

THE KONDO LATTICE

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{J}{\mathcal{N}} \sum_j \vec{S}_j \cdot c_{\mathbf{k}\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{R}_j}$$

T. Kasuya (1951)

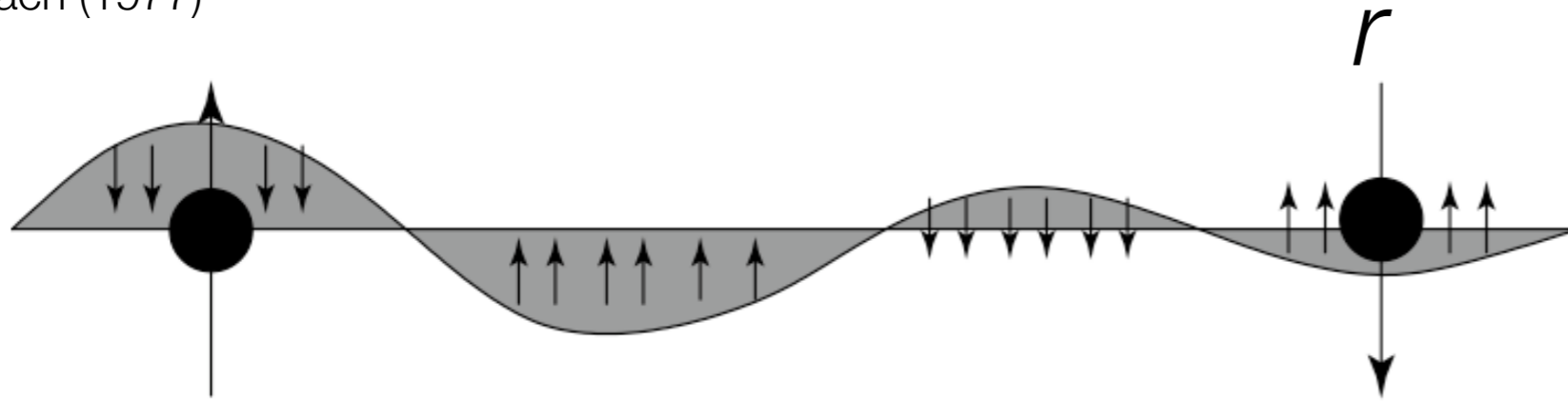
- Doniach's Kondo Lattice Hypothesis
- Large Fermi Surface
- Exhaustion Myth
- Large N expansions and Gauge Theories
- Large N Expansion for the Kondo Lattice
- Conclusions and questions.

- Cotunneling and copairing

<http://physics.rutgers.edu/~coleman/talks/Boulder3.pdf>

DONIACH'S Hypothesis.

Doniach (1977)

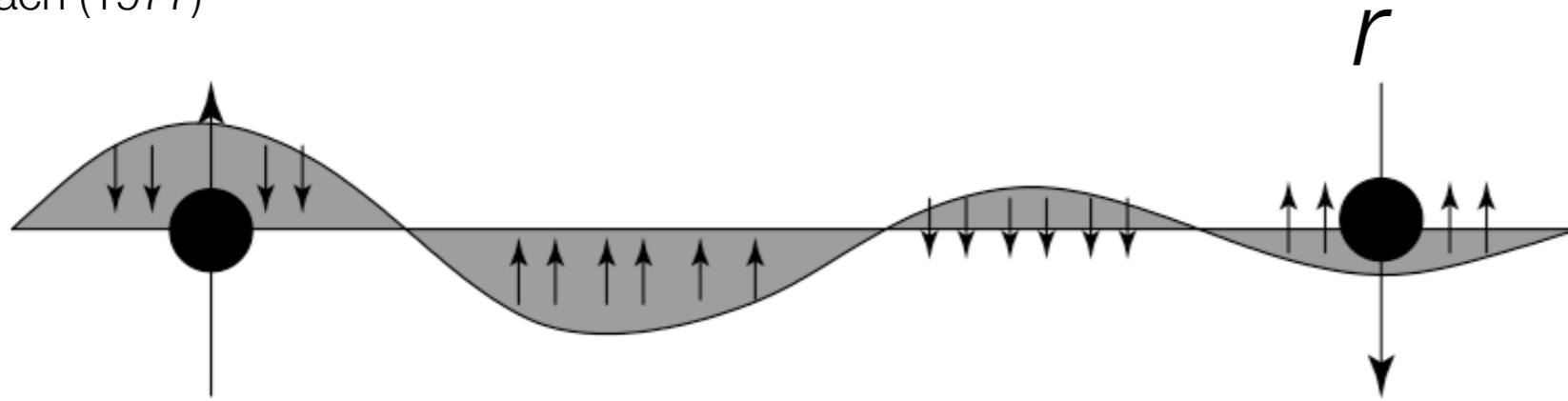


Friedel Oscillations

$$\langle \vec{\sigma}(r) \rangle \sim -J\rho \frac{\cos 2k_F r}{|k_F r|^3}$$

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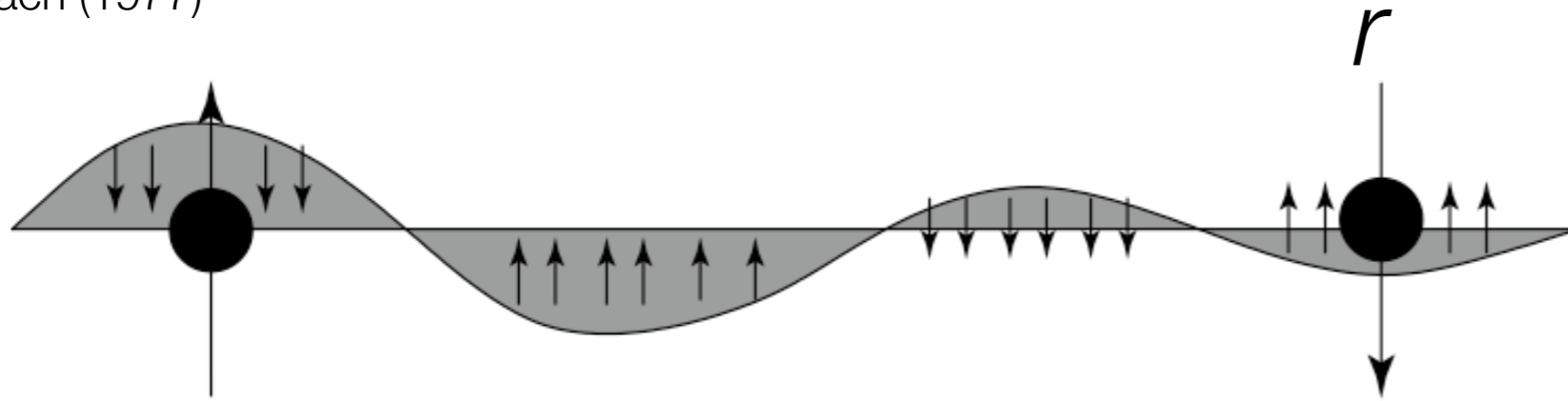
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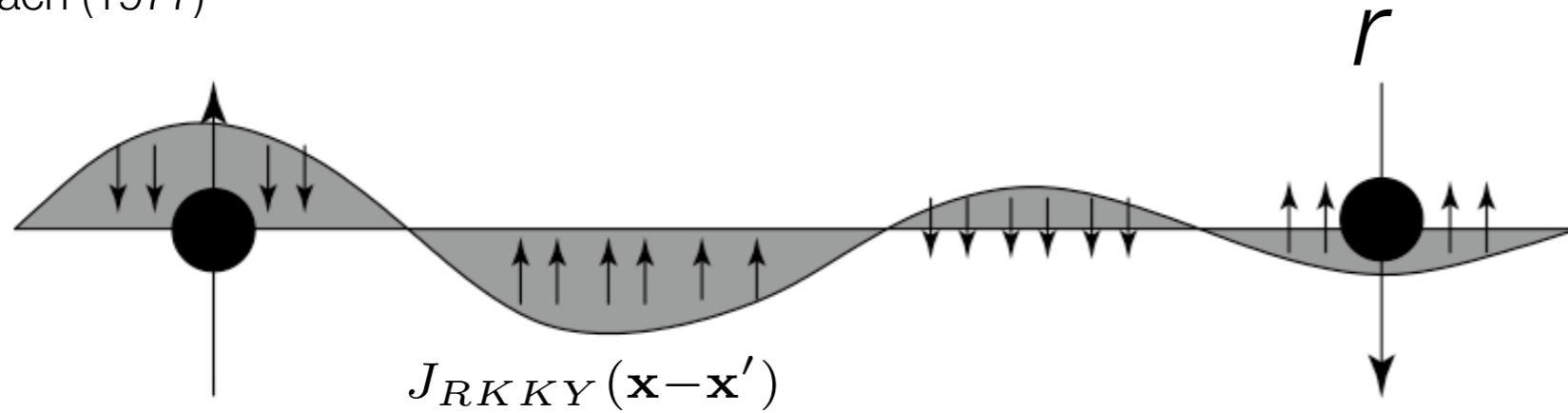
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$$\chi(x) = 2 \sum_{\mathbf{k}, \mathbf{k}'} \left(\frac{f(\epsilon_{\mathbf{k}}) - f(\epsilon_{\mathbf{k}'})}{\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}}} \right) e^{i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{x}} \sim \rho \frac{\cos 2k_F r}{|k_F r|^3}$$

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$$H_{RKKY} = \underbrace{-J^2 \chi(\mathbf{x} - \mathbf{x}')}_{J_{RKKY}(\mathbf{x} - \mathbf{x}')} \vec{S}(\mathbf{x}) \cdot \vec{S}(\mathbf{x}').$$

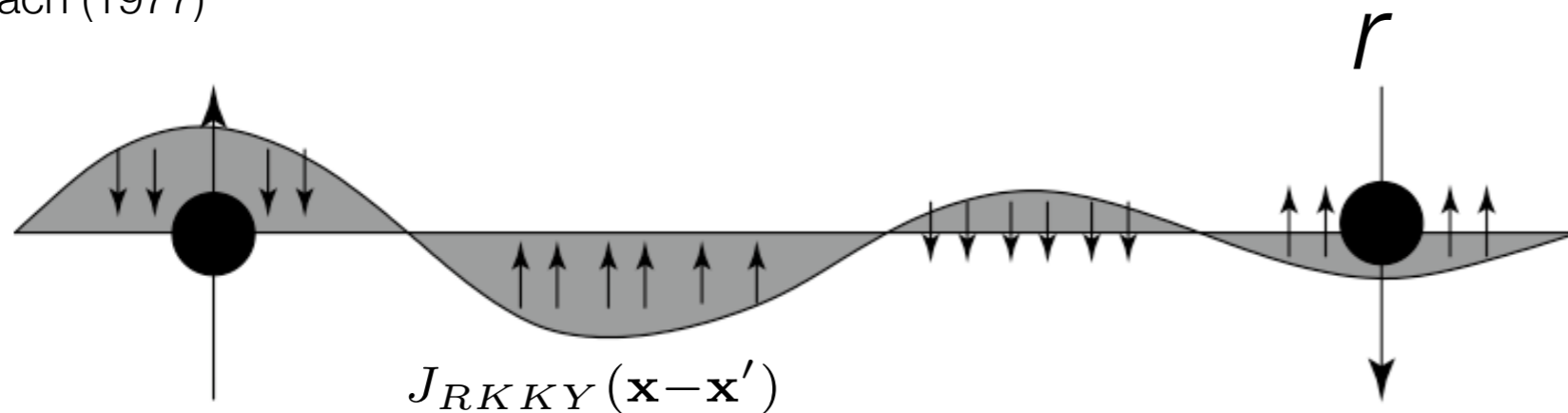
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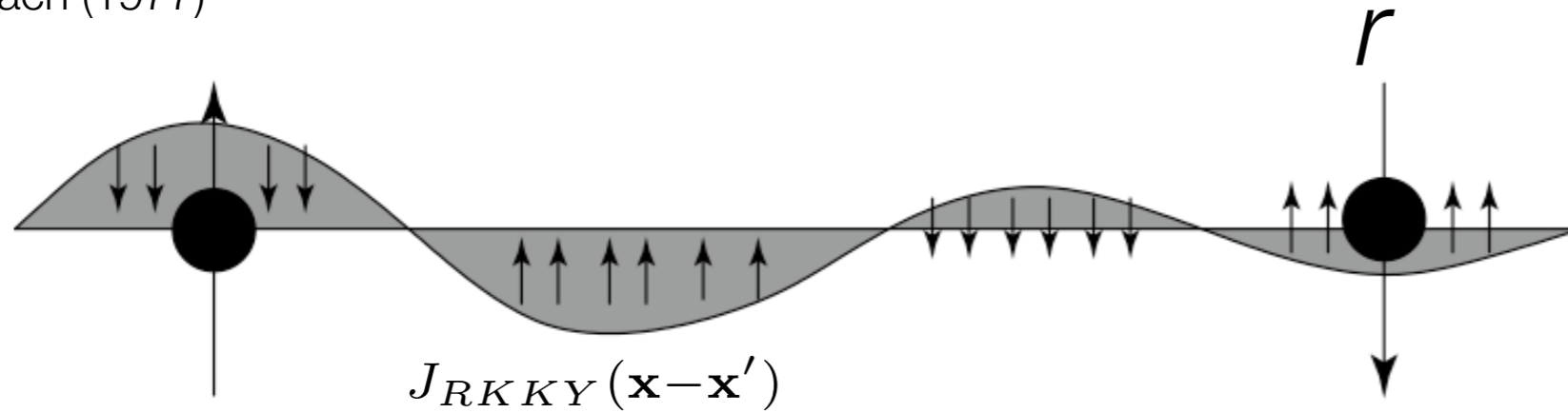
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$$\begin{aligned} T_K &= D e^{-1/(2J\rho)} \\ T_{RKKY} &= J^2 \rho \end{aligned}$$

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Friedel Oscillations



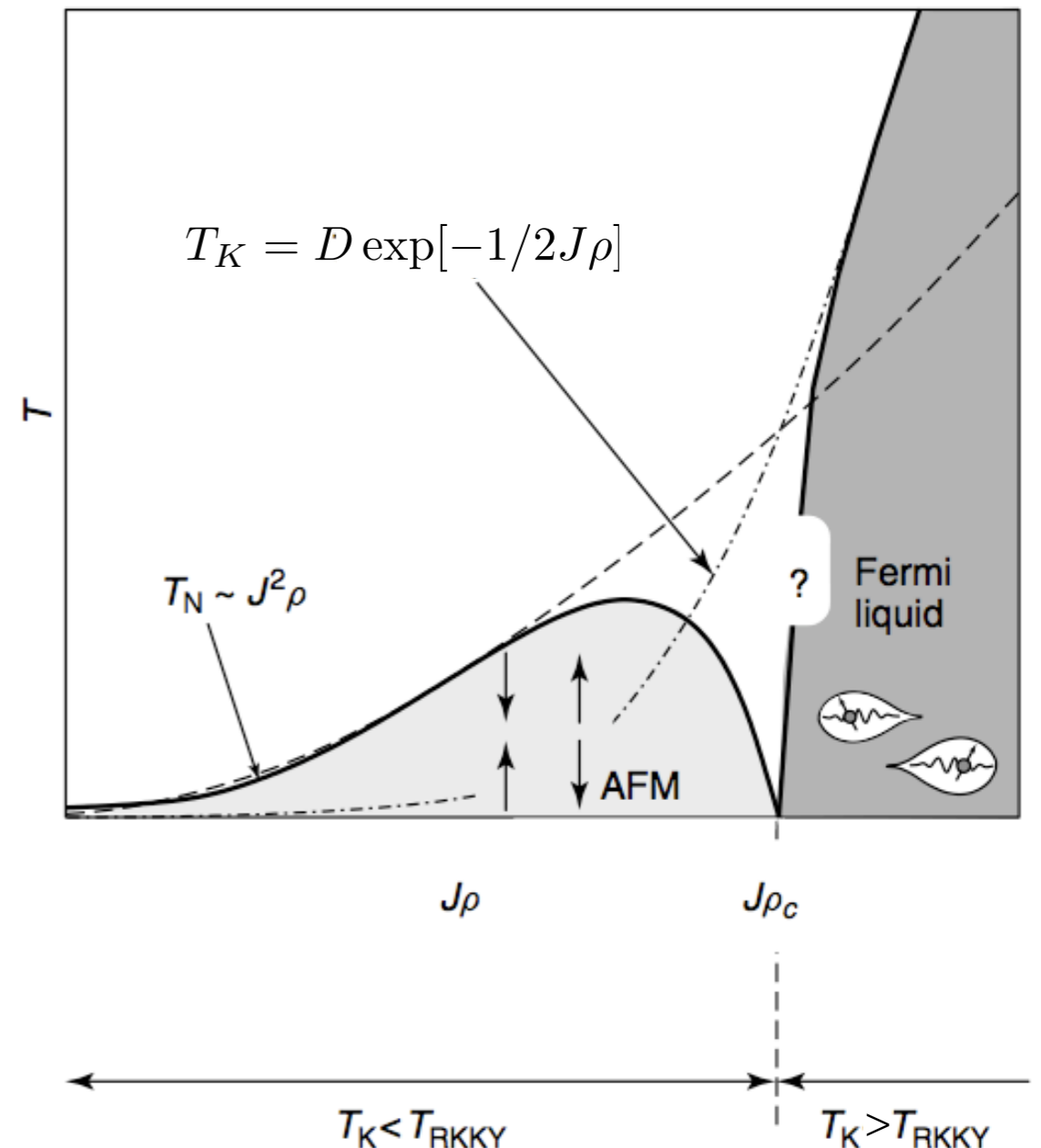
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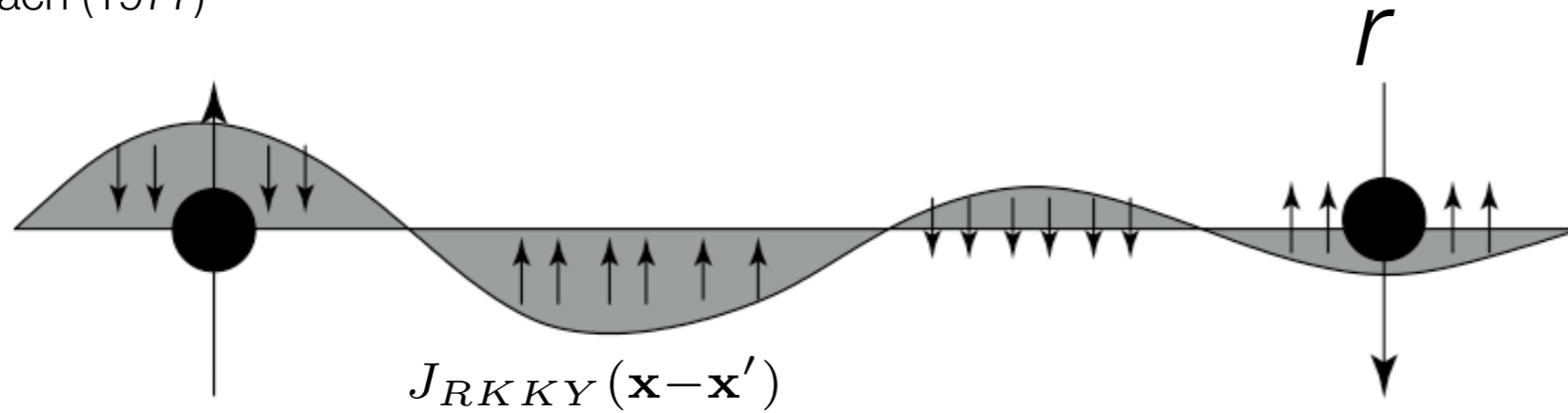
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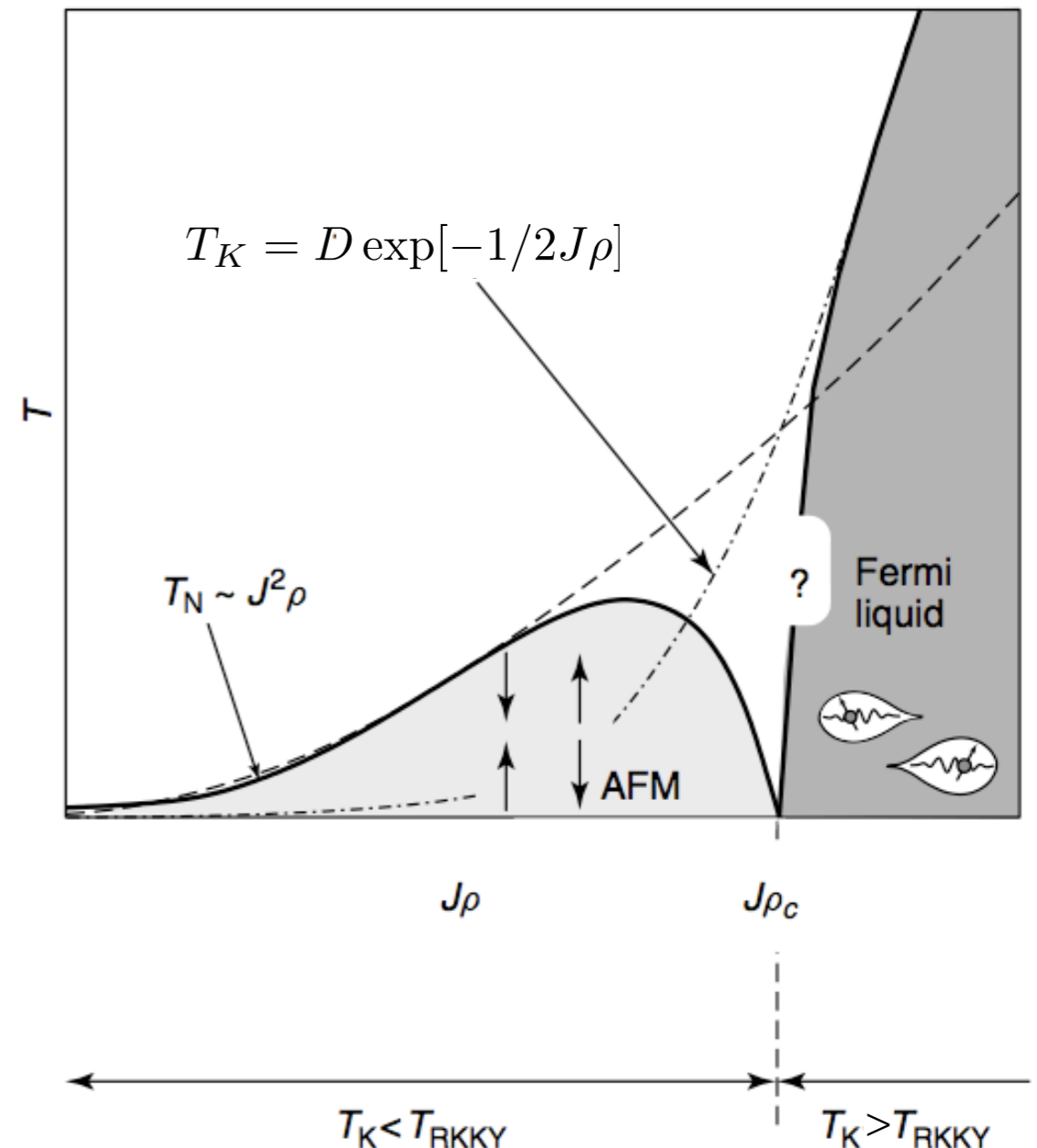
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Increasing U: $J \sim V^2/U$



