Exotic phases and unconventional quantum criticality

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Conventional condensed matter physics: Two central ideas

1. Integrity of the electron as a ``quasiparticle” in phases of matter

(Fermi liquid metals, band insulators, BCS superconductors, spin density wave states, ……)

2. Notion of ``order parameter’’ to describe phases of matter

- related notion of spontaneously broken symmetry
- basis of phase transition theory

Modern quantum condensed matter physics

In the last 25 years both of these ideas have been challenged enormously by discoveries such as the fractional quantum Hall effect, high temperature superconductivity, and other phenomena.

Many fundamental questions have been raised (and some answered).

A sample of some basic conceptual questions

1. Does the electron have to survive as a quasiparticle in a phase of matter?

2. Does every quantum phase have `elementary excitations'?
(i.e does the low energy excitation spectrum admit a particle-like description)?

3. Does a clean metal need to have a sharp Fermi surface at T = 0 ?

4. Can interacting bosons have a metallic ground state?

5. Can a solid with odd number of electrons per unit cell have an insulating ground state that does not break any symmetry?

6. Is the order in a phase necessarily captured by a Landau order parameter?
⇔ Is symmetry breaking the only route to ordering?

7. Is it always correct that singularities at phase transitions are due to slow fluctuations of the order parameter?

In these lectures I will describe recent theoretical advances in understanding phenomena that defy some aspect of conventional condensed matter physics.

Plan:

1. `Exotic’ Mott insulators (popularly known as quantum spin liquids)
2. Quantum criticality beyond the Landau-Ginzburg-Wilson paradigm
3. Ideas on non-fermi liquid metals (if time permits)

Reading material: ``Quantum matters: Physics beyond Landau’s paradigms”, TS, Int Journal of Mod. Phys., ’06 (arxiv: 04xxxxx).

Some simple observations on correlated systems

In general $$H = \underbrace{\hat{T}}_{\text{K.E}} + \underbrace{\hat{V}}_{\text{Interaction energy}}$$

Example : (i) $$H = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2} \sum_{i,j} V(\vec{x}_i - \vec{x}_j)$$

appropriate for He-4 or He-3

(ii) Lattice systems: Hubbard model

$$H = - \sum t_{ij} (c_i^{\dagger} c_j + h.c) + U \sum_i n_i (n_i - 1)/2$$

Two extreme limits

$\hat{T}$ dominates: diagonalize in $\vec{k}$-space

"Wave" picture.

$\Leftrightarrow$ Fermi / Bose liquid

$\hat{V}$ dominates: diagonalize in real space

"Particle" picture

$\Leftrightarrow$ Solid

Many interesting phenomena happen in `intermediate’ correlation regime where neither kinetic or interaction energy is clearly dominant.

Neither `particle’ nor `wave’ picture clearly superior in this regime.

The physics described in these lectures is due to the competition between kinetic and interaction energies in this regime.

Delocalized limit: Correlated bose and fermi liquids – a brief discussion

Bose liquid

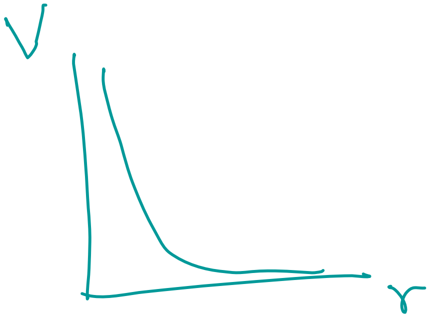
Good guess for ground state wavefunction

$$\psi(\vec{r}_1, \ldots, \vec{r}_N) \propto \prod_{i<j} f(\vec{r}_i - \vec{r}_j) = e^{-\frac{1}{2} \sum_{i<j} u(\vec{r}_i - \vec{r}_j)}$$

"Jastrow wavefunction"

Ideal gas: f = constant.

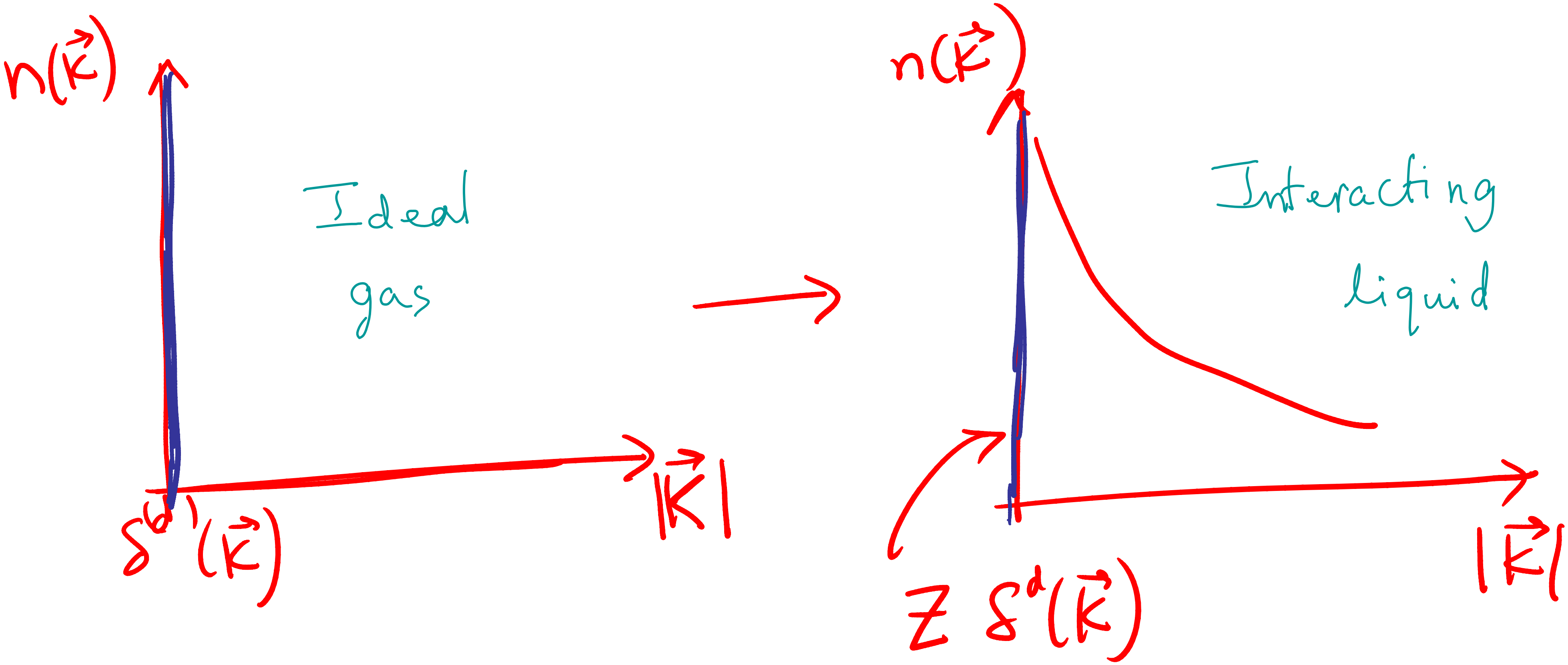
Repulsive "hard core": $f(\vec{r} \to 0) \to 0$



