Driven Colloids II: Reversible/Irreversible Transitions

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This talk in 1 slide

- Irreversibility: appearance of entropy, creation of time's arrow
- By tuning the parameters of a system in the right way, can we cause irreversibility to suddenly appear, as if through a phase transition? (Answer: Yes)
- If so, what kind of phase transition is it? (Answer: Consistent with absorbing)
- Can an irreversible phase transition of this type be generalized to any other phenomena? (Answer: Yes, to plastic depinning)

Road map

- We consider three systems:
  - Colloids sheared in what was supposed to be a Stokes flow regime (NYU experiment, simulation)
  - Vortices driven back and forth over quenched disorder
  - Plastic depinning transition of colloids on quenched disorder

This talk in more than 1 slide

- Let's begin by considering a situation where irreversibility would naively be expected to occur… but it does not.

Reversibility in Viscous Flows:
No particles

Kinematic_reverse

G.I. Taylor: Reversibility in low Reynolds number flows in a viscous liquid under shear

Stokes flow

Navier-Stokes equation:

$$\overbrace{\rho\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\substack{\text{Unsteady}\\\text{acceleration}}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\substack{\text{Convective}\\\text{acceleration}}}\Big)}^{\text{Inertia (per volume)}} = \overbrace{\underbrace{-\nabla p}_{\substack{\text{Pressure}\\\text{gradient}}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}}$$

For Re << 1, neglect unsteady and convective terms to obtain Stokes equation:

$$\boldsymbol{\nabla} p = \mu \nabla^2 \boldsymbol{u} \,.$$

Linear in velocity and pressure
Instantaneous: no dependence on time except through boundary conditions
Reversible: a time-reversed Stokes flow solves the Stokes equations

Reynolds Number = Re

$$\text{Re} = \text{ratio} = \frac{\text{Inertia Force}}{\text{Viscous Force}}$$

$$\text{Re} = \frac{\rho V \; dV/dx}{\mu \; d^2V/dx^2}$$

$$\text{Re} = \frac{\rho \; V \; V/L}{\mu \; V/L^2}$$

$$\text{Re} = \frac{\rho V L}{\mu}$$

ρ = density
μ = bulk viscosity
V = velocity
L = length

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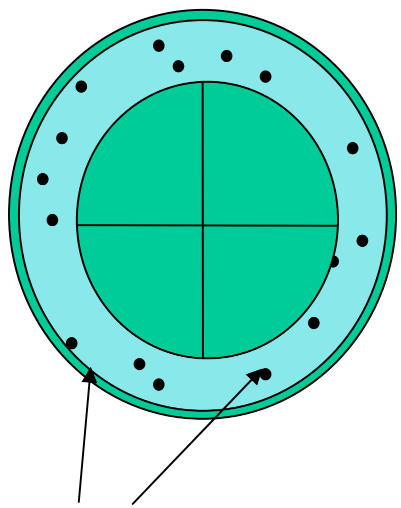
Reversibility in Viscous Flows:
With particles

Top view of Couette cell



Dyed
Colloidal
Particles

Pine et al [Nature 438, 997 (2005)] investigated reversibility in viscous flow of colloidal suspensions

Cell is sheared periodically at a chosen strain amplitude.

Position of colloids is measured at the end of each shear cycle.

When the behavior is reversible, colloids return to the same location after each cycle.

Experimental image of shearing

D.J. Pine et al

Continuous_small

Continuous_small

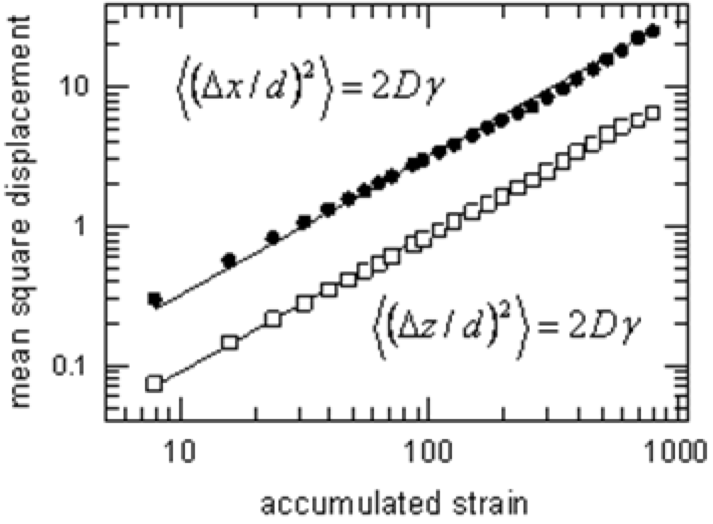
Continuous movie (without strobe)

Reversible

Higher strain amplitude

D.J. Pine et al

Irreversible_small

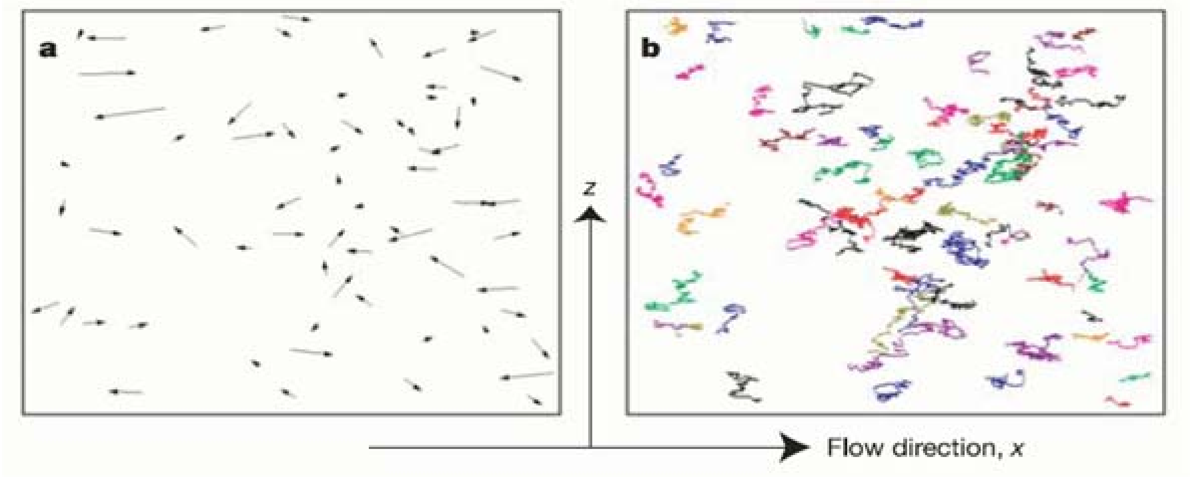


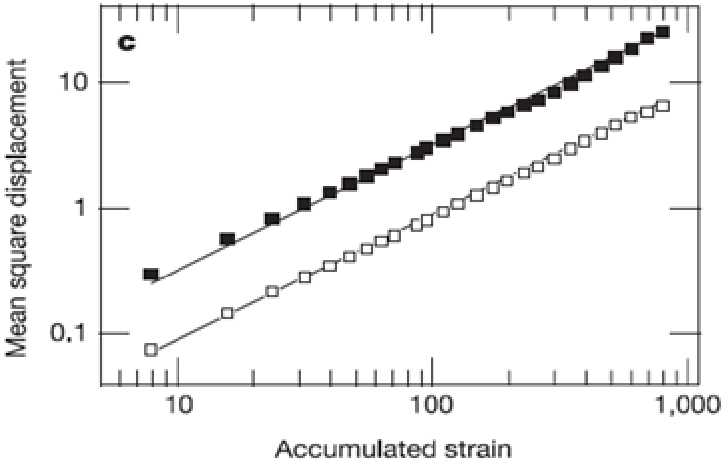
Irreversible!

Mean square displacements in the two directions versus accumulated strain, which is proportional to the number of cycles

Experimental image of shearing

D.J. Pine et al, Nature 438, 997 (2005)

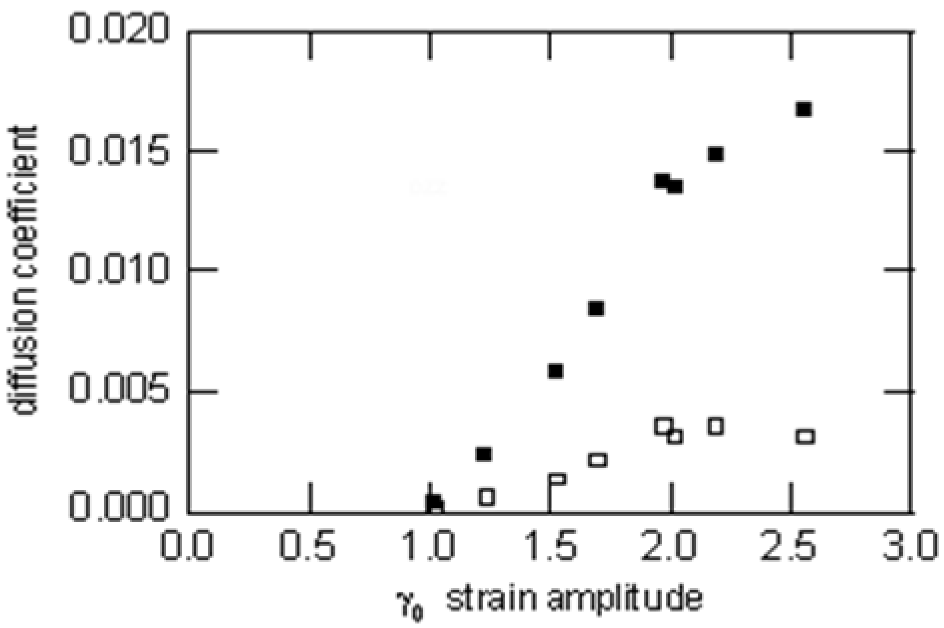


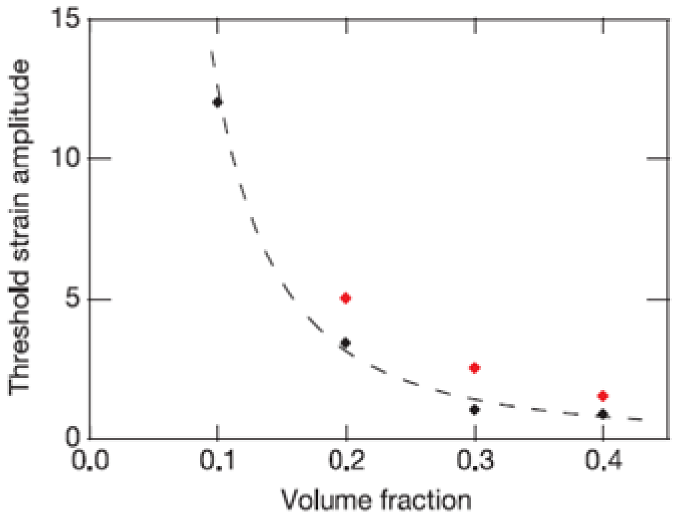


Violation of Stokes flow. Re did not change; what did?