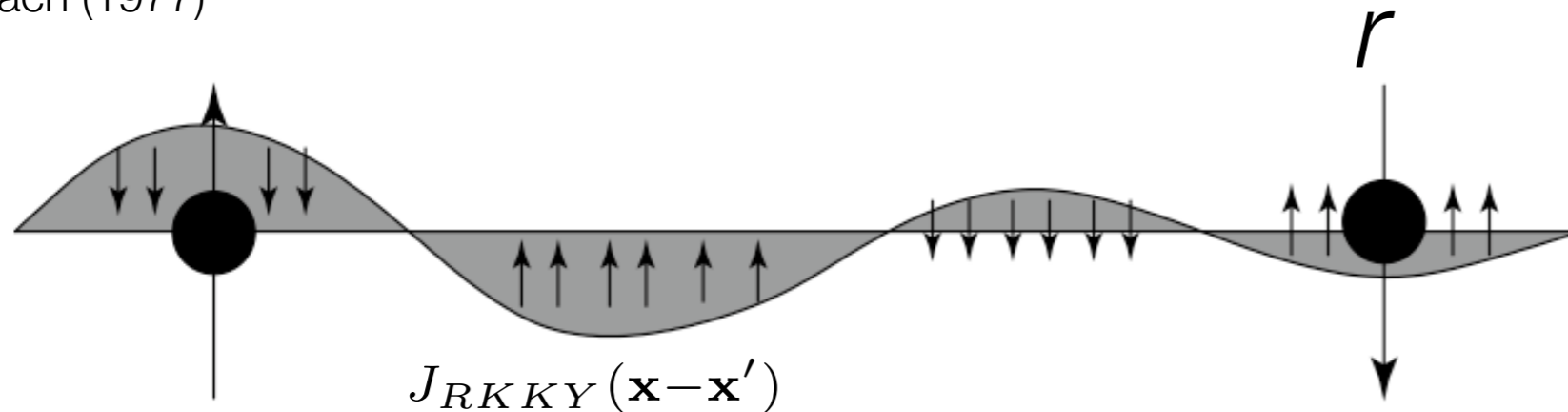
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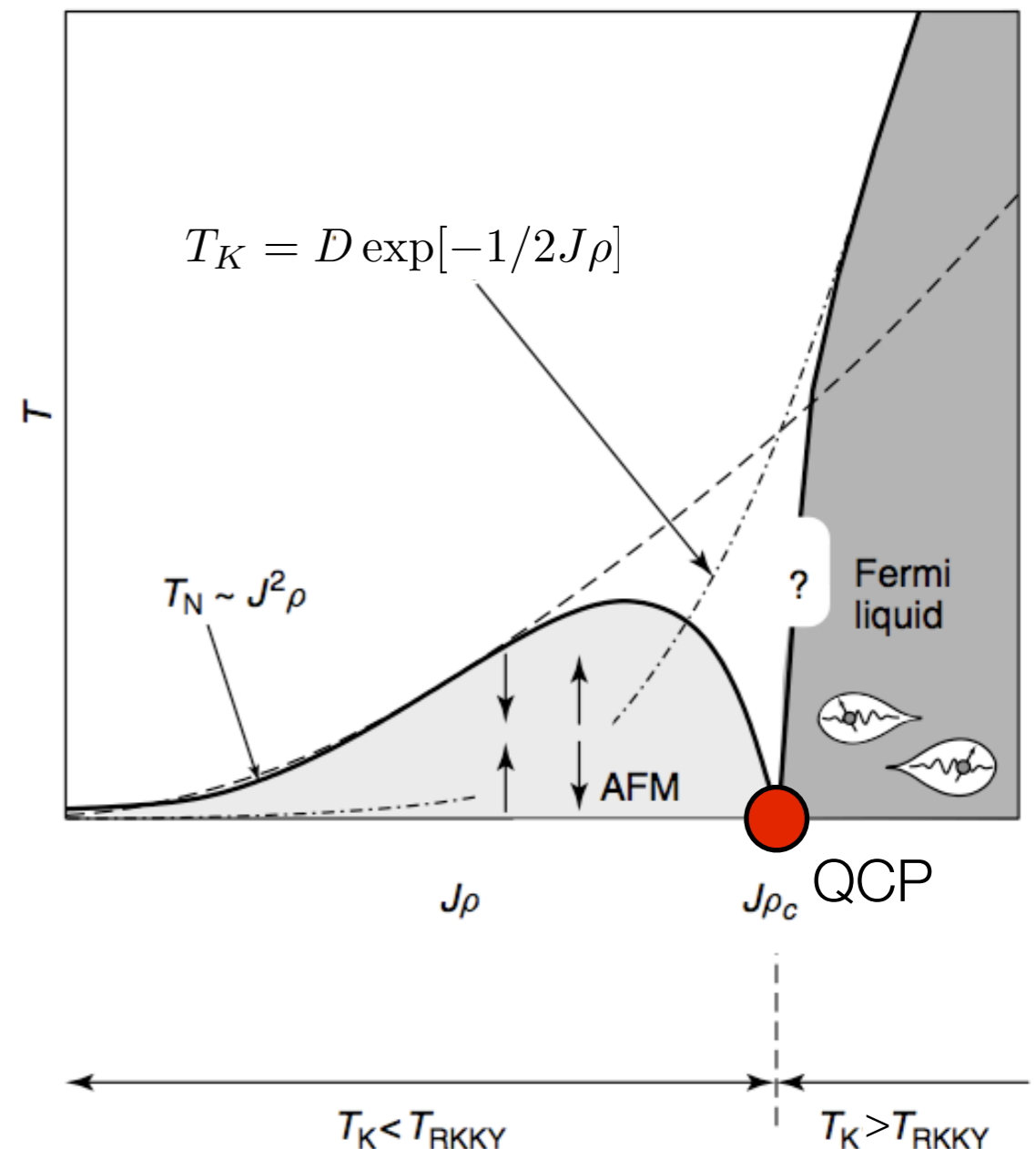
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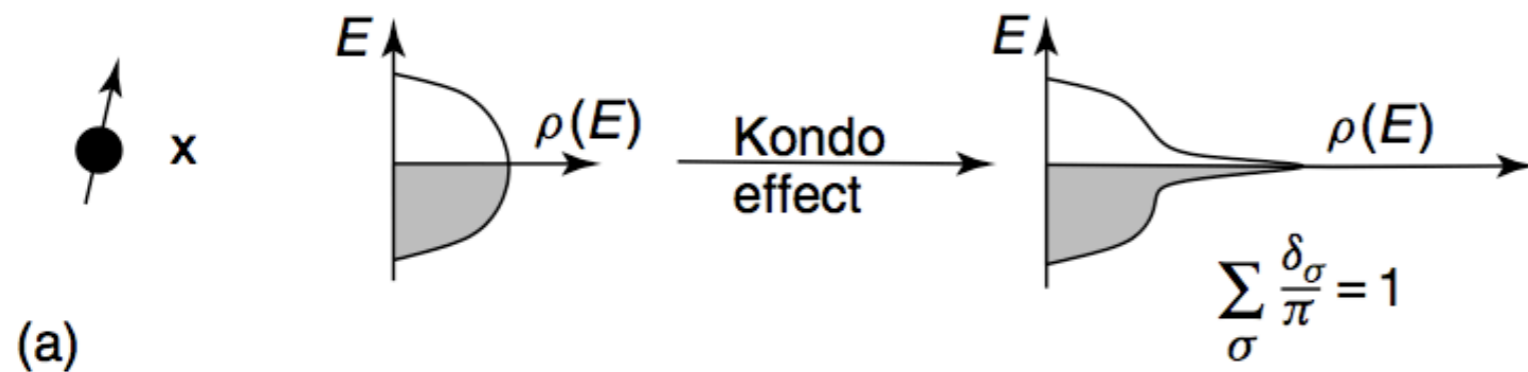
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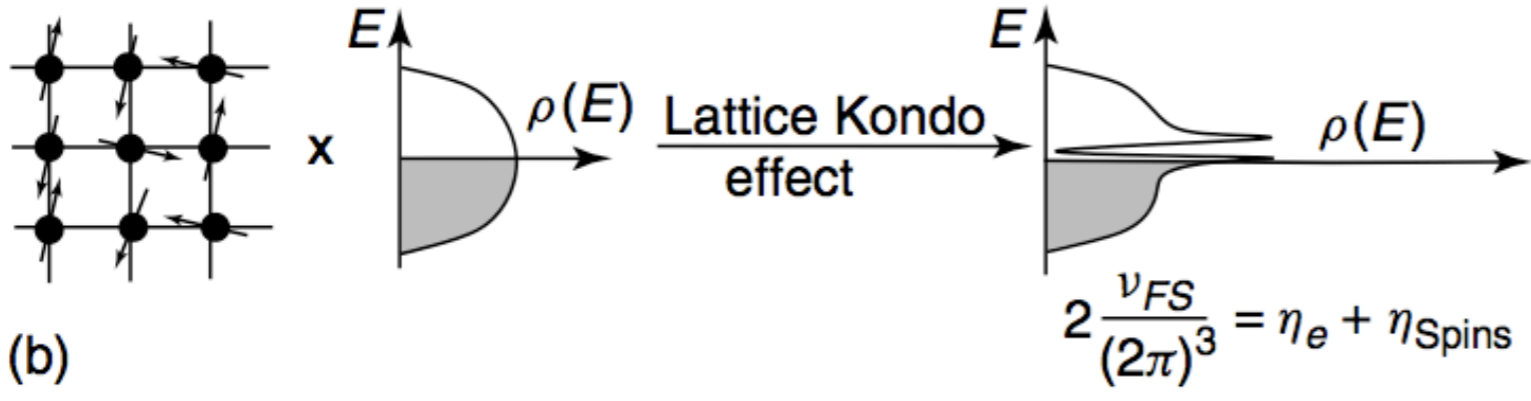
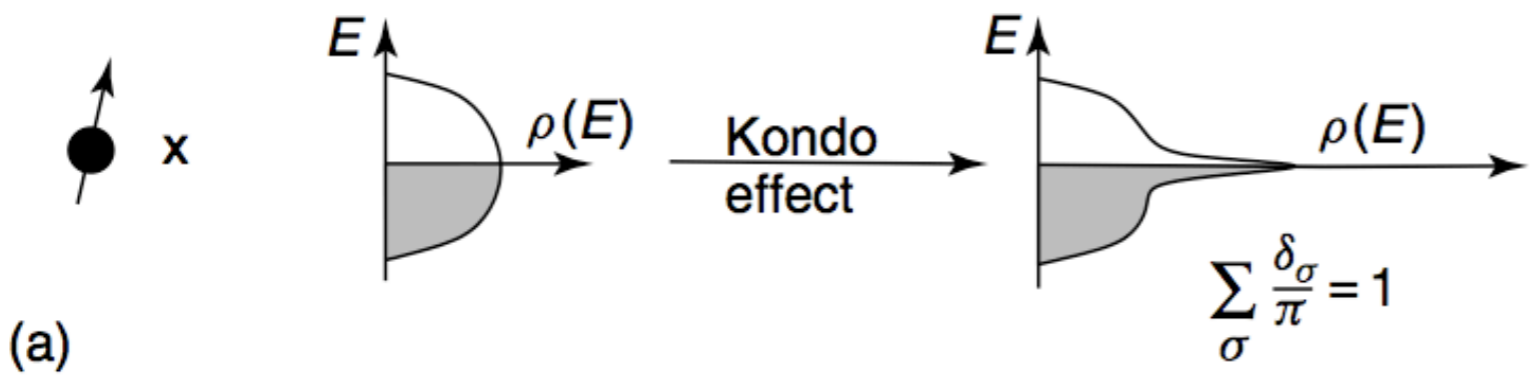


Large Fermi Surface of the Kondo Lattice. Martin (1982)



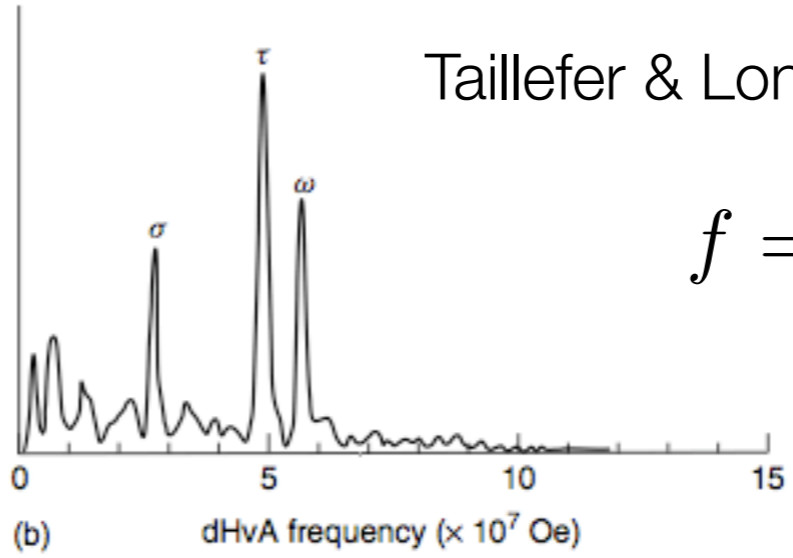
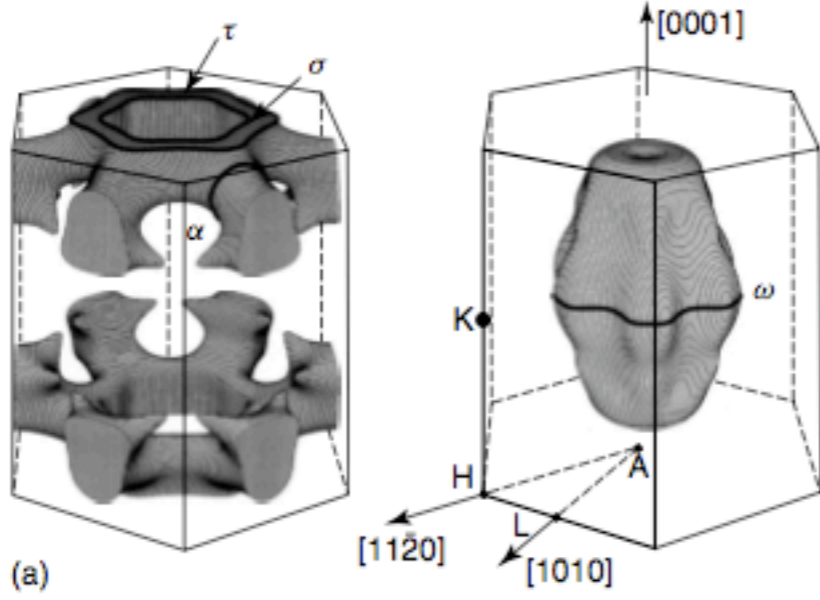
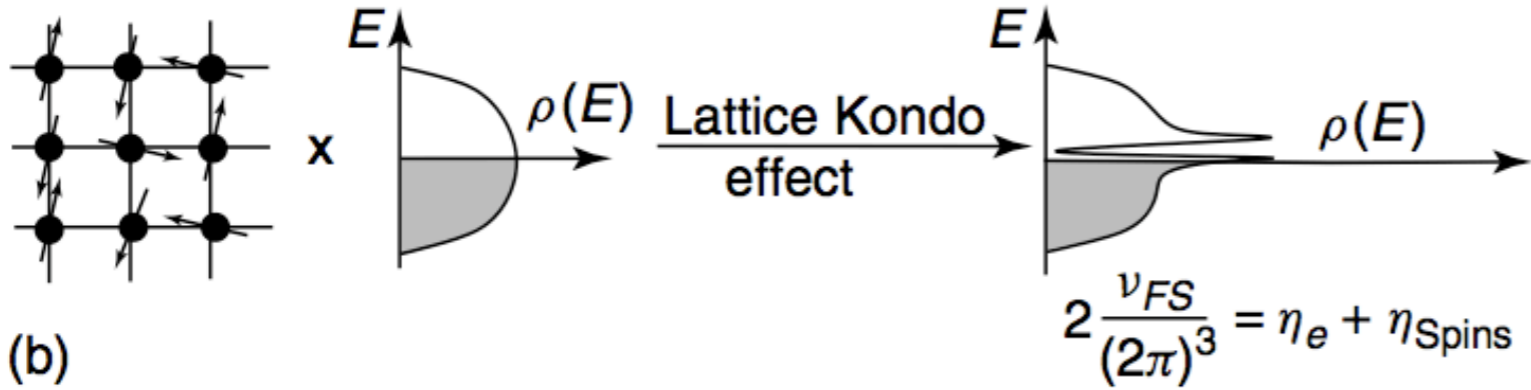
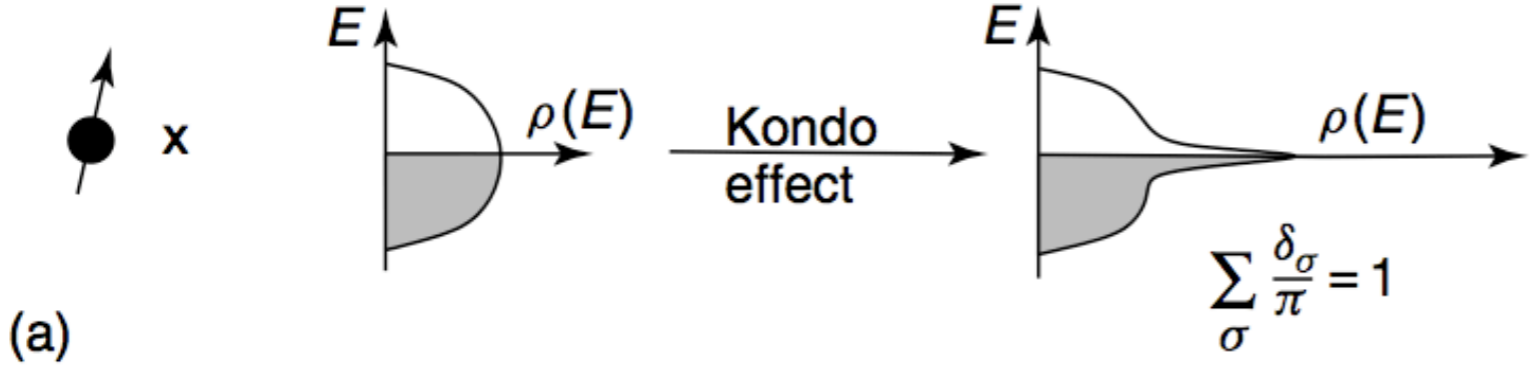
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Taillefer & Lonzarich (1985)

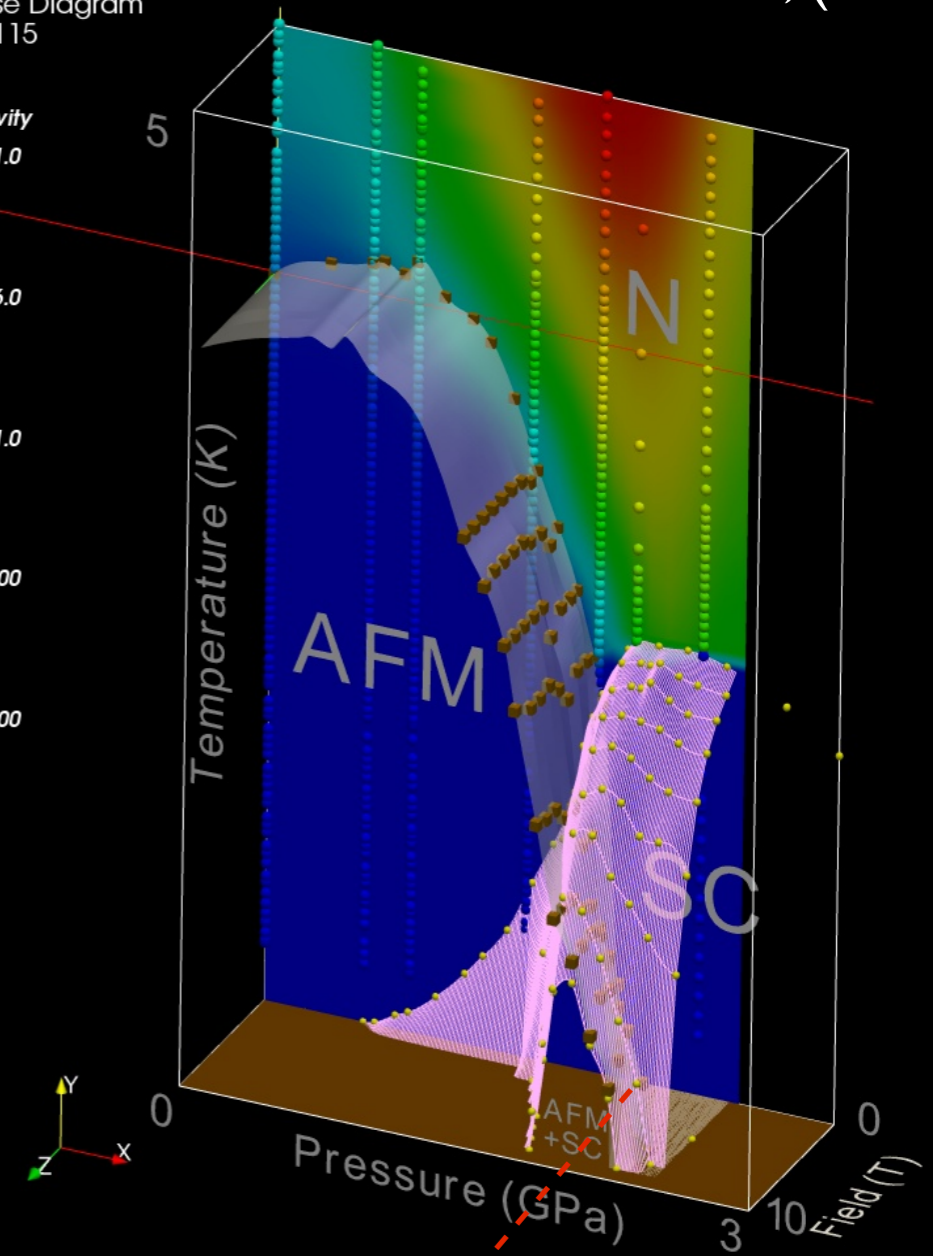
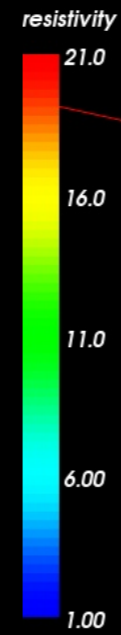
$$f = A \left(\frac{\hbar^2}{2\pi e} \right)$$

Figure 9. (a) Fermi surface of UPt₃ calculated from band theory assuming itinerant 5f electrons (Oguchi and Freeman, 1985; Wang *et al.*, 1987; Norman, Oguchi and Freeman, 1988), showing three orbits (σ , ω and τ) that are identified by dHvA measurements. (After Kimura *et al.*, 1998.) (b) Fourier transform of dHvA oscillations identifying σ , ω , and τ orbits shown in (a). (Kimura *et al.*, 1998.)

Reconstruction of the Fermi Surface and mass divergence

Tuson Park, (2007).

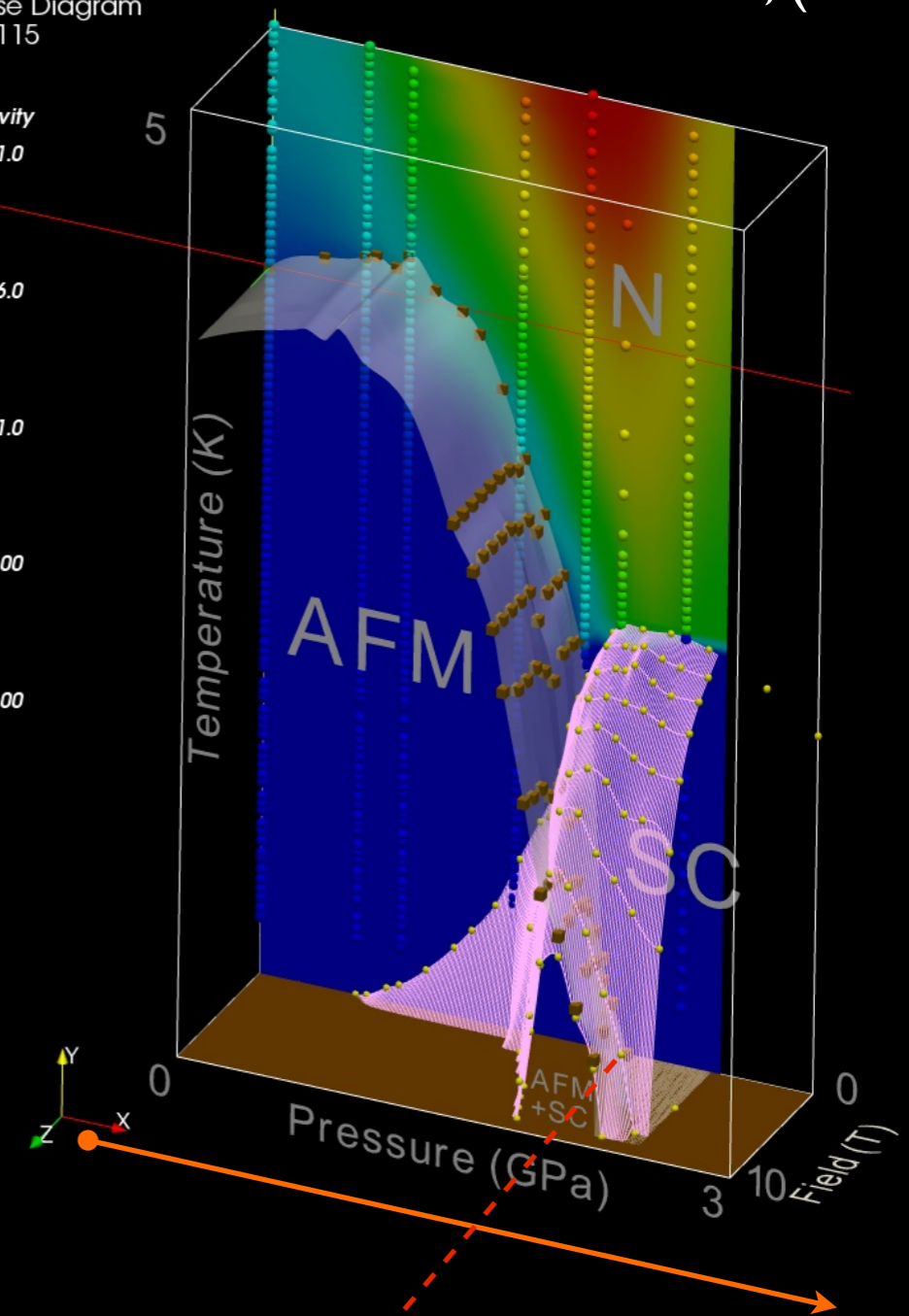
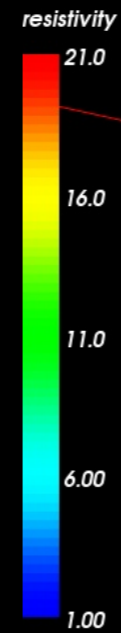
Phase Diagram
Ce-115



