



Nonequilibrium dynamics in Coulomb glasses near the metal-insulator transition

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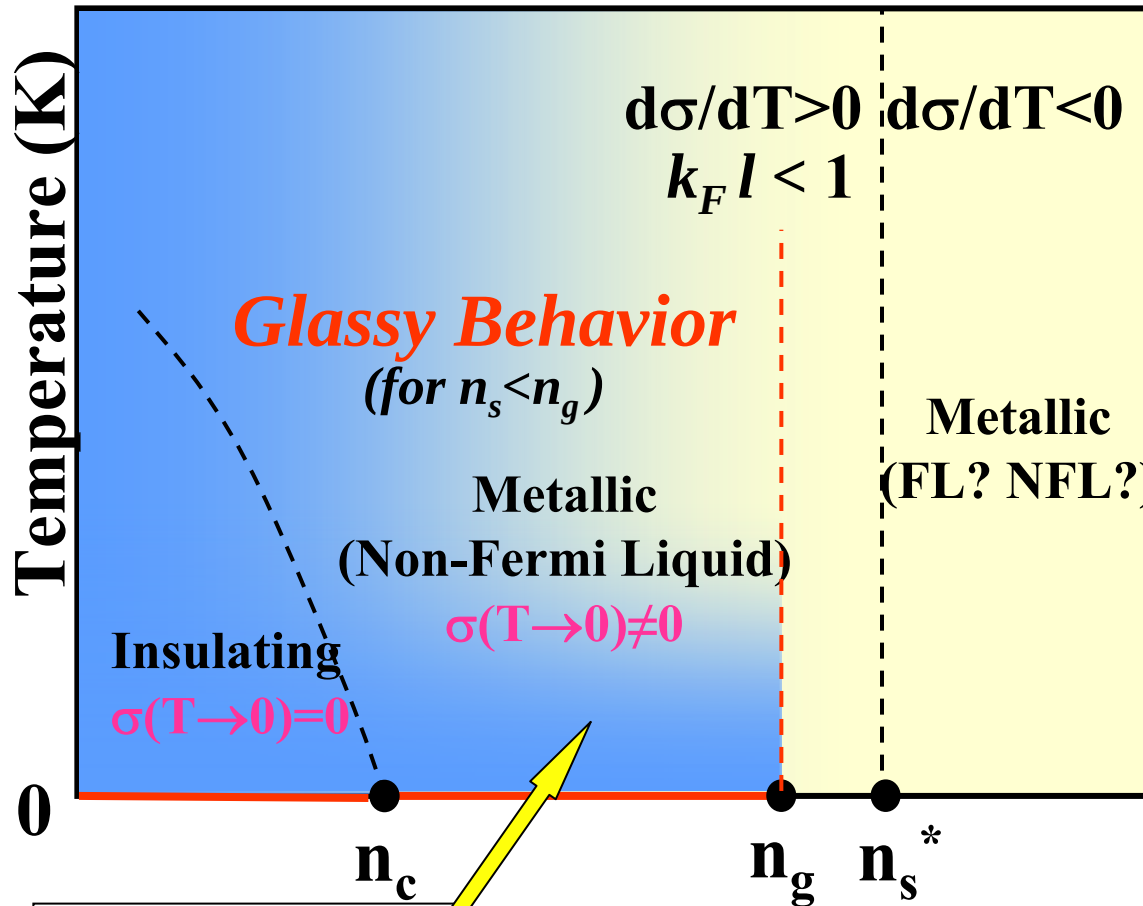
Lecture I: Metal-insulator transition and complexity in electronic systems

Lecture II: Studies of the electron dynamics near the 2D MIT: Relaxations of conductivity

Lecture III: Studies of the electron dynamics near the 2D MIT: Fluctuations of conductivity

- Phase diagram and aging in a 2DES in Si – brief **summary**
- **Noise measurements as a probe of glassiness** in low- and high-mobility samples:
 - **probability density functions**
 - **noise power spectrum**
 - **second spectrum**
- Noise measurements in a **magnetic field**
- Theory, simulations, experiments in other systems

Phase diagram of a 2DES in Si



intermediate
glassy phase

Glassy regime:

- slow, nonexponential relaxations
- diverging τ_{eq} ($T_g = 0$)
- **aging** (change at the MIT)

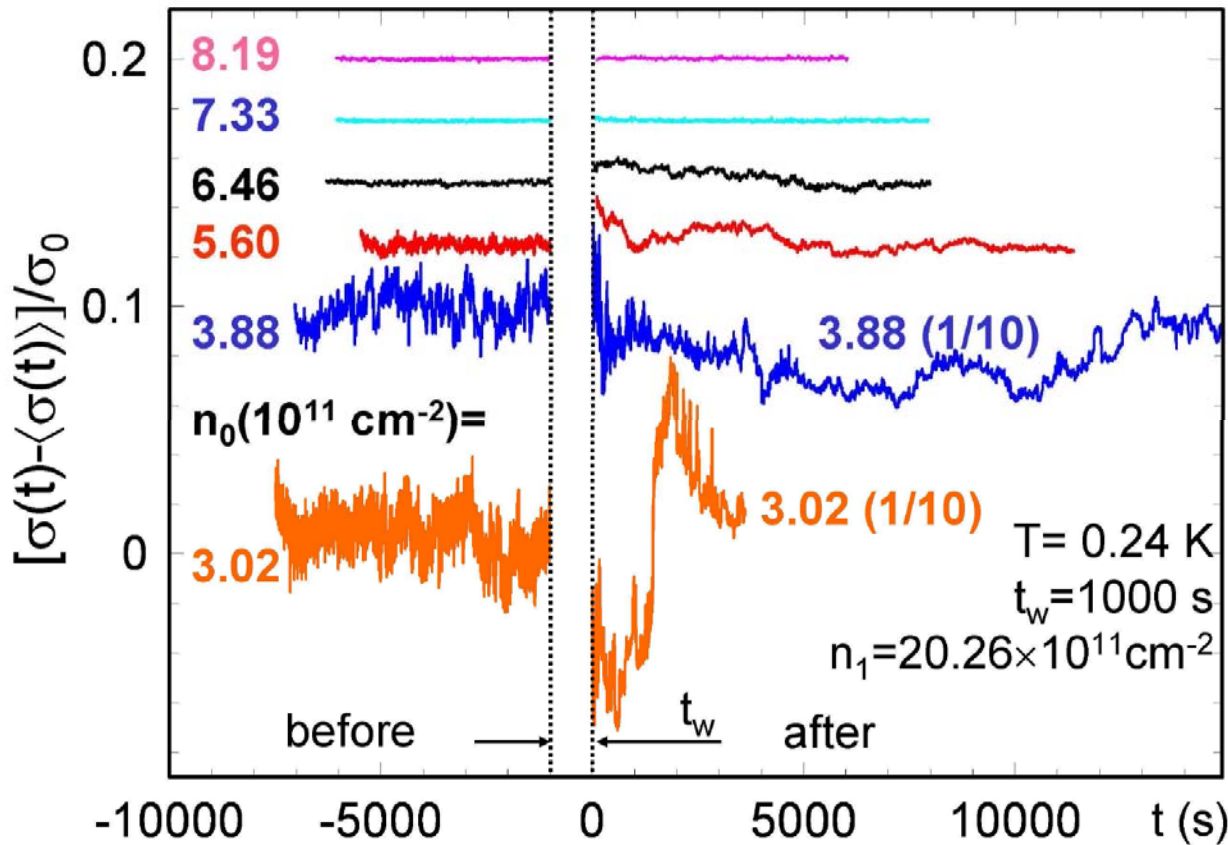
Waiting time protocol
for each density n_s :

- fix n_s at 10 K
- cool down to meas. T
- change n_s to a different value during t_w
- back to the initial density and measure $\sigma(t)$
- warm up to 10 K; repeat for different t_w

T=1 K in the waiting time protocol

What about aging at lower T?

Waiting time protocol at T=0.24 K



} slowly relaxing
< $\sigma(t)$ > subtracted
(only top 3 traces)

