



Spins Dynamics in Nanomagnets

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Lecture 1



Outline

I. Magnetic Interactions

- Exchange, Anisotropy and Dipolar Interactions*
- Zeeman Interaction*

II. Micromagnetic Energy

- Energy, Field and Length Scales*
- Examples of Magnetic Domain Structure*

III. Magnetic Nanostructures

- Single Domain Model*
- Experiments on Individual (classical) Nanomagnets*
- Thermal Activation and Quantum Tunneling of Magnetization*

IV. Classical Magnetization Dynamics

- Landau-Lifshitz-Gilbert Equation*

References

Magnetic Interactions

- Exchange

- Coulomb Interactions + Pauli Exclusion Principle



locks spins together

- Magnetocrystalline Anisotropy

- Spin-orbit interactions

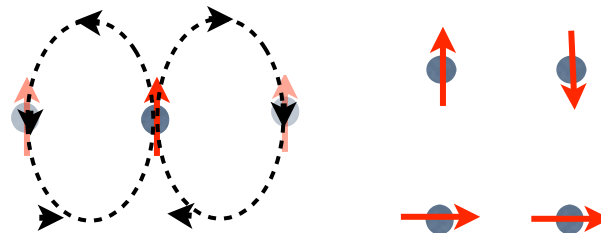


preferred direction in space

- Magnetic Dipole Interactions

non-local

- Magnetic field associated with a magnetic moment



Magnetic Interactions

Microscopic

• Exchange

$$H_{\text{exc}} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$J < 0$ Ferromagnetic interactions



• Anisotropy

$$H_{\text{anis}} = -d \sum_i S_{iz}^2$$

$d > 0$ Easy axis anisotropy

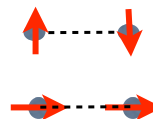
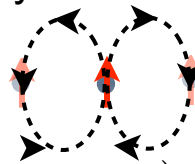
$d < 0$ Easy plane type anisotropy



• Dipolar Interactions

$$H_{ij} = \frac{\mu_0}{4\pi r^3} (3(\vec{m}_i \cdot \hat{r}_{ij})(\vec{m}_j \cdot \hat{r}_{ij}) - \vec{m}_i \cdot \vec{m}_j)$$

$$\vec{m} = \gamma \vec{S} \quad \gamma = g\mu_B/\hbar$$



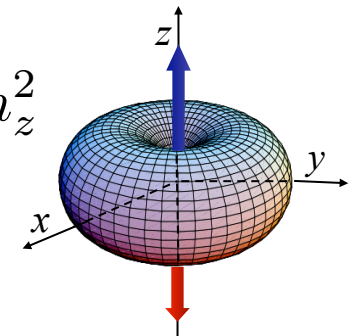
Continuum

$$E_{\text{exc}} = A(|\nabla m_x|^2 + |\nabla m_y|^2 + |\nabla m_z|^2)$$

$$A = -JS^2c/a$$

$A \simeq kT_c/a$ a =lattice constant
 c (lattice): $c=1$ (sc), 2 (bcc), 4 (fcc)

$$E_{\text{anis}} = -Km_z^2$$



Magnetostatic Energy

$$\vec{H}_d = -\tilde{D}\vec{m}$$

$$E_{\text{ms}}^{\text{sample}} = -\frac{\mu_0}{2} M_s \vec{m} \cdot \vec{H}_d$$

$$E_{\text{ms}}^{\text{all space}} = \frac{\mu_0}{2} H_d^2$$

Magnetic Interactions

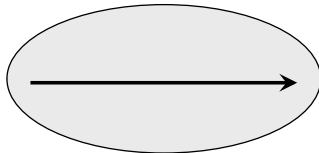
•Zeeman Energy

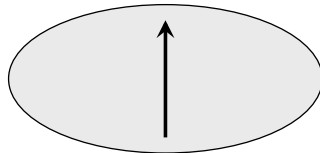
$$H_z = g\mu_B \sum_i \vec{S}_i \cdot \vec{H}_e$$

$$E = -\mu_0 \vec{M} \cdot \vec{H}_e$$

Magnetostatic Energy

May sometimes be approximated by a *local* contribution to the energy

$E_z = \frac{\mu_0}{2} D_z M_s^2$


$E_y = \frac{\mu_0}{2} D_y M_s^2$


$E_{ms} = \frac{\mu_0}{2} (D_z - D_y) M_s^2 m_z^2$

Thin film
 $D_z = 1, D_x = D_y = 0$

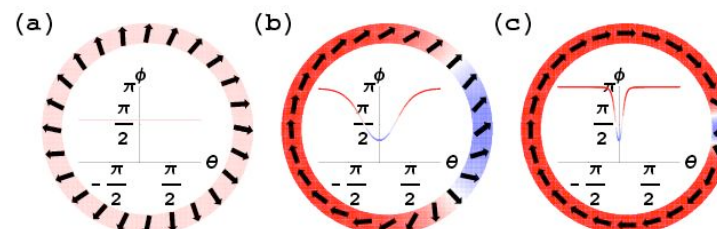
Infinite Cylinder
 $D_z = 0, D_x = D_y = 1/2$

Sphere
 $D_x = D_y = D_z = 1/3$

Examples:

- 1) single domain particles
- 2) thin films with in-plane magnetization

Analytic solutions for the micromagnetic energy, saddle states, rate of thermally activated reversal, etc. are then possible.



