Talk 2

# Nernst effect in vortex-liquid state of cuprates

1. Introduction to the Nernst effect
2. Vortex signal above Tc
3. Loss of long-range phase coherence
4. The Upper Critical Field
5. The cuprate phase diagram

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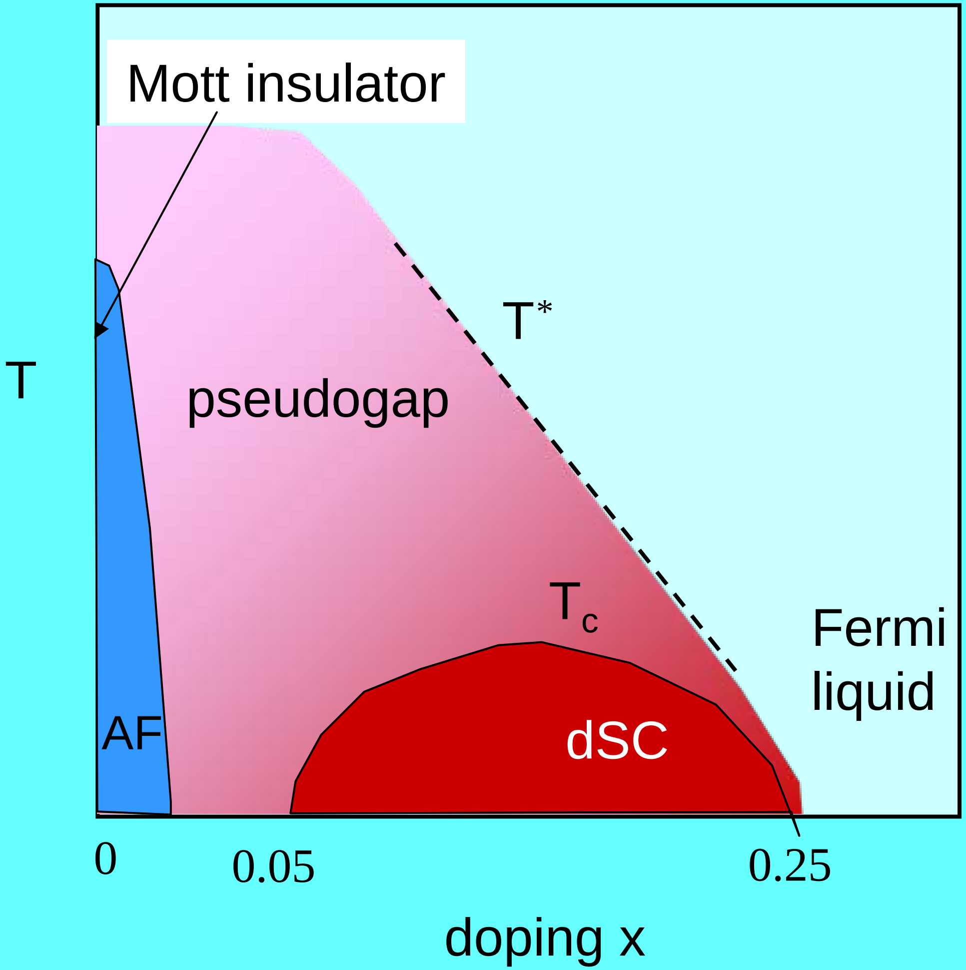
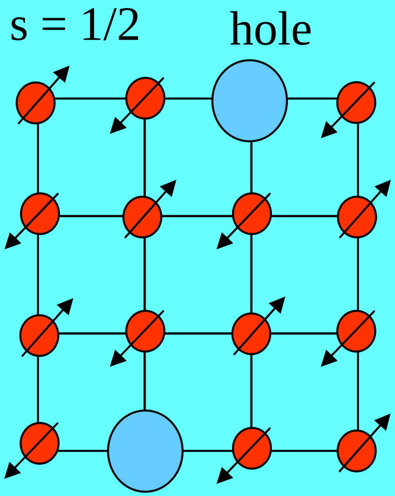
Supported by NSF-MRSEC, ONR

Boulder, July 2008

# Phase diagram of Cuprates

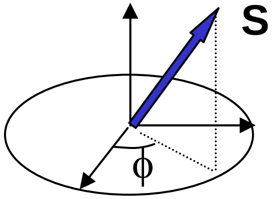




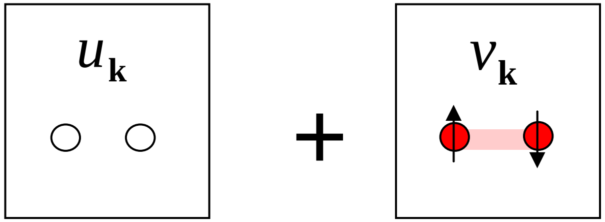
# The *phase* of macroscopic pair-wave function

**BCS wave function**

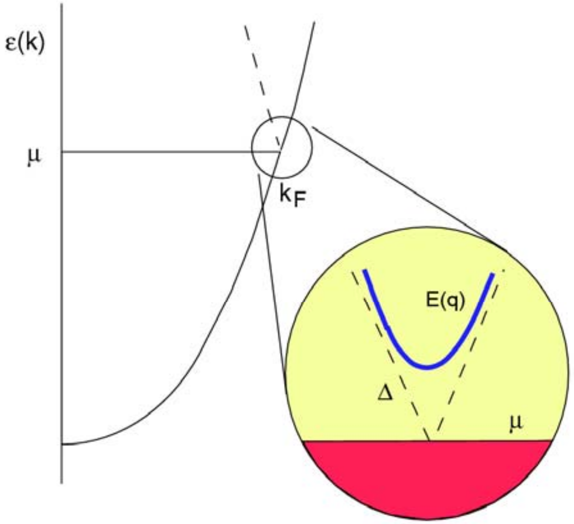
$$\Psi_{BCS} = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + e^{i\phi} v_{\mathbf{k}} c^*{}_{-\mathbf{k}\downarrow} c^*{}_{\mathbf{k}\uparrow}\right) \left|0\right\rangle$$



Anderson
pseudospin



$$u_{\mathbf{k}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + v_{\mathbf{k}} e^{i\phi} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Phase $\phi$ fixed (phase representation); *N* fluctuates

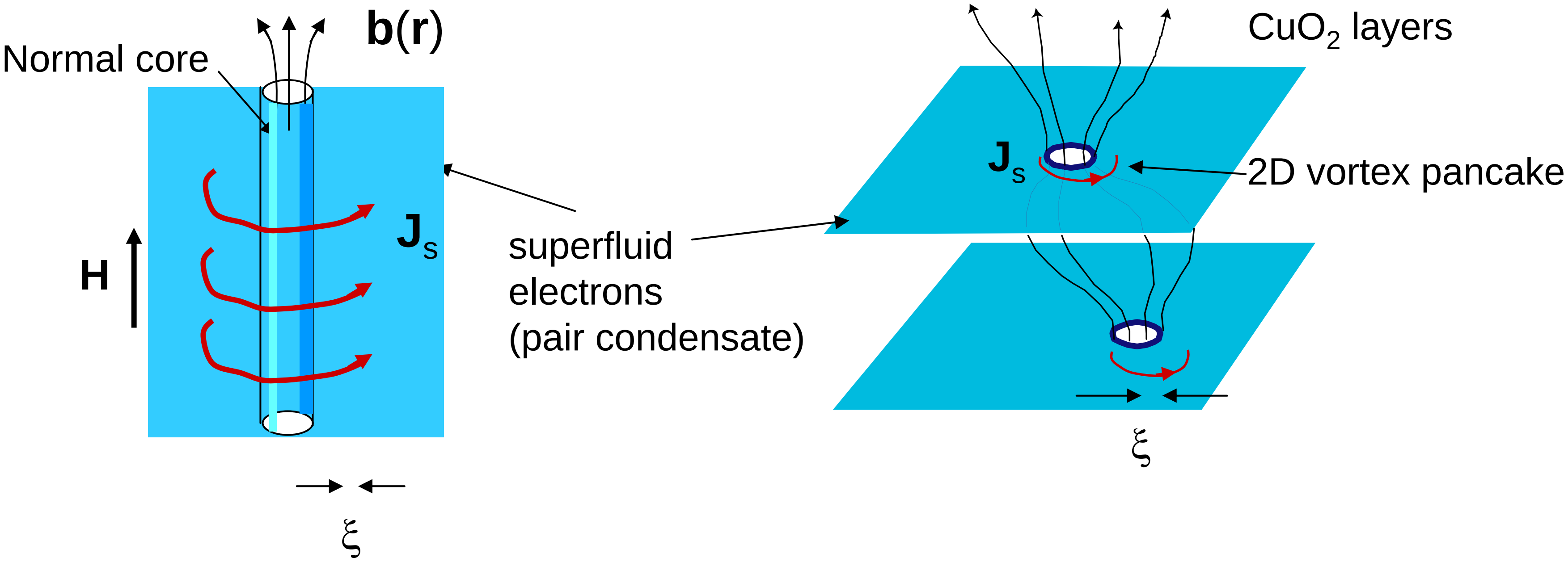
$$\hat{\Psi}(\mathbf{r}) = |\Psi| e^{i\phi(\mathbf{r})}$$

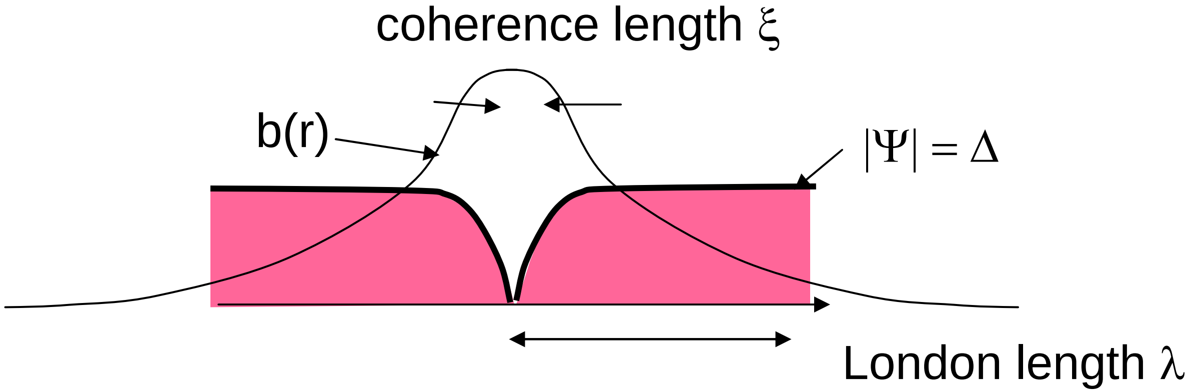
$$[N, \phi] = 1$$

# Vortices in type-II superconductors

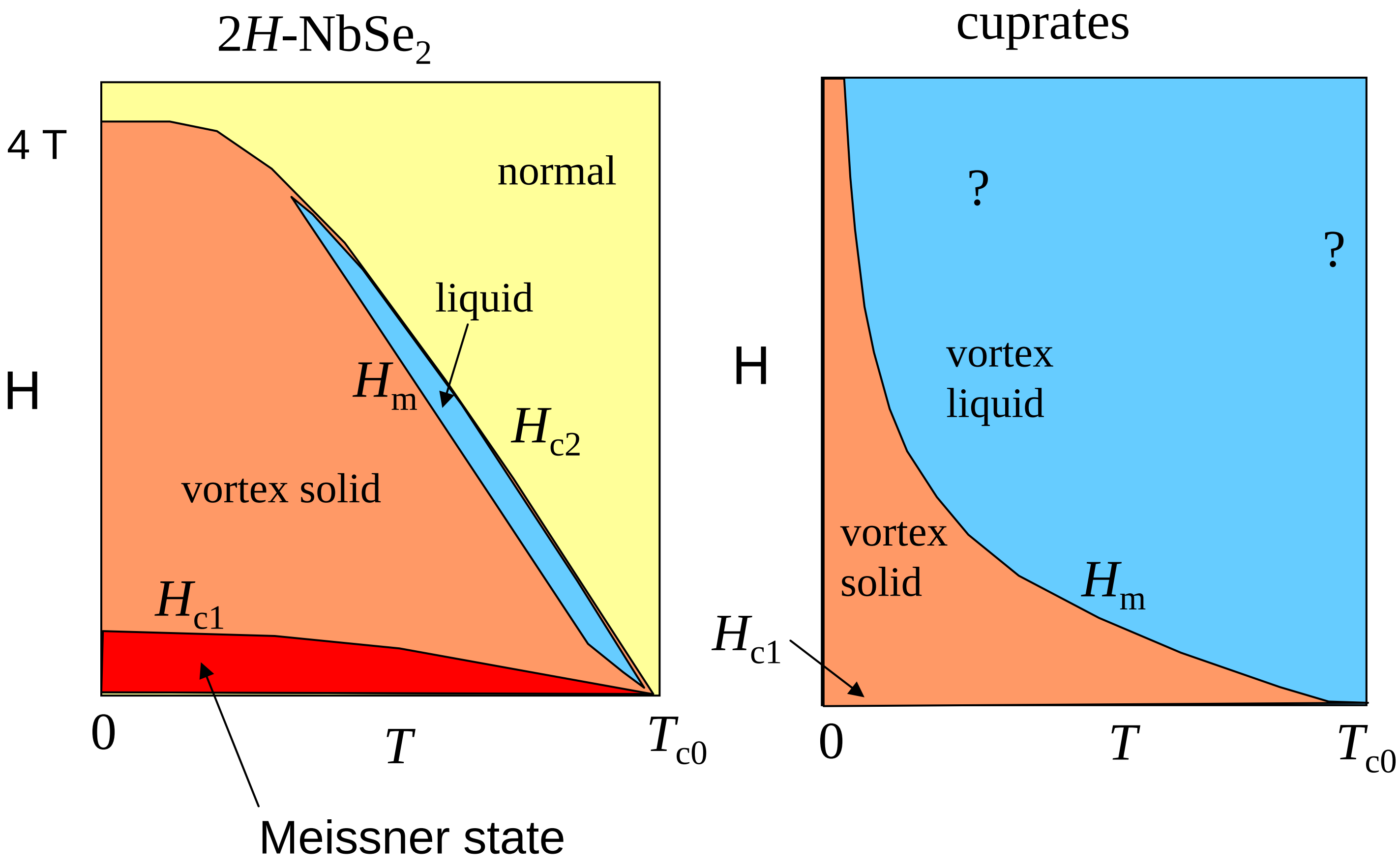
## Vortex in Niobium

## Vortex in cuprates





# Phase diagram of type-II superconductor

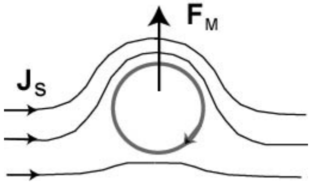


2$H$-NbSe$_2$

cuprates

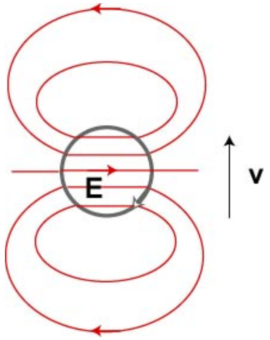
# Vortex motion in type II superconductor

(Bardeen Stephen,  Nozieres Vinen)

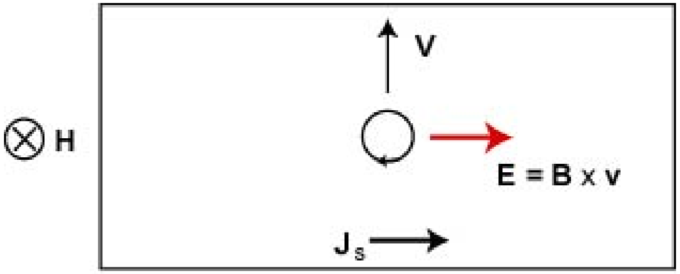


Applied supercurrent $J_s$ exerts magnus force on vortex core

$$\mathbf{F}_M = \mathbf{J}_s \times \vec{\Phi}_0$$



Velocity gives *induced E*-field in core (Faraday effect)
Current enters core and dissipates (damping viscosity)

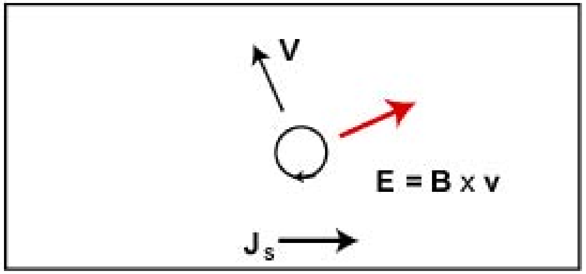


Motion of vortices generates *observed E*-field

$$\boxed{\mathbf{E} = \mathbf{B} \times \mathbf{v}}$$

$$\rho_{xx} = \rho_N \frac{H}{H_{c2}} = B\Phi_0 / \eta$$

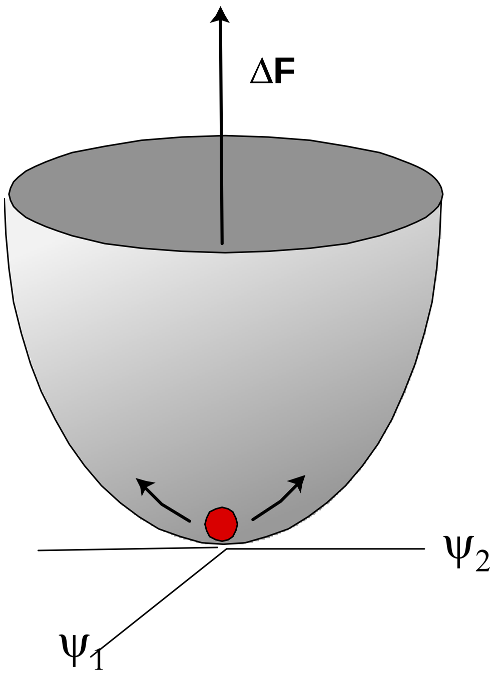
Consequence of Josephson equation



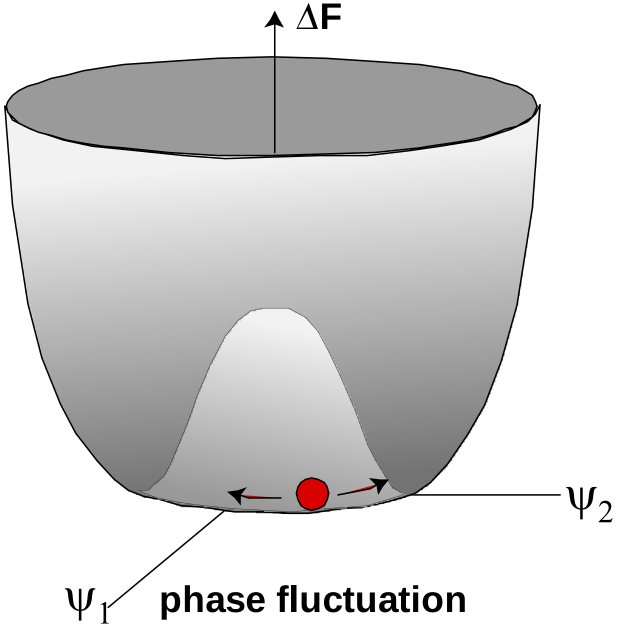
Tilt angle of velocity gives negative vortex Hall effect

In clean limit, vortex v is || - $J_s$

# Anderson-Higgs mechanism and phase rigidity



**amplitude fluctuation**



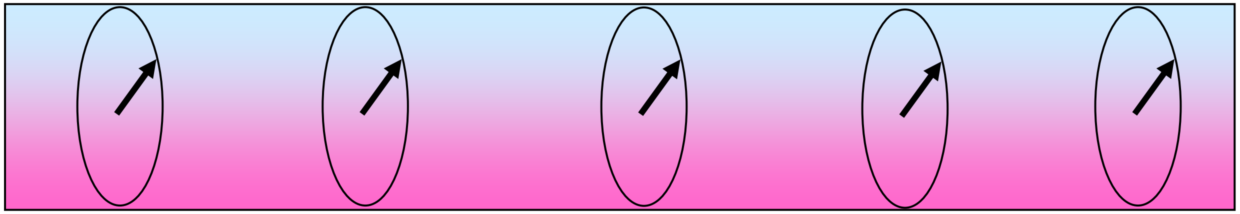
Anderson-Higgs mechanism: ⇨ Phase stiffness
singular phase fluc. (excitation of vortices)

