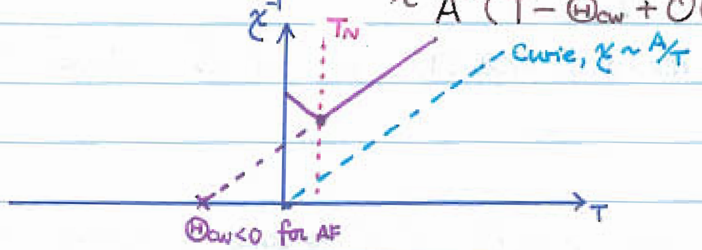
Frustrated Magnets (I) [Balents]

- Focus on system with well-defined local moments.
  - ▲ approx. isolated ions with partially filled shell.
  - ▲ Magnetism arise from Hund's rule, e.g. Mn^{2+} $\uparrow\uparrow$ e_g, $\uparrow\uparrow\uparrow$ t_{2g}
  - ▲ Local atomic physics (ionization state, crystal field, spin-orbit) allows for wide variety of magnetism.
  - ▲ Experimentally, magnetism is detected through:
    - (1) electrically insulating, $\rho \sim e^{E_g/T}$
    - (2) Curie susceptibility $\chi \sim \dfrac{A}{T-\Theta_{cw}}$, $A = \dfrac{N g^2 \mu_B^2 S(S+1)}{3k_B}$

$$\chi^{-1} \sim A^{-1}\left(T - \Theta_{cw} + \mathcal{O}(1/T)\right)$$



  - ▲ For high-temperature Heisenberg MFT,

$$\mathcal{H} = \frac{1}{2}\sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j \implies \Theta_{cw} = \frac{-(\sum_j J_{ij})\, S(S+1)}{3k_B}$$

  - ▲ For simple cubic, $T^{MF} = |\Theta_{cw}|$
  - ▲ Empirically, frustration occurs if (Ramirez):

$$f = |\Theta_{cw}|/T_N \gg 1. \qquad (\text{e.g. } \sim 5\text{–}10).$$

- Conceptually, frustration $\longleftrightarrow$ competing interactions
  - ▲ Cannot minimize $J_{ij}\, \vec{S}_i \cdot \vec{S}_j$ simultaneously.
  - ▲ Many states are as good as each other.
    (i.e. many approx. degenerate ground state)
  - ▲ The disordered phase at $T_N < T < |\Theta_{cw}|$ is often referred to as "spin liquid".

- Interesting Features of Frustrated Magnets
  - ▲ Emergent low energy scale $T_N \ll |\Theta_{cw}|$
    - $\Longrightarrow$ interesting (universal ?) low-energy physics at spin liquid ph
  - ▲ Degeneracy — many (approx) ground states
    - $\Longrightarrow$ system is very sensitive to interactions.
    - $\Longrightarrow$ perturbation are always strong in the degenerate manifold (c.f. quantum Hall in 2DEG under external field)
    - $\Longrightarrow$ systems can be tuned by weak (lab) fields, hence is controlable.
    - $\Longrightarrow$ interesting states (e.g. non-collinear magnetism, dimer and other non-magnetic states) and phenomena (reduced dimensio can be found.

- Start with simple Ising model:

$$\mathcal{H} = \frac{1}{2}\sum_{i,j} J_{ij}\, \sigma_i^z \sigma_j^z \qquad [\sigma_i = \pm 1]$$

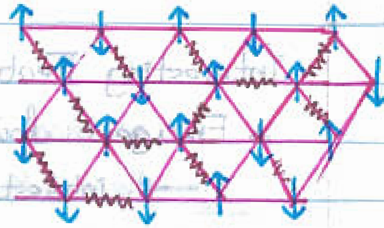
  - ▲ The practical reason why Ising model arise is complex. e.g. need strong single-ion anisotropy (from crystal field + spin orbit)
  - ▲ Partial filled f shells tend to be more Ising like.
  - ▲ In real material, Ising axes are in general different for different spin, i.e. $\vec{\sigma}_i = \hat{n}_i \sigma_i^n$
  - ▲ Moreover, exchange is weak $\Longrightarrow$ dipolar interaction is strong.

  true in "spin ice"

  - ▲ Since $[\sigma_i^z, \mathcal{H}] = 0$, the Ising model is essentially classical. To add quantum effects:
    - (1) Weaker anisotropy: $\Delta\mathcal{H} = \frac{1}{2}\sum_{ij} J_{ij}^{xy}(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+)$
    - (2) Adding transverse field: $\Delta\mathcal{H} = \sum_i (h_i^+\sigma_i^- + h_i^-\sigma_i^+)$
  - ▲ Consider triangular geometry:

    - $\Longrightarrow$ 6 ground states in 3-site model.