Martens, Stein, ADK, PRB 2006
 Chaves, ADK, Stein, PRB 2009

Micromagnetic Energy

$$E[\mathbf{m}(\mathbf{r})] = \underbrace{A \int_{\Omega} |\nabla \mathbf{m}|^2 d^3r}_{\text{Exchange}} - \underbrace{K \int_{\Omega} m_z^2 d^3r}_{\text{Anisotropy}} - \underbrace{\mu_0 M_s \int_{\Omega} \mathbf{H}_e \cdot \mathbf{m} d^3r}_{\text{Zeeman}} + \underbrace{\frac{\mu_0}{2} \int_{\mathbf{R}^3} |\nabla U|^2 d^3r}_{\text{Magnetostatic}}$$

$\Omega \equiv \text{ferromagnet}$

$$E[\mathbf{m}(\mathbf{r})] = \frac{\mu_0 l_{\text{ex}}^2}{2} \int_{\Omega} |\nabla \mathbf{m}|^2 d^3r - K \int_{\Omega} m_z^2 d^3r - \mu_0 M_s \int_{\Omega} \mathbf{H}_e \cdot \mathbf{m} d^3r + \frac{\mu_0}{2} \int_{\mathbf{R}^3} |\nabla U|^2 d^3r$$

Exchange length $l_{\text{ex}} = \sqrt{2A/(\mu_0 M_s^2)}$

Domain wall width $\lambda = \sqrt{2A/K}$

Anisotropy/Magnetostatic $Q = \frac{2K}{\mu_0 M_s^2}$

Magnetostatic Potential

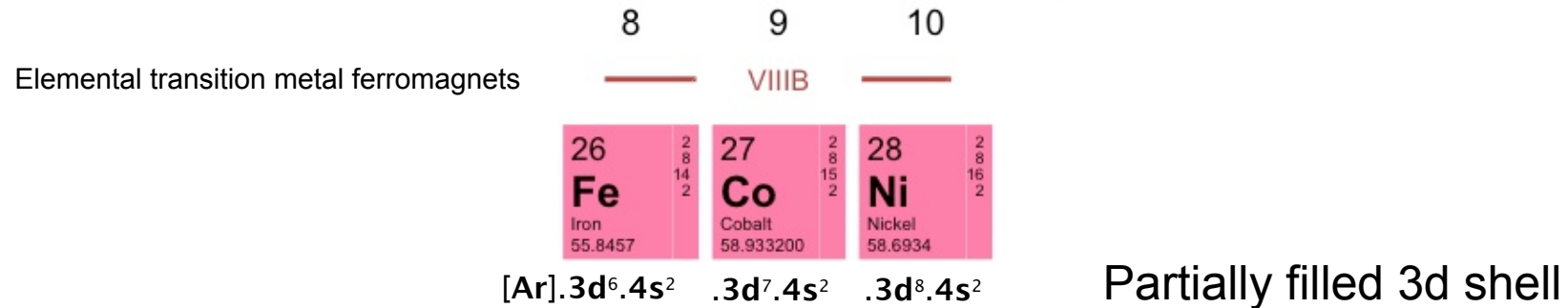
$$\mathbf{H} = -\nabla U$$

$$\nabla^2 U = \nabla \cdot \mathbf{M}$$

+ boundary conditions

$$U_{\text{in}} = U_{\text{out}} \quad \frac{\partial U_{\text{in}}}{\partial n} - \frac{\partial U_{\text{out}}}{\partial n} = \mathbf{M} \cdot \hat{n}$$

Energy, Field and Length Scales



- Exchange ~1000 K, 100 T
Curie Temperature
- Anisotropy ~10 K, 0.01 to 10 T
- Dipolar Interactions ~1 K, mT

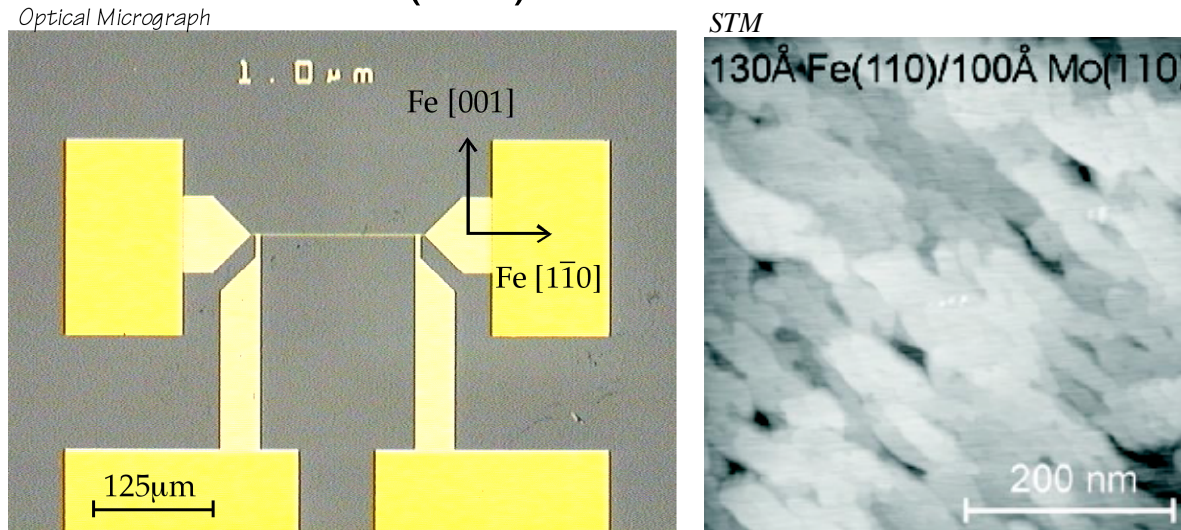
Exchange length $l_{\text{ex}} = \sqrt{2A/(\mu_0 M_s^2)} \sim 4 \text{ nm}$

Domain wall width $\lambda = \sqrt{2A/K} \sim 5 \text{ to } 100 \text{ nm}$

Examples

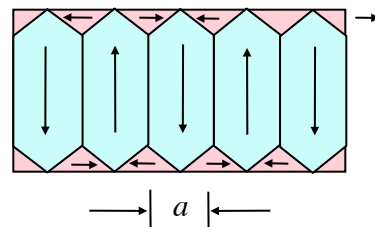
- Competing interactions lead to the formation of magnetic domains: exchange, anisotropy and magnetostatic

