Aging phenomena in magnetic systems

Michel Pleimling

Department of Physics
Virginia Tech

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Content

Defining characteristics and symmetry properties of aging:

- slow dynamics (i.e. non-exponential relaxation)
- breaking of time-translation invariance
- dynamical scaling

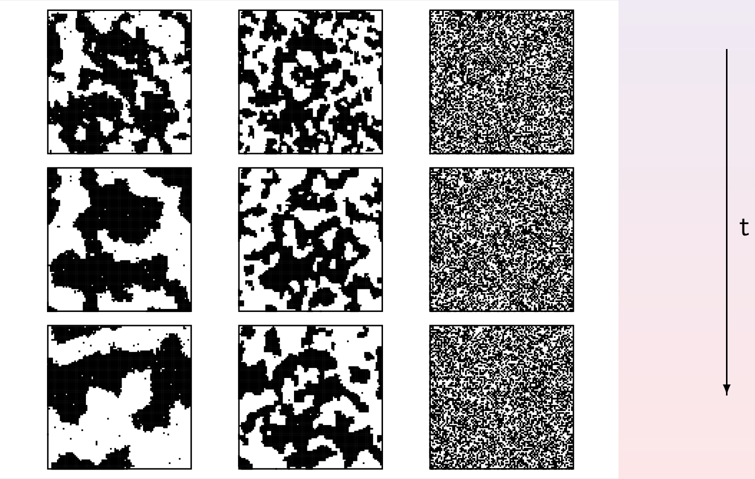
General discussion of scaling functions

- autocorrelation
- integrated responses
  $\longrightarrow$ presence of correction terms!

Today:

study of specific systems:

perfect magnets, disordered magnets, critical spin glasses perfect magnets disordered magnets spin glasses

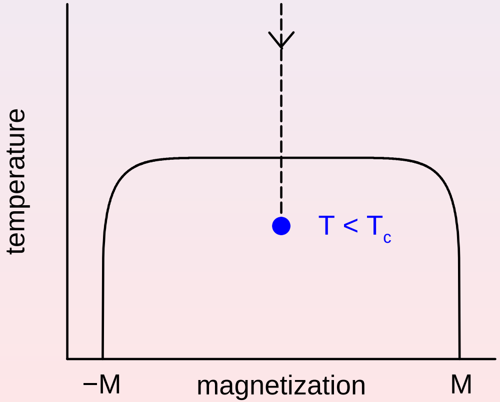


quench from high to low temperatures

Perfect magnets

Systems with competing equilibrium states

Example: uniaxial ferromagnet quenched below its critical point



Systems with competing equilibrium states

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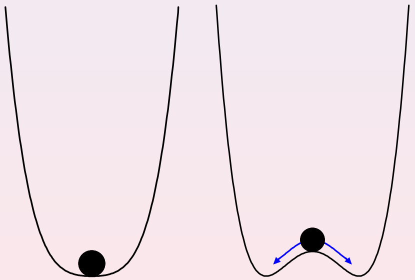
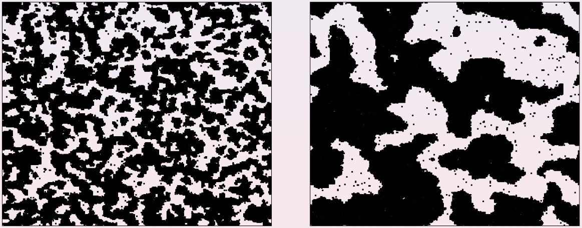


Systems with competing equilibrium states

Example: uniaxial ferromagnet quenched below its critical point

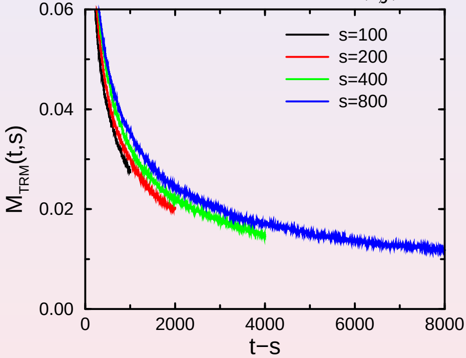


Equilibrium is never reached in the infinite system

$$L(t) \sim t^{1/z}$$

dynamical exponent for pure systems with nonconserved dynamics:
$z = 2$

Simplest model: Ising model with $\mathcal{H} = -J \sum_{<i,j>} S_i S_j$ and $S_i = \pm 1$