$f(\vec{r} \to \infty) \to$ constant

$$\left(u(\vec{r} \to \infty) \sim \frac{1}{r^{d-1}} \right)$$

Ground state momentum distribution



"Condensate fraction" $Z = |\langle b \rangle|^2 < 1$.

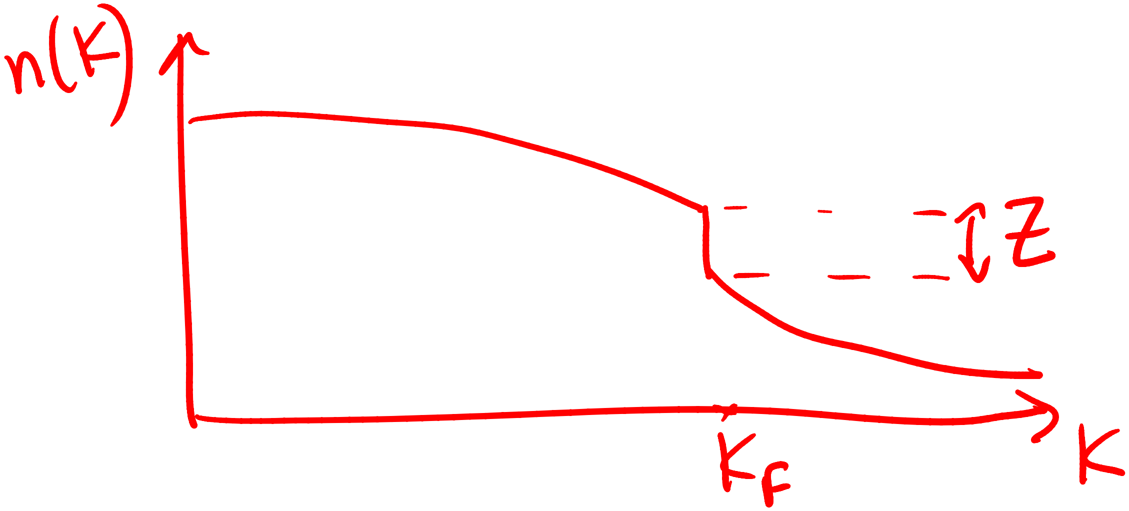
Interacting Fermi liquid

Interacting Fermi fluid :

Incorporate correlations with a Jastrow factor

$$\psi_F(\vec{r}_1\sigma_1, \ldots, \vec{r}_N\sigma_N) = e^{-\frac{1}{2}\sum_{i<j} u(\vec{r}_i - \vec{r}_j)} \psi_{\text{slater}}(\{r_\ell \sigma_\ell\})$$

Momentum distribution



Jump discontinuity at Fermi surface $Z < 1$

(analagous to boson condensate fraction)

Special case: Gutzwiller wavefunction

For lattice Hubbard model, special choice $f_{ij} = g\,\delta_{ii}$ with $g<1$

$$\Rightarrow \quad \psi_{\text{Gutzwiller}} = \underbrace{\left[\prod_i \left(1-(1-g)\,n_{i\uparrow}n_{i\downarrow}\right)\right]}_{\text{Jastrow factor}} \psi_{\text{slater}}$$

Can write $\psi_F = g^D \psi_{\text{slater}}$ (D = # of doubly occupied sites)

g^D factor suppresses weight for large D.

An interesting point of view

Can think of $\psi_F = (\text{Jastrow}) \times \psi_{\text{slater}}$

as $\psi_F = \underbrace{\psi_b(\vec{r}_1, \ldots, \vec{r}_N)}_{\text{Boson wave fn}} \psi_{\text{slater}}(\vec{r}_1\sigma_1, \ldots, \vec{r}_N\sigma_N)$

Clearly any choice of ψ_b will give a legitimate fermion wavefunction.

Choosing ψ_b as wavefunction of superfluid leads to the Fermi liquid wavefunction ψ_F.

A interesting point of view (cont'd)

Spatial coordinates of bosons $\vec{r}_i$ are same as those of fermions irrespective of spin

$\Rightarrow$ (i) Bosons are "slaved" to the fermions.

(ii) Bosons should be viewed as spinless.

Motivates a "slave boson" mean field theory of correlated metal.