(110) Fe Thin Films

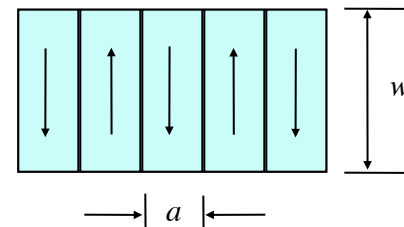


$$Q = \frac{2K}{\mu_0 M_s^2}$$

$Q \ll 1$ flux closure domains



$Q \gg 1$ stripe domains



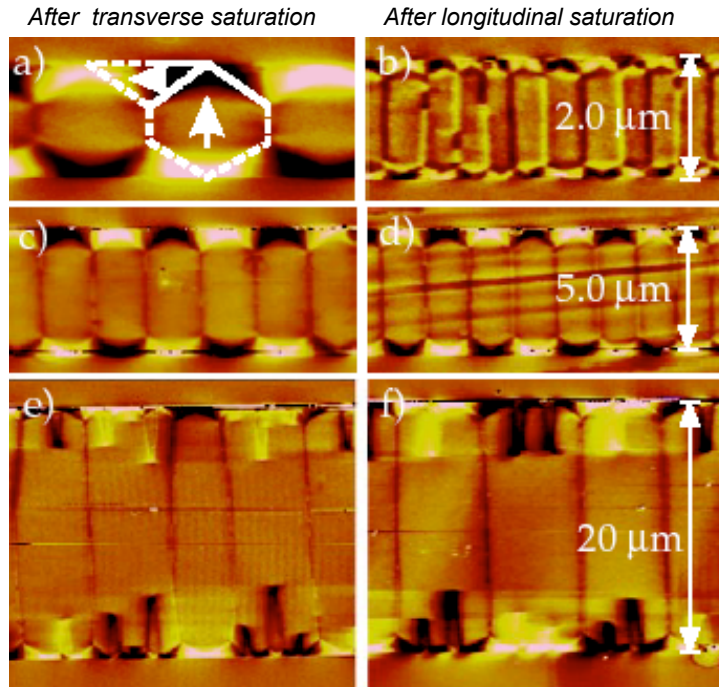
easy axis

film thickness t

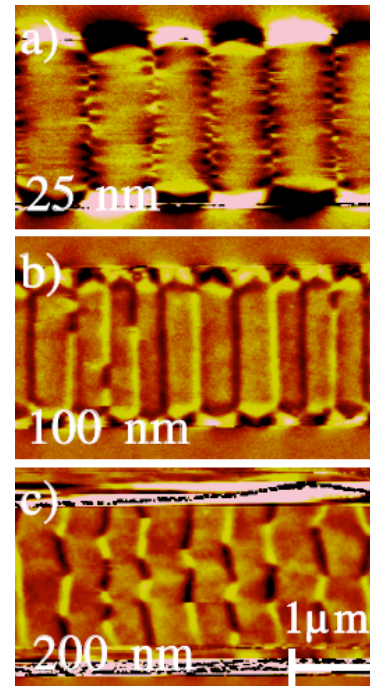
Examples

- Domain configurations as function of:
 - Linewidth
 - Magnetic history

$H=0$



Domain wall structure vs film thickness



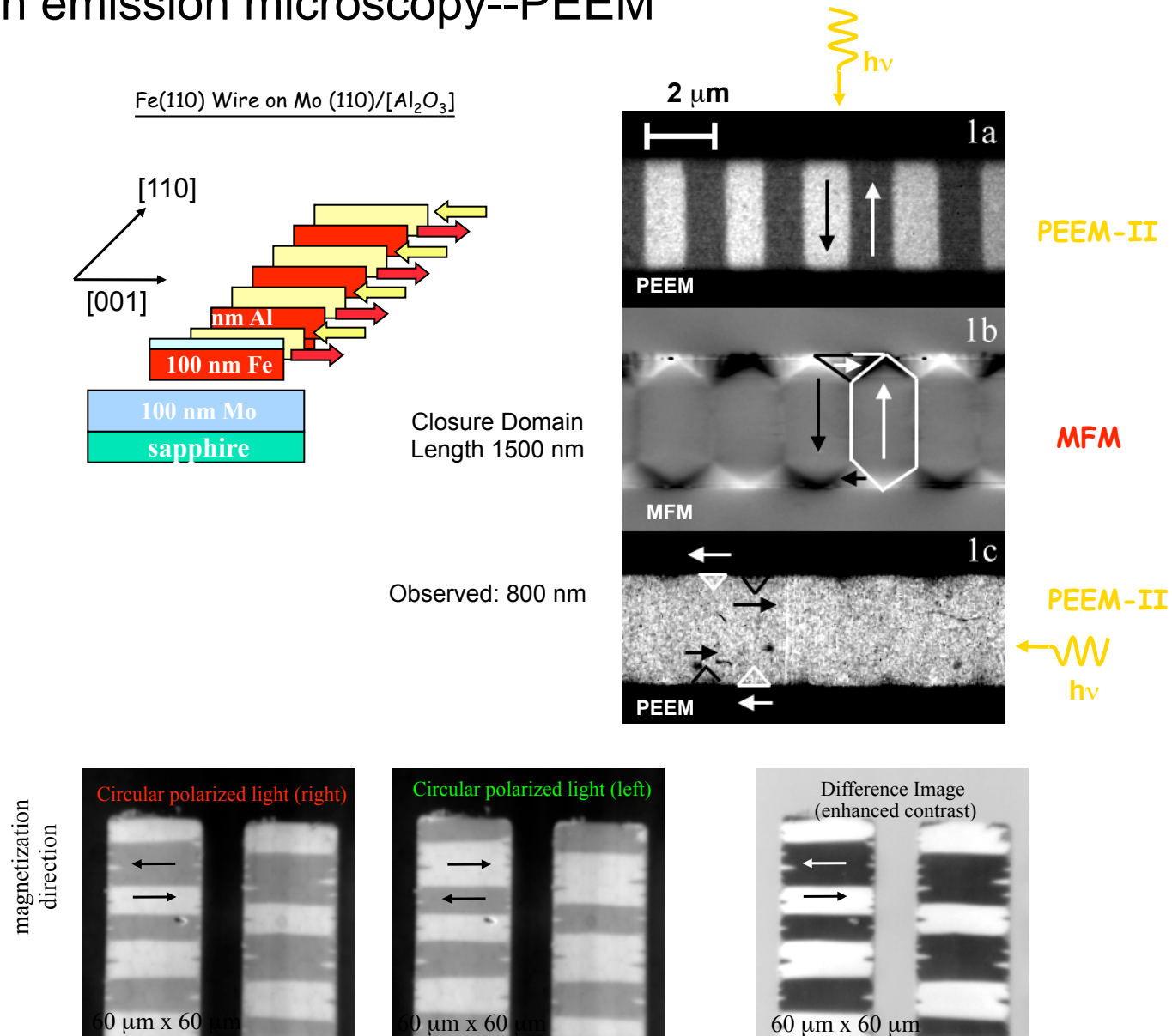
Neel-cross-tie walls

(Asymmetric) Bloch walls

Canted Bloch walls

Examples

Photoelectron emission microscopy--PEEM



Ni/Co (111) Single Crystalline Superlattices

Magneto-optical images

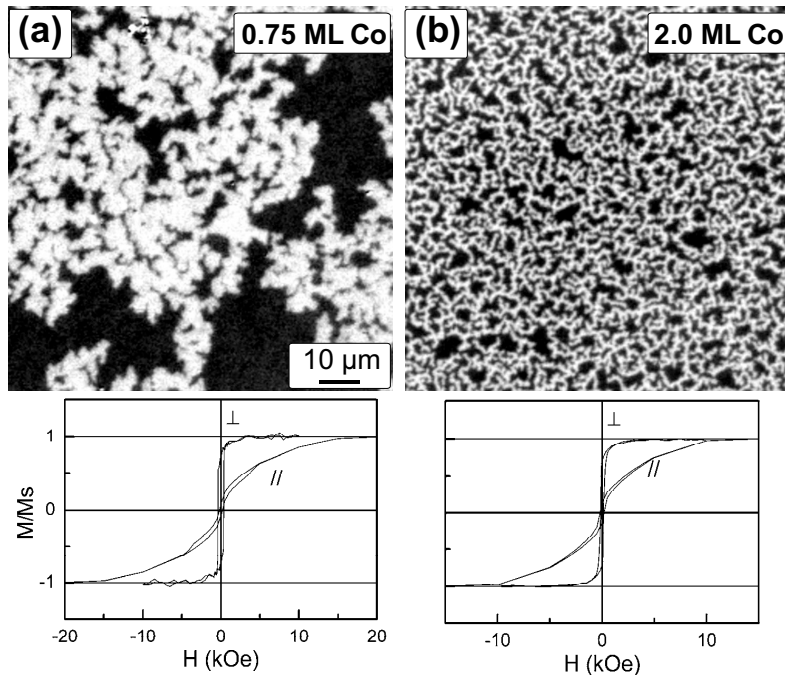


FIG. 2. (top) Kerr images obtained during magnetization reversal close to the coercive field for a (a) SL with Co coverage equal to 0.75 ML and (b) for a SL with Co coverage equal to 2 ML. (bottom) Corresponding hysteresis loops measured by SQUID on these samples.