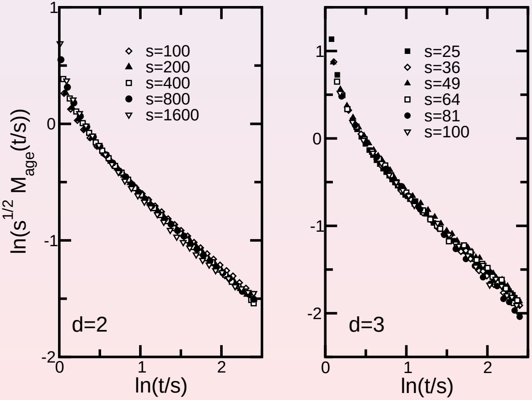
Integrated answers are easily accessible

thermoremanent magnetization

$$M_{TRM}(t,s)/h = \int_0^s du\, R(t,u) = r_0\, s^{-a}\, f_M(t/s)$$

nonequilibrium exponents:

$$a = 1/z = 1/2\ , \quad \lambda_R = \begin{cases} 1.26 & d = 2 \\ 1.60 & d = 3 \end{cases}$$

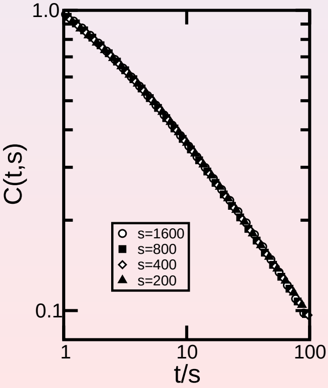


Phys. Rev. E **68**, 065101(R) (2003)

autocorrelation function (Europhys. Lett. **68**, 191 (2004))

nonequilibrium exponents:

$$b = 0 \ , \quad \lambda_C = \begin{cases} 1.26 & d = 2 \\ 1.60 & d = 3 \end{cases}$$



A successful theoretial approach: local scale invariance

Theoretical description after coarse graining: Langevin equation for non-conserved order parameter $\phi(t,\mathbf{r})$ (model A dynamics)

$$2\mathcal{M}\frac{\partial\phi}{\partial t} = \Delta\phi - \frac{\delta\mathcal{V}\left[\phi\right]}{\delta\phi} + \eta$$

$\mathcal{V}$: Ginzburg-Landau potential

η: Gaussian white noise, with
$\langle\eta(t,\mathbf{r})\eta(t',\mathbf{r}')\rangle = 2T\delta(t-t')\delta(\mathbf{r}-\mathbf{r}')$

first:
free-field case at $T=0$

$\longrightarrow$ free diffusion equation (Schrödinger equation)

$$\frac{\partial \phi(t,\mathbf{r})}{\partial t} = D\,\Delta\phi(t,\mathbf{r})$$

Global scale transformation $\mathbf{r} \longrightarrow b\mathbf{r}$:

$$\phi(b^2 t, b\mathbf{r}) = b^{-x}\phi(t,\mathbf{r})$$

Niederer '72, Hagen '72:
generalization of dynamical scaling with $b =$ const towards time and space dependent scaling factors $b = b(t,\mathbf{r})$
$\longrightarrow$ Schrödinger invariance

Schrödinger group:
maximal kinematical invariance group which leaves the free Schrödinger equation invariant:

$$t \to t' = \frac{\alpha t + \beta}{\gamma t + \delta} \qquad \mathbf{r} \to \mathbf{r}' = \frac{\mathrm{R}\mathbf{r} + \mathbf{v}t + \mathbf{a}}{\gamma t + \delta}, \quad \alpha\delta - \beta\gamma = 1$$

- global scale transformation
- time translation
- spatial translation
- Galilei transformation
- special Schrödinger transformation

$$t' = t/(1 + \gamma\, t) \ \ , \ \ \mathbf{r}\,' = \mathbf{r}/(1 + \gamma\, t)$$

