Heavy Fermion Physics (III) [Coleman]

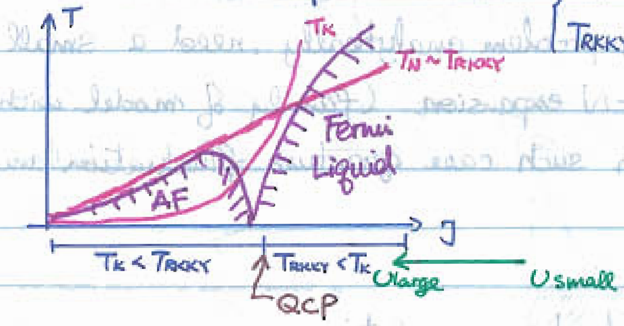
- Kondo Lattice (Kasuya, 1951):

$$\mathcal{H} = \sum \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{J}{N} \sum \vec{S}_j \cdot c^\dagger_{k\alpha} \vec{\sigma}_{\alpha\beta} c_{k'\beta}\, e^{i(\vec{k}'-\vec{k})\cdot \vec{R}_j}$$

▲ Doniach's hypothesis: recall that local moments produce Friedel oscillation $\langle \vec{\sigma}(r) \rangle \sim -J\rho \frac{\cos k_F r}{|k_F r|^3}$. This induces interaction between the local moments, $H_{RKKY} = -J^2 \chi(\vec{x}-\vec{x}')\, \vec{S}(\vec{x}) \cdot \vec{S}(\vec{x}')$

(The interaction induced is often antiferro...)

Thus we have 2 temperature scale: $\begin{cases} T_K = D e^{-1/(2J\rho)} \\ T_{RKKY} = J^2 \rho \end{cases}$



▲ "Large Fermi surface" (Martin, 1982)

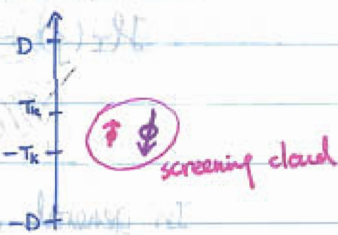
Fermi surface volume

$$\frac{2\Omega_{FS}}{(2\pi)^3} = n_e + n_{spins}$$

- ▶ Each spin contribute one unit of e^- to Fermi surface (!)
- ▶ At QCP, Fermi surface size seems to jump, Fermi mass diverges, but the transition is 2nd order.

▲ "Exhaustion" vs. "Composite Fermion"

$\xi \sim (D/T_K)\, a \cdot 100 \cdot a$

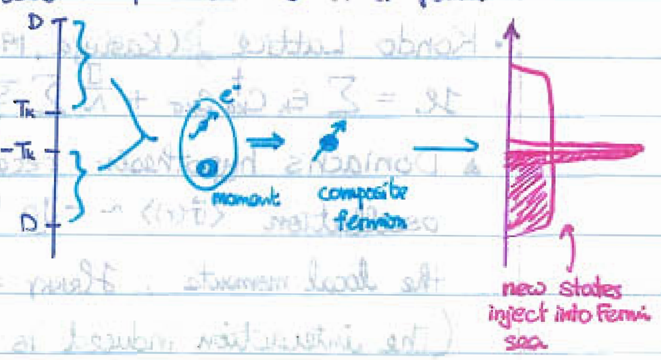


- ▶ Picture is that local moment & conduction e^- bind to form singlet.

When local moment concentration ↗, there won't be enough e^- to fully screen the local moment

⟹ system is always magnetic?

▶ Alternative, may imagine all e^- & local moments form composite heavy fermion:



▲ To analyse the problem analytically, need a small parameter. Do so by large-N expansion (family of model with N spin components). In such case quantum fluctuation amplitude may be reduced

▲ Gauge theory and strong correlation

Recall that in large U limit part of proj space is projected away. This give rise to constraint and hence gauge fields

i.e. strong correlation $\longleftrightarrow$ constriant Hilbert space $\longleftrightarrow$ gauge theory

e.g. $\vec{S}_j = f^\dagger_{j\alpha}\left(\frac{\vec{\sigma}}{2}\right) f_{j\beta}$ reflecting there is no charge degree of freedom

$\Longrightarrow f_j \longmapsto e^{i\phi_j} f_j$ a local $U(1)$ symmetry

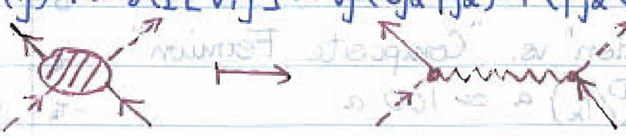
In large N, take $S_{\alpha\beta} = f^\dagger_\alpha f_\beta - \delta_{\alpha\beta} n_f/N$

Lagrage multiplier constriant

$$\Longrightarrow \mathcal{H} = \sum \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \frac{J}{N}\sum (c^\dagger_{j\beta} f_{j\beta})(f^\dagger_{j\alpha} c_{j\alpha}) + \lambda_j (n_f(j) - Q)$$

Decomple using Hubbard-Stravocrsh:

$$\mathcal{H}_I(j) \mapsto \mathcal{H}_I[V,j] = \tilde{V}_j (c^\dagger_{j\alpha} f_{j\alpha}) + (f^\dagger_{j\alpha} c_{j\alpha}) V_j + N\frac{\tilde{V}_j V_j}{J}$$



In general, $V(j)$ is a strongly fluctuating field. But in large N, $V(j)$ becomes a smooth field.

