Landau Fermi Liquid and Heavy Fermion [Coleman]

- Overview

▲ Increasing localization: $\underset{\text{localization}}{\xrightarrow{5d \;\; 4d \;\; 3d \;\; 5f \;\; 4f}}$

Ce ⋯ (A, Z ↗) ⋯ Yb

▲ Interesting physics occur at the crossover between iterant and localized system.

▲ f-spin are always localized

$\begin{cases} \text{high-}T : \text{local moment metal} \\ \text{low-}T : \text{spins "quench" to form heavy fermions.} \end{cases}$

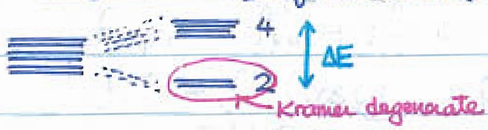
▲ Spin ≡ localized moment, when e^- lost all its charge degree of freedom

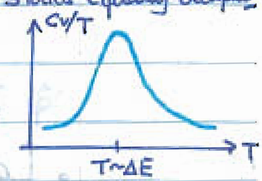
e.g. Ce^{3+} $(4f^1)$: $L = 3,\ S = 1/2,\ J = L - S = 5/2$

At high-T, $\chi = \frac{n_i M^2}{3T}$, $M^2 = g_J^2 \mu_B^2 J(J+1)$.

$S_Q = k_B \ln(2J+1)$ [unquenched ≡ all states equally occupied]

▲ Under crystal field,





▲ At low-T, these materials form Fermi liquid :

$E_p = \frac{p^2}{2m^*}$; $N^*(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$; $\chi = \frac{\mu_B^2 N^*(0)}{1 + F_0^a}$, $\gamma = \lim_{T\to 0}\left(\frac{C_V}{T}\right) = \frac{\pi^2 k_B^2}{3} N^*(0)$

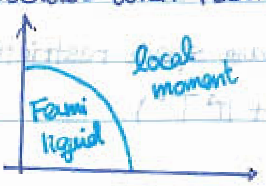
$\Longrightarrow W = \frac{\chi}{\gamma} = 3\left(\frac{\mu_B^2}{2\pi k_B}\right)\frac{1}{1 + F_0^a}$

▲ Although 4f/5f metals vary 3 order of magnitudes in both χ and γ, the ratio W seems to fit on a line

$\Longrightarrow$ most effects are account for by $N^*(0)$

NOTE : actual material below the ideal line because of correction by g_J and F_0^a

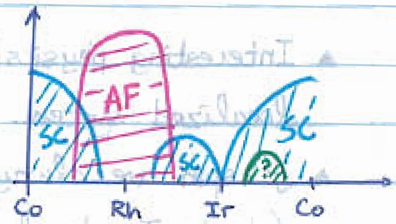
▲ Local moment interact with iterant e^- to form resonant states



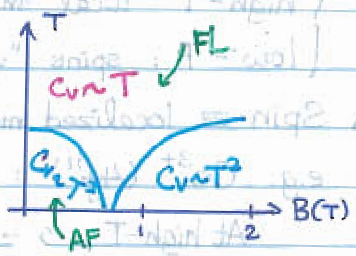
$T_K \sim \varepsilon_F e^{-\varepsilon_F / J}$

- Possible Phases of heavy fermion system
  - ▲ metal, superconductor, Kondo insulator, quantum critical pts, "others"
  - ▲ UBe_{13} : local moment ⟶ metal ⟶ superconductor

    But traditionally (~BCS) local moment is thought to destroy s.c.
  - ▲ $Ce_3Pt_4Bi_3$: Kondo insulator
  - ▲ Ce(Co, Rh, Ir) In [Pagliuso et al]



  - ▲ $YbRh_2Si_2$: QCP



  - ▲ UPd_2Al_3 : SC + AF coexistence

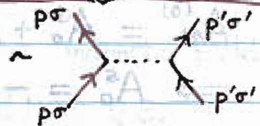
- Landau Fermi Liquid
  - ▲ Turn on interaction adiabatically : Pauli exclusion ⟶ Fermi surface robust ; excitation spectrum preserved.