$\Longrightarrow$ Lie algebra

local scale transformations for anisotropic systems generated by an infinite set of generators: $X_{-1}, X_0, X_1, \cdots$ and $Y_{-1/2}, Y_{1/2}, Y_{3/2}, \cdots$

commutator relations:

$$
\begin{aligned}
[X_n, X_m] &= (n-m) X_{n+m} \\
[X_n, Y_m] &= \left(\frac{n}{2} - m\right) Y_{n+m}
\end{aligned}
$$

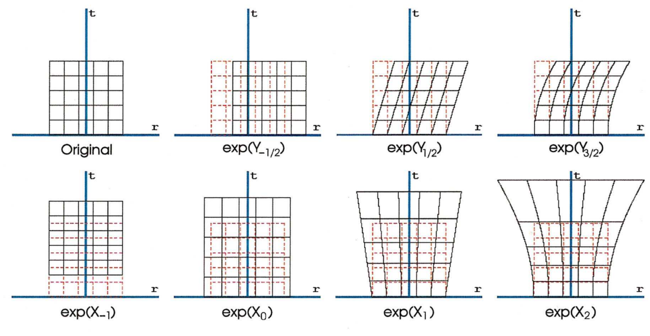
simplest representation of the generators (in one dimension):

$$
\begin{aligned}
X_n &= -t^{n+1}\partial_t - \frac{n+1}{2} t^n \, r \, \partial_r \\
Y_{k-1/2} &= -t^k \, \partial_r
\end{aligned}
$$

each of these generators corresponds to a geometrical transformation in space-time

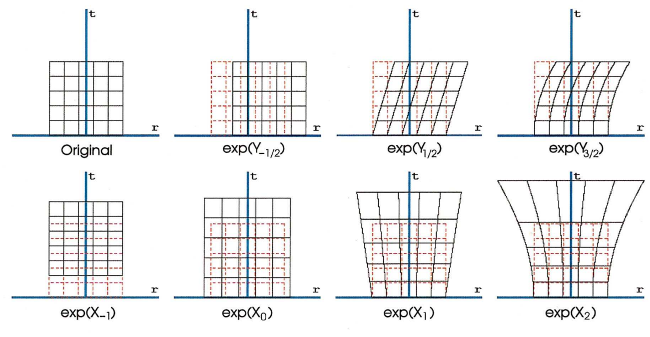
$X_{-1} = -\partial_t$ generates translations in time:

$$\exp(\tau X_{-1}) f(t, r) = f(t - \tau, r)$$



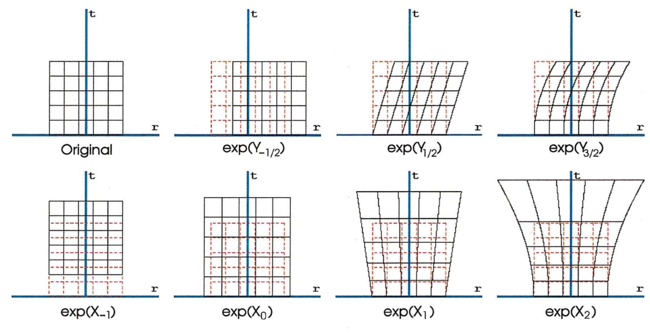
$Y_{-1/2} = -\partial_r$ generates translations in space:

$$\exp(\rho\, Y_{-1/2}) f(t, r) = f(t, r - \rho)$$



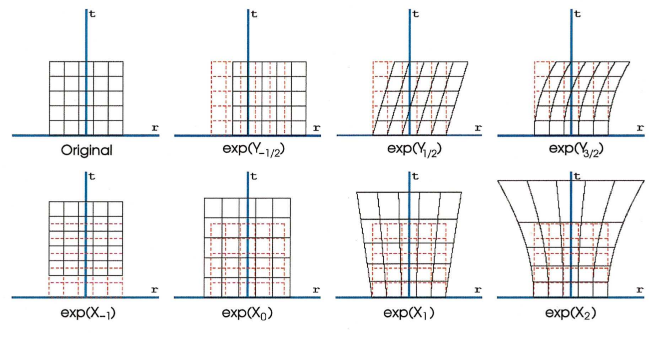
$X_0 = -t\,\partial_t - \frac{1}{2}\,r\,\partial_r$ generates anisotropic dilatations:

$$\exp(\lambda X_0) f(t,r) = f(t/b, r/b^{1/2}) \qquad \text{with} \quad b = e^{\lambda}$$



