Driven Colloids III: Jamming

Cynthia Reichhardt

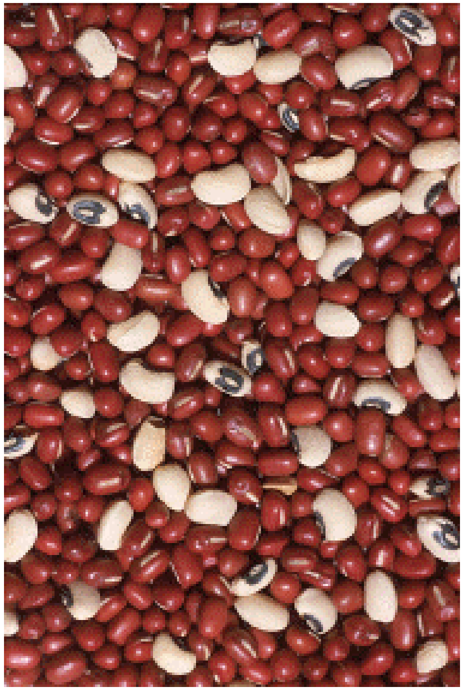
Theoretical Division

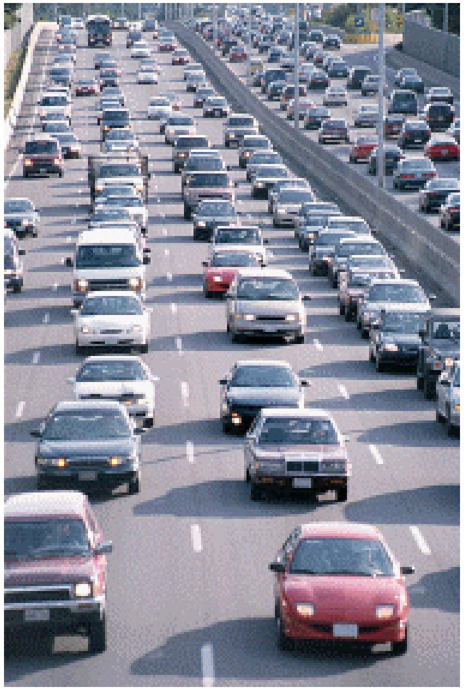
Los Alamos National Laboratory

Outline

• Granular media and the jamming phase diagram. Is “Point J” a type of phase transition? Is it related to glassiness?
• Our approach: Simulation of a disordered 2D packing of disks.
• Evidence of a diverging length scale when a single grain is driven through a packing
• Velocity fluctuations change character significantly as the jamming transition is approached
• Quenched disorder as a fourth axis of the jamming phase diagram: Clogging vs jamming transition

Jamming





Jamming in granular media

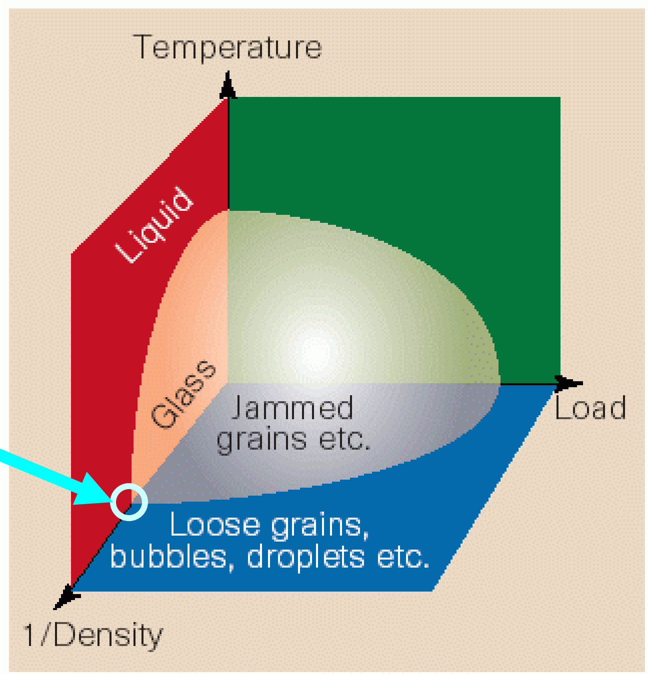
• Jamming: Development of a resistance to shear

• Grains jam but things like superconducting vortices (usually) do not. Why? Interaction range.

- Medium to long range: particles sense each other’s approach and shift out of the way
- Short range (granular): particles do not know they are going to interact until it is too late

Jamming Phase Diagram and "Point J"

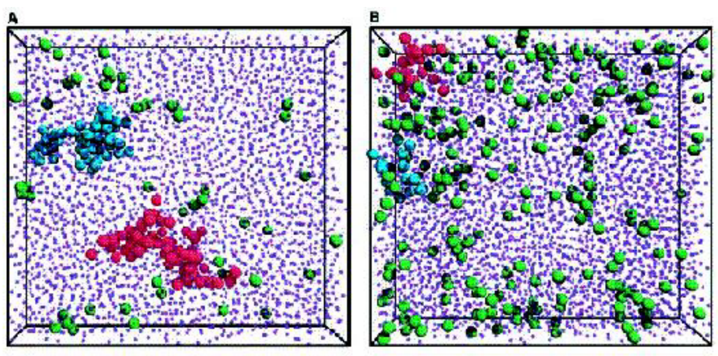
Point J



A.J. Liu, S.R. Nagel,
Nature 396, 21 (1998)

Are jamming and the glass transition the same?

"The deepest and most interesting unsolved problem in solid state theory." –PW Anderson, Science 267, 1615 (1995)

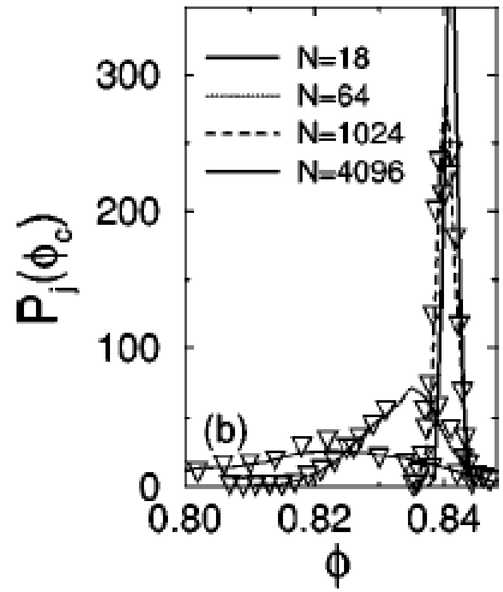


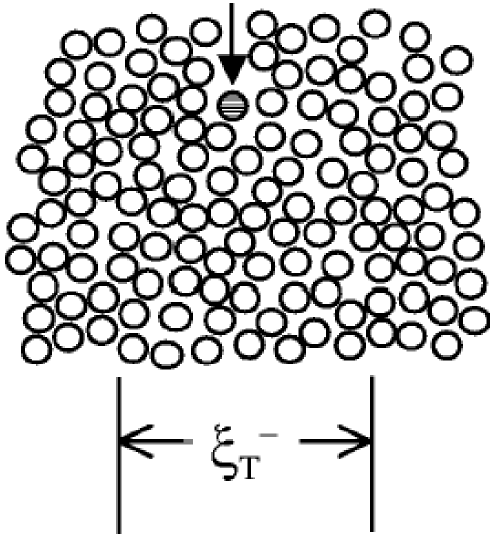
Finite temperature colloidal glass

ER Weeks et al, Science 287, 627 (2000)

The glass transition is at finite temperature where relaxation will be extremely important. Point J is nonthermal; however, Point J is well-defined.

Diverging length scale at jamming transition?





$$\phi_0 - \phi^* = \delta_0 L^{-1/\nu}.$$

$$\xi = (\phi_c - \phi)^\nu$$

ν=0.71

C.S. O'Hern, L.E. Silbert, A.J. Liu, S.R. Nagel,
PRE 68, 011306 (2003)

Tabletop Experiment

Low density

High density

Simulation

2D Granular dynamics simulation

Binary mixture of disks of radii $r_A : r_B = 1.4 : 1$.

$L = 24$ to 60; 2600 disks at $\phi = 0.8395$.

Initialization: Thermalize while adding disks.

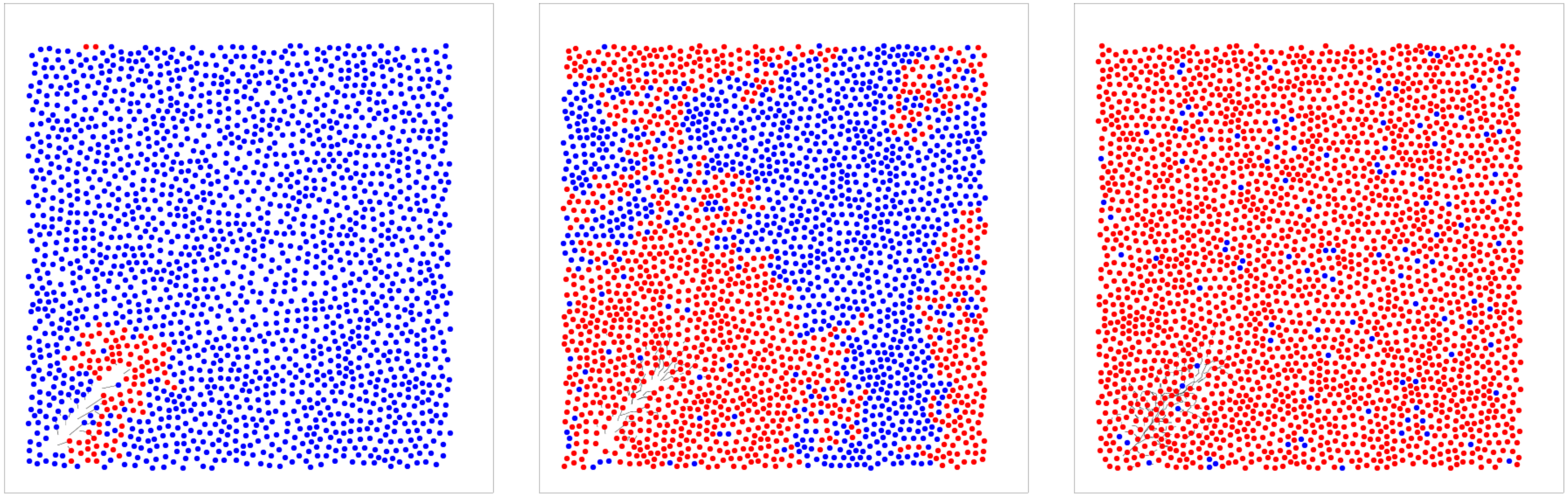
$$\eta \mathbf{v}_i = \sum_{i \neq j} k_g (|\mathbf{r}_{ij}| - r_{eff}) \frac{\mathbf{r}_{ij}}{|\mathbf{r}_{ij}|} + \mathbf{f}_d$$