Reconstruction of the Fermi Surface and mass divergence

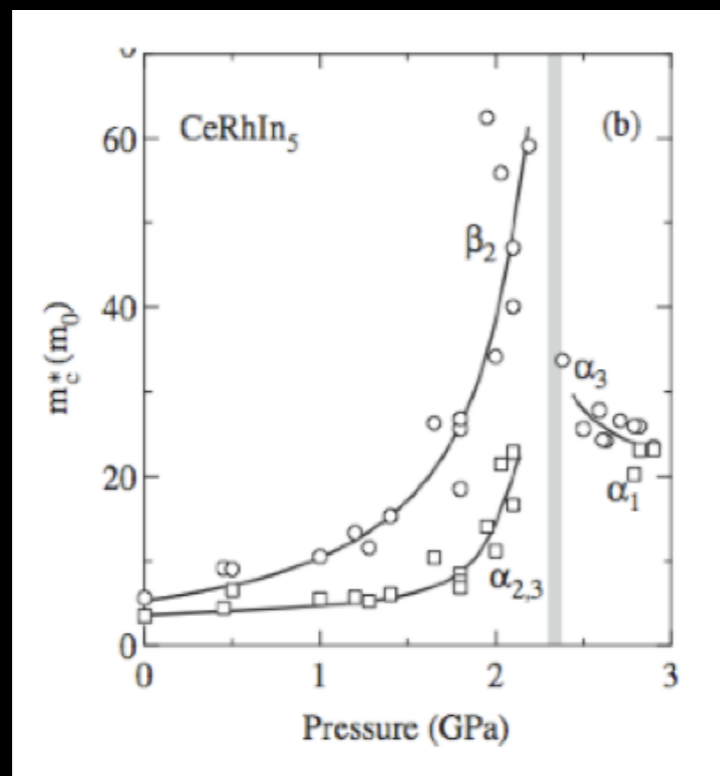
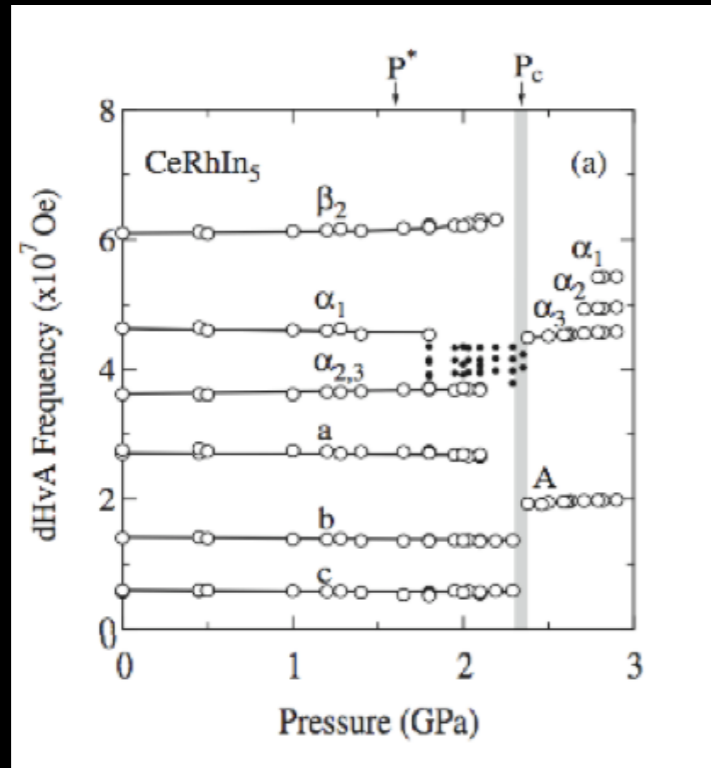
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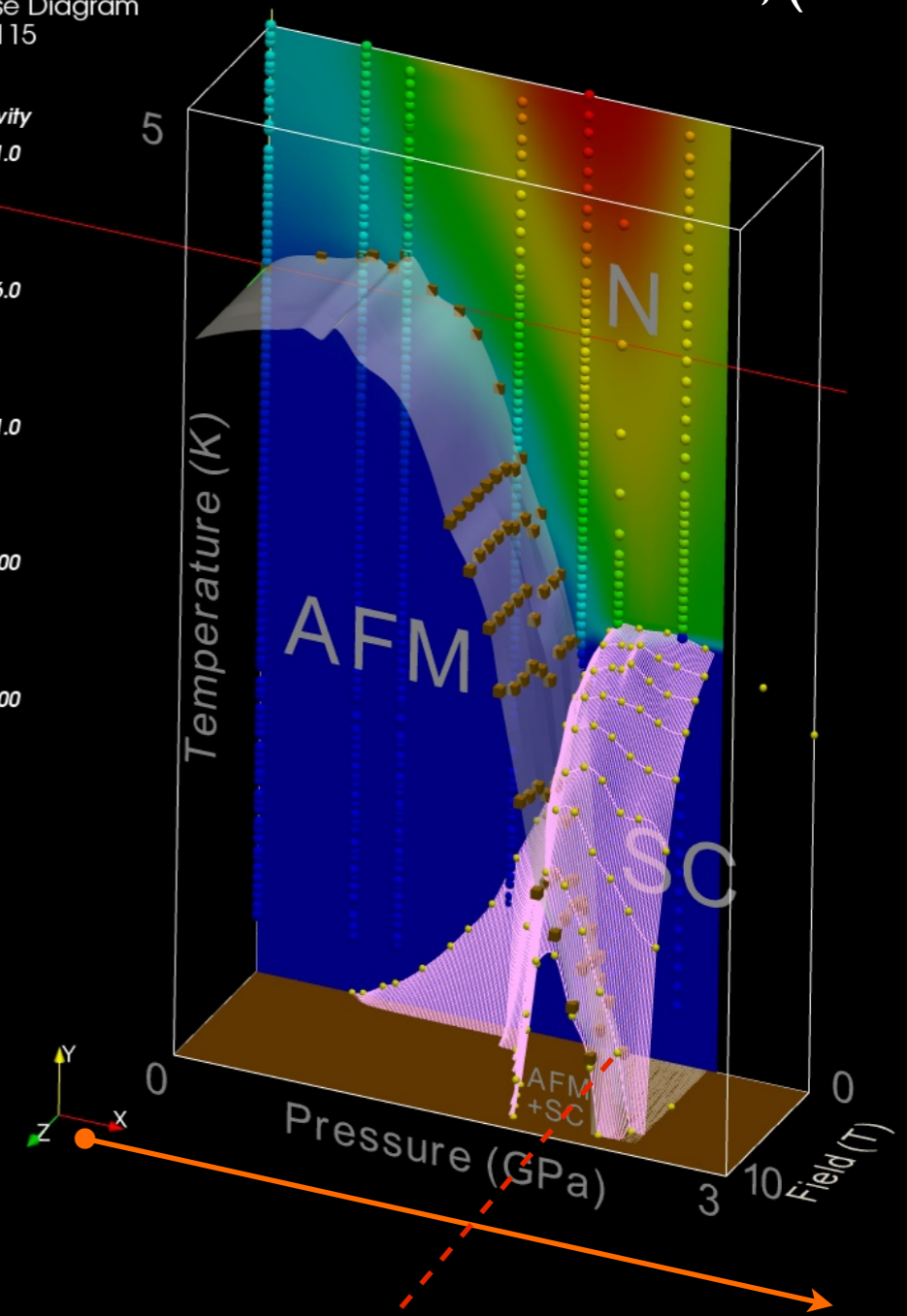
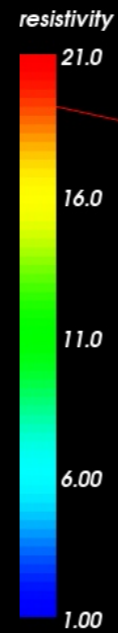
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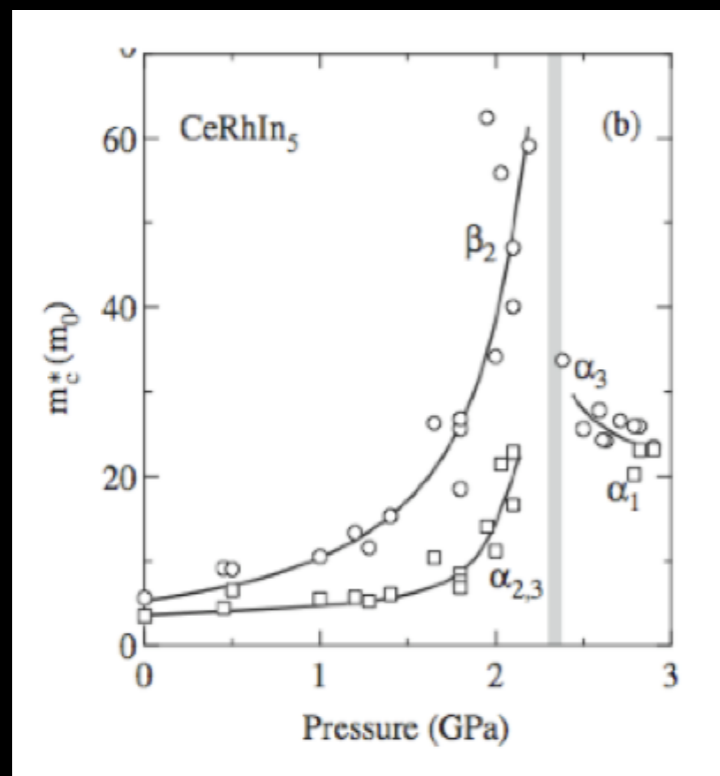
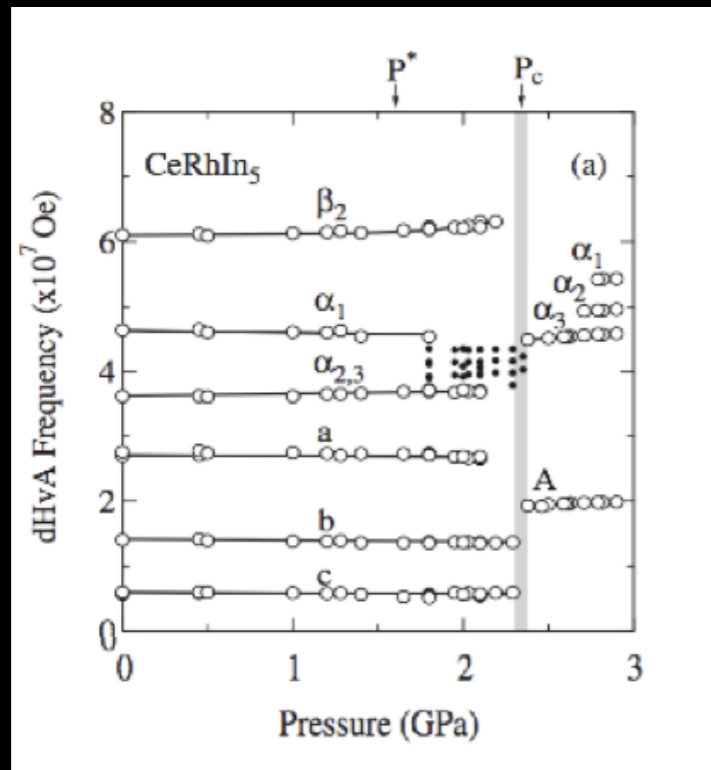
Shimuzu et al (2006)

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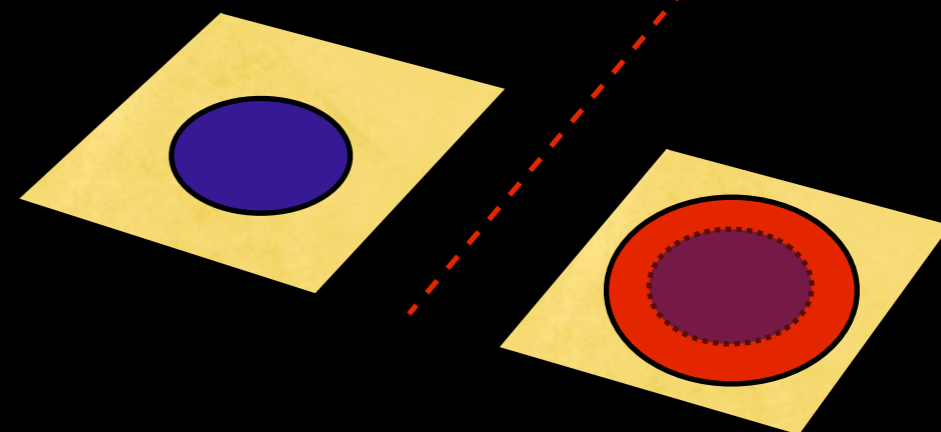
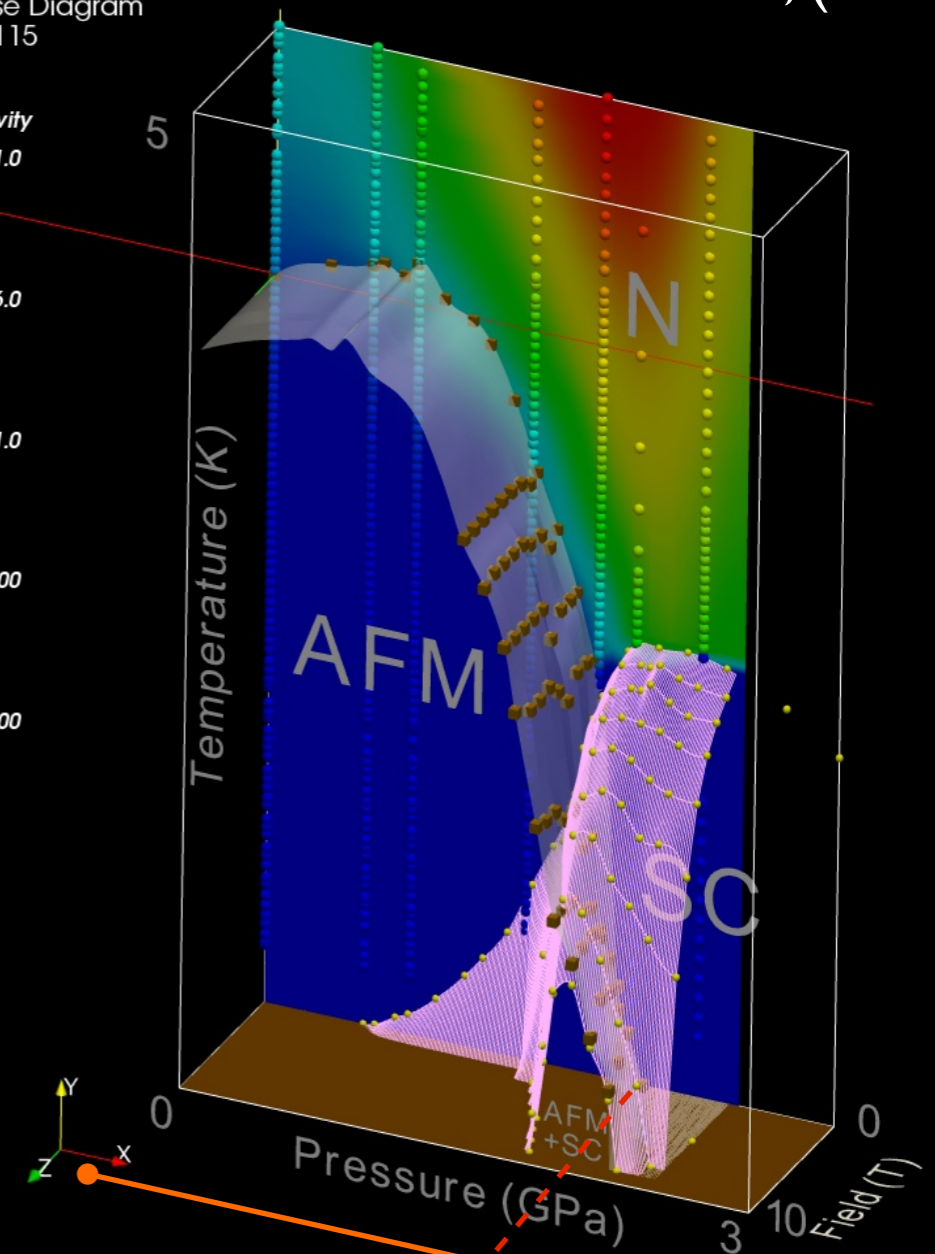
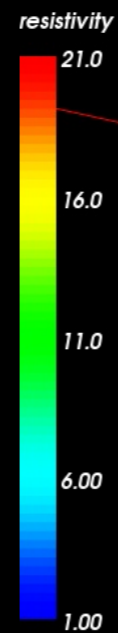


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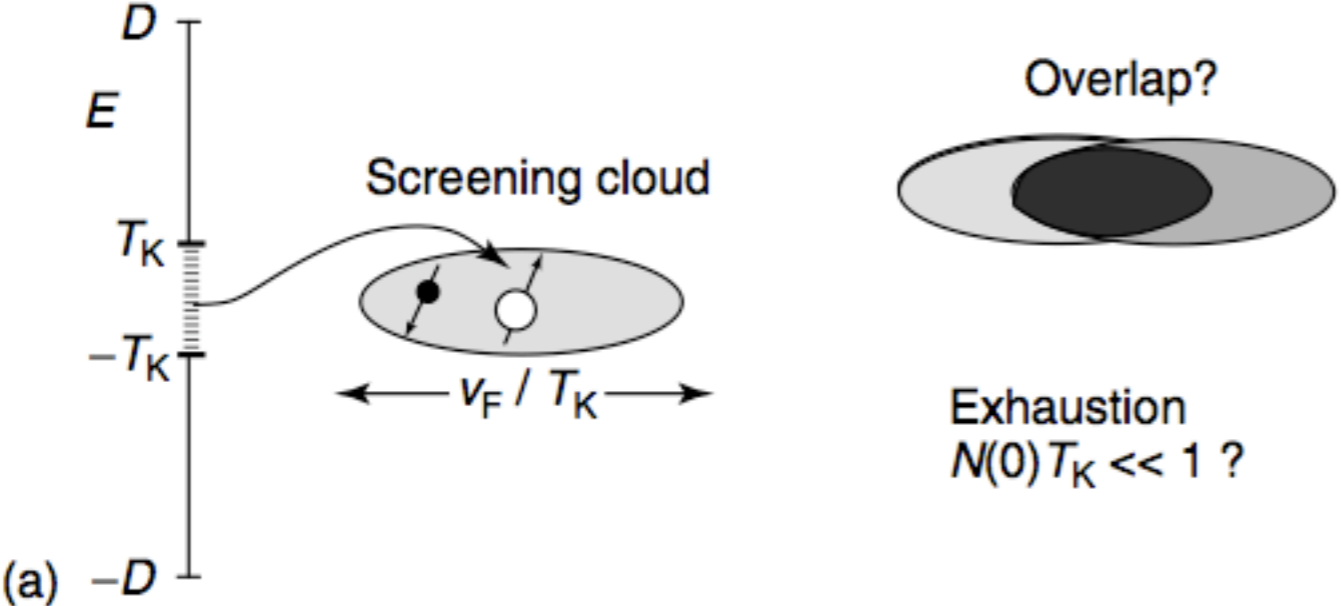


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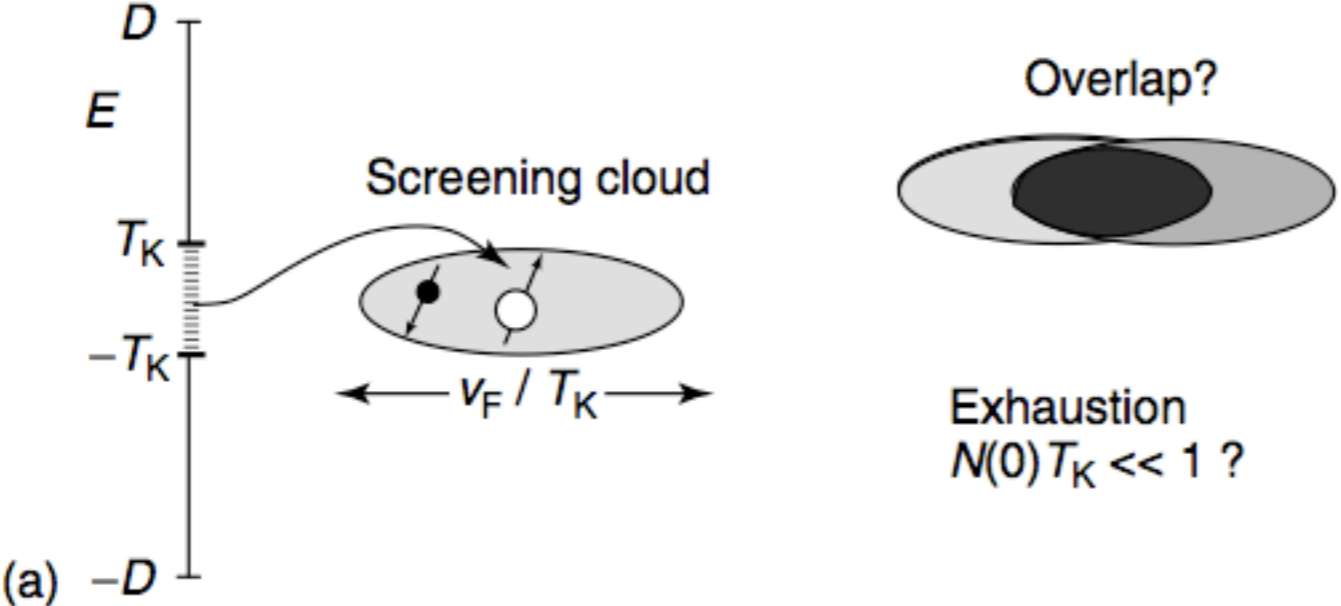
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“Exhaustion” vs. Composite fermion.



Nozieres(1985)

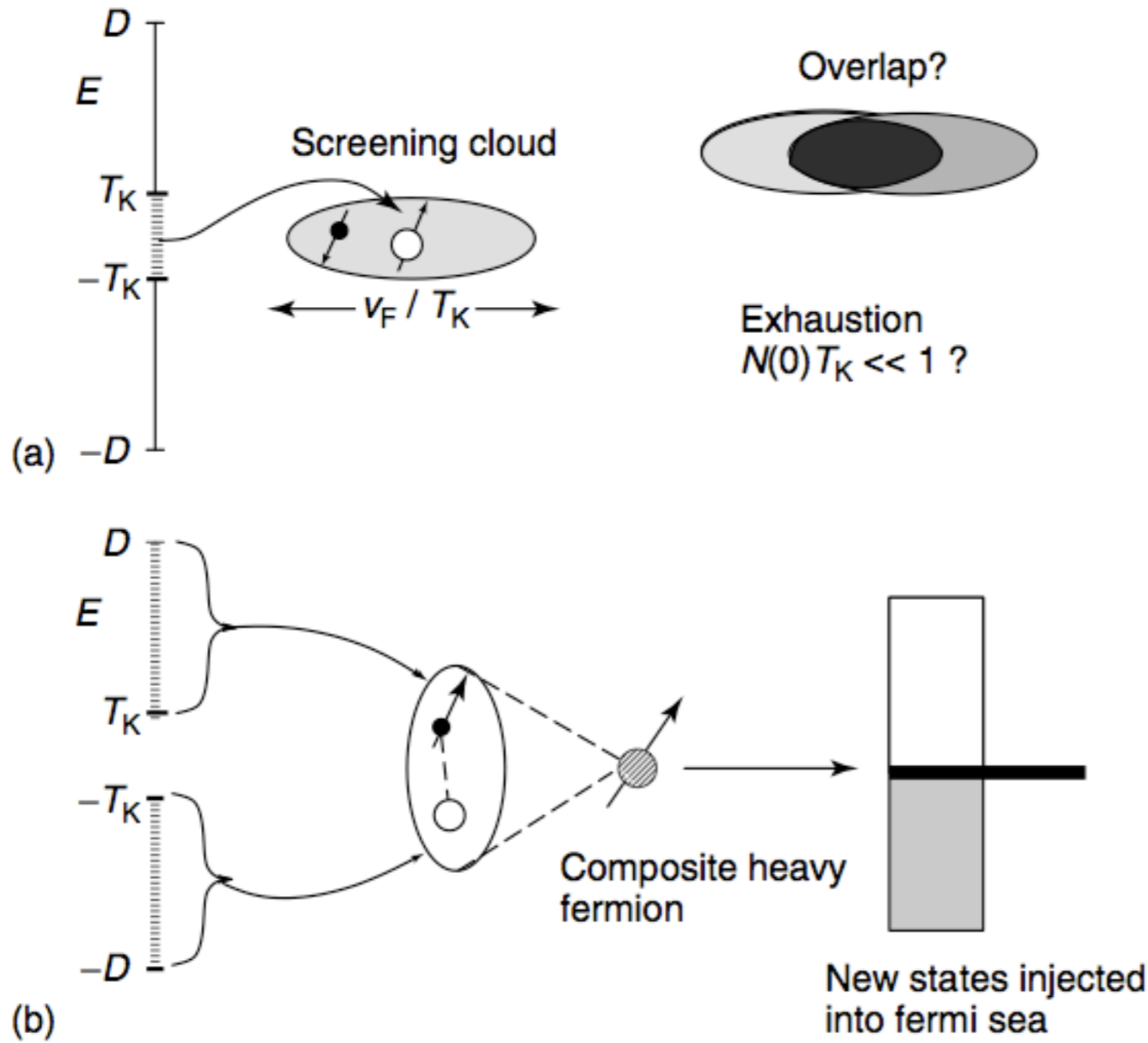
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Strongly correlated electron physics: no small parameter

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Large N : family of models with “ N ” spin components, which retain the key physics and can be solved in the large N limit.