Clue: Threshold strain needed for irreversible behavior

D.J. Pine et al, Nature 438, 997 (2005)





Threshold decreases with increasing colloid volume fraction
Colloid-colloid interactions are playing a role.

Questions

- Does a similar reversible to irreversible transition exist in other particle-filled viscous media? How universal is the behavior?
- Is the R-IR transition a nonequilibrium phase transition? What is the nature of this phase transition?
- Can similar behavior occur in vortex matter in type II superconductors?

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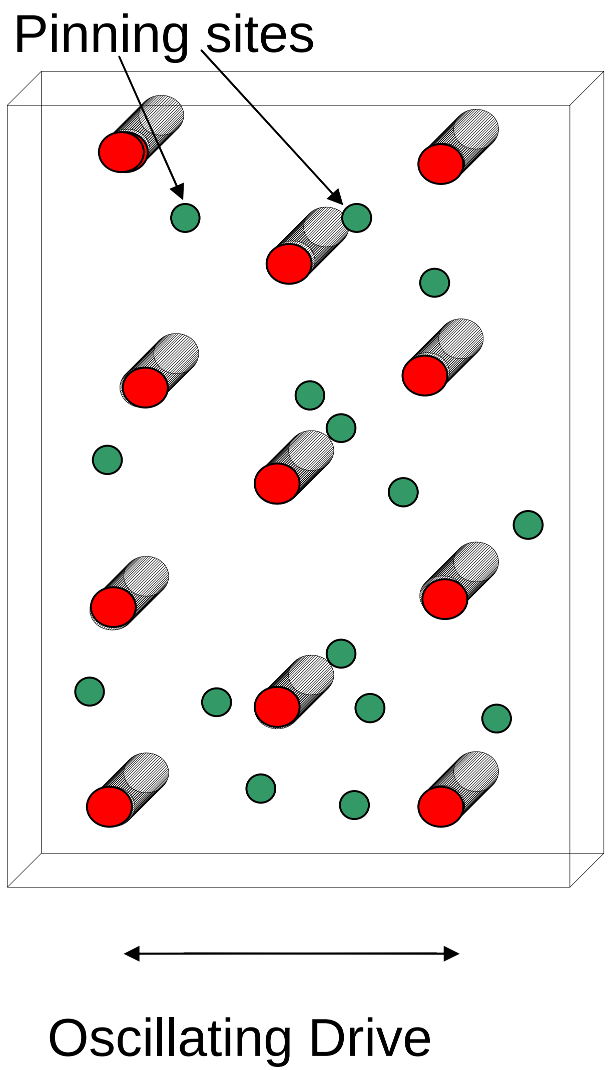
Overdamped Dynamics



Equation of Motion: $$\eta \frac{d\mathbf{R}_i}{dt} = \mathbf{F}_i^{vv} + \mathbf{F}_i^{p} + \mathbf{F}_i^{d}$$

Vortex-Vortex Interaction:

$$\mathbf{F}_i^{vv} = \sum_{j \neq i}^{N_v} f_0 K_1\left(\frac{R_{ij}}{\lambda}\right)\hat{\mathbf{R}}_{ij},$$

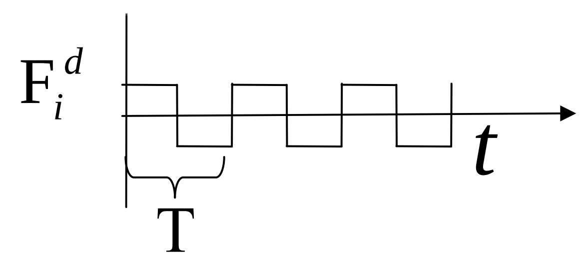
Modified Bessel Function with exponential decay

Vortex-Pin Interaction:

Attractive Parabolic Wells

$$\mathbf{F}_i^{p} = -\sum_{k=1}^{N_p} \frac{f_p}{R_p} R_{ik}^{(p)} \Theta\left(\frac{R_p - R_{ik}^{(p)}}{\lambda}\right)\hat{\mathbf{R}}_{ik}^{(p)}.$$

Square Wave Driving Force:



$$\mathbf{F}_i^d = \begin{cases} +\mathrm{F}_i^d, & 0 < t \bmod \mathrm{T} < \mathrm{T}/2 \\ -\mathrm{F}_i^d, & \mathrm{T}/2 < t \bmod \mathrm{T} < 0 \end{cases}$$

Simulation Measurements

Vary displacement per cycle $d = F_d T/2$

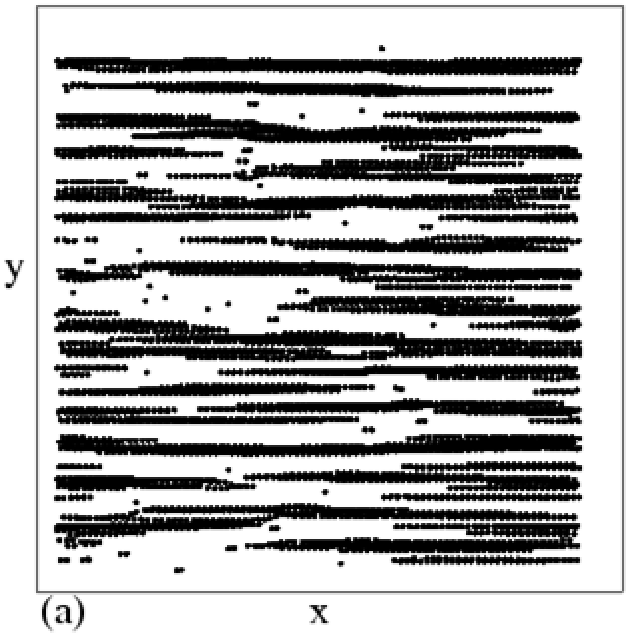
Measure:

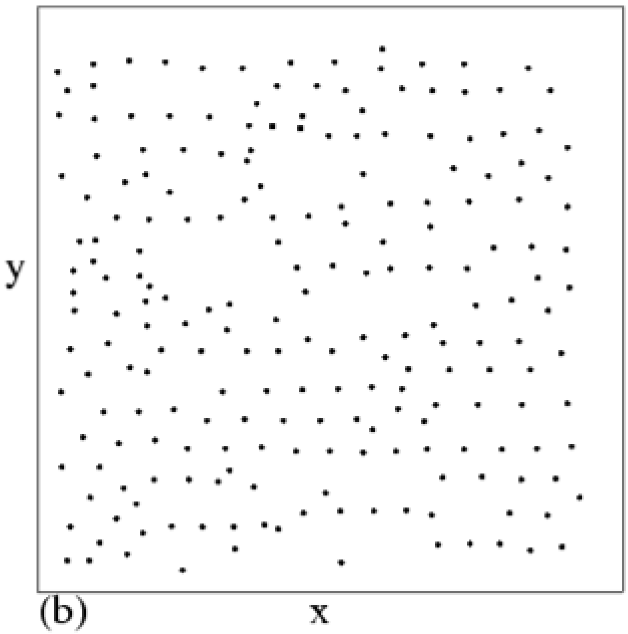
$$R_\alpha^d(n\tau)=N_v^{-1}\,\Sigma[(\mathbf{R}_i(t_0+n\tau)-\mathbf{R}_i(t_0))\ .\ \alpha]^2$$