$$|\mathbf{r}_{ij}| < r_{eff} = r_g^{(i)} + r_g^{(j)} \quad ; k_g = 200$$

Increasing Density

Unjammed

Jammed

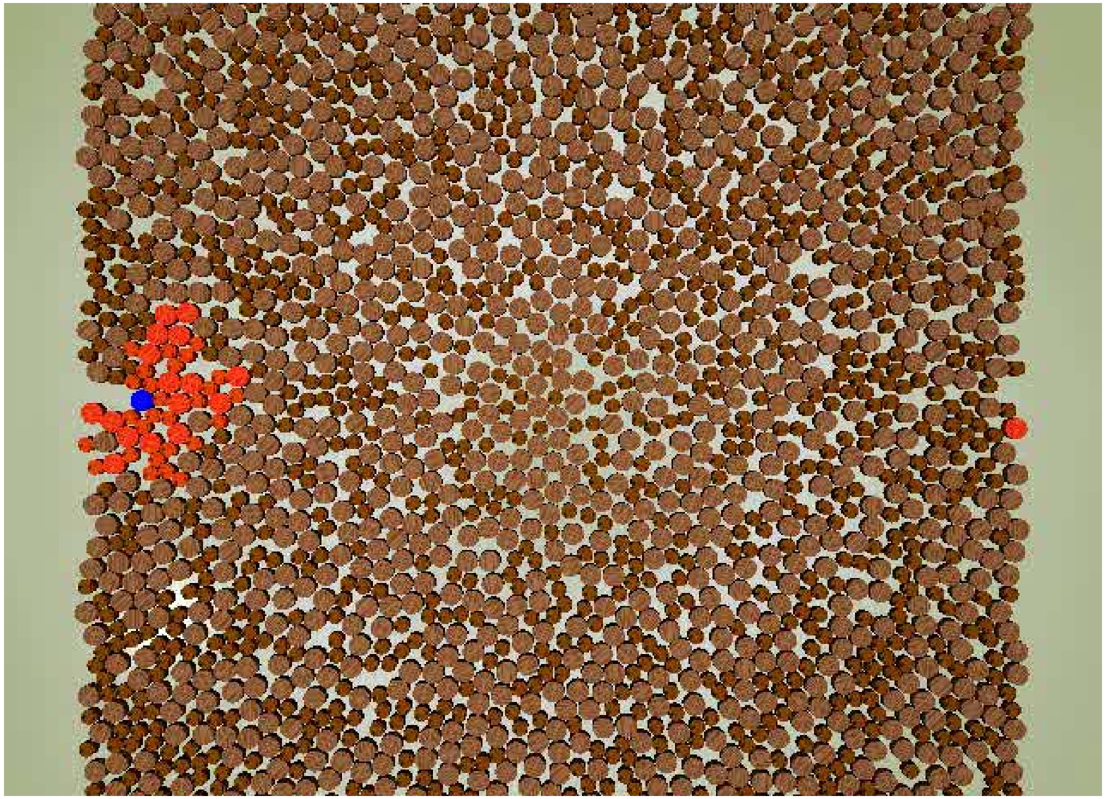


0.656 0.811 0.837

Blue: not moving. Red: force contact with driven disk.

Well below jamming

Brown disks: stationary
Red disks: in force contact with driven disk
Blue disk: driven



ϕ=0.67

Close to jamming

Brown disks: stationary

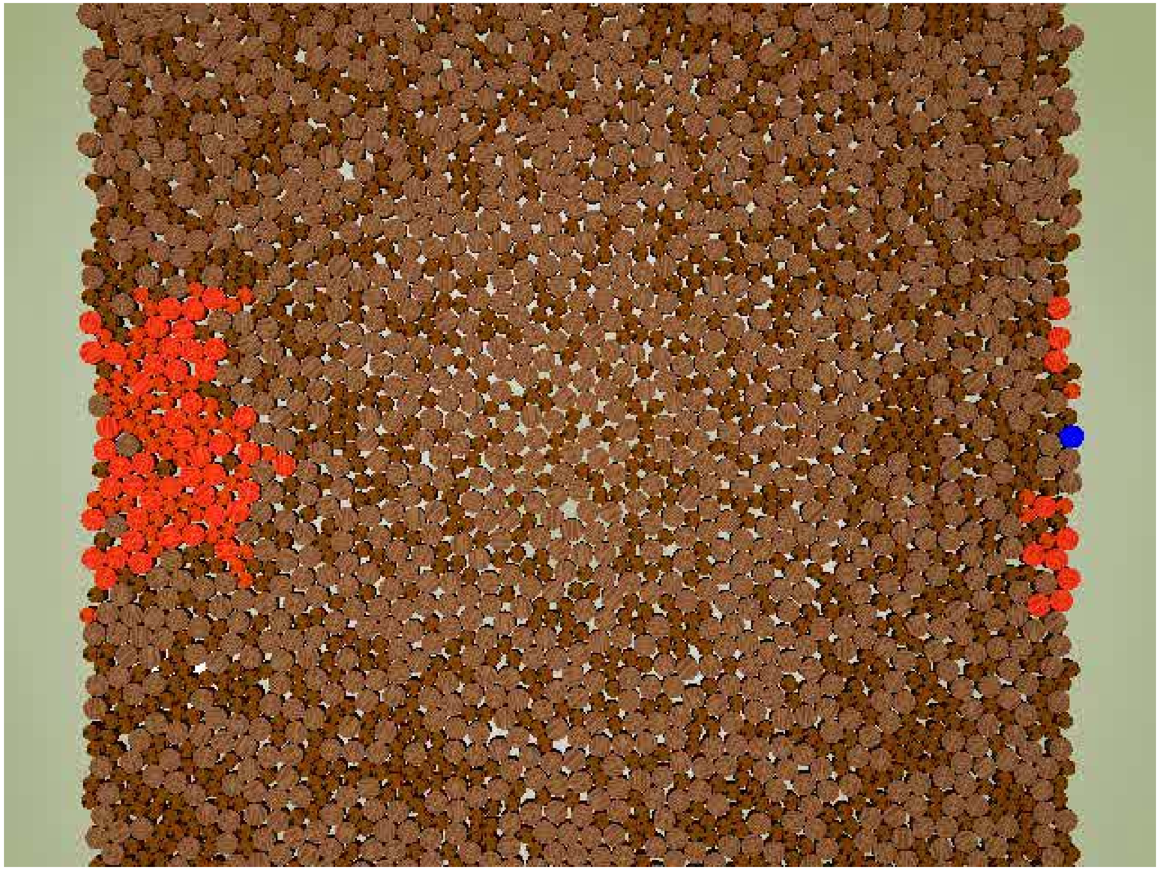
Red disks: in force contact with driven disk

Blue disk: driven

$\phi=0.801$

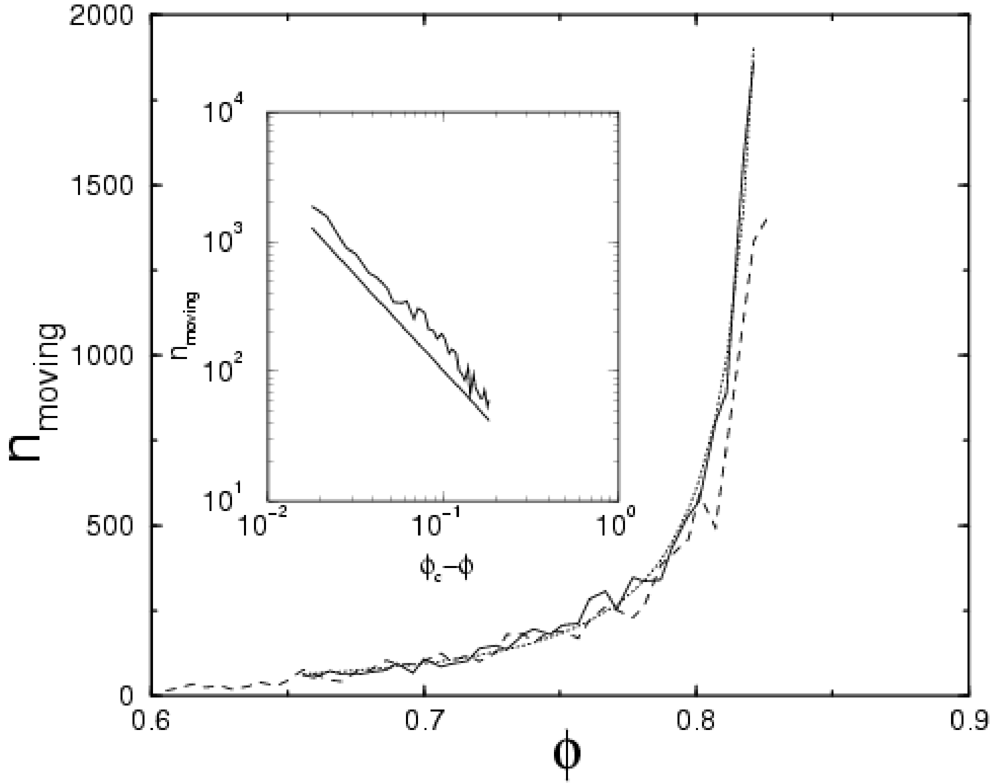
At jamming

Brown disks: stationary
Red disks: in force contact with driven disk
Blue disk: driven



ϕ=0.839

Number of moving grains



$$n_{\text{moving}} \propto (\phi_c - \phi)^{-\tau}$$

$$\tau = 1.2 \text{ to } 1.46$$

$$\phi_c = 0.839$$

$$\xi = (\phi_c - \phi)^{\nu}$$

$$d = 2\ ;\tau = \nu d$$

$$\nu = 0.6 \text{ to } 0.7$$

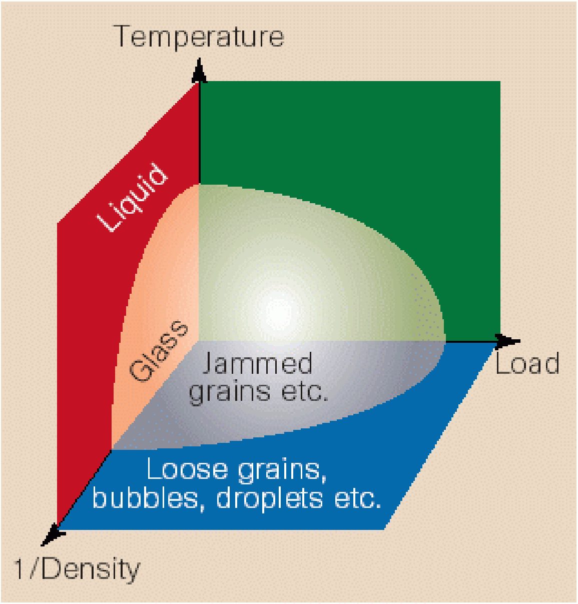
PRL 95, 088001 (2005)

Jamming: a phase transition?

