Heisenberg Magnets

In many cases, with half-filled shells especially, one obtains isotropic spins w/ Heisenberg interaction

$$H = \frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Even classically, frustration is rather different than in Ising systems.

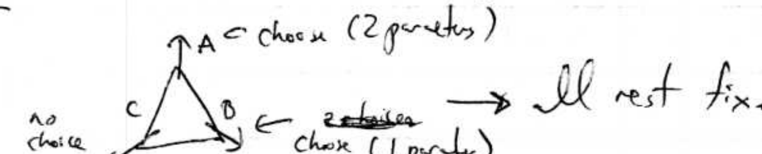
$\sum_i \vec{S}_i = 0$ can be achieved on a triangle

c.f. non-collinear Mostovoy

of course $\vec{S}_i \cdot \vec{S}_j \neq -1$

Still, one can find significant <u>ground state degeneracy</u> in some cases.

<u>Triangular AF</u> $\rightarrow$ "No" G.S. degeneracy



ordered at $T=0$. No order $T>0$ due to Mermin-Wagner

<u>Kagome</u>

still have choices!

turns out to have extensive entropy

However $\rightarrow$ orders as $T \rightarrow 0$ due to "order by disorder" phenomena.

~~Note presence of~~

A nice method which is useful "Luttinger-Tisza" method.

Idea: Minimize H relaxing constraint $|S_i| = S$.
instead just force $\sum_i |S_i|^2 = NS^2$

Guaranteed to get energy $E_{min} \leq E_0$ of $|S_i| = S$ states.

If you can find such a sol. which then satisfies $|S_i| = S$, you are done.

$$\text{Min} \quad \sum_{ij} \frac{1}{2} J_{ij} \vec{S}_i \cdot \vec{S}_j \; - \overset{\text{Lagrange}}{\frac{\lambda}{2}} \sum_i \left(|S_i|^2 - S^2 \right)$$

Quadratic! $\quad J_{ij} \vec{S}_j = \lambda \vec{S}_i \quad$ eigenvalues problem for J_{ij}!

energy $= N \lambda S^2 \rightarrow$ want min eigenvalue.

w/ Translational Invariance, eigenvectors satisfy Bloch Form

$$\vec{S}_i = e^{i\vec{q}\cdot\vec{r}_i} u_i \hat{n}$$

$\vec{n}$ arbitrary
u_i depends only upon basis site (i.e. periodic)

energies $\lambda \rightarrow \varepsilon_n(\vec{q})$ bands.

$\rightarrow$ Look for band minima.

* In general, for Bravais lattice, can always satisfy $|S|^2 = S^2$ constraint by

$$\vec{S}_i = S\left(\cos \vec{q}\cdot\vec{r}\; \hat{e}_1 + \sin \vec{q}\cdot\vec{r}\; \hat{e}_2 \right) = S \,\mathrm{Re}\left(\vec{d}\, e^{i\vec{q}\cdot\vec{r}} \right)$$

$$\vec{d} = \hat{e}_1 + i\hat{e}_2$$

These are coplanar spirals. → another reason for ferroelectricity!

In some other circumstances, can make this work for non-Bravais lattices. Some examples of band minima

kagome	1 flat band
pyrochlore	2 " ",
FCC	minima along lines
diamond $J_2 \geq J_1/8$	" along surface

OK, but how good is this classical picture?
Simplest gauge is spin-wave theory ($1/S$ expansion)

e.g. triangular lattice: ordered moment ($T=0$)

$$M = |\langle \vec{S} \rangle| = S - 0.2613 + 0.0055/S$$

$$= \begin{cases} 0.7442 & S=1 \quad (74\%) \\ 0.2497 & S=\frac{1}{2} \quad (\simeq 50\%) \end{cases}$$

Evidently for $S > 1$ fairly clearly good, and perhaps not so bad for smaller S?

~~* In fact, strong evidence from Exact Diago~~

Other methods?

* Numerics — Exact Diagonalization (size limited)
  - DMRG (~~...~~ mostly 1d...)
  - Series expansion (biased)
  - QMC (Sign Problem)

Recent DMRG, Series, $M_{S=\frac{1}{2}\Delta} \simeq 0.205 \pm 0.015$
(ED also strongly suggests LRO)

Strongly suggests SW good for Isotropic triangle AF

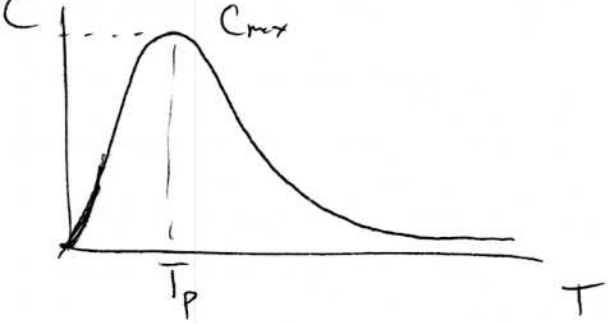
* Other approaches? Talk more 3rd Lecture
  - Slave-Particles : MFT + Variational WFs
  - Exact Solutions (1d + special cases)

Turn now to survey of experiments I find interesting. Start w/ better understood & move to less understood.

~~Finite~~ T > 0 Properties? How good is classical?

e.g. ~~Q:~~ ~~How does~~ Where is specific heat peak?

Spin-S tri-lat



$$\tau = \frac{T_P}{|\Theta_{CW}|} \qquad \tau(S\to\infty) \simeq 0.17$$

Classical MC

Q: Is τ larger or smaller for small S?

(Answer for $S=\frac{1}{2}$ known from Series exp.) $\tau_{1/2} \simeq 0.27$