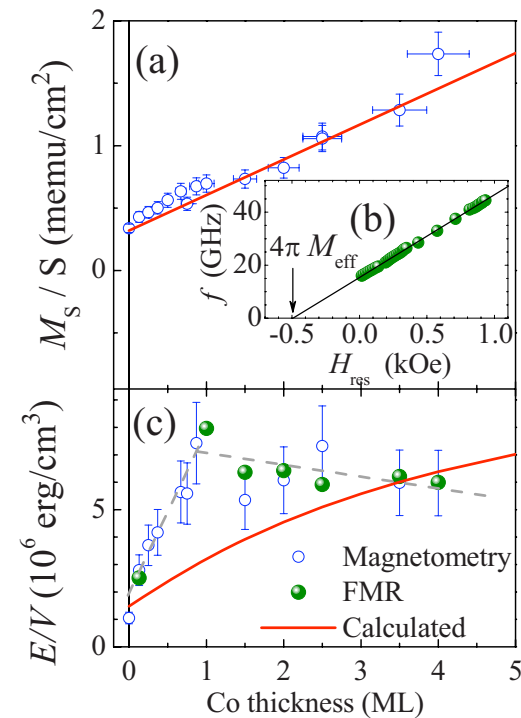
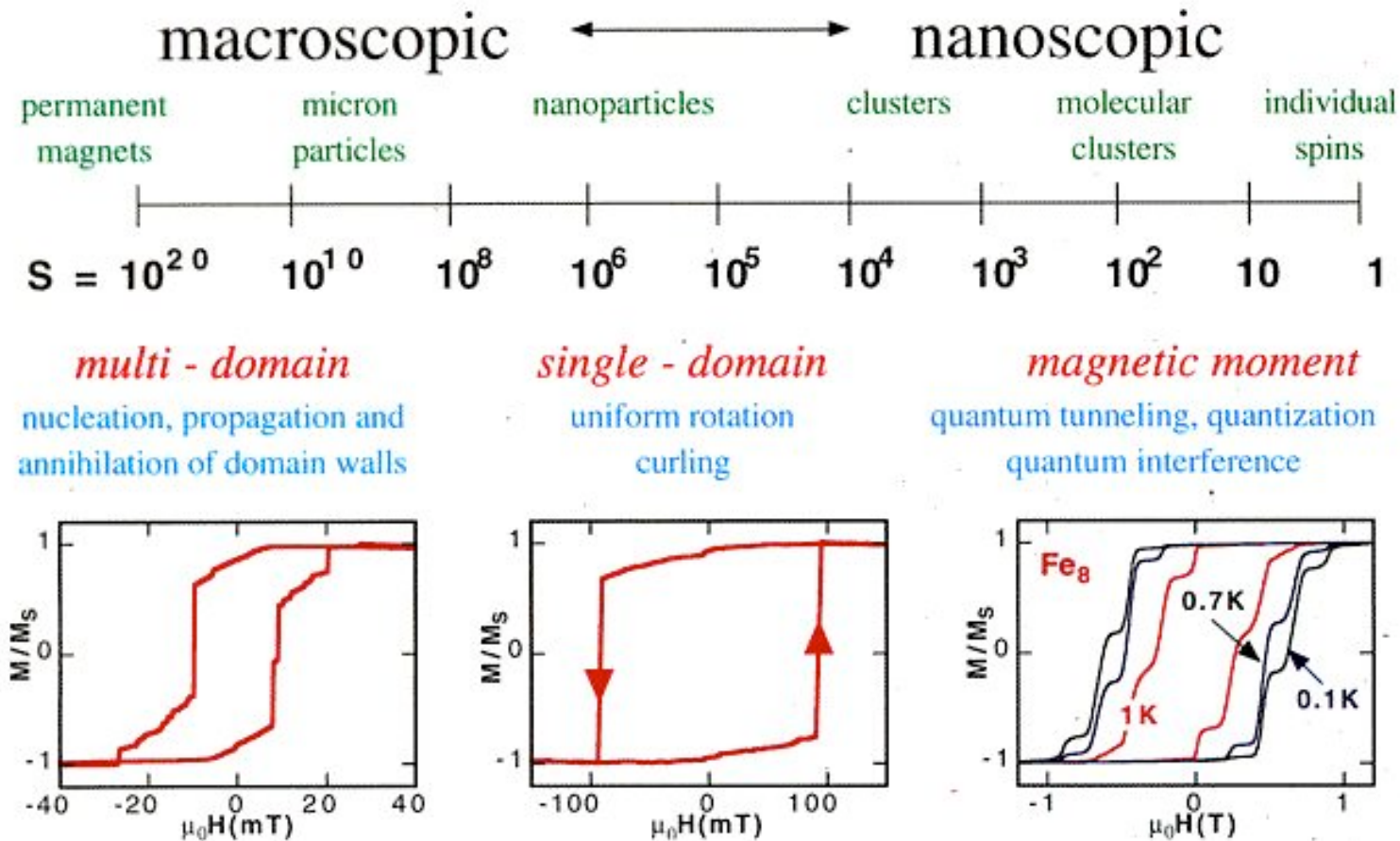


FIG. 3. (Color online) Data for a series of SLs with 3 ML of Ni, Co coverage varying from 0 to 4 ML, repeated ten times. (a) Saturation magnetization per unit surface area versus Co layer thickness from magnetometry (open circles) compared to the calculated value assuming magnetization value of the bulk Co and Ni—(solid line) (b) Frequency dependence of the resonance field for the multilayer film with 2.5 ML Co. The x -intercept gives the perpendicular anisotropy constant. (c) Energy density (Perpendicular magnetocrystalline anisotropy constant and demagnetization energy density) vs Co layer