Slave boson mean field theory

Write electron operator as $c_{i\alpha} = b_i f_{i\alpha}$

spinless boson

spin-$\frac{1}{2}$ fermion ("spinon")

Replace microscopic H by equivalent approximate non-interacting H_{MF} for holons & spinons with self-consistently determined parameters.

$$H_{MF} = \underbrace{H[b]}_{\text{Repulsive bosons}} + \underbrace{H[f]}_{\text{Non-interacting fermions}}$$

$$H[b] = -\sum_{ij} t^c_{ij} (b^\dagger_i b_j + h.c) + V_{int}[b^\dagger b]$$

$$H[f] = -\sum_{ij} t^s_{ij} (f^\dagger_{i\alpha} f_{j\alpha} + h.c).$$

f_α form a Fermi surface.

Metallic phase: b condensed, $\langle b \rangle \neq 0$

$$\Rightarrow \quad c_{i\alpha} = \langle b \rangle f_{i\alpha}$$

Electron Green function $\langle c \bar{c} \rangle \simeq |\langle b \rangle|^2 \langle f \bar{f} \rangle$

$\Rightarrow$ Quasiparticle residue $Z = |\langle b \rangle|^2$

(= "condensate fraction" of boson)

Correlated superconductors

Wavefunction $\Psi_F^{SC} = \underbrace{\Psi_b(\vec{r}_1, \ldots, \vec{r}_N)}_{\text{Jastrow}} \Psi_{BCS}(\vec{r}_1\sigma_1, \ldots, \vec{r}_N\sigma_N)$

Within slave boson mean field theory

$$H_{MF} = H[b] + H[f]$$

Allow $H[f]$ to have ff pairing terms

$$H[f] = -\sum_{ij} t^s_{ij} \left(f^\dagger_{i\alpha} f_{j\alpha} + h.c\right) + \Delta_{ij} f_{i\uparrow} f_{j\downarrow} + h.c$$

Cooper pair order parameter $\langle cc \rangle \approx |\langle b \rangle|^2 \langle f f \rangle$

Slave boson/Gutzwiller mean field theories are a useful way to incorporate correlations into a description of metals/superconductors.

Examples: good description of heavy electron metal, success in capturing some of the phenomenology of the cuprates.

Localized limit: Mott insulators and conventional quantum magnetism – a brief discussion

Electronic Mott insulators

Prototype: ½-filled Hubbard model at large-U

$$H = -\sum t_{ij} \left(c^{\dagger}_{i\alpha} c_{j\alpha} + h.c \right) + U \sum_{i} \frac{n_i (n_i - 1)}{2}$$

Large-U : Charges localize below some temperature $\sim o(U)$

Active low energy degree of freedom is electron spin.

Describe by $H_{eff} \approx J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \ldots\ldots$

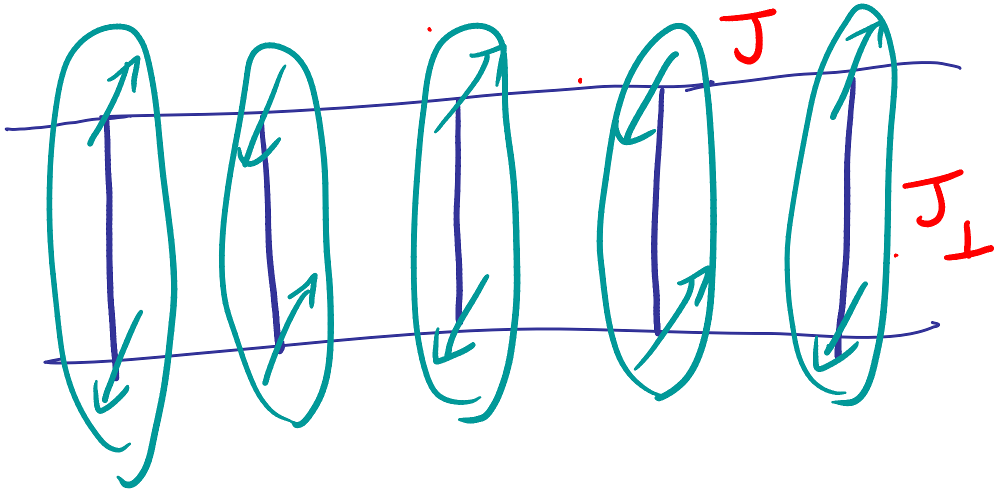
$\left(J \sim t^2/U > 0 \right)$

Common fate: Neel magnetic ordering into some frozen spin pattern

Interesting situations when Neel ordering is "geometrically" frustrated by lattice structure and/or low dimension.

Can get quantum paramagnet ground states with no magnetic LRO even at $T=0$.

Spin ladders: A simple example of a quantum paramagnet



$J_\perp \gg J$:

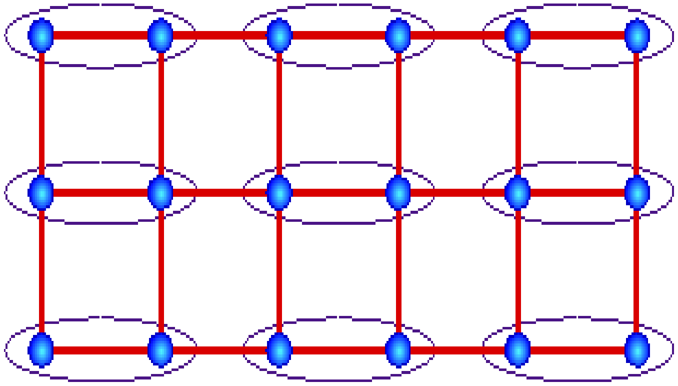
Form rung singlets

$\Rightarrow$ Paramagnetic ground state

Smoothly connected to $J_\perp \ll J$!