$\alpha = x, y$

Effective diffusivity $D_\alpha = R_\alpha^d/2t$

Reversible regime





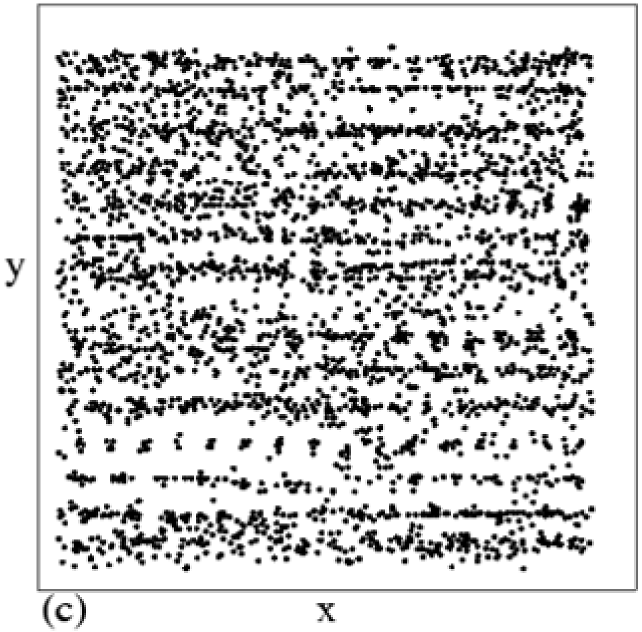
25 cycles marked every 0.01τ

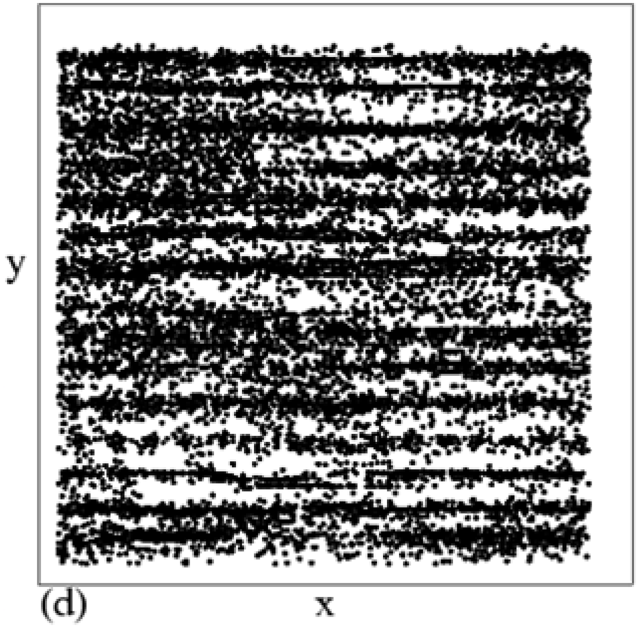
25 cycles marked every τ

$n_v = 0.052/\lambda^2$, displacement per cycle=48λ

Irreversible regime

For larger displacements per cycle



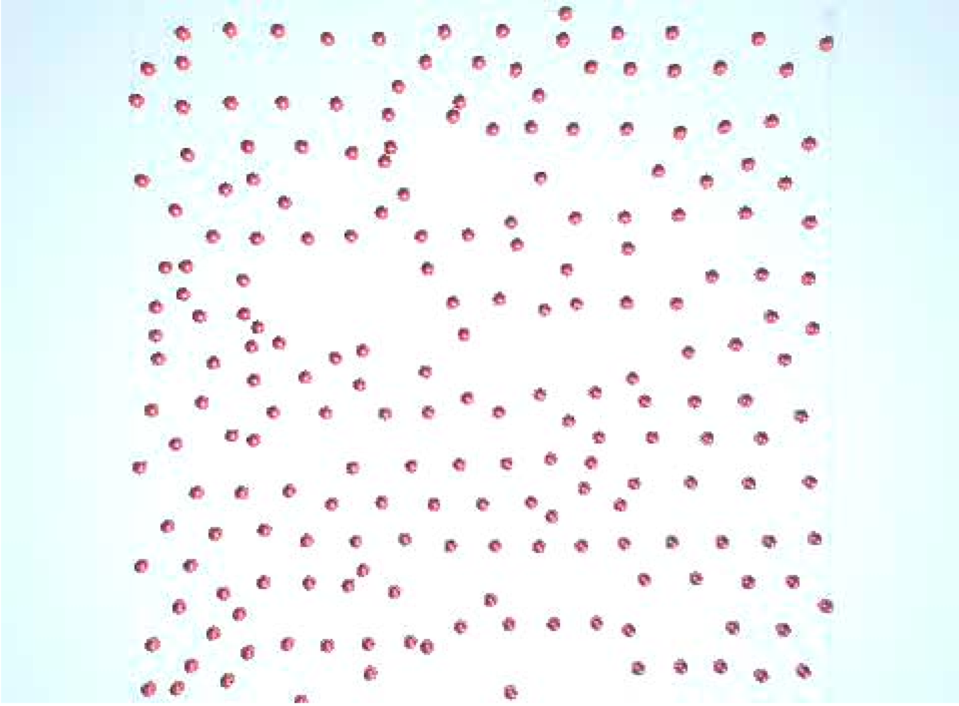


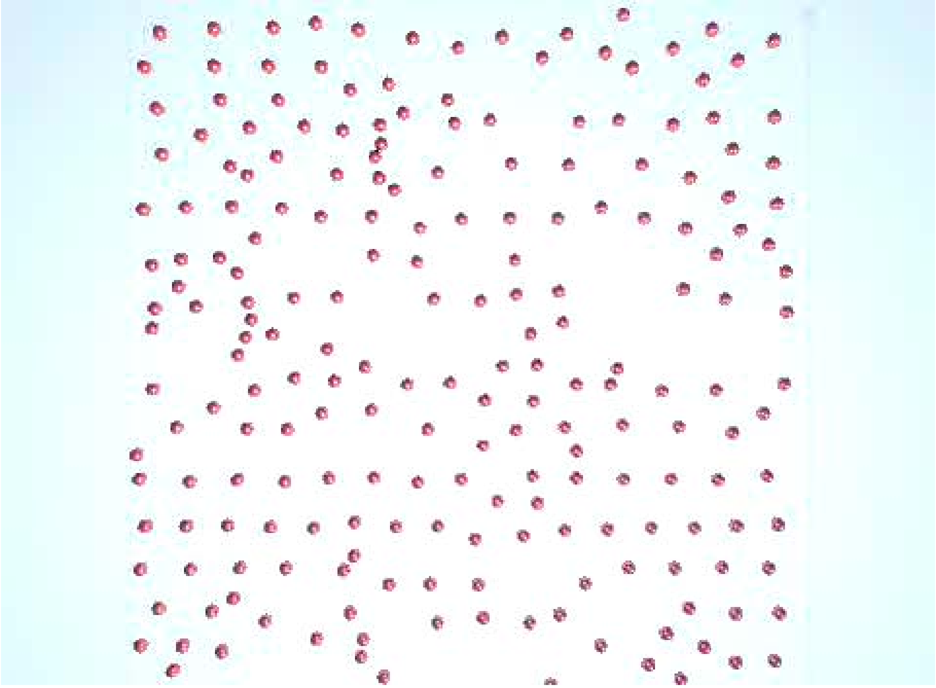
25 cycles marked every τ

100 cycles marked every τ

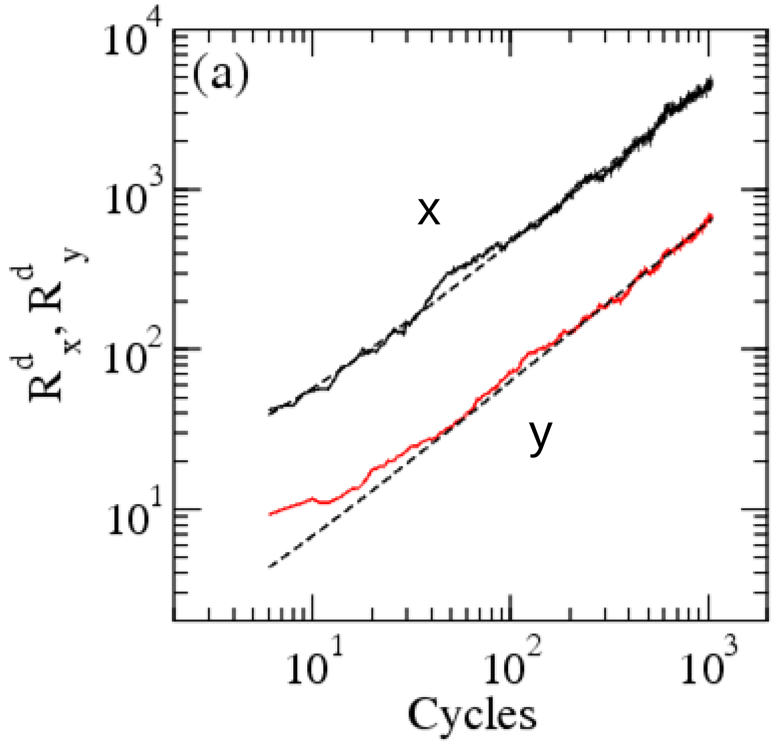
$n_v=0.052/\lambda^2$, displacement per cycle=160λ