**Phase mode $\theta$ + EM $F_{\mu\nu}$ = Massive mode (Meissner effect)**

P.W. Anderson  Phys. Rev. 1959, RMP 1966

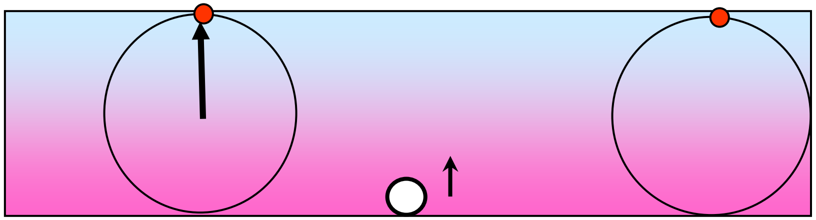
$$|\Psi|\, e^{i\theta(\mathbf{r})}$$

**Phase rigidity → uniform phase $\theta$**



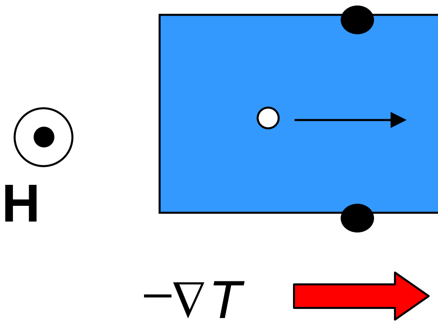
$$H_\rho = \tfrac{1}{2}\int d^3r\, \rho_S (\nabla\theta)^2$$ phase rigidity measured by $\rho_s$

**But phase coherence destroyed by mobile vortices**



$$\Delta\theta = 2\pi$$

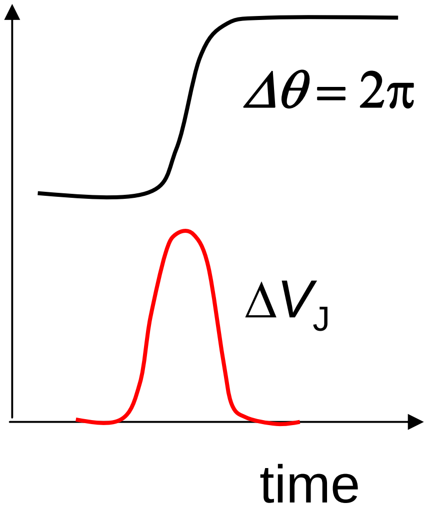
# phase-slip and Nernst signal



**Passage of a vortex →**
**Phase diff. $\theta$ jumps by $2\pi$**

Josephson Eq.

$$2eV_J = \hbar\dot{\phi} = 2\pi\hbar\dot{n}_V$$



**Integrate $V_J$ to give *dc* signal prop. to $\dot{n}_v$**

- Baskaran, Zou, Anderson (Sol. St. Comm. 1987)
- Doniach, Inui (PRB 1989)
- Uemura plot (Nature 1989)

- Emery, Kivelson (Nature 1995)
  *low hole density and high Tc*
  *cuprates highly suscep. to phase fluctuations*

- Corson, Orenstein (Nature 1999)
  *Kinetic inductance meas. at THz freq extends above Tc*
  *KT physics in ultra-thin film BSCCO*

- M. Franz and Z. Tesanovic (1999)
  *Vortex-charge duality, QED3 model*

- S. Sachdev (2005)
  *Quantum vortices*

# Theories on phase fluctuation in cuprates

**Baskaran, Zou, Anderson (Solid State Comm. 1987)**
$\Delta$ vs. x and the loss of phase coherence in underdoped regime

**Emery Kivelson (Nature 1995)**
Phase fluctuation and loss of coherence at Tc in low (superfluid) density SC's

**M. Renderia et al. (Phys. Rev. Lett. '02)**
Cuprates in strong-coupling limit, distinct from BCS limit.

**Tesanovic and Franz (Phys. Rev. B '99, '03)**
Strong phase fluctuations in d-wave superconductor treated by dual mapping to Bosons in Hofstadter lattice --- vorticity and checkerboard pattern

**Balents, Sachdev, Fisher et al. (2004)**
Vorticity and checkerboard in underdoped regime

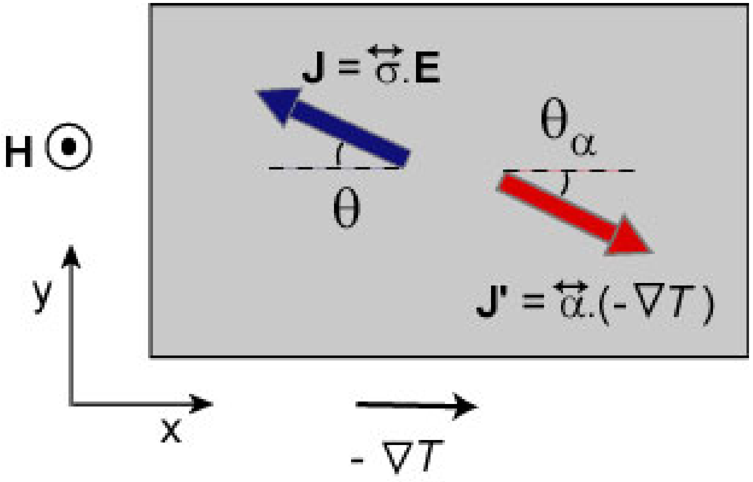
**P. A. Lee, X. G. Wen. (PRL, '03, PRB '04)**
Loss of phase coherence in tJ model, nature of vortex core

**Lee, Nagaosa, Wen, Rev. Mod. Phys. (cond-mat/0410***)**
Good review of phase stiffness, phase fluctuation issues

**P. W. Anderson (cond-mat '05)**
Spin-charge locking occurs at $T_{\mathrm{onset}} > T_{\mathrm{c}}$

# The Nernst effect of carriers

**Wang et al. PRB '01**



$$\mathbf{J} = \overleftrightarrow{\sigma}.\mathbf{E} + \overleftrightarrow{\alpha}.(-\nabla T)$$

Open boundaries, so set **J** = 0.

$$\mathbf{E} = -\overleftrightarrow{\rho}.\overleftrightarrow{\alpha}.(-\nabla T)$$

$$E_y = -\left(\rho\alpha_{yx} + \rho_{yx}\alpha\right)(-\partial_x T)$$

**Off-diag. Peltier cond.**

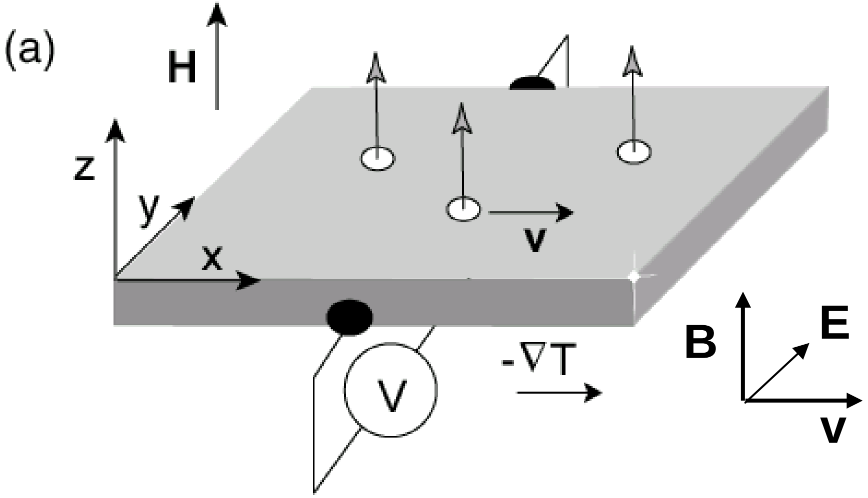
$$\alpha_{xy} = 2e^2 \sum_{\mathbf{k}} \left(-\frac{\partial f_{\mathbf{k}}^0}{\partial \varepsilon}\right) \frac{\varepsilon_{\mathbf{k}} - \mu}{T} \ell_y \, \mathbf{v} \times \mathbf{B} . \frac{\partial}{\partial \mathbf{k}} (\ell_x)$$

**Measured Nernst signal**

$$e_N \equiv \frac{E_y}{|\nabla T|} = \frac{\pi^2}{3} \frac{k_B^2 T}{e} \frac{\partial \theta}{\partial \varepsilon}$$

**Generally, very small because of cancellation between $\alpha_{xy}$ and $\sigma_{xy}$**

# The vortex Nernst effect; dominant in vortex liquid state



Moving vortex produces
**E = B x v**

Gradient drives vortex current
with velocity **v** || **x**

Force exerted on vortex line by grad *T*

$$\mathbf{F} = s_\phi(-\nabla T)$$

Line entropy $s_\phi$

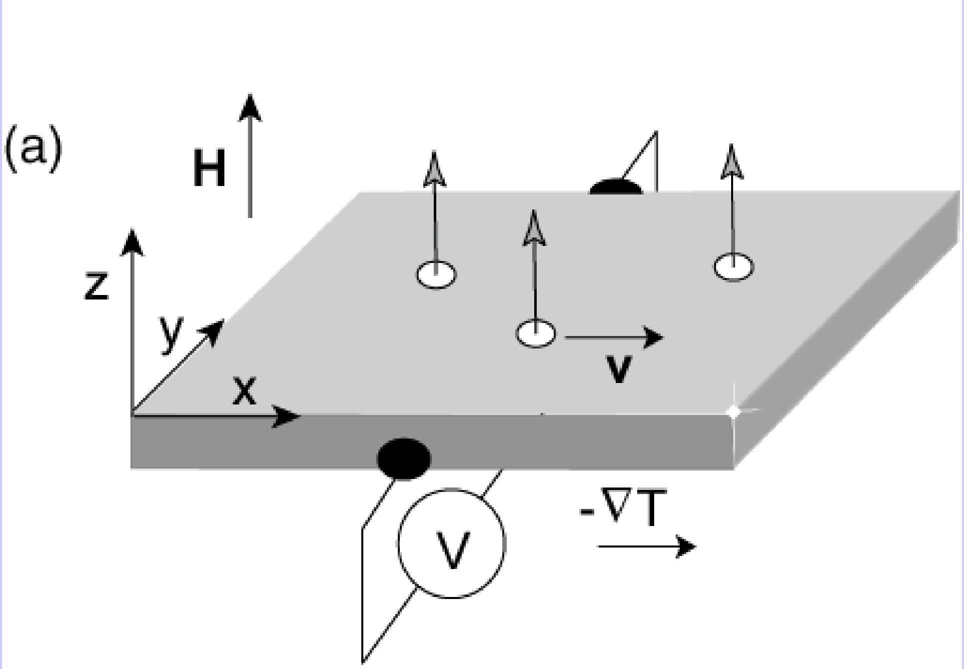
Balance **F** by viscous damping

$$\eta \mathbf{v} = s_\phi(-\nabla T)$$

**Nernst signal $e_N$**

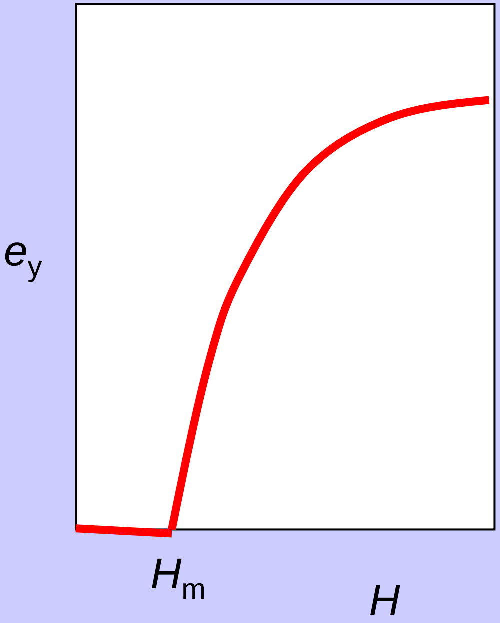
$$e_N = \frac{E}{|\nabla T|} = \frac{B s_\phi}{\eta}$$

# Nernst experiment



Vortices move in a temperature gradient
Phase slip generates Josephson voltage

$$2eV_J = 2\pi\hbar\,\dot{n}_V$$
$$\mathbf{E}_J = \mathbf{B} \times \mathbf{v}$$

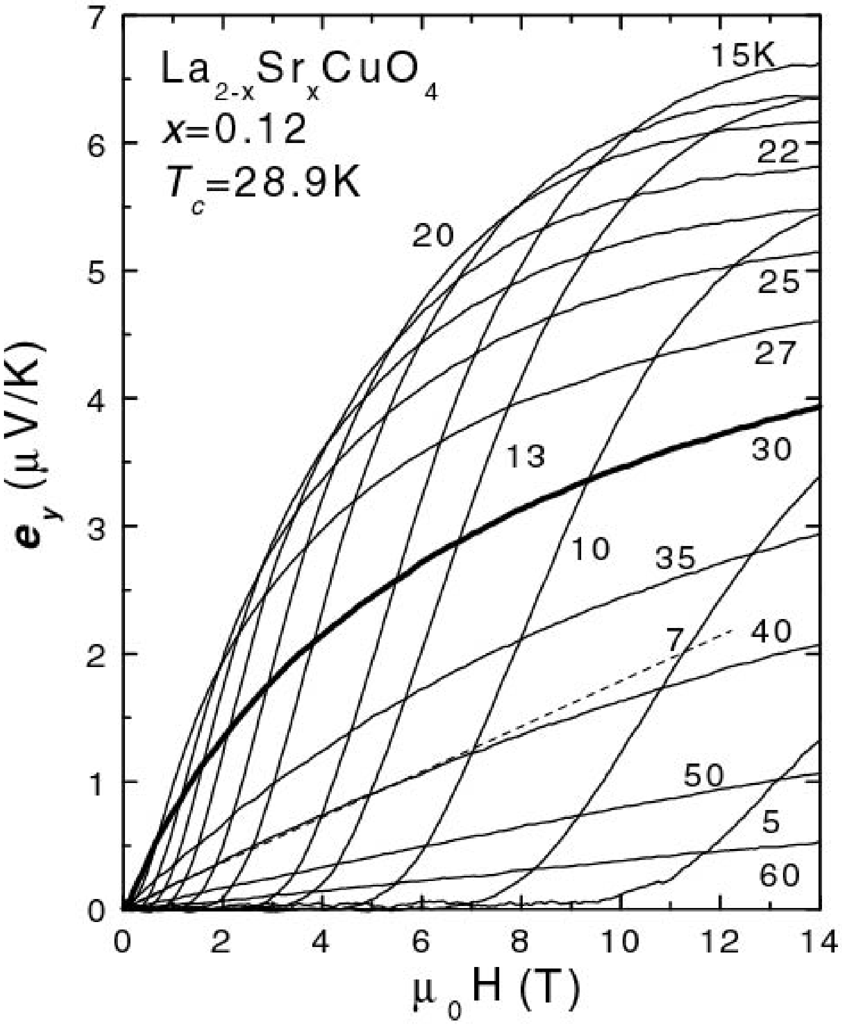


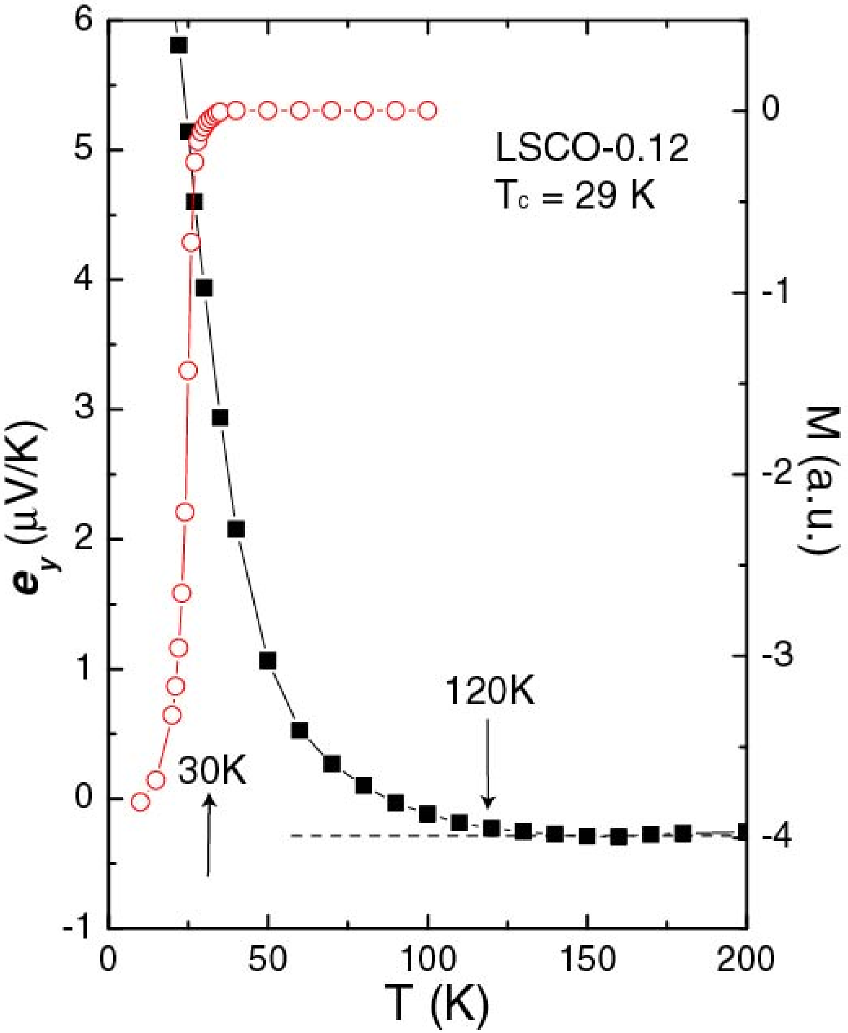
Nernst signal

$$e_y = E_y / |\nabla T|$$

# Nernst effect in LSCO-0.12

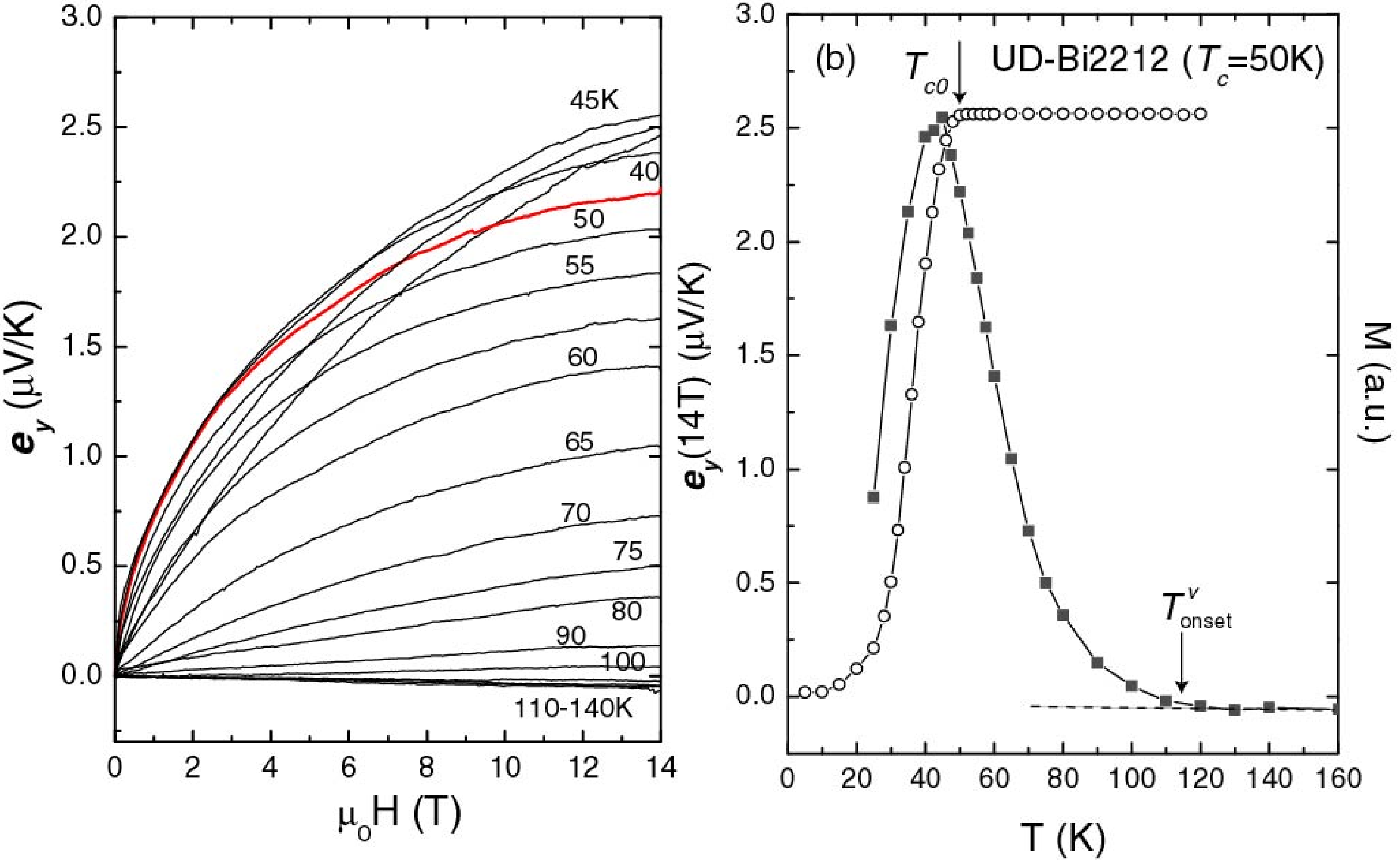
Xu et al. Nature (2000)
Wang et al. PRB (2001)