thickness. The anisotropy constant is deduced from magnetometry (open circles) and FMR measurements (full circles). The dash lines are guide to the eyes. The demagnetization energy density (full line) is calculated assuming magnetization value of the bulk Co and Ni.

sapphire substrate/bcc V (110)/fcc Au (111)-10/fcc (111) $\times\{\text{Ni}_3/\text{Co}_x\}_{10}/\text{Au}$

S. Girod et al. APL 2009

Magnetic Nanostructures



Single Domain Model

Uniform M--no exchange energy

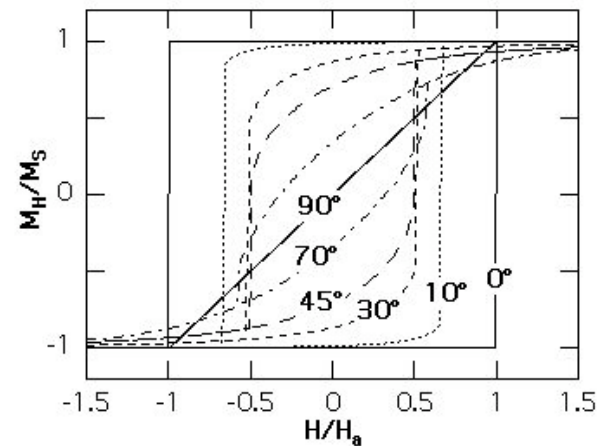
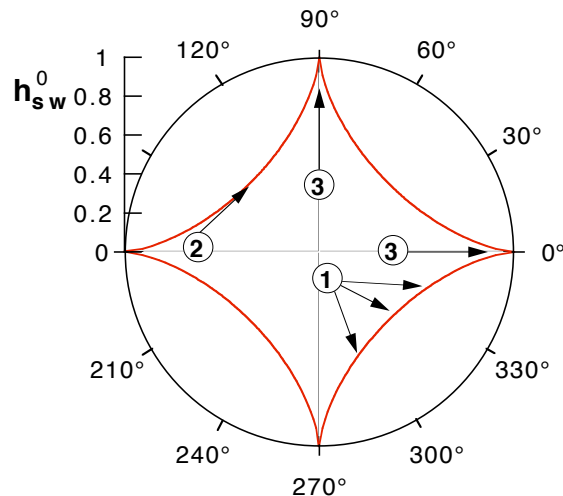
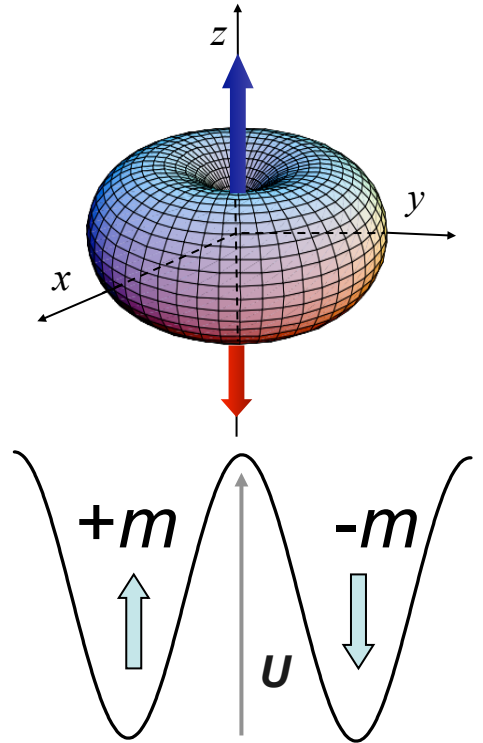
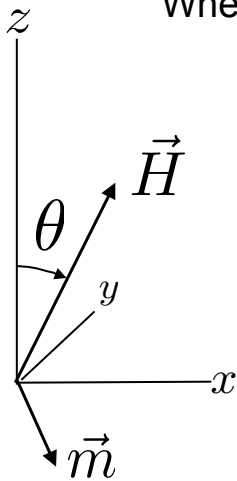
$$E = -K m_z^2 - \mu_0 M_s \vec{m} \cdot \vec{H}$$