Reversible and irreversible regimes

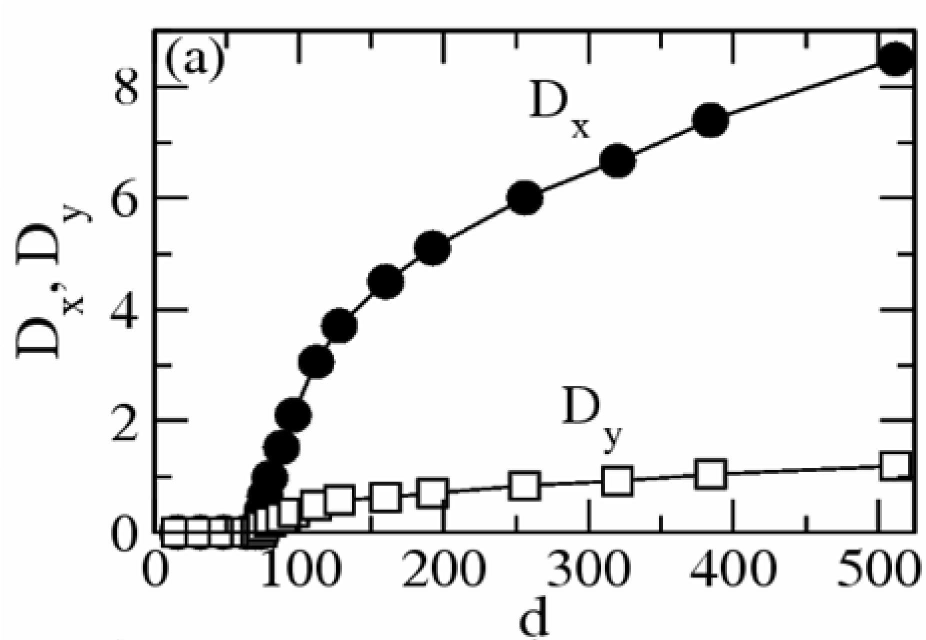




Anisotropic random walk behavior and threshold displacement



Anisotropic random walk

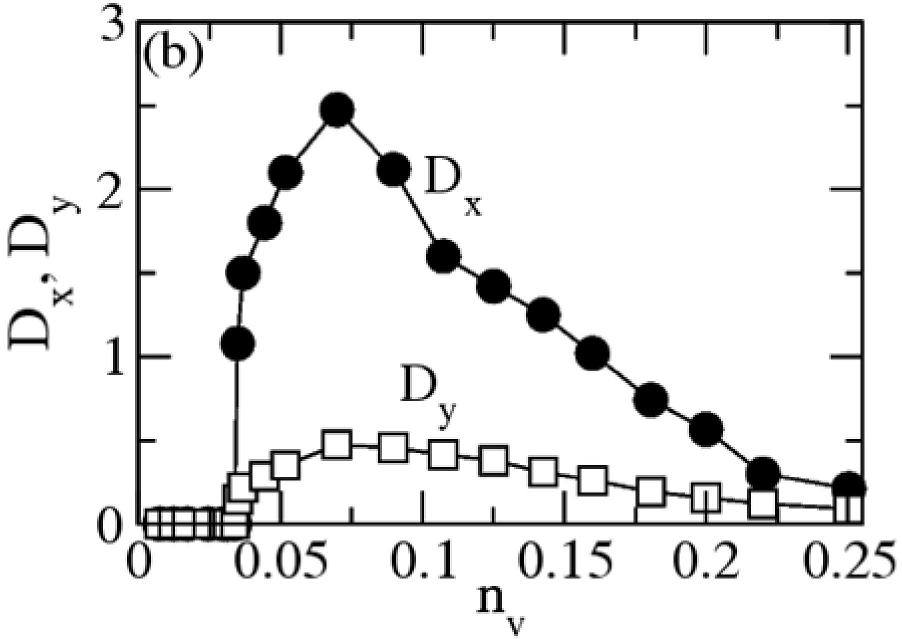


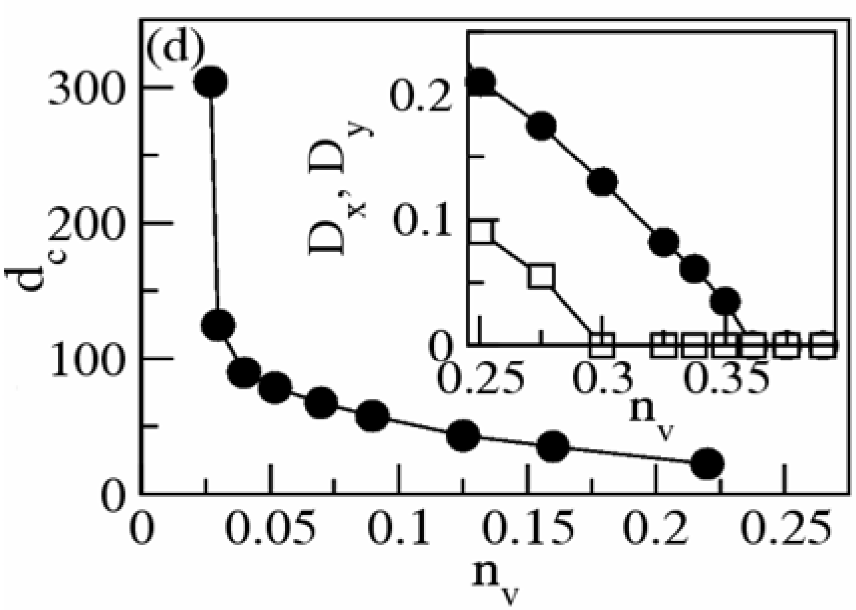
Threshold behavior for fixed density and increasing displacement per cycle

n_v=0.052/λ^2, displacement per cycle=160λ

Effective diffusivities D_x and D_y are obtained from slopes of R^d_x, R^d_y

Threshold as a function of vortex density; Jamming behavior





Displacement per cycle=96λ

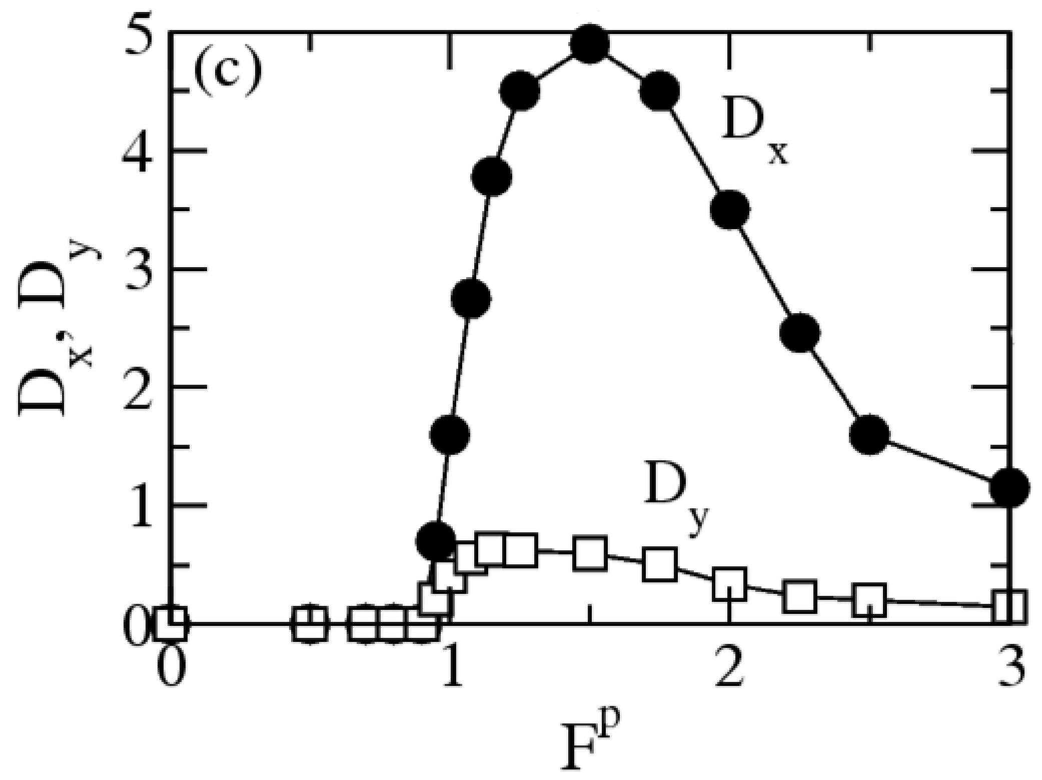
At higher vortex density, we find a smectic phase followed by a high-density, jammed (reversible) phase.

Displacement threshold for irreversible behavior decreases with vortex density.

NNSA

Los Alamos

Pinning is necessary for irreversible behavior

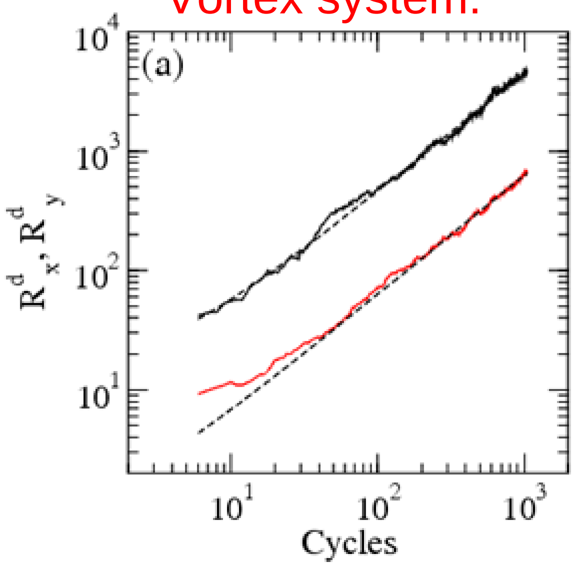


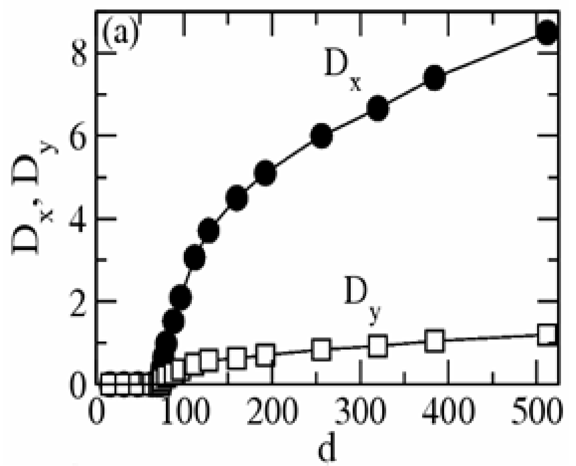
For weak pinning, vortices act elastically (jammed)

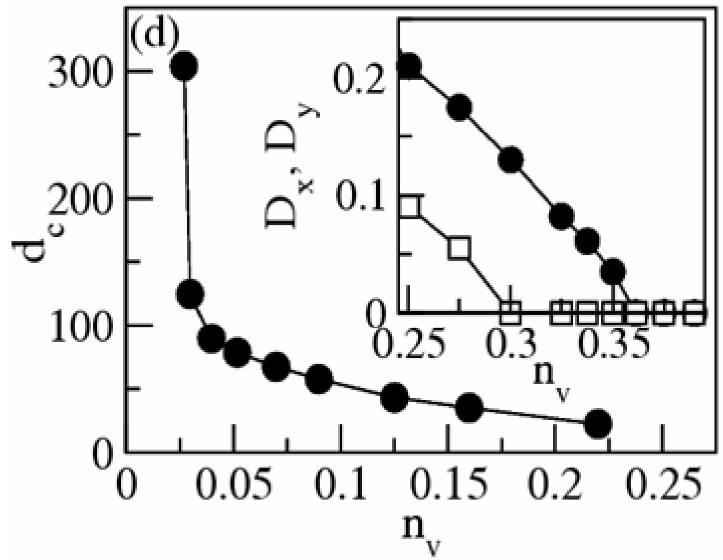
n_v=0.052/λ^2, displacement per cycle=160λ

