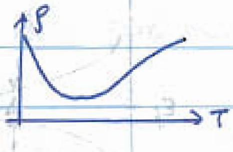
Heavy Fermion — Anderson & Kondo Model [Coleman]

- Moment formation
  - $Nb_{1-x}Mo_x$ $\quad$ $x > 0.4 \Rightarrow$ local moment (e.g. Curie law)
  $\quad x < 0.4 \Rightarrow$ no local moment
    - Also observe resistivity minimum



- Anderson Model
  - $$\mathcal{H} = \sum \epsilon_k n_{k\sigma} + \underbrace{\sum V(k)\left[c^\dagger_{k\sigma} f_\sigma + f^\dagger_\sigma c_{k\sigma}\right]}_{\text{resonance}} + \underbrace{\varepsilon_f n_f + U n_{f\uparrow} n_{f\downarrow}}_{\text{atomic}}$$
  $V(k) = \langle k | V_{\text{atomic}} | f \rangle$; $\int V(\vec{x}-\vec{x}') \, |\psi_f(\vec{x})|^2 \, |\psi_f(\vec{x}')|^2$
  $= 4\pi i^l \int r^2 dr \, j_l(kr) V(r) R_f(r)$
  - Focus on the atomic part, we have 4 states:

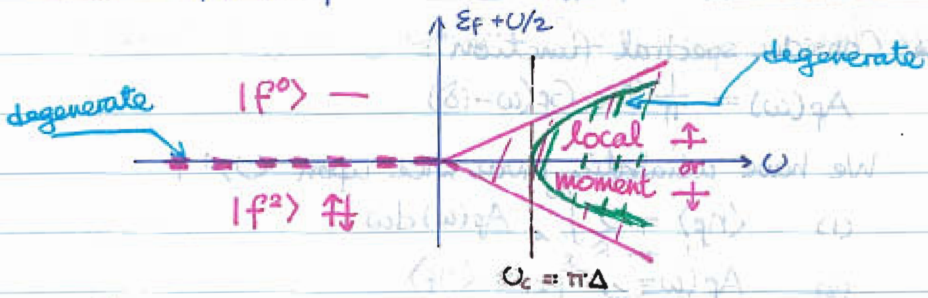
non-magnetic: $\begin{cases} |f^2\rangle : \ \mathcal{E}(f^2) = 2\varepsilon_f + U \\ |f^0\rangle : \ \mathcal{E}(f^0) = 0 \end{cases}$

magnetic $\begin{cases} |f^\uparrow\rangle : \ \mathcal{E}(f^\uparrow) = \varepsilon_f \\ |f^\downarrow\rangle : \ \mathcal{E}(f^\downarrow) = \varepsilon_f \end{cases}$

For stable magnetic state

$\begin{cases} \mathcal{E}(f^0) - \mathcal{E}(f^\sigma) = -\varepsilon_f > 0 & \text{stable under } e^- \text{ removal} \\ \mathcal{E}(f^2) - \mathcal{E}(f^\sigma) = U + \varepsilon_f > 0 & \text{stable under } e^- \text{ addition} \end{cases}$

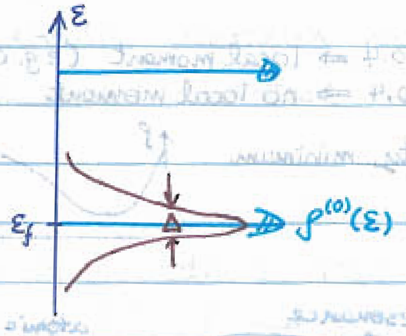
$\Longrightarrow \quad U/2 > \varepsilon_f + U/2 > -U/2$



$\Rightarrow$ mean-field predicted region where local moment forms when coupling is turned on.

[mean-field $\equiv U n_{f\uparrow} n_{f\downarrow} \mapsto U\langle n_{f\uparrow}\rangle n_{f\downarrow} + U n_{f\uparrow}\langle n_{f\downarrow}\rangle - U\langle n_{f\uparrow}\rangle\langle n_{f\downarrow}\rangle$]

▲ Next start with resonance term and neglect atomic part



/ $\rho^{(0)}(\varepsilon)$ [without $V(k)$]

/ $\rho(\varepsilon)$ [with $V(k)$]

$$\Delta = \sum_k |V_k|^2 \delta(\varepsilon_k - \varepsilon_f) \cdot \pi$$

▲ Two approaches

(1) atomic : start with $V(k)=0$, dial up $V(k)$

$\Rightarrow$ local momenta

(2) adiabatic : start with $U=0$, dial up U

$\Rightarrow$ Friedel oscillation (Fermi liquid state ...)

