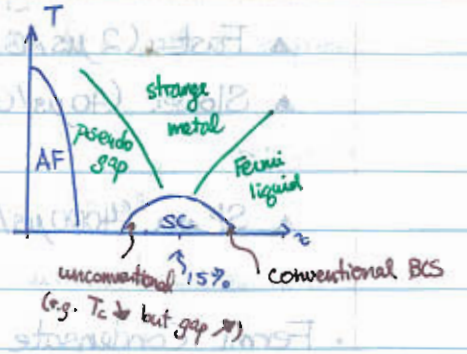
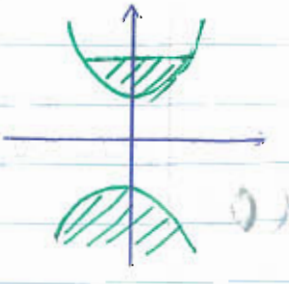
1D – 2D Dimensional Crossover [Le Hur]

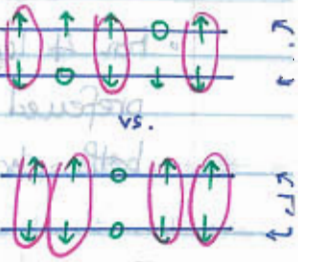
- High-T_c Cuprate
  - Coupled CuO_2 layer
  - Antiferro $\xrightarrow{\text{dope}}$ superconductor
  - NOT phonon mediated?
  - From LDA+U, $U \sim 2eV$, $J \sim 4t^2/U$
  - $\uparrow$ $3d_{x^2-y^2}$ —
    $\uparrow\downarrow$ $3d_{3z^2-r^2}$ $\uparrow\downarrow$
    (undoped) (doped)
  - In principle it's more energetically favored for hole to go to O-site. But then the $O^- - Cu^{2+}$ & $O^{2-} - Cu^{3+}$ states together form singlet. The effect of singlet is essentially the same as Cu^{3+} state.
    So it is possible to describe using a one-band model.
  - Doped system still has residual AF correlation, but it becomes short-ranged.
  - Doped system also has remanent of the $U \sim 2eV$ charge gap
  - Gap opens in anti-nodal points and persist beyond T_c (hence the name pseudo gap).
  - Many channels, approximation needed.



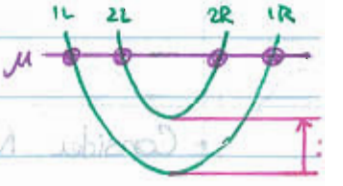
- In 1D, it is known that for half-fill and $U<0$, Umklapp process produces Mott gap $\sim e^{-\epsilon_F/U}$
  - The doped case can be analysed by mapping to doped band insulator
    $g \sim 1/2$ and $v \sim \delta$ for all U.



- Now consider 2-chain,,



  - ▲ At large $J_\perp$ holes form singlet
    $\Longrightarrow$ charge gap and spin gap
  - ▲ The conclusion remains true for small $J_\perp$.



- Next consider weak coupling

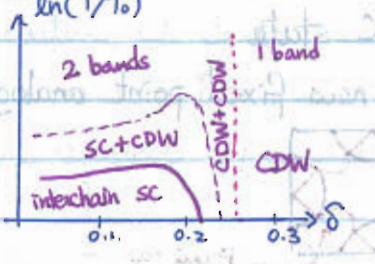
$$\epsilon_j(k) = \mp t_\perp - 2t\cos(k)$$

$$\Longrightarrow \begin{cases} V_1 = V_2 \\ k_{F1} + k_{F2} = \pi \end{cases} \quad \text{at half-filling.}$$

$$\Longrightarrow V_2 \ll V_1 \quad \text{for large doping}$$

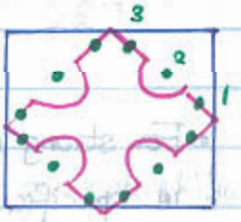
  - ▲ Interaction channel can be interpret in 2D picture:

  - ▲ RG analysis: neglecting Umklapp, forward scattering flows the Cooper interaction to strong coupling regime.

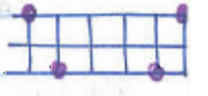


- Can repeat the exercise in 3-leg ladder:

$$\epsilon_j(k) = -2t\cos(k) - 2t_\perp \cos(k_{Fj}^y)$$



  - ▲ Mott gap is opened, but charge gap does not exists.
  - ▲ In real-space the hole resides only on outer legs

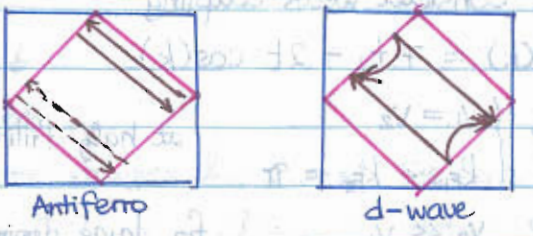


  - ▲ The Mott gap flows to strong coupling faster on band 1 & 3 than band 2.

- For 4 legs, pairing between outer leg (1-4) is prefered to between inner leg (2-3). And we see both charge and spin gap.

- In general, { N even ⇒ spin gap
  N odd ⇒ no spin gap

- Consider N bands.

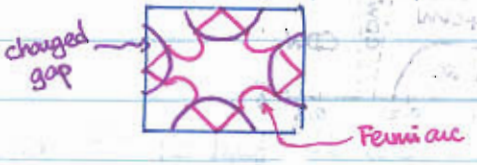


▲ N large ⟹ AF channel prefered to d-wave SC channel. at half-filling.

▲ When doped, the AF channel are cutoff for $\delta > (t_\perp/t)/N$ ⟹ SC become stabilized.

- For a pure t-U model, renormalization gives only 2 fixed points, namely AF state & d-SC state.

▲ Adding t′ produces a new fixed point analogous to pseudo-gap.



- For strong coupling we have t-J model. If we decouple the J-term in specific way we find d-SC state

e.g. $J\sum(\vec{S}_i\cdot\vec{S}_j - n_i n_j/4) \longrightarrow -J\sum b_{ij}^\dagger b_{ij}$

$b_{ij}^\dagger = \frac{1}{\sqrt{2}}(c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger - c_{i\downarrow}^\dagger c_{j\uparrow}^\dagger)$

▲ But the specific way of decoupling is biased. One should keep as many channels as possible.