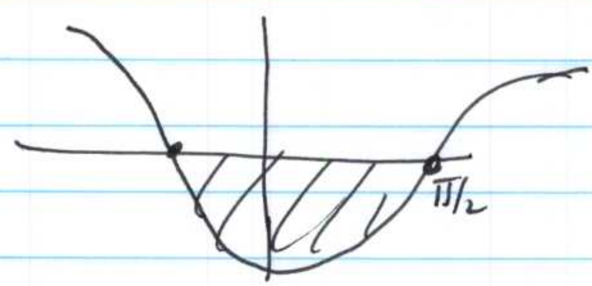
Lecture II Shankar July 2, 08

A trial run with RG.

Spinless fermions at half-filling in $d=1$

$$H_0 = -\sum \psi_n^\dagger \psi_{n+1} + hc$$

$$= -\int dK \; \psi^\dagger(K)\,\psi(K)\cos K$$



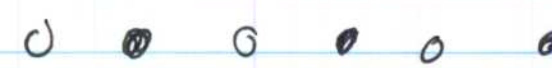
is a gapless system. Now add

$$H_1 = U \sum \psi_n^\dagger \psi_n \, \psi_{n+1}^\dagger \psi_{n+1}$$

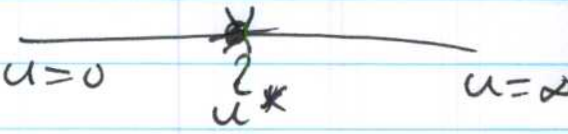
$U = 0$ conductor

$U \to \infty$ CDW

or



When does it change



Mean field theory:

$$\chi_{CDW} = \infty \qquad \text{CDW forms at } U = 0^+$$

$$\chi_{BCS} = \infty \qquad \text{SC " " } U = 0^-$$

What actually happens? We use RG.

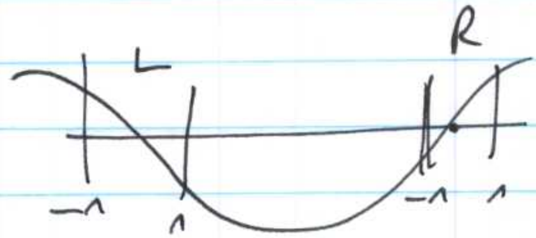
<u>Tactic</u> $\quad H_0 \to \int d\bar{\Psi}\, d\Psi\, e^{S_0}$

Find RG such that $\{ S_0 \to S_0$ (a fixed point) $\}$

now add <u>any</u> $H_1 \to S_1$ and see

if S_1 is rel, irrel. or marginal

$$H_0 = \int dk\, \psi^\dagger(k)\psi(k) \cos k$$



Keep just a band $k_F \pm \Lambda$ at L & R

Fermi points. Linearize there

$$\boxed{H_0 = \sum_{\alpha = L,R} \psi_\alpha^\dagger(k)\psi_\alpha(k)\, k}$$

Recall that if

$$H = \omega_0 \psi^\dagger \psi$$

$$Z = \int [d\bar\psi d\psi]\, e^{\int \bar\psi(\omega)[i\omega - \omega_0]\psi(\omega)\frac{d\omega}{2\pi}}$$

Now we have an osc. at each k and α with $\omega_0 = k$. Thus

$$Z = \int (d\bar\psi d\psi)\, e^{S_0}$$

$$S_0 = \sum_\alpha \int_{-\infty}^{\infty}\int_{-\Lambda}^{\Lambda} \bar\psi(\omega k)[i\omega - k]\psi(\omega k)\frac{d\omega\, dk}{(2\pi)^2}$$

is a Gaussian action.

RG $\Lambda \to \Lambda/s$ → change cut-off

$$\left.\begin{aligned} k' &= sk \\ \omega' &= s\omega \end{aligned}\right) \text{ rescale momenta, freq}$$

$$S_0 \to \sum_\alpha \int_{-\infty}^{\infty}\int_{-\Lambda}^{\Lambda} \bar\psi\left(\tfrac{\omega'}{s}, \tfrac{k'}{s}\right)\left[\frac{i\omega' - k'}{s}\right]\psi\left(\tfrac{\omega'}{s}, \tfrac{k'}{s}\right)\frac{d\omega'\, dk'}{s^2}$$

So we let $\underbrace{\psi\left(\tfrac{\omega'}{s}, \tfrac{k'}{s}\right) = \psi'(\omega', k')\, s^{3/2}}_{\text{Rescale field}}$

Then $S_0 \to S_0$

Why we must ensure $S_0 \to S_0$: apples → apples.