$Y_{1/2}$ generates a Galilei transformation:

$$\exp(c\, Y_{1/2}) f(t,r) = f(t, r - ct)$$



Langevin equations do not have non-trivial dynamic symmetries!
noise η does break Galilei-invariance!

stochastic field-theory:
partition sum of the stochastic process defined as functional integral over all realizations of the field $\phi(t,\mathbf{r})$ and the noise $\eta(t,\mathbf{r})$ which satisfy the Langevin equation

$$\Longrightarrow Z \sim \int \mathcal{D}\phi\,\mathcal{D}\tilde{\phi}\,\exp\left(\mathcal{J}\left[\phi,\tilde{\phi}\right]\right)$$

Janssen-De Dominicis action ($\tilde{\phi}$: response field)

$$\mathcal{J}\left[\phi,\tilde{\phi}\right] = \underbrace{\int \tilde{\phi}\,(2\mathcal{M}\partial_t - \Delta)\,\phi + \tilde{\phi}\mathcal{V}'[\phi]}_{\mathcal{J}_0[\phi,\tilde{\phi}]:\ \text{deterministic}} \underbrace{- T\int \tilde{\phi}^2 - \int \tilde{\phi}_{t=0}\,C_{init}\,\tilde{\phi}_{t=0}}_{\mathcal{J}_b[\tilde{\phi}]:\ \text{noise}}$$

$$C(t,s) = \langle \phi(t)\phi(s) \rangle \quad , \quad R(t,s) = \langle \phi(t)\tilde{\phi}(s) \rangle$$

expectation value: $\langle A \rangle := \int \mathcal{D}\phi\, \mathcal{D}\tilde{\phi}\, A\left[\phi, \tilde{\phi}\right] \exp\left(\mathcal{J}\left[\phi, \tilde{\phi}\right]\right)$

consider also the average taken with the deterministic action $\mathcal{J}_0$ alone: $\langle A \rangle_0 := \int \mathcal{D}\phi\, \mathcal{D}\tilde{\phi}\, A\left[\phi, \tilde{\phi}\right] \exp\left(\mathcal{J}_0\left[\phi, \tilde{\phi}\right]\right)$

Theorem:
if $\mathcal{J}_0$ is Galilei- and spatially translation-invariant, then Bargman superselection rules hold true:

$$\left\langle \phi_1 \cdots \phi_n \tilde{\phi}_1 \cdots \tilde{\phi}_m \right\rangle_0 \sim \delta_{n,m}$$

response function

$$\begin{aligned} R(t,s) &= \langle \phi(t)\tilde{\phi}(s)\rangle = \langle \phi(t)\tilde{\phi}(s)\exp(\mathcal{J}_b[\tilde{\phi}])\rangle_0 \\ &= \langle \phi(t)\tilde{\phi}(s)\rangle_0 = R_0(t,s) \end{aligned}$$

$\Longrightarrow$ response function does not depend on noise!
$\Longrightarrow$ response function can be computed by exploiting the local scale-symmetry of the deterministic equation!

application to aging

$$R(t,s,\vec{r}) = r_0 s^{-1-a} \left(\frac{t}{s}\right)^{1+a'-\lambda_R/z} \left(\frac{t}{s}-1\right)^{-1-a'} \exp\left(-\frac{\mathcal{M}}{2}\frac{r^2}{t-s}\right)$$

correlation function

$$C(t,s) = \langle \phi(t)\phi(s)\rangle = \langle \phi(t)\phi(s)\exp(\mathcal{J}_b[\tilde{\phi}])\rangle_0$$

contains four-point responses (Bargman rule!)

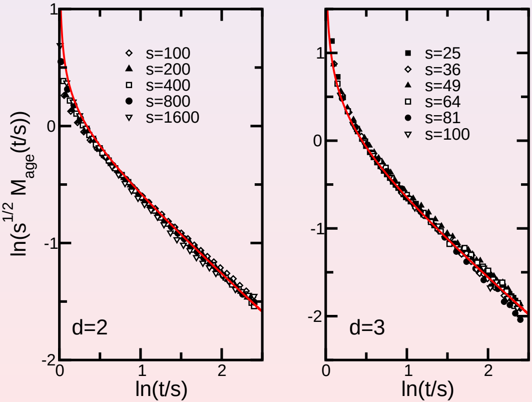
tests of linear Langevin equations

- spherical model with non-conserved ($z = 2$) and non-conserved ($z = 4$) dynamics (Henkel '02, Baumann/Henkel '06)
- Edwards-Wilkinson ($z = 2$) and Mullins-Herring models ($z = 4$) for surface growth (Phys. Rev. E **74**, 061604 (2006))
- spherical model with long-range interactions ($0 < z < 2$) (Baumann/Dutta/Henkel '07)

How about tests for non-linear Langevin equations?

