Plan: (1) Intro / Ising magnets (2) Heisenberg systems (3) Quant Spin Liquids

~~Mott~~

Frustrated Magnets

- Magnets → I will focus on nearly localized e^-'s
  $\approx$ atomic eigenstates w/ weak overlap

  usually Mott Insulators

  - Get magnetism when partially filled shells

    e.g. Mn^{2+} $S = 5/2$

    lots of complexity + variation due to many different atomic conditions

- How do you know?
  - Usually good insulator $\rho \sim 1/T^\alpha$
  - Curie Law at high-Temp

    $$\chi \sim A/T \qquad A = \frac{N g^2 \mu_B^2 S(S+1)}{3 k_B} \quad \text{Curie Constant}$$

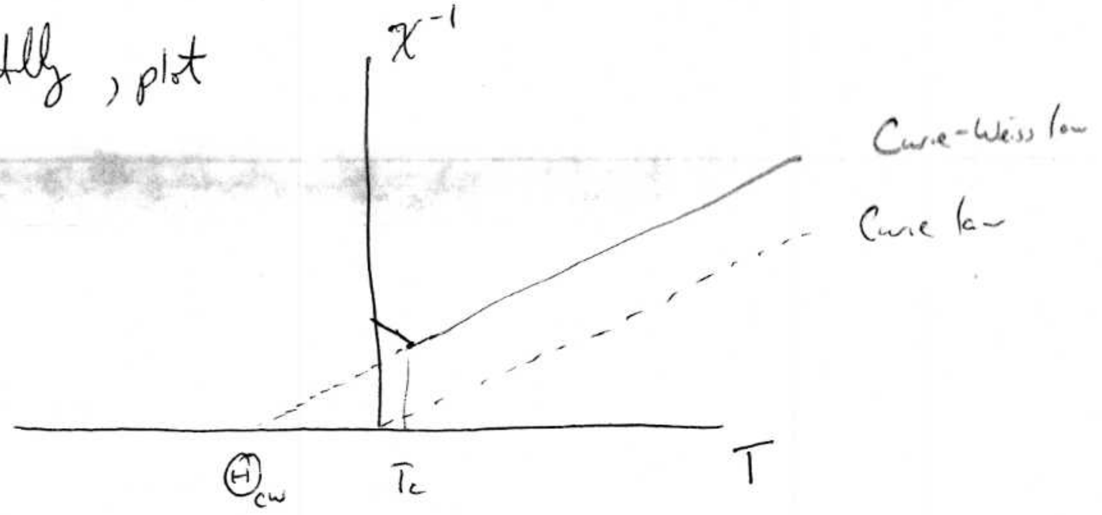
    signature of free moments.

- Interactions between spins e.g. $H = \frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$

  (note I choose $J > 0$ for AF)

  - AF interactions → generally suppresses χ.

Experimentally, plot



$$\chi \sim \frac{A}{T - \Theta_{cw}} \qquad \Theta_{cw} < 0 \quad \text{for AFs}$$

- This form is obtained in MFT above T_c
- Also in high-T expansion

$$\chi^{-1} \sim A^{-1}\left(T - \Theta_{cw} - O(1/T)\right)$$

$$\Theta_{cw} = -\frac{\left(\sum_j J_{ij}\right) S(S+1)}{3 k_B}$$

- $|\Theta_{cw}|$ Gives a measure of strength of magnetic interactions

  - For ~~unfrustrated~~ e.g. ~~sq~~ cubic lattice AF,
  $$|\Theta_{cw}| = T_c^{MF}.$$

- "Frustration Fingerprint"

$$f = \frac{|\Theta_{cw}|}{T_c} \gg 1 \longrightarrow \text{ordering is anomalously suppressed. "frustrated"}$$

empirically $f \gtrsim 5$-10 usually accepted as frustrated.

Why $f \gg 1$?

- Frustration $\hat{=}$ competing interactions

→ Cannot satisfy all $J_{ij} \vec{S}_i \cdot \vec{S}_j$ terms simultaneously

→ Get many almost equally good compromises

= "degeneracy"

→ System fluctuates between these degenerate states instead of ordering — Quantum / Thermal ?

Before getting to any details, talk about why this is interesting

- Emergent low energy scale $T_c \ll \Theta_{cw}$

  - Expect some new (universal?) behavior $T_c < T < \Theta_{cw}$?
  - Here spins are correlated but not ordered. "spin liquid"

- Quasi-degeneracy

  - Perturbations that break degeneracy are only intrinsic energy scale

    → Similar to FQHE → project V_c into LLL

  - Very sensitive
    - relatively small effects can control G.S.
    - controllable ?