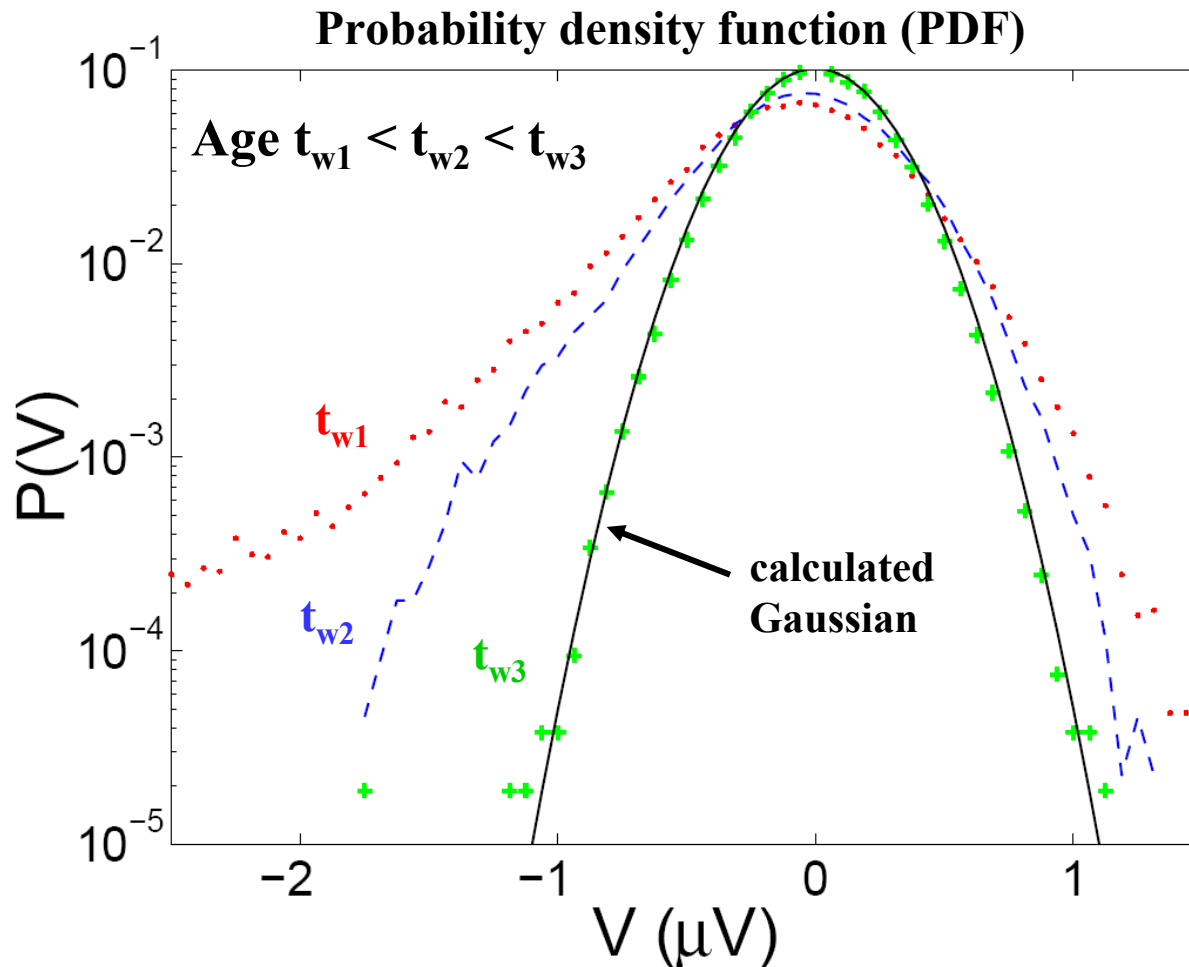
**Much more noise
after t_w (during
aging) than before**

Before: signal **noisier**
at low n_s

Study fluctuations (noise) at low T

provides **complementary information**;
so far, only **mean values of conductivity**

Noise in Laponite (colloidal suspension)



non-Gaussian



Gaussian

as it ages and
approaches
equilibrium

(Buisson *et al.*, Proc. SPIE 5469, 95 (2004);
edited by Popović, Weissman, Rácz)



Noise protocol:

- start from a fixed n_s at high T (e.g. 10 K)
- cool down to measurement T (< 1 K)
- measure $\sigma(t)$
- warm up to ~ 1 K;
change V_g (density n_s) by a small amount – no observable relaxations
- cool down to measurement T (< 1 K)
- measure $\sigma(t)$ for this new density
- repeat (warm up to ~ 1 K, change n_s in a small step, cool down, measure $\sigma(t)$)

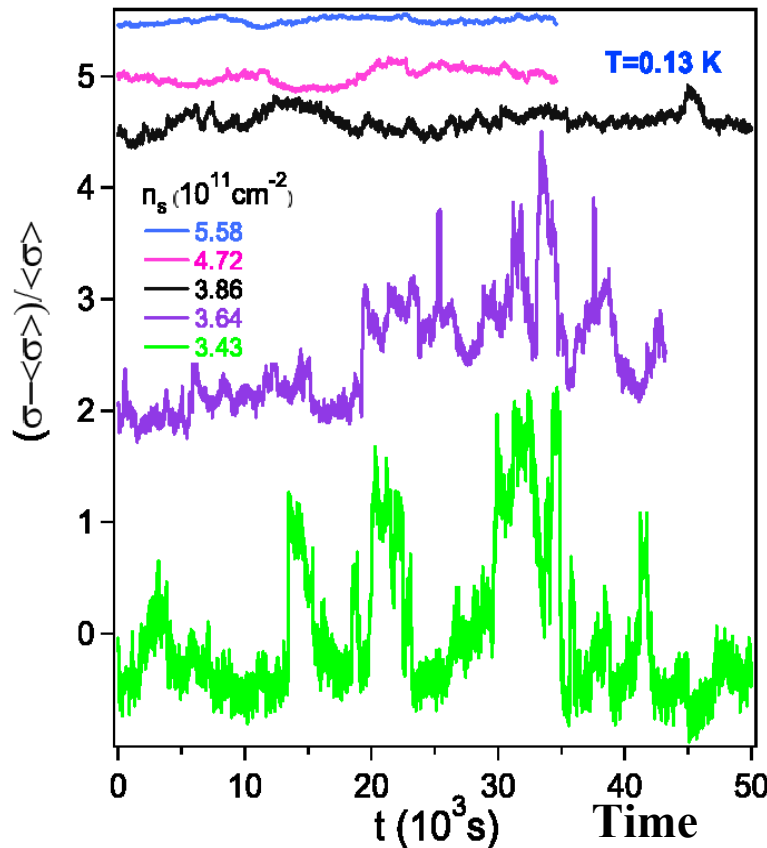
Noise protocol: density changed in tiny steps at ~ 1 K (not at 10 K)

Noise in low-mobility Si MOSFETs

[S. Bogdanovich and D. Popović, PRL 88, 236401 (2002)]



Conductivity



- relative fluctuations are of the order of **100%** at low n_s and T
- noise decreases with increasing n_s and T
- **non-Gaussian**
- the **character** of the noise **changes** dramatically as n_s is varied: at high n_s , the variance no longer varies with time

$\langle \sigma \rangle$ - time averaged conductivity
variance:

$$\delta\sigma^2 = \langle (\sigma - \langle \sigma \rangle)^2 \rangle$$

Large, slow, and non-Gaussian at low n_s and T !