vortex Nernst signal onset from T = 120 K, ~ 90K above $T_c$`1

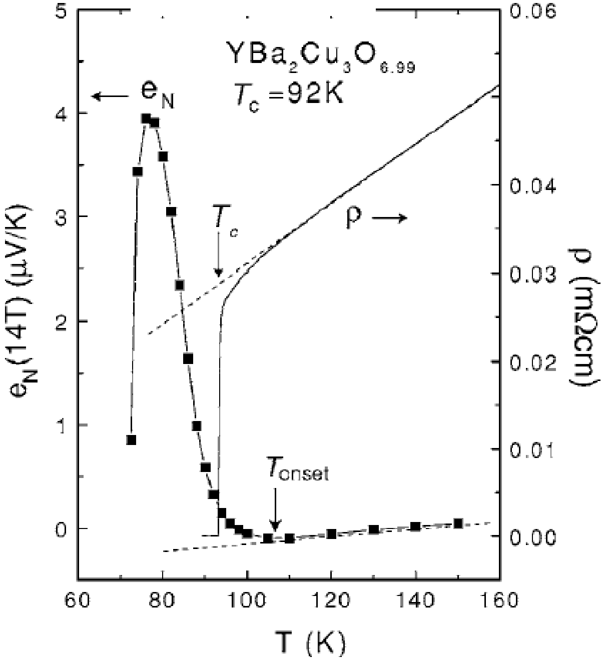
# Nernst effect in underdoped Bi-2212 ($T_c = 50$ K)



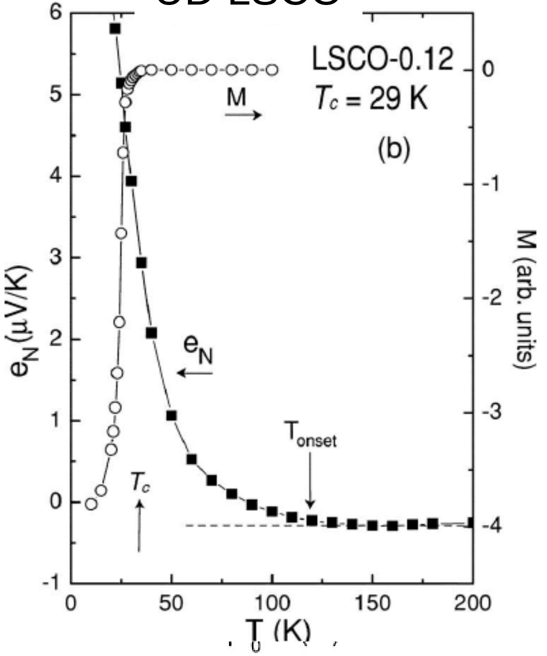
Vortex signal persists to 70 K above $T_c$.

Wang, Li, NPO
PRB (2006)

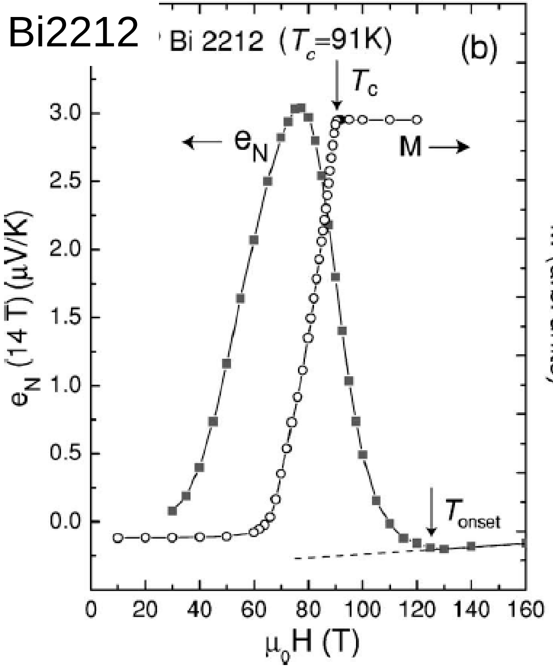
OP YBCO



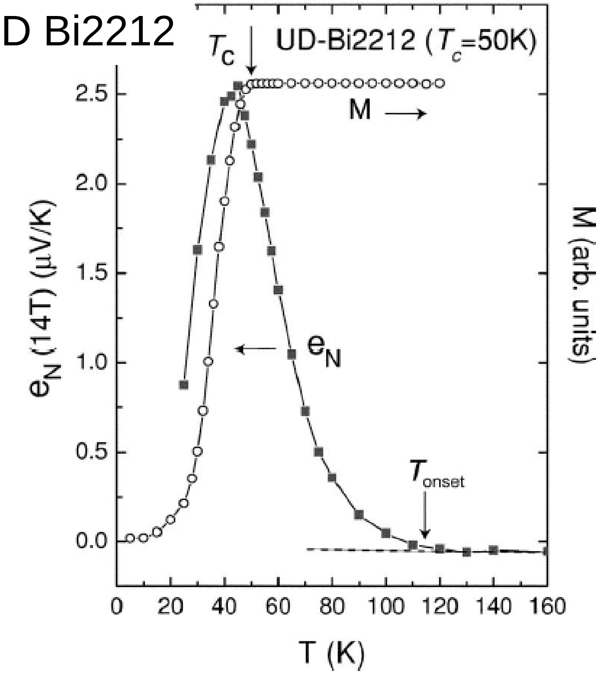
UD LSCO

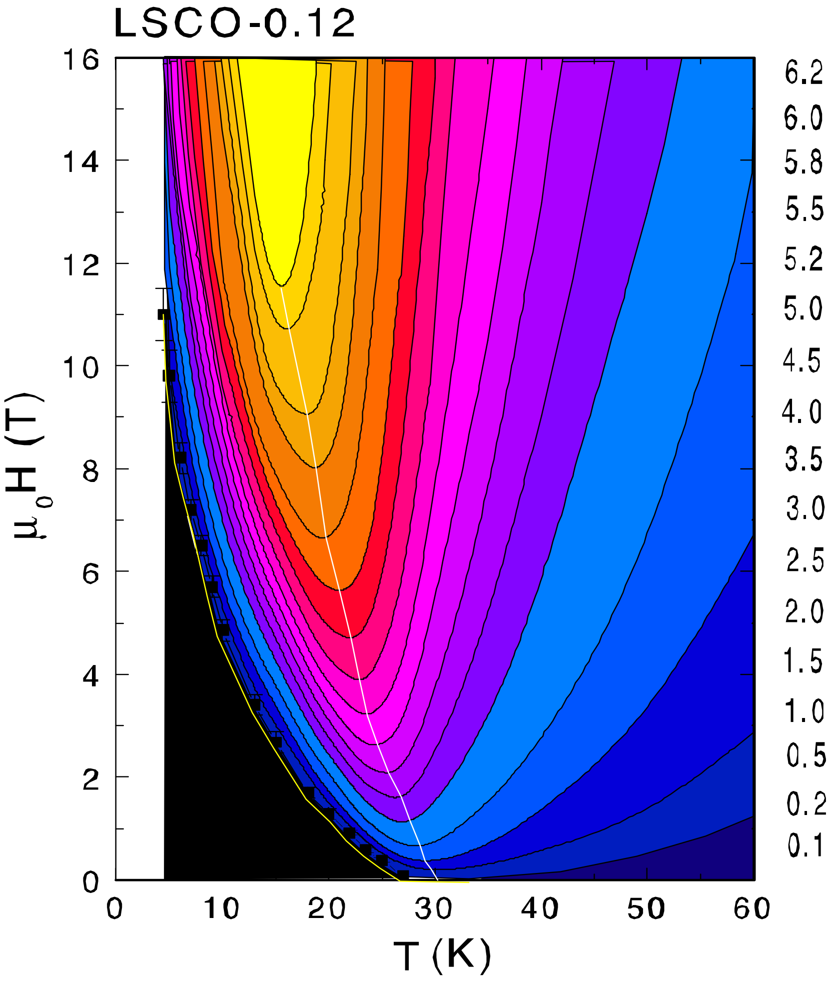


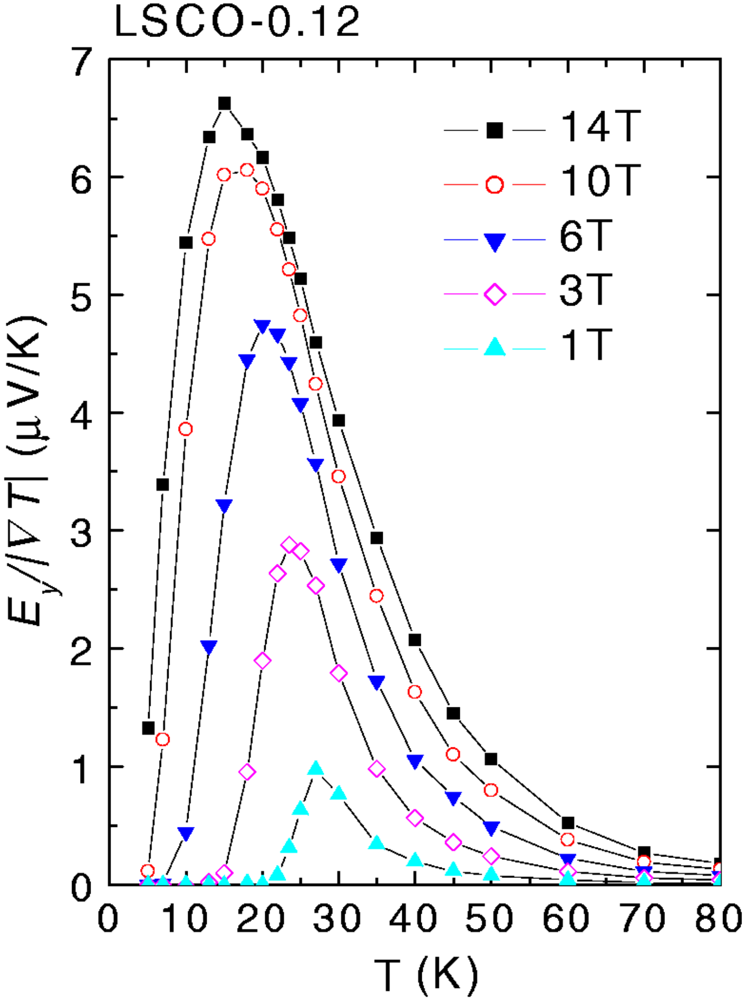
OP Bi2212



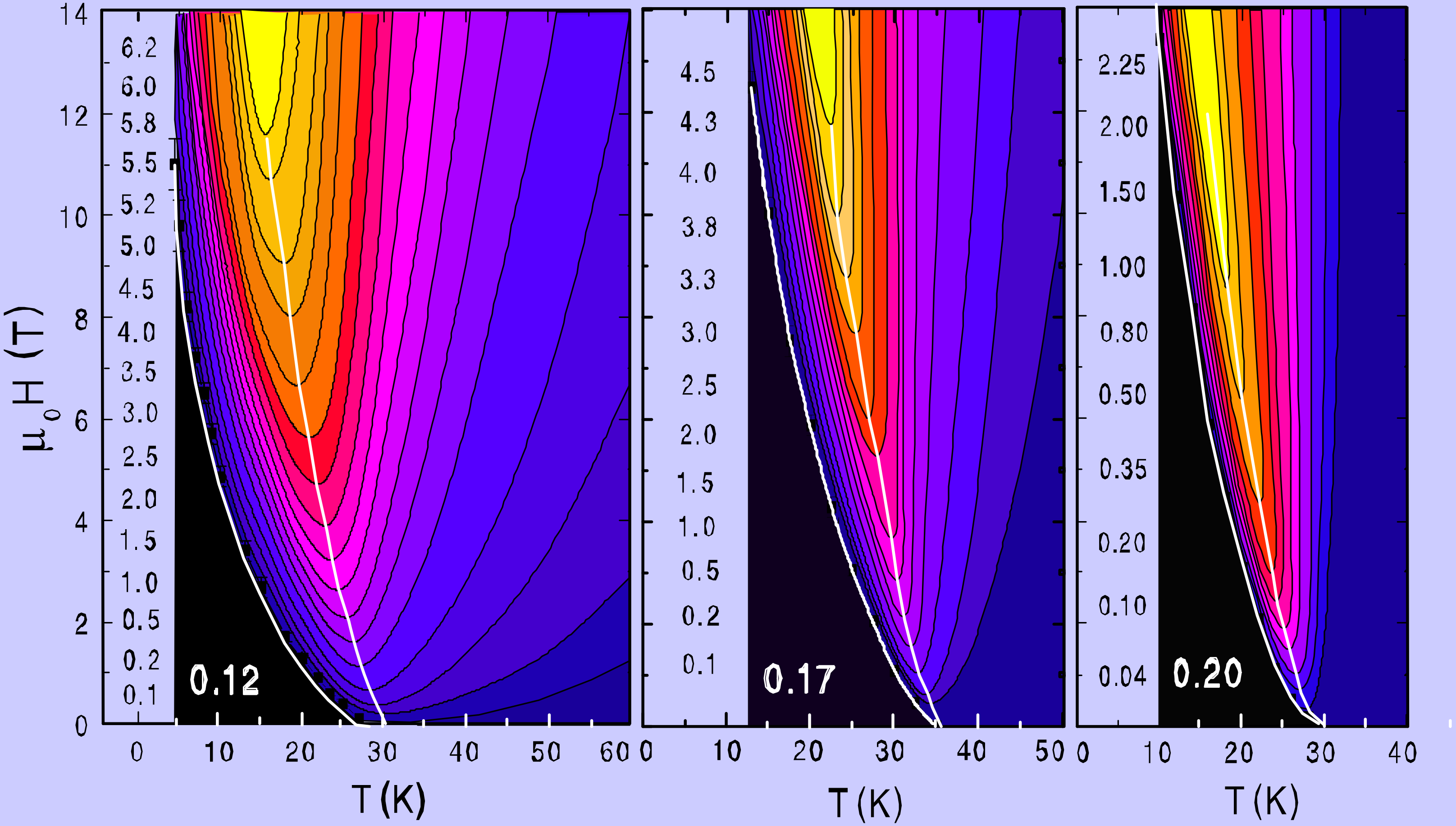
UD Bi2212



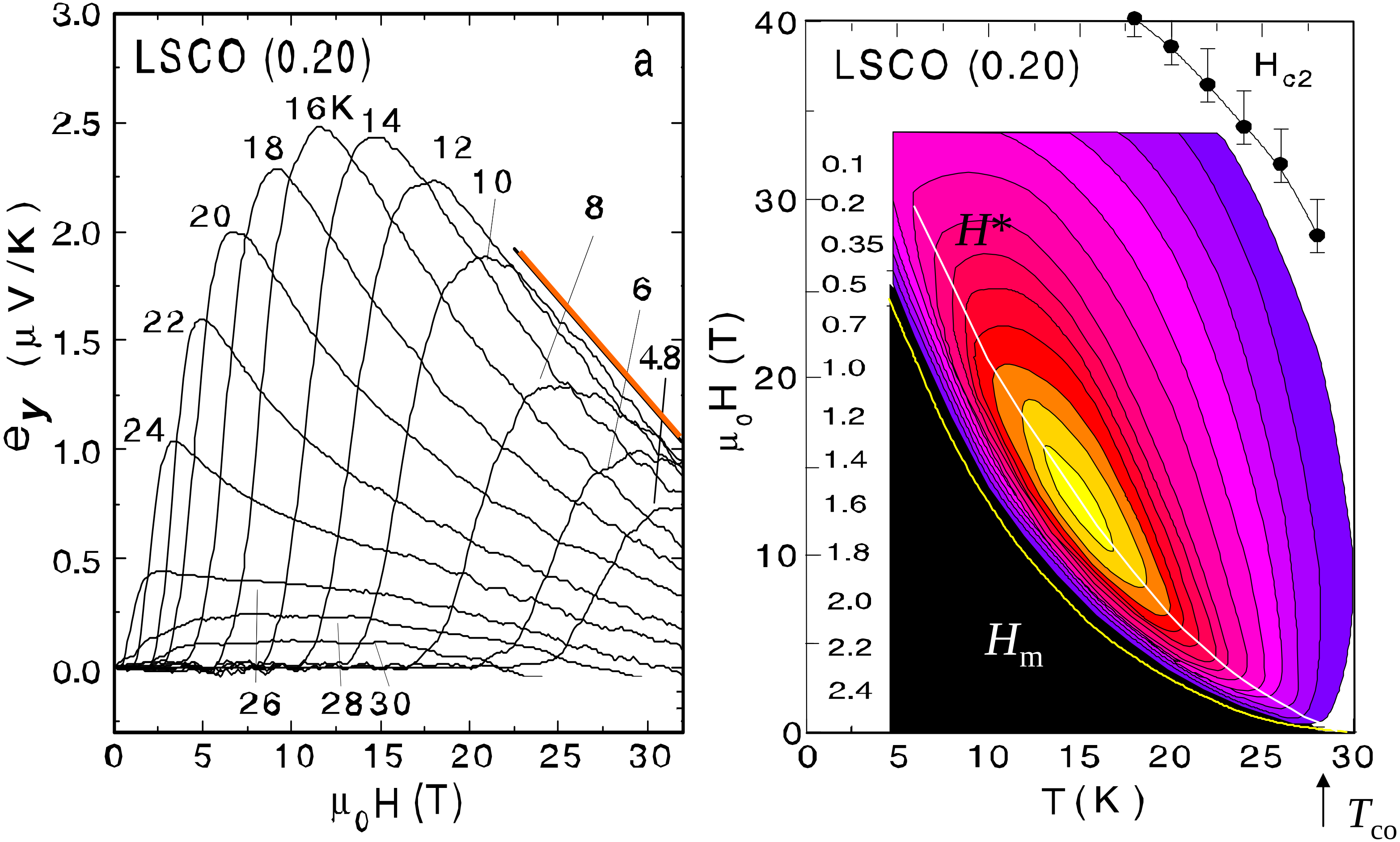

LSCO-0.12
$\mu_0H$ (T)
0
2
4
6
8
10
12
14
16
T (K)
0
10
20
30
40
50
60
6.2
6.0
5.8
5.5
5.2
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.2
0.1