Exponents measured through scaling:
Density axis, O'Hern et al: 0.71
PRE 68, 011306 (2003)

Load axis, Olsson and Teitel: 0.6 $\pm$ 0.1
PRL 99, 178001 (2007)

Exponent measured directly:
Drocco et al: 0.6 to 0.7
PRL 95, 088001 (2005)



Suggests: Jamming is second order phase transition ~~in directed percolation class.~~ (Exponent out of range)

Glass transition could be a true phase transition?

Values of critical exponents for DP

	D=1	D=2	D=3	
β	0.276486(8)	0.584(4)	0.81(1)	
$\nu_{\perp}$	1.096854(4)	0.734(4)	0.581(5)	(time)
$\nu_{\|\|}$	1.733847(6)	$\boxed{1.295(6)}$	1.105(5)	(space)
z	1.580745(10)	1.76(3)	1.90(1)	

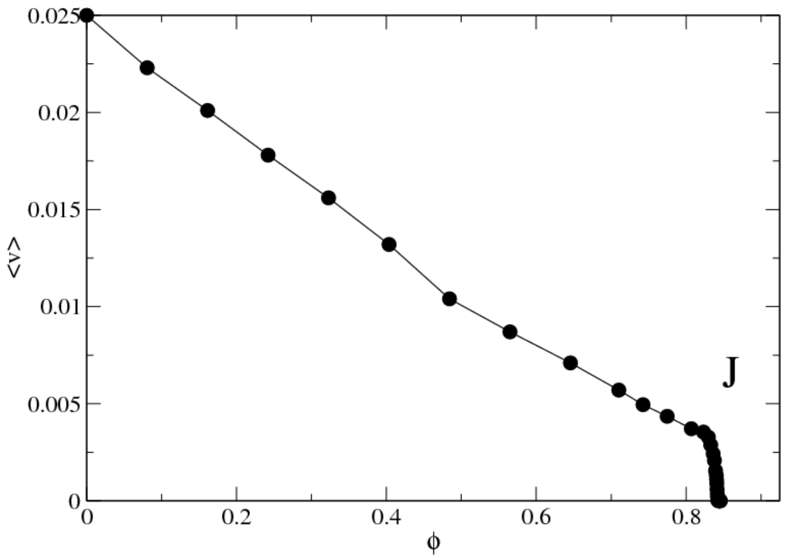
H. Hinrichsen
Adv. Phys. 49 815 (2000)

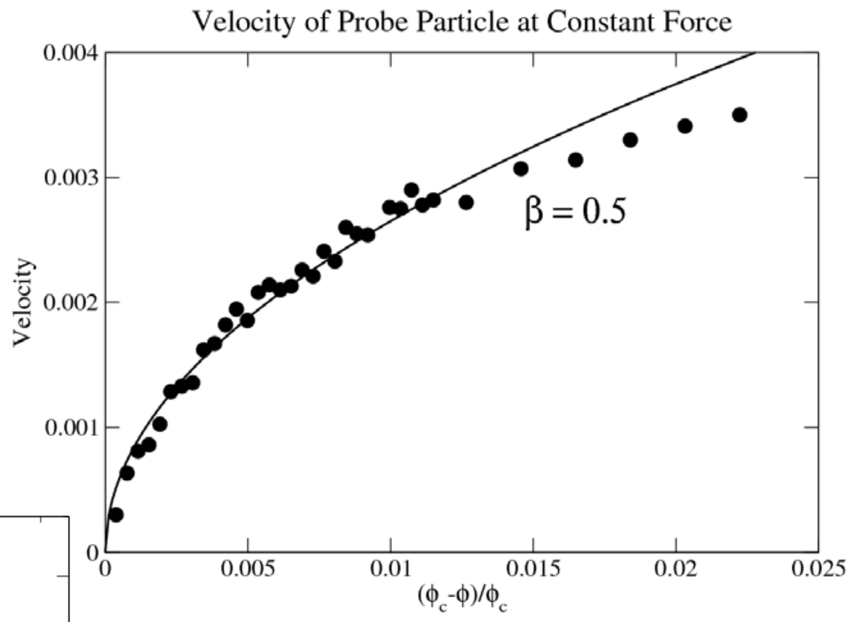
Velocity information from local probe

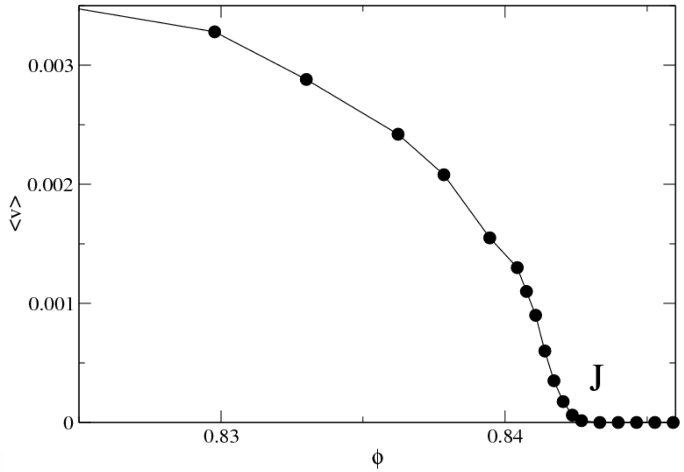
We can measure velocity signature of the local probe for fixed driving force

What does this tell us about the system?

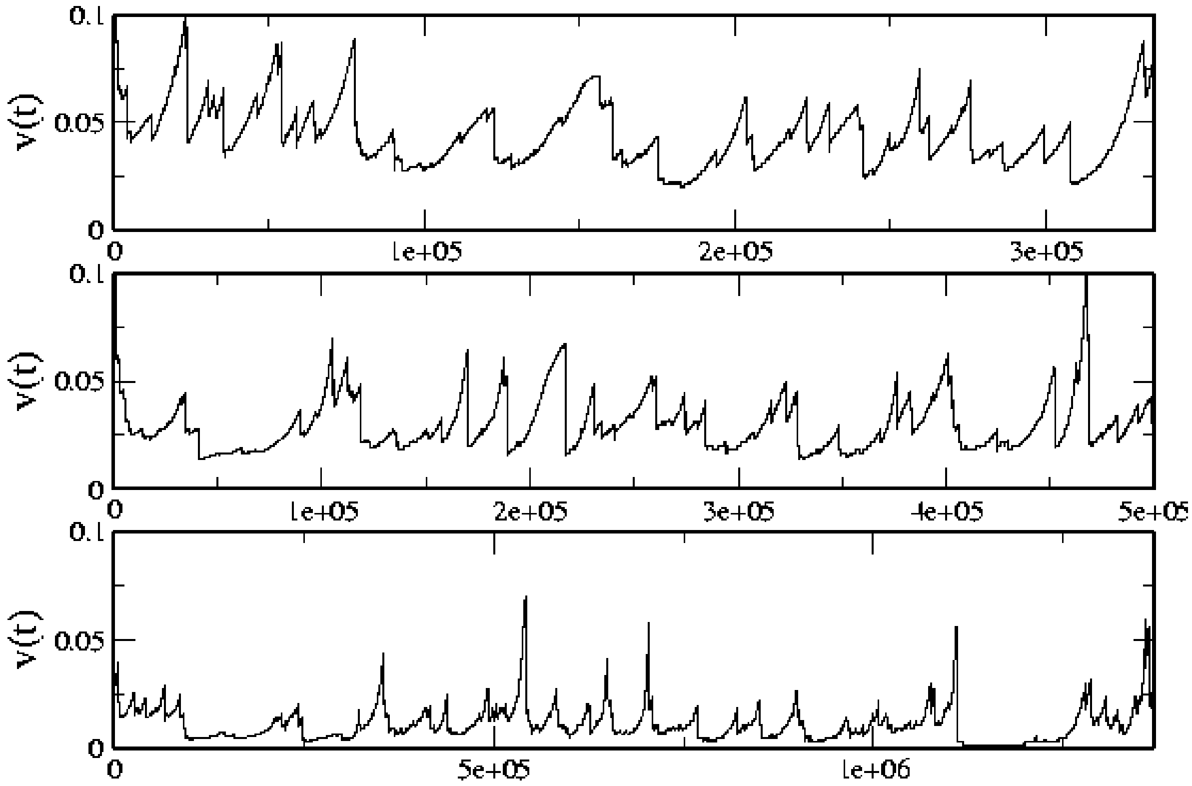
Scaling of probe velocity near jamming







Velocity Fluctuations



0.656

0.747

0.837

Multiscaling (or Multi-fractal scaling)

Probability of measuring a given velocity at fixed ϕ: $p(v)dv$

Define the q-th inverse moment of the velocity:

$$M(q) = \int \mathrm{d}v p(v) v^{-q}$$

Define a set of multifractal exponents $\tau(q)$:

$$M(q) = \int \mathrm{d}v\, p(v) v^{-q} \propto (\phi_c - \phi)^{-\tau(q)}.$$

We compute M and extract $\tau(q)$ by fitting the moments to the form

$c * (\phi_c - \phi)^{-\tau(q)}$, with $\phi_c \approx 0.839$.