$T=0.130$ K

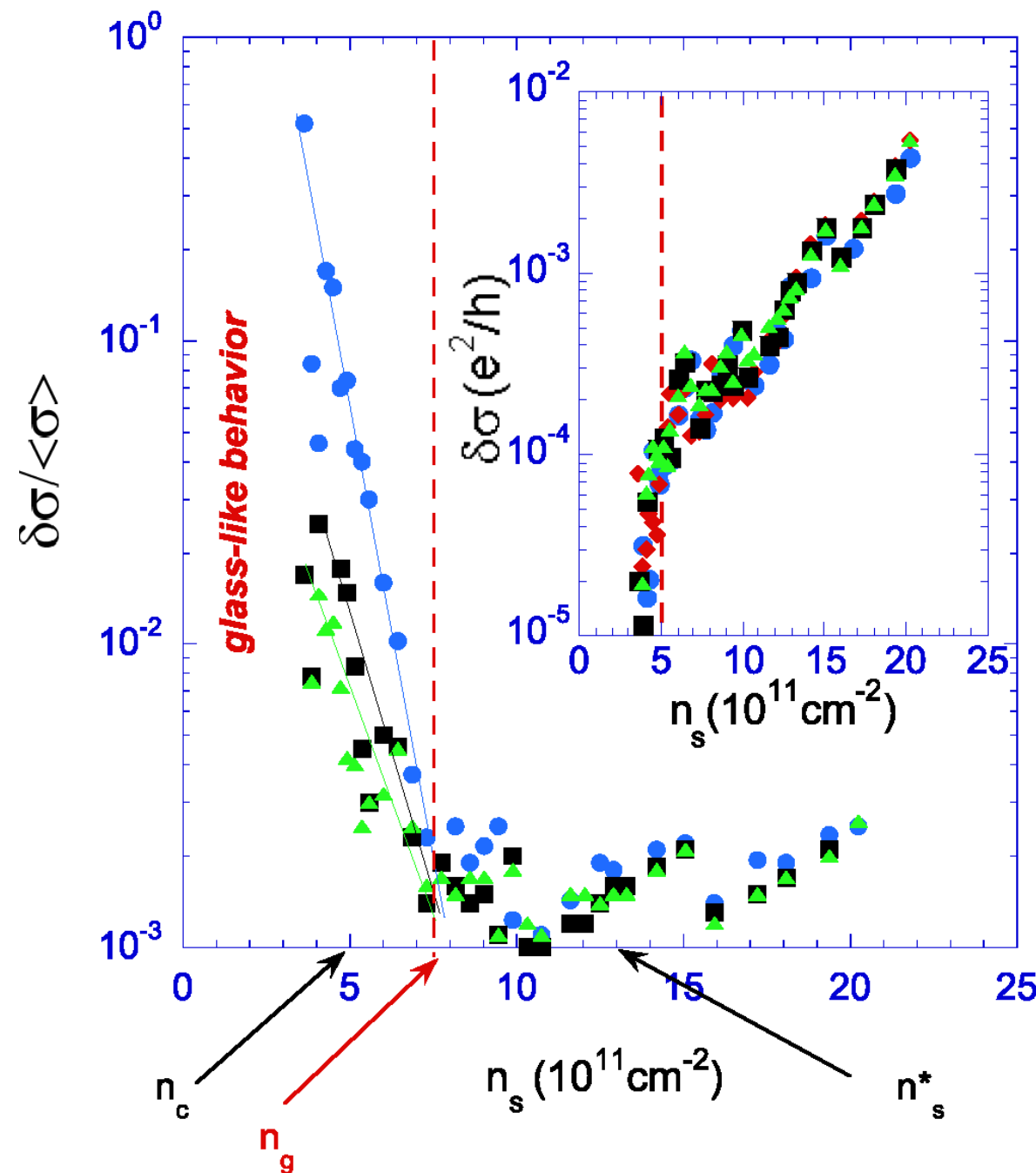
$T=0.196$ K

$T=0.455$ K

$T=0.805$ K

$$\delta\sigma = \langle (\sigma - \langle \sigma \rangle)^2 \rangle^{1/2}$$

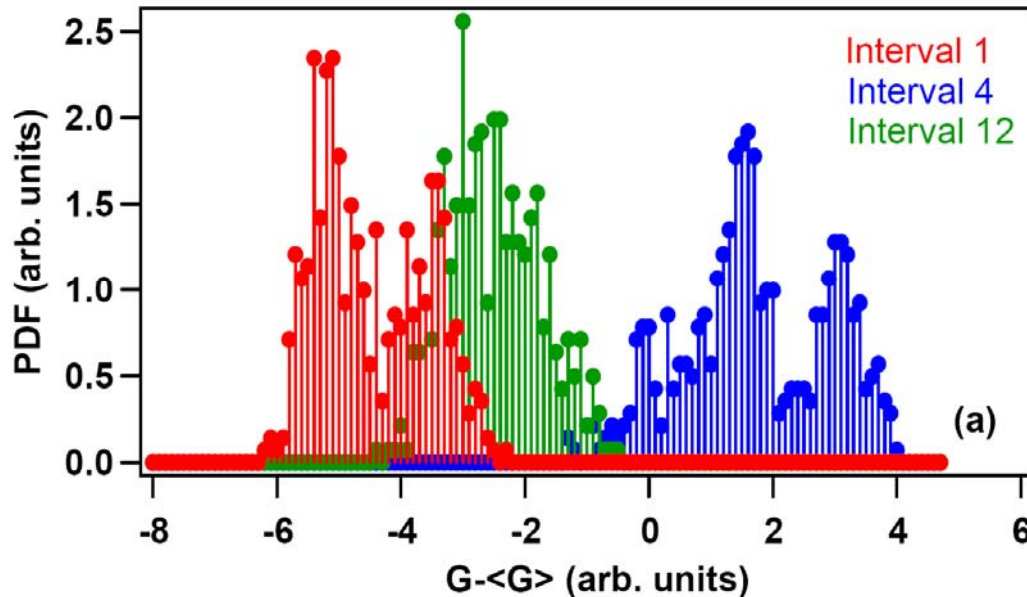
- a dramatic **change** of $d(\delta\sigma)/dn_s$ at n_c (**2D MIT**)
- a dramatic **increase** of noise ($\delta\sigma/\langle\sigma\rangle$) at n_g (**glass transition**)
- **glassy phenomena observed below n_g**



Probability density functions (PDF) of fluctuations



- PDF: - Gaussian for $n_s > n_g$
- non-Gaussian for $n_s < n_g$



Low-mobility sample

$T = 0.13$ K

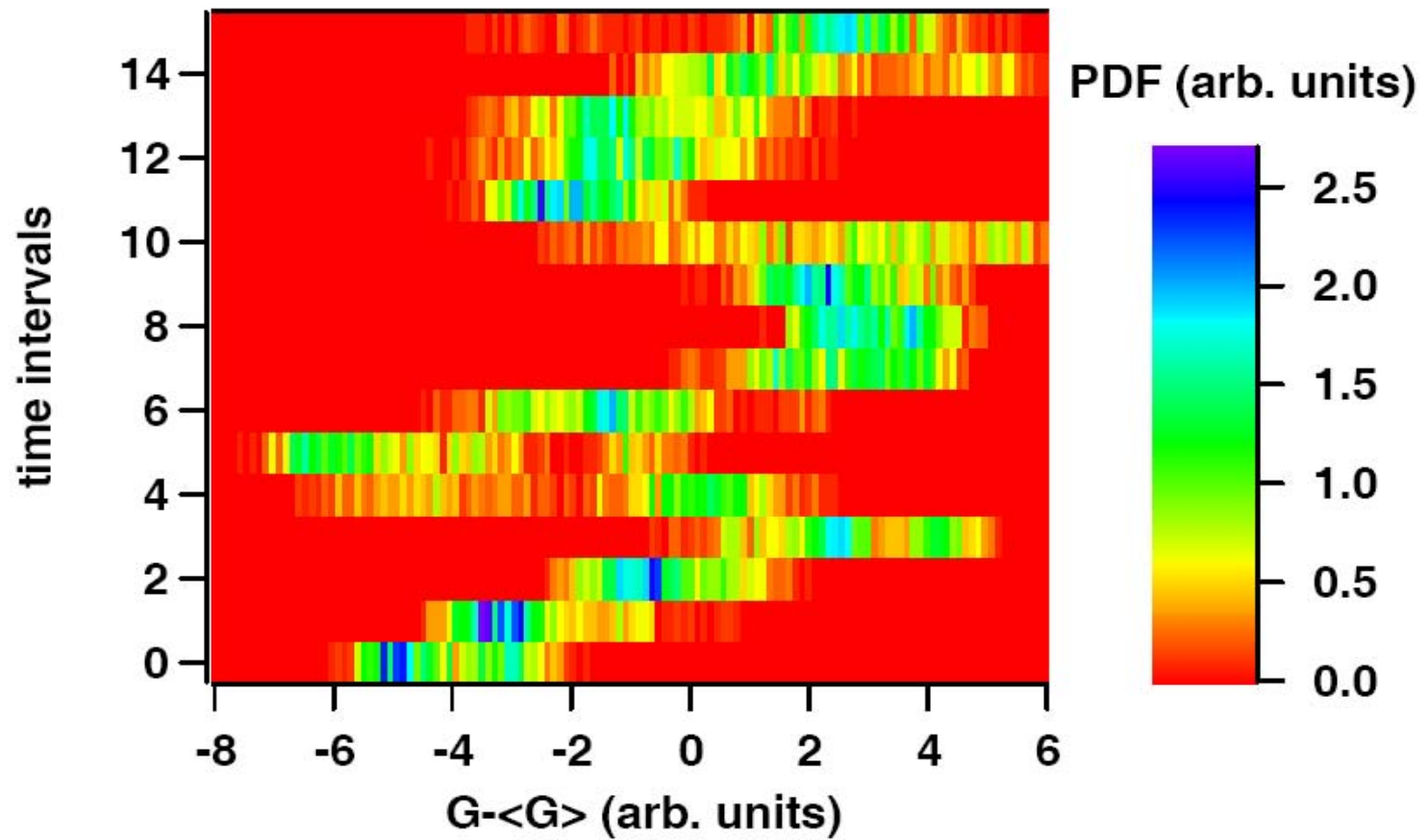
$n_c < n_s (10^{11} \text{ cm}^{-2}) = 5.58 < n_g$
(metallic glassy phase)

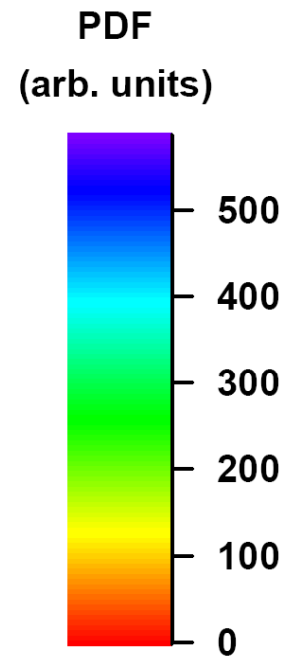
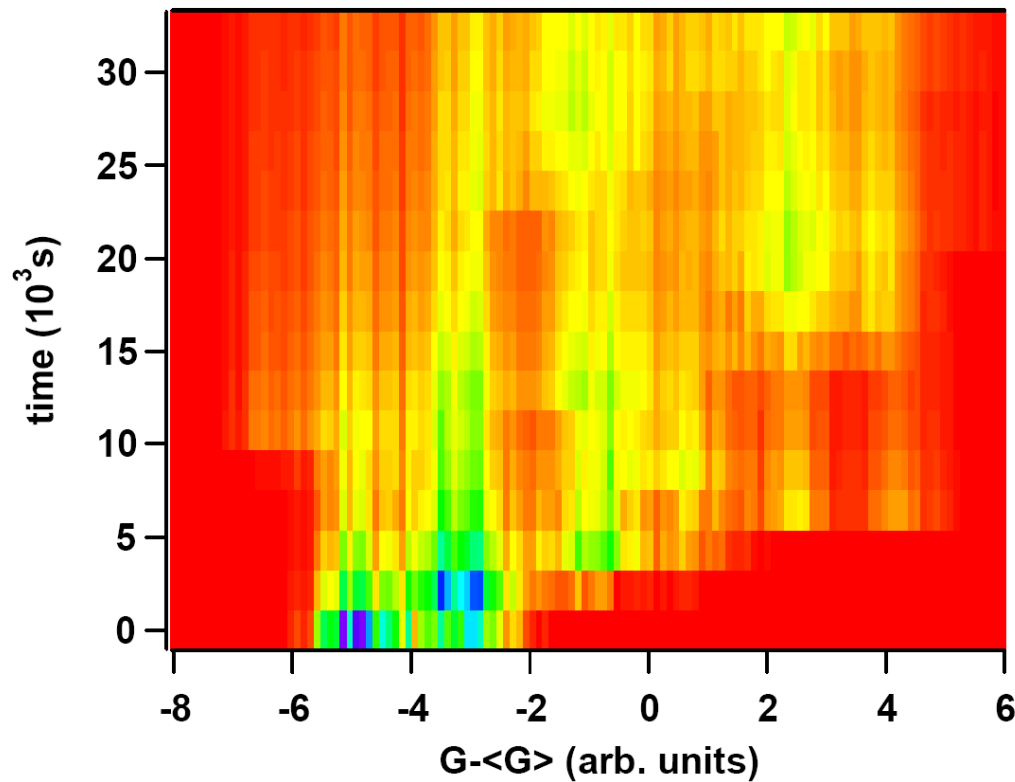
Each time interval ≈ 35 minutes