LSCO-0.12
$E_y/|\nabla T|$ (μV/K)
0
1
2
3
4
5
6
7
T (K)
0
10
20
30
40
50
60
70
80
14T
10T
6T
3T
1T

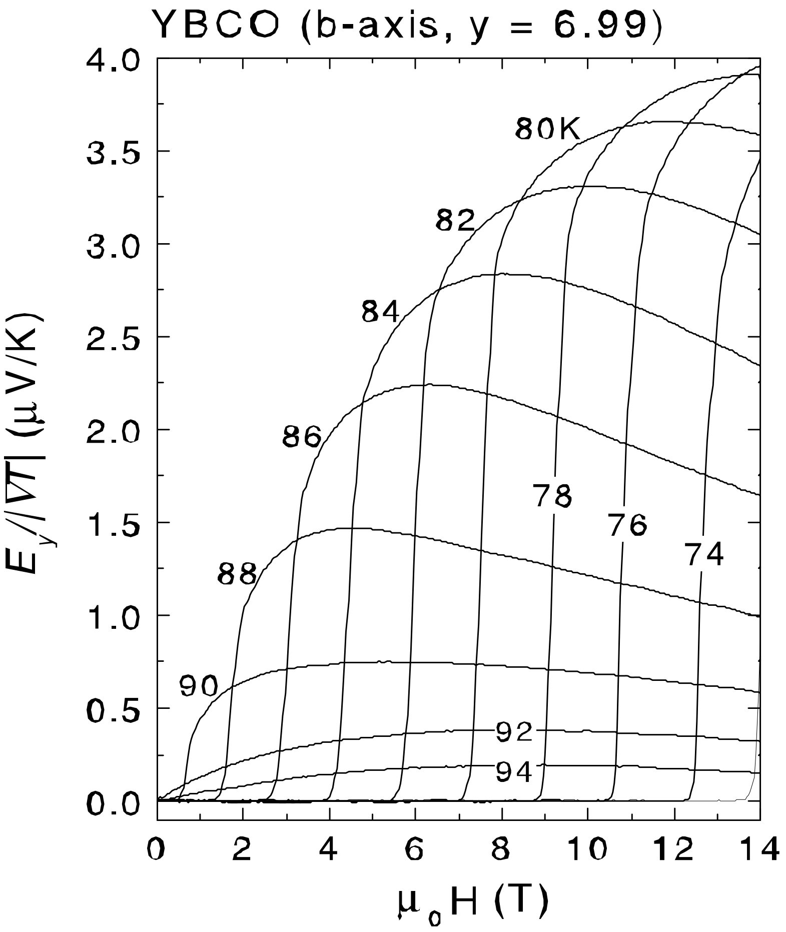
# Nernst contour-map in underdoped, optimal and overdoped LSCO

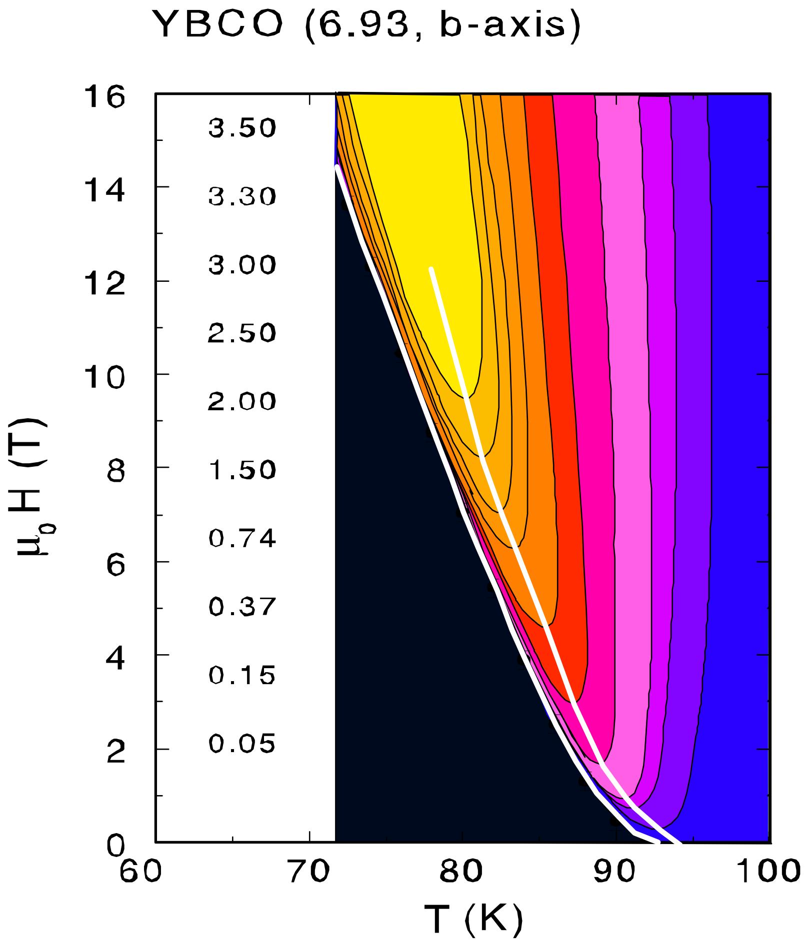


# Overdoped LaSrCuO x = 0.20



# Optimal, untwinned BZO-grown YBCO



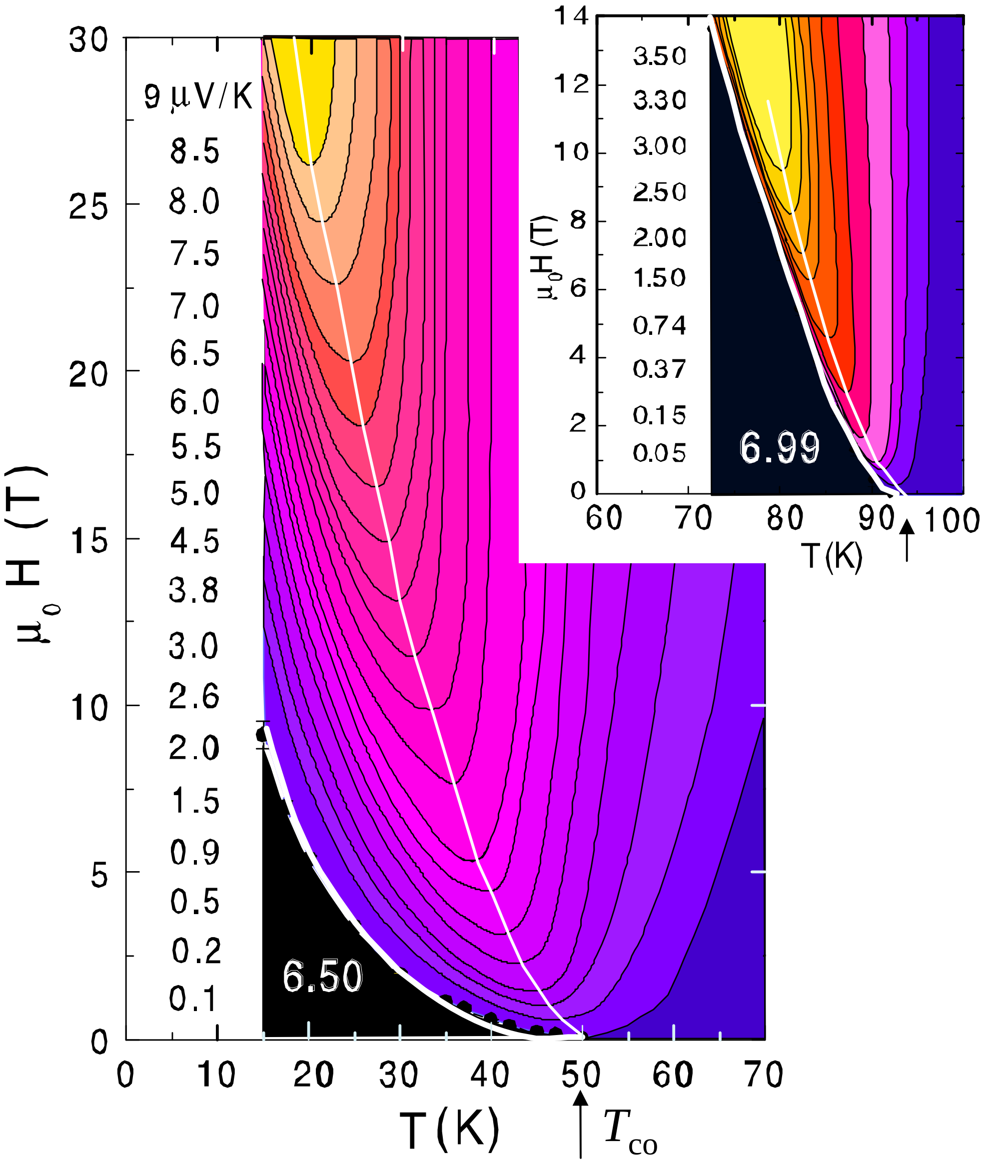


# Nernst contour maps in UD YBCO and OP YBCO

Contour plots in underdoped YBaCuO$_{6.50}$ (main panel) and optimal YBCO$_{6.99}$ (inset).

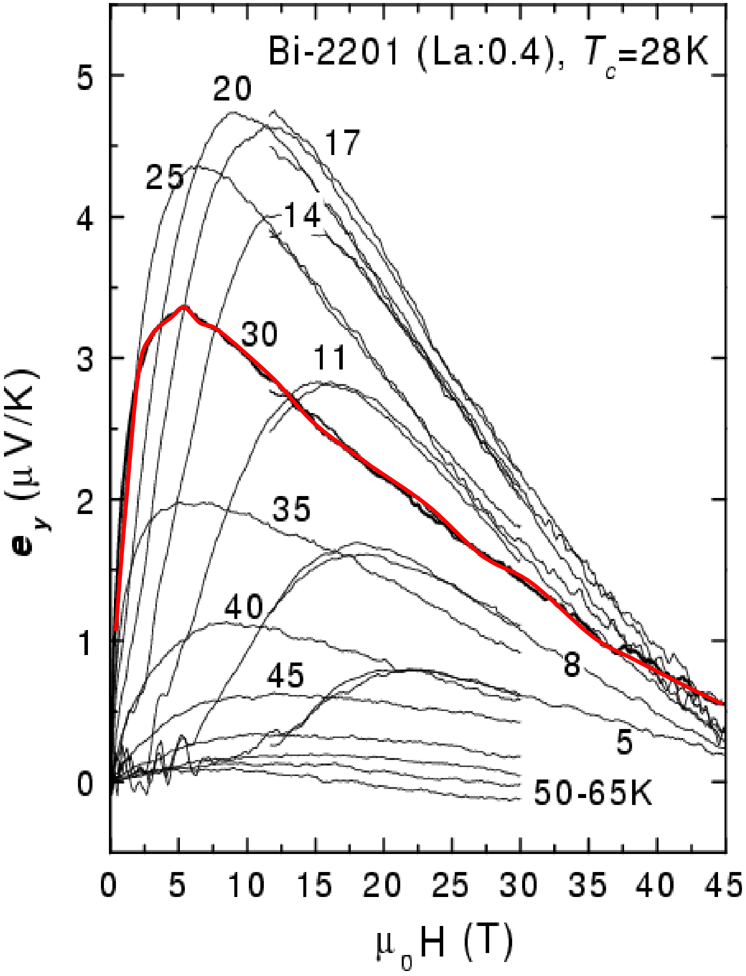
- Vortex signal extends above 70 K in underdoped YBCO, to 100 K in optimal YBCO

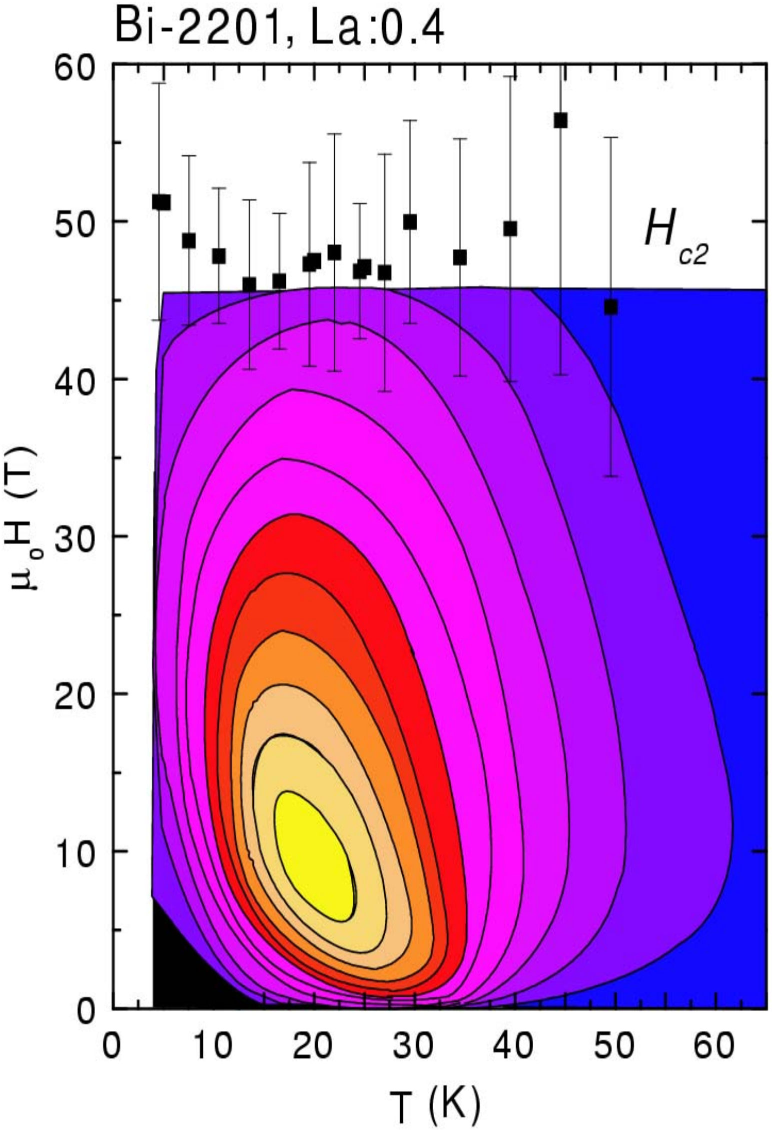
- High-temp phase merges continuously with vortex liquid state



**Wang et al., PRL '02**

# Contour Map of Nernst Signal in Bi 2201





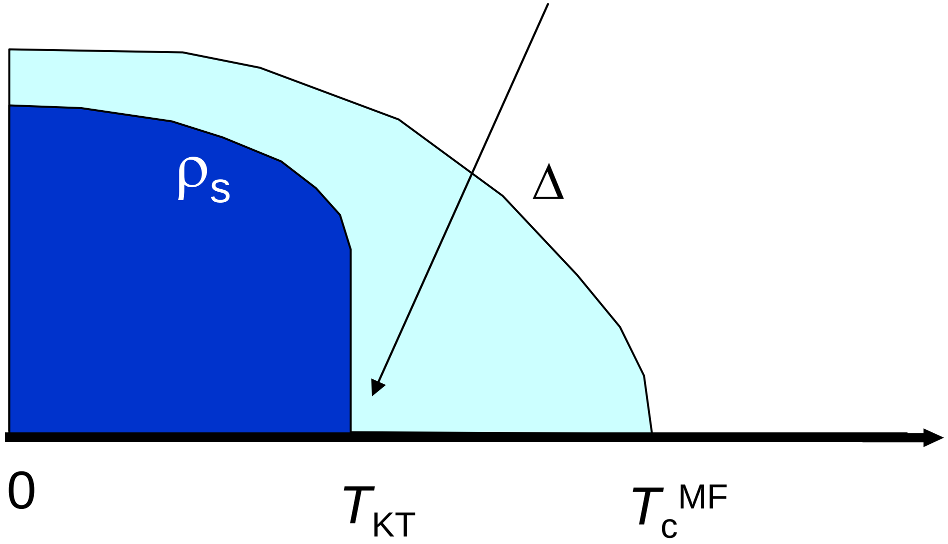
# Kosterlitz-Thouless transition

Spontaneous vortices destroy superfluidity in 2D films

Change in free energy ΔF to create a vortex

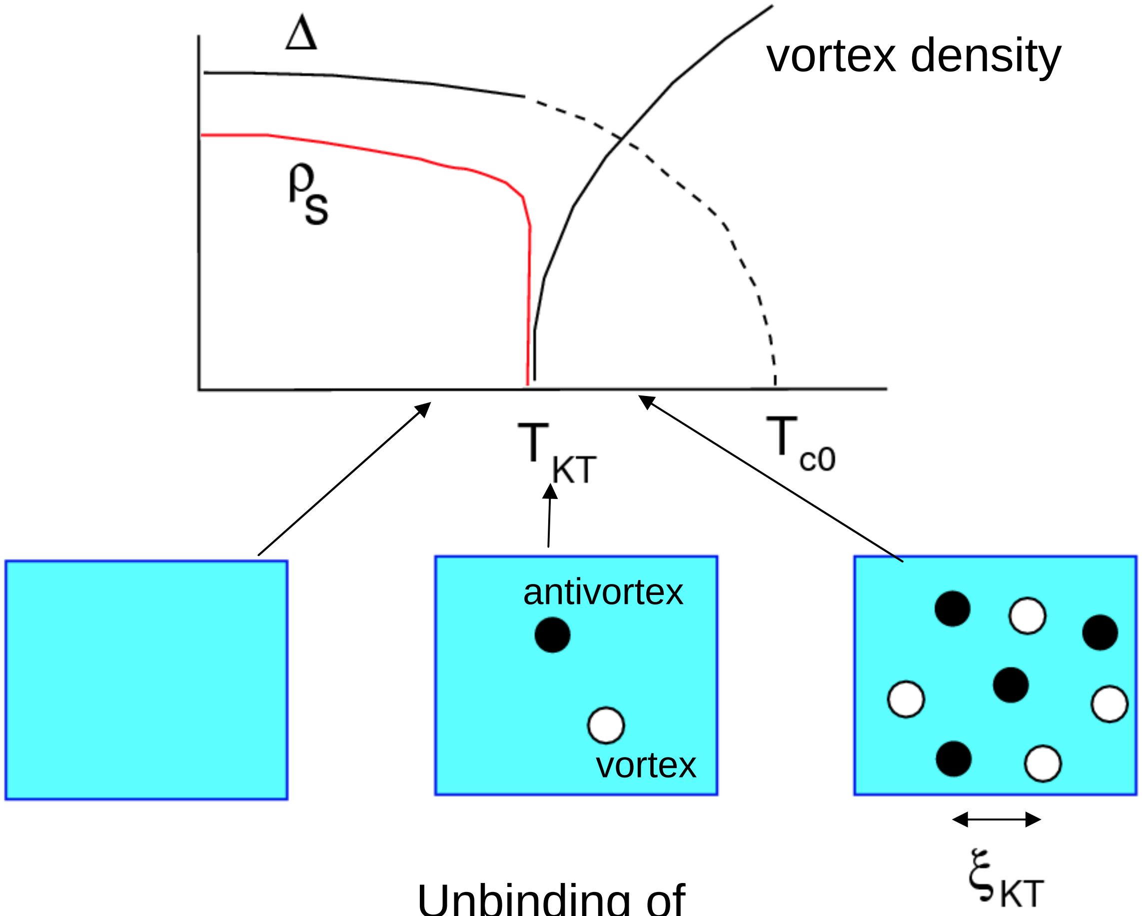
$$\Delta F = \Delta U - T\Delta S \ = \ (\varepsilon_c - k_B T) \log (R/a)^2$$

$\Delta F < 0$ if $T > T_{KT} = \varepsilon_c/k_B$ ➡ vortices appear spontaneously



3D version of KT transition in cuprates?