Strongly correlated electron physics: no small parameter

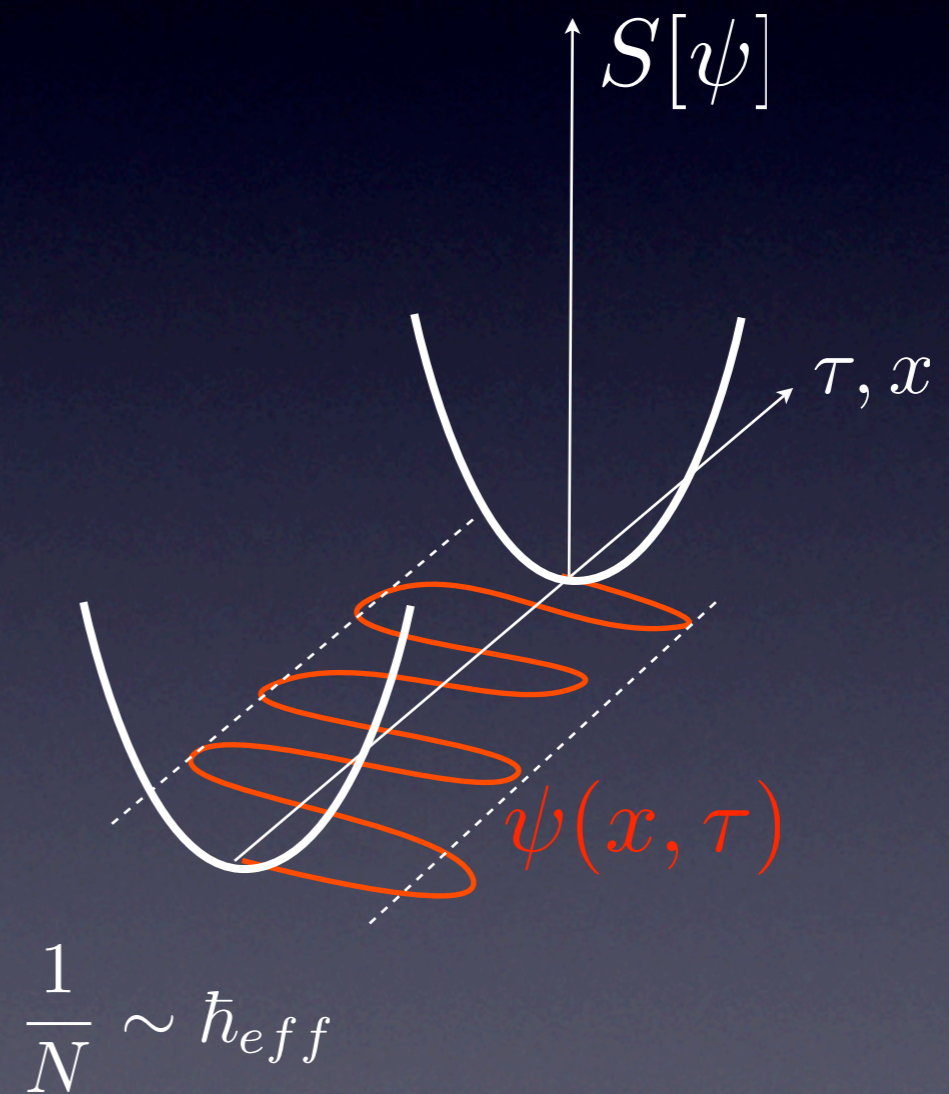
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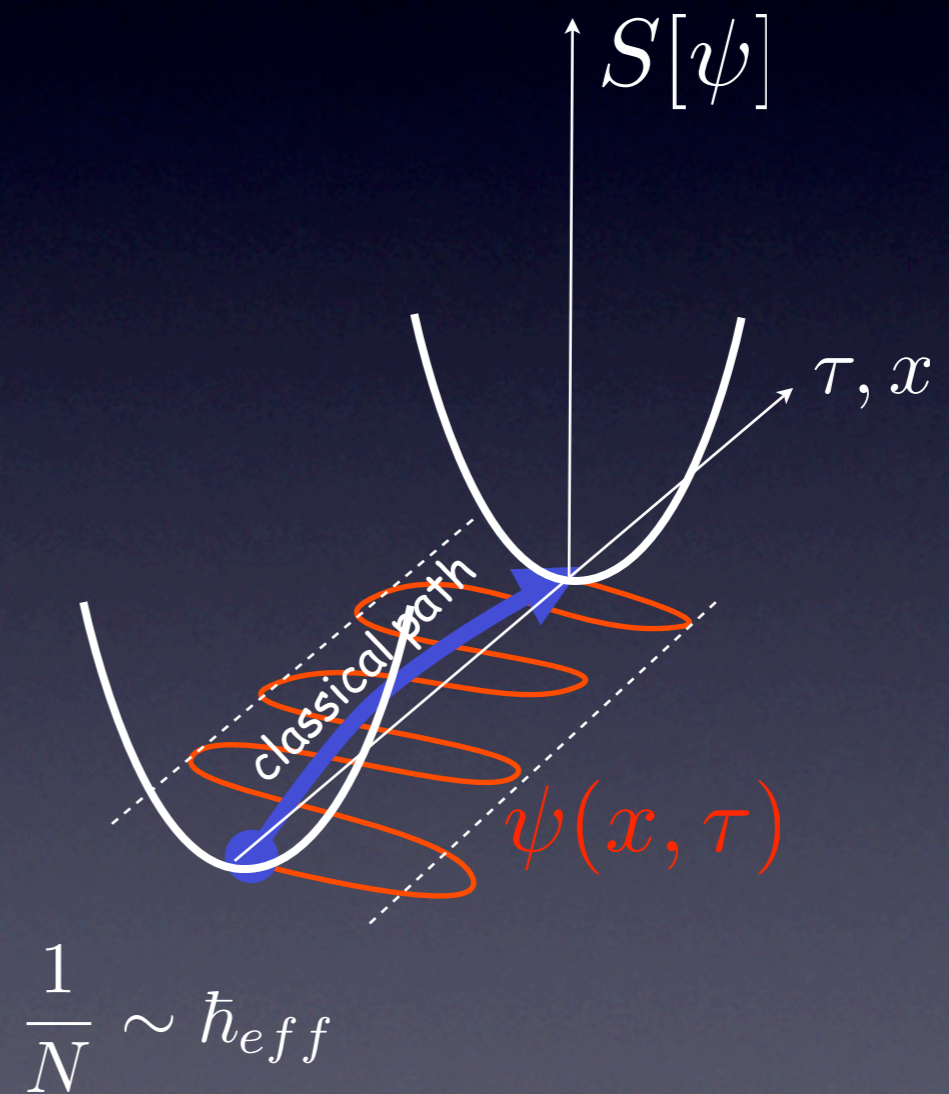
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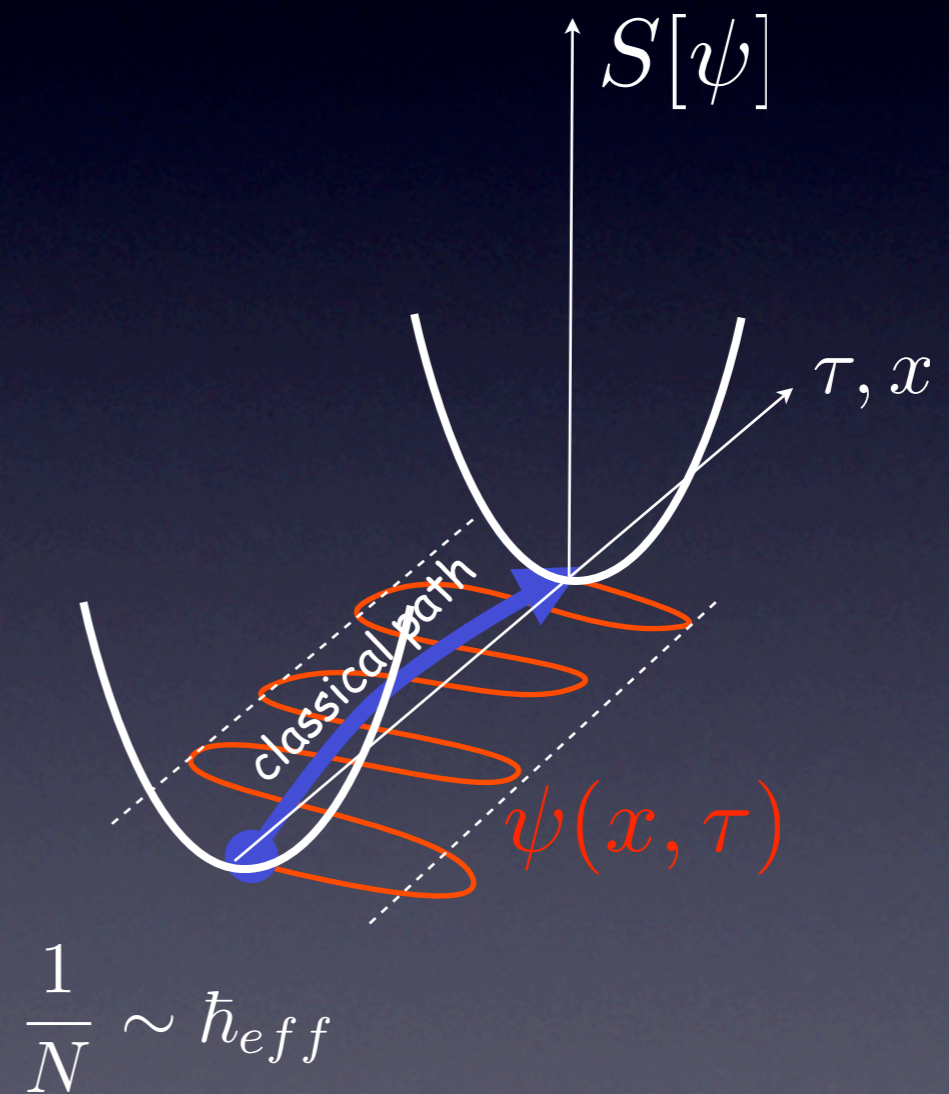
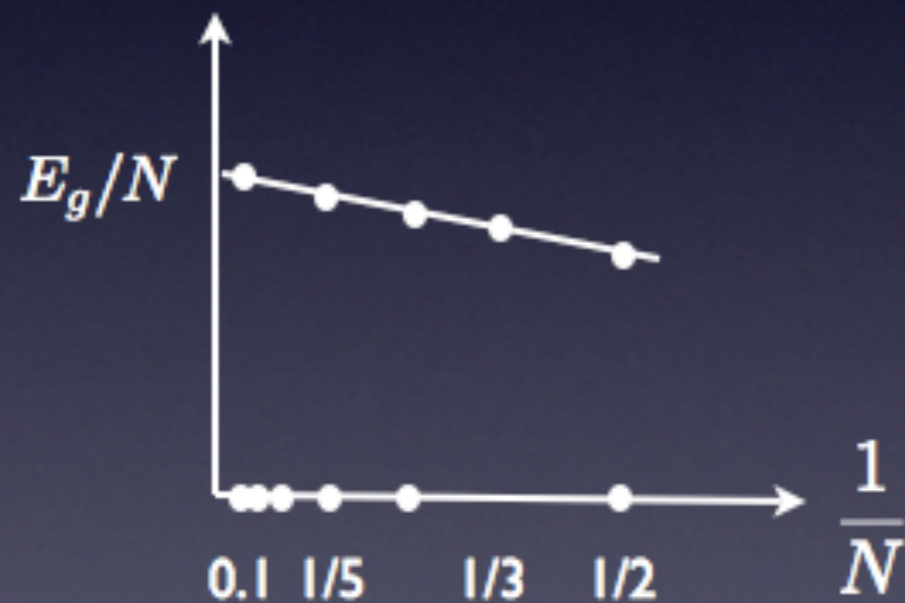
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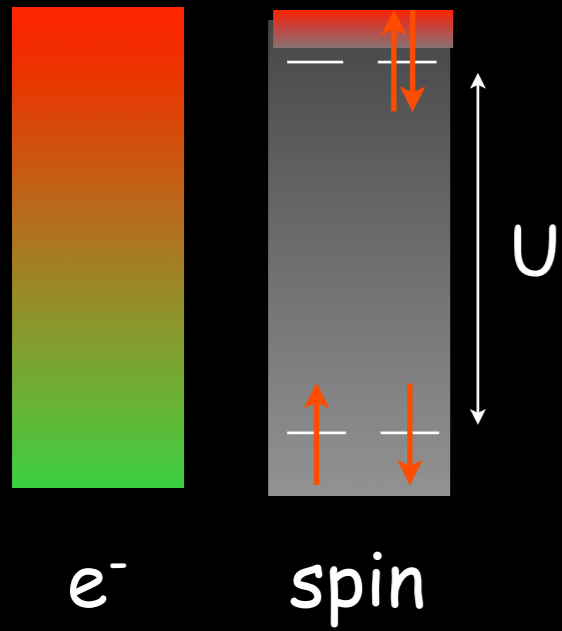
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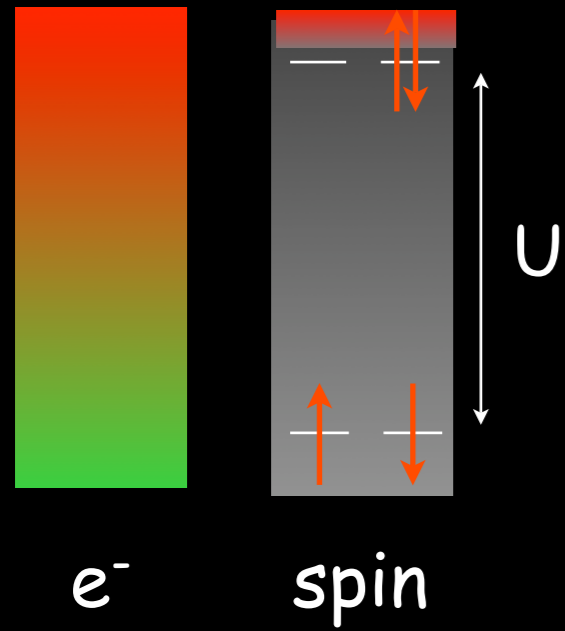
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Gauge Theories and Strong Correlation.



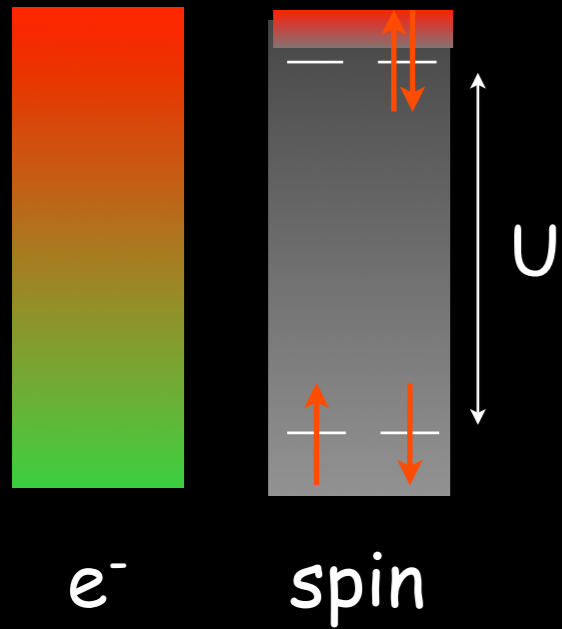
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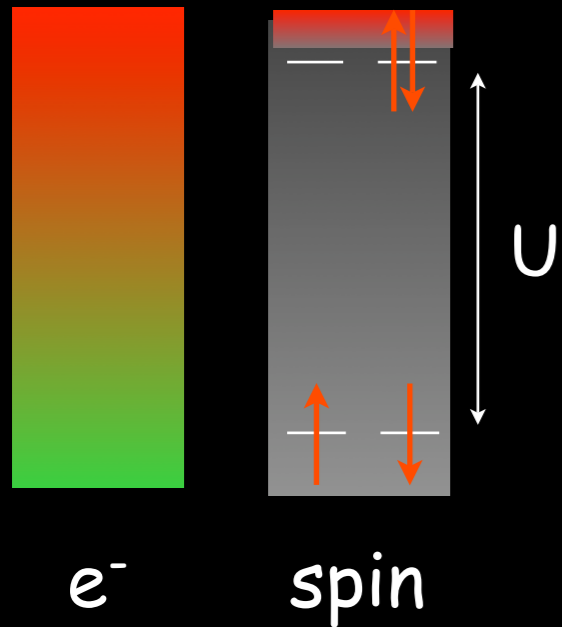
e.g. $\vec{S}_j = f_{j\alpha}^\dagger \left(\frac{\vec{\sigma}}{2} \right)_{\alpha\beta} f_{j\beta},$

$$f_j \rightarrow e^{i\phi_j} f_j,$$

$U(1)_{\text{local}}$

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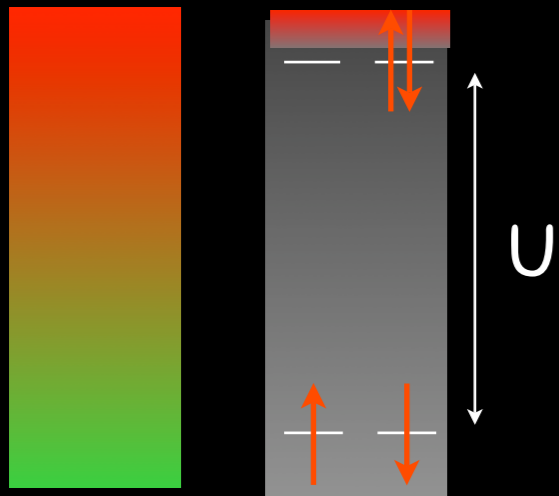
$U(1)_{\text{local}}$

$$H = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{J}{N} \sum_j c_{j\alpha}^\dagger c_{j\beta} S_{\beta\alpha}(j) + H_g$$

$$S_{\alpha\beta} = f_\alpha^\dagger f_\beta - \delta_{\alpha\beta} n_f / N$$

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 $U(1)_{\text{global}} \times U(1)_{\text{local}}$

spin
 λ
 $U(1)_{\text{local}}$

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$$H_g = (\Phi - \mu) c_j^\dagger c_j + \lambda_j (f_j^\dagger f_j - Q),$$

$$(Q = qN = 1)$$

Large N Approach.

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \sum_j H_I(j)$$

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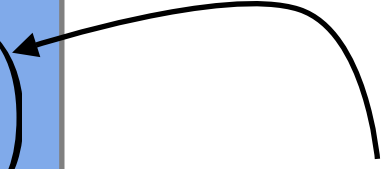
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Large N Approach.

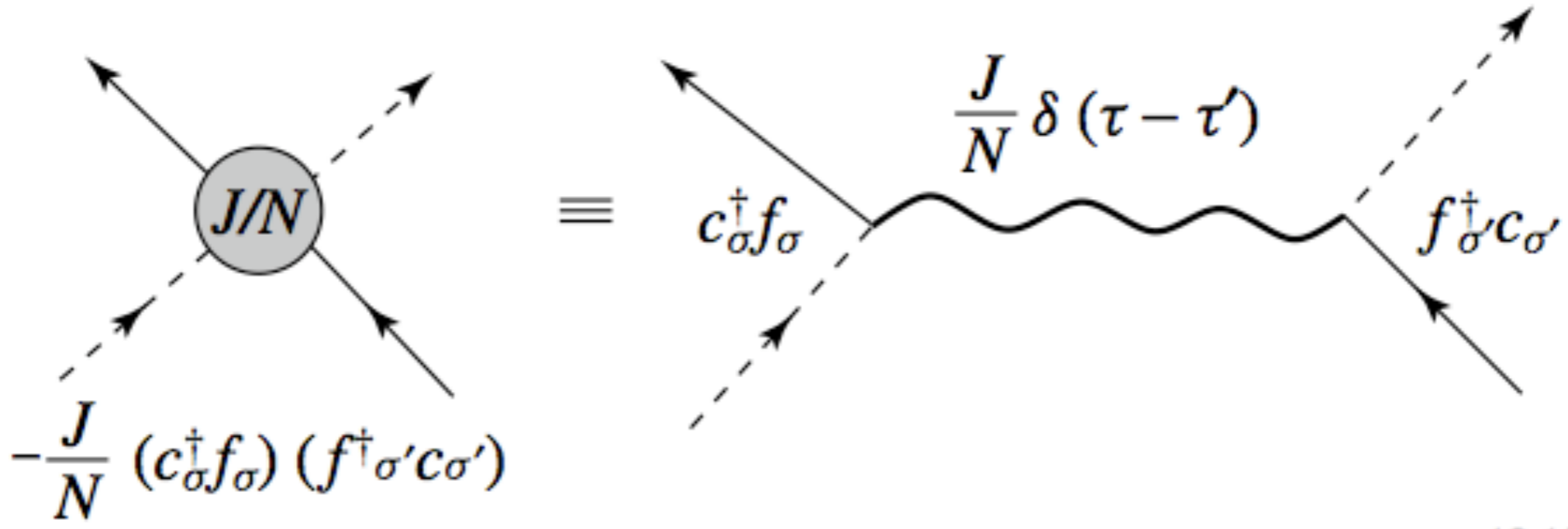
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$-gA^\dagger A \rightarrow A^\dagger V + \bar{V}A + \frac{\bar{V}V}{g}$

$$H_I(j) \rightarrow H_I[V, j] = \bar{V}_j \left(c_{j\alpha}^\dagger f_{j\alpha} \right) + \left(f_{j\alpha}^\dagger c_{j\alpha} \right) V_j + N \frac{\bar{V}_j V_j}{J}.$$



(81)

Large N Approach.

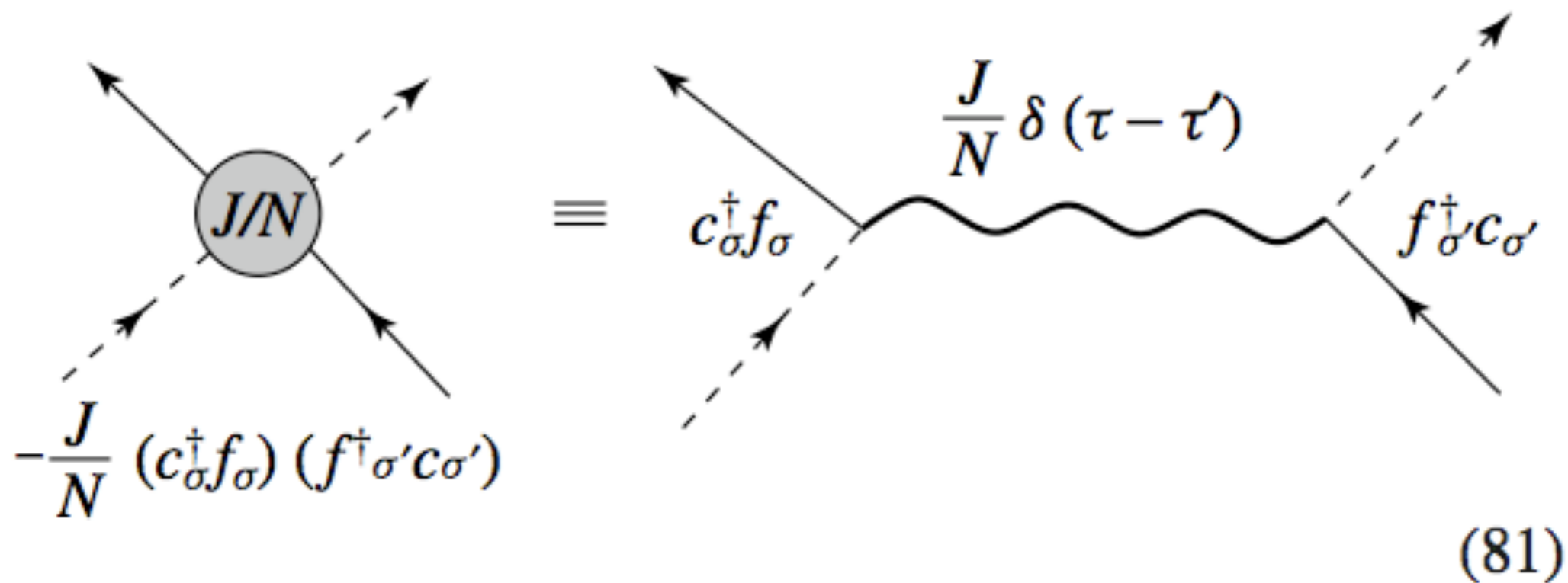
The diagram shows an equivalence between two representations of a vertex. On the left, a central grey circle labeled J/N is connected to four external lines: two solid lines and two dashed lines. Below this vertex is the expression $-\frac{J}{N} (c_{\sigma}^{\dagger} f_{\sigma}) (f_{\sigma'}^{\dagger} c_{\sigma'})$. On the right, a wavy line representing a propagator is shown, labeled $\frac{J}{N} \delta(\tau - \tau')$. The wavy line connects two vertices. The left vertex is connected to two external lines (one solid, one dashed) and is labeled $c_{\sigma}^{\dagger} f_{\sigma}$. The right vertex is connected to two external lines (one solid, one dashed) and is labeled $f_{\sigma'}^{\dagger} c_{\sigma'}$. The two diagrams are separated by an equivalence symbol \equiv .

$$-\frac{J}{N} (c_{\sigma}^{\dagger} f_{\sigma}) (f_{\sigma'}^{\dagger} c_{\sigma'}) \equiv \frac{J}{N} \delta(\tau - \tau') (c_{\sigma}^{\dagger} f_{\sigma}) (f_{\sigma'}^{\dagger} c_{\sigma'}) \quad (81)$$

Large N Approach.

$$H[V, \lambda] = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_j (H_I[V_j, j] + \lambda_j [n_f(j) - Q]),$$

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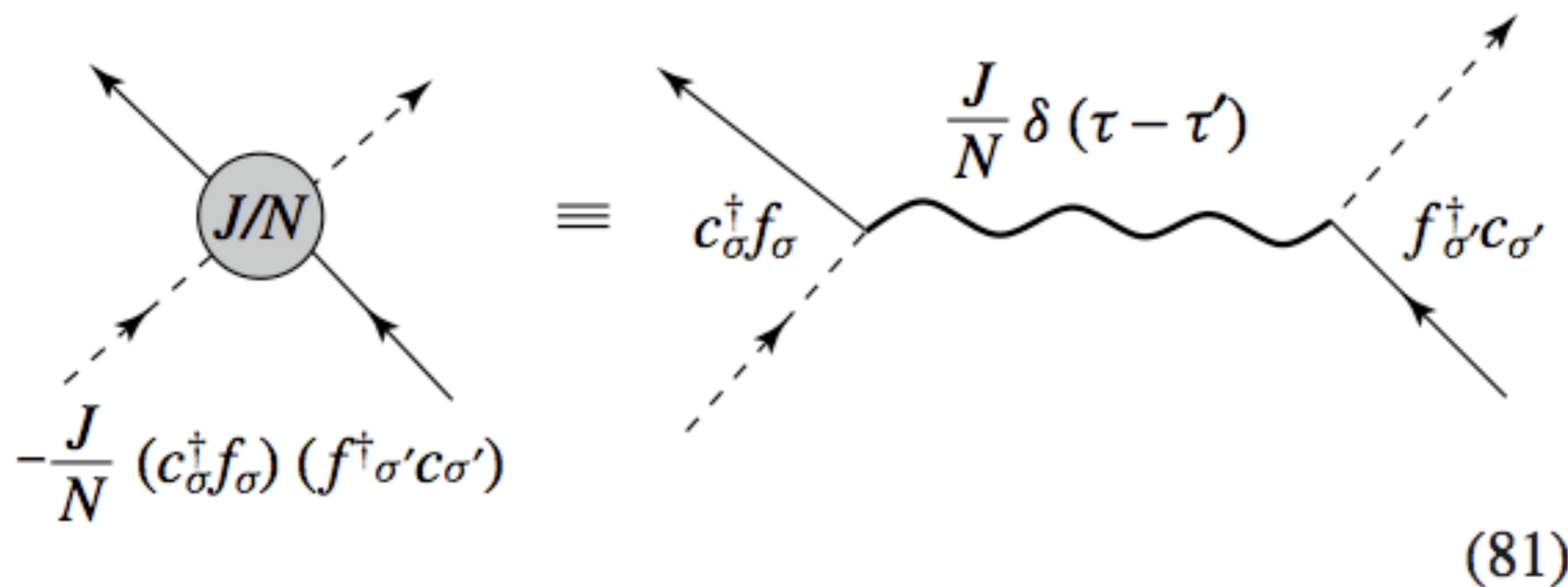


