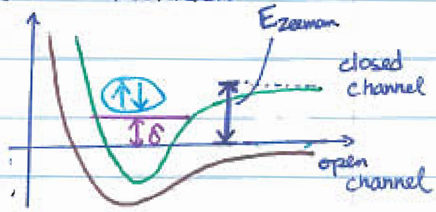
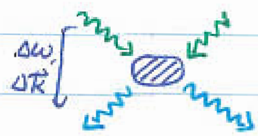
Resonant Atomic Gases [Radzihovsky]

- Overview of atomic physics
  - degenerate Bose and Fermi atomic gas ('95)
  - optical lattices (potential from AC-Stark effect)
    - Superfluid (SF) — Mott Insulator (MI) transition
  - Feshbach resonance.
    - The two channels have different spin contents, & coupled by hyperfine interaction



    - The detuning δ can be tuned by magnetic field
      $\Longrightarrow$ interactions can be weak/strong, and attractive/repulsive
    - paired superfluidity and BCS — BEC crossover

- Experimental Probes
  - Time-of-flight measurement
    - momentum distribution
    - temperature (fitting tail to non-interacting gas)
    - noise $\Longrightarrow$ pair correlation
    - vortices.
  - Thermodynamics (e.g. $C-T$ curve)
  - Transport
  - Bragg spectroscopy (input two counter-propagating lasers)
  - RF spectroscopy
  - k-resolved photoemission



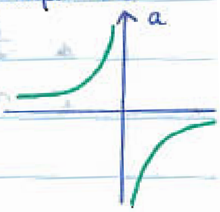
- S-wave Feshbach resonance 

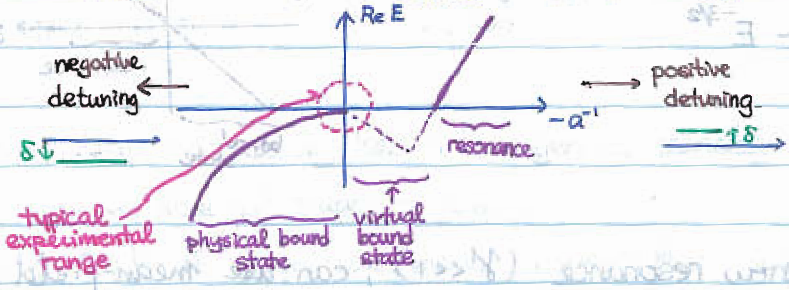
▲ $\mathcal{H} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m}\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$

atom field; molecular field; ∴ mass = 2m; "bare" detuning (δ is renormalized detuning); Feshbach resonance

$\Longrightarrow f_s(p) = \dfrac{1}{-a^{-1} + (r_0/2)p^2 - ip}$ is scattering amplitude

where $a \sim -g^2/\omega_0 \propto \dfrac{\Delta B}{B_0 - B}$; $r_0 \sim -1/g^2$

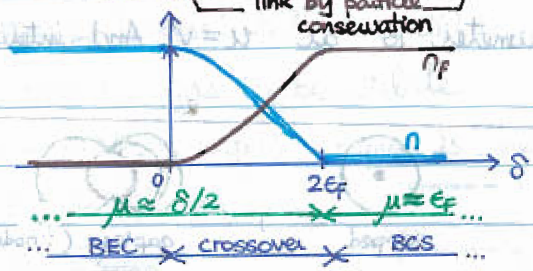




▲ For manybody at finite density,

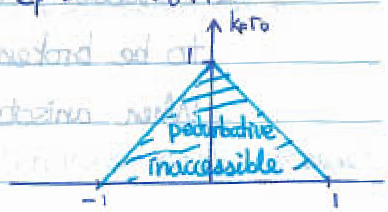
$\mathcal{H} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$

link by particle conservation



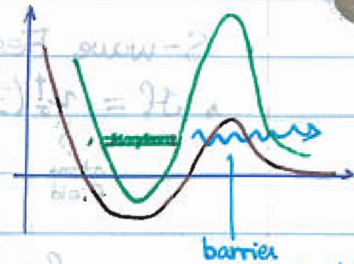
▲ The dimensionless coupling is $\gamma \sim \left(\frac{g\sqrt{n}}{\epsilon_F}\right)^2 \sim \frac{1}{r_0 n^{1/3}}$

- narrow resonance when $\gamma \ll 1$
  - $\Rightarrow$ mean-field valid
  - $\Rightarrow$ can replace $\phi(x) = B$.
- broad resonance when $\gamma \gg 1$
  - $\Rightarrow$ ϕ, ψ strongly coupled
  - $\Rightarrow$ mean-field not valid.



$[\gamma \approx \dfrac{|I_{k_F}| \cdot n}{\epsilon_F} \approx \dfrac{1}{(k_F a)^{-1} - k_F r_0}$

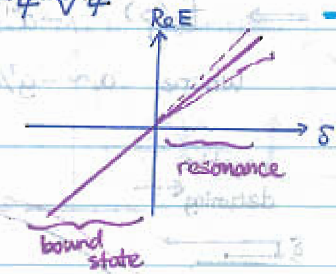
- P-wave Feshbach Resonance



$$\mathcal{H} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m}\right)\psi_\sigma + \vec{\phi}^\dagger\left(\frac{\hat{p}^2}{4m} - \epsilon_0\right)\vec{\phi} - ig\vec{\phi}\cdot\psi^\dagger\vec{\nabla}\psi$$

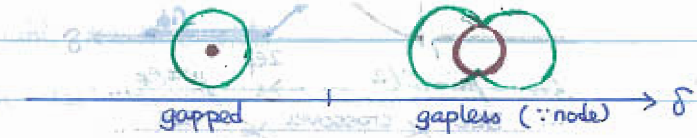
▲ $f_p = \dfrac{q^2}{-v^{-1} + (r_0/2)q^2 - iq^3}$

▲ $\tau \sim E^{-3/2}$



▲ For narrow resonance $(\gamma \ll 1)$, can use mean-field,
$\phi(x) = \vec{B} = \vec{u} + i\vec{v}$.

▲ In isotropic resonance, in both BEC and BCS limit, the order parameter is at $u = v$. And interaction is always ferromagnetic.



▲ However, rotation invariance in Feshbach resonance turn out to be broken. Thus p_x may be preferred.
When anisotropy is moderate