Now we add not $u\,\psi_n^+\psi_n\,\psi_{n+1}^+\psi_{n+1}$ but most general ~~term~~ term

$$S_4 = \int_{-\infty}^{\infty}\!\!\int\!\!\int_{-\Lambda}^{\Lambda} d\omega_1 \ldots d\omega_3\, dk_1 - dk_3 \;\; u_{\alpha\beta\gamma\delta}(4,3,2,1)\, \bar{\psi}_\alpha(4)\, \bar{\psi}_\beta(3)\, \bar{\psi}_\gamma(2)\, \psi_\delta(1)$$

$4 = k_4, \omega_4$ etc.

$u_{\alpha\beta\gamma\delta}(4,3,2,1)$ = coupling function.

Do tree level RG

$$\int \frac{d\omega_1' \ldots d\omega_3'\, dk_1' \ldots dk_3'}{s^6}\; \bar{\psi}_\alpha(4') \ldots \psi_\delta(1')\, s^6$$

$$u_{\alpha\beta\gamma\delta}\left(\frac{4'}{s}, \frac{3'}{s} \ldots\right)$$

So

$u'(k') = u(k'/s)$ $\qquad k = k, \omega$

$\alpha\beta\gamma\delta$ suppressed

$u(k) = u_0 + u_1 k + u_2 k^2 \ldots$

$u_0' + u_1' k' + u_2' k'^2 \ldots = u_0 + u_1 \frac{k'}{s} + u_2 \frac{k'^2}{s^2}$.

$u_0' = u_0$

$\left.u_1' = \frac{u_1}{s} \text{ etc}\right\}$ irrelevant

all 6 pt coupling irrelevant even ~~at~~ leading term.

Summary: Only survivor $u_{\alpha\beta\gamma\delta}$ with no ω or k dep. There seem to be 2^4 of them. Actually there is just 1.

$$\int u_{LLRR} \quad \bar{\psi}_L(4)\,\bar{\psi}_L(3)\,\psi_R(2)\,\psi_R(1) \quad dk_3\,d\omega_3 \cdots dk_1\,d\omega_1$$