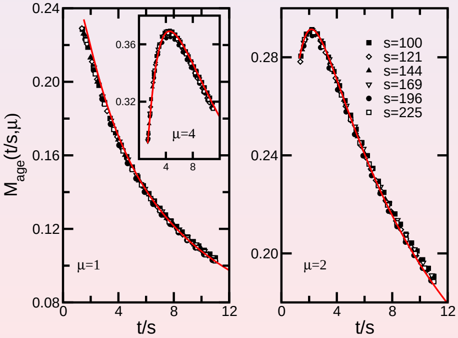
Ising model

thermoremanent magnetization (Phys. Rev. E **68**, 065101(R) (2003))



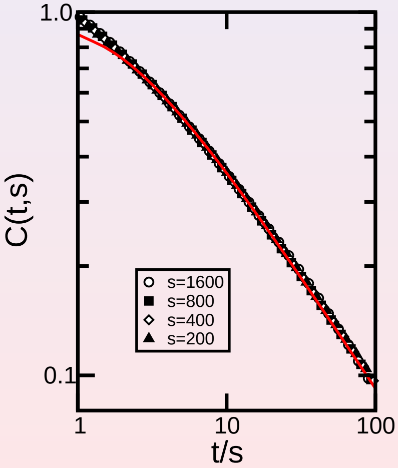
space-time response (Phys. Rev. E **68**, 065101(R) (2003))

$$R(t,s;\mathbf{r}) = R(t,s)\,\exp\left(-\frac{\mathcal{M}}{2}\frac{r^2}{t-s}\right)$$



μ: range of spatial integration

autocorrelation function (Europhys. Lett. **68**, 191 (2004))



Disordered magnets

Domain walls are trapped close to disorder-induced defects
$\Longrightarrow$ slowing down of the coarsening process

impurities act as energy barriers E_B to the domain growth

Huse/Henley '85: $E_B(L) \simeq E_0 \, L^\psi \Longrightarrow L(t) \sim (\ln(t/t_0))^{1/\psi}$

Example: two-dimensional ferromagnetic random bond Ising model

$$\mathcal{H} = -\sum_{(i,j)} J_{ij}\sigma_i\sigma_j \quad ; \quad \sigma_i = \pm 1$$

$J_{ij} > 0$ are random variables drawn from some distribution

$\Longrightarrow \psi = 1/4$

However: unclear experimental situation
some experiments in agreement with Huse/Henley, some find an algebraic growth law with $z = z(T)$

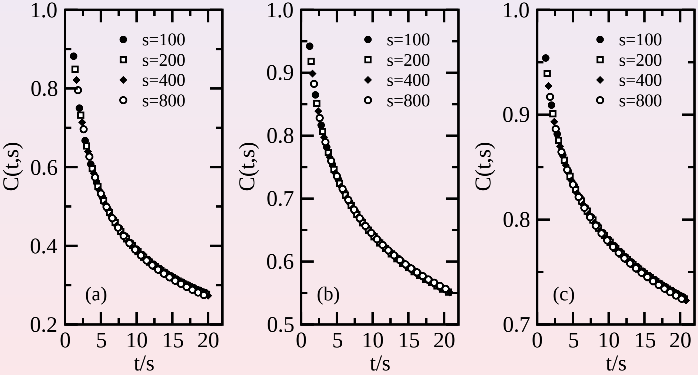
Paul/Ruri/Rieger '04:
study of random bond Ising model with J_{ij} equally distributed over the interval $[1 - \epsilon/2, 1 + \epsilon/2]$ and $0 < \epsilon \leq 2$