    Hence, $\underset{m}{|e^-\rangle} \xrightarrow{\text{same quantum number}} \underset{m^*}{|qp^-\rangle}$ $\quad \dfrac{m^*}{m} = \dfrac{N(0)^*}{N(0)} = 1 + \dfrac{F_1^s}{3}$
  - ▲ NOTES: From 1950 → 1960, it is realized that one can drop the long-ranged part of Coulomb and capture the essential physics by remnant short-ranged interaction (e.g. Hubbard model, Anderson model, etc.) This is faciliated by discovery / experiments on ^3He
  - ▲ By Pauli exclusion, phase space restriction

    $\Longrightarrow \tau^{-1}(\varepsilon) \propto (\varepsilon^2 + \pi^2 T^2)$

▲ Thus we have Landau energy functional:

$$\mathcal{E}_1 = \mathcal{E}_0 + \sum_{p,\sigma} (E^{(0)}_{p,\sigma} - \mu)\,\delta n_{p\sigma} + \underbrace{\frac{1}{2}\sum f_{p\sigma p'\sigma'}\,\delta n_{p\sigma}\,\delta n_{p'\sigma'}}$$

$\sim$ 

▲ Landau energy functional $\sim$ "fixed point" Hamiltonian (Shankar, RMP, 94')

▲ Warning: $\epsilon_{p\sigma} = \epsilon^{(0)}_{p\sigma} + \sum_{p'\sigma'} f_{p\sigma p'\sigma'}\,\delta n_{p'\sigma'}[\epsilon_{p'\sigma'}] \neq \epsilon^{(0)}_{p\sigma}$

This is a feedback (aka self-consistent) condition & lead to renormalization of excitation energy.

▲ Entropy: $S = -k_B \sum_{p,\sigma} \left(n_{p\sigma}\ln n_{p\sigma} + (1-n_{p\sigma})\ln(1-n_{p\sigma})\right)$

$$\Longrightarrow \; n_{p\sigma} = \frac{1}{e^{\beta\epsilon_{p\sigma}}+1} = f(\underbrace{\epsilon_{p\sigma}}_{\text{renormalized energy.}})$$

As $T \to 0$, $\delta n_p \to 0$, $n_{p\sigma} = f(\epsilon^{(0)}_{p\sigma})$

$$V_F = \frac{\partial \epsilon_{p\sigma}}{\partial p} = \frac{p_F}{m^*}$$

▲ This give rise to linear heat capacity.

▲ By rotation invariance, $\begin{cases} f_{p\sigma p'\sigma'} = f^s_{pp'} + f^a_{pp'}\,\sigma\sigma' \\ f^{sa} = f^{sa}(\underbrace{\cos\theta}_{=\hat{p}\cdot\hat{p}'}) \end{cases}$

$$\Longrightarrow f^{s,a}(\cos\theta) = \frac{1}{N^*(0)}\sum_{\ell=0}^{\infty}(2\ell+1)\underbrace{F^{s,a}_\ell}_{\text{Landau parameters}} P_\ell(\cos\theta)$$

▲ Let $\delta\epsilon^{(0)}_{p\sigma} = b_\ell P_\ell(\cos\theta) \Longrightarrow$

$\delta\epsilon_{p\sigma} = a_\ell P_\ell(\cos\theta)$

▲ From this we find $\begin{cases} \chi_s = \overbrace{\dfrac{\mu_B N^*(0)}{1+F^a_0}}^{\text{large in heavy fermion}} = \mu_B^2 N^*(0)(1-A^a_0) \\ \chi_c = \dfrac{N^*(0)}{1+F^s_0} = N^*(0)(1-A^s_0) \; [= N(0) \text{ in heavy fermio} \end{cases}$

Where $A^{s,a}_0 = \dfrac{F^{s,a}_0}{1+F^{s,a}_0}$

(The A's can be interpret as T-matrix amplitude for s-wave scatteri

▲ Since χ_s large while χ_c unrenormalized, $1 - A^s_0 \approx 0$.

- Heavy Fermion $\approx$ Local Landau Fermi Liquid
  - $A_{p\sigma p'\sigma'} = \frac{1}{N^*(0)}(A_0^s + A_0\,\sigma\sigma')$

    $A_{\uparrow\uparrow}^{(0)} = A_0^s + A_0^a \approx 0$ and $\chi_c \approx 0$ locality of moments

    $\Longrightarrow A_0^s = -A_0^a \approx 1$
  - $\begin{cases} \rho(T) = \rho_0 + AT^2 \\ C_V(T) = \gamma T \end{cases}$; $\frac{A}{\gamma^2} = \alpha_{KW}$ is approx. const., & is universal