Comparison between two systems

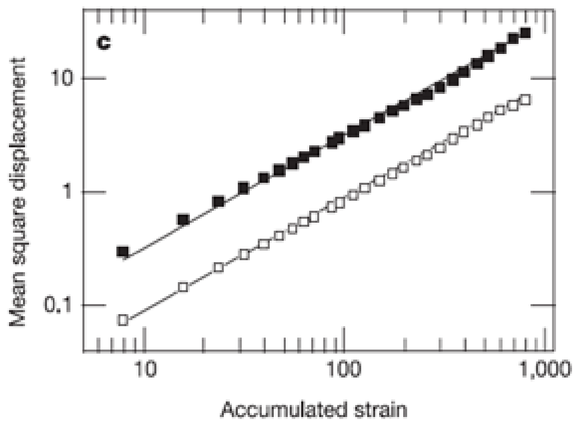
Vortex system:

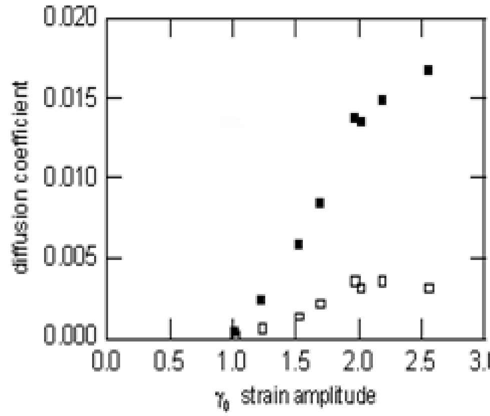


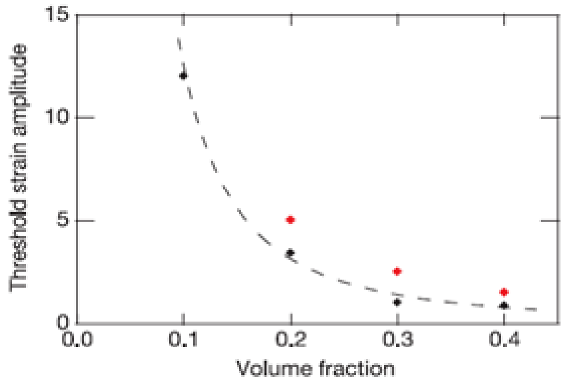




Colloidal system:







Irreversibility transitions

- Displacement per cycle: reversible for low displacements, irreversible for larger displacements

- Vortex density: reversible at both low and high densities, irreversible for intermediate density

- Quenched disorder strength: reversible for weak quenched disorder, irreversible for stronger quenched disorder

Next we show that the reversible-irreversible transition is consistent with an absorbing phase transition

Absorbing states

Absorbing state: configuration which can be entered by the system but cannot be left by the system.

Most important universality class of this type: Directed percolation

Example: Percolation through sites which have directed bonds

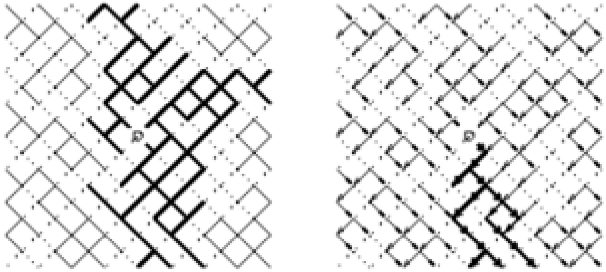
Example: Spreading of infectious disease (SIR)

Endemic infection: fluctuating state

Infection dies out: absorbing state

Continuous phase transition between two states w/ diverging correlation lengths

DP systems do NOT obey detailed balance; DP is a nonequilibrium process.



For a review, see H. Hinrichson, Adv. Phys. 49, 815 (2000)

Critical exponents for DP

Order parameter: Density of active sites $\rho(t)$

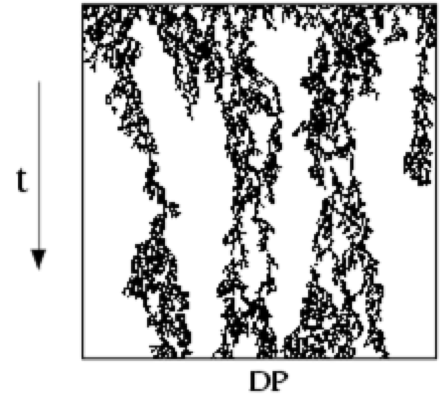
Close to the transition at $p=p_c$, density varies as a power law:

$$\rho \sim (p - p_c)^{\beta}$$

Two independent correlation lengths, spatial $\xi_{\perp}$ and temporal $\xi_{||}$

$$\xi_{\perp} \sim |p-p_c|^{-\nu_{\perp}} \qquad \xi_{||} \sim |p-p_c|^{-\nu_{||}}$$

Dynamical critical exponent $z = \nu_{||}/\nu_{\perp}$



No theory since DP is not conformally invariant: no symmetry between space and time

Values of critical exponents for DP

	D=1	D=2	D=3
β	0.276486(8)	0.584(4)	0.81(1)
$\nu_{\perp}$	1.096854(4)	0.734(4)	0.581(5)
$\nu_{\|\|}$	1.733847(6)	1.295(6)	1.105(5)
z	1.580745(10)	1.76(3)	1.90(1)

H. Hinrichsen
Adv. Phys. 49 815 (2000)

Experimental realization of DP

Few DP experimental confirmations, until recently

K.A. Takeuchi, M. Kuroda, H. Chate, M. Sano, PRL 99, 234503 (2007), U. Tokyo

Transition between two topologically different turbulent states.

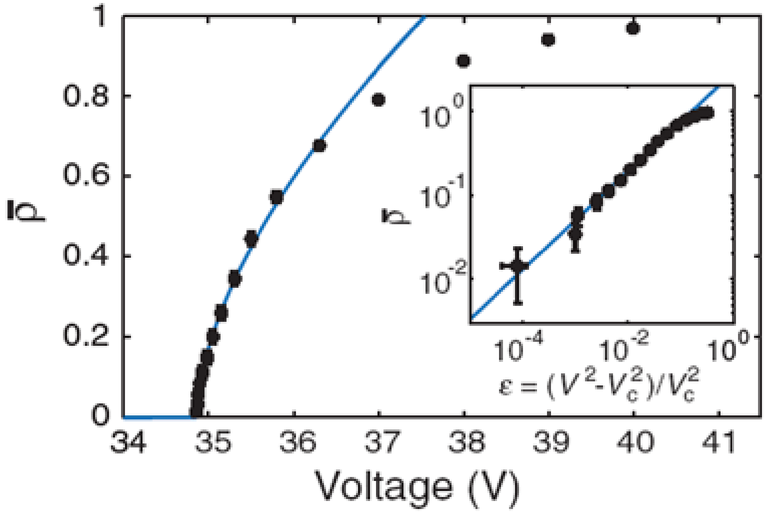
Electrohydrodynamic convection in nematic liquid crystals
Transition between two dynamic scattering modes DSM1 (few disclinations), DSM2 (many elongated disclinations)

Measurement of active phases as they tune through the transition with voltage.

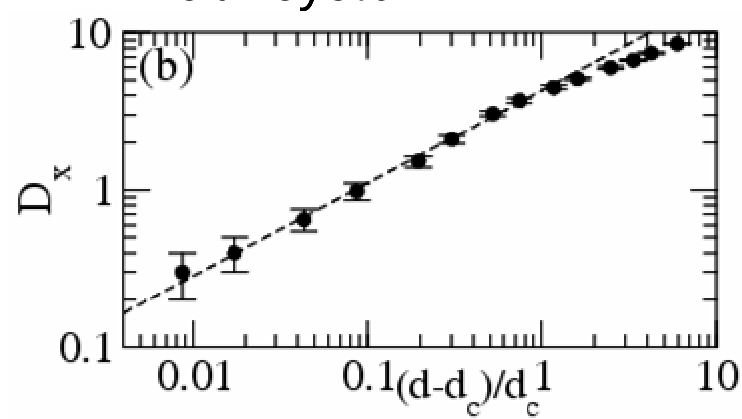
System with extremely low disorder

Directed Percolation Order Parameter

ρ: fraction of system in DSM2 state



Our system



$$\bar{\rho} \sim (V^2 - V_c^2)^\beta$$

$$D_x \propto (d - d_c)^\beta$$

β=0.59

β=0.59

(Dielectric torque driving convection is proportional to V^2)

K.A. Takeuchi, M. Kuroda, H. Chate, M. Sano, PRL 99, 234503 (2007)

Theory predicts β=0.584
H. Hinrichsen
Adv. Phys. 49 815 (2000)

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H. Hinrichsen
Adv. Phys. 49 815 (2000)

R-IR = DP?

- Our shearing system shows behavior that is consistent with an absorbing phase transition.
- Can we find additional evidence supporting the nature of the transition? Can we generalize the transition?