$\Longrightarrow$ growth law for quenches inside the ordered phase

$$L(t) \sim t^{1/z} \quad , \quad z = z(T, \epsilon) = 2 + \epsilon/T$$

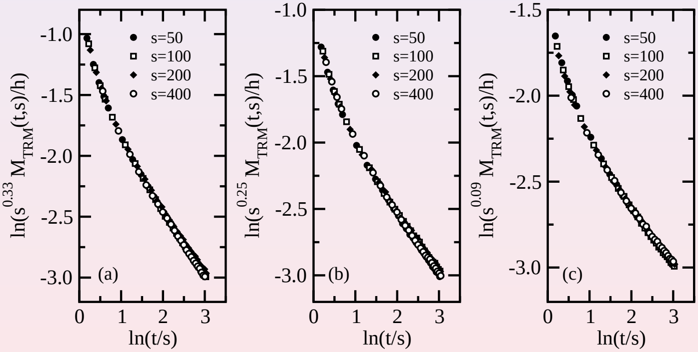
Explanation: energy barriers $E_B \simeq \epsilon \ln(1 + L)$

autocorrelation (Phys. Rev. B **79**, 224419 (2008))



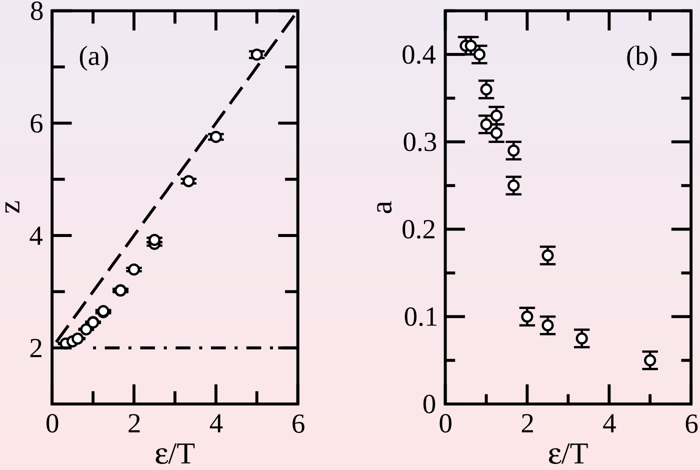
$\varepsilon = 0.5,\ T = 1$ $\varepsilon = 1,\ T = 0.4$ $\varepsilon = 2,\ T = 0.4$

thermoremanent magnetization (Phys. Rev. B **79**, 224419 (2008))

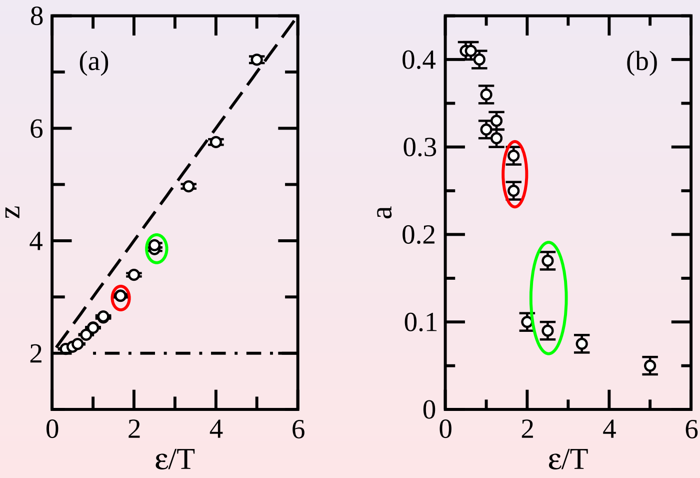


$\varepsilon = 0.5,\ T = 0.4$ $\varepsilon = 1,\ T = 0.6$ $\varepsilon = 2,\ T = 0.8$

