasiparticle residue $Z = |\langle b\rangle|^2$

▲ If instead we're interested in correlated superconductor:

$$\psi_F^{SC} = \psi_B \cdot \psi_{BCS}$$

or $\mathcal{H}[f] = \Delta f_{i\uparrow} f_{i\downarrow} + \Delta^* f^\dagger_{i\uparrow} f^\dagger_{i\downarrow}$

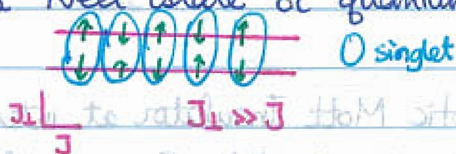
▲ These capture physics of heavy electron metal & phenomenlogy of curprates.

- Next consider localized limit
  - ▲ Consider Bose-Hubbard:

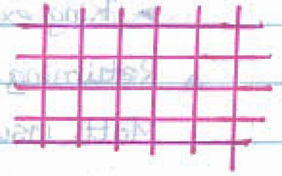
$$\mathcal{H} = -\sum t_{ij}(b_i^\dagger b_j + h.c.) + \frac{1}{2}\sum_i n_i(n_i - 1)$$

  - ▲ These often end up as Neel state or quantum paramagnet.

    e.g. spin ladder



    e.g. Valence bond solid: bond pattern that spontaneously breaks lattice translation symmetry (can be caused by both magnetic or phonon coupling)



- Quantum Spin Liquid
  - ▲ Mott insulator whose ground state is not smoothly connected to band insulator [technical defn]
  - ▲ OR: spin-1/2 quantum system whose ground state does not break any underlying symmetries [rough defn]
  - ▲ From theoretical considerations, quantum spin liquid CAN exists in $d > 1$.
  - ▲ Experimentally, it seems that quantum spin liquid DO exists in $d > 1$ (e.g. organics, kagome, hyper-kagome)
  - ▲ Interesting properties of quantum spin liquid

    (1) exotic excitations (with fractional spin, non-local interactions described through gauge fields, etc.)

    (2) ordering not captured by broken symmetry (e.g. "topological order" that capture global property of wavefunction)

    (3) platform for onset of many unusual phenomena (e.g. superconductivity from spin liquid state?)

    (4) great experimental setting for violating "conventional" condensed matter.

- Possible systems where quantum spin liquid can be found
  - geometrically frustrated magnets (e.g. Kagome)
  - Intermediate correlation regime (e.g. organics)

- Exotic Mott Insulator at intermediate correlation
  - Recall if $t/U \nearrow \Longrightarrow \mathcal{H}_{eff} = \sum \underbrace{J_{ij}}_{\text{more than nearest neighbor}} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} (P_{1234} + P_{1234}^{-1})$
    - Ring exchange tends to favor spin liquid.
  - Returning to wavefunction formulation: $\Psi_F = \Psi_B \cdot \Psi_{slater}$

    Mott insulator ~ freezing of charge motion

    $\Longrightarrow$ use Ψ_B of localized Bose solid wf.
    - But then spin correlation would be roughly the same as in ordinary metal.

      coming from Ψ_{slater}
  - A competing state would be $\Psi_F = \Psi_B \cdot \Psi_{BCS}$, with Ψ_B again a localized Bose solid wf. The resulting state is still a Mott insulator, but may be a spin singlet.