Consider time-dependent quantities

Particle density starting from fully occupied lattice

$\rho(t) \sim t^{-\alpha} f(\Delta t^{1/\nu_{||}})$

Survival probability that cluster is still active after t steps

$P(t) \sim t^{-\delta} g(\Delta t^{1/\nu_{||}})$

Probability that a randomly chosen site belongs to infinite cluster

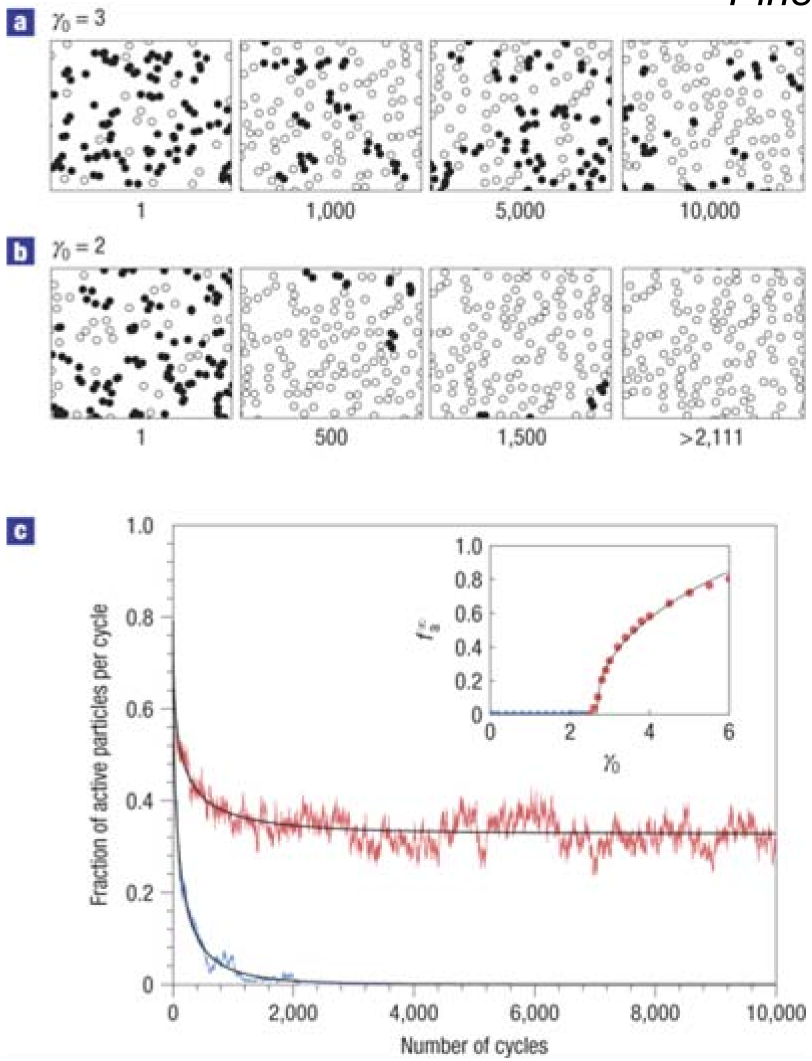
$P_{inf} \sim (p-p_c)^{\beta'}$

$\alpha=\beta/\nu_{||}$, $\delta=\beta'/\nu_{||}$, and in DP, $\beta=\beta'$

Random Organization observed in a multi-particle system with oscillatory shear

Simulation

L. Corte, P.M. Chaikin, J.P. Gollub, D.J. Pine, Nature Phys. ***4****, 420 (2008)*



Irreversible state (above transition)

Black dots: Active particles

White dots: Reversible particles

Reversible state (below transition)

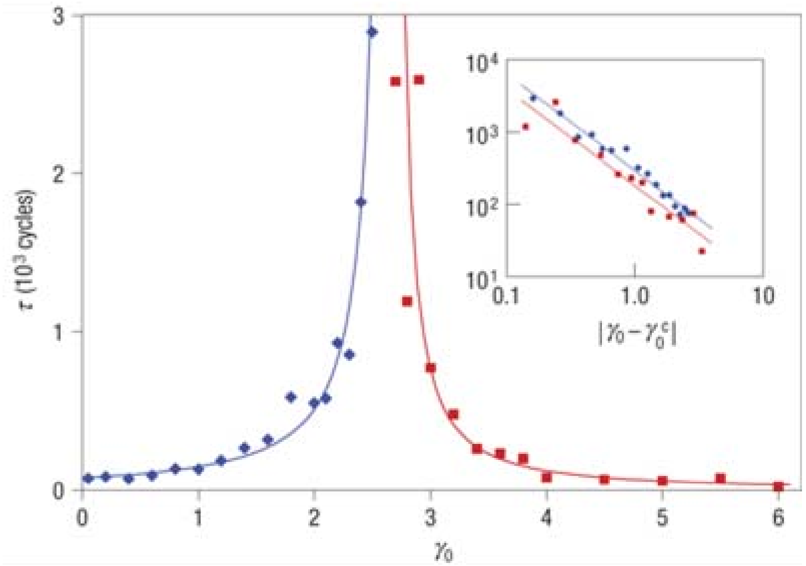
Near the transition, the system settles into a steady state that is either reversible or irreversible.

Time to reach a steady state is $\exp(-t/\tau)+C$

Also seen in experiment.

Diverging time scale to reach steady state at transition

Simulation:



Reversible (absorbing)

Irreversible

Exponent: 1.33

L. Corte, P.M. Chaikin, J.P. Gollub, D.J. Pine, Nature Phys. ***4****, 420 (2008)*

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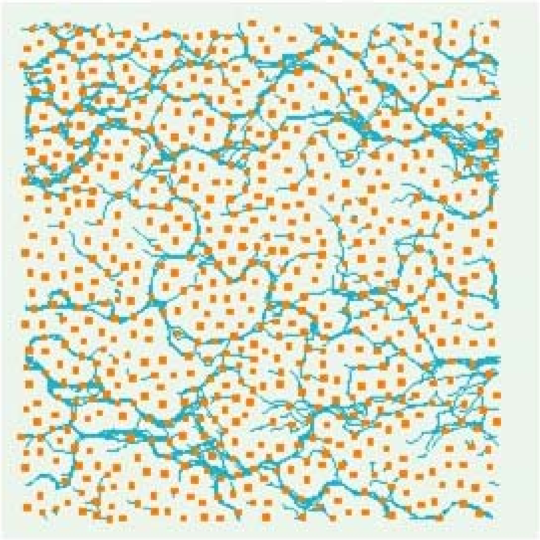
What is the nature of the plastic depinning transition? Is it connected to the reversible-irreversible phase transition?



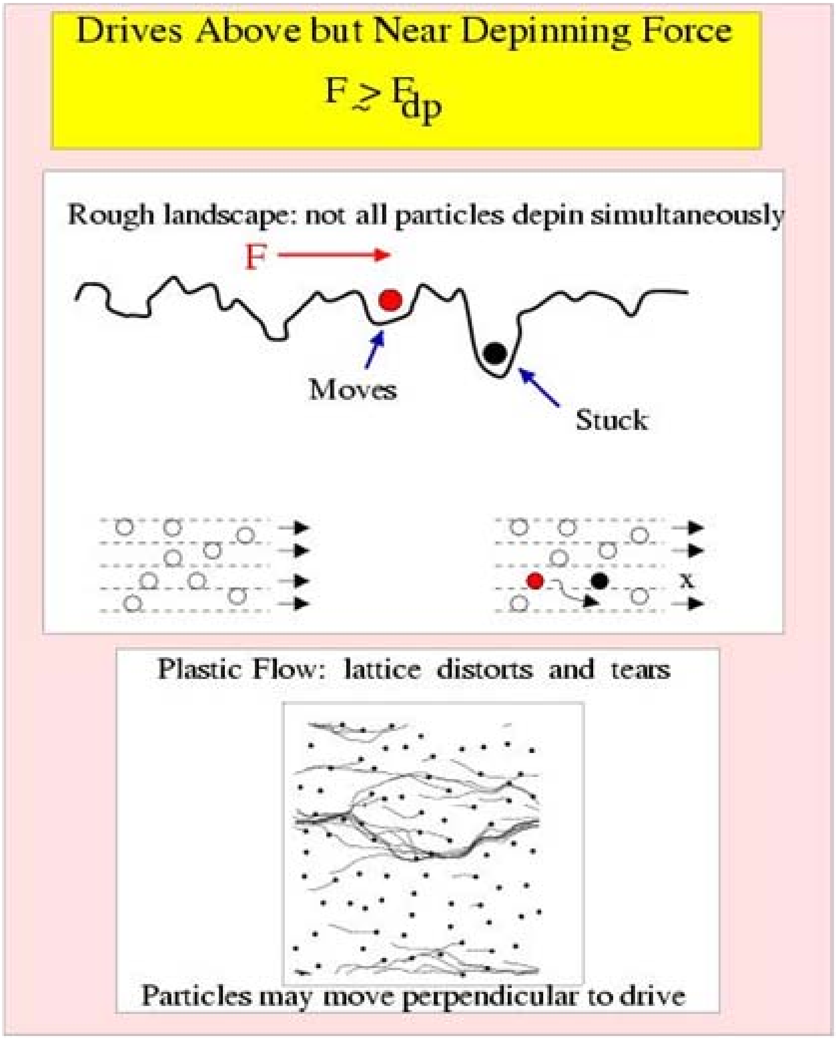
C. Reichhardt, C.J. Olson, PRL 89, 078301 (2002)

Plastic depinning of colloids on a random substrate
Orange dots: colloids
Blue lines: colloid traejctories