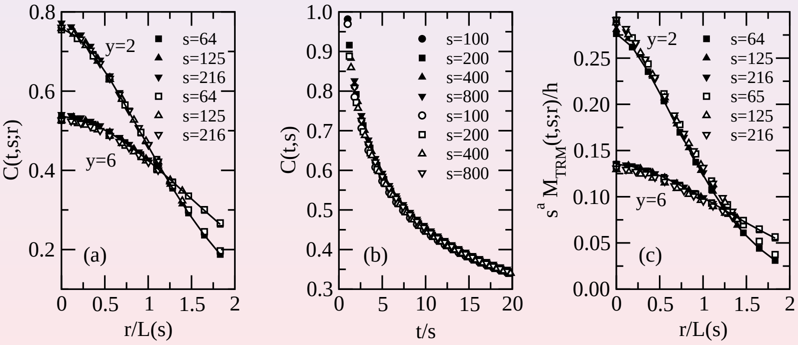
$$z \neq 2 + \epsilon / T$$



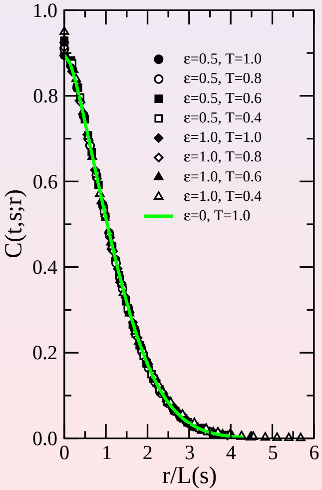
$z \neq 2 + \epsilon/T,$ $\qquad$ $a \neq 1/z$



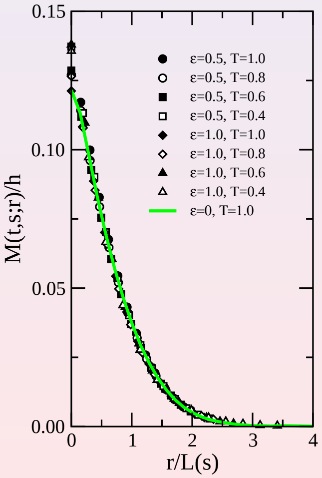
$\varepsilon = 1,\ T = 1$ (filled symbols); $\varepsilon = 0.5,\ T = 0.5$ (open symbols)



superuniversality: space-time correlation



superuniversality: space-time response



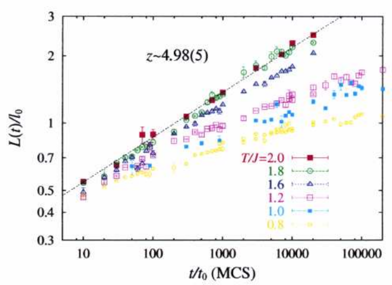
Critical spin glasses

Edwards-Anderson spin glass ($S_i = \pm 1$):

$$\mathcal{H} = - \sum_{<i,j>} J_{ij} \, S_i \, S_j$$

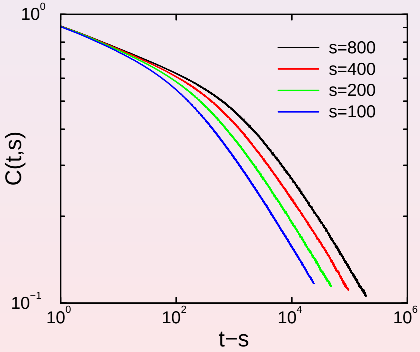
J_{ij}: random couplings with mean value zero
frustration effects due to competing interactions

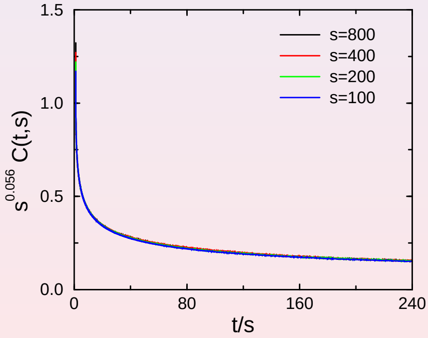
four-dimensional spin glass with a bimodal distribution of the couplings (Yoshino et al. '02)



Three-dimensional critical spin glass with a bimodal distribution of the couplings