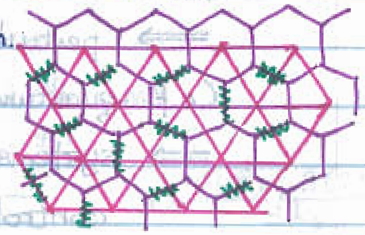
▲ Focus on triangular anti-ferromagnet:



It's known that the number of ground states is exponentially large:

$$\Omega = e^{S/k_B} \; ; \; S = 0.34 N k_B \quad \text{[Wannier 1950]}$$

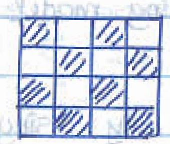
We may construct the dimer representation. Label a dimer link if it cross a bond in direct lattice with frustrated spins.

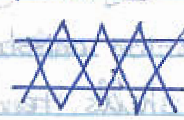


▲ There is at least one frustrated bond per sites. So naively $S \sim 3^N$. But this is over-counting:

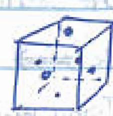
Thus, $S \sim \left(\left(\frac{2}{3}\right)^2 \frac{1}{3} \cdot 3\right)^N \approx (4/9)^N$

▲ Other examples:

(1) Checkerboard  $S = \frac{3}{4}\left(\ln \frac{4}{3}\right) N k_B \approx 0.216 N k_B$

(2) kagome  $S \approx 0.5 N k_B$

(3) pyrochlore  $S \approx 0.203 N k_B$

(4) FCC  $S \approx c N^{1/3} k_B$

• Pyrochlore system is related to spin ice.

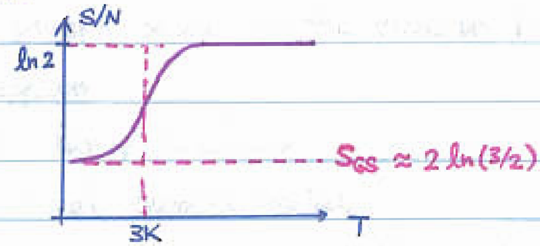
e.g. $Dy_2Ti_2O_7$; $Ho_2Ti_2O_7$

These have strong local Ising anisotropy along (111) axis connecting tetrahedra.

▲ In spin-ice the strongest exchange is nearest neighbor ferromagnet

▲ The best state is to have 2 spins in and 2 spins out

▲ The entropy can be measured (Ramirez et. al.) by $\int_{T_0}^{T} \frac{C}{T} dT = \Delta S$



$|\Theta_{cw}| = 0.5K$

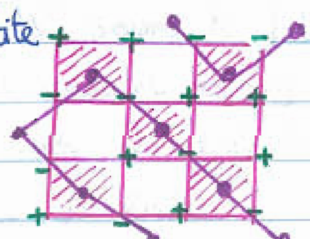
- General Comparison:

lattice	T_N	Correlation at $T \ll |\Theta_{cw}|$
FCC	1.6 J	long-ranged
checkerboard	0	power law
triangular	0	power law
pyrochlore	0	power law
kagome	0	very short ranged

} non-trivial classical spin liquid (checkerboard, triangular, pyrochlore)

- For the intermediate states, power law can be understood as:
  (1) ground state has a dimer representation
  (2) dual lattice is bipartite

▲ NOTE: for checkerboard



2 links per ▨

$\Longrightarrow$ loops are closed

▲ These ground state have a representation similar to magnetic field lines. $n_{ij} = \begin{cases} 1 & \text{if dimer} \\ 0 & \text{no dimer} \end{cases}$

And bipartite allow us to define $b_{ij} = \begin{cases} n_{ij} & i \in A,\ j \in B \\ -n_{ij} & i \in B,\ j \in A \end{cases}$

▲ Then dimer gives constraint:

$$\underbrace{\sum_j b_{ij}}_{= (\mathrm{div.}\,\vec{b})_i} = q\,\varepsilon_i \qquad \varepsilon_i = \begin{cases} +1 & i\in A \\ -1 & i\in B \end{cases}$$

$\left[\begin{array}{ll} q=1 & \text{for triangular} \\ q=2 & \text{for checkerboard} \end{array}\right]$