When does the metastable state become unstable?

$$h_{sw}^0 = \frac{H_{sw}^0}{H_a} = \frac{1}{\left(\sin^{2/3} \theta + \cos^{2/3} \theta \right)^{3/2}}$$

$$H_a = 2K/(\mu_0 M_s)$$

$$h_x^{2/3} + h_y^{2/3} = 1$$



Single Domain Model

Experimental study of an individual Co nanoparticle

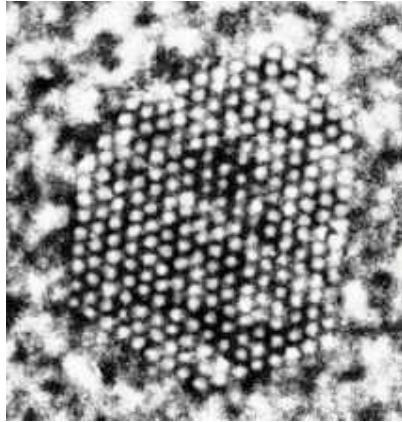


Fig. 2.4 High Resolution Transmission Electron Microscopy observation along a [110] direction of a 3 nm cobalt cluster exhibiting a *f.c.c.*-structure.

Wernsdorfer, et al. 1996

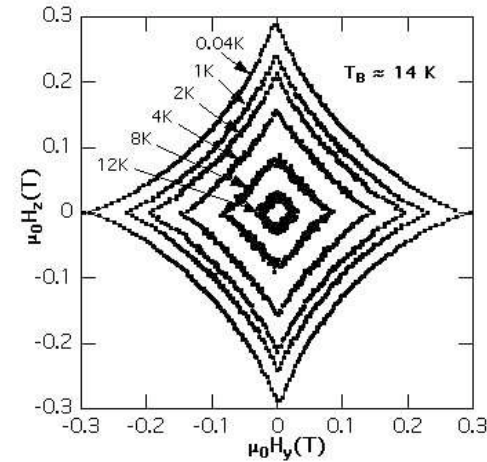


Fig. 3.3 Temperature dependence of the switching field of a 3 nm Co cluster, measured in the plane defined by the easy and medium hard axes ($H_y - H_z$ plane in Fig. 2.6). The data were recorded using the blind mode method (Sect. 1.2.6) with a waiting time of the applied field of $\Delta t = 0.1$ s. The scattering of the data is due to stochastic and in good agreement with Eq. 3.10.

Prob, of not switching in a time t :

$$P(t) = e^{-t/\tau}$$

$$\tau = \tau_0 \exp(U/k_B T)$$

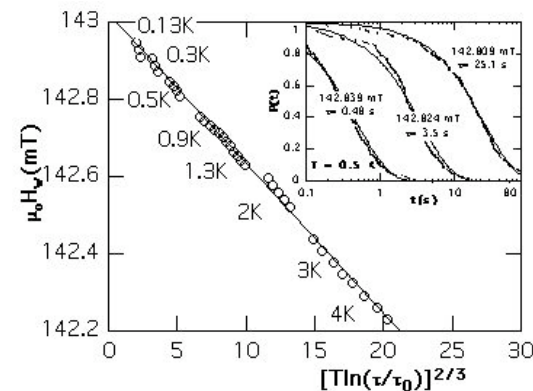


Fig. 3.2 Scaling plot of the mean switching time $\tau(H_w, T)$ for several waiting fields H_w and temperatures ($0.1 \text{ s} < \tau(H_w, T) < 60 \text{ s}$) for a Co nanoparticle. The scaling yields $\tau_0 \approx 3 \times 10^{-9} \text{ s}$. *Inset*: examples of the probability of not-switching of magnetization as a function of time for different applied fields and at 0.5 K. *Full lines* are data fits with an exponential function: $P(t) = e^{-t/\tau}$

Thermal Activation and Quantum Tunneling

VOLUME 60

22 FEBRUARY 1988

NUMBER 8

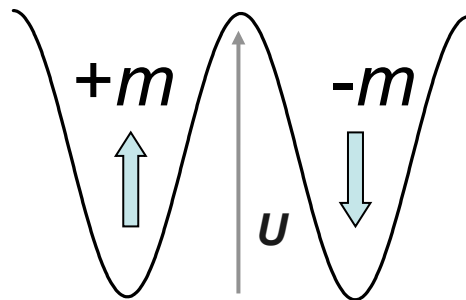
Quantum Tunneling of Magnetization in Small Ferromagnetic Particles

E. M. Chudnovsky and L. Gunther

Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155

(Received 29 October 1987)

The probability of tunneling of the magnetization in a single-domain particle through an energy barrier between easy directions is calculated for several forms of magnetic anisotropy. Estimated tunneling rates prove to be large enough for observation of the effect with the use of existing experimental techniques.