- structure changes with time



PDF for sixteen different 35 minute intervals:





Low-mobility sample

$T=0.13$ K

$n_c < n_s(10^{11} \text{ cm}^{-2})=5.58 < n_g$

Each time interval ≈ 35 min

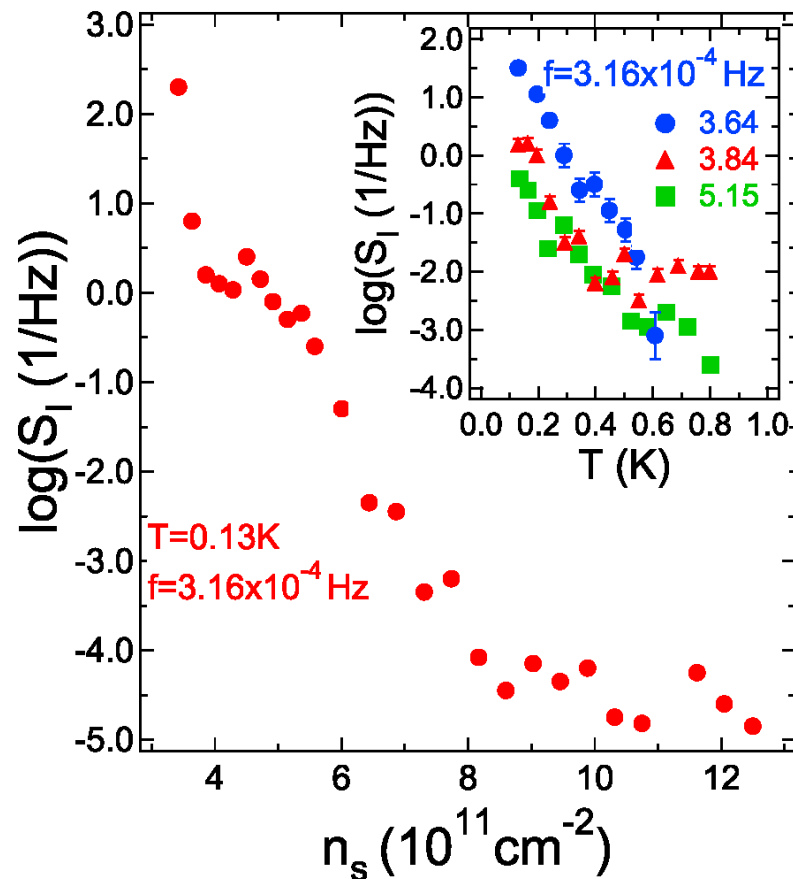
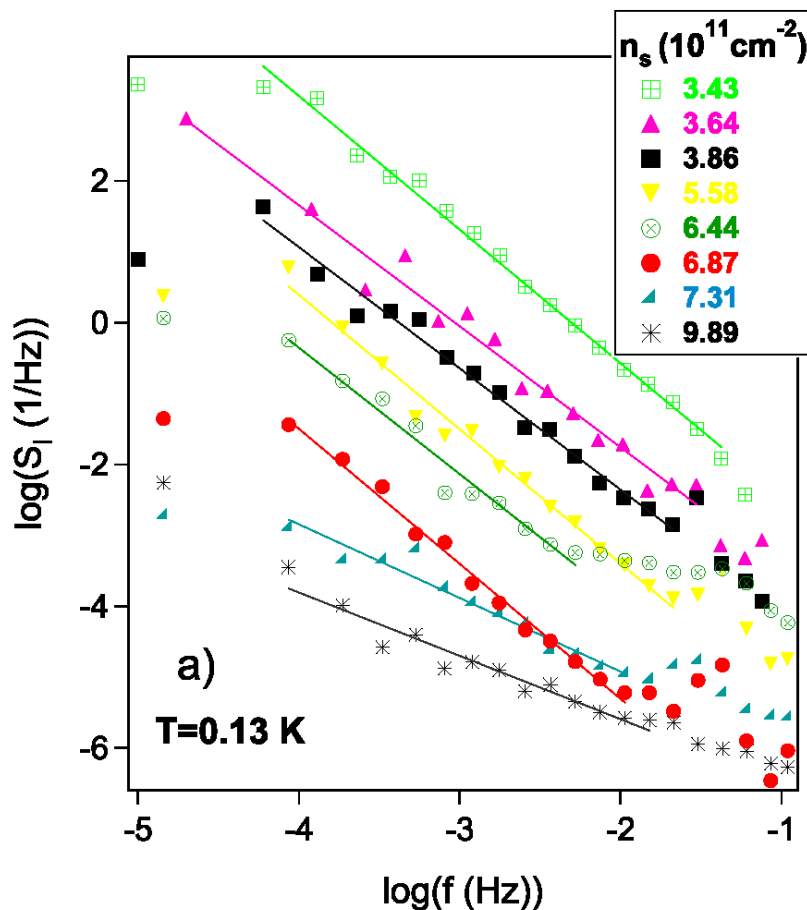
- broadens with the increasing sampling time but **never becomes Gaussian**
- **nonergodic** on experimental time scales



Noise power spectrum (low-mobility samples):

Power spectrum of $[\sigma(t) - \langle \sigma \rangle] / \langle \sigma \rangle$:

$$S_I \propto 1/f^\alpha$$

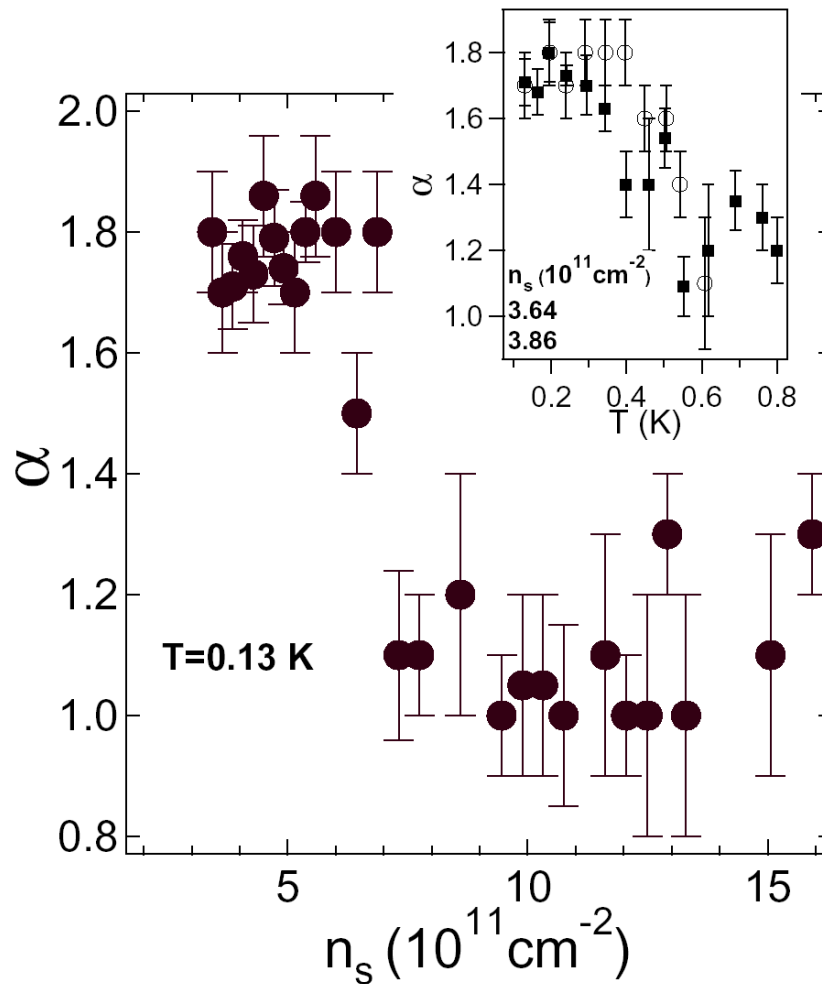


- a sudden and dramatic slowing down of the electron dynamics for $n_s < n_g \approx 7.5 \times 10^{11} \text{ cm}^{-2}$