But these two do not seem to agree

▲ What happens is that we have tunneling :

$$e^- + f^{\uparrow} \begin{array}{c} \rightleftarrows f^2 \rightleftarrows \\ \rightleftarrows f^0 \rightleftarrows \end{array} e^- + f^{\downarrow}$$

This gives characteristic rate τ_{eff}.

$k_B T < \frac{\hbar}{\tau_{eff}} = k_B T_K \Longrightarrow$ local moment description invalid

Found $T_K = \sqrt{\frac{2U\Delta}{\pi^2}}\, e^{-\pi U/(8\Delta)}$

▲ Consider spectral function :

$$A_f(\omega) = \frac{1}{\pi} \operatorname{Im} G_f(\omega - i\delta)$$

We have adiabatic invariance upon $U \nearrow$:

(1) $\langle n_f \rangle = 2\int_{-\infty}^{0} A_f(\omega)\, d\omega$

(2) $\frac{1}{\pi} \sum_\sigma \delta_{f\sigma} = \langle n_f \rangle$

(3) $A_f(\omega = 0) = \frac{\sin^2 \delta_f}{\pi\Delta}$

For (3), note $G_f(\omega - i\delta) = \frac{1}{\omega - \varepsilon_f - i\Delta(\omega) - \Sigma(\omega)}$

$\operatorname{Im}\Sigma(\omega)|_{\omega=0} = 0$

$\because$ no inelastic scattering on Fermi surface

$\longrightarrow \operatorname{Im} G_f(\omega=0)$ unrenormalized

Then, use $G_f(0 - i\eta)^{-1} = |G_f^{-1}(0)|\, e^{-i\delta_f}$ to get (3)

- We then have hierachy of scale.
  For $-\varepsilon_f < \omega < \varepsilon_f + U$, we can use the infinite-U Anderson model. (~ project out double occupied states)
  For $\omega < -\varepsilon_f$, can also project out zero occupied state (Kondo model)

- Can approach by renormalization group

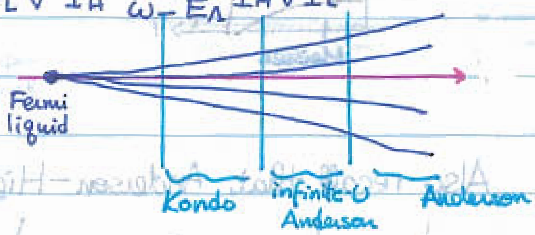
$$\mathcal{H}(\Lambda) \sim \begin{bmatrix} H_L & V^\dagger \\ V & H_H \end{bmatrix}$$

$$\mathcal{H}(\Lambda) \mapsto U\mathcal{H}(\Lambda)U^\dagger = \begin{bmatrix} \tilde{H}_L & 0 \\ 0 & \tilde{H}_H \end{bmatrix}$$

$$\mathcal{H}(\Lambda') = \tilde{\mathcal{H}}_L = \mathcal{H}_L + \delta\mathcal{H}$$

By pertubation, $\delta\mathcal{H} \approx P_L V^\dagger P_H \dfrac{1}{\omega - E_\Lambda} P_H V P_L$

- In our problem



- Using this approach, $\mathcal{H}_{\text{Anderson}} \mapsto \mathcal{H}_{\text{Kondo}} = \sum \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J\vec{\sigma}(0)\cdot\vec{S}_f$
  where $J \approx V^2 \Big[\underbrace{\dfrac{1}{\varepsilon_f + U}}_{f^1 + e^- \leftrightarrow f^2} + \underbrace{\dfrac{1}{-\varepsilon_f}}_{f^1 \leftrightarrow f^0 + e^-}\Big] > 0$ [antiferro!]

- In the traditional RG sense, J is marginal operator. But if Fermi surface has unusual co-dimensional, J may become irrelevant / relevant (e.g. in graphene, J is irrelevant, local moment always form)