- Interesting states arise
  - non-collinear magnetic order (c.f. Mostovoy)
  - non-magnetic states: dimers, spin-liquids
  - reduced dimensionality

Ising Magnets

theory $\vec{S}_i \longrightarrow \sigma_i^z \equiv \sigma_i \qquad \sigma_i = \pm 1$ (

practice: need strong ~~single-ion anisotropy~~ single-ion anisotropy
{crystal field + spin-orbit} effects

most Ising-like $\rightarrow$ f e^-'s in rare earth elements.

but also some d e^-s in asymmetric environments

~~Difficulty: one of Ising-like~~

Difficulties

- Ising axes often not collinear
  $\vec{S}_i \longrightarrow \hat{n}_i \sigma_i \qquad \hat{n}_i$ different for different sites
- For f e^-'s, dipolar interactions important.

(both effects are present in famous "spin ice" materials I will discuss)

NN Ising AFs

$$H = J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

Note: this is purely classical problem since $[\sigma_i^z, \sigma_j^z] = 0$.

QMs arises if one includes transverse components, e.g.

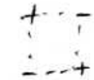
$$J_\perp \left(S_i^x S_j^x + S_i^y S_j^y \right)$$

terms.

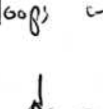
or $\quad h_\perp S_i^x$

Let's ignore for now.

On bipartite lattice, H has unique G.S. up to symmetry

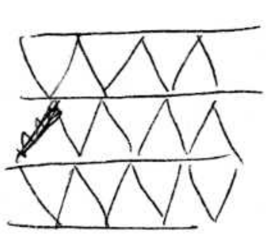


If odd-site loops are present, get frustration

 $\longrightarrow$ $\begin{matrix} & - & \\ + & & + \end{matrix} \quad \begin{matrix} & + & \\ - & & - \end{matrix}$ + perms

$\longrightarrow$ 6 G.S. of triangle

Triangular (hexagonal) lattice



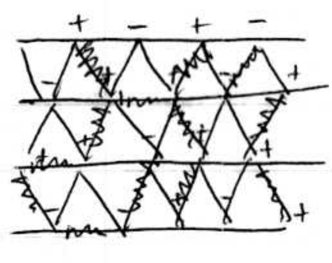
one ~~frustrated~~ dissatisfied bond per triangle.

sometimes call these frustrated bonds.

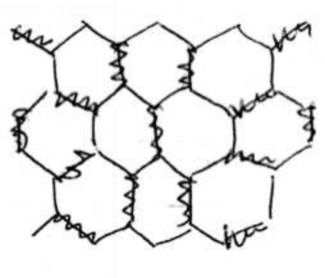
The number of Ground States is exponential in the # of spins!

$$\Omega = e^{S/k_B} \qquad S \simeq .34 k_B N \quad \left(\begin{array}{l}\text{Wannier}\\ \text{Houtappel}\end{array} 1950\right)$$

Picture $\rightarrow$ color dissatisfied bonds



Draw dual lattice $\longrightarrow$ color bonds passing through dissatisfied bonds on direct lattice



"hardcore" dimer covering
= 1 dimer / site
(= 1 bad bond / $\triangle$)

Fairly clear here that G.S. entropy is extensive.

Actually we can make a crude estimate.

Consider these dual sites. Y $\rightarrow$ 3 configurations

$$\rightarrow 3^{N_Y} = 3^{N} \text{ states.}$$

But must be sure that λ are ok $=$

Prob ($\wr$) = $\frac{1}{3}$ Prob (|) = $\frac{2}{3}$ So Prob (λ = good) = $\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot 3 = \frac{4}{9}$

total # of states =

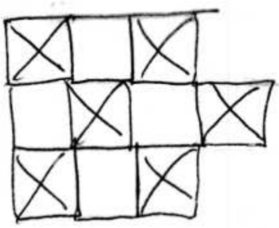
$$\Omega \simeq 3^N \left(\frac{4}{9}\right)^N = \left(\frac{4}{3}\right)^N = e^{N \ln 4/3} \simeq e^{S/k_B}$$

$$S \simeq .29\, N k_B$$

Other lattices w/ frustration

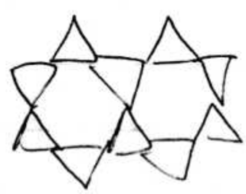
(Lieb)