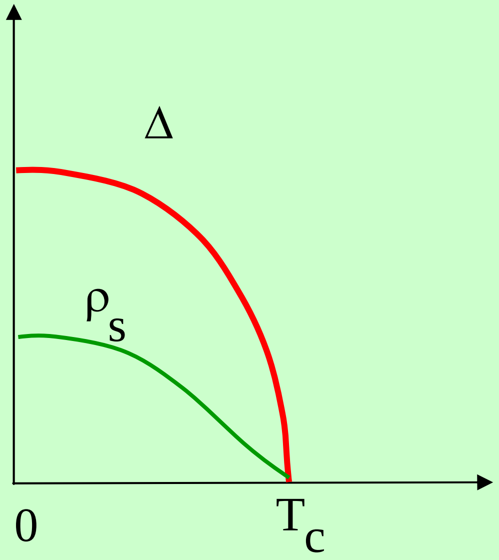
# Kosterlitz Thouless transition in 2D superconductor



Unbinding of
vortex-antivortex

**Free energy gain** $\Delta F = U - TS$

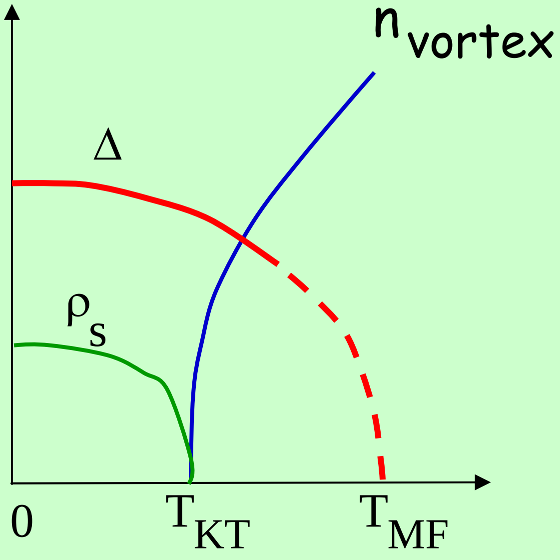
## BCS transition



$$H = \tfrac{1}{2}\,\rho_s \int d^3r\,(\nabla\,\phi)^2$$

$\rho_s$ measures phase rigidity

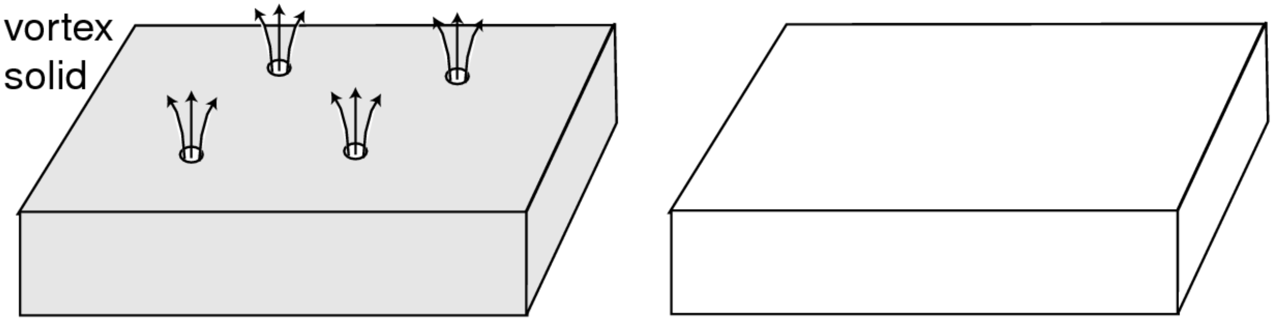
## 2D Kosterlitz Thouless transition



Phase coherence destroyed at $T_{KT}$ by proliferation of vortices

High temperature superconductors?

## Low Tc superconductor

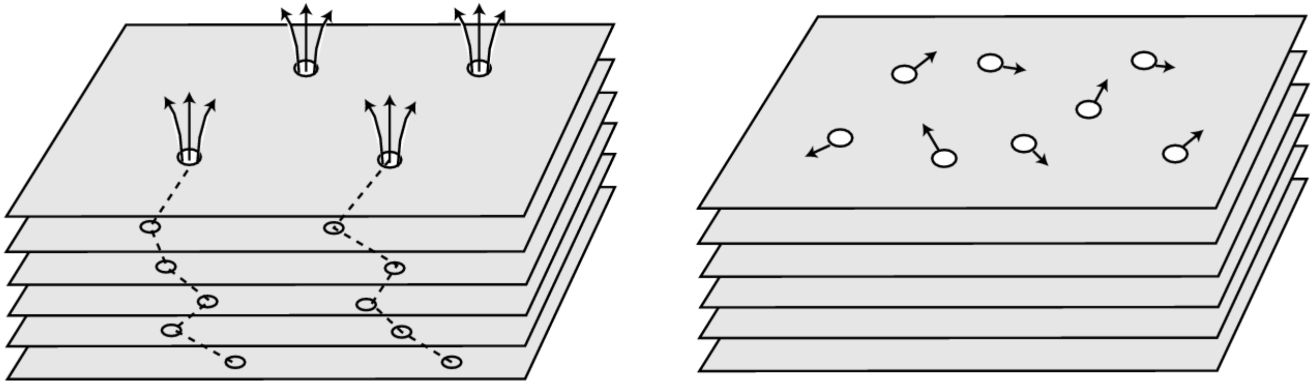


normal state

vortex solid

No pair condensate

## High-Tc superconductor



2D vortices

anisotropic vortex solid

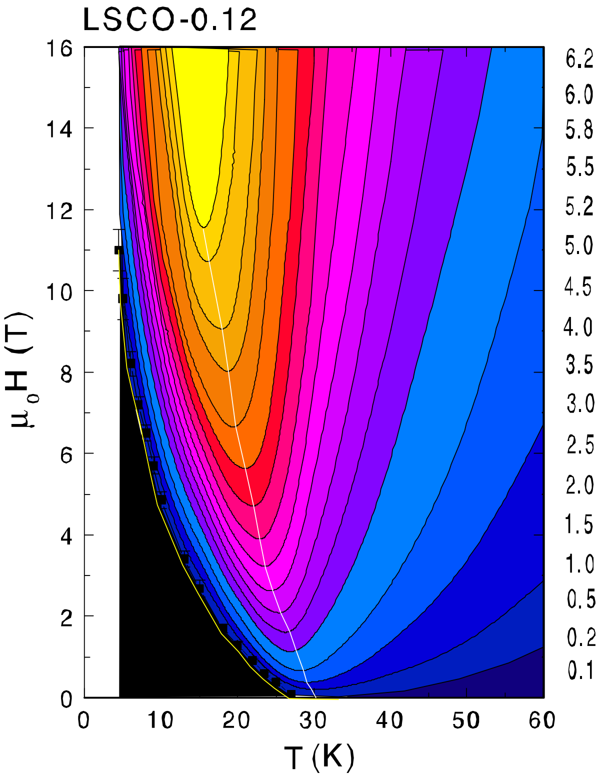
vortex liquid

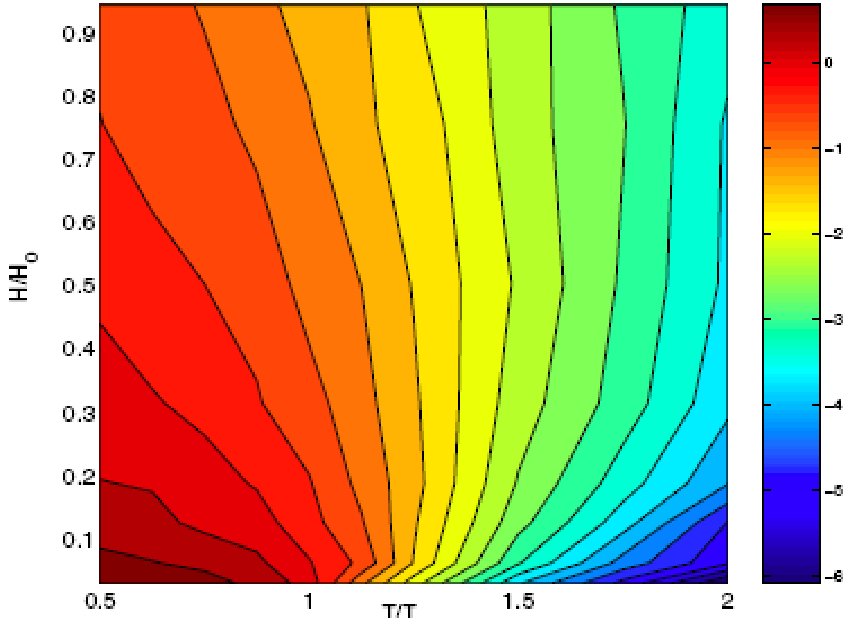
pair condensate survives

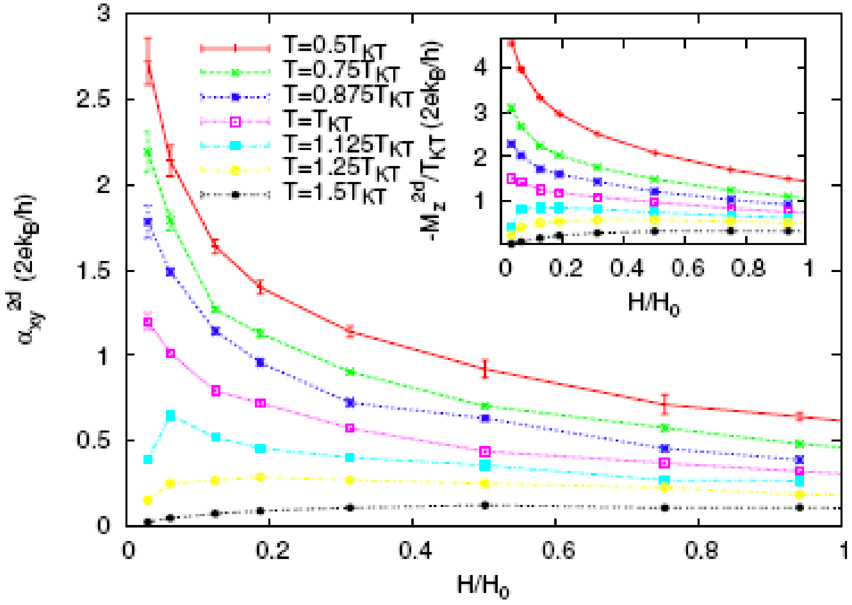
# Simulation Nernst effect in 2D XY model

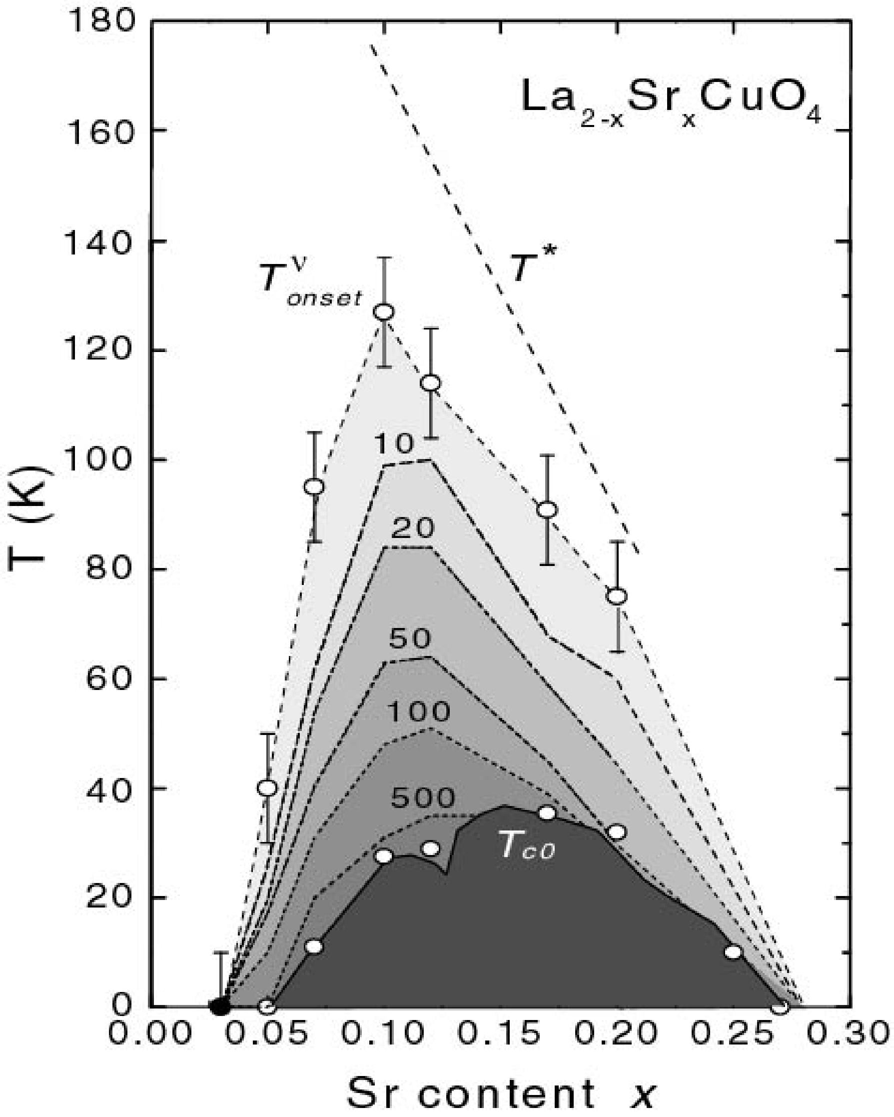
**Podolsky, Raghu, Vishwanath PRL (2007).**

$$\alpha_{xy}^{2d} = \lambda \frac{2ek_B}{h}\left(\frac{J}{T}\right)^{\mu} \sin\frac{H}{H_0}$$



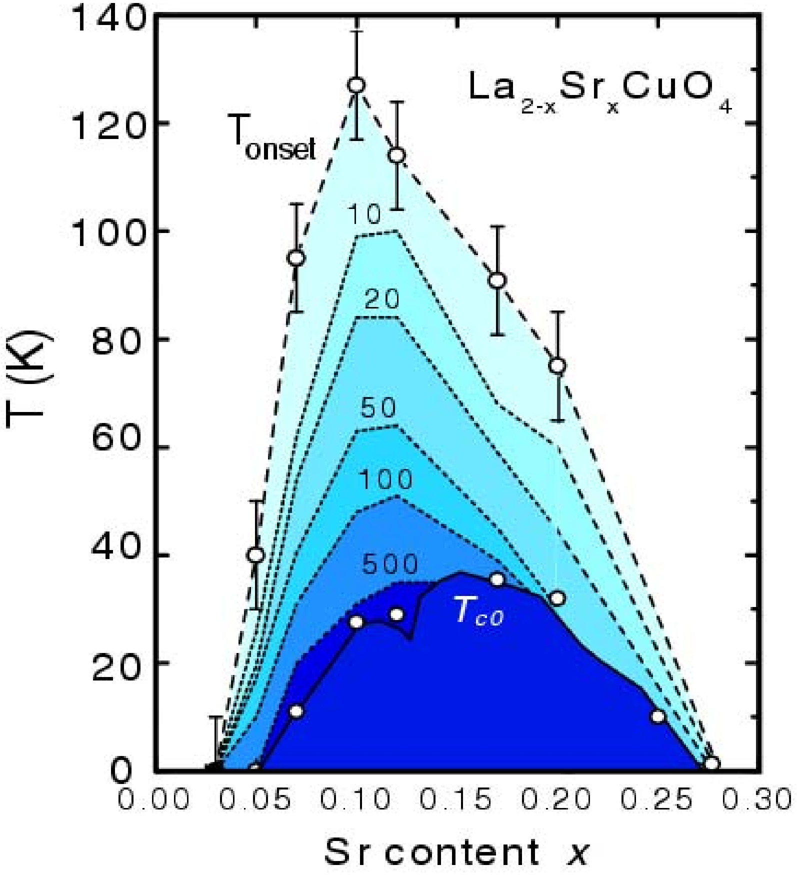


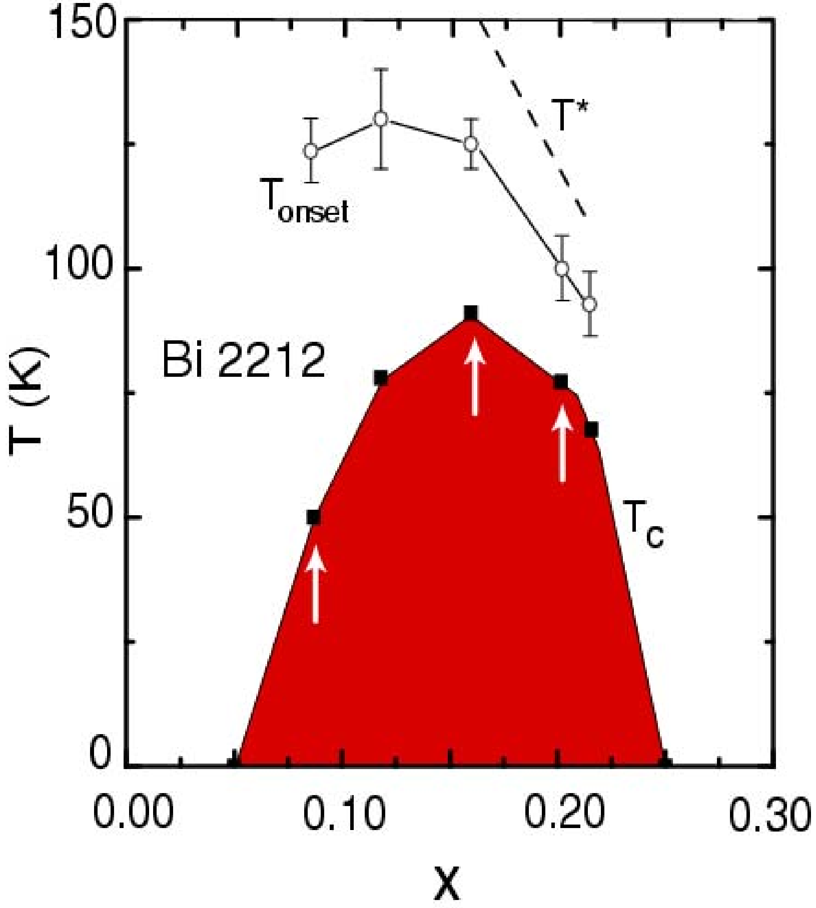




Wang et al. PRB (2001),
NPO et al. Ann. Phys (2004)

- Loss of phase coherence determines Tc
- Condensate amplitude persists T>Tc

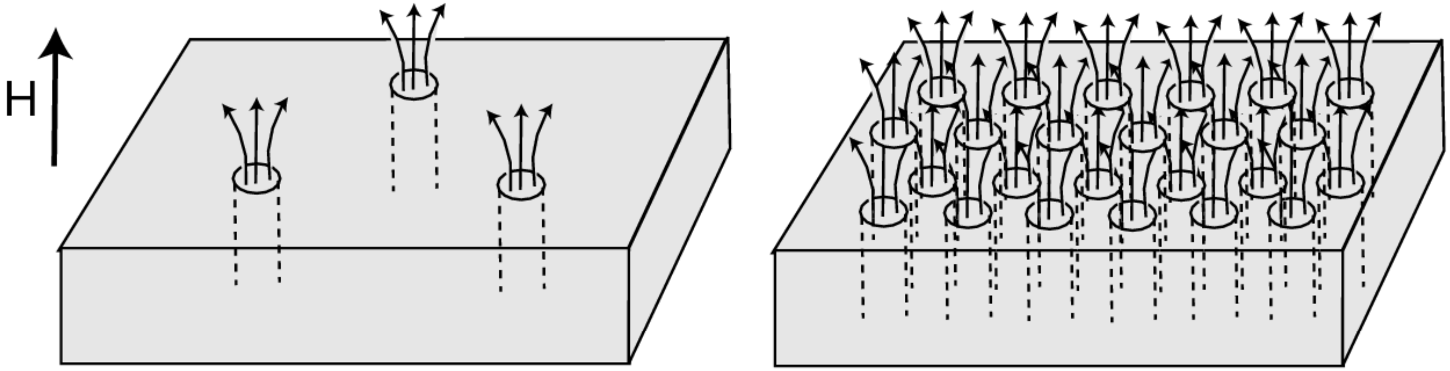




- **Condensate amplitude persists to $T_{\mathrm{onset}} > T_{\mathrm{c}}$**
- **Nernst signal confined to SC dome**
- **Vorticity defines Nernst region**

