Frustrated Magnets (II) [Balents]

From last time:

frustrated bonds $\longrightarrow$ dimer representation $\longrightarrow$ "magnetic fields" B_{ij}

When B_{ij} satisfies $\sum_i B_{ij} = q\,\varepsilon_i$, $(\mathrm{div}\,\vec{B})_i = q\,\varepsilon_i$ $\qquad \varepsilon_i = \begin{cases} +1 & \text{on } A \\ -1 & \text{on } B \end{cases}$

Write $B_{ij} = \bar{b} + b$, where $(\mathrm{div}\,\bar{b})_i = q\varepsilon_i$ and $(\mathrm{div}\, b)_i = 0$

Now coarse-grain b to $\vec{b}(\vec{r})$ $\quad \left[\text{now } \vec{b} : \mathbb{R}^2 \longrightarrow [-1,1]^2\right]$

case (1): $\vec{b}$ does not fluctuate much $\Longrightarrow$ order

case (2): $\vec{b}$ fluctuate substantially.

Then we can write down an effective free energy:

$S[\vec{b}]$, $\beta F[\vec{b}] = \int d^d\vec{r}\ \frac{c}{2}|\vec{b}|^2$ $\quad [c \sim o(1) \text{ number}]$

The constraint $\mathrm{div}\,\vec{b} = 0 \Longrightarrow \vec{b} = \begin{cases} \hat{z} \times \vec{\nabla}\theta & (2+1)\text{-D} \\ \vec{\nabla} \times \vec{A} & (3+1)\text{-D} \end{cases}$

Then, $\langle b_\mu(0)\, b_\nu(0)\rangle = \frac{1}{c}\,\varepsilon_{\mu\lambda}\varepsilon_{\nu\gamma}\,\langle \partial_\lambda\theta(r)\,\partial_\gamma\theta(0)\rangle$

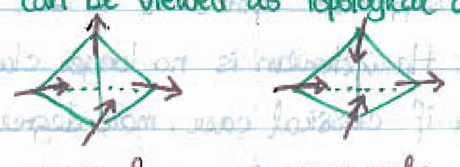
$\sim -\frac{1}{c}\,\varepsilon_{\mu\lambda}\varepsilon_{\nu\gamma}\,\frac{\hat{r}_\lambda \hat{r}_\gamma}{r^2}$ $\qquad$ in (2+1)-D

And similarly $\langle b_\mu(r)\, b_\nu(0)\rangle \sim \frac{-1}{c}\underbrace{(\delta_{\mu\nu} - 3\hat{r}_\mu\hat{r}_\nu)\frac{1}{r^3}}_{\text{dipolar form} \Rightarrow \text{"pinch pt"}}$ $\qquad$ in (3+1)-D

From this into spin correlation $\langle \sigma(r)\,\sigma(0)\rangle$ is difficult ....

REMARK: Power law is cutoff by correlation length $\xi \sim e^{|\theta_{cw}|/T}$, which corresponds to defects in which the ground state constraint is broken

States that break constraint can be viewed as topological defects, e.g. "magnetic monopole" in spin ice:



normal $\qquad\qquad$ monopole

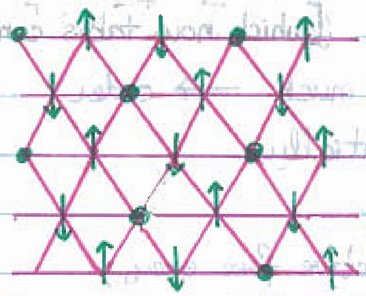
- Quantum Ising can be introduced by anisotropy or transverse fields:

$$\mathcal{H} = \frac{1}{2}\sum_{ij} J_{ij}\left(\sigma_i^z\sigma_j^z + \alpha(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y)\right) - h\sum_i \sigma_i^x$$

such that $[\sigma_i^z, \mathcal{H}] \neq 0$. $\Longrightarrow$ quantum fluctuation at $T=0$

▲ The problem at $T=0$ can be solved by degenerate perturbation theory. e.g., in transverse field case,

$$\mathcal{H}_{eff} = \hat{P}_{GS}(-h\textstyle\sum_i \sigma_i^x)P_{GS}$$ → projector onto ground state



• = superposition of ↑ & ↓

Hence quantum fluctuation $\Longrightarrow$ long-ranged order

▲ In XXZ model, $\mathcal{H}_{eff} \approx P\alpha(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y)P$

$\Longrightarrow$ supersolid with similar order as above, $\langle\sigma_i^+\rangle \neq 0$