Checkerboard



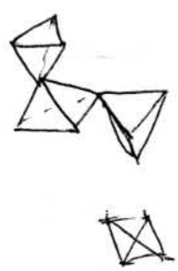
$$S = N k_B \left(\tfrac{3}{4} \ln \tfrac{4}{3}\right) \simeq 0.216\, k_B N$$

kagome



$$S = 0.502\, N k_B$$

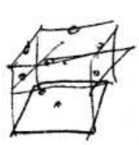
pyrochlore



hard to draw

$$S \simeq 0.203\, N k_B$$

fcc



essentially can choose 2 states for each (100) plane

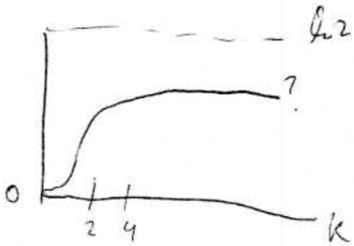
$$S \simeq \text{const}\; k_B N^{1/3}$$

(!)

Aside on spin ice

$\theta_{cw} \simeq .5K$

$\Delta S = \int_{T_0}^{T} \frac{C}{T}\, dT$



Ramirez

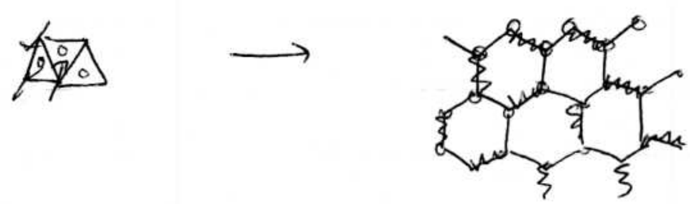
$Dy_2Ti_2O_7$

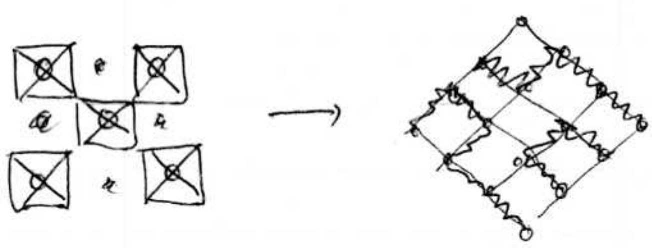
What happens at $T > 0$? Effect of thermal fluctuations

Model	Transition	Correlations ($T << \Theta_{cw}$)
fcc	yes! $\tilde{T} \approx .155$	LRO*
kagome	no	very short-range
triangular	no	power law
checkerboard	no	"
pyrochlore	no	"

<u>Trend</u>: less entropy → more ~~ordered~~ correlated

? What is nature of power-law correlations in these cases?
- Use Dimer Picture

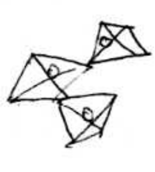
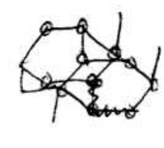
Recall  checkerboard

Checkerboard  square

 2 up/2 down spins per

 ~~dimer~~ + → dimer

2 dimers/site

Pyrochlore  + → dimer  diamond

2 dimers/site

Common feature → G.S.s are represented as coverings of a bipartite lattice with a fixed # $q=1,2$ of dimers per site.

Simple way to think about fixed # constraint → mapping to magnetostatics.

$n_{ij} = 0, 1$: # of dimers on bond ij

define $$B_{ij} = \begin{cases} n_{ij} & i\in A \text{ sublattice},\ j\in B \\ -n_{ij} & i\in B \quad '' \quad ,\ j\in A \end{cases}$$

B_{ij} is an integer valued vector field

$$B_{ij} = +1 \qquad\qquad B_{ij} = -1$$

Constraint $$\sum_j B_{ij} = q\,\varepsilon_i \qquad \varepsilon_i = \begin{cases} +1 & i\in A \\ -1 & i\in B \end{cases}$$

This is lattice equivalent of $(\operatorname{div} B)_i = q\,\varepsilon_i$

Fluctuations of B_{ij} are constrained to be dipole only.

$$B_{ij} = \bar{B}_{ij} + b_{ij} \qquad (\operatorname{div} \bar{B})_i = q\,\varepsilon_i$$

$\operatorname{div} b = 0$

Fluctuations of b are similar to those of classical magnetostatics

Consider "coarse-grained" spatially averaged $\vec{b}$

- either discreteness of b_{ij} is important $\rightarrow$ LRO
  b does not fluctuate significantly

or

- $\vec{b}$ can be regarded as a continuous field $\rightarrow$?