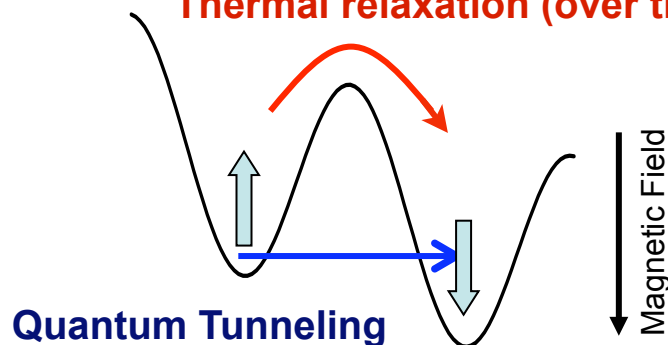


Thermal $\Gamma \sim e^{-U/k_B T}$

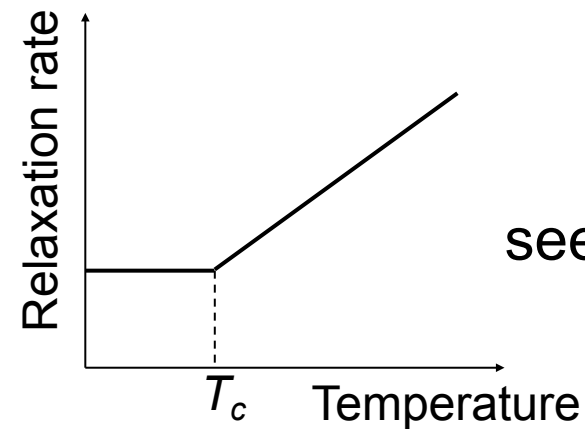
Quantum $\Gamma \sim e^{-B(0)} = e^{-U/k_B T_C}$

$$T_C = U/k_B B(0)$$

Thermal relaxation (over the barrier)



Quantum Tunneling



see lecture 3

also, Enz and Schilling, van Hemmen and Suto (1986)

Classical Magnetization Dynamics

Effective Field $\mu_o \mathbf{H}_{\text{eff}} = -\delta E / \delta \mathbf{M}$

Single Domain Model

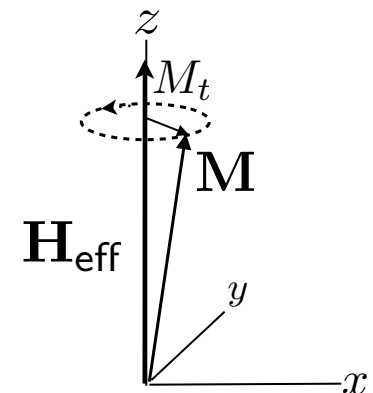
$$E = -K m_z^2 - \mu_0 M_s \vec{m} \cdot \vec{H}$$

$$\vec{H}_{\text{eff}} = \vec{H} + \frac{2K}{M_s} m_z$$

$$\frac{d\vec{L}}{dt} = \text{Torque} = \mu_0 \vec{M} \times \vec{H}_{\text{eff}}$$

$$\vec{M} = -\gamma \vec{L} \quad \gamma = \left| \frac{g\mu_B}{\hbar} \right| = \frac{g|e|\hbar}{2m}$$

$$\frac{d\vec{M}}{dt} = -\gamma \mu_o \vec{M} \times \vec{H}_{\text{eff}}$$



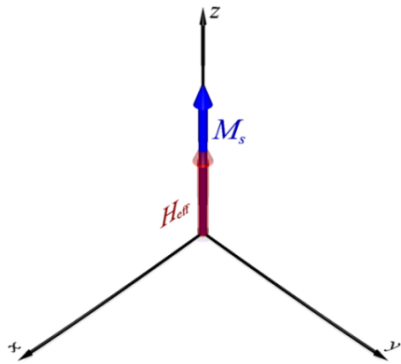
Magnetization Dynamics

Landau Lifshitz Gilbert Equation

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mu_o \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}$$

Gyroscopic term

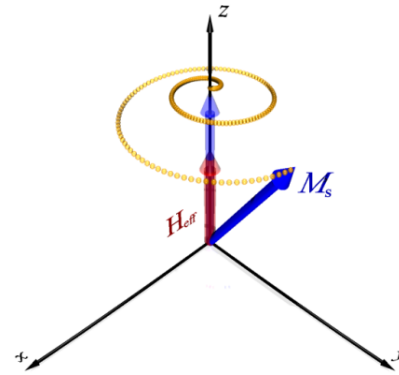
AT EQUILIBRIUM



$\mathbf{M}$ is aligned with the effective field.

Damping term

DISTURBED FROM EQUILIBRIUM



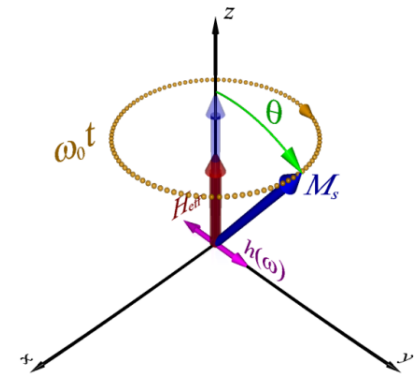
- $\mathbf{M}$ precesses about H_{eff} at the Larmor frequency:

$$\omega_0 = \gamma H_{\text{eff}}$$

- Decrease of the angle of precession due to damping.