# The Upper Critical Field -- destruction of pair condensate

Applied flux $B = n\,\phi_0$



low field

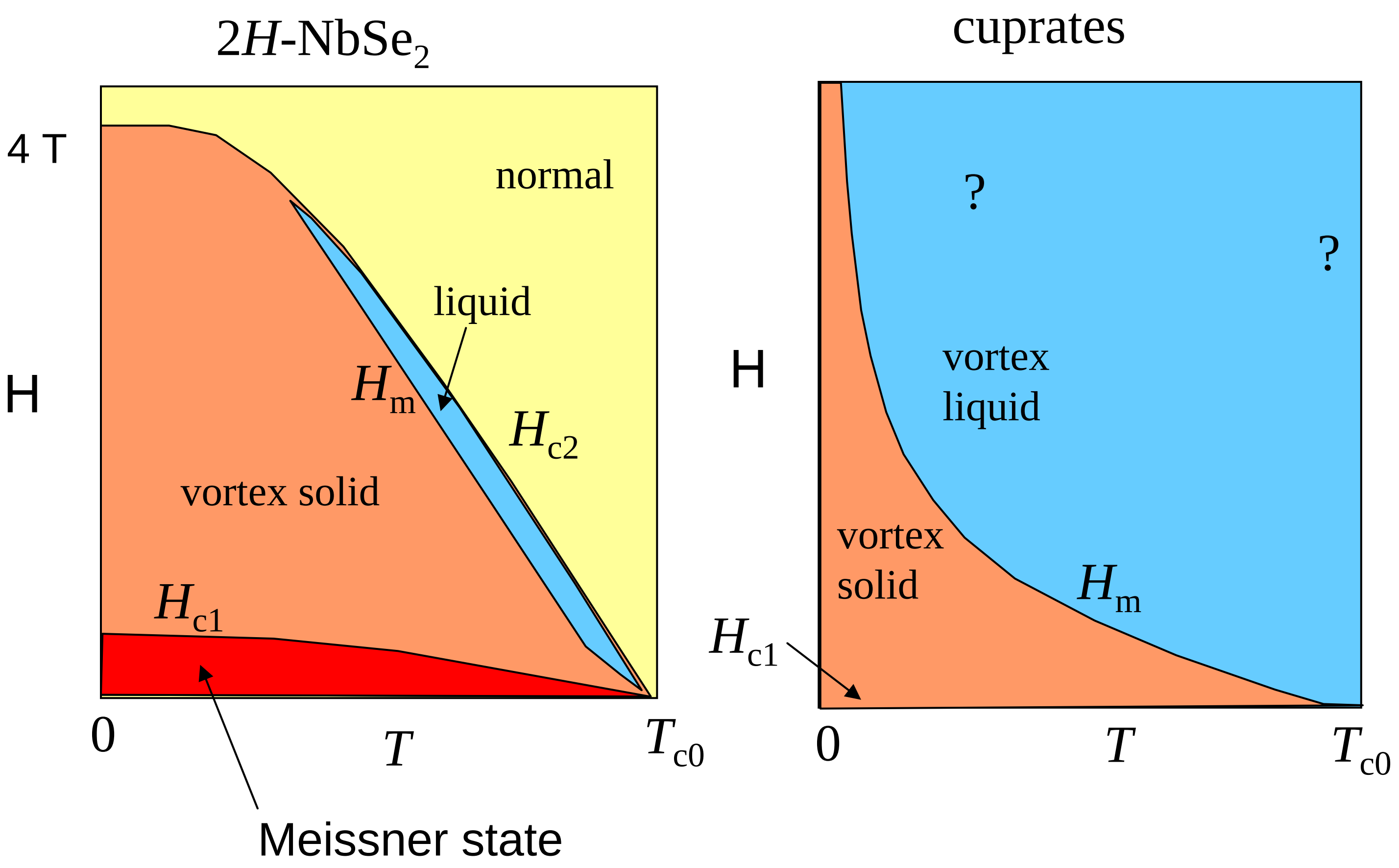
$H \sim H_{c2}$

Close packing of vortex cores →

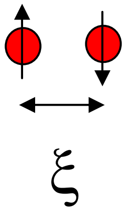
$$H_{c2} = \phi_0 / (2\pi\xi_0^2)$$

upper critical field

# Phase diagram of type-II superconductor

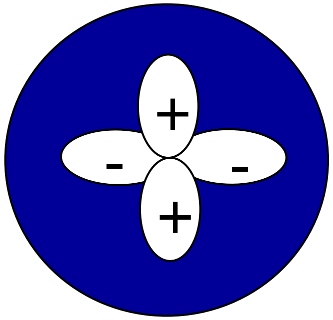


# Cooper pairing in cuprates



coherence length

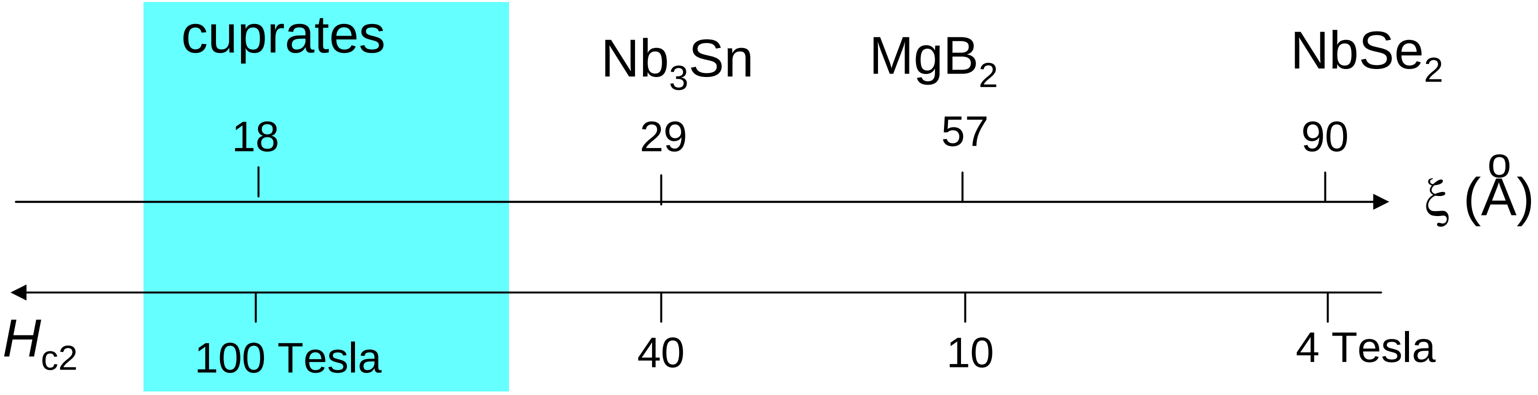
*d*-wave symmetry



$$\Delta(\theta) = \Delta_0 \cos 2\theta$$

Upper critical field

$$H_{c2} = \frac{\phi_0}{2\pi\xi^2}$$


cuprates

$Nb_3Sn$

$MgB_2$

$NbSe_2$

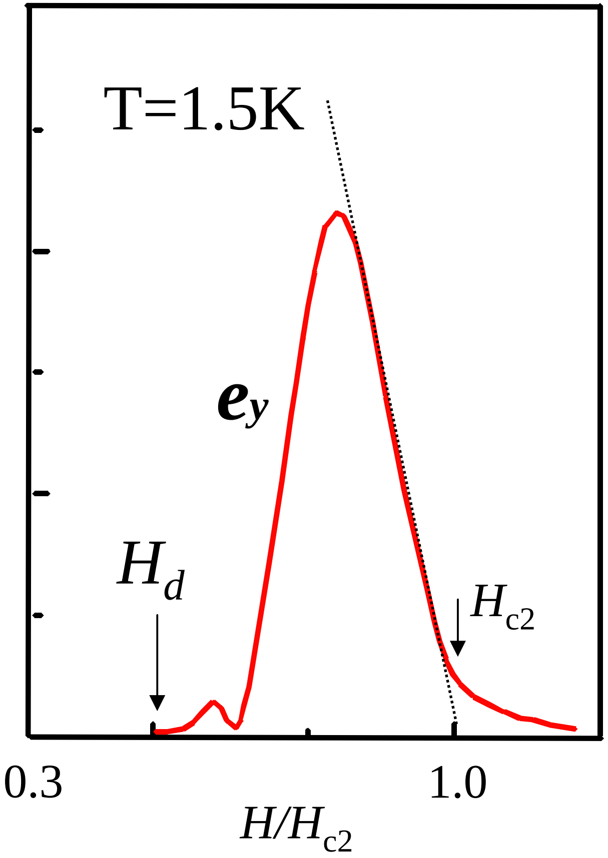
18
29
57
90

ξ (Å)

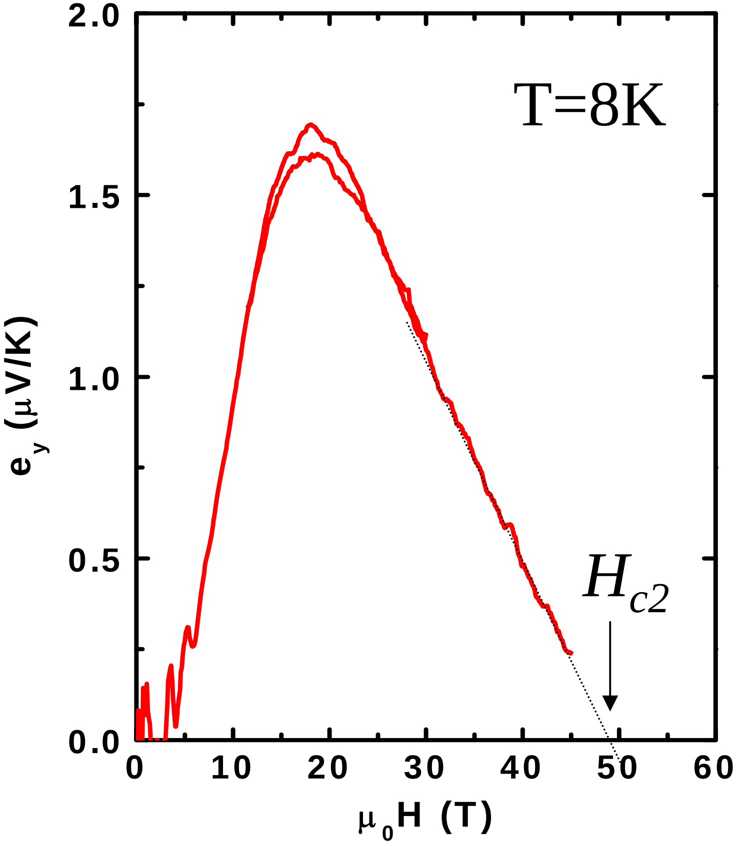
$H_{c2}$

100 Tesla
40
10
4 Tesla

PbIn, $T_c = 7.2$ K (Vidal, PRB ’73)



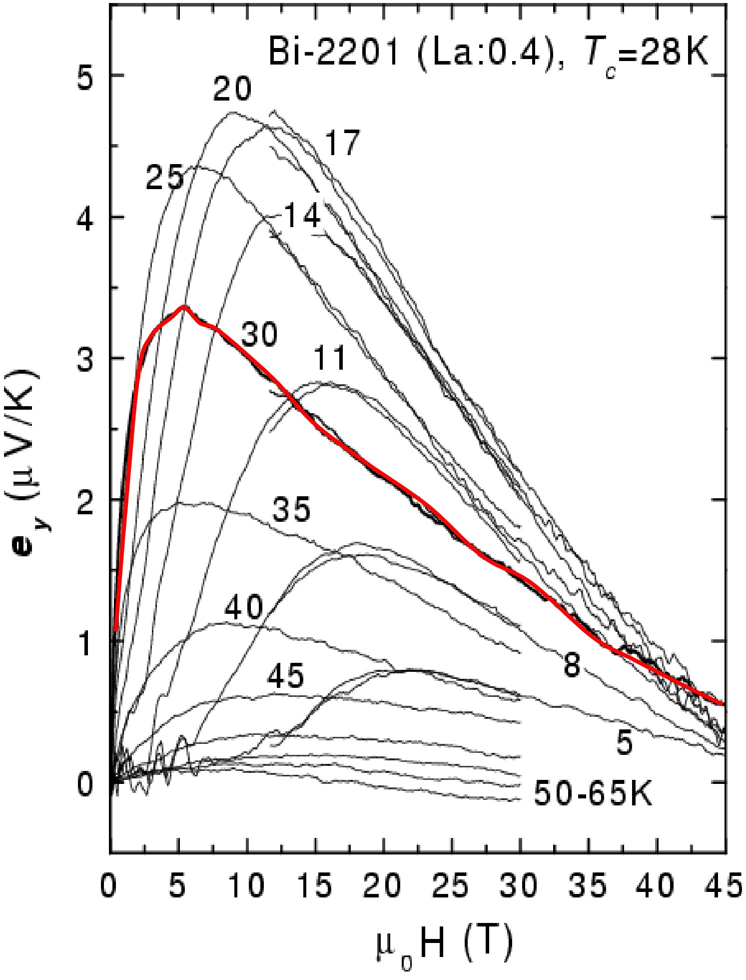
Bi 2201 ($T_c = 28$ K, $H_{c2} \sim 48$ T)

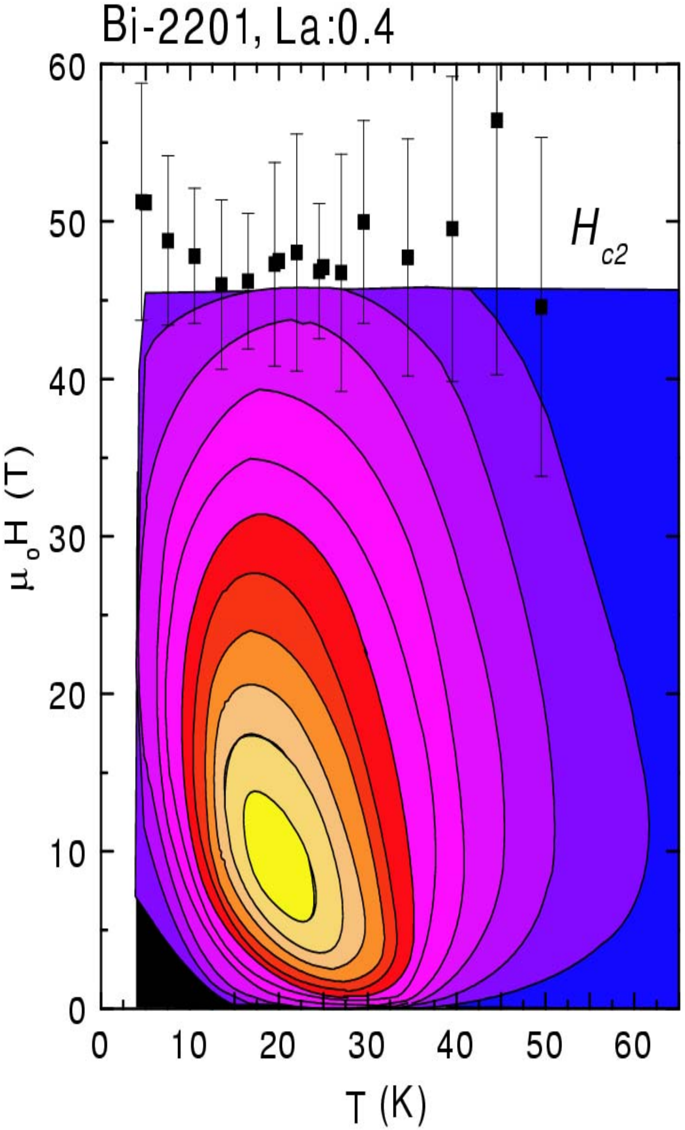


- Upper critical Field $H_{c2}$ given by $e_y \to 0$.
- Hole cuprates --- Need *intense* fields.

Wang et al. Science (2003)