Multiscaling example: No fluctuations

The q-th inverse moment of the velocity: $\overline{v^{-q}}$

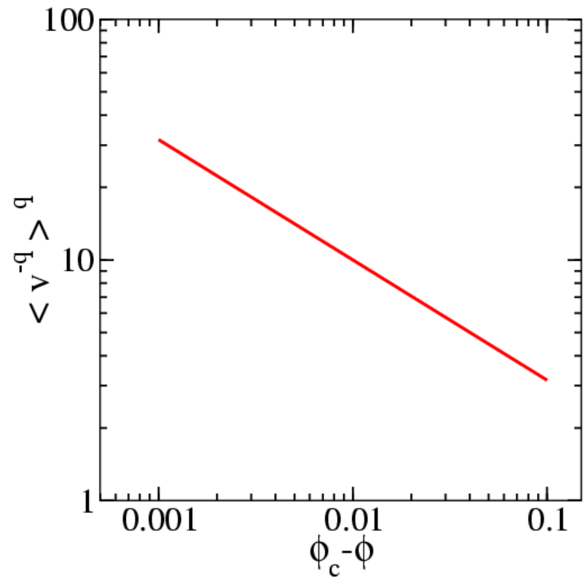
$$\overline{v^{-q}} \sim (\phi_c - \phi)^{-\tau(q)}$$

$$v(t) = V_0(\phi)$$

$$V_0 \sim (\phi_c - \phi)^{-\alpha}$$

$$\overline{v^{-q}} = V_0^{-q} \sim (\phi_c - \phi)^{q\alpha}$$

$$\tau(q) = -q\alpha$$



Multiscaling example: Simple scaling

Avg. velocity over some time interval: $p(v)dv$

$$p(v)dv = g\left(\frac{v}{V_0}\right)\frac{dv}{V_0}$$

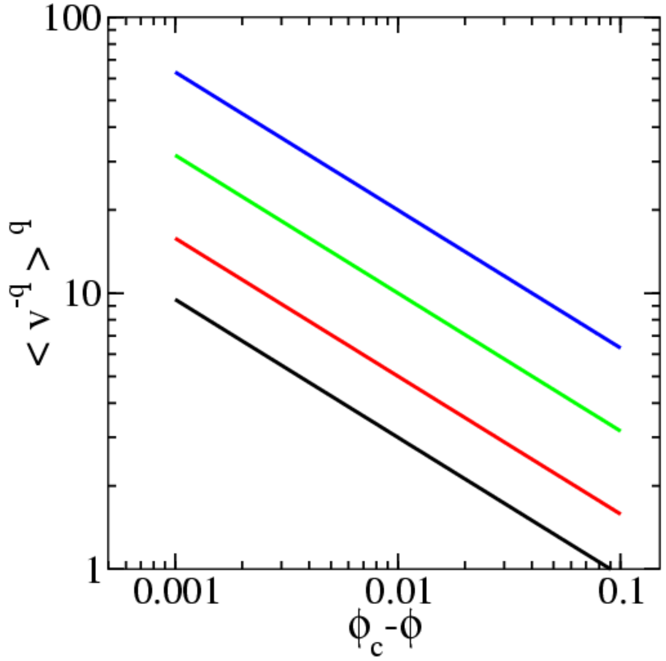
$$V_0 \sim (\phi_c - \phi)^{-\alpha}$$

$$\overline{v^{-q}} = \int v^{-q} p(v)dv$$

$$= \int \left(\frac{v}{V_0}\right)^{-q} g\left(\frac{v}{V_0}\right)\frac{dv}{V_0}\, V_0^{-q}$$

$$\overline{v^{-q}} = G(-q)V_0^{-q} \sim (\phi_c - \phi)^{q\alpha}$$

$$\tau(q) = -q\alpha$$



Multiscaling example: Scaling

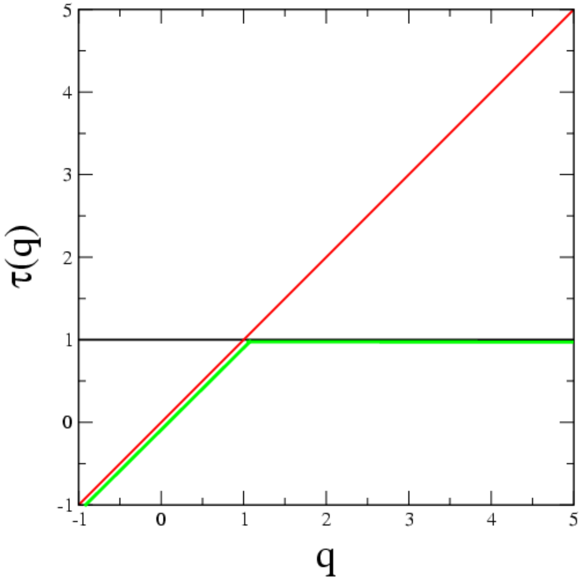
$$p(v)dv = \delta(v-1)(\phi_c - \phi)^{f(0)} + \delta(v - (\phi_c - \phi)^{-\alpha})$$

$$\text{where } (\phi_c - \phi)^{\mathrm{f}(0)} \ll 1$$

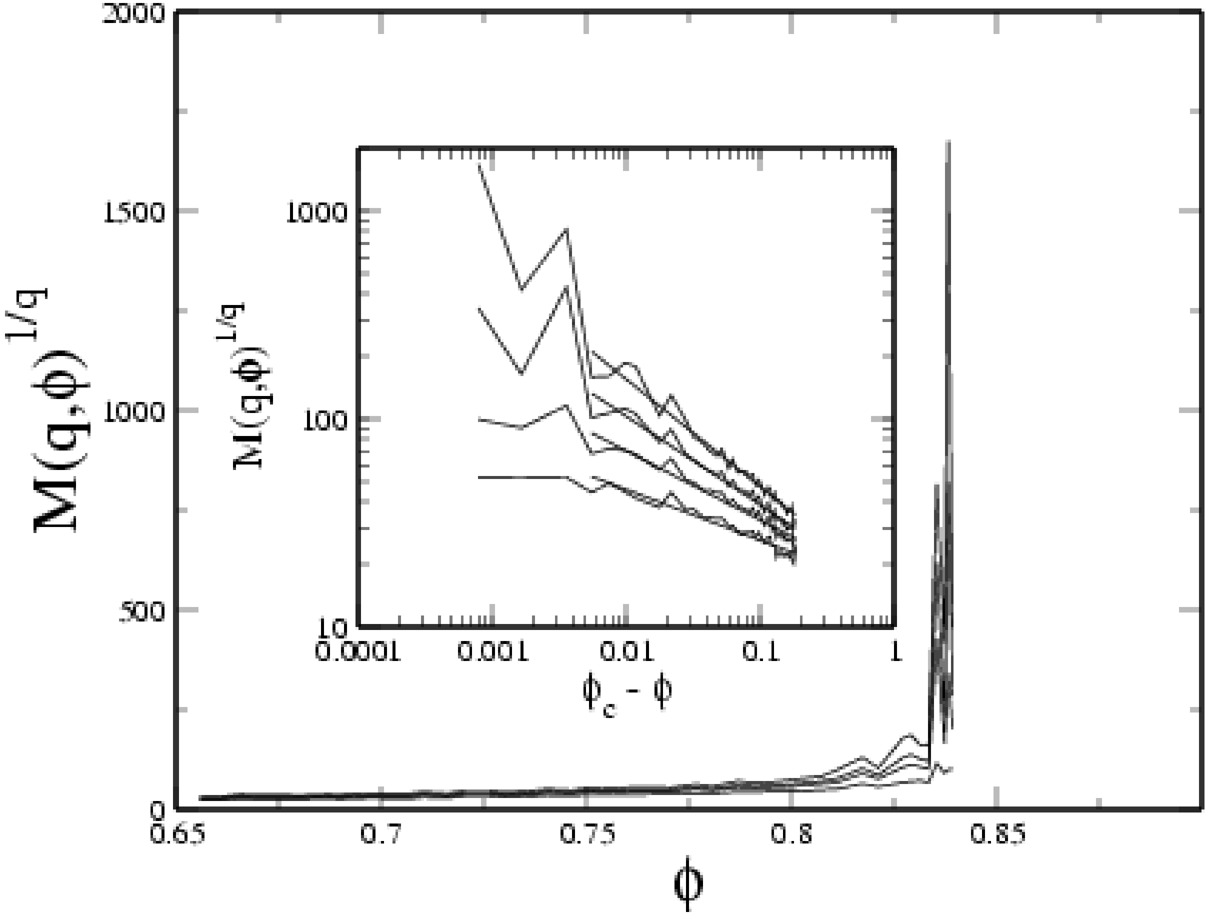
This PDF does not have simple scaling. We obtain:

$$\overline{v^{-q}} = \begin{cases} (\phi_c - \phi)^{f(0)} \\ (\phi_c - \phi)^{q\alpha} \end{cases}$$