Resonance

DRIVEN PRECESSIONAL MOTION



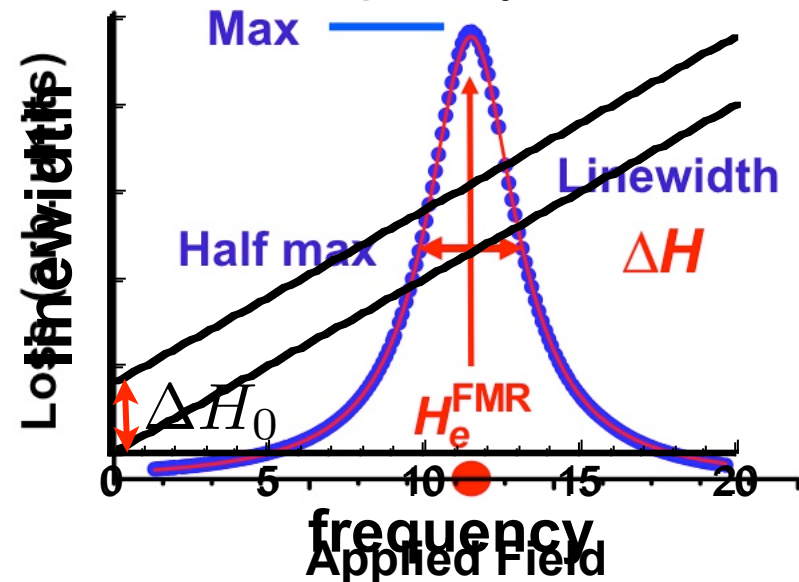
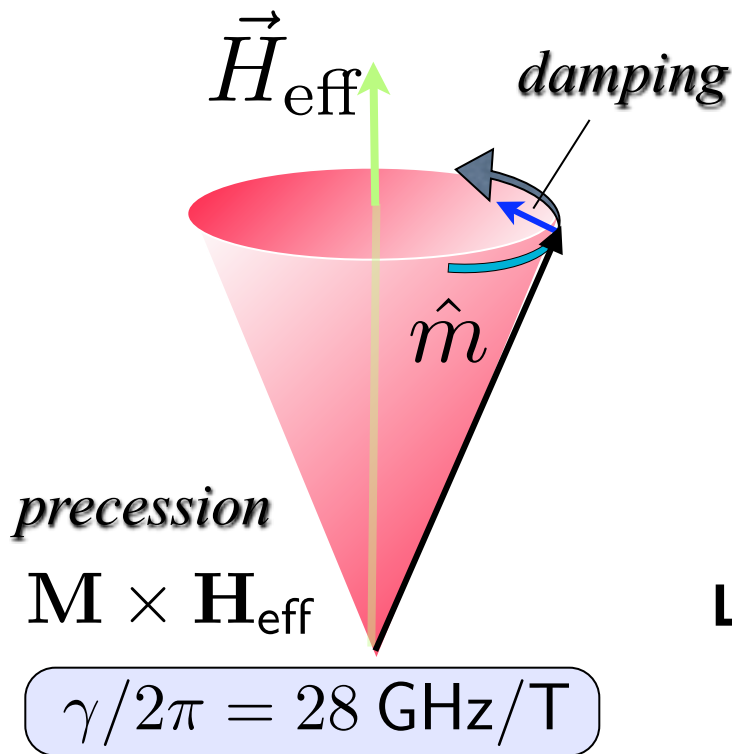
- Maintain the precessional motion with rf field (ω):
- When $\omega = \omega_0 \rightarrow \theta$ is max.

FERROMAGNETIC RESONANCE

FMR: Landau-Lifshitz-Gilbert Dynamics

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mu_o \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}$$

FMR: fixed frequency rf field $\perp \mathbf{H}$

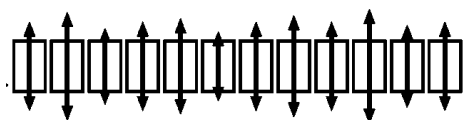


Linewidth: Homogeneous or intrinsic linewidth

$$\Delta H = 4\pi\alpha f / (\mu_o\gamma)$$

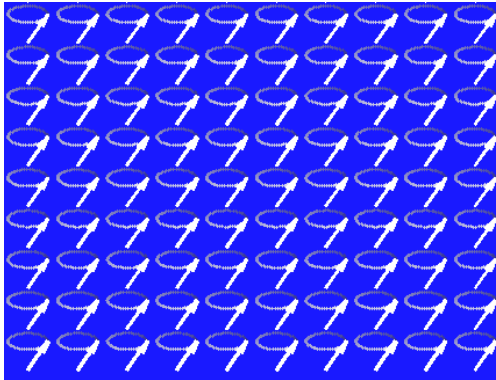
with inhomogeneous broadening

$$\Delta H = \Delta H_0 + \frac{4\pi\alpha}{\mu_o\gamma} f$$



$$\Delta H_{inh.}^{\Delta K_1} = \left| \frac{dH_{res}}{dK_1} \right| \Delta K_1$$

Spin Waves

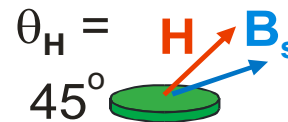


Uniform Precession



Finite k spin waves

Example: $\theta_H = 45^\circ$ situation



Spin wave band
(at FMR)

Spin wave wave
frequency (GHz)

FMR

16
14
12
10
8
6

$\theta_k = 90^\circ$

Band of modes
in between

$\theta_k = 0^\circ$

Spin wave wave
number k (rad/cm)

0 1 2 3 4 5 6 7 8 $\times 10^5$

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References

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