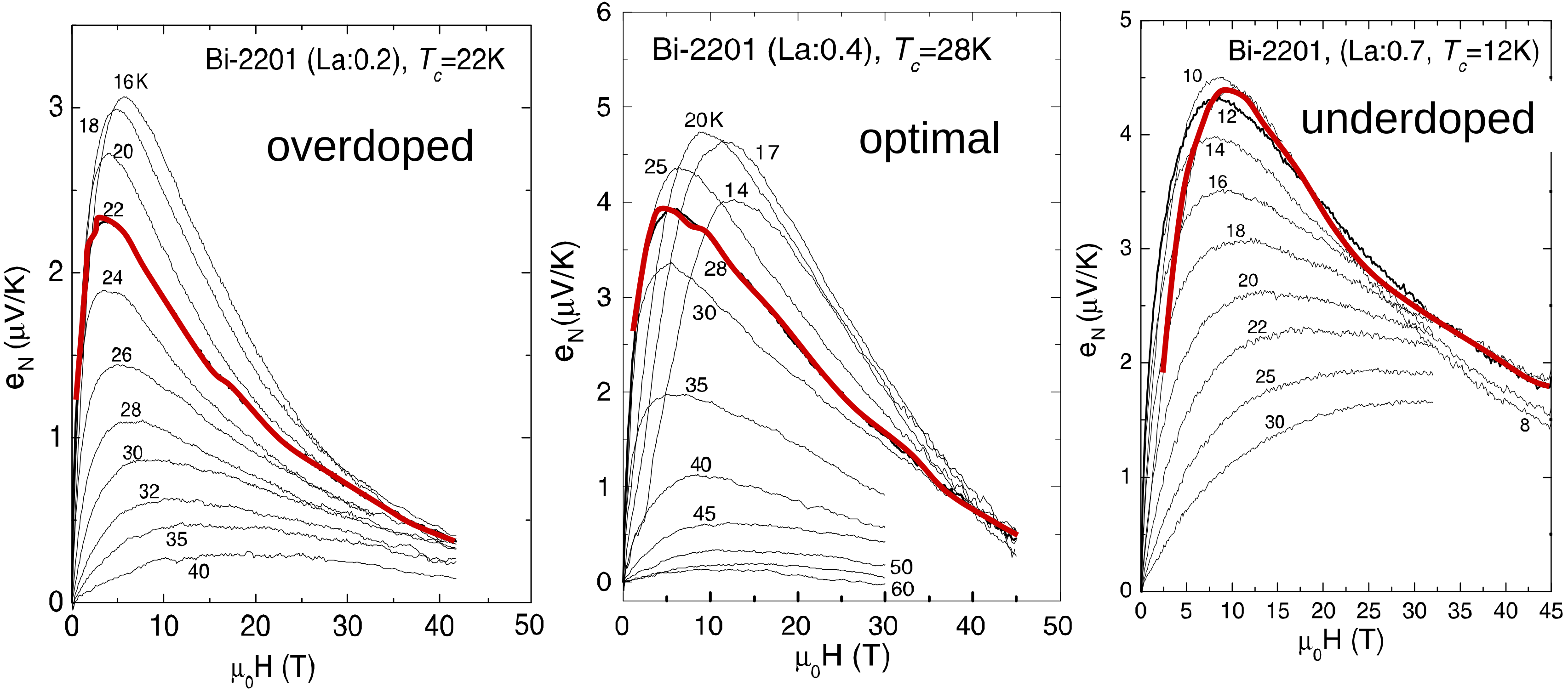
# Vortex-Nernst signal in Bi 2201

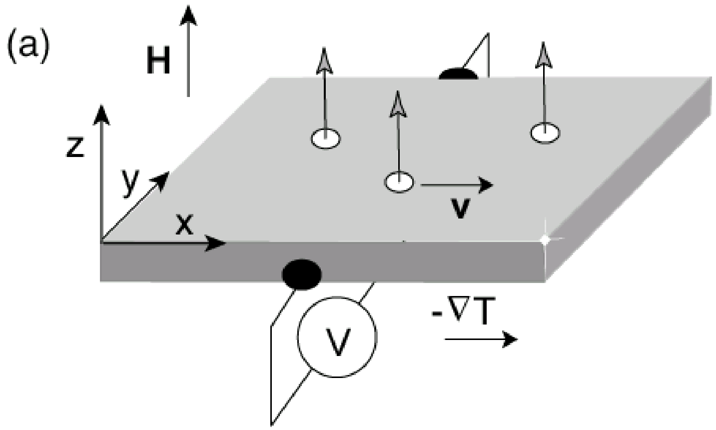




# Vortex Nernst signal in overdoped, optim. and underdoped regimes

**Wang, Li, NPO PRB 2006**

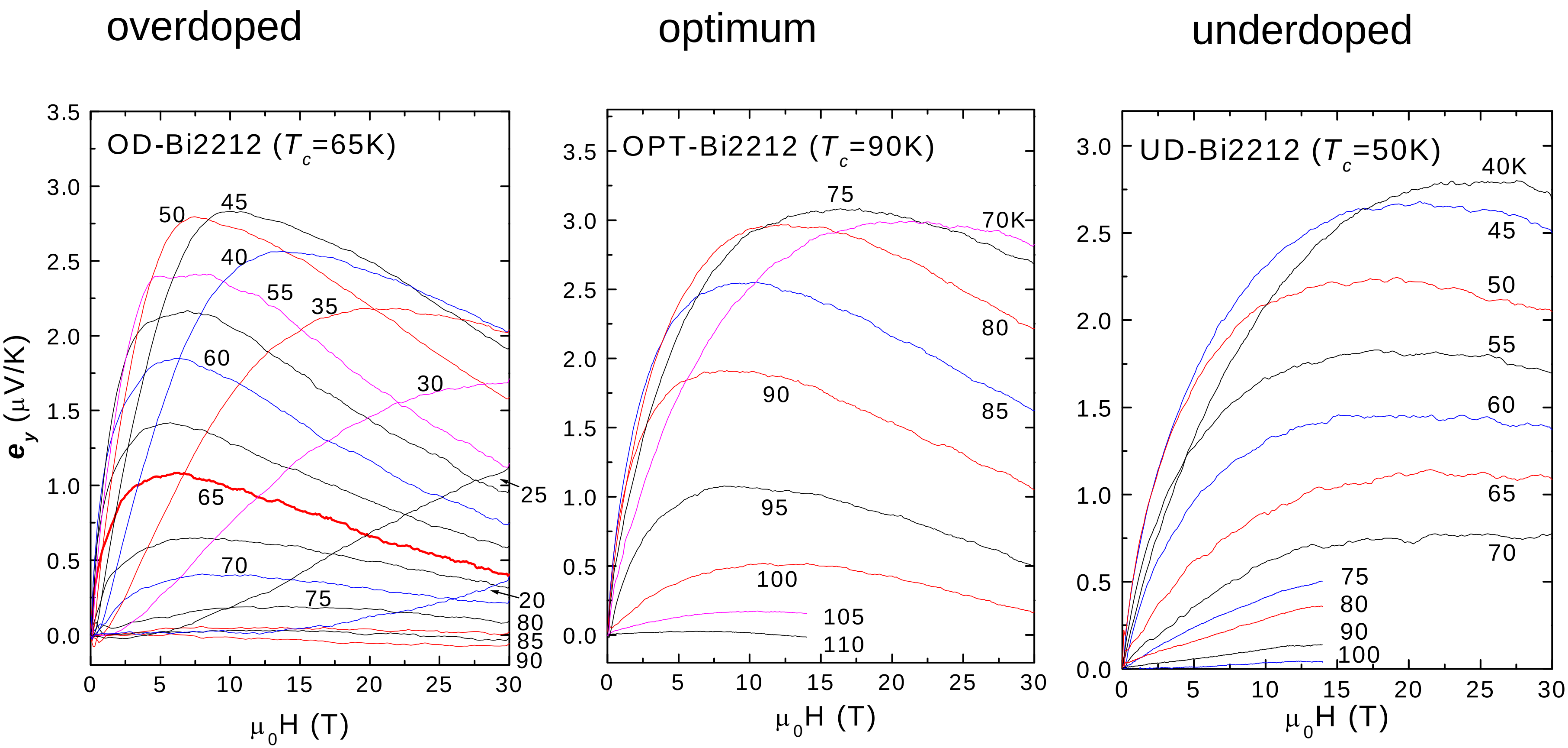




**Nernst signal**

$$e_N = E_y / |\nabla T|$$

Wang et al. Science (2003)


overdoped

optimum

underdoped

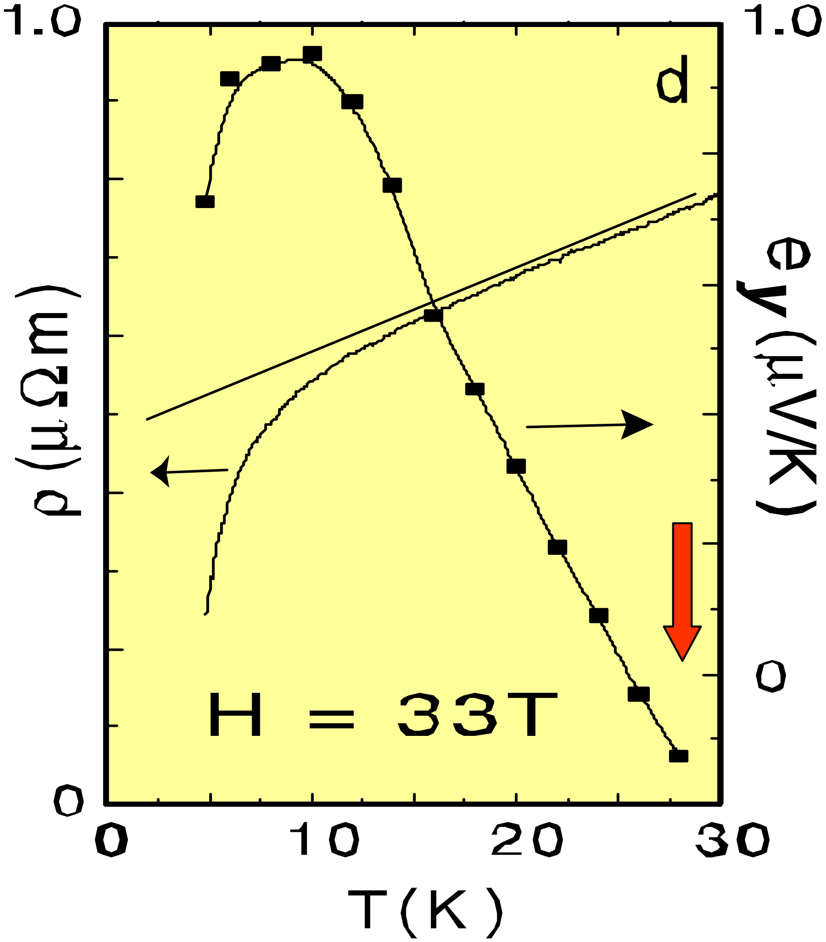
Field scale increases as x decreases

# Resistivity a bad probe for absence of pair amplitude

Plot of ρ and $e_y$ versus $T$ at fixed $H$ (33 T).

Vortex signal is large for $T < 26$ K, but ρ is close to normal value $\rho_N$ above 15 K.

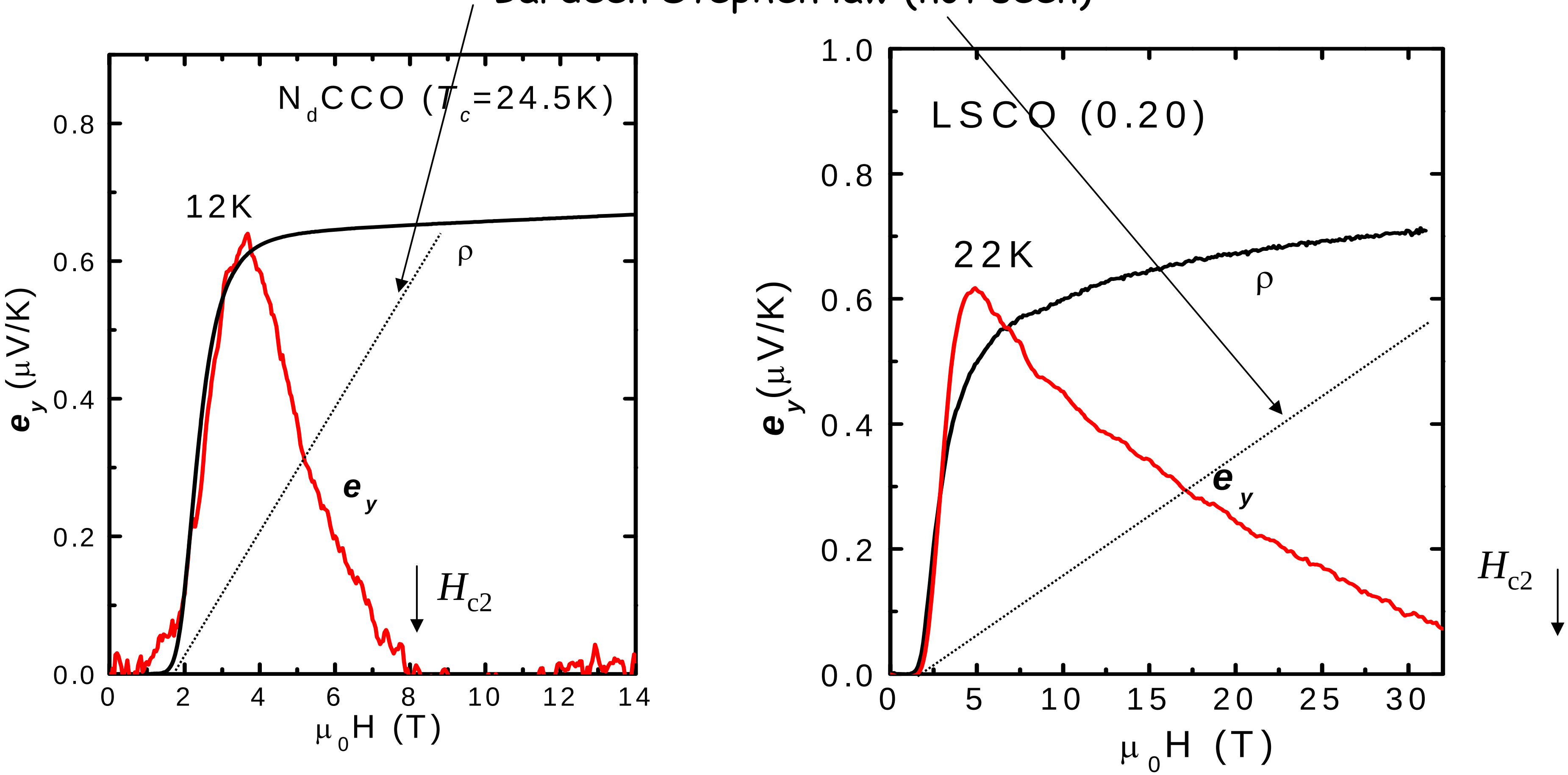
**LaSrCuO**



# Problems with Flux-flow Resistivity

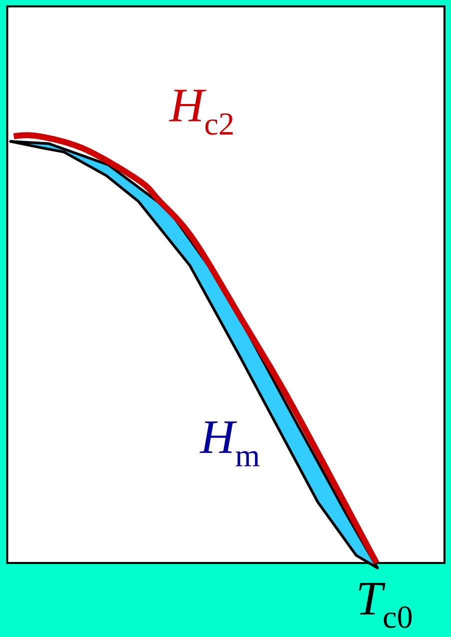
**Wang, Li, NPO PRB '06**

Bardeen Stephen law (not seen)



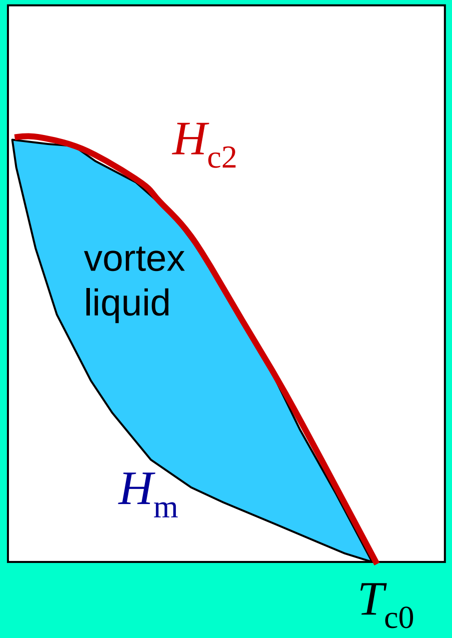
**Resistivity does not distinguish vortex liquid and normal state**

## NbSe$_2$



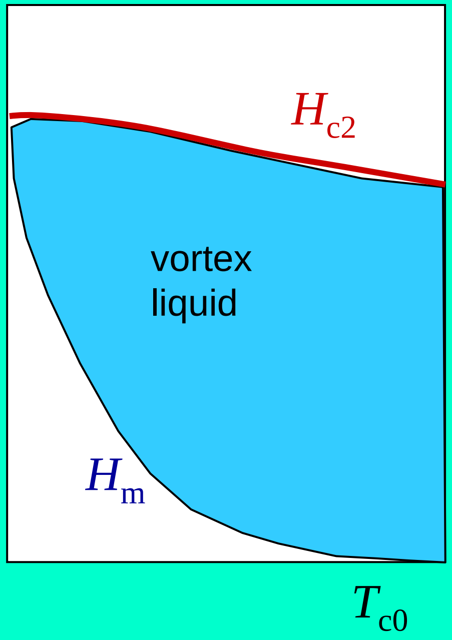
Conventional SC
Amplitude vanishes
at $T_{c0}$ (BCS)

## NdCeCuO



Expanded vortex liquid
Amplitude vanishes at
$T_{c0}$

## Hole-doped cuprates



Vortex liquid dominant.
Loss of phase coherence
at $T_{c0}$ (zero-field melting)

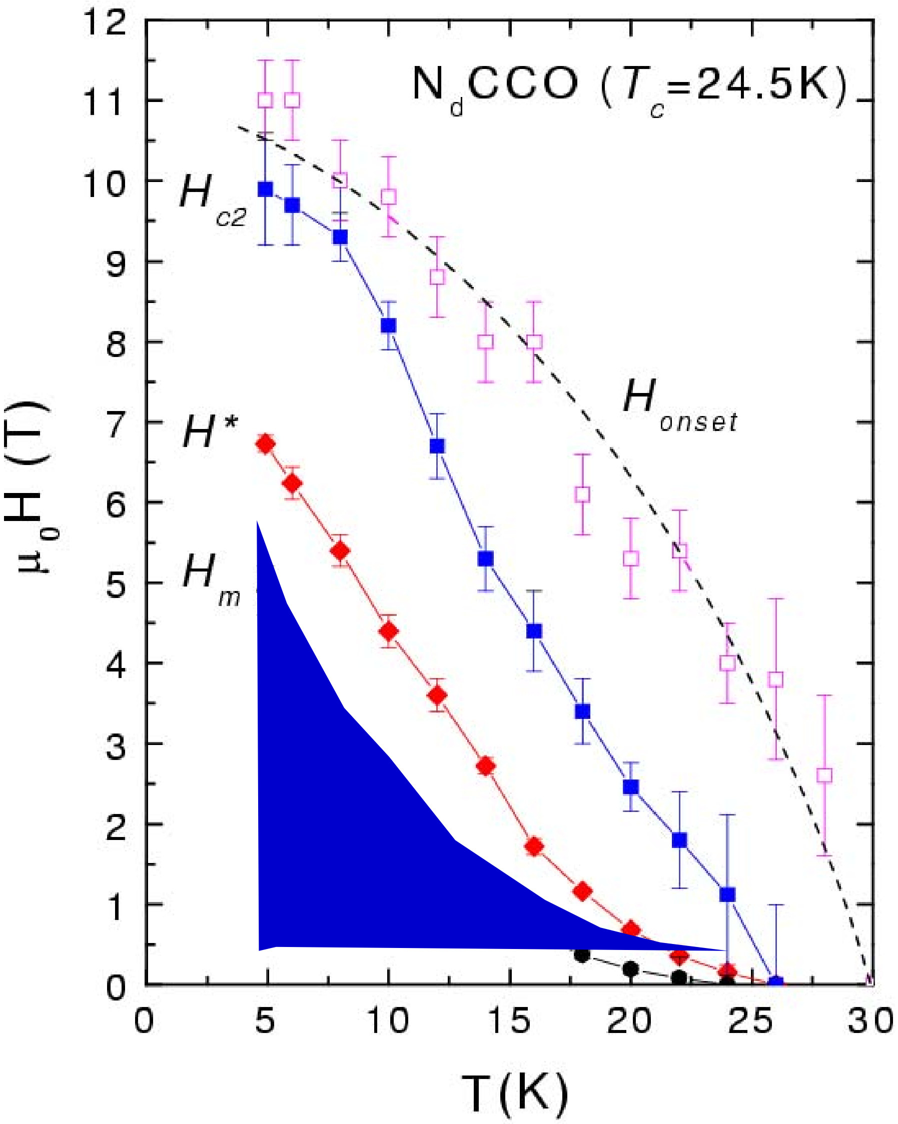
# Vortex Nernst signal in NdCeCuO -- mean-field scenario

**Plot of $H_m$, $H^*$, $H_{c2}$ vs. $T$**

- **$H_m$ and $H^*$ similar to hole-doped**

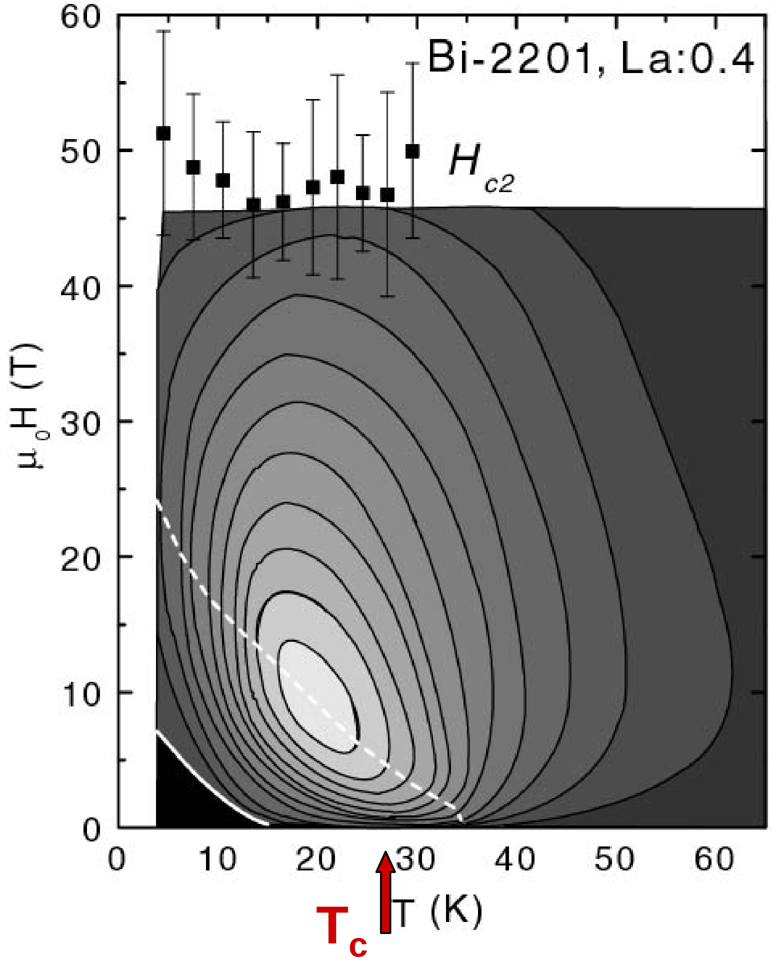
- **However, $H_{c2}$ has mean-field form**