In path integral formulation,

$$Z = \int \mathcal{D}V\mathcal{D}\lambda \int \mathcal{D}c\mathcal{D}f \; \exp\left[-\int_0^\beta \sum c^\dagger_{k\sigma}\partial c_{k\sigma} + \sum f^\dagger_{j\sigma}\partial f_{j\sigma} + \mathcal{H}[V,\lambda]\right]$$

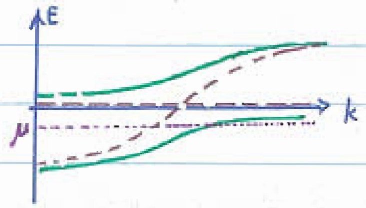
In large N we can just replace V & λ by their mean-field value.
And we absorb the phase of V into the $U(1)$ of f.
This give rise to mean-field Hamiltonian:

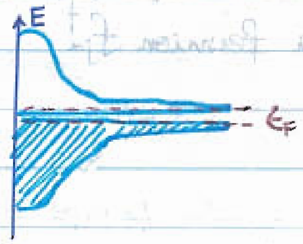
$$\mathcal{H}_{MFT} = \sum \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum \left[V(c^\dagger_{j\sigma} f_{j\sigma} + f^\dagger_{j\sigma} c_{j\sigma}) + \lambda f^\dagger_{j\sigma} f_{j\sigma}\right] + N\mathcal{N}_s\left(\frac{\bar{V}V}{J} - \lambda q\right) \qquad \left[q = \frac{Q}{N} \text{ fixed}\right]$$

$$= \sum (a^\dagger_{k\sigma}, b^\dagger_{k\sigma})\begin{bmatrix} \mathcal{E}_{k+} & \\ & \mathcal{E}_{k-}\end{bmatrix}\begin{pmatrix} a_{k\sigma} \\ b_{k\sigma}\end{pmatrix} + N\mathcal{N}_s\left(\frac{\bar{V}V}{J} - \lambda q\right)$$

$$\left[a^\dagger_{k\sigma} = u_k c^\dagger_{k\sigma} + v_k f^\dagger_{k\sigma} \; , \; b^\dagger_{k\sigma} = v_k c^\dagger_{k\sigma} - u_k f^\dagger_{k\sigma} \right]$$

Solving, $\mathcal{E}_{k\pm} = \frac{\epsilon_k + \lambda}{2} \pm \sqrt{\left(\frac{\epsilon_k - \lambda}{2}\right)^2 + V^2}$



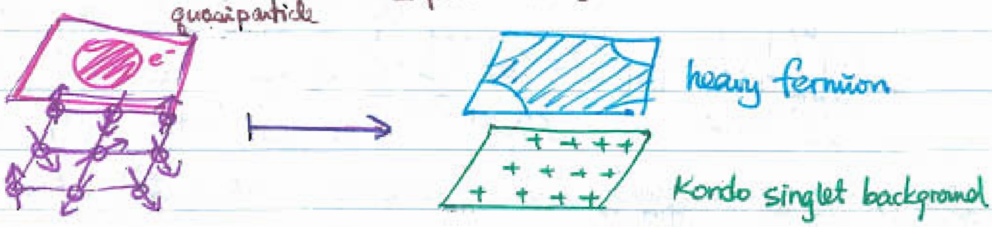


NOTE: V can be thought of as a field that capture the high-freq. charge oscillation at the local site (which previously give rise to Kondo J)

Now, the size of Fermi surface is:

$$N \cdot a^D \frac{\Omega_{FS}}{(2\pi)^D} = \frac{1}{\mathcal{N}_s}\sum_{k\sigma}\langle a^\dagger_{k\sigma} a_{k\sigma} + b^\dagger_{k\sigma} b_{k\sigma}\rangle = \overbrace{n_e}^{\text{density of conduction } e^-} + Q$$

$$\longrightarrow \quad n_e = \underbrace{\frac{N\Omega_{FS}}{(2\pi)^D}}_{\text{quasiparticle}} - \underbrace{\frac{Q}{a^D}}_{\text{positive background}}$$



At $T=0$, the ground state energy is:

$$\frac{E_0}{N\mathcal{N}_s} = -\frac{\rho D^2}{2} + \frac{\Delta}{\pi}\ln(\Delta\rho \quad - \lambda q \qquad \begin{bmatrix} T_K = De^{-1/J\rho} \\ \Delta = \pi\rho V \end{bmatrix}$$

Minimization of E_0 w.r.t. λ gives:

$$\frac{\partial E}{\partial \lambda} = \langle n_f \rangle - Q = \frac{\Delta}{\pi \lambda} - q \implies \frac{\Delta}{\lambda} = \pi q$$

Then, $\frac{E_0}{N N_s} = \frac{\Delta}{\pi} \ln\left(\frac{\Delta \rho}{}\right.$

And the renormalized density of state is $\rho^*(\epsilon) = \rho(\epsilon)\left[1 + \frac{V^2}{\lambda^2}\right] = \overset{=1/D}{\rho} + \frac{q}{T_K}$

$$\implies m^*/m = \rho^*/\rho = (Dq/T_K) \gg 1.$$

▲Note that in the problem we have replaced the Kondo term by hybridization term:

$$\underbrace{\frac{J}{N} S_{\alpha\beta}(j)\, c^\dagger_{j\beta}}\, c_{j\alpha}$$

$$= V f^\dagger_{j\alpha} \; ; \; V \sim \langle f_{j\beta} c^\dagger_{j\beta} \rangle$$

Thus in mean-field we cluster a boson $S_{\alpha\beta}$ and a fermion $c^\dagger_{j\beta}$ into a fermion $f^\dagger_{j\alpha}$