Then $\beta F[b] \propto \int d^d r \; \frac{c}{2} |\vec{b}|^2 + \text{h.o.t.'s}$

$$\operatorname{div} \vec{b} = 0 \longrightarrow \quad \vec{b} = \begin{cases} \hat{z} \times \vec{\nabla}\phi & d=2 \\ \vec{\nabla} \times \vec{a} & d=3 \end{cases}$$

(N.N. $T \to 0$ no intrinsic energy scale $\to c = O(1)$)

$$\beta F \propto \begin{cases} \int d^2 r \; \frac{c}{2} |\nabla \phi|^2 \\ \int d^3 r \; \frac{c}{2} |\nabla \times a|^2 \end{cases}$$

both cases $\rightarrow$ power-law "dipolar" correlations

e.g. 2d $\quad \langle b_\mu(r) b_\nu(r') \rangle \sim \varepsilon_{\mu\lambda} \varepsilon_{\nu\gamma} \frac{\partial}{\partial x_\lambda} \frac{\partial}{\partial x'_\gamma} \underbrace{\langle \phi(r) \phi(r') \rangle}_{\propto \ln |r-r'|}$

$$\langle b_\mu(r) b_\nu(0) \rangle \sim -\varepsilon_{\mu\lambda} \varepsilon_{\nu\gamma} \frac{\hat{r}_\lambda \hat{r}_\gamma}{r^2}$$

3d $\quad \langle b_\mu(r) b_\nu(0) \rangle \sim -\frac{(\delta_{\mu\nu} - 3\hat{r}_\mu \hat{r}_\nu)}{r^3}$

This form also implies "pinch points" in magnetic structure factor, which reflect singular dipolar dependence in q-space

Hints of this structure (certainly consistent with it) have been seen in neutron scattering on a few materials such as $Dy_2Ti_2O_7$, $ZnCr_2O_4$

? How can this power-law correlation exist w/o a phase transition to the high-T phase, where spin correlations are surely exponential?

A: They are always exponential beyond some length ξ, set by deviations from constraint that spins lie in G.S. configurations.

For $T \ll \Theta_{cw}$, costs energy of $O(J) \sim O(\Theta_{cw})$.

So get $\xi \sim e^{c(\Theta_{cw}/T)}$ very long.

These defects (e.g. tetrahedron with  state) appear as points where

$$(\operatorname{div} B)_i = \pm 1 \rightarrow \operatorname{div} \vec{b} \sim \pm \delta(r)$$

"magnetic monopole"

In spin ice, these have a real physical magnetic charge.

Other stories

• Quantum effects

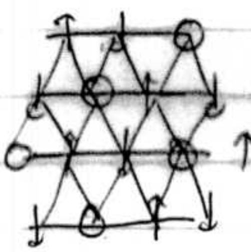
~~• No clear indication of these in expts.~~

• Theoretical studies of

① Transverse field Ising Models $H = \frac{1}{2}\sum_{ij} J_{ij}\sigma_i^z\sigma_j^z - \sum_i h\sigma_i^x$

now $[\sigma_i^z, H] \neq 0 \rightarrow$ quantum fluctuations.

→ leads to "order by disorder" for $h>0$ in the 2d models with classical power-law correlations.



(Moessner – Sondhi)

→ Can be understood also from dipolar picture of G.S. manifold.

(I am not clear on where/how one might arrange such a model experimentally)

② XXZ models

$$H = \frac{1}{2}\sum_{ij} J_{ij}\left(\sigma_i^z\sigma_j^z + \alpha(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y)\right)$$

$|\alpha| << 1.$

These have been studied for triangular, kagome, some other cases beyond N.N. models.

$\alpha < 0$ FM XY exchange has no sign problem in QMC

e.g. Δ lattice → get "Super-Solid" order with same z-axis pattern as above but also XY magn. non-zero.

XXZ on pyrochlore lattice

Hermele et al
YBKim, Isakov..
Banerjee

→ evidence favors **QSL state** for small α.

in which spins form superposition of classical G.S.'s.

(I'll probably mention related work on XXZ in a field which is relevant to ACr_2O_4 spinels) in a later lecture.

→ In magnetic analogy, XY terms induce an extra term

$$\mathcal{H}_{eff} = \int d^3r \left\{ \frac{c}{2}|b|^2 + \frac{\tilde{c}}{2}|e|^2 \right\}$$

where $[e^\mu, a^\nu] = i\delta^{\mu\nu}$

~~electrodynamics~~ QSL = "Coulomb Phase" of this gauge theory.

· Effects of perturbations

- "Classical" perturbations can destabilize these spin liquid states by breaking the degeneracy. → get $T_c > 0$

- For $f = \frac{|\Theta_{cw}|}{T_c} >> 1$, can well approx. behavior by assuming rigid constraint

→ Ordering transitions out of spin liquid state

* Argue these are unconventional