$$\psi_R(2)\,\psi_R(1) = -\psi_R(1)\,\psi_R(2)$$

$$= -\psi_R(2)\,\psi_R(1) \qquad 2 \leftrightarrow 1$$

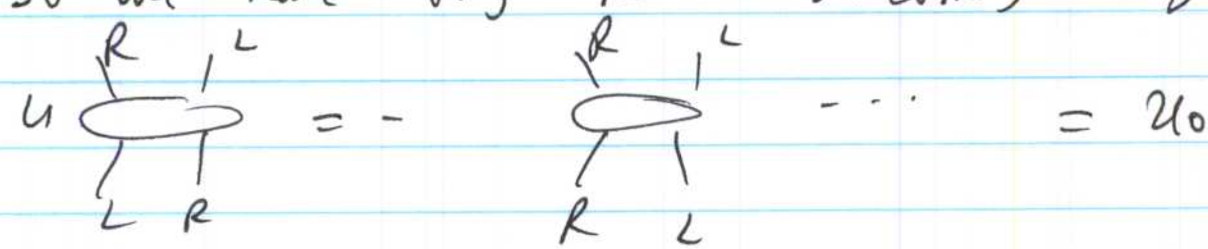
needs $u_{LLRR}(k_4 - k_1)$

$$\to -u_{LL}(k_4 k_3,\ k_1 k_2)$$

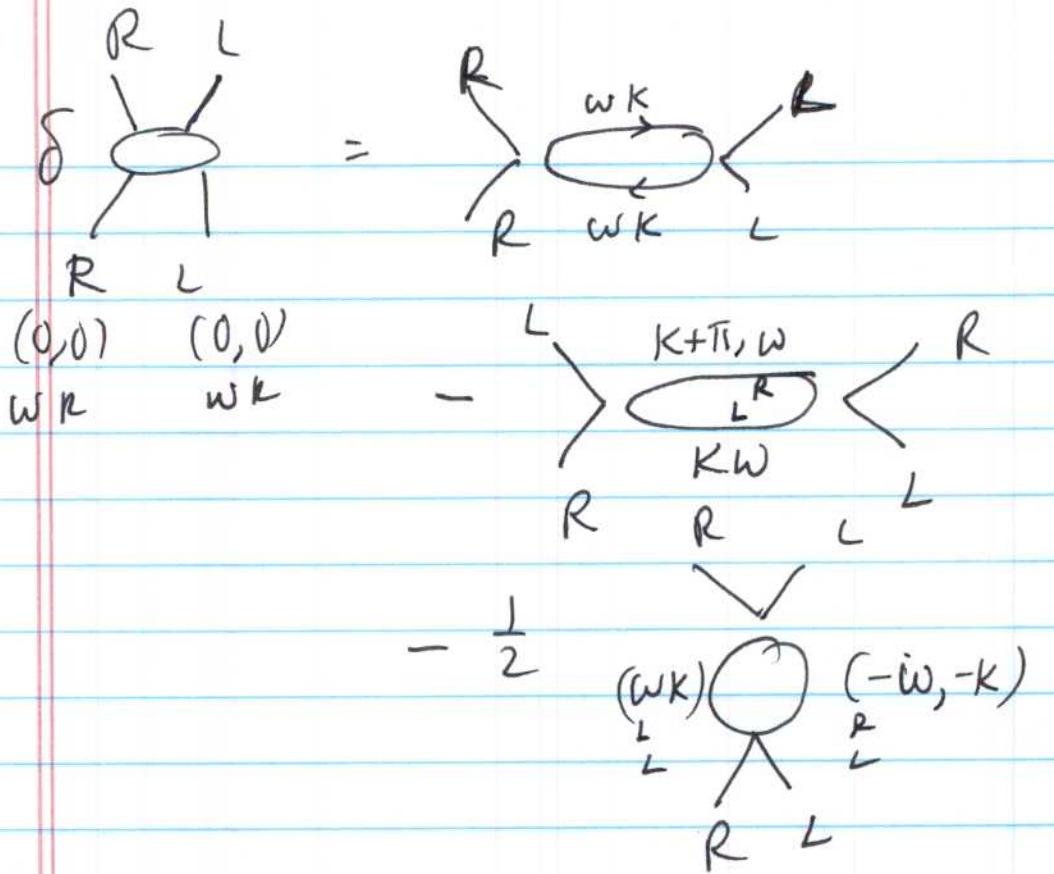
But u has no dependence.

Eg $u = (k_4 - k_3)(k_2 - k_1)\,\bar{u}$ is fine but has too many k's $\left(\frac{1}{s^2}\right)$

So we have only R L incoming & outgoing



Need to go to 1 loop to see fate of u_0



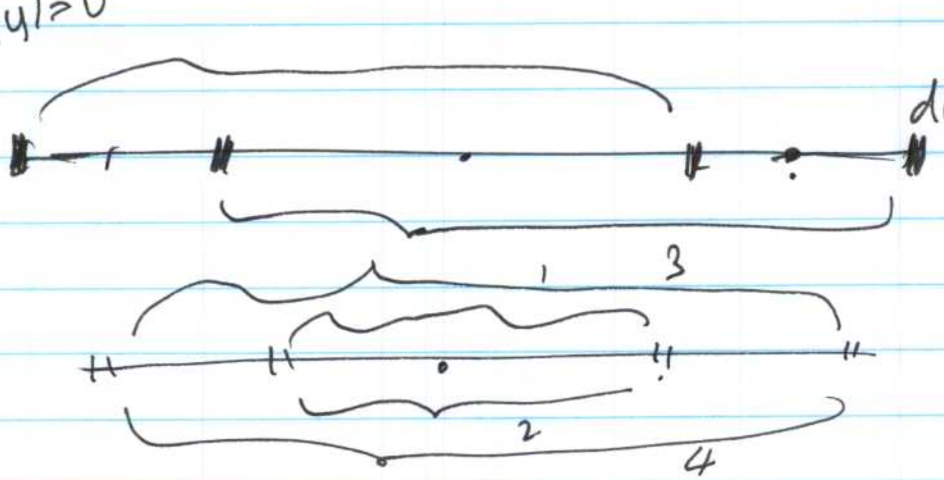
$$E(k+\pi) = -E(k)$$

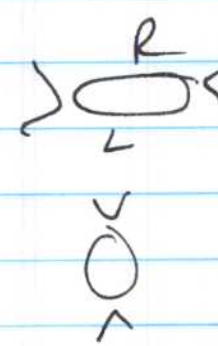
$$\delta u_0 = \cancel{u_0^2}\, u^2 \iint \frac{dk\, d\omega}{(i\omega - E(k))^2}$$

$$- u^2 \iint \frac{dk\, d\omega}{[i\omega - E(k)]\,[i\omega + E(k))}$$

$$- 2 \times \frac{1}{2} u^2 \iint \frac{dk\, d\omega}{[i\omega - E(k))\,(-i\omega - E(-k))}$$