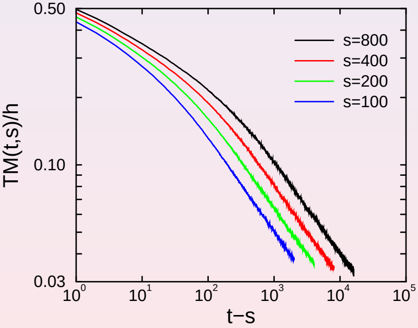
autocorrelation

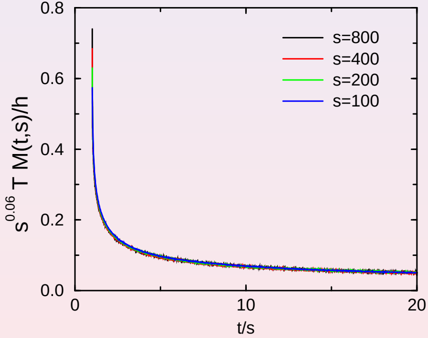




Europhys. Lett. **69**, 524 (2005)

thermoremanent magnetization





$\Longrightarrow$ Same phenomenology as for the critical ferromagnets

Europhys. Lett. **69**, 524 (2005)

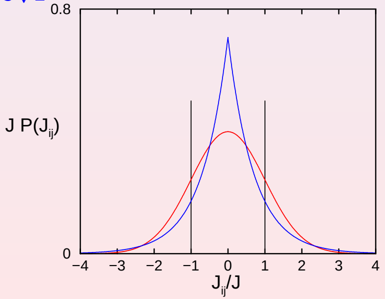
Universality?

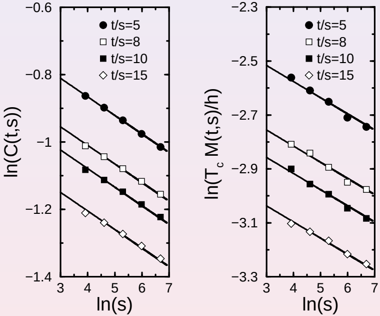
Consider different distributions of the couplings:

- bimodal distribution: $P_B(J_{ij}) = \frac{1}{2J}(\delta(J_{ij}/J-1)+\delta(J_{ij}/J+1))$
- gaussian distribution: $P_G(J_{ij}) = \frac{1}{J\sqrt{2\pi}}\exp(-(J_{ij}/J)^2/2)$
- laplacian distribution: $P_L(J_{ij}) = \frac{1}{J\sqrt{2}}\exp(-\sqrt{2}|J_{ij}/J|)$

$< J_{ij} >= 0$ and $< J_{ij}^2 > /J^2 = 1$ for all three distributions

different values of the kurtosis $< J_{ij}^4 > / < J_{ij}^2 >^2$

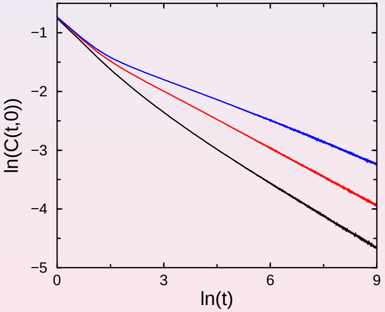




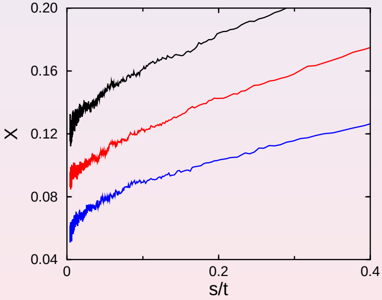
	bimodal	gaussian	laplacian
$a = b$	0.056(3)	0.043(1)	0.032(2)

$$C(t,s) \sim s^{-b} f_C(t/s)$$

Europhys. Lett. **69**, 524 (2005)
Phys. Rev. B **72**, 184429 (2005)



	bimodal	gaussian	laplacian
$a = b$	0.056(3)	0.043(1)	0.032(2)
λ_C/z	0.362(5)	0.320(5)	0.259(2)



fluctuation-dissipation ratio

$$X(t,s) = T\,R(t,s)\left(\frac{\partial C(t,s)}{\partial s}\right)^{-1}$$

	bimodal	gaussian	laplacian
$a=b$	0.056(3)	0.043(1)	0.032(2)
λ_C/z	0.362(5)	0.320(5)	0.259(2)
X_∞	0.12(1)	0.09(1)	0.055(2)

Conclusion

Aging phenomena and dynamical scaling behaviour are encountered in many out-of-equilibrium systems. These systems are often governed by a simple growth law for the dynamical correlation length

$$L(t) \sim t^{1/z}$$

Systems discussed in this talk:

- simple magnets: phase ordering and critical dynamics
- disordered magnets
- critical spin glasses