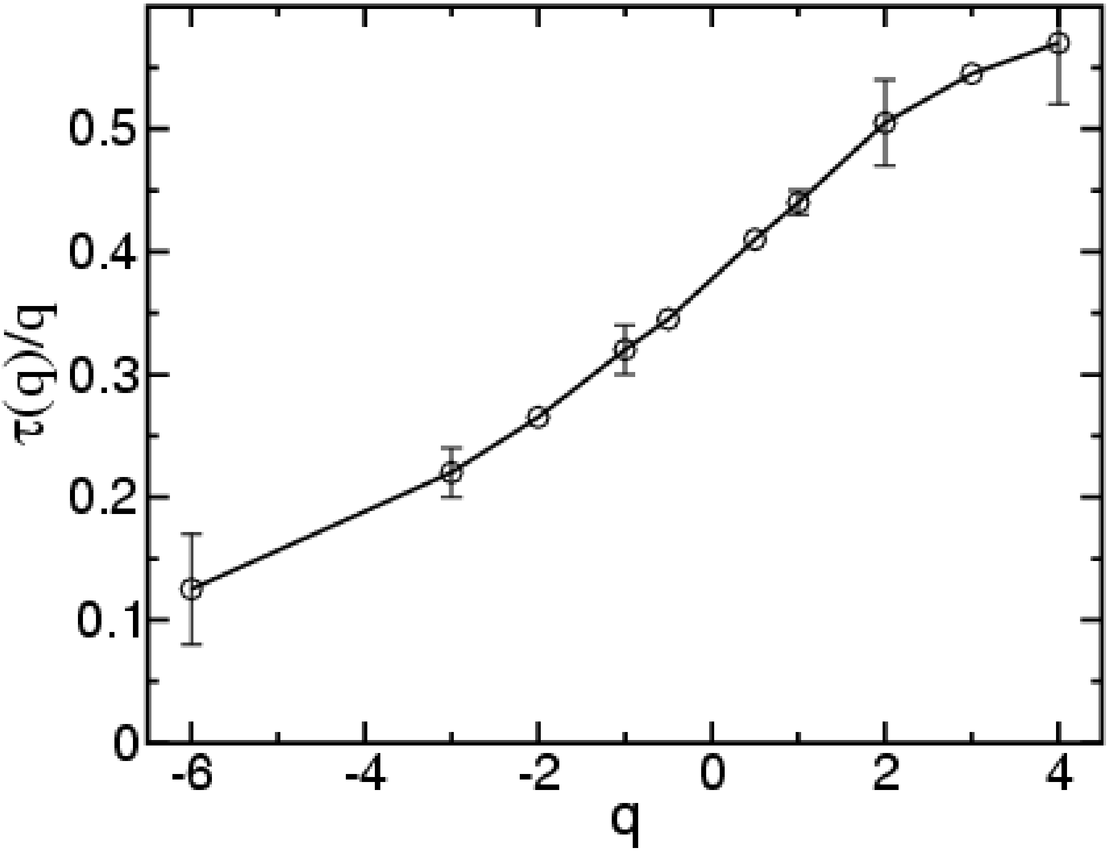
$$\tau(q) = \min(f(0), -q\alpha)$$



Multiscaling



Multiscaling

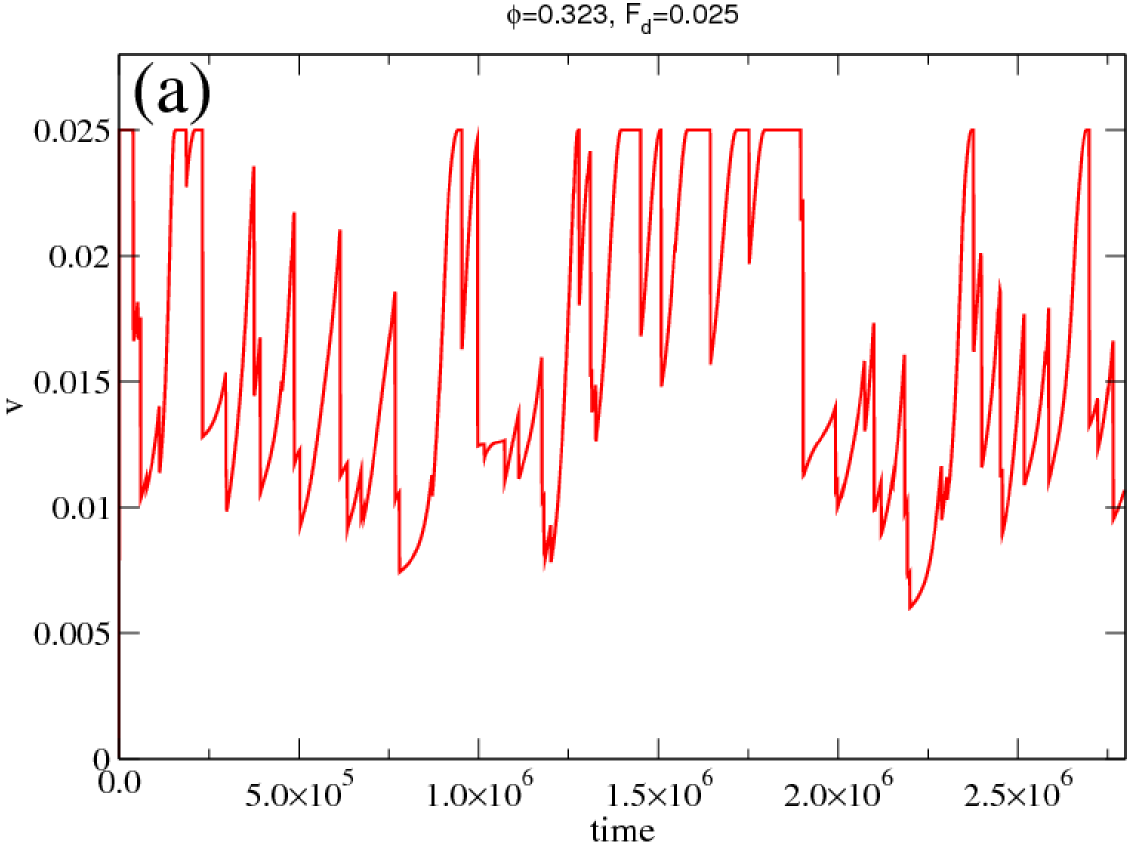


Multiscaling and velocity distribution

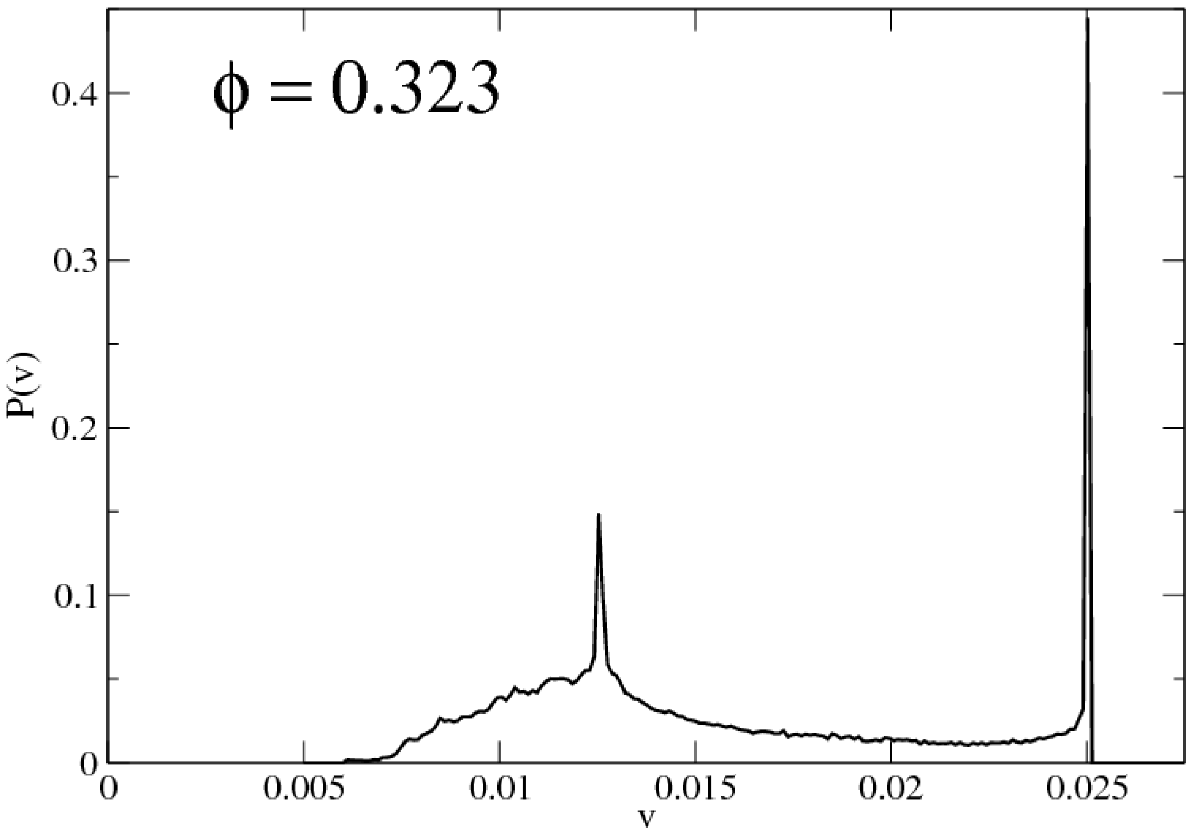
We find evidence for multiscaling of the velocity of our probe particle. The velocity distributions do not scale in a simple fashion as we approach the jamming transition.

So what are the actual velocity distributions?

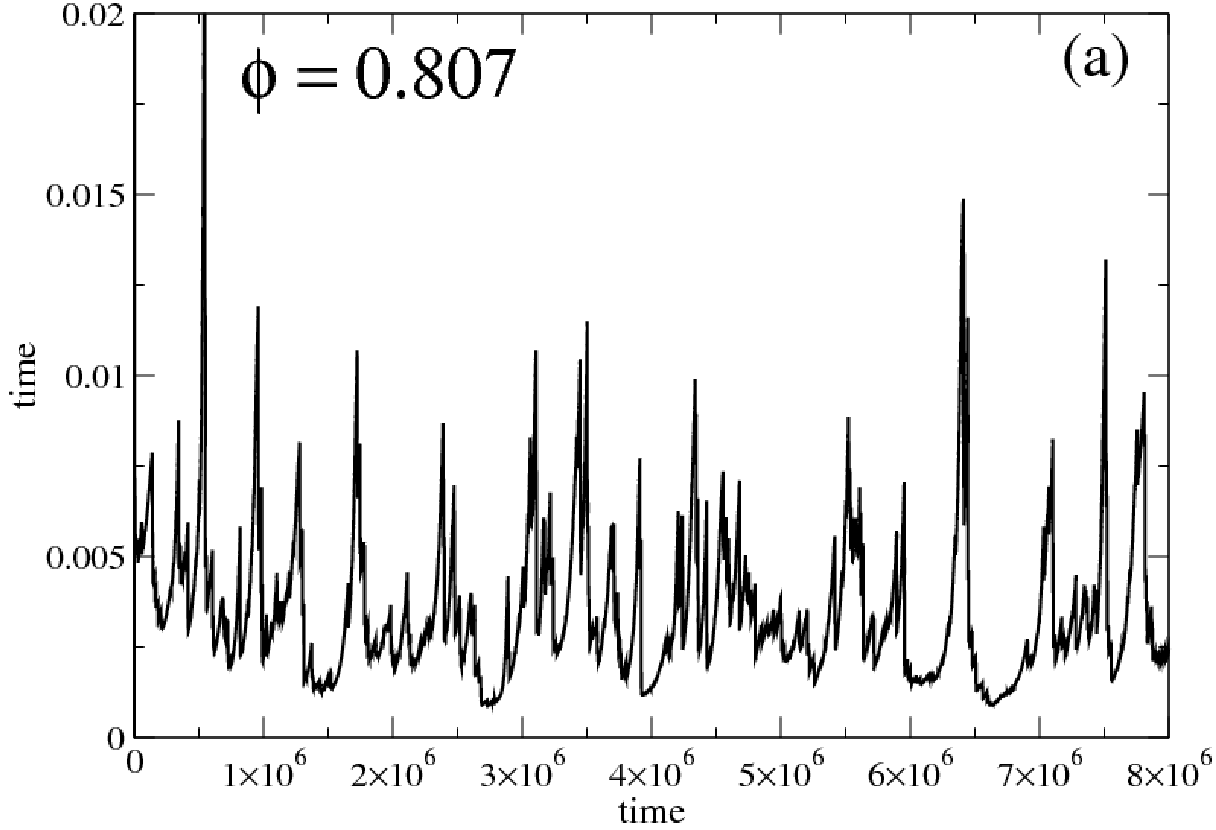
Probe velocity at low density over a long time