▲ For 3D pyrochlore under XXZ, need 3rd order perturbation:

$$\mathcal{H}_{eff} \sim (PVQVQVP)/|\hbar\omega| \quad [Q = 1-P]$$

It turns out that $\mathcal{H}_{eff} \approx \int d^3\vec{r}\left[\frac{c}{2}|\vec{b}|^2 + \frac{\tilde{c}}{2}|e|^2\right]$

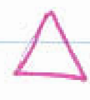
G.S. preserving operation — break G.S. manifold $[e_\mu, b_\nu] = i\delta_{\mu\nu}$

$\mathcal{H}_{eff}$ does not order but has gapless photon $\Longrightarrow$ quantum spin liquid

- Heisenberg Magnets: $\mathcal{H} = \frac{1}{2}\sum_{ij} J_{ij}\vec{S}_i\cdot\vec{S}_j$

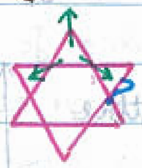
▲ Usually occurs in half-filled d-shell (or half filled e_g, t_{2g}), which suppresses spin-orbit to 2nd order.

▲ The Hamiltonian is no longer classical

▲ Even if classical case, more degree of freedom $\Longrightarrow$ more choices to minimize energy. e.g., triangular  $\Sigma\theta = 0$

▲ However, there is also groundstate degeneracy:

$\Longrightarrow$ 2+1 parameters



$\Longrightarrow$ ground state entropy $\propto N$

But as $T \to 0$ spin becomes long-ranged order ($\sqrt{3} \times \sqrt{3}$ state coplaner)

▲ Luttinger — Tisza method for classical ground state.

▸ Idea: $|\vec{S}_i| = s$ makes energy mimization difficult.

Instead may minimize w.r.t. $\sum |\vec{S}_i|^2 = NS^2$, then assemble these into a state satisfying original constriant

▸ Then, can use Lagrage multiplier:

$$\mathcal{F} = \sum \left[\frac{1}{2} J_{ij} \vec{S}_i \cdot \vec{S}_j - \frac{\lambda}{2}(|S_i|^2 - s^2) \right] \Longrightarrow J_{ij} \vec{S}_j = \lambda \vec{S}_i$$

▸ In such case we have Bloch's thm:

$$\vec{S}_i = u_i \vec{n}\, e^{i \vec{q} \cdot \vec{r}_i}$$

▸ For Bravais lattice, $\vec{S}_i = s\big(\underbrace{\hat{e}_1 \cos(\vec{q}\cdot\vec{r}) + \hat{e}_2 \sin(\vec{q}\cdot\vec{r})}_{\text{coplaner spiral}}\big)$ $\left[\hat{e}_1^2 = \hat{e}_2^2 = 1,\ \hat{e}_1 \cdot \hat{e}_2 = 0 \right]$

▸ This gives energy bands $\mathcal{E}_n(\vec{q})$. The system is degenerate iff minima of $\mathcal{E}_n(q)$ are degenerate.

Examples:

lattice	minima
FCC	lines
diamond	surfaces
kagome	flat

▲ Need to know the quality of classical picture.

▸ Simplest approach —— spin-wave theory ($\sim 1/s$ expansion)

For triangular lattice, $M_s = |\langle S_i \rangle| = s - 0.2613 + \frac{0.0055}{s} + \dots$

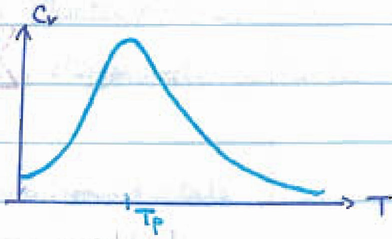
Thus for $s > 1$ the system is ordered & spin-wave is good approximation.

▸ Alternatively can use numerical approaches, e.g. exact diagonalization, $\underbrace{\text{DMRG}}_{\text{mostly 1D, now beyond}}$, $\underbrace{\text{QMC}}_{\text{sign problem}}$, series expansions

- For triangular system of $s = 1/2$, $M_s \approx 0.205 \pm 0.015$, and exact diagonalization indicates long-ranged order.

▲ However, for intermediate s at $T > 0$, spin wave does not provide good answer.

e.g. consider spin-s on triangular lattice



Let $\tau_p = T_p / |\Theta_{cw}|$

$\tau_{s\to\infty} = 0.17$

$\tau_{s=1/2} \approx 0.27$

▲ Spin-wave theory for frustrated magnets is non-trivial because of classical degeneracy.

- naively there are zero-energy spin-wave modes $\Longrightarrow$ "small denominator".
- To circumvent one needs non-linear SWT, in which the corrections are non-analytic in $1/s$.
- e.g. $M_s \approx \begin{cases} s - c \ln s & \text{for FCC} \\ s - c\, s^{\alpha} \quad [0 < \alpha < 1/2] & \text{for kagome/pyrochlore} \end{cases}$