Large N Approach.

$$Z = \int \mathcal{D}[V, \lambda] \int \mathcal{D}[c, f] \exp \left[- \int_0^\beta \left(\sum_{k\sigma} c_{k\sigma}^\dagger \partial_\tau c_{k\sigma} + \sum_{j\sigma} f_{j\sigma}^\dagger \partial_\tau f_{j\sigma} + H[V, \lambda] \right) \right] = \text{Tr} \left[T \exp \left(- \int_0^\beta H[V, \lambda] d\tau \right) \right]$$

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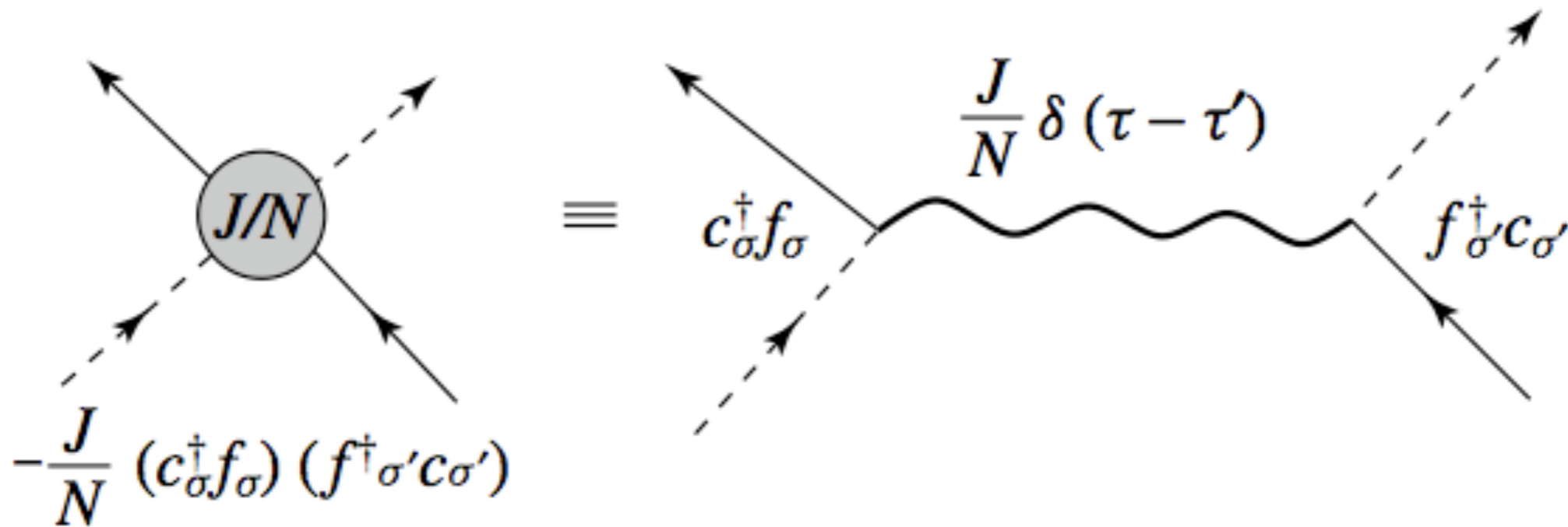


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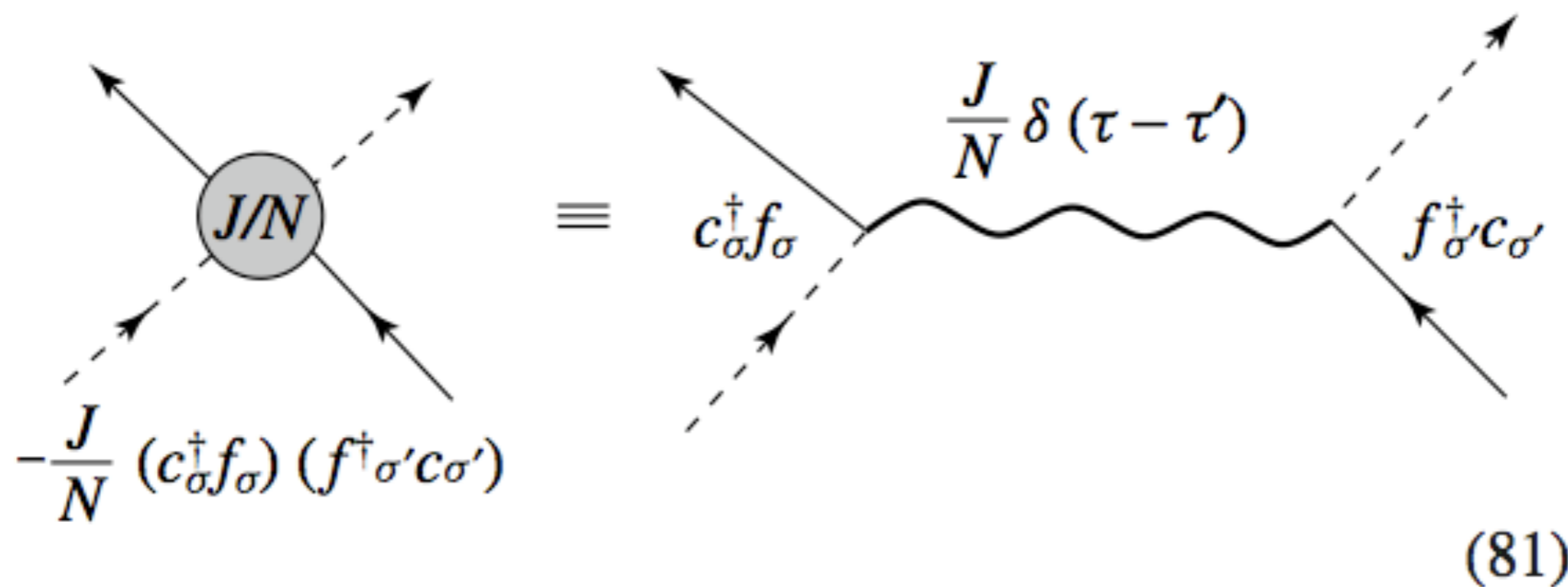


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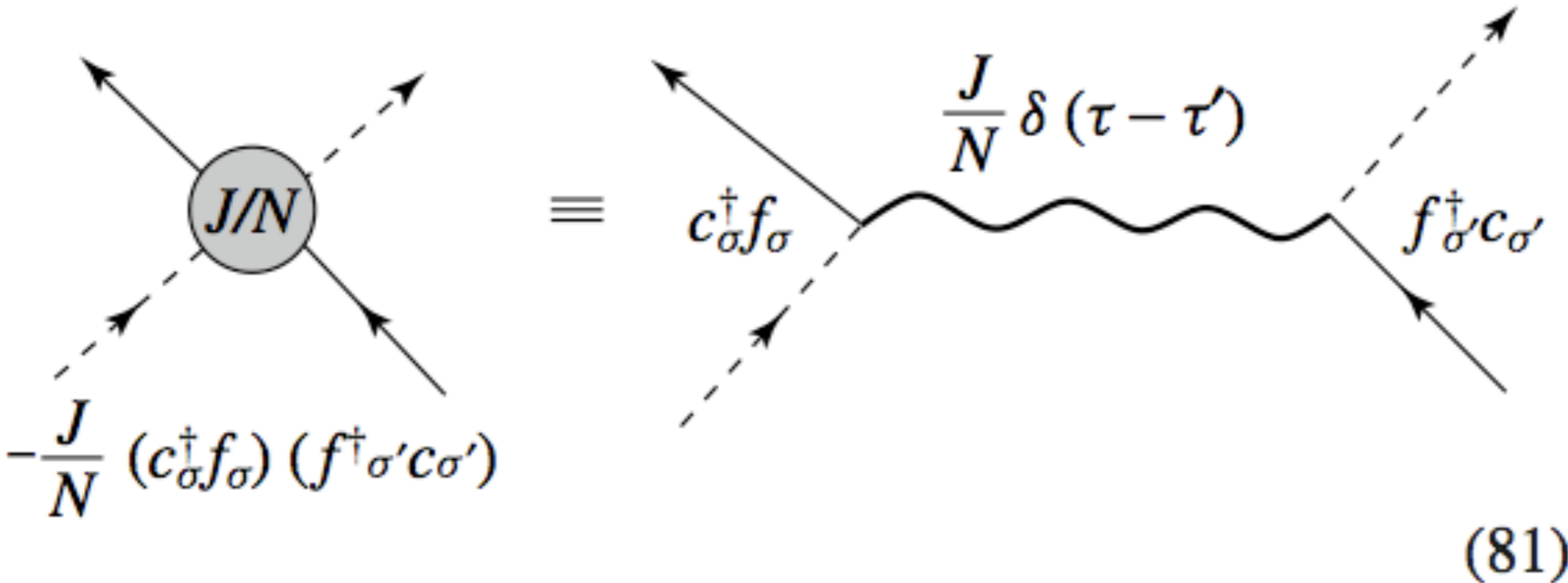


Large N Approach.

$$Z = \text{Tr} e^{-\beta H_{MFT}}, \quad (N \rightarrow \infty)$$

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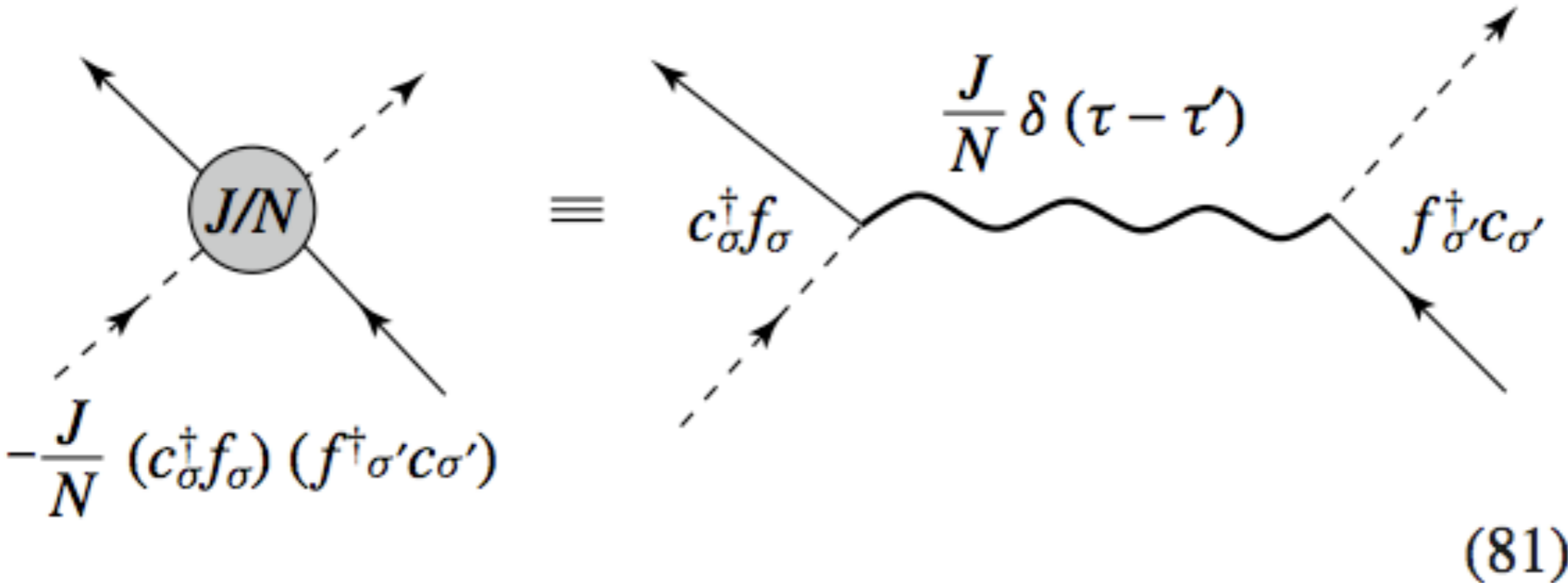


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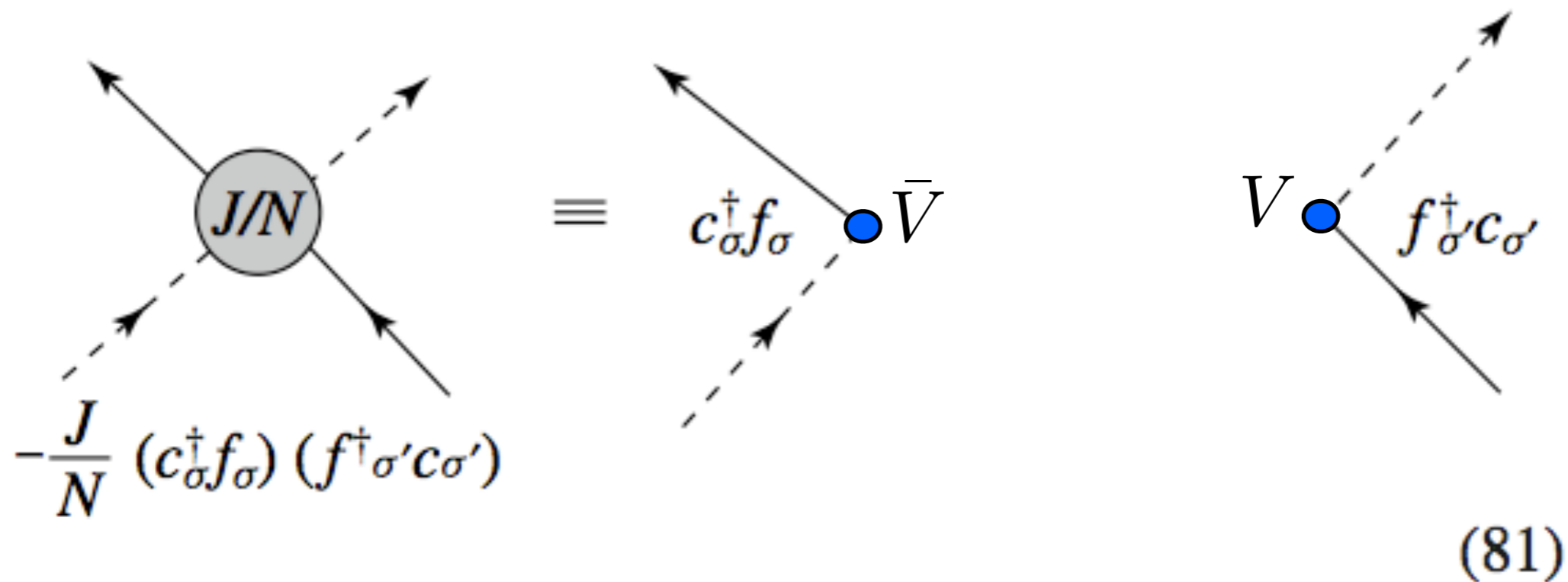


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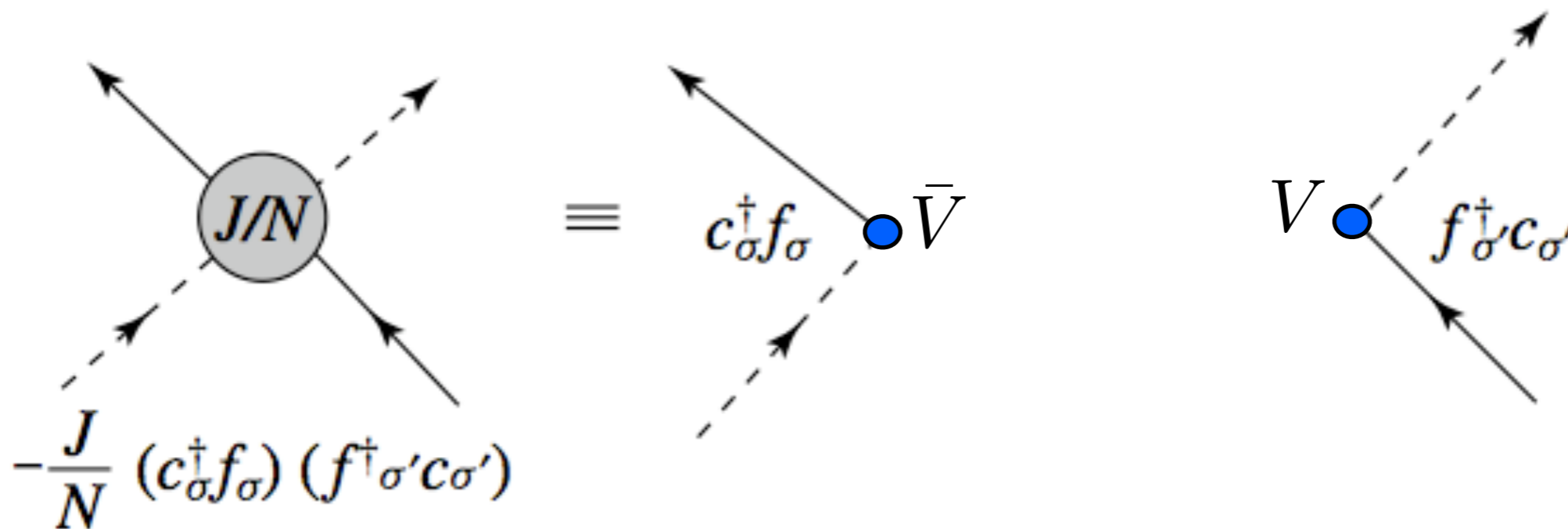


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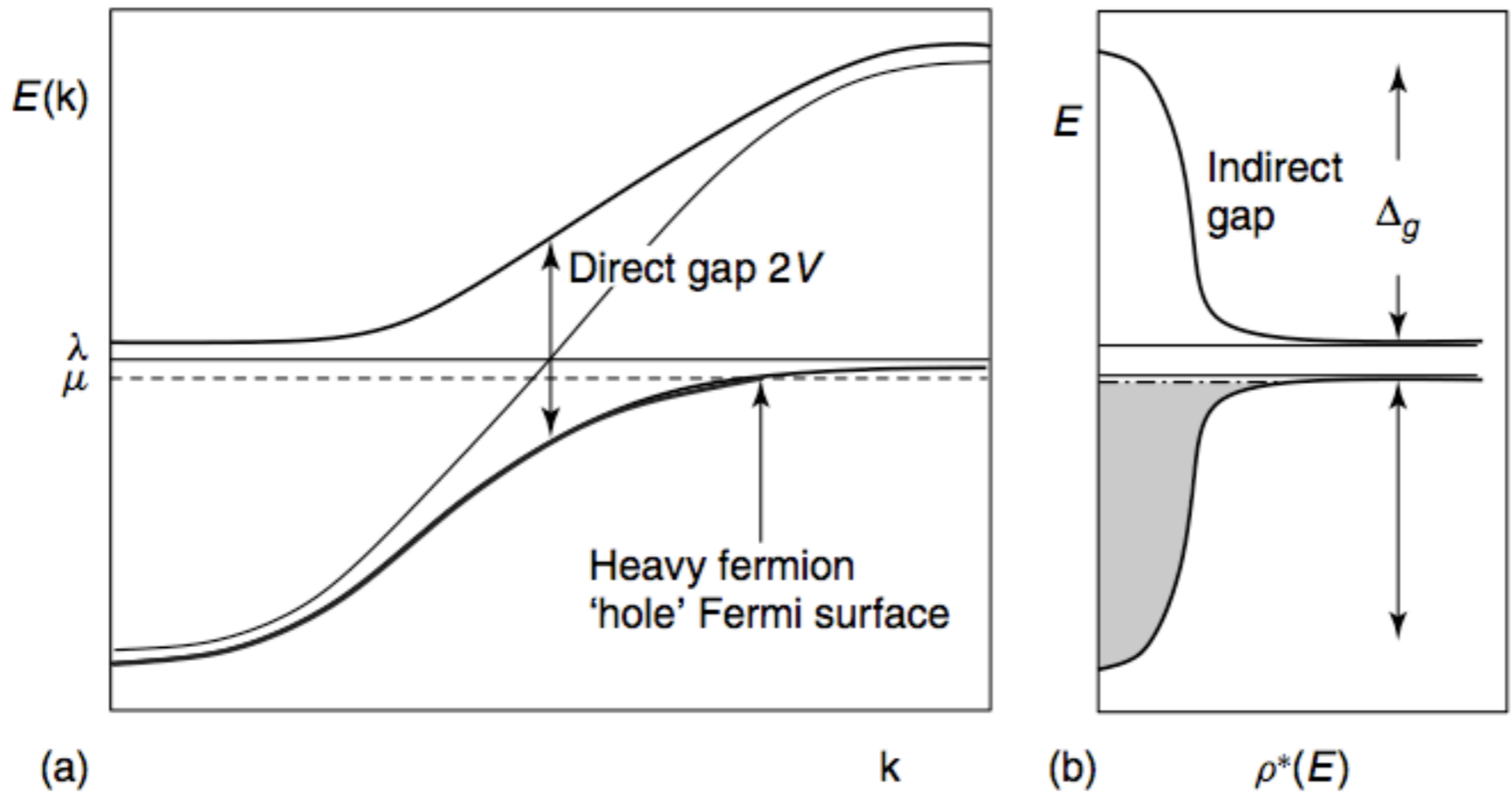
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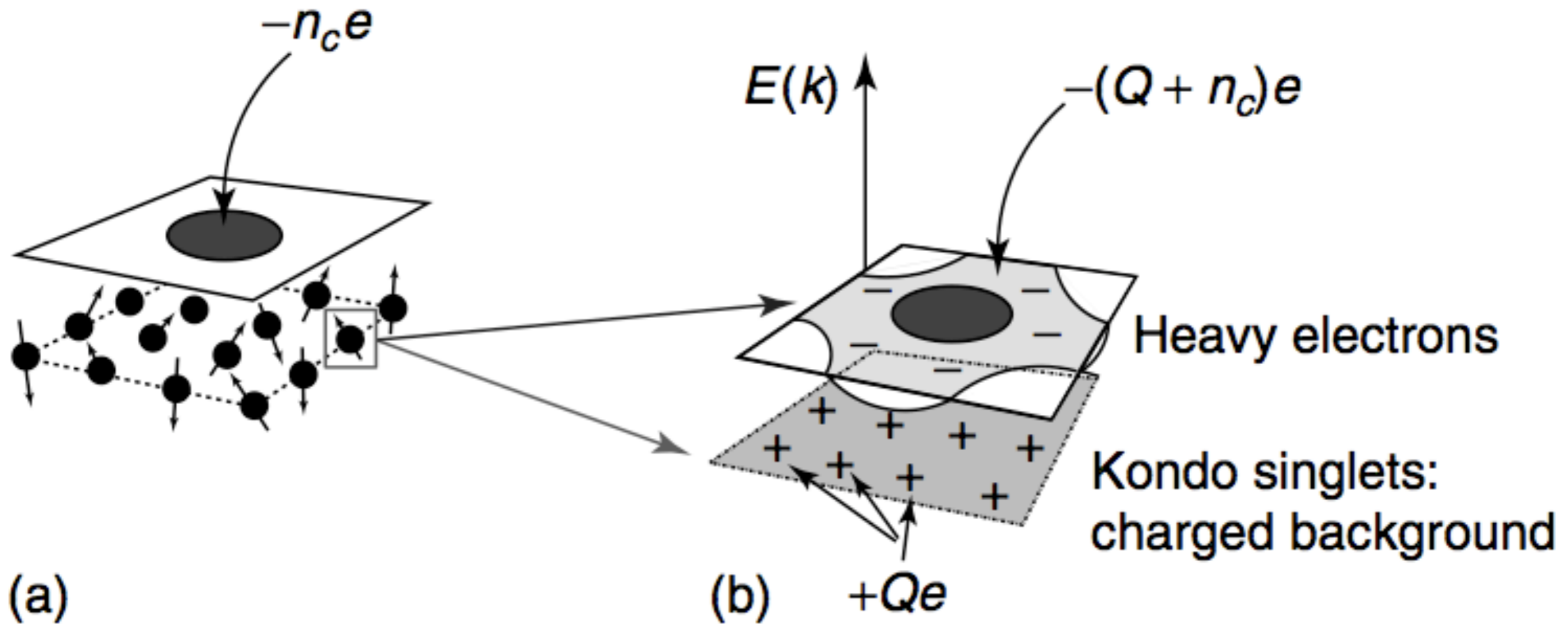
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Large N Approach.



Large Fermi surface and the charge of the f-electron



Conclusions / Open Questions.

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Spin \rightarrow Composite electron + background charge.

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Spin \rightarrow Composite electron + background charge.

- What is the nature of the zero modes (fermionic?) at a heavy electron QCP.
- How do we unify the Kondo effect with (i) Superconductivity and (ii) Magnetism.