Many examples ($SrCu_2O_3$, ...)

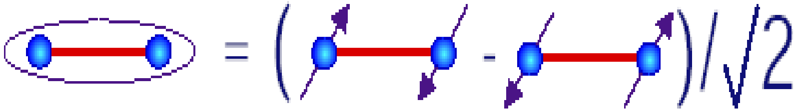
Other quantum paramagnets: Valence Bond Solid(VBS) (alias spin-Peierls) states



- Ordered pattern of valence bonds breaks lattice translation symmetry.

- Ground state smoothly connected to band insulator

- Elementary spinful excitations have S = 1 above spin gap.



Seen in many model calculations,
(Eg: $J_1 - J_2$ model on □ lattice with $S = 1/2$)

Real materials - strong coupling to phonons ($CuGeO_3$, $TiOCl$, $ZnCr_2O_4$, ...)

Most interesting possibility: quantum spin liquids

What is a quantum spin liquid?

Rough definition: Quantum paramagnet with $S = 1/2$ per unit cell that does not break any symmetries.

More precise and general: Mott insulator with ground state not smoothly connected to band insulator

(Spin ladder paramagnets, VBS states do not count).

Some natural questions

Can quantum spin liquids exist in d > 1? (Anderson '73)

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Theoretical question

Do quantum spin liquids exist in d > 1?

Experimental question

Some natural questions

Can quantum spin liquids exist in d > 1? (Anderson '73)

Theoretical question: YES!!

Do quantum spin liquids exist in d > 1?

Experimental question: Remarkable new materials possibly in spin liquid phases

Organics $\kappa\text{-}(ET)_2\,Cu(CN_3)_2$; Kagome $ZnCu(OH)_2Cl_2$,

$EtMe_3Sb[Pd(dmit)_2]_2$; Hyper Kagome $Na_4Ir_3O_8$

2d solid He-3 .

Why are quantum spin liquids interesting?

1. Exotic excitations

Excitations with fractional spin (spinons),

non-local emergent interactions described through gauge fields

As rich in possibility if not richer than the fractional quantum Hall systems……….but requires less extreme conditions (eg: no strong B-fields)

Why are quantum spin liquids interesting?

2. Ordering not captured by concept of broken symmetry

- new concepts of `topological order' and generalizations

Order is a global property of ground state wavefunction

Possibility of encoding information non-locally.

?? Useful for computing?? (Kitaev)

Why are quantum spin liquids interesting?

3. Platform for onset of many unusual phenomena

Eg: (i) Superconductivity in doped Mott insulators

?? Relevant to cuprates ??

(Anderson '87 ; Kivelson, Rokhsar, Sethna '88)

(ii) Non-fermi liquid phenomena in correlated d or f-electron metals.

Why are quantum spin liquids interesting?

4. Excellent experimental setting for exploration of violation of long cherished notions of condensed matter physics

-quasiparticles with fractional quantum numbers and unusual statistics

- the very existence of a quasiparticle description

- inadequacy of Landau order parameter to describe phases and phase transitions of correlated matter

Stability of quantum spin liquids

1. Solution of concrete quantum spin models within 1/N expansions (Read, Sachdev ’91)

2. Effective field theory descriptions (Wen ’91; Balents,Fisher,Nayak, ’99; TS, Fisher’00,…..)

3. Solution of effective models of quantum dimers (Moessner,Sondhi’01; Misguich,Serban,Pasquier’02;….)

4. Solution of various quantum spin/boson models (Kitaev’97,’06;Balents,Fisher,Girvin ’02, Motrunich, TS’02, ……)

5. Numerical calculations on simpler models (Misguich,Lhuillier’98; Sheng, Balents, ’05; Isakov,Paramekanti,Kim,Sen,Damle’07)

Where might we find quantum spin liquids?

- Geometrically frustrated quantum magnets

- ``Intermediate'' correlation regime

Eg: Mott insulators that are not too deeply into the insulating regime

Where might we find quantum spin liquids?

- Geometrically frustrated quantum magnets

Balents lectures ; experiments ? Kagome magnets ?

- ``Intermediate'' correlation regime

Eg: Mott insulators that are not too deeply into the insulating regime

I will mainly focus on these.

Perhaps more promising in experiments ?

Organics $\kappa(ET)_2Cu_2(CN_3)_2$; solid He-3 ? Hyperkagome $Na_4Ir_3O_8$?
$EtMe_3Sb[Pd(dmit)_2]_2$

Exotic Mott insulators at intermediate correlation

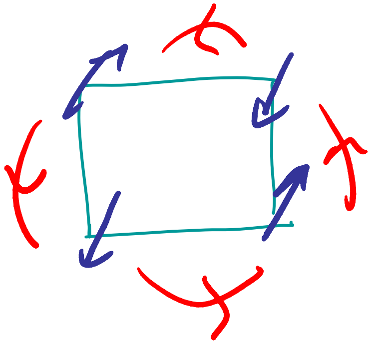
Approach from insulator

t/U ↗ $\Rightarrow$ Build in more virtual charge fluctuations in ground state wave function

$$H_{\text{eff}}[\{\vec{S}_i\}] = \sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \left(P_{1234} + P^{-1}_{1234} \right) + \ldots$$

longer range exchange

$P_{1234} =$



$=$ 4-particle ring exchange, etc

Various arguments: Ring exchange promotes spin liquids

Alternate view from the metallic side

Start with wavefunction of correlated metal

$$\Psi_F(\vec{r}_1\sigma_1, \cdots, \vec{r}_N\sigma_N) = \underbrace{\Psi_b(\vec{r}_1, \cdots, \vec{r}_N)}_{\text{superfluid}} \Psi_{Slater}(\{\vec{r}_\ell \sigma_\ell\})$$

How to get a Mott insulator?