$\beta(u) = 0$

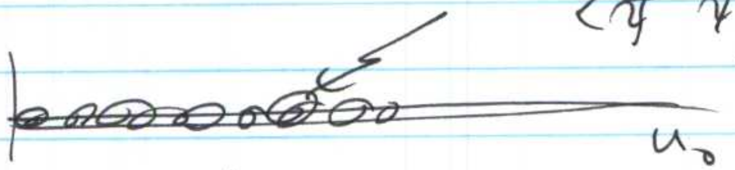




$\beta(u) = 0$ unbiased result

neither CDW nor SC wins

$\langle \bar{\psi} \psi \rangle_{\substack{red \\ \omega}} \sim \frac{1}{(\omega - k + i\varepsilon)^{\alpha}}$ no pole in red ~~lam~~



Luttinger liquid

seems like RG invariant state $\forall\ u_0$

But we know a gap must develop.

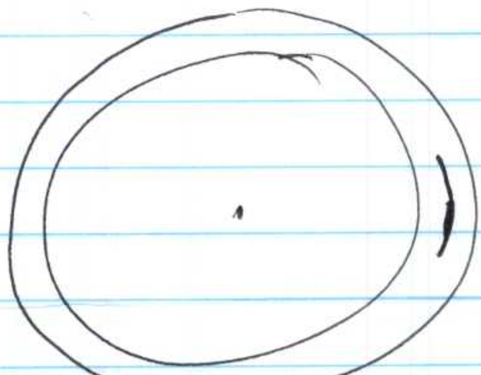
Umklapp $RR \to LL$ highly irrel.

becomes marginal

and then relevant at some u^*.

now go on to $d=2$

The question in old days: Is $d=2$ like $d=1$, where there is no Fermion pole?



Use same trick

$$S_0 = \int\int\int d\theta_1\, dk\, d\omega\; \bar{\psi}(\omega k \theta)\,(i\omega - k)\,\psi(\omega k \theta)$$

$\Lambda \to \Lambda/s \quad k' = sk, \quad \omega' = s\omega \quad \psi = s^{3/2}\psi'$

$u'(k') = u(k'/s)$ only u_0 lives

but now $\underline{\underline{u = u(\theta_1 \cdots \theta_4)}}$

<u>No need for $\bar{\psi}\bar{\psi}\bar{\psi}\psi\psi\psi$</u>

$u(\theta_1\, \theta_2\, \theta_3\, \theta_4) = u(\theta_1\, \theta_2\, \theta_3)$ due to mom cons

$= u(\theta_1, \theta_2)$ 

$= u(\theta_1 - \theta_2)$ rot-inv 

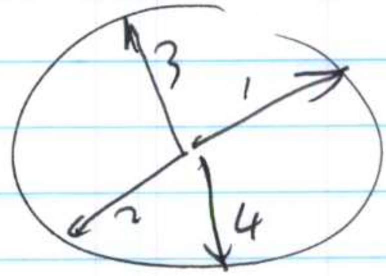
$= F(\theta)$ - Landau.

Low energy physics of Fermion is determined by $F(\theta)$. (determined by microscopic theory).

$$\begin{matrix} 3=1 \\ 2=4 \end{matrix} \longrightarrow u(\theta_1, \theta_2)$$

One exception is $1+2=0$

then we need $3+4=0$ cooper



$v(\theta_3-\theta_1)$

$= v(\theta)$

Landau set this $=0$.

He wrote

$$E(\delta n) = \sum_k k\,\delta n_k + \sum_{kk'} f(k,k')\,\delta n_k\,\delta n_{k'}$$

why stop at $(\delta n)^2$?

and $(\delta n)^3$

These two are marginal.

$(\delta n)^3$ irrelevant