Possible close links with (a) atom traps and (b) quantum dots.

$$H = \sum_k \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \sum_j (\psi_j^\dagger \vec{\sigma} \psi_j) \cdot \vec{S}_j$$

"THE BATTLEGROUND"

Kondo Lattice Model

(Kasuya, 1951)

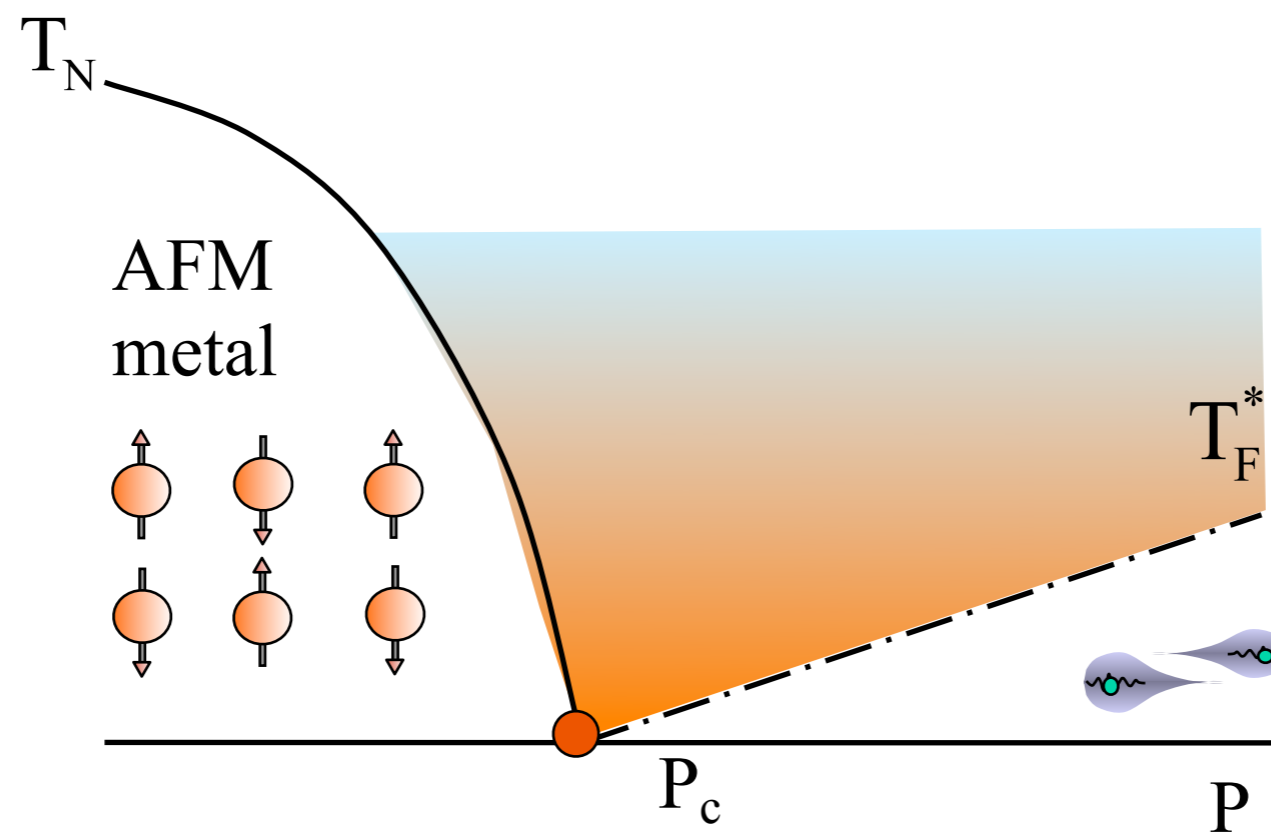
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Heavy Fermion
Materials



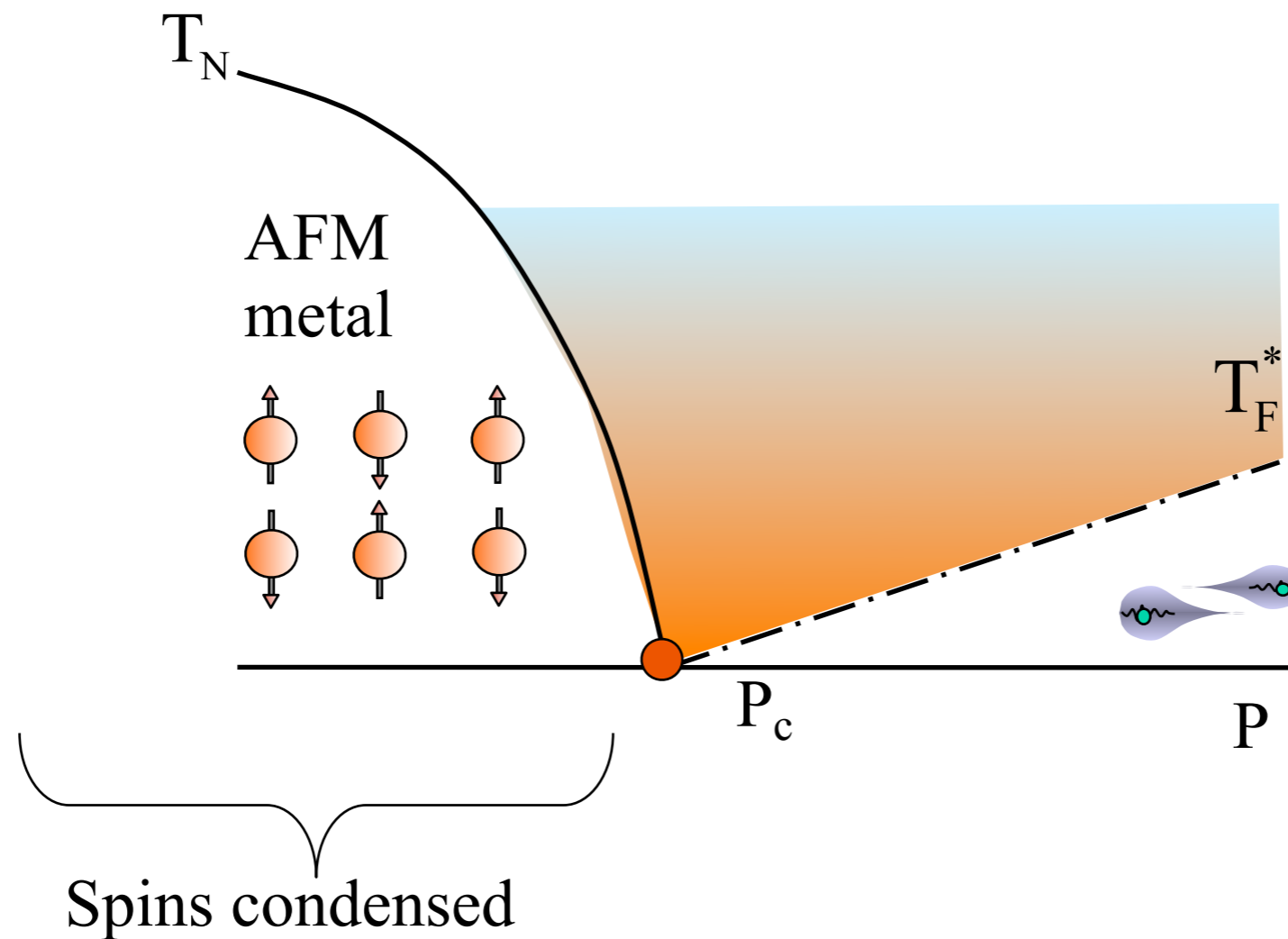
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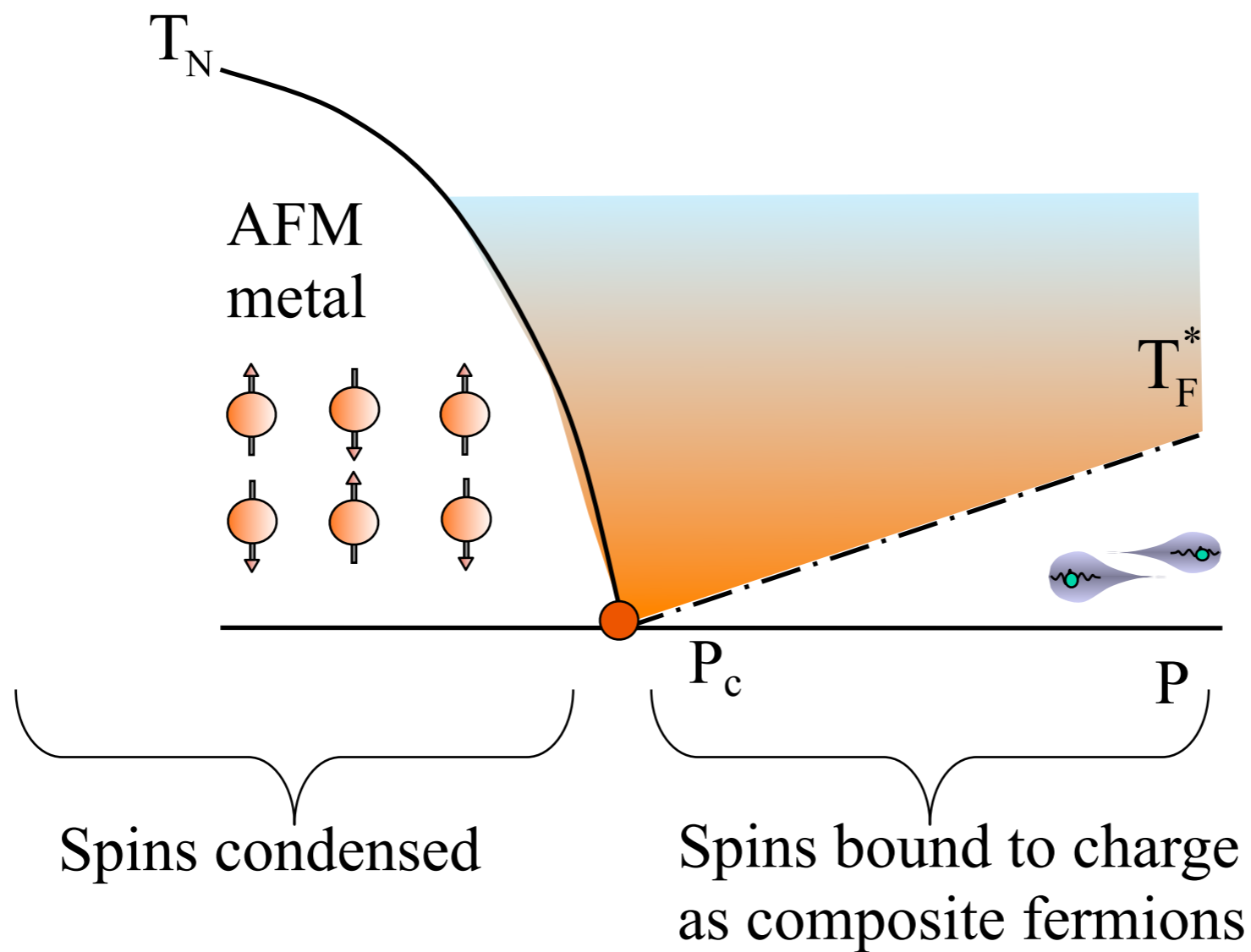
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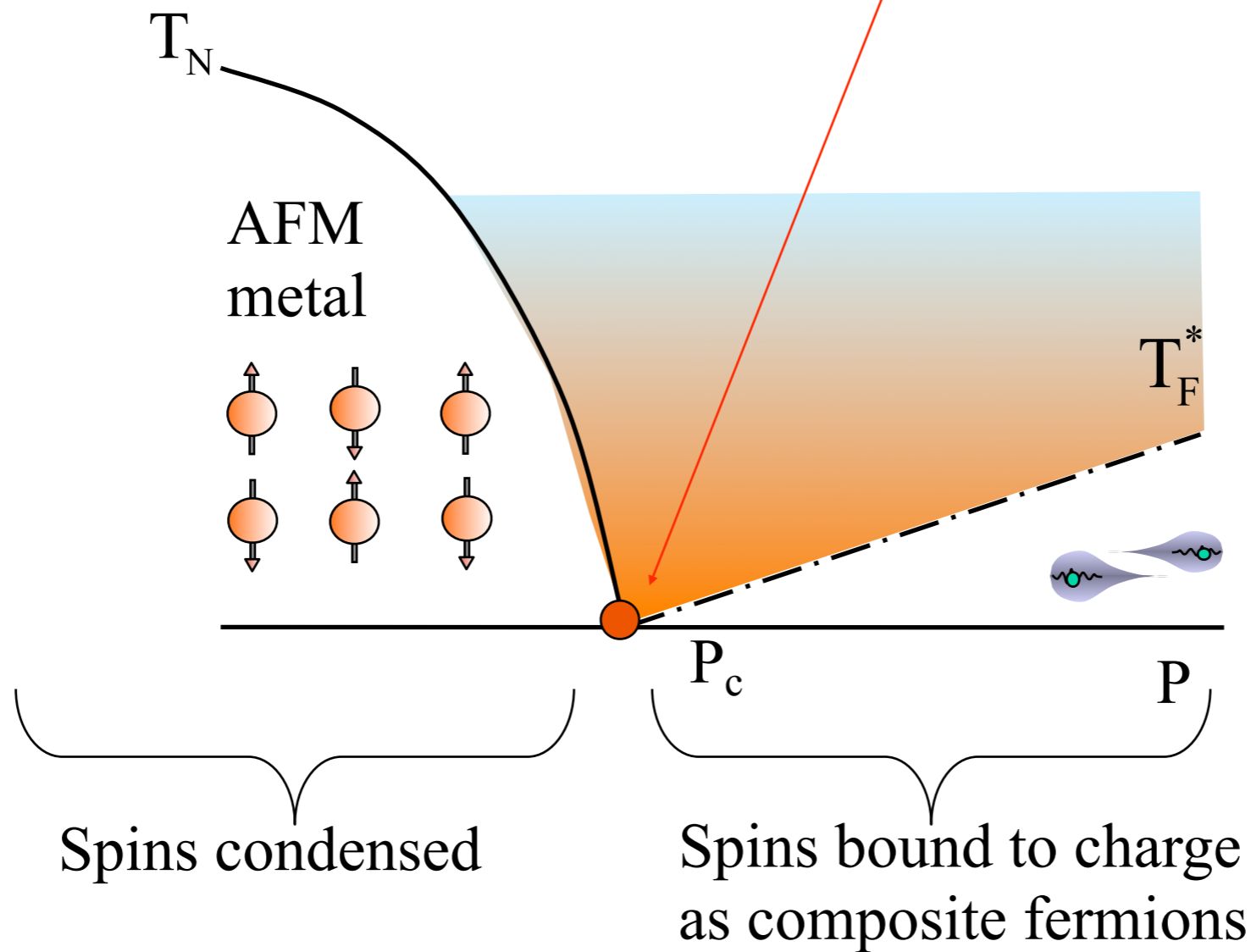
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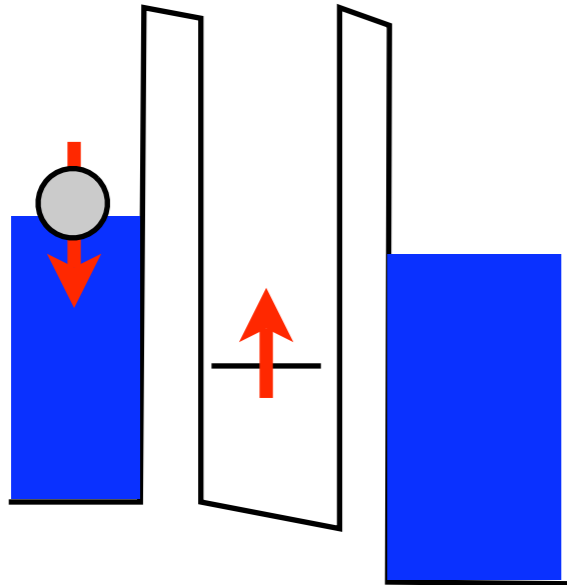
(Kasuya, 1951)

Deconfinement of spin:
fundamentally new kind of
zero mode.

Heavy Fermion
Materials



Cotunneling and copairing:

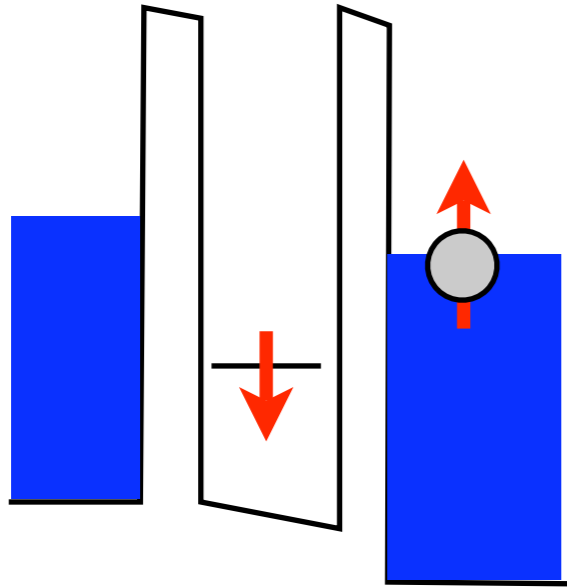


Cotunneling

(Glazman +Pustilnik)

(quantum dots)

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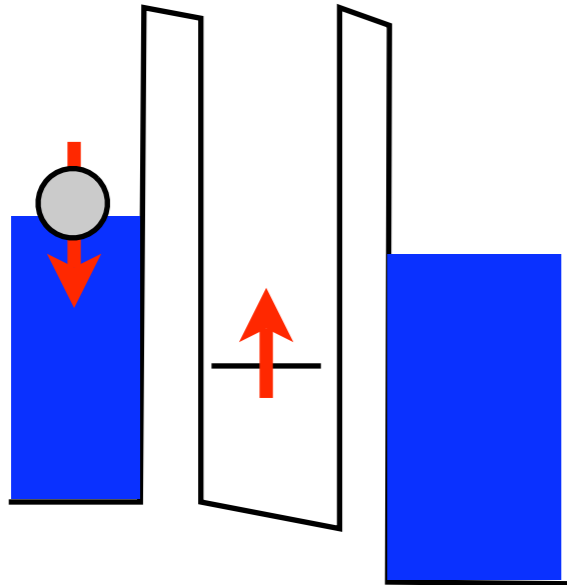


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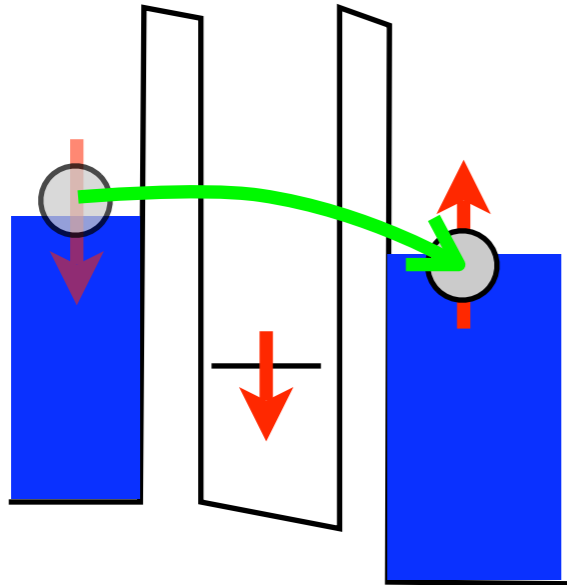


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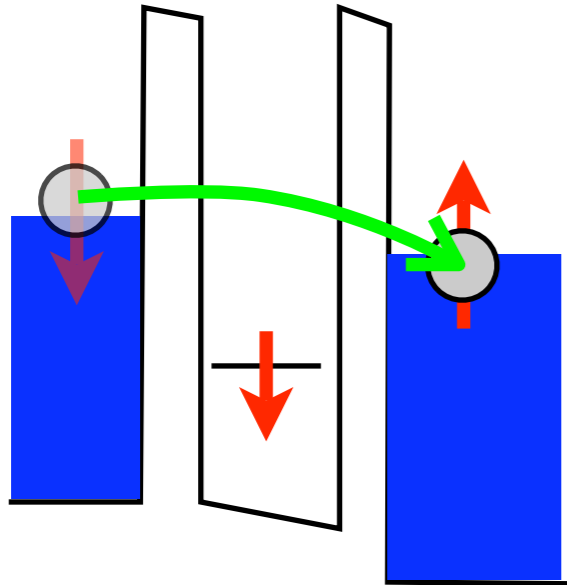


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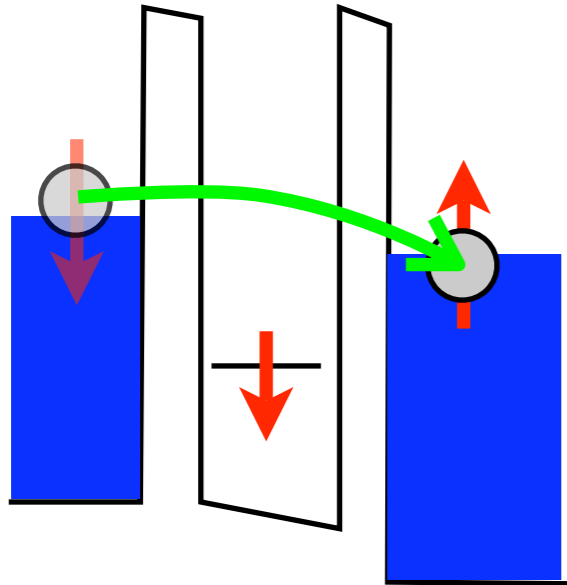
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$$f_{\downarrow}^{\dagger} = c_{\uparrow}^{\dagger} S_{-}$$

Cotunneling and copairing:

van der Wiel et al, Science (2000)

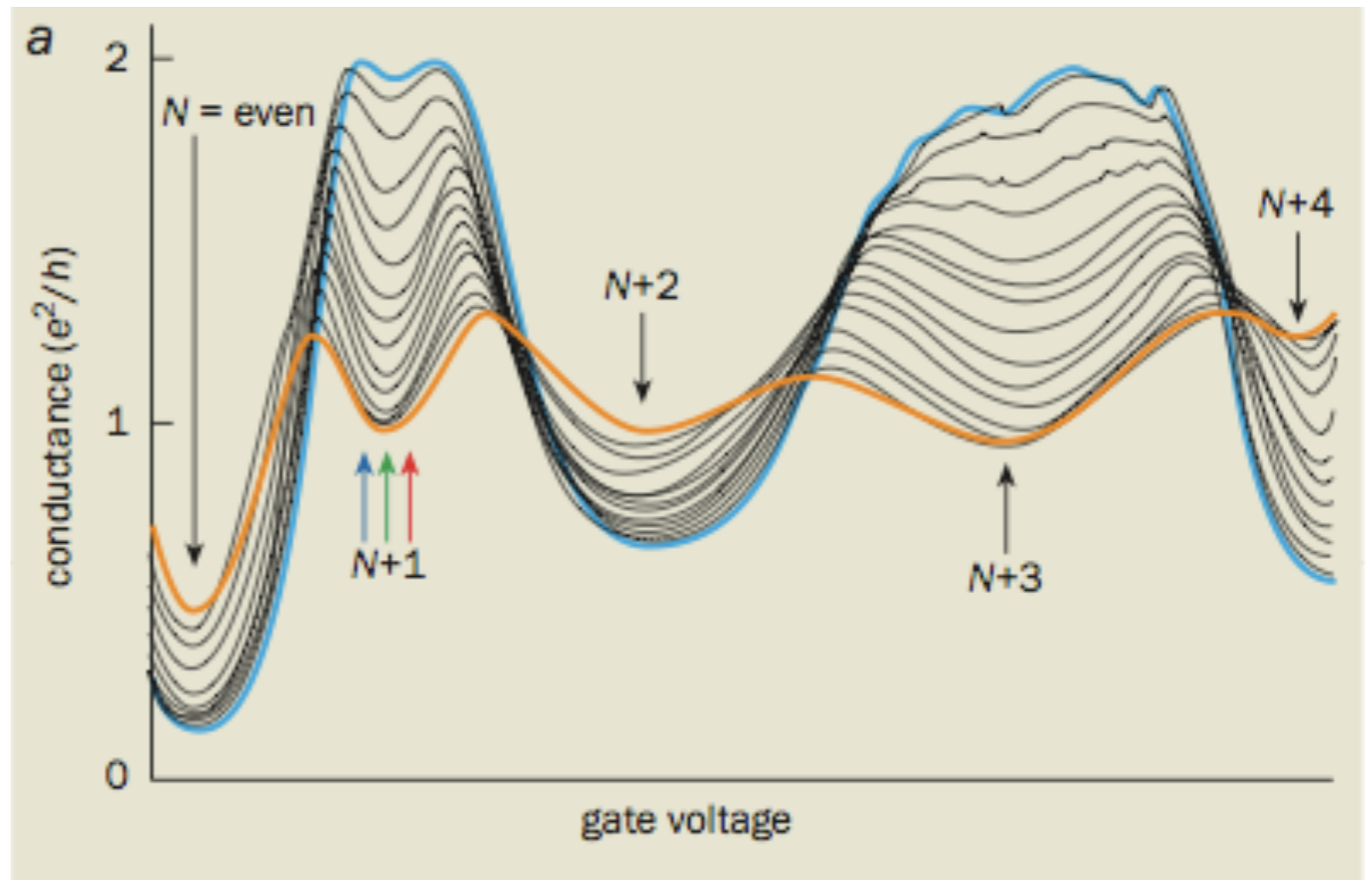


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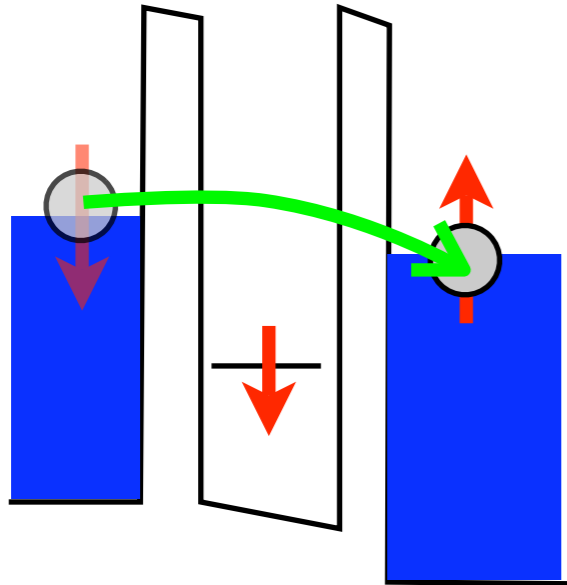
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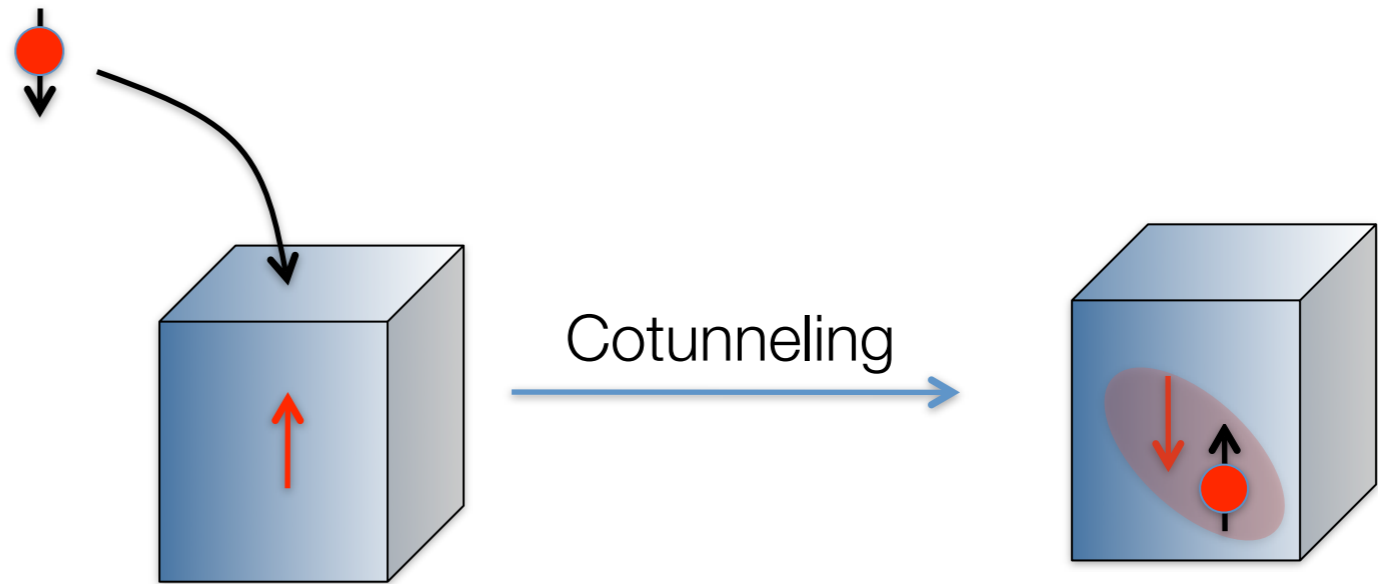