Let $\Psi_b \rightarrow$ wavefunction of localized solid of bosons

$\Rightarrow$ freeze out charge motion

$\Psi_F = \Psi_b^{solid} \Psi_{Slater}$ is wavefunction for fermionic Mott insulator!

Comments

(i) In solid boson condensate fraction $|\langle b \rangle|^2 = 0$

Consistent with expected loss of Fermi surface discontinuity in $n(k)$ in Mott state

(ii) $\psi_F = \psi_b^{solid} \psi_{slater}$ is a spin singlet wavefunction

Expect spin correlations roughly similar to metal?

Reasonable guess for a highly exotic spin liquid Mott state.

Different choices for ψ_{slater} give distinct kinds of spin liquid wave functions.

Particularly interesting choice:

$\psi_F = \psi_b^{solid} \psi_{BCS}$ - still a Mott insulator due to ψ_b^{solid}

Expect to inherit spin physics of superconductor?

Reasonable construction of a simple spin liquid state.

Extreme limit: Gutzwiller projection

Extreme limit of ψ_b^{solid} : Completely freeze out all charge fluctuations $\Rightarrow$ $n_i = 1 \quad \forall i$

$$\psi_F = \psi_b^{solid} \, \psi_{slater} \longrightarrow P_G \, \psi_{slater}$$

P_G = "Gutzwiller projector" = $\prod_i (1 - n_{i\uparrow} n_{i\downarrow})$

ψ_F is a pure spin wave function ; can be tested variationally on ring exchange spin models derived in t/U expansion.

Structure of wavefunction

Illustrate with $\psi = \psi_b^{\text{solid}} \, |BCS\rangle$

$|BCS\rangle \Rightarrow$ spins are paired into singlets

But $\psi_b^{\text{solid}} \Rightarrow$ No phase coherence

Gutzwiller limit: $P_G = \prod_i \int \frac{d\phi_i}{2\pi} \, e^{-i\phi_i} \, e^{i\hat{n}_i\phi_i}$

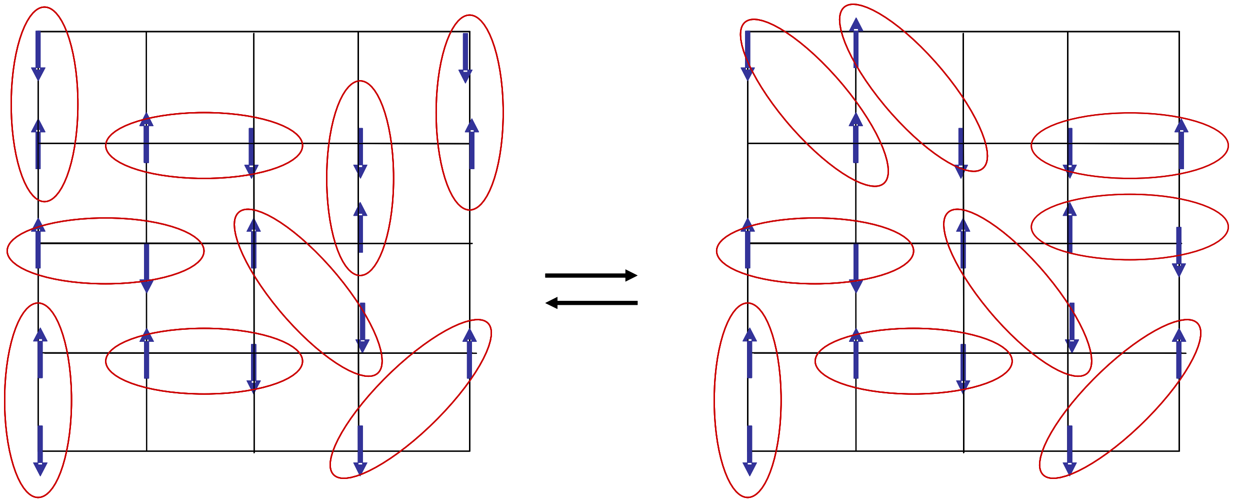
$(\hat{n}_i = c_i^\dagger c_i)$

Integral over phase ϕ_i:

$\Rightarrow$ Phase coherence lost as expected in insulator.

Picture of wavefunction in spin model limit

- ``Resonating valence bond liquid'' (RVB)
- Can think of a singlet valence bond as a Cooper pair formed between two localized electrons.

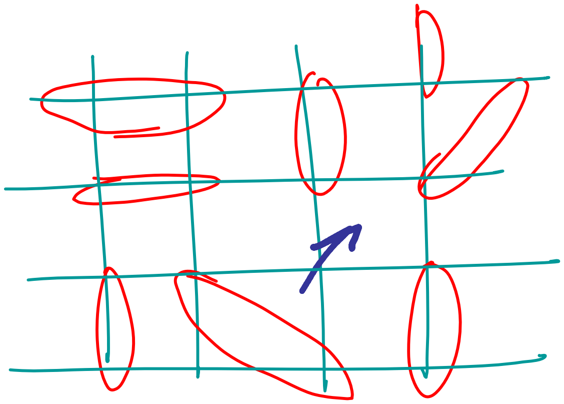


Excitations - spinons

$P_G \,|\, \text{unpaired BCS quasiparticle at site } i \rangle$

$\Rightarrow$ Spin at i not part of valence bond pairing

$\Rightarrow$ Spin-$\frac{1}{2}$ moment at site i = "spinon" excitation



Other excitations

Excitations of a superconductor - (i) quasiparticles

(ii) $hc/2e$ vortices

$$P_G |BCS\rangle \longrightarrow |\text{spin liquid}\rangle$$

$$P_G |\text{quasiparticle}\rangle \longrightarrow |\text{spinon}\rangle$$

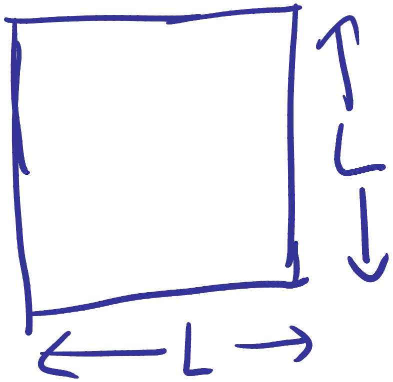
$$P_G |hc/2e \text{ vortex}\rangle = |v\rangle = ??$$