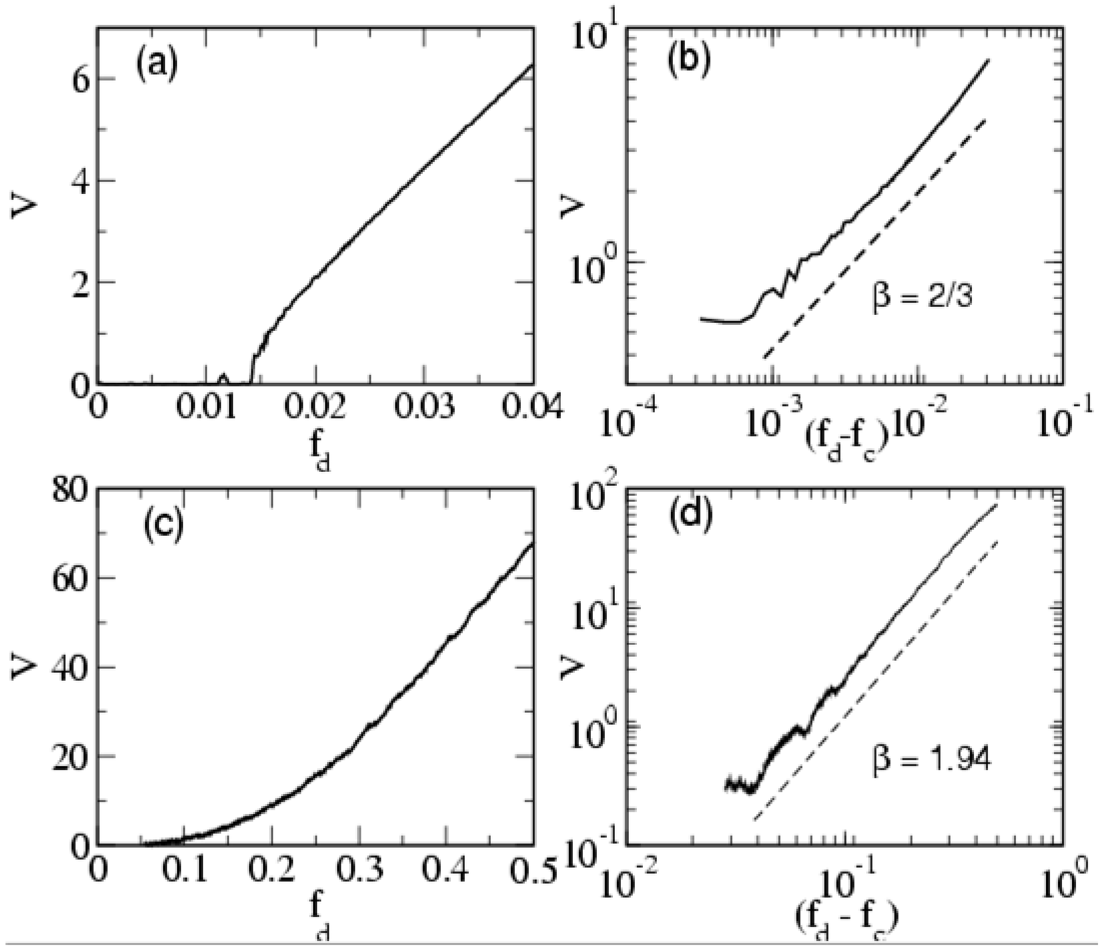
Plastic depinning occurs for vortices in type-II superconductors, friction, charge density waves, Wigner crystals, etc.

There is currently no theory for the nature of the transition.

Plastic Flow



Elastic and plastic depinning



Elastic

Plastic

Colloid simulation near plastic depinning transition

Overdamped equations of motion

$$\eta \frac{d\mathbf{R}_i}{dt} = \mathbf{F}_i^{vv} + \mathbf{F}_i^{p} + \mathbf{F}_i^{d}$$

Colloid-colloid interaction

$$\mathbf{F}_i^{vv} = \nabla V(r_{ij}) \qquad V(r_{ij}) = \frac{\exp(-\kappa r_{ij})}{r_{ij}}$$

Pinning force: parabolic traps

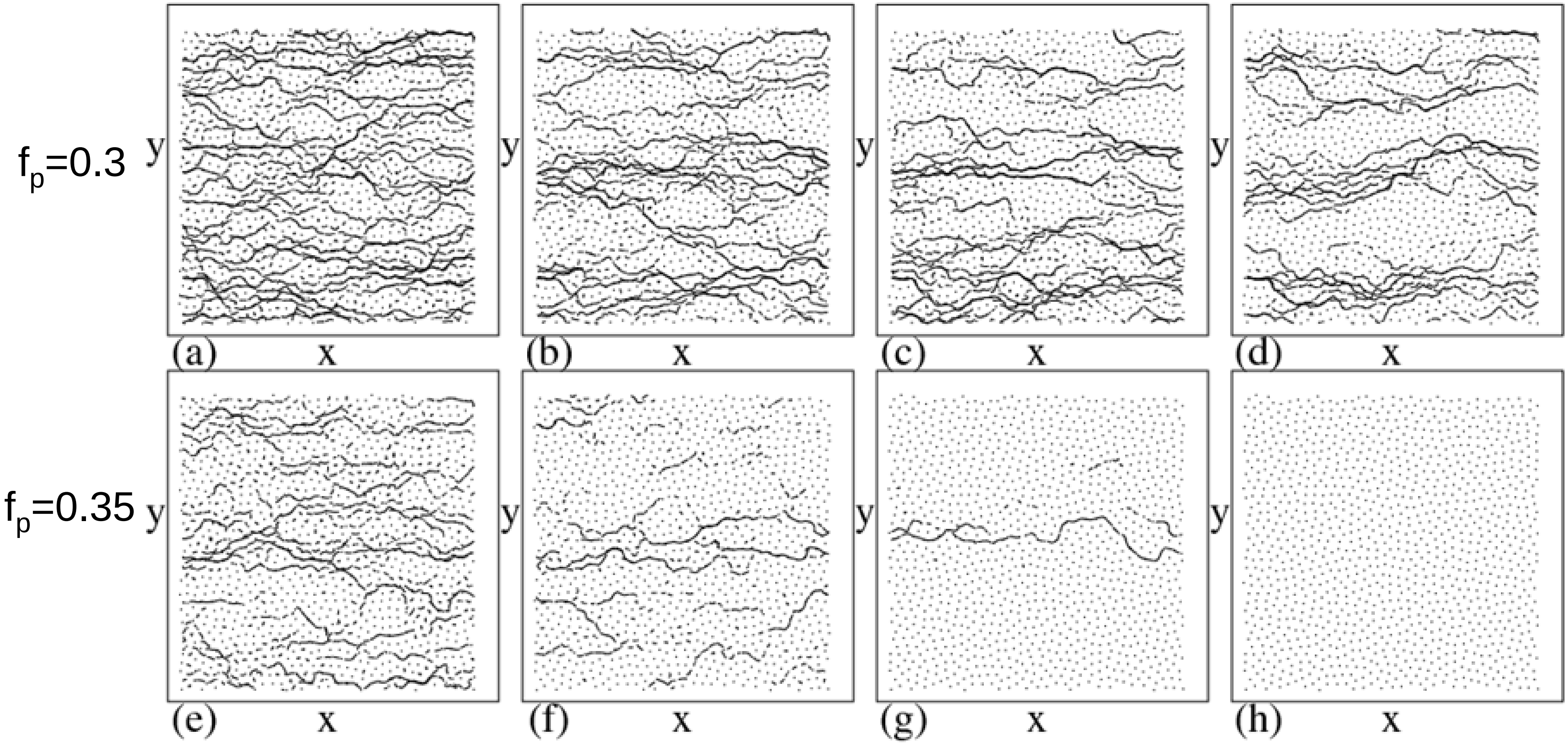
Fix Fd, vary Fp

$$\mathbf{F}_i^{p} = -\sum_{k=1}^{N_p} \frac{f_p}{R_p} R_{ik}^{(p)} \Theta\left(\frac{R_p - R_{ik}^{(p)}}{\lambda}\right) \hat{\mathbf{R}}_{ik}^{(p)}.$$

Driving force: in x direction $\mathbf{F}_i^{d}$

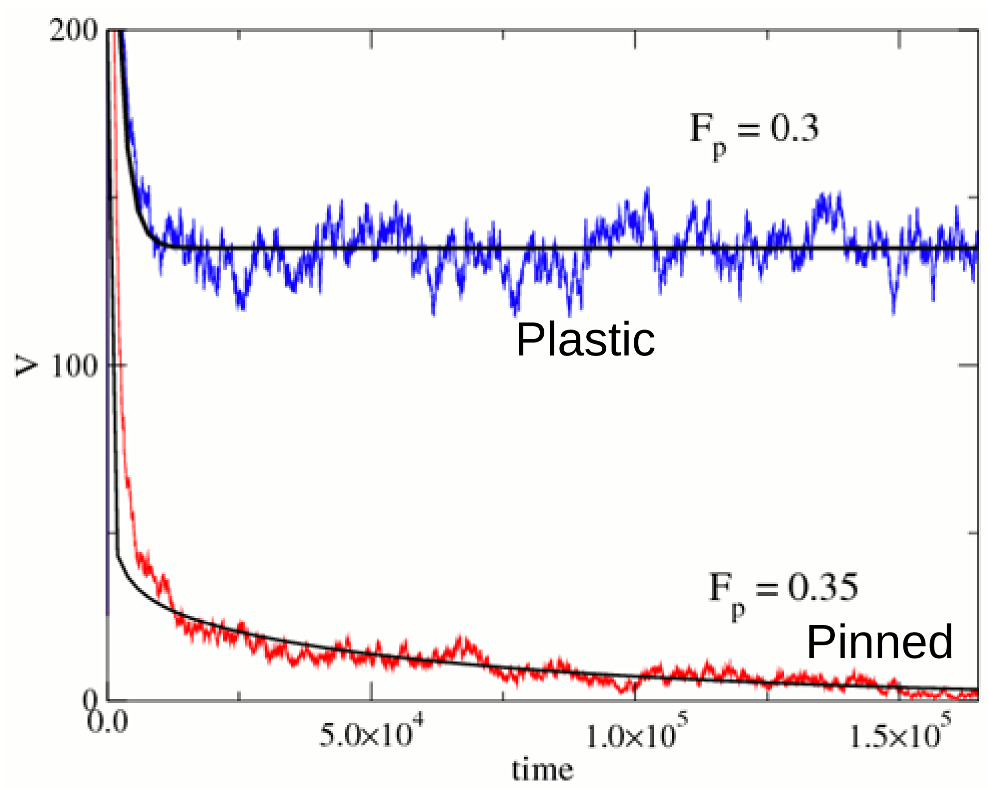
Up to 17,000 colloids

Sudden application of a force for different disorder strengths produces a pinned (absorbing) or flowing (fluctuating) state