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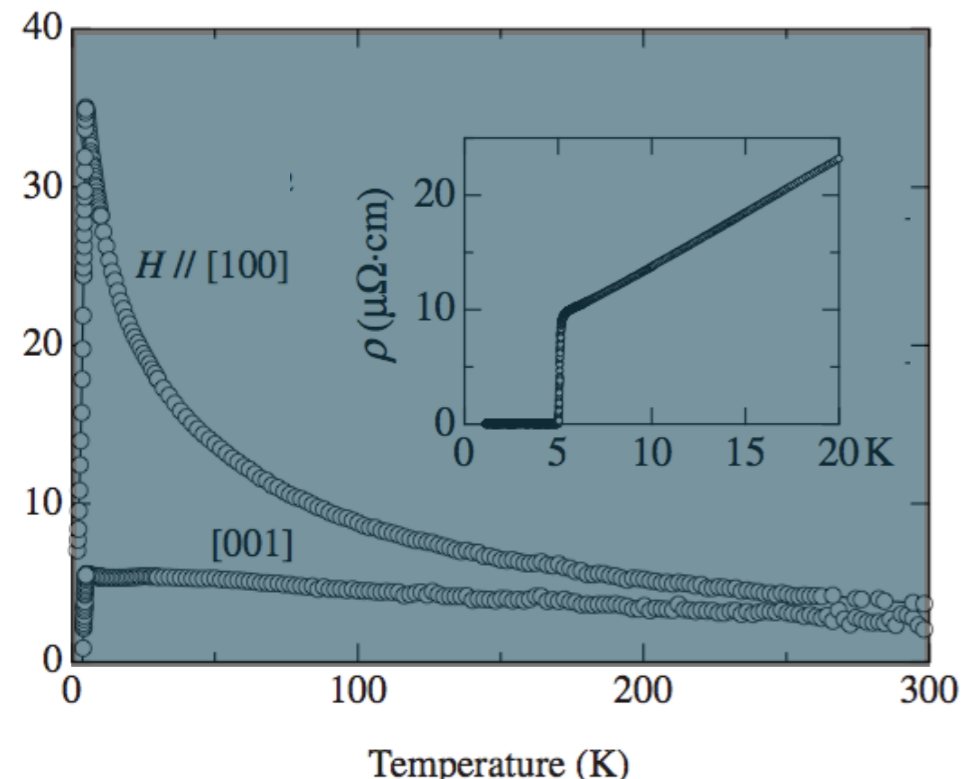
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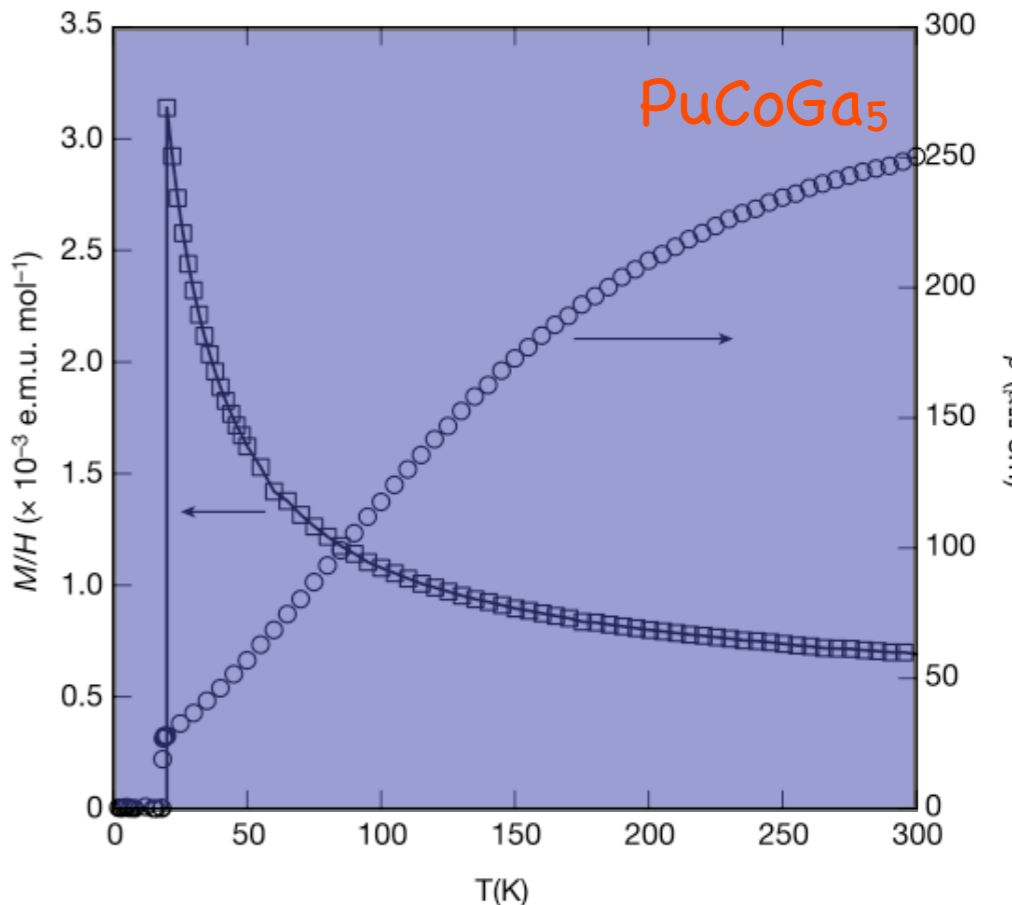


Heavy electron = (electron x spinflip)

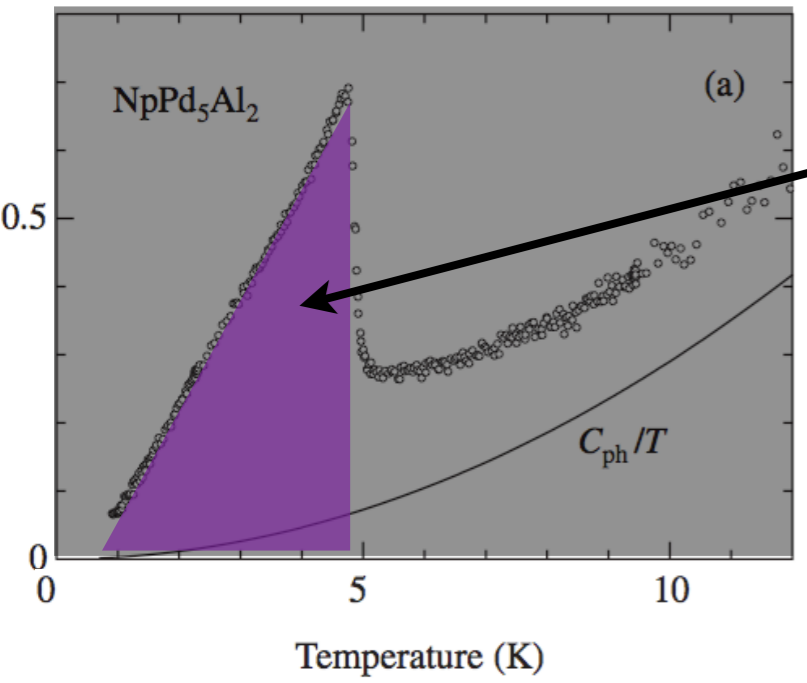
New HF Superconductors: PuCoGa₅ & NpAl₂Pd₅



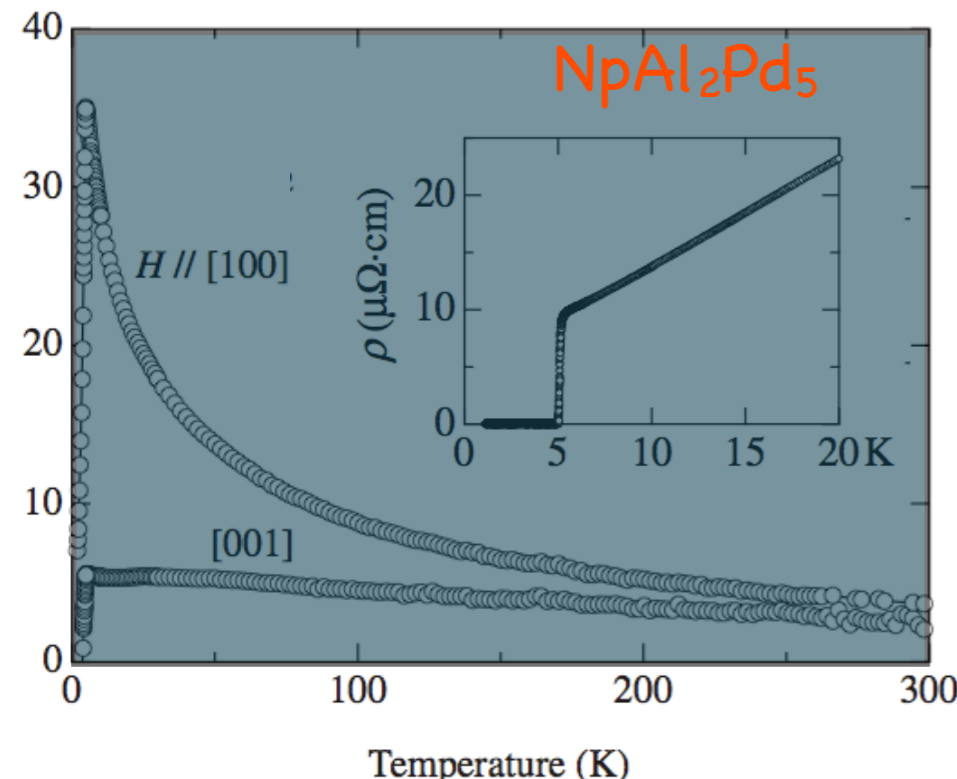
Directly Curie PM to SC



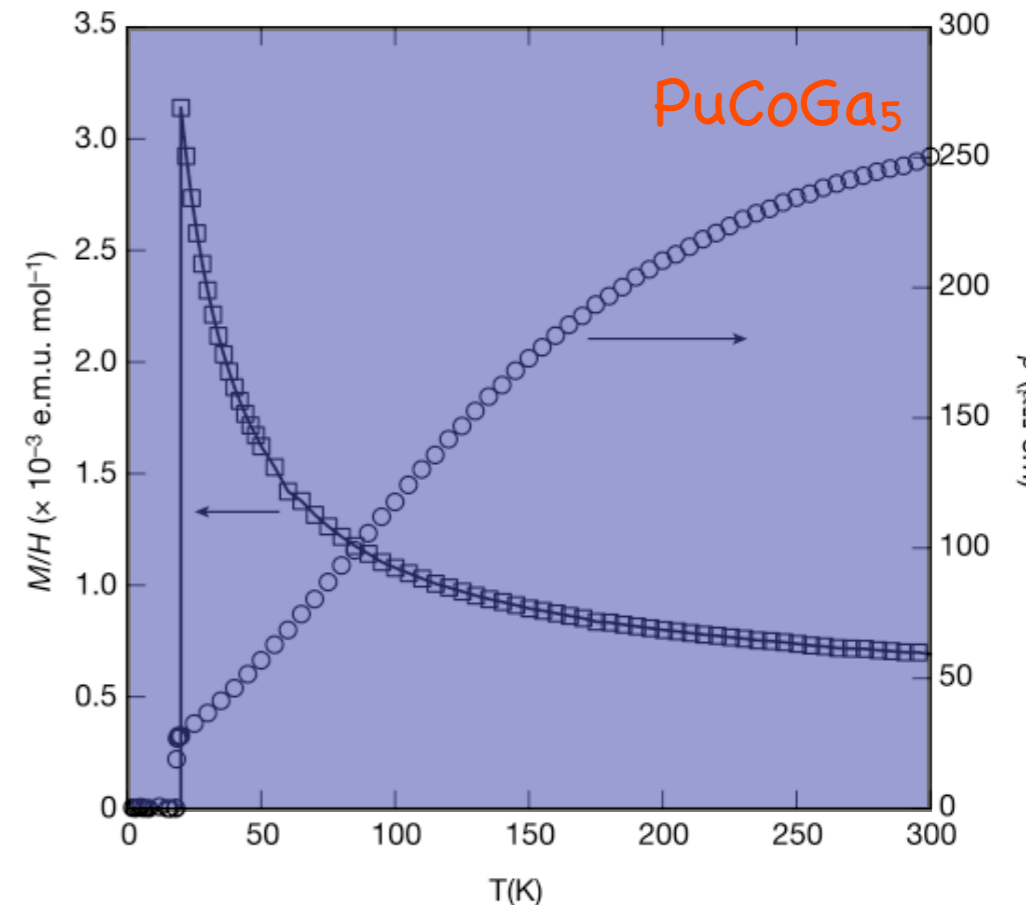
Substantial fraction of spin entropy.
 $\frac{1}{3} R \ln 2$



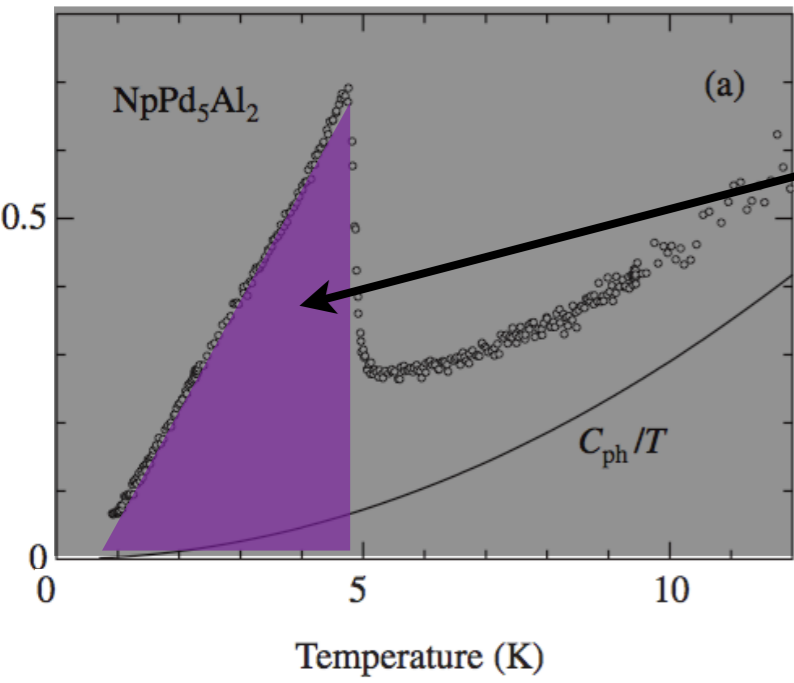
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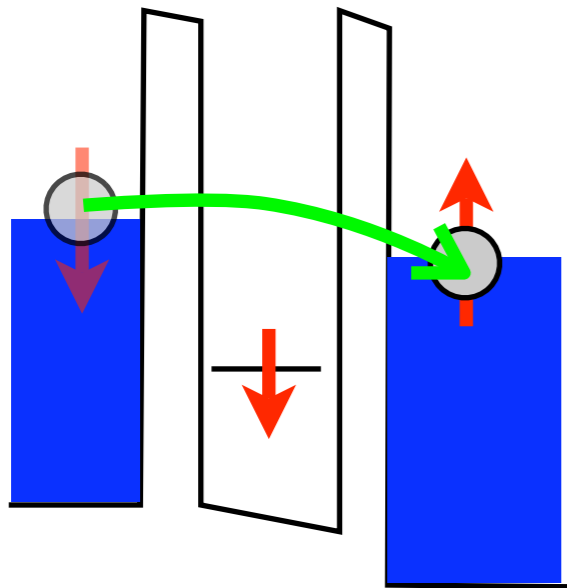


Substantial fraction of spin entropy.
 $1/3 R \ln 2$



- Suggests Kondo spin quenching and superconductivity develop simultaneously.
- Spin is not the glue, but the *fabric*

Cotunneling and copairing:

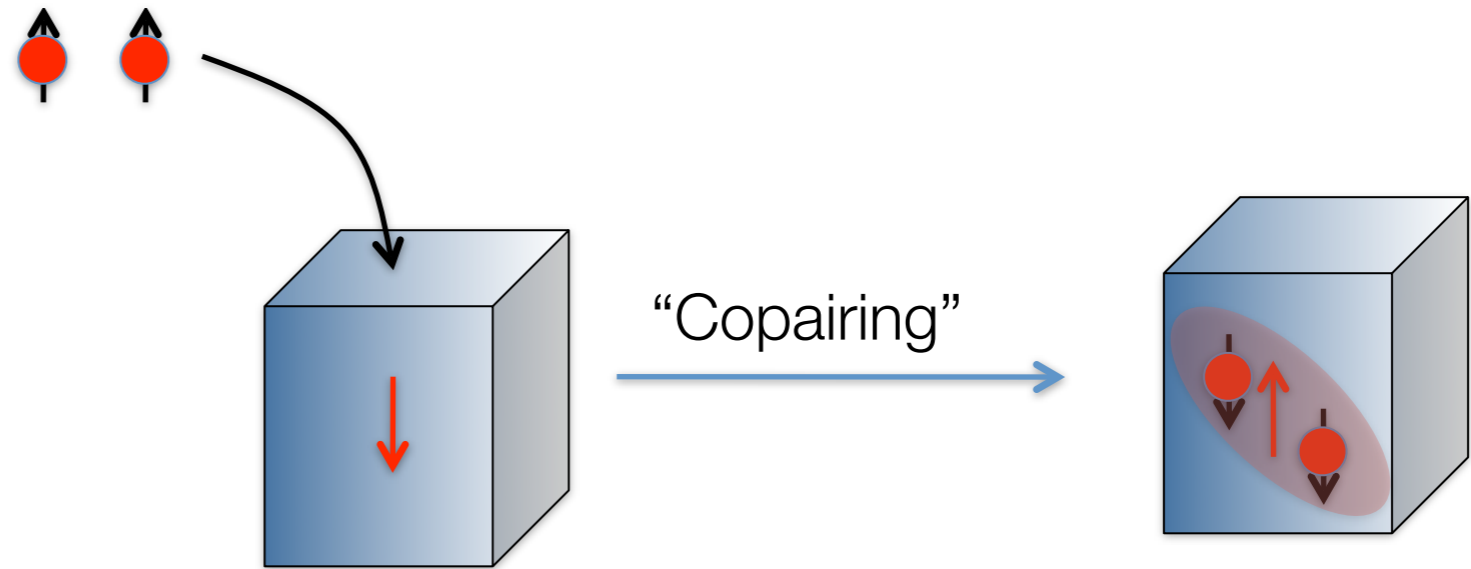


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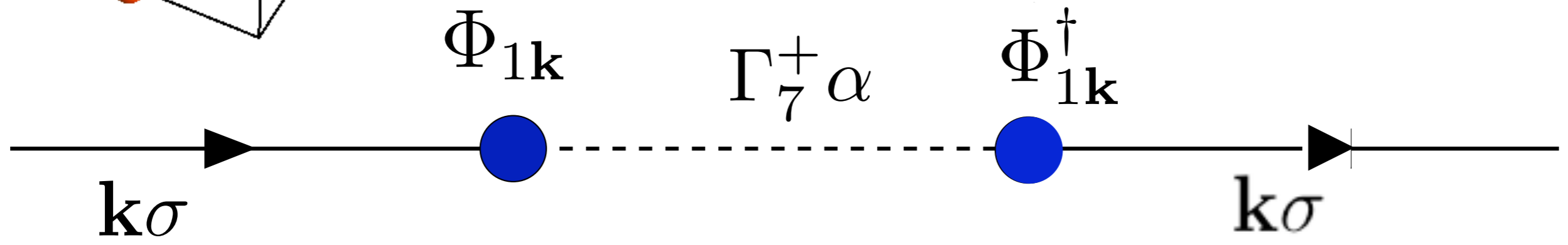
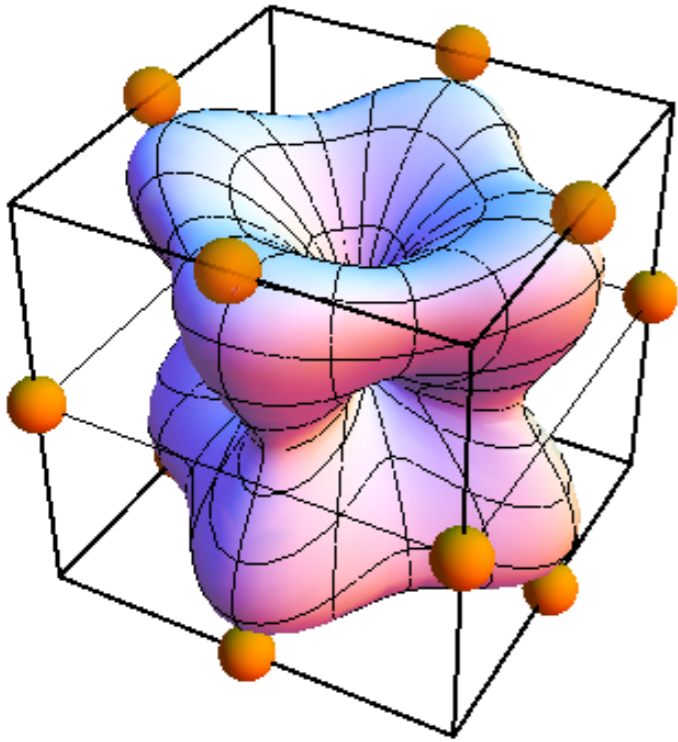
$$f_{\downarrow}^{\dagger} = c_{\uparrow}^{\dagger} S_{-}$$

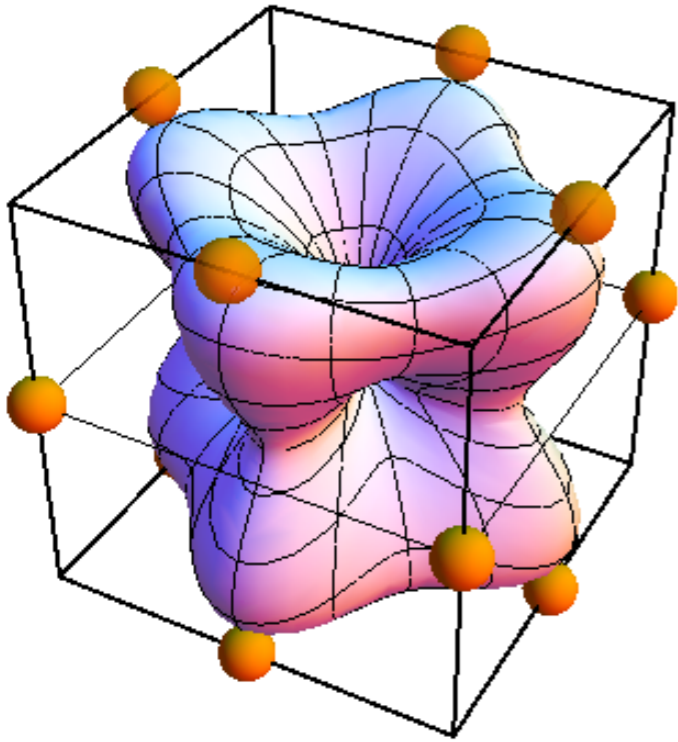


Heavy Cooper pair = (pair x spinflip)

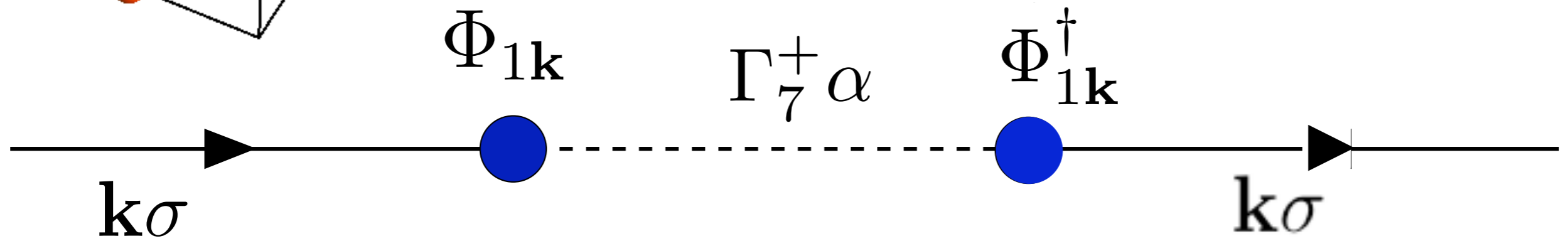
$$\Psi^{\dagger} = c_{1\downarrow}^{\dagger} c_{2\downarrow}^{\dagger} S_{+}$$

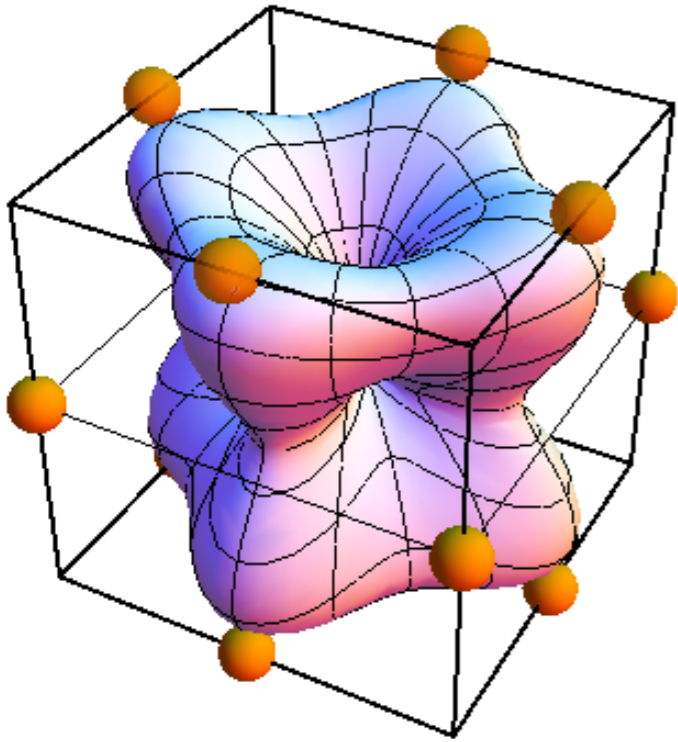
R. Flint et al, Nature Physics, 11 July, 2008.