- **Vortex-Nernst signal vanishes just above $H_{c2}$ line**



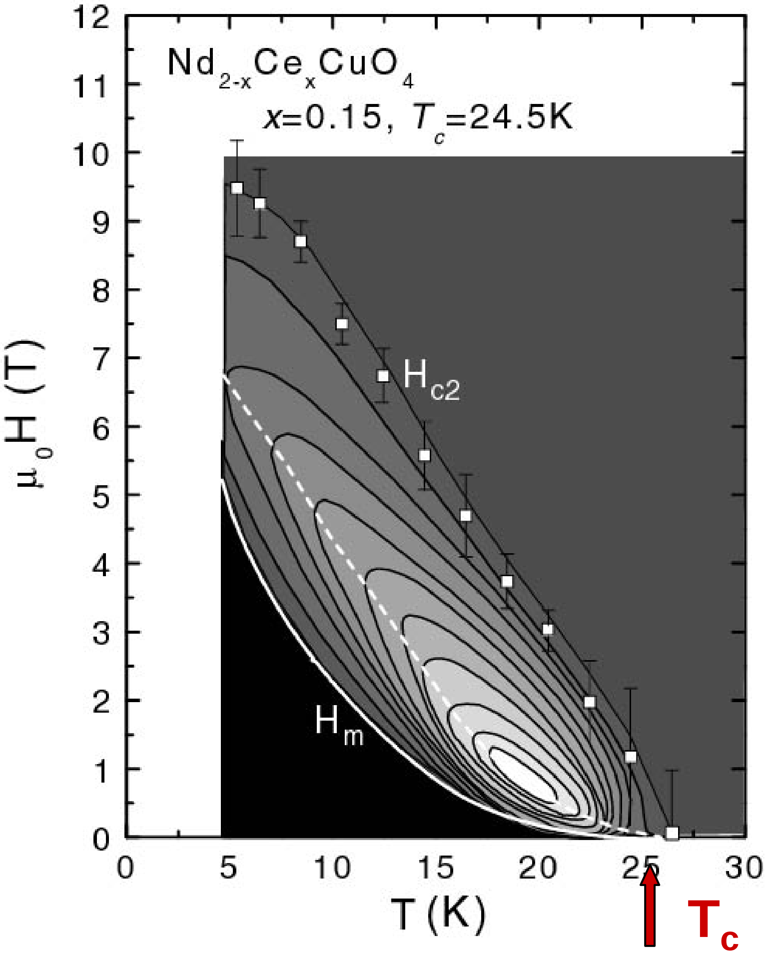
# Anomalous Nernst signal only in pseudogap state

Hole-doped optimal



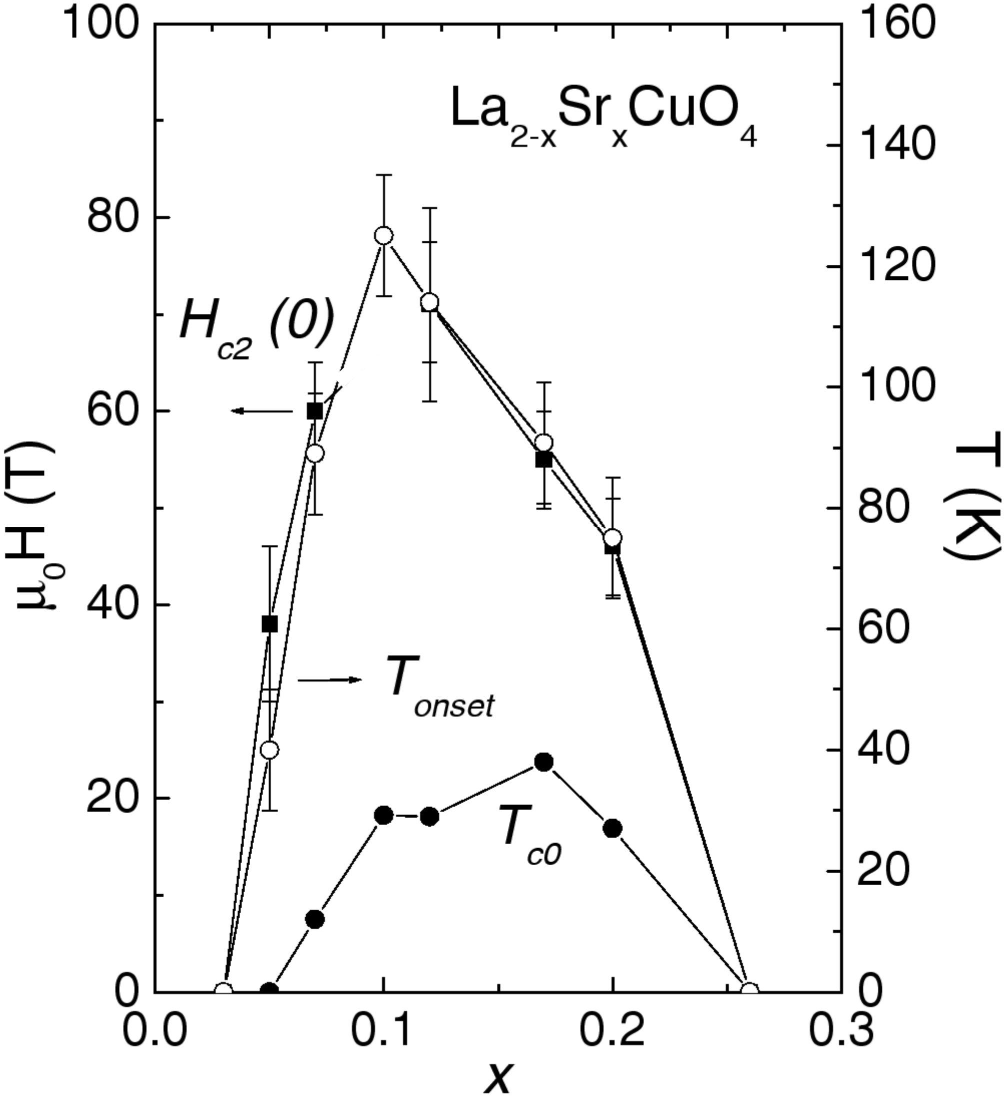
Strong phase fluctuations (non Gaussian)

Electron-doped optimal



Mean-field like (Gaussian fluctuations above Tc)

# $H_{c2}(0)$ vs x matches $T_{onset}$ vs x



# Summary

1. Existence of large vortex-Nernst region above $T_c$ dome

2. Transition at ***Tc*** not mean-field BCS, but loss of long-range phase correlation

3. Vortex liquid state extends high above ***Tc*** in UD regime

4. Upper critical field $H_{c2}$ is very large, 80-150 Tesla

*5.* $H_{c2}$ vs. $T$ dependence not of mean-field BCS form

# References (Talk 2)

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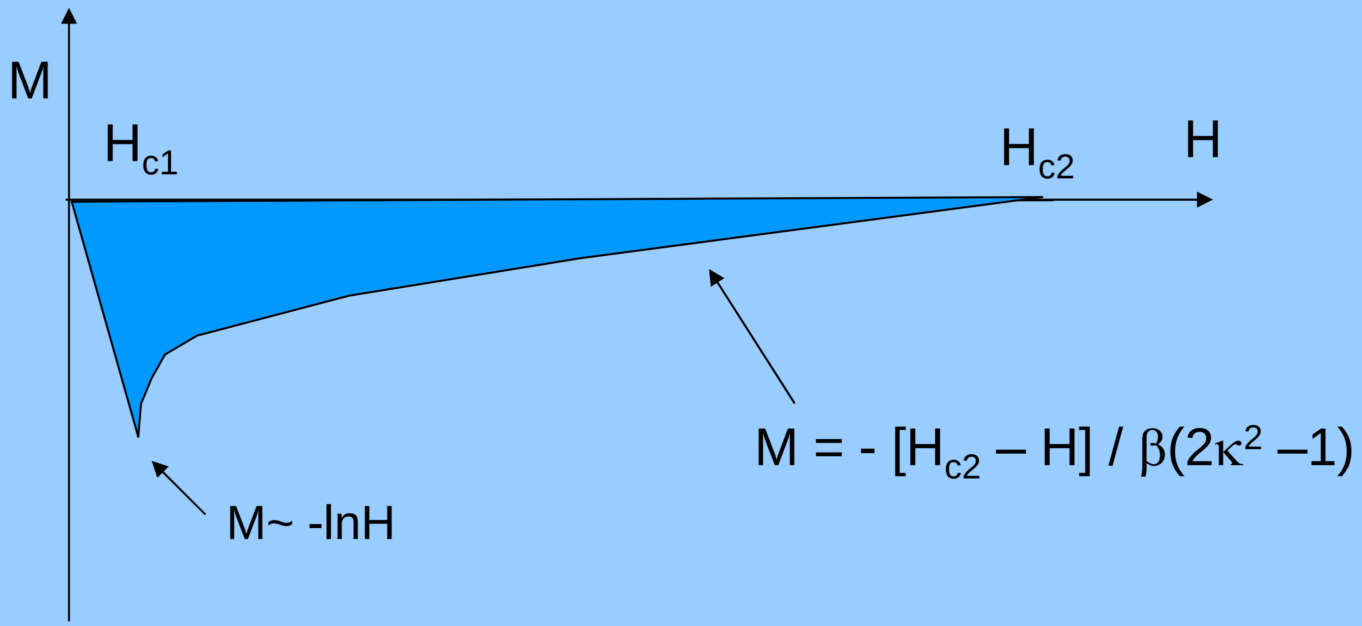
Phys. Rev. Lett. 96, 067002 (2006)

# Magnetization in Abrikosov state



$M = -[H_{c2} - H] / \beta(2\kappa^2 - 1)$

$M \sim -\ln H$

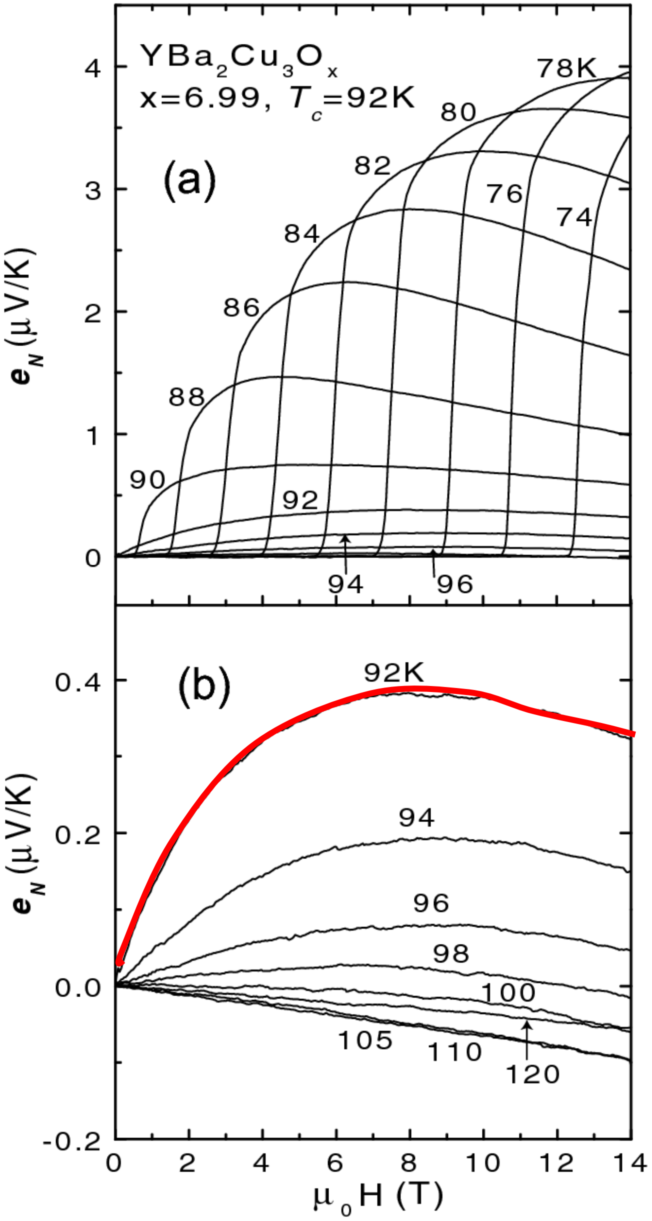
In cuprates, $\kappa$ = 100-150, $H_{c2}$ ~ 50-150 T

M < 1000 A/m (10 G)

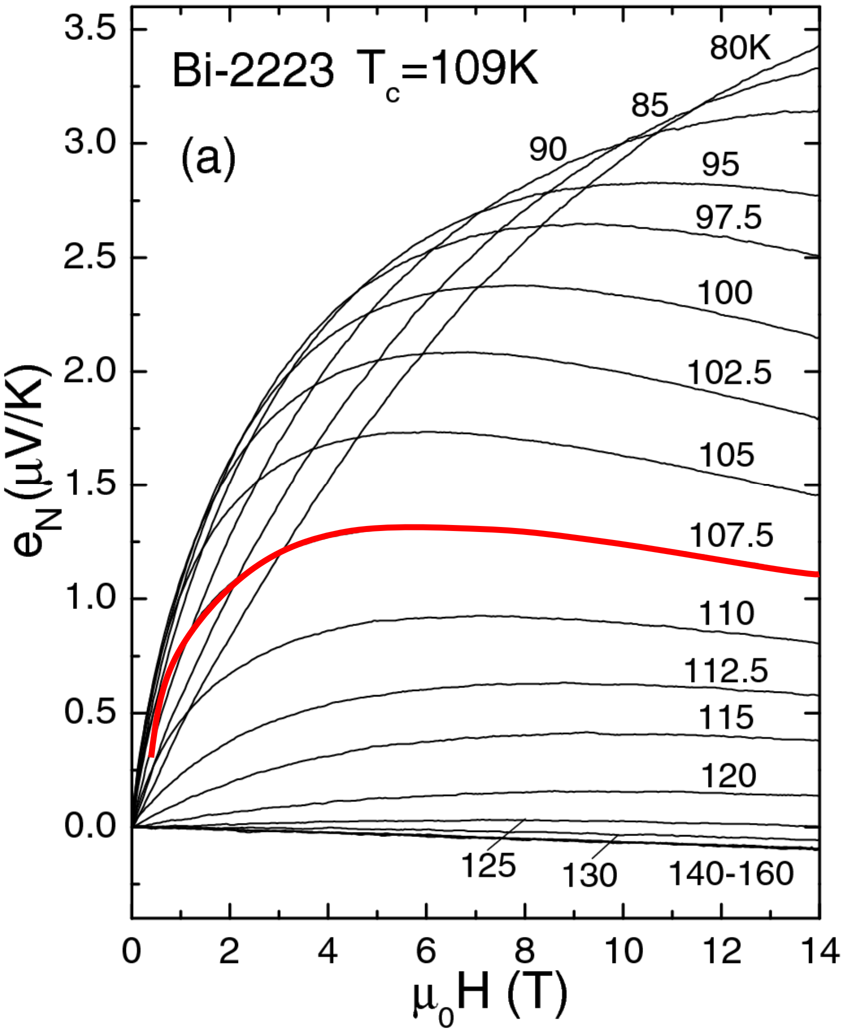
Area = Condensation energy U

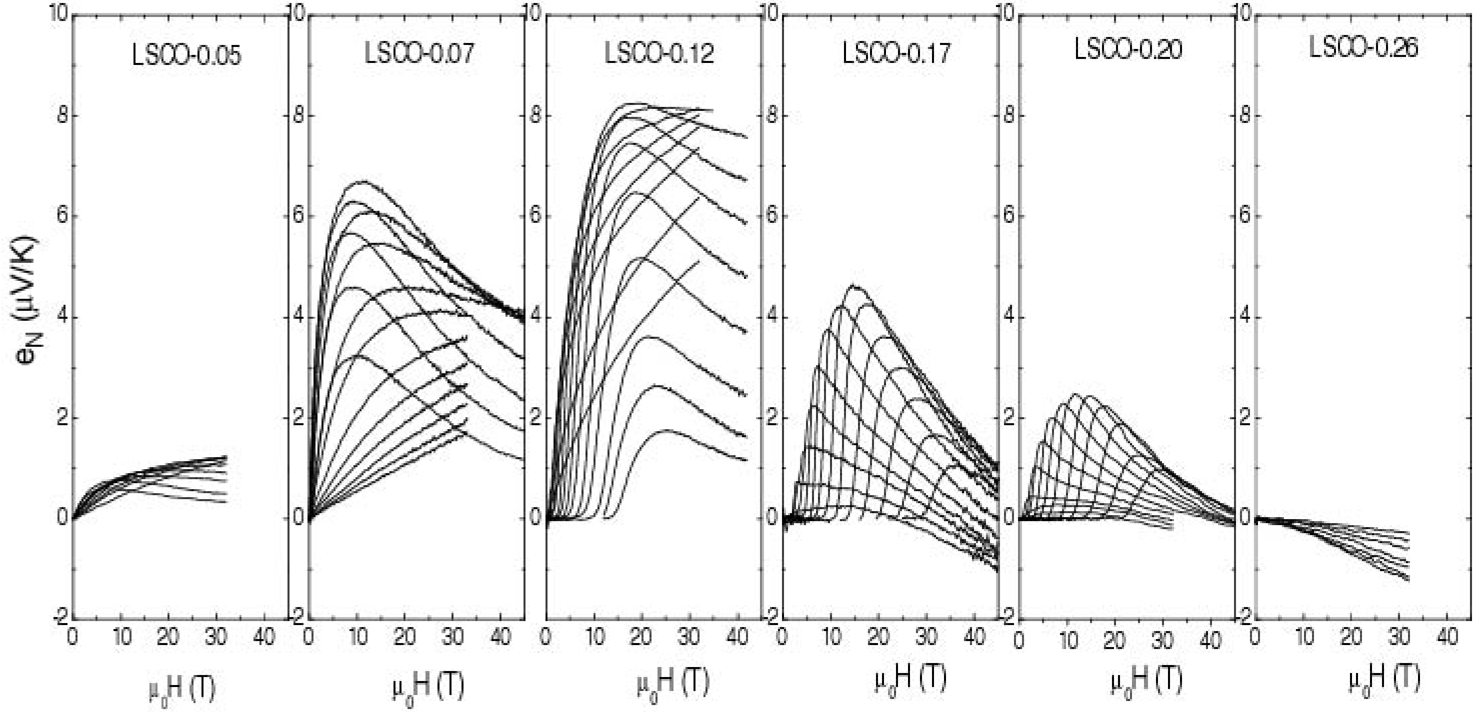
Wang, Li and NPO, PRB 06

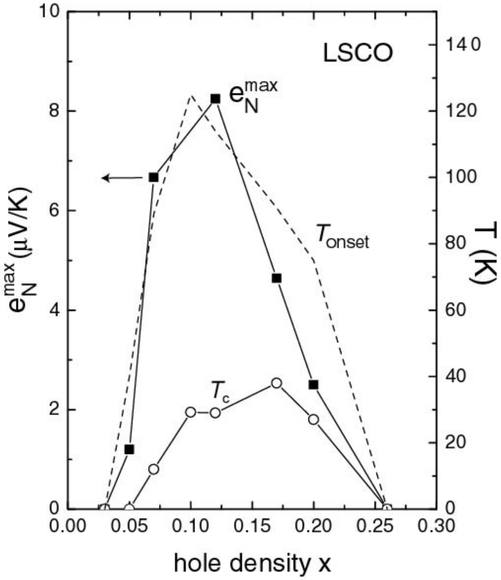
Optimal YBCO



3-layer Bi 2223




LSCO-0.05
LSCO-0.07
LSCO-0.12
LSCO-0.17
LSCO-0.20
LSCO-0.26
e_N (μV/K)
-2
0
2
4
6
8
10
0
10
20
30
40
μ_0H (T)


LSCO
e_N^max
T_onset
T_c
e_N^max (μV/K)
T (K)
hole density x
0
2
4
6
8
10
0
20
40
60
80
100
120
140
0.00
0.05
0.10
0.15
0.20
0.25
0.30