$|v\rangle$: a "topological" excitation of the spin liquid ("vison")

Remarks on BCS wavefunctions - I

$$|BCS\rangle = \prod_{k} \left(u_k + v_k \, c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}\right) |0\rangle$$

$$\propto \left(e^{\sum_k g_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}}\right) |0\rangle \qquad \left(g_k = \frac{v_k}{u_k}\right)$$



(With periodic boundary conditions,

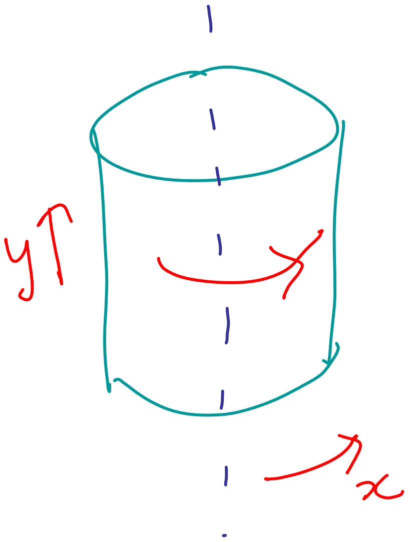
$$c_{\vec{r}+L\hat{x}} = c_{\vec{r}+L\hat{y}} = c_{\vec{r}}\,)$$

For fixed even # of particles = 2N

$$|BCS\rangle_{2N} \propto \left(\sum_k g_k \, c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}\right)^N |0\rangle = \left(\sum_{ij} g_{ij} \, c^{\dagger}_{i\uparrow} c^{\dagger}_{j\downarrow}\right)^N |0\rangle$$

Vortices in BCS wavefunctions

$$|hc/e \text{ vortex}\rangle \propto e^{i\sum_i \theta_i \hat{n}_i} |BCS\rangle_{2N}$$



$$\theta_i = \frac{2\pi x}{L_x}$$

$hc/2e$ vortex must be constructed differently

Naive guess $e^{i\sum_i \frac{\theta_i}{2}\hat{n}_i}|BCS\rangle_{2N}$ not single valued!

Modify naive guess by taking $c_{\vec{r}}$ <u>antiperiodic</u> $c_{\vec{r}+L\hat{x}} = -c_{\vec{r}}$

gives correct $|hc/2e\rangle = e^{i\sum_i \theta_i/2\, \hat{n}_i}|BCS^{(AP)}\rangle_{2N}$

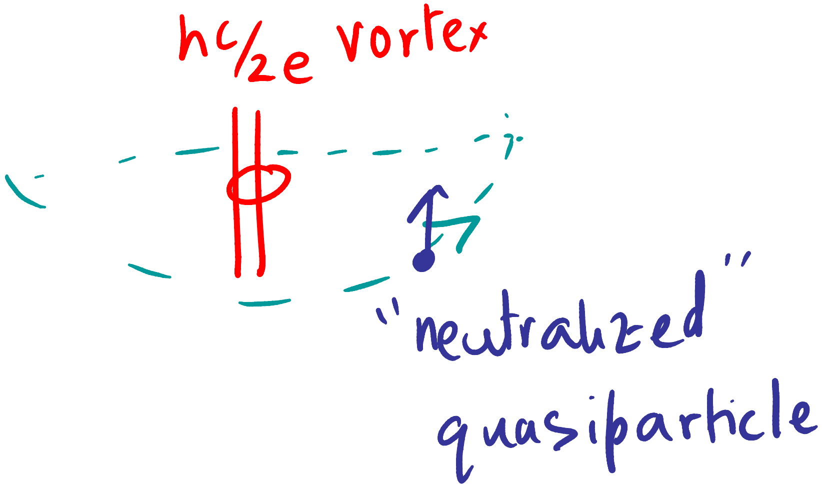
Projected vortices

Overall phase factors unimportant in insulator

$\Rightarrow$ Non-trivial new state obtained by projecting vortices is $P_G \left| {}^{h}\!/\!_{2e} \right\rangle = P_G \left| BCS^{(A \cdot P)} \right\rangle$

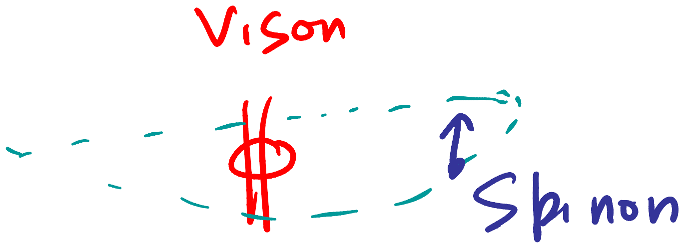
$\Rightarrow$ Spinons see anti-periodic boundary conditions on going around the "projected" vortex.

Full description of excitation spectrum



: Phase of π

$\Rightarrow$



: Phase of π

Full excitation spectrum: Spin-$1/2$ spinons, spin-0 visons with infinitely non-local "stastical" interaction.