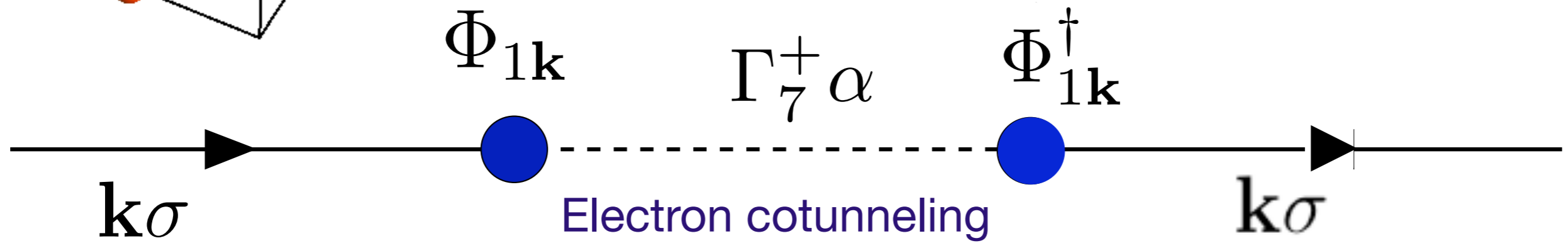


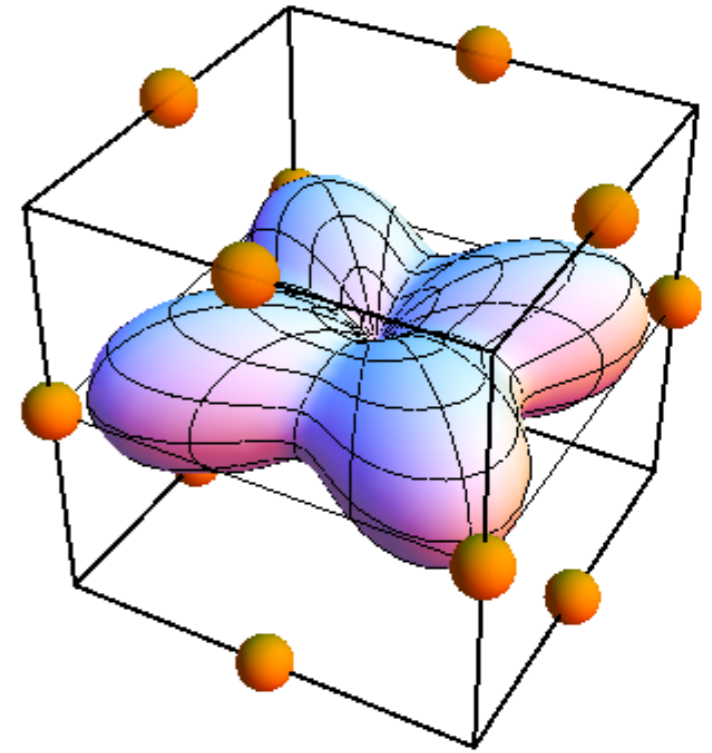
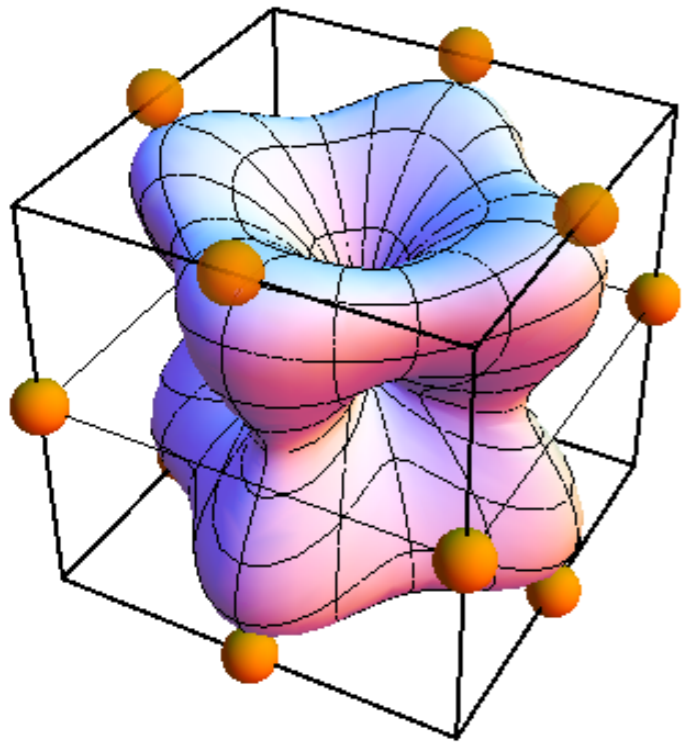
$$= \Phi_{1\mathbf{k}}^\dagger \cdot \Phi_{1\mathbf{k}}$$

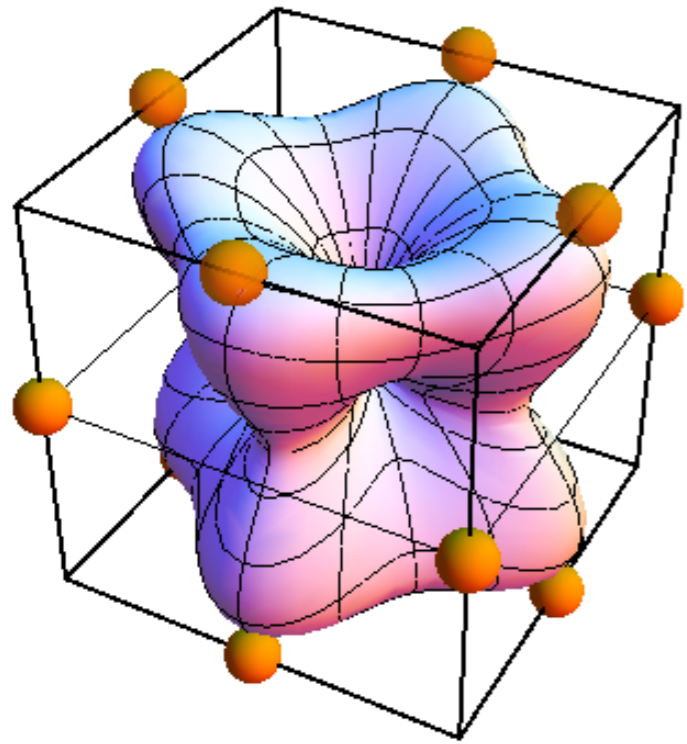




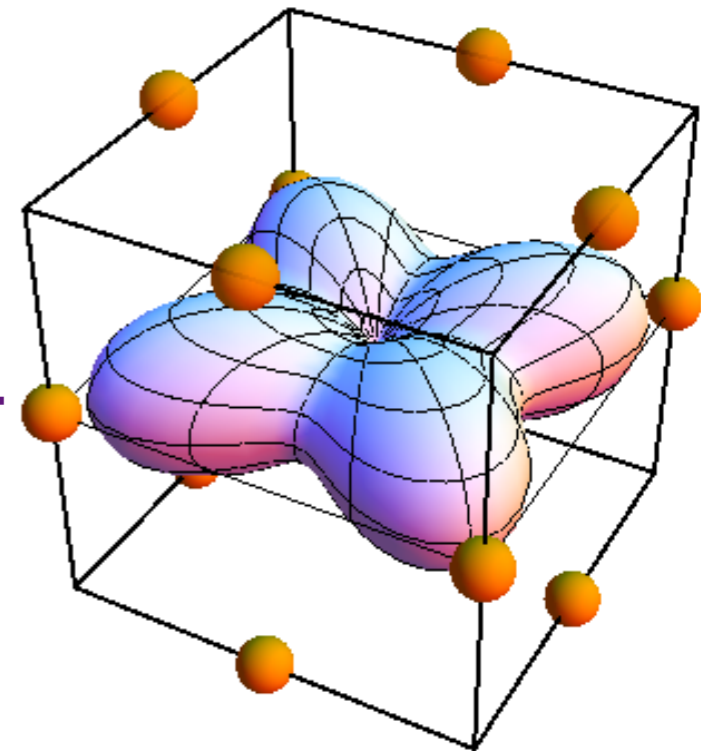
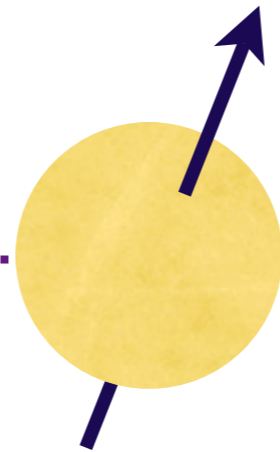
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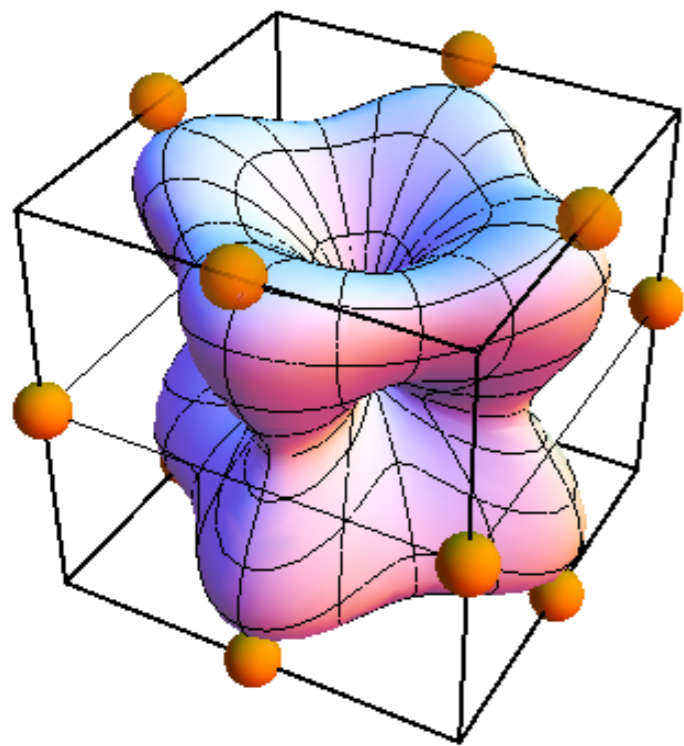




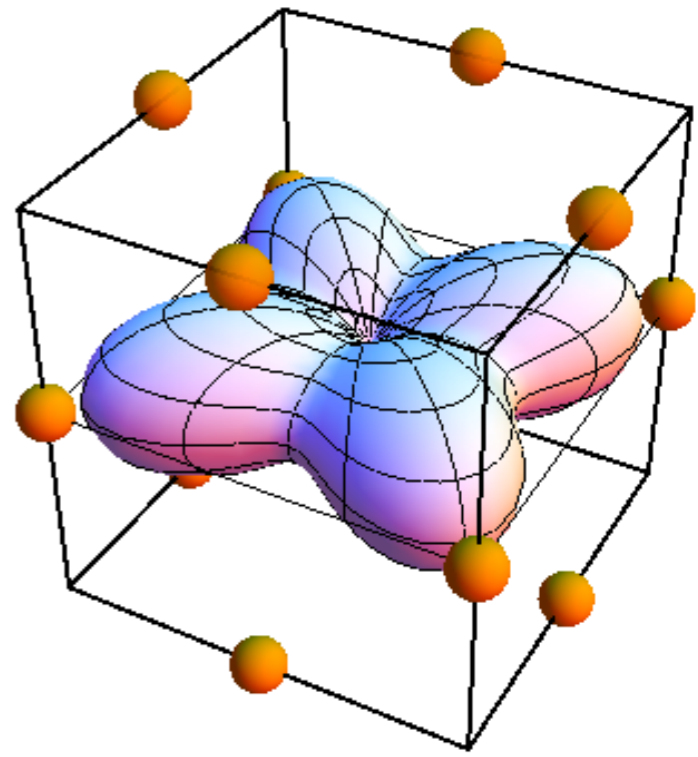
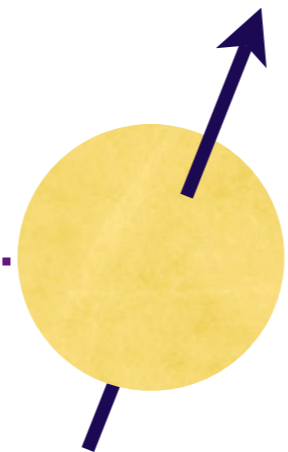


$$\langle \psi_2^\dagger \vec{\sigma} \epsilon \psi_1^\dagger \cdot \vec{S} \rangle$$

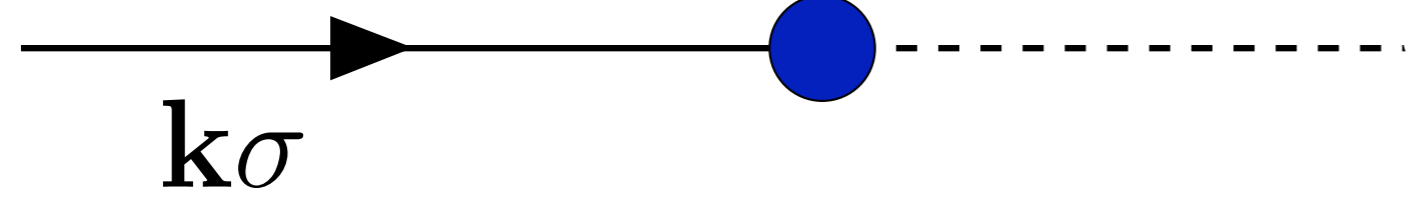
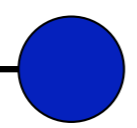


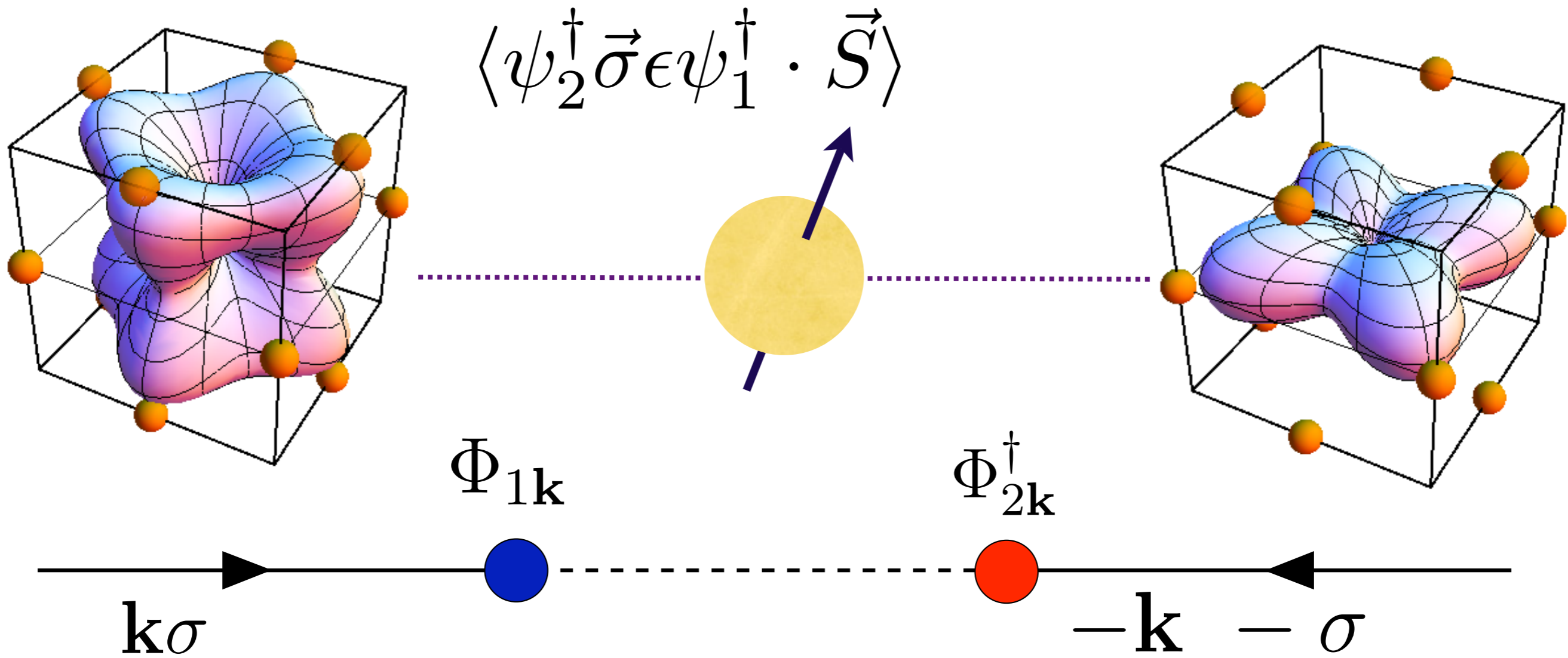


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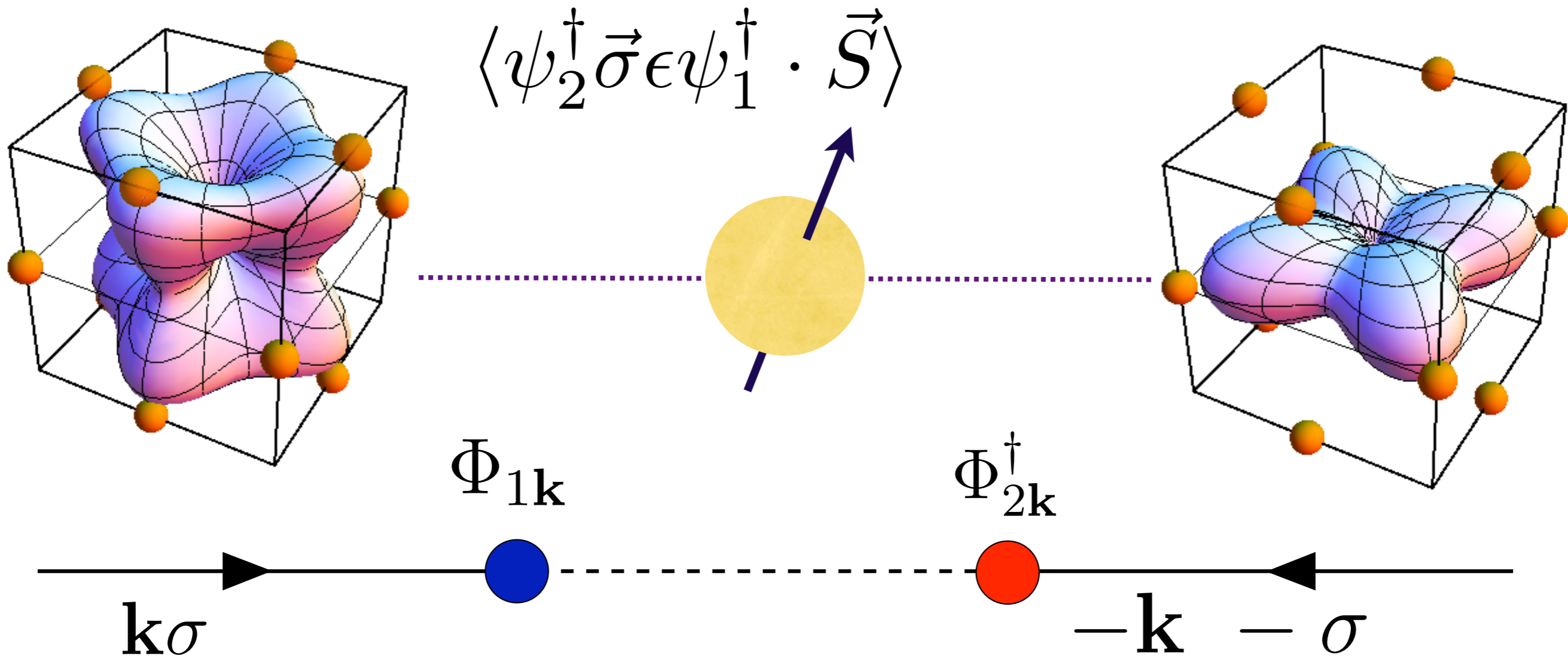


$$\Phi_{1\mathbf{k}}$$



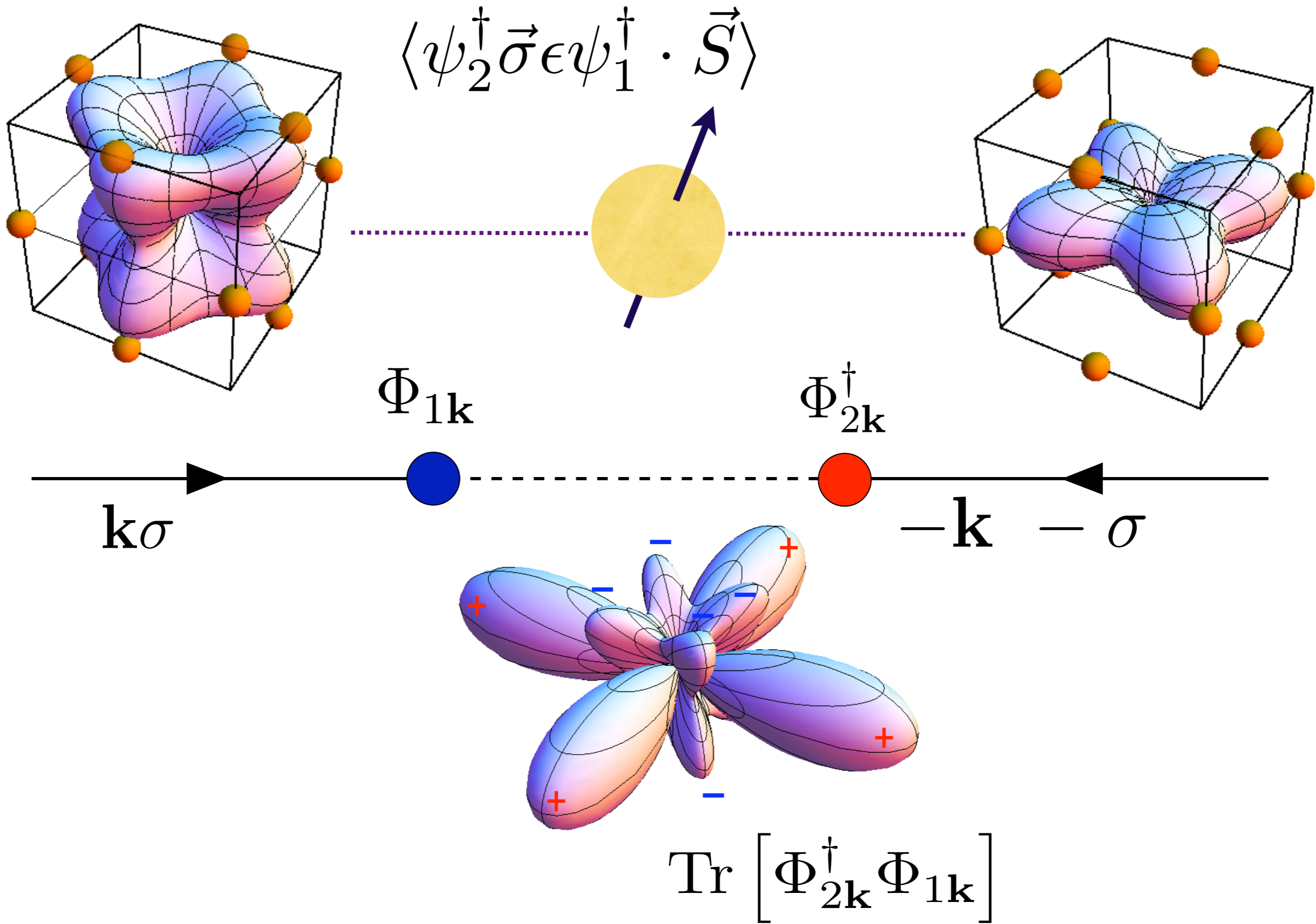


Resonant Andreev Reflection



$$\text{Tr} \left[\Phi_{2\mathbf{k}}^\dagger \Phi_{1\mathbf{k}} \right]$$

Resonant Andreev Reflection



Resonant Andreev Reflection