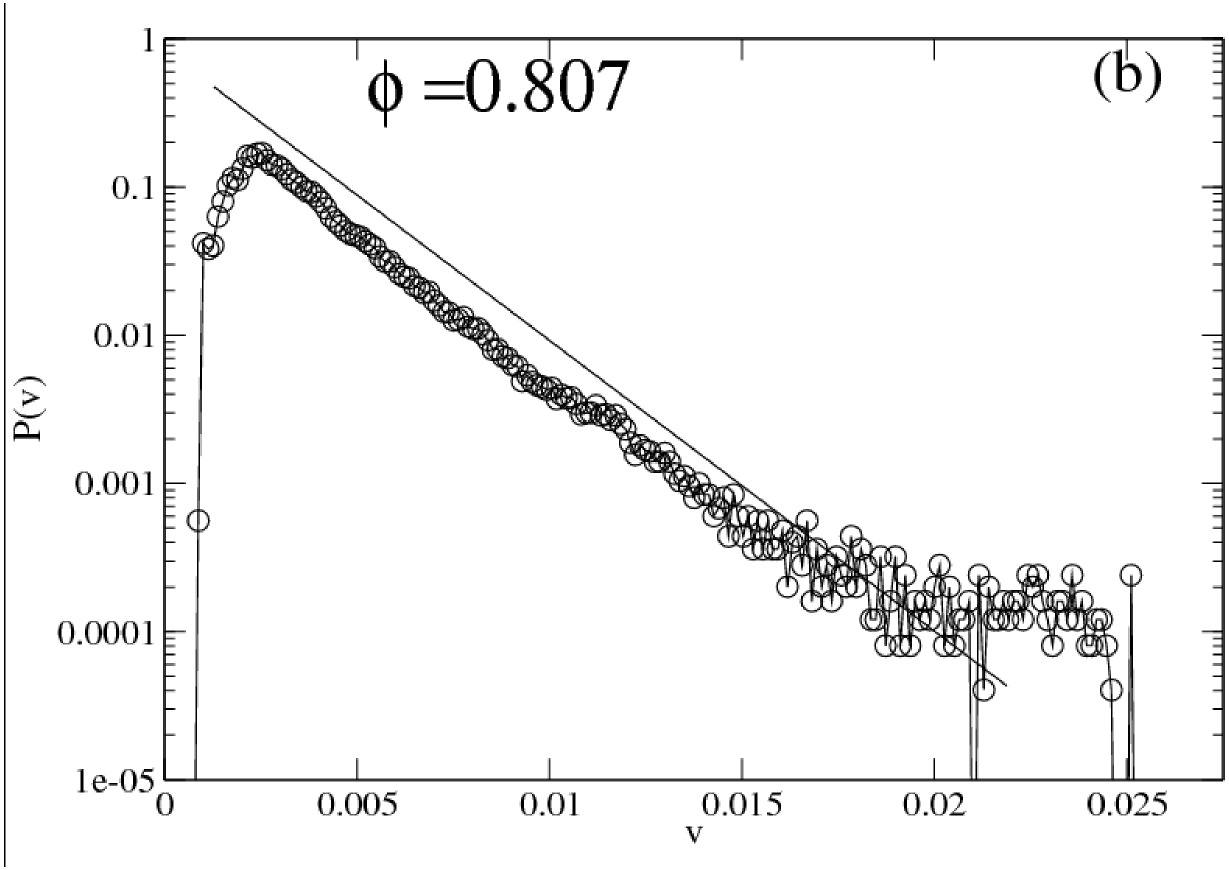
Velocity distribution function of probe particle



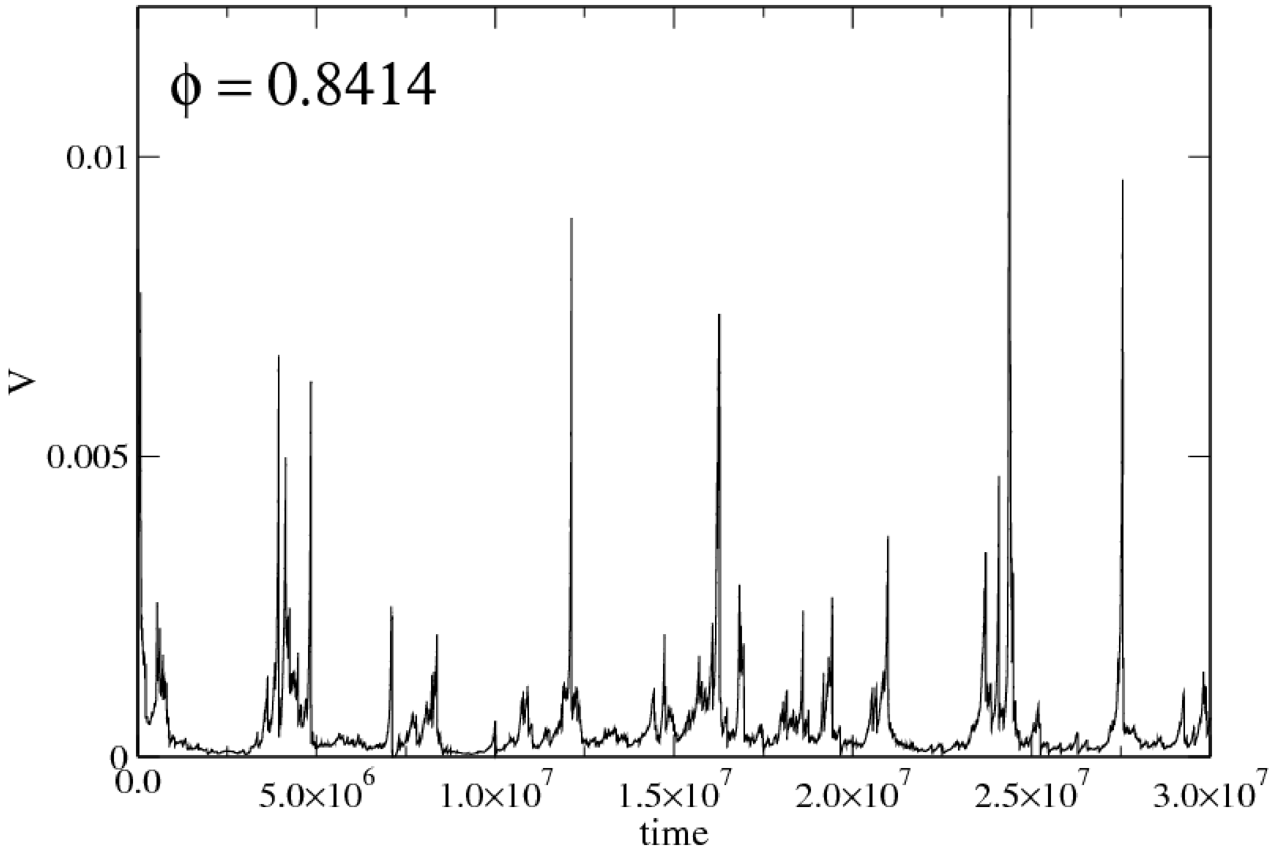
Probe velocity as jamming transition is approached