(+ charge e holons above charge gap)

Utility of gauge theory

Convenient mathematical formulation of non-local interaction :

Spinons – Ising "electric charge"

Visons – Ising "magnetic flux"

Non-locality : Aharanov-Bohm interaction

Gauge theory forced upon us as natural language for describing spin liquids .

Formal treatment: slave particle theory

Start with slave boson mean field theory of correlated superconductor

$$H_{MF} = H_b + H_f$$

$$H_b = -t_c \sum_{\langle ij \rangle} \left(b_i^\dagger b_j + h.c \right) + U_b \sum_i \frac{n_i (n_i - 1)}{2}$$

$$H_f = -\sum_{ij} t_{ij}^s \left(f_i^\dagger f_j + h.c \right) + \sum_{ij} \Delta_{ij} f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger + h.c$$

In SC, $t_c \gg U_b \Rightarrow \langle b \rangle \neq 0$

Slave particle mean field theory of Mott insulator

If $U_b \gg t_c$, bosons will form a Mott insulator

$\Rightarrow$ electronic Mott insulator.

If charge gap is large, low energy spin physics described by H_f = BCS Hamiltonian for the spinons f_α.

Fluctuations: gauge theory

Slave particle representation $c_{i\alpha} = b_i f_{i\alpha}$

invariant under $b_i \to b_i e^{i\theta_i}$, $f_{i\alpha} \to f_{i\alpha} e^{i\theta_i}$

$\Rightarrow$ U(1) "gauge" redundancy

Mean field with spinon pairing ff terms

only invariant under Z_2 subgroup $b_i \to -b_i$

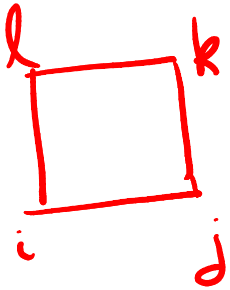
$f_{i\alpha} \to -f_{i\alpha}$

$\Rightarrow$ Theory of fluctuations must include a Z_2 gauge field.

Preliminaries: Z2 gauge fields

Ising version of electromagnetism

(= U(1) gauge theory)



Z_2 "electric" field $= \pm 1$ ($\Leftrightarrow \sigma^x_{ij}$)

Z_2 "vector potential" $A = 0$ or π

$\Rightarrow e^{iA} = \pm 1$ ($\Leftrightarrow \sigma^z_{ij}$)

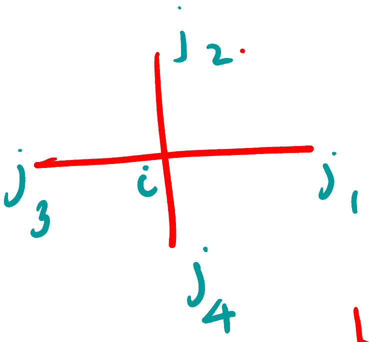
Z_2 "magnetic flux" $\Phi = \sigma^z_{ij} \sigma^z_{jk} \sigma^z_{kl} \sigma^z_{li}$

E-field σ^x_{ij} flips the magnetic flux Φ.

`Pure' Z2 gauge theory (Ising ``electrodynamics")

$$H_{gauge} = \underbrace{-K \sum_{\substack{\text{plaquettes} \\ P}} \sigma^z_{ij}\,\sigma^z_{jk}\,\sigma^z_{kl}\,\sigma^z_{li}}_{\text{analog of } B^2} \underbrace{- h \sum_{\langle ij \rangle} \sigma^x_{ij}}_{\text{analog of } E^2}$$

$$+ \text{ Gauss law constraint } \quad \underbrace{\prod_{j \in i} \sigma^x_{ij}}_{\text{analog of } \vec{\nabla}\cdot\vec{E}} = +1 \quad \forall\, i$$



H invariant under $\sigma^z_{ij} \to -\sigma^z_{ij}$ for all links $\langle ij \rangle$ connected to site i

Effective theory of fluctuations beyond slave particle mean field theory

b_i, $f_{i\alpha}$ carry 'Ising' gauge charge

$\Rightarrow$ couple 'minimally' to σ^z

$$H_{eff} = -\sum_{ij} t^c_{ij}\,\sigma^z_{ij}\left(b_i^\dagger b_j\right) + V_{int}[n_b]$$

$$- \sum_{ij} t^s_{ij}\,\sigma^z_{ij}\left(f_i^\dagger f_j\right) + \sigma^z_{ij}\,\Delta_{ij}\, f_{i\uparrow} f_{j\downarrow} + h.c$$

$$+ H_{gauge}$$

$$+ \text{ Gauss law } \quad \prod_{\substack{\star \\ j\in i}} \sigma^x_{ij} = \underbrace{(-1)}_{Z_2 \text{ 'electric' charge at } i}{}^{n_i + f_i^\dagger f_i}$$

A different spin liquid state

Example: State $\Psi = \Psi_b^{solid} \underbrace{\Psi_{slater}}_{\text{Metal with Fermi surface.}}$

Slave particle mean field theory has no ff pairing terms

$\Rightarrow$ U(1) gauge redundancy unbroken.

Low energy spin physics below charge gap

$$H = - \sum t^s_{ij} \left(e^{i a_{ij}} f^\dagger_{i\alpha} f_{j\alpha} + h.c \right) \left(+ \text{constraint } \vec{\nabla} \cdot \vec{E} = f^\dagger f \right)$$

= U(1) gauge theory of spinons with Fermi surface + gauge field.

Stability of spin liquids

Lattice gauge theories with such structure have several phases

(i) "Confined" : f, b disappear from spectrum; only gauge neutral composites can propagate

(ii) "Deconfined" : f, b useful degrees of freedom

(iii) "Higgs" : $\langle b \rangle \neq 0$ (as in Fermi liquid or BCS states)

Stability of spin liquids

A question of whether gauge structure of mean field state admits a deconfined state.