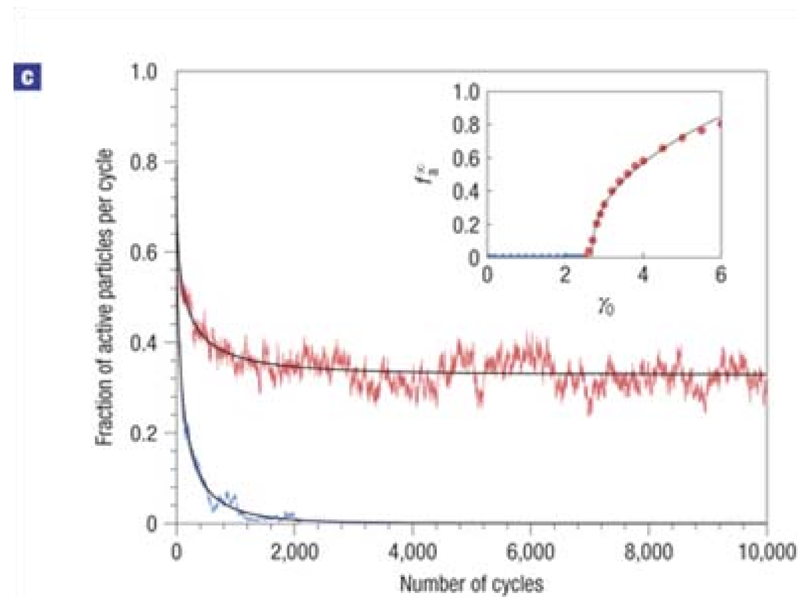


Time to reach steady state after a suddenly applied drive

External drive F_d=0.1



Colloid simulation (Corte et al)

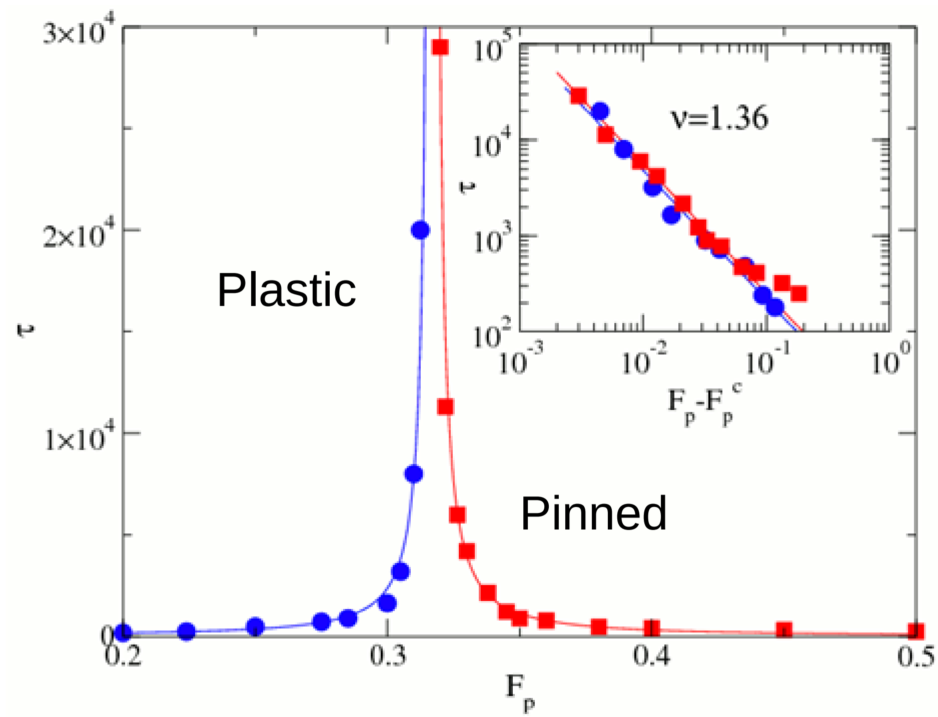


$$V(t) = (V^0 - V^s)\exp(-t/\tau)/t^\alpha + V^s$$

Plastic flow = irreversible regime
Pinned regime = reversible regime or absorbing state

Diverging time scale to reach steady state at transition

Our results:



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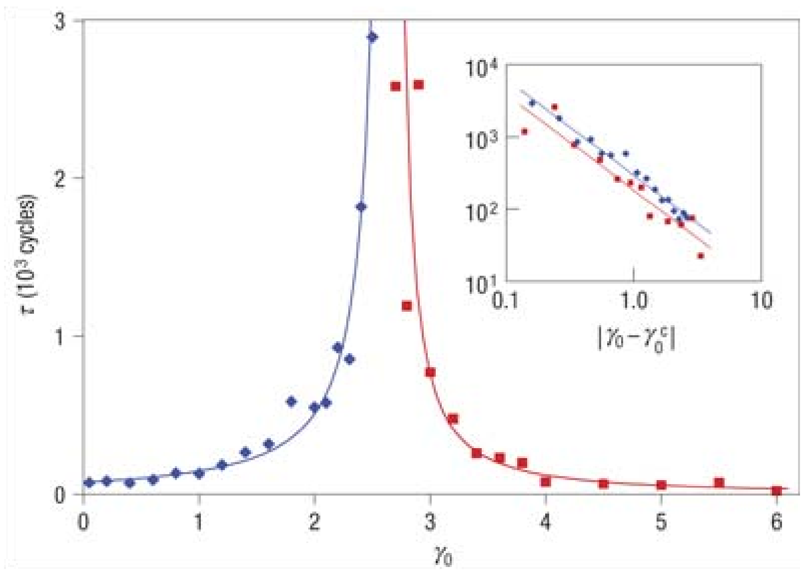

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β	0.276486(8)	0.584(4)	0.81(1)	
$\nu_\perp$	1.096854(4)	0.734(4)	0.581(5)	(time)
$\nu_{\parallel}$	1.733847(6)	$\boxed{1.295(6)}$	1.105(5)	(space)
z	1.580745(10)	1.76(3)	1.90(1)	

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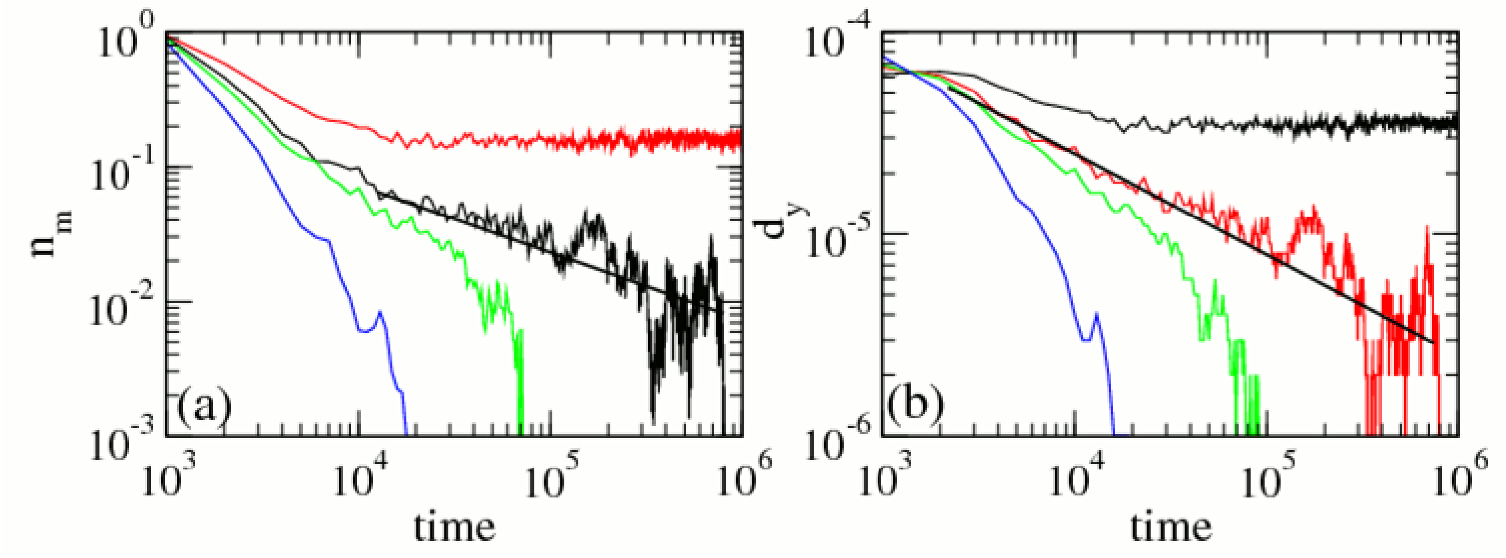
Power law decay at transition



F_p/F_p^c = 0.927, 1.008, 1.05, and 1.2 (top to bottom)

Fit: α=1/2

Nature of transition

- In our simulations we find ν=1.36. Corte et al. simulations find ν=1.33. For conserved directed percolation (C-DP) in 2D, $\nu_{||}$=1.29; for directed percolation (DP), $\nu_{||}$=1.295.
- We find α=0.5. C-DP predicts α=0.5, DP predicts α=0.45.
- The nature of the phase transition is consistent with certain directed percolation models in 2+1 dimensions.

- Summary: Plastic depinning also exhibits the same random organization phenomenon found in the colloidal systems, suggesting that plastic depinning falls into the universality class of the absorbing phase transition of conserved directed percolation.
- Other systems where similar effects could occur include superconducting vortices, charge density waves, Wigner crystals, and plastic flow.

Summary

- We show that vortices interacting with random pinning can exhibit a nonequilibrium phase transition from reversible to irreversible behavior.
- This is the same behavior observed in recent colloidal shearing experiments.
- The nature of the phase transition is consistent with certain directed percolation models in 2+1 dimensions.
- Study of transient effects shows that both the colloidal shearing experiments and the behavior of colloids near a plastic depinning transition are consistent with the same absorbing phase transition
- Possible insight into the nature of plastic depinning?