Exponential tail of velocity distribution near jamming

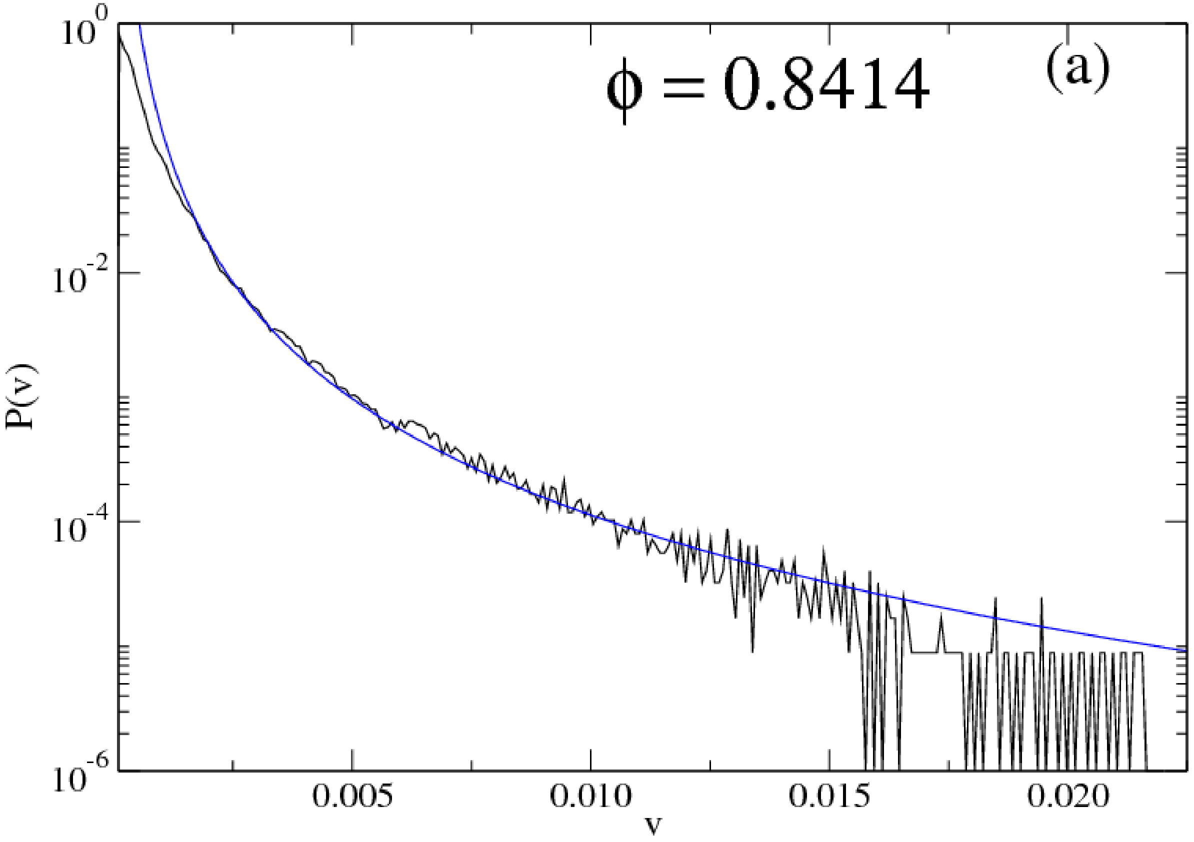


Probe velocity very close to jamming

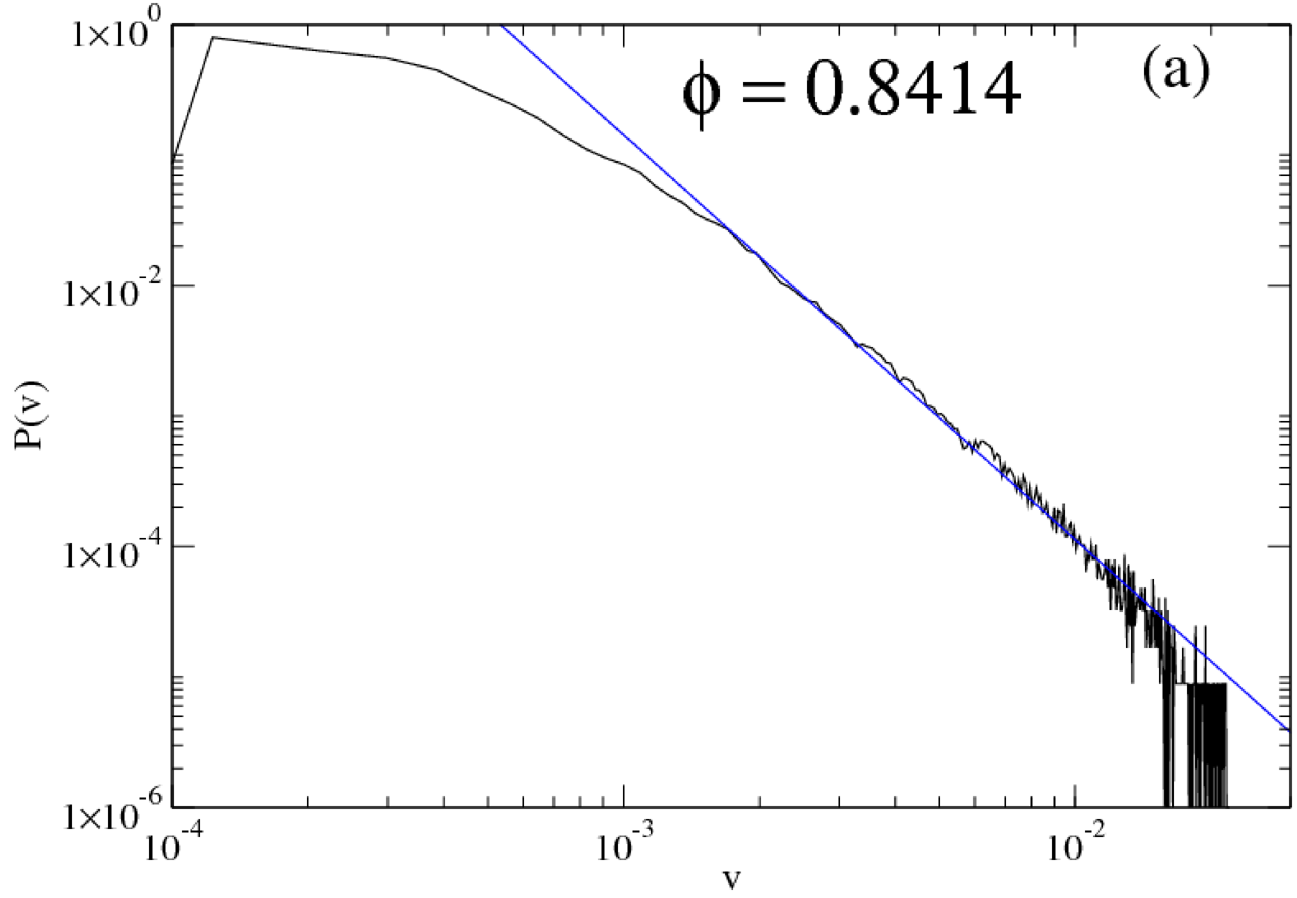


Los Alamos

Velocity distribution very close to jamming



Velocity distribution very close to jamming



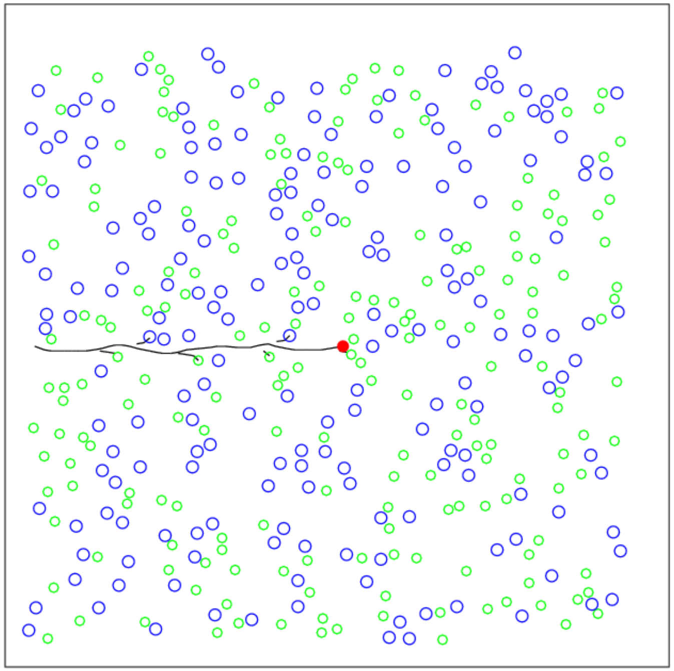
What we have learned

• There appears to be diverging length scale as jamming transition is approached, but it is not consistent with any obvious known phase transition

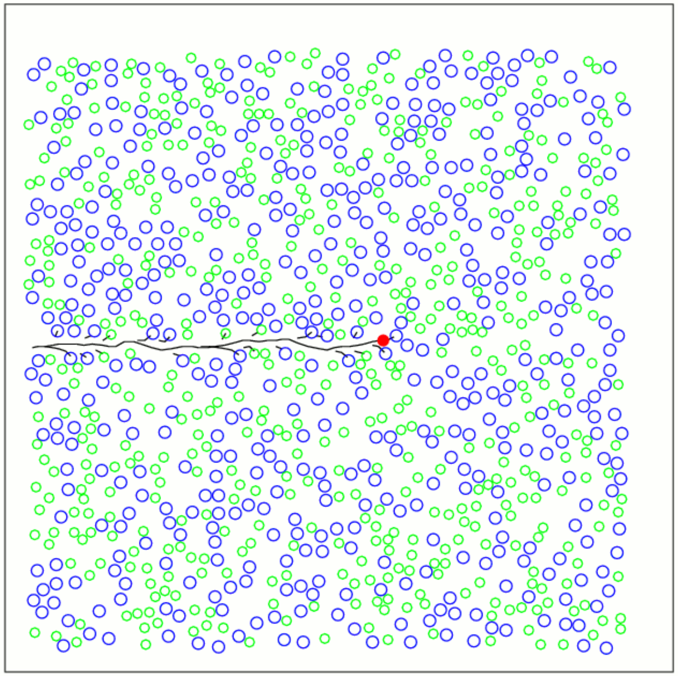
• Velocity becomes strongly intermittent near jamming and the velocity fluctuations exhibit multiscaling, passing from bimodal to exponential tail to power law tail.

• Multiple regimes? Let’s take another look at the displacement of the background media by our driven grain.

Low densities



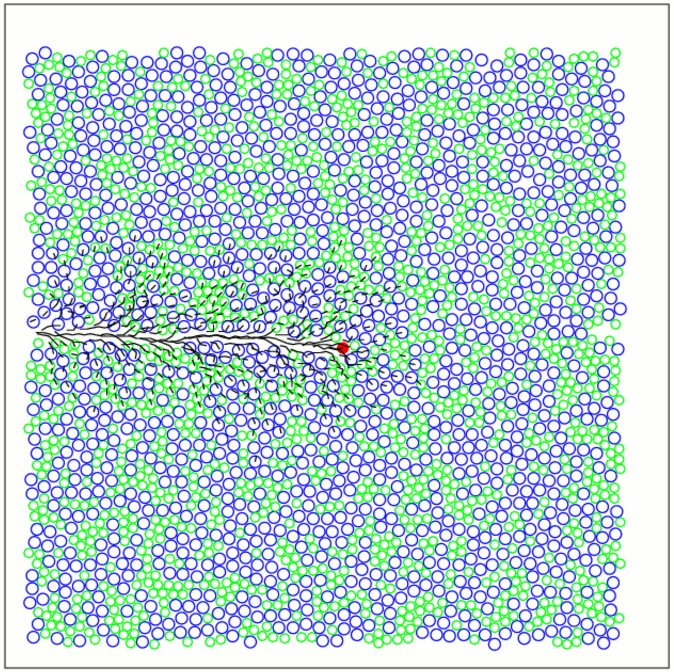
0.13



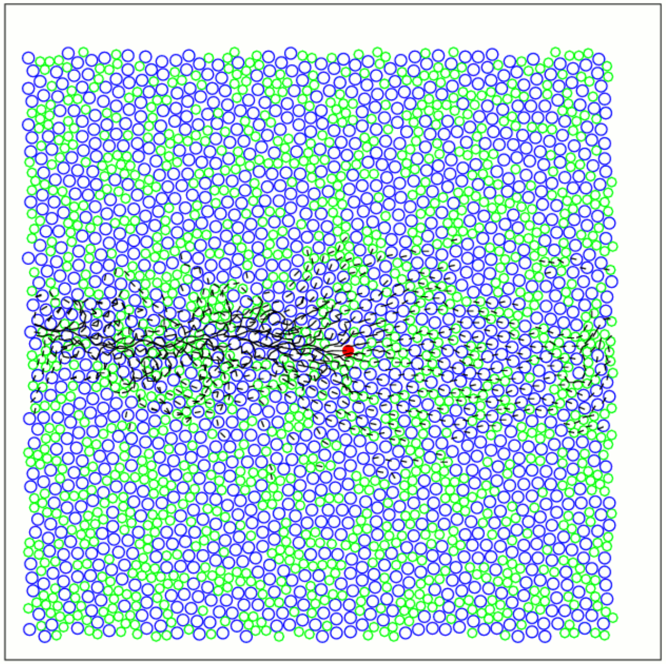
0.323

Simple linear displacements of background grains

High densities



0.807



0.8414

Appearance of “T1”-like events as displaced zone increases in extent

What the single probe measurement may really mean

• Diverging length scale may not fit to known models because it may not actually be a single length scale

Instead:

• Length scale 1 (short lengths): Linear displacements of surrounding grains

• Length scale 2 (longer lengths): Displacements get too large for linear regime and plastic rearrangement events of surrounding grains occur

Our probe lumped these regimes together

Multiple length scales?

• If multiple length scales are present, is there a way to separate them with our probe?

One possibility: Change of reference frame

Create immobile grains and drive all remaining grains along the x direction

Simulation

2D Granular dynamics simulation

Binary mixture of disks of radii $r_A : r_B = 1.4 : 1$.