Differing answers depending on

(i) gauge group (Z_2, $U(1)$, ...)

(ii) Spatial dimensionality (analog of "lower critical dimension")

(iii) excitation spectrum of charged matter fields (gapped / gapless with Fermi points/surfaces, etc).

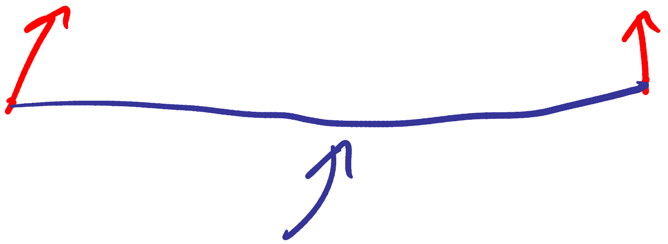
Confinement versus deconfinement

Illustrate with example of Z_2 gauge theory.

Spinon carries Z_2 gauge charge

$\Rightarrow$ source of Z_2 'electric' fields $\left(\sigma^x_{ij} = \pm 1\right)$

2 well separated spinons



"electric" field line.

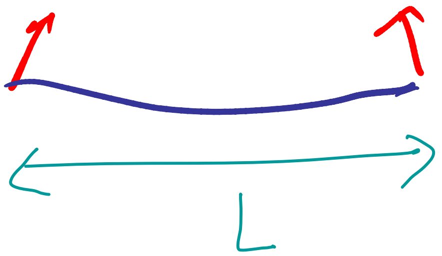
Crucial question: does the E-field line have a tension?

Confinement versus deconfinement (cont'd)

Non-zero line tension ⇒ energy to separate spinons

by length L is ~ L

"Linear confinement"



(E-field does not fluctuate

much ⇒ large B-field fluctuations).

Opposite limit : B-field non-fluctuating

⇒ large fluctuations of E-field lines

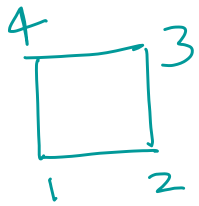
⇒ Zero line tension ⇒ liberate spinons! "Deconfinement"

Deconfinement in pure gauge theory

For simplicity assume both b and f_α gapped & integrate them out.

Effective gauge Hamiltonian

$$H = -K \sum_{P} \underbrace{\sigma^z_{12}\sigma^z_{23}\sigma^z_{34}\sigma^z_{41}}_{\text{B-flux}} - h \sum_{\langle ij\rangle} \underbrace{\sigma^x_{ij}}_{\text{E-field}}$$



(+ constraint)

$h \gg K$: Small σ^x fluctuations $\Rightarrow$ confined

$K \gg h$: Large σ^x fluctuations $\Rightarrow$ Deconfined

What favors deconfinement?

Depressed charge gap in Mott insulator

$\Rightarrow$ K increases

$\Rightarrow$ Increase chances of deconfinement.

Increasing itinerancy may stabilize spin liquid in a Mott insulator.

Varieties of spin liquids

Different kinds of spin liquids distinguished by their emergent gauge structure

Examples: ① Z_2 spin liquids, stable for $d \geqslant 2$

② $U(1)$ spin liquids with gapped spinons & holons, stable for $d \geqslant 3$

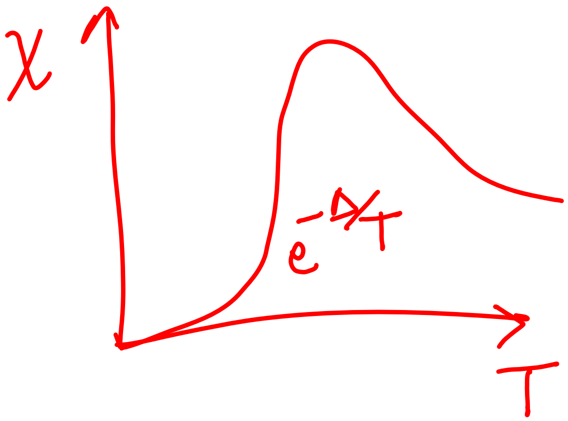
③ $U(1)$ spin liquds with gapless spinons

- potentially stable for $d = 2$ (depending on details)

A useful distinction:

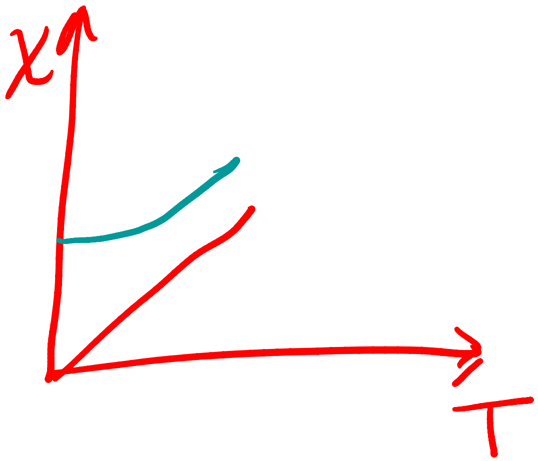
Gapped versus gapless spin spectrum

(i) Spin liquids with spin gap



Best understood theoretically

(ii) Spin spectrum may be gapless



Spinons with Fermi statistics

that have Fermi points or

Fermi surfaces ($\Rightarrow$ Nontrivial low-T thermal transport)

In general different quantum spin liquid phases will have very different universal low temperature properties

Clear cut predictions for experiments which can distinguish between different spin liquid states.

Summary/outlook

- Maturing theoretical understanding of quantum spin liquid phases in d > 1
- Theoretically demonstrable violations of long cherished notions of condensed matter physics
- Interesting candidate materials exist – exciting times ahead!
- Important general lessons for correlated metallic systems