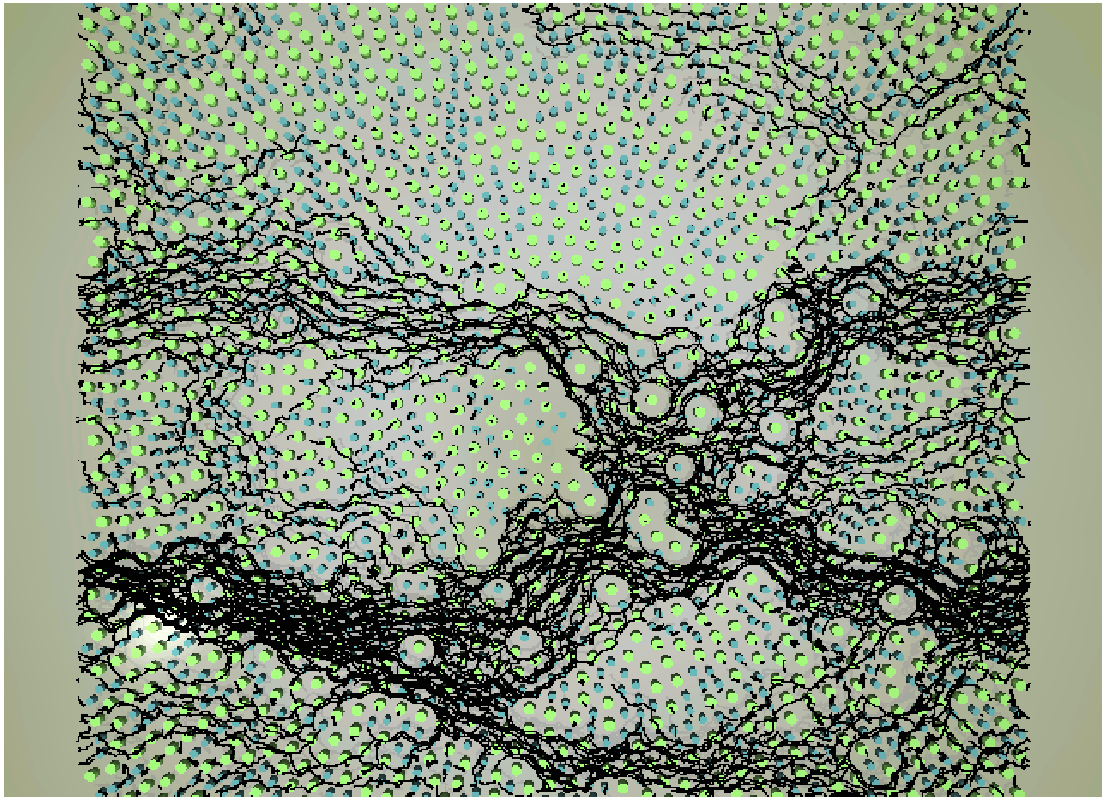
$L = 24$ to 60; 2600 disks at $\phi = 0.8395$.

Initialization: Thermalize while adding disks.

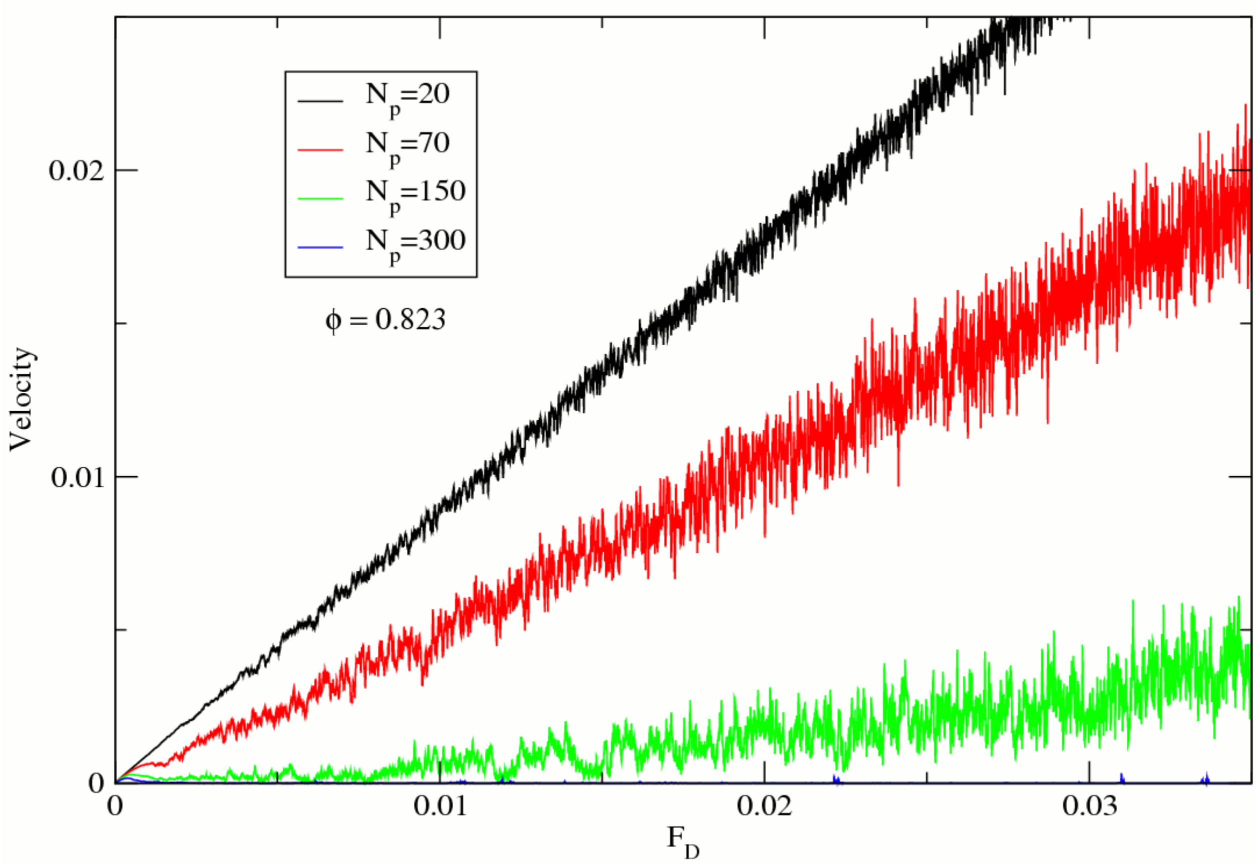
$$\eta \mathbf{v}_i = \sum_{i \neq j} k_g (|\mathbf{r}_{ij}| - r_{eff}) \frac{\mathbf{r}_{ij}}{|\mathbf{r}_{ij}|} + \mathbf{f}_d$$

$$|\mathbf{r}_{ij}| < r_{eff} = r_g^{(i)} + r_g^{(j)} \quad ; k_g = 200$$

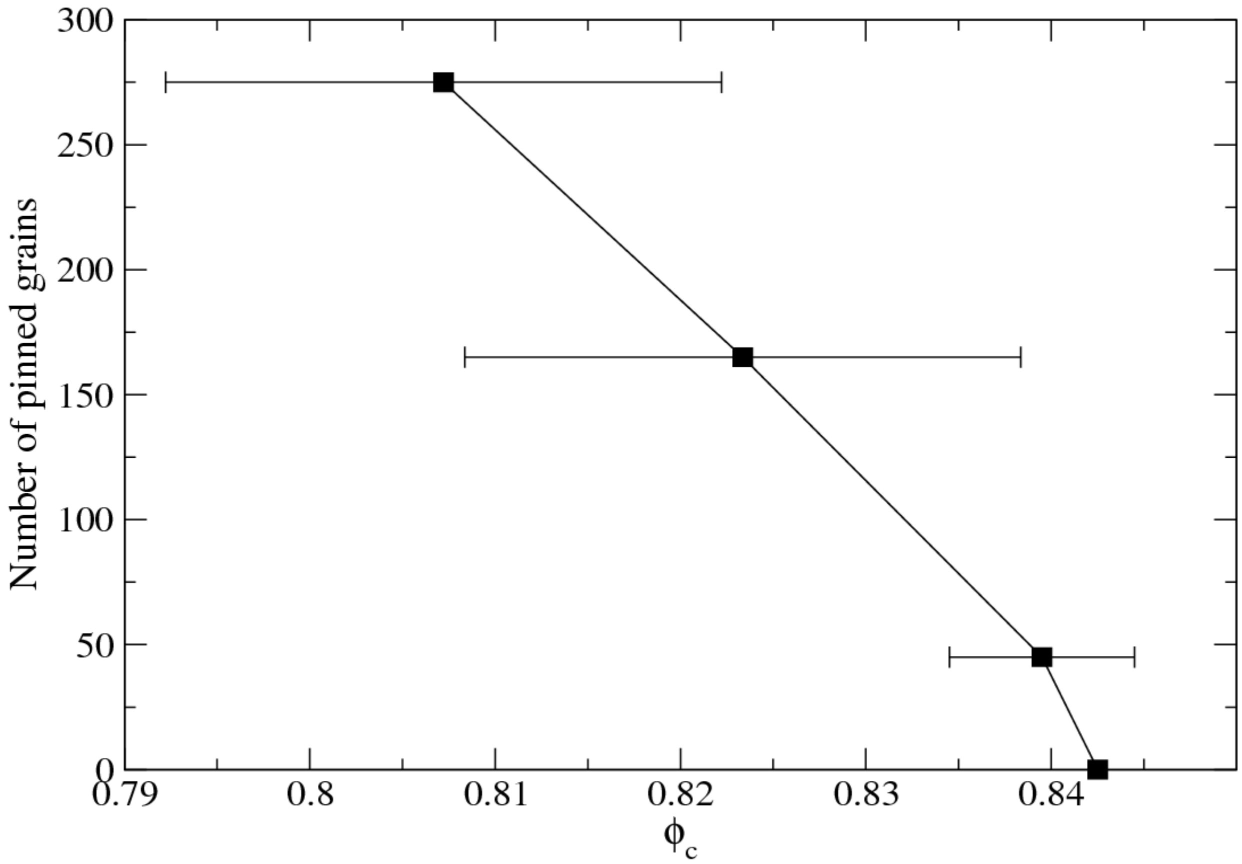
Heterogeneous flow over quenched disorder



Approach to jamming by adding disorder

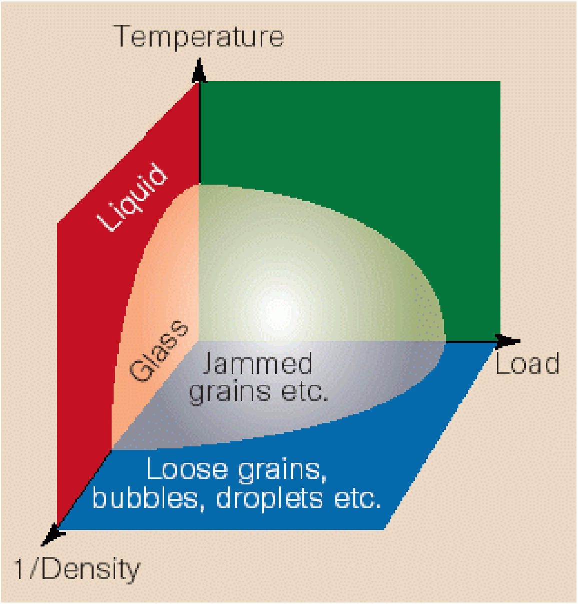


Depression of jamming density by quenched disorder



“Fourth axis” of jamming phase diagram

- Quenched disorder: new axis.



Clogging

Summary

• We simulate a 2D disordered system of disks, and drag a single particle through a packing of increasing density.

• The number of moving disks diverges as a power law, providing evidence that the jamming transition is a second order phase transition of undetermined type.

•The velocity fluctuations increase as we approach the jamming transition, and show multiscaling

• Possible existence of two, rather than one, length scales

• Quenched disorder: new axis of jamming phase diagram.