Noise exponent α has a sudden jump at $n_s = n_g$!



$$S_I \propto 1/f^\alpha$$

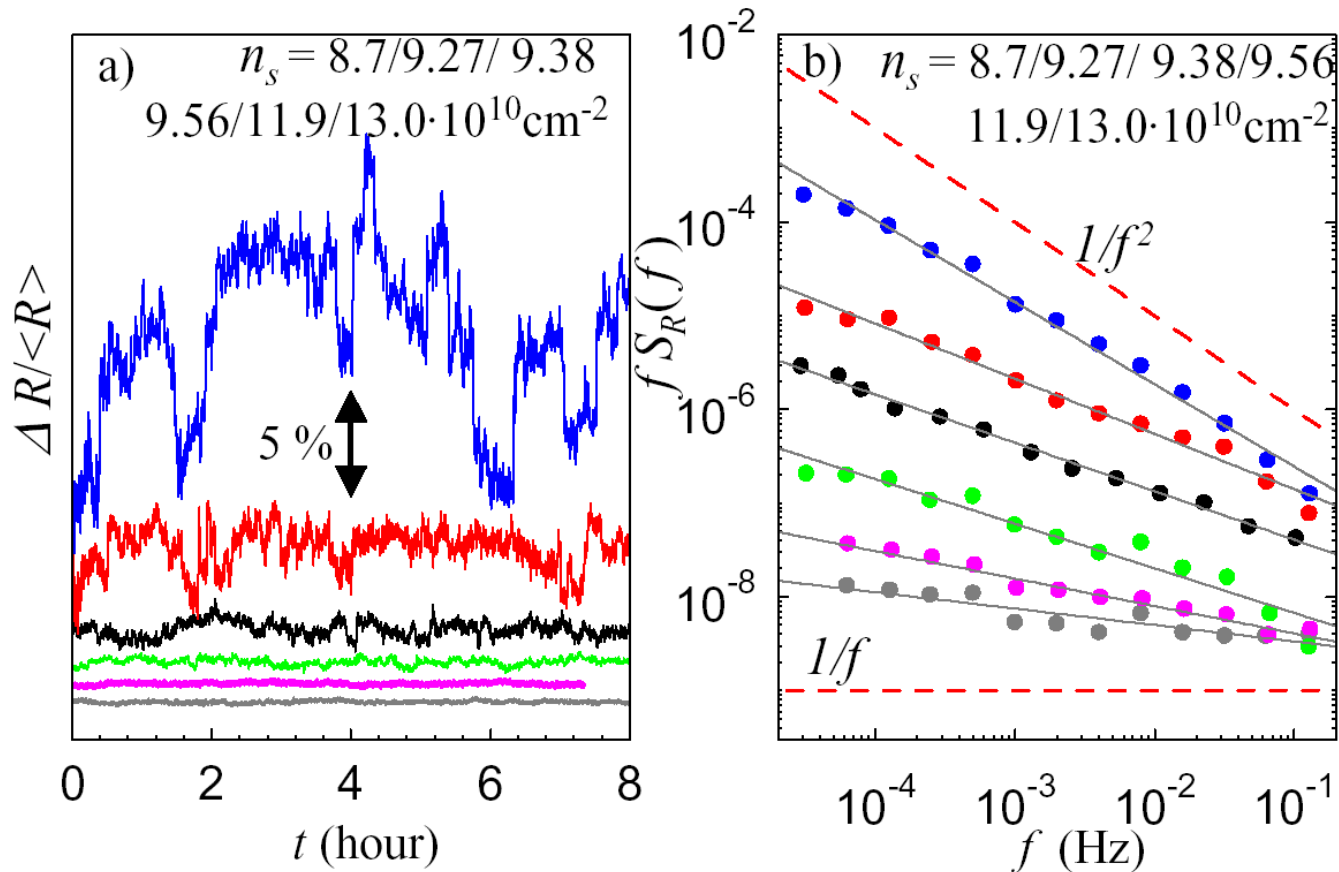


- a sudden shift of the spectral weight towards lower f indicates a sudden (sharp) and dramatic slowing down of the electron dynamics at n_g
- observable only at low enough temperatures

Use noise to identify the glass transition!

[Bogdanovich and Popović, PRL 88, 236401 (2002)]

Resistance noise and power spectra in high-mobility (low disorder) Si MOSFETs



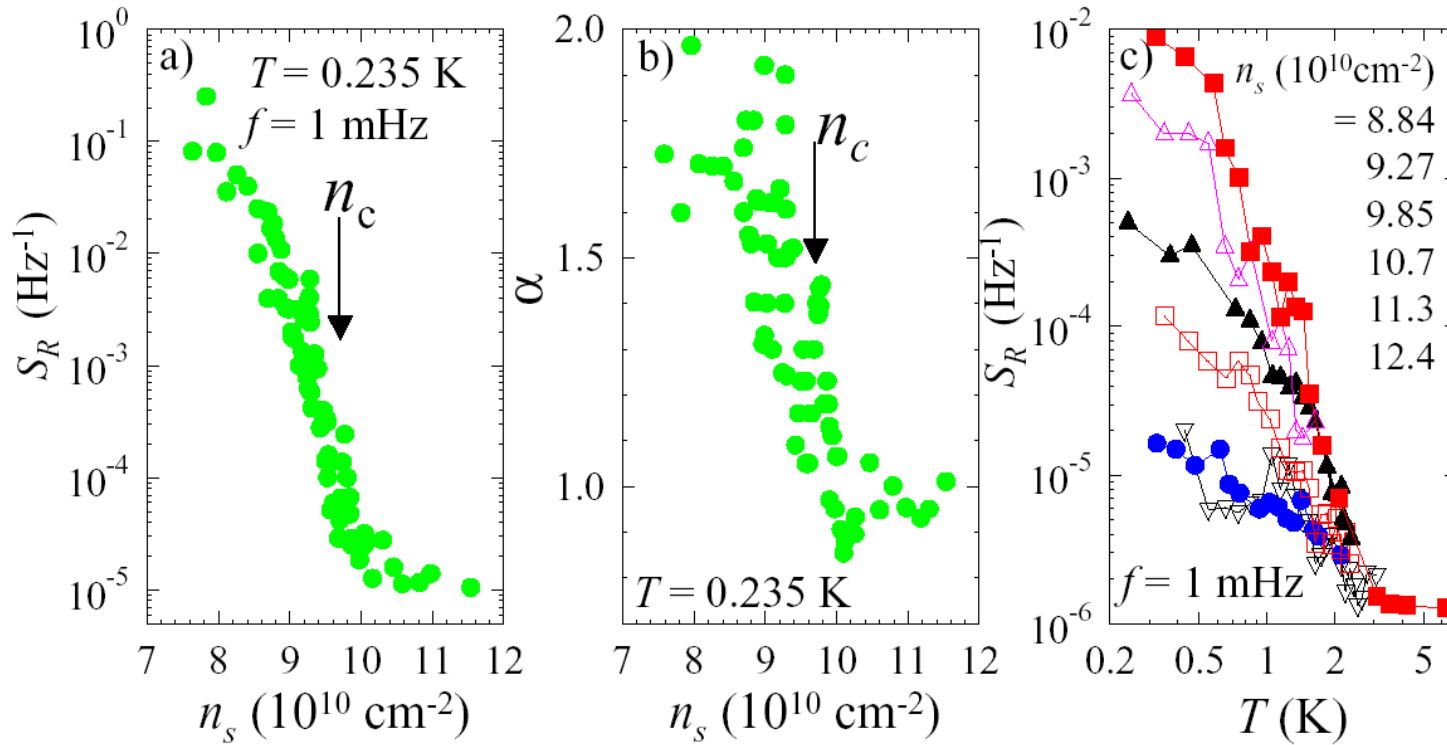
T=0.24 K

$S_R \propto 1/f^\alpha$,
 $1 \leq \alpha \leq 2$

[Jaroszyński, Popović, Klapwijk, PRL 89, 276401 (2002)]



Noise in samples with low disorder:

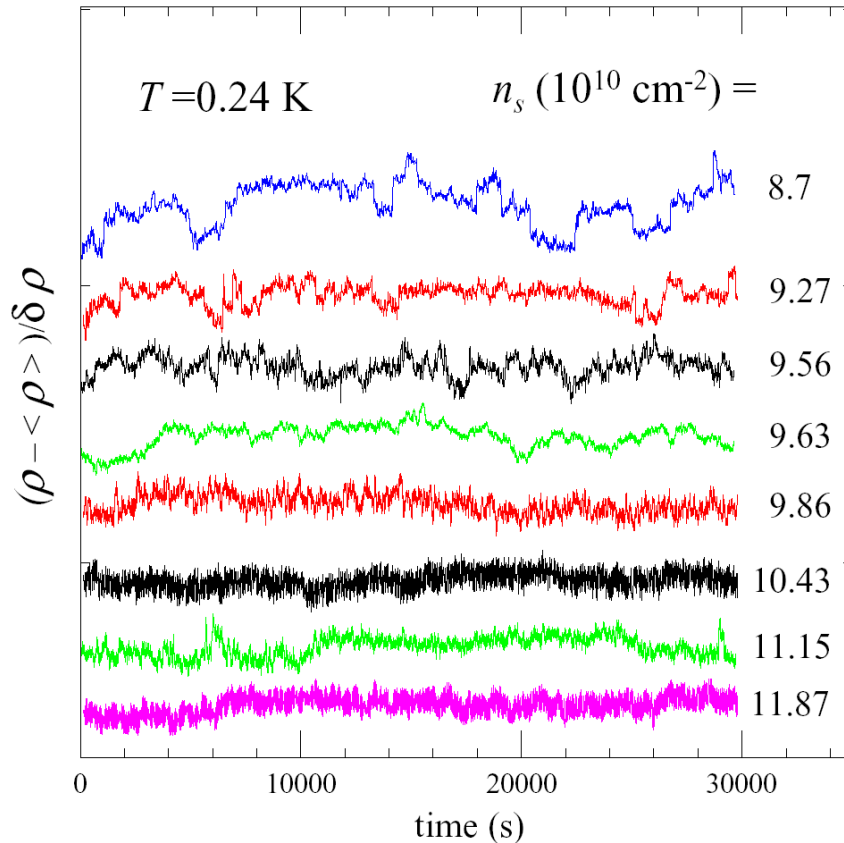


- a dramatic increase of the low-frequency noise at low n_s and T
- a rapid increase of the exponent α with decreasing n_s

$\Rightarrow$ Glass transition at $n_g \approx 10 \times 10^{10} \text{ cm}^{-2} \geq n_c$

Glass transition and the MIT almost coincide – no intermediate phase

Noise in high-mobility Si MOSFETs:



ρ - resistivity

$\langle \rho \rangle$ - time-averaged resistivity

$\delta \rho^2$ = variance

- non-Gaussian at low n_s and T

- variance changes with time



check higher order correlations

[Weissman:

Rev. Mod. Phys. 60, 537 (1988);

Rev. Mod. Phys. 65, 829 (1993)]

Second spectrum – “noise of the noise”

(Voss&Clarke, Restle&Weissman, Seidler&Solin)



- $S_2(f_2, f)$: power spectrum of the fluctuations of $S_R(f)$ with time

[i.e. Fourier transform of the autocorrelation function of the time series of $S_R(f)$; fourth-order noise statistic]

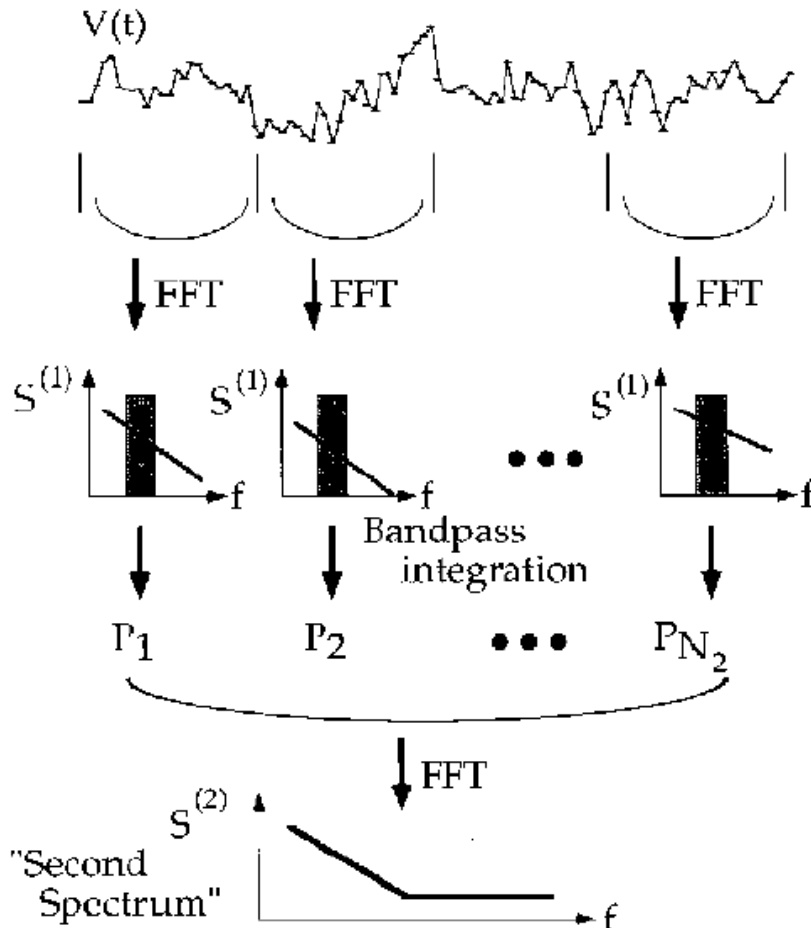
- probes correlations between fluctuators:

uncorrelated fluctuators

$\Rightarrow S_2(f_2, f)$ white;

correlated fluctuators

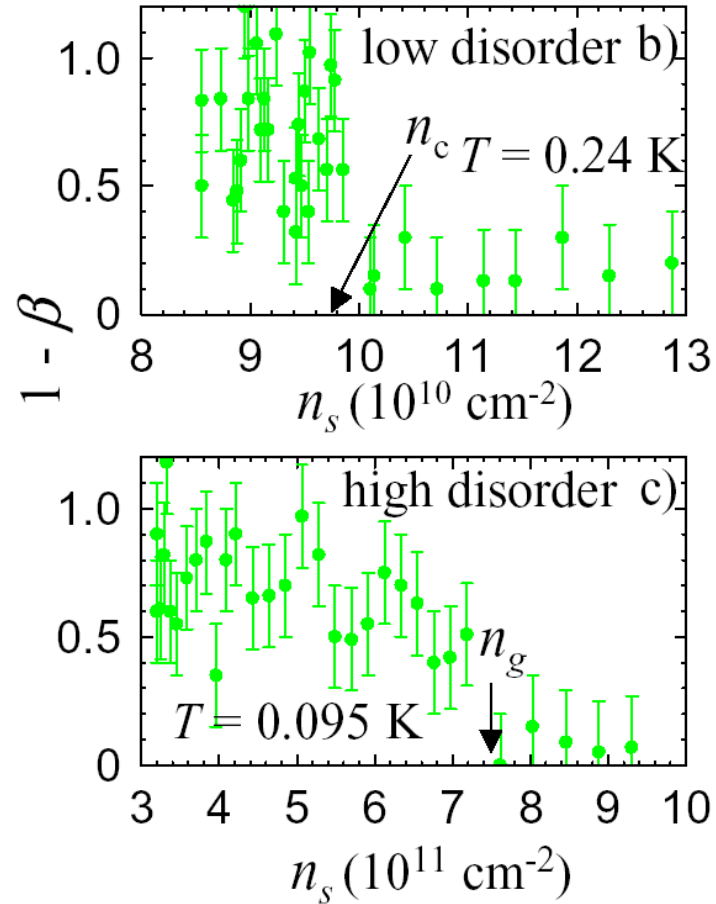
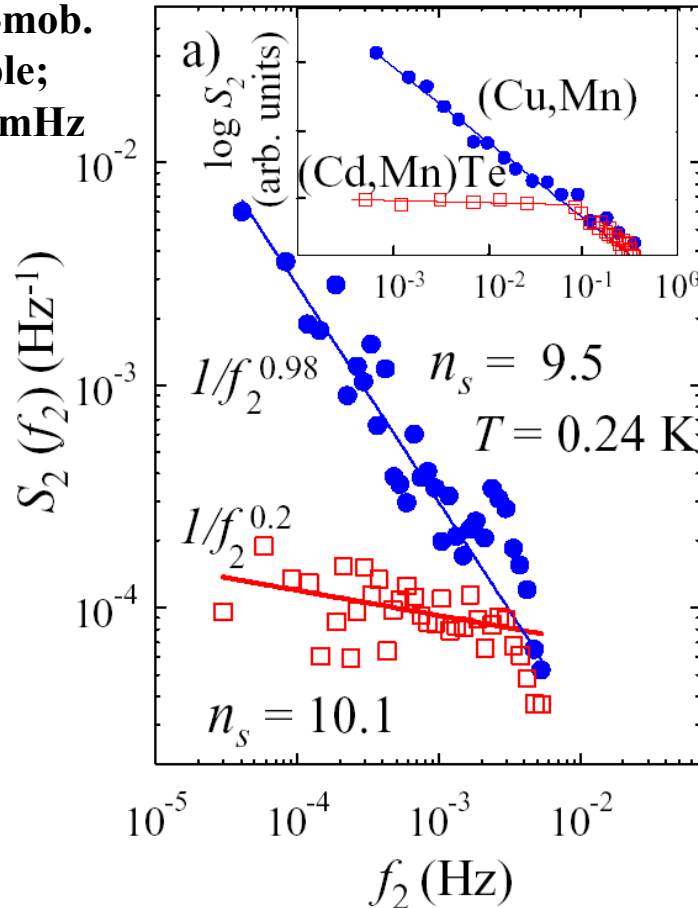
$\Rightarrow S_2 \propto 1/f_2^{1-\beta}$ (non-Gaussian)



Second spectrum in a 2DES in Si:



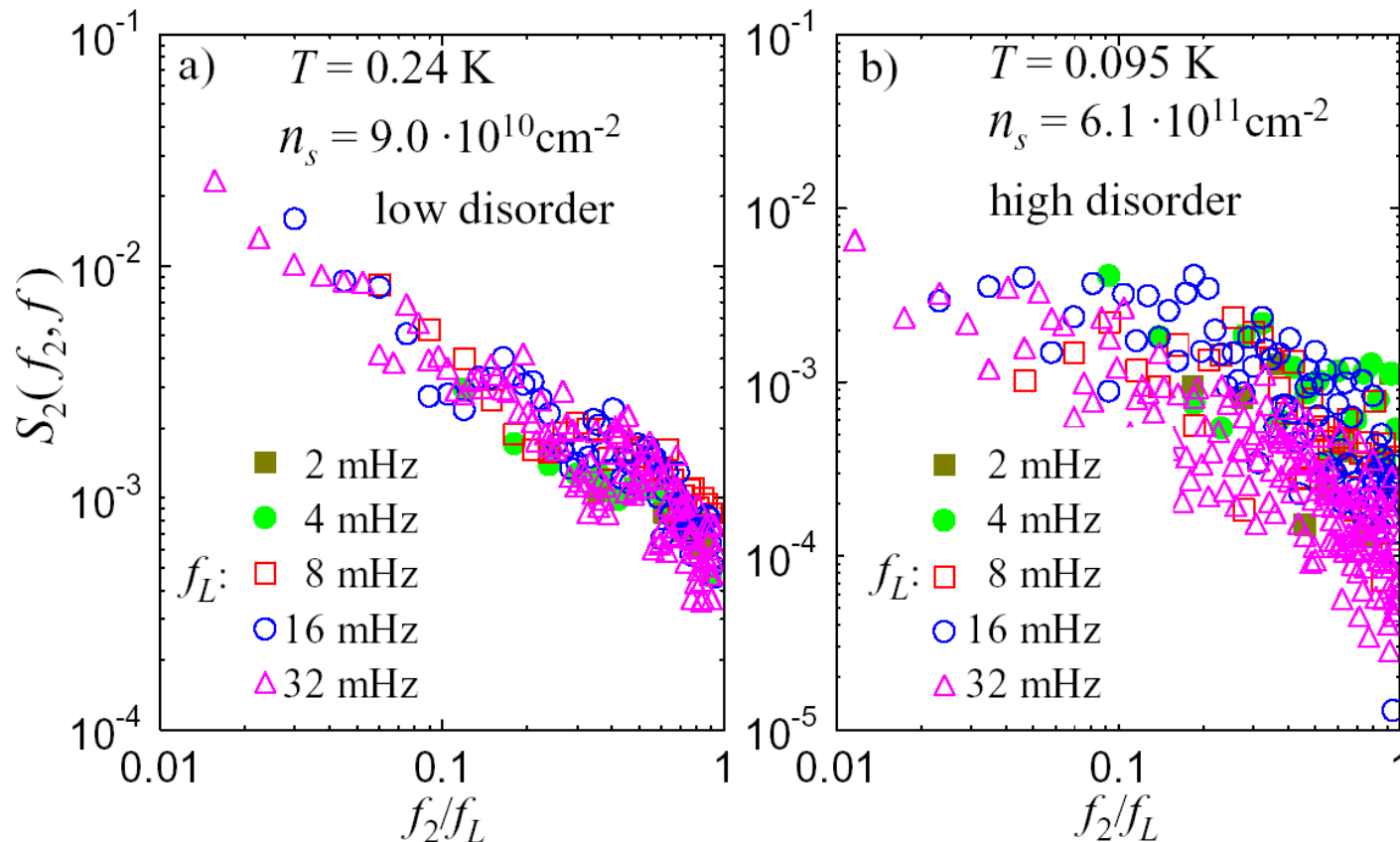
high-mob.
sample;
 $f_L = 1$ mHz



- a dramatic change in S_2 near n_g : S_2 white – uncorrelated at high n_s , S_2 nonwhite – correlated below n_g
- high-mobility samples: $n_c \lesssim n_g \sim n_s^*$ (no intermediate phase)
- low-mobility samples: $n_c < n_g \approx 1.5n_c < n_s^*$ (wide intermediate phase)

Scaling of the second spectra

[measured at $f = (f_L, 2 f_L)$]

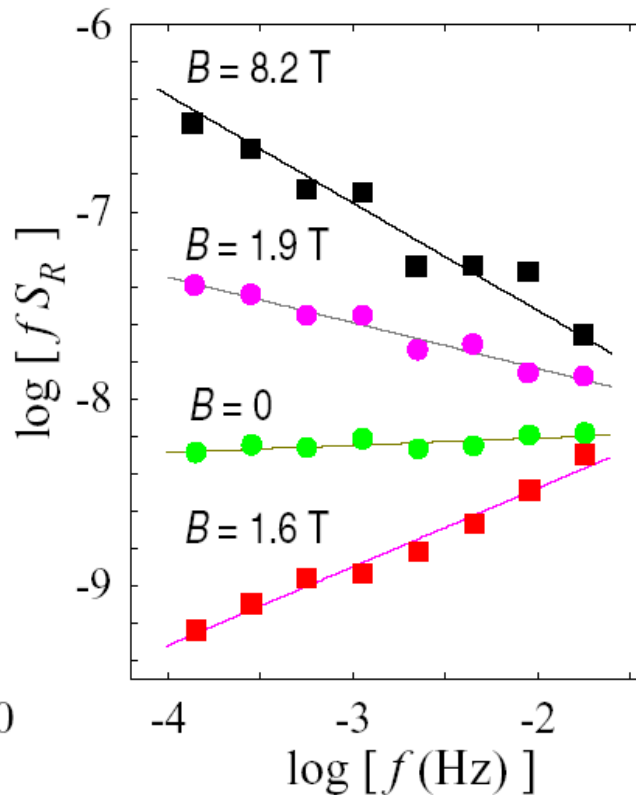
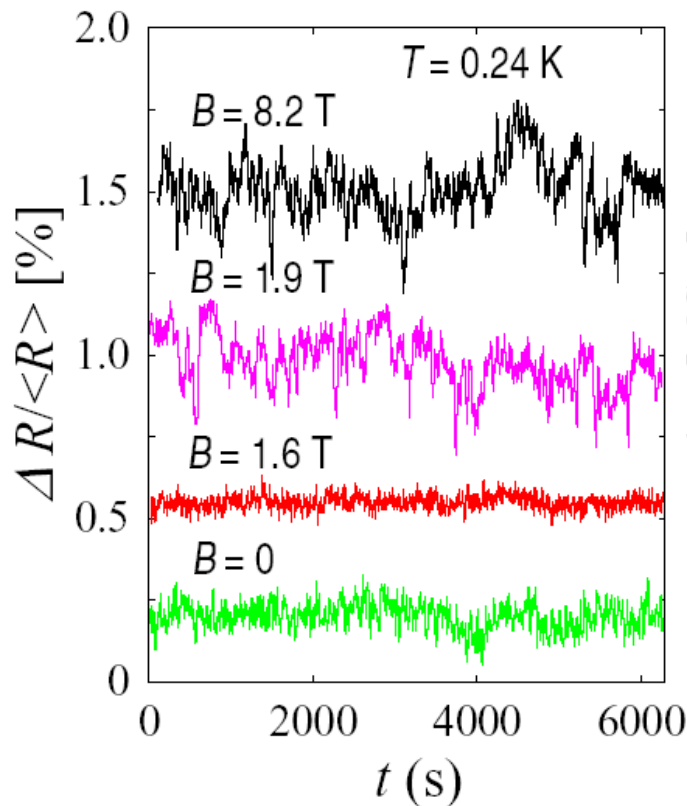


- no systematic dependence on $f \Rightarrow$ **scale invariance**
- consistent with the **hierarchical picture** of glassy dynamics
- rules out interacting two-state systems (droplets, clusters, defects, ...) as possible sources of noise

Noise measurements in parallel magnetic fields: spin or charge?



- a sufficiently strong magnetic field is expected to destroy the spin glass order; **2DES fully spin polarized for $B \approx 2-4$ T**



$$S_R \propto 1/f^\alpha$$

$$\alpha = 1.58$$

$$\alpha = 1.24$$

$$\alpha = 0.96$$

$$\alpha = 0.58$$

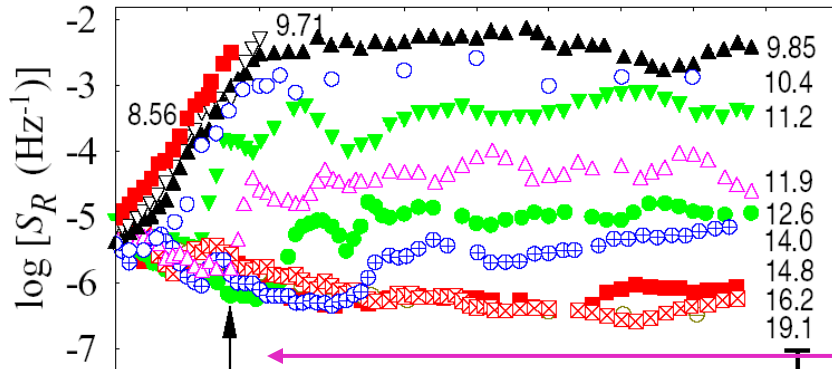
(high-mobility samples)

$$n_s = 11.9 \times 10^{11} \text{ cm}^{-2} > n_c(B=0)$$

Same analysis as before:



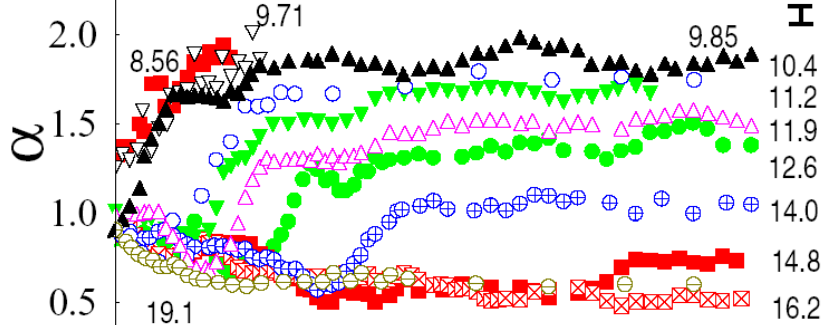
$f=1$ mHz



$n_s(10^{10} \text{ cm}^{-2})$

B_g for $n_s=11.9 \times 10^{10} \text{ cm}^{-2}$

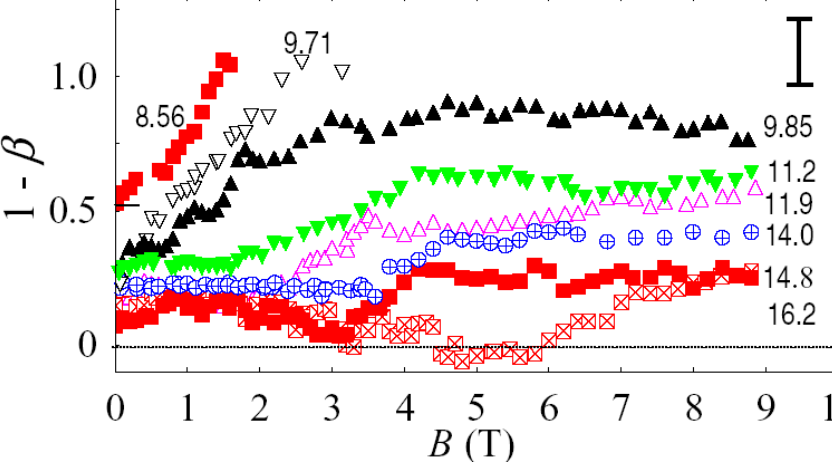
$S_R \propto 1/f^\alpha$



The same criteria for glass transition:

- a sudden and dramatic increase of S_R , a rapid rise of α , and a change of $(1-\beta)$ at $B_g(n_s)$ at low B and $n_g(B)$ at high B

$S_2 \propto 1/f^{1-\beta}$



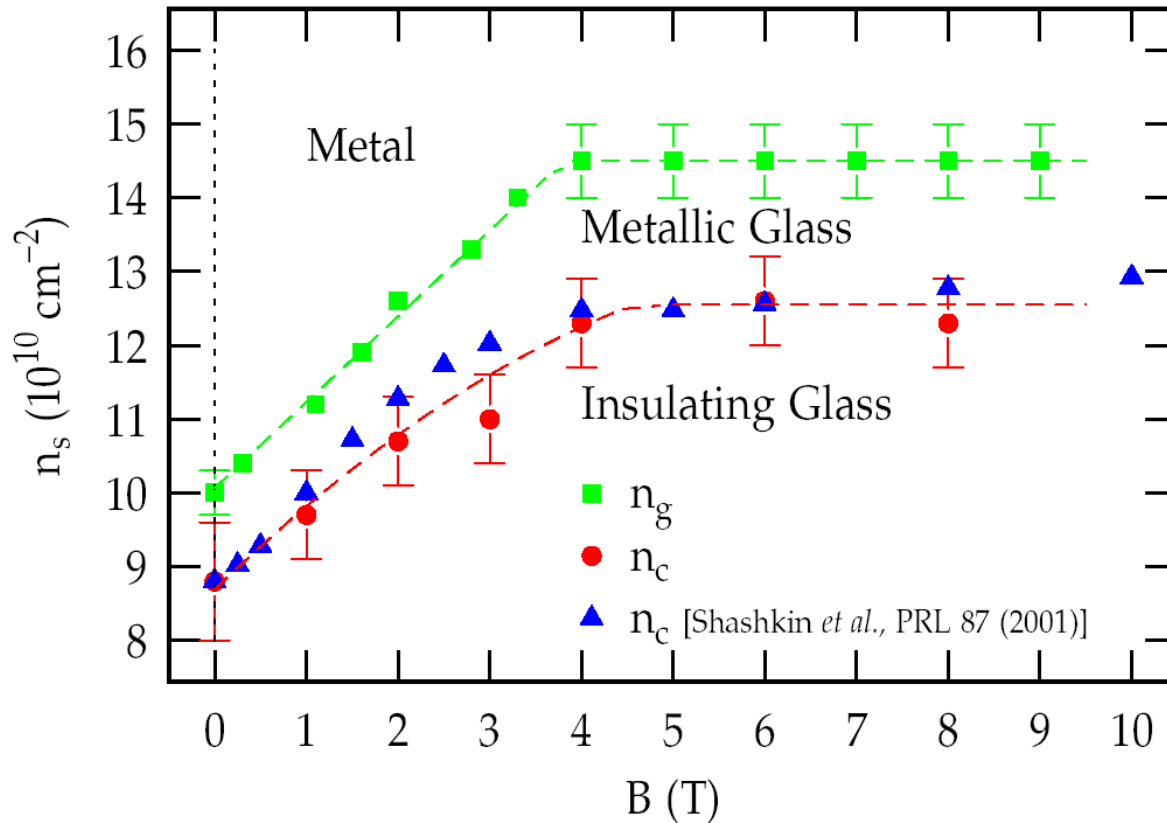
glass transition at $B_g(n_s)$ and $n_g(B)$

Phase diagram in B

[Jaroszyński, Popović, Klapwijk,
PRL 92, 226403 (2004)]

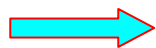


B → 2DES spin-polarized



n_g – glass transition

- glass transition persists in high parallel B, where the 2DES is spin polarized

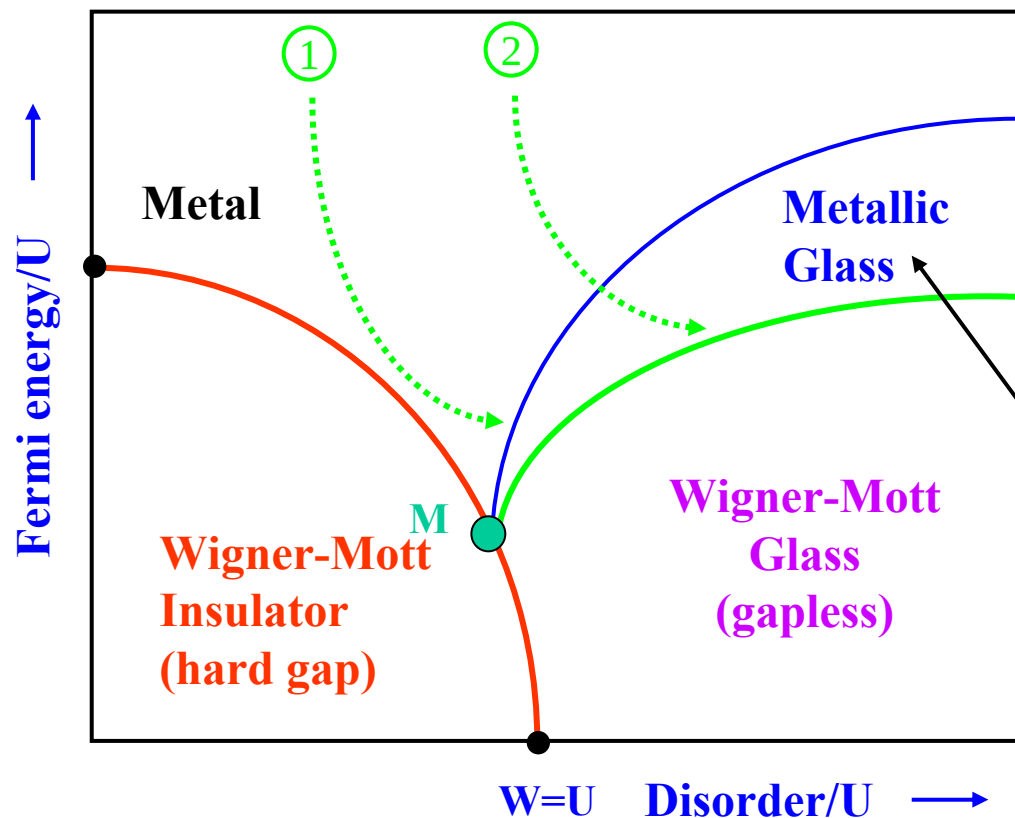


charge, as opposed to spin, responsible for glassy ordering

Coulomb glass

[V. Dobrosavljevic *et al.*: PRL 83, 4642 (1999); PRB 66, 081107 (2002); PRL 90, 016402 (2003); PRL 91, 066603 (2003); EPL 67, 226 (2003); PRL 94, 046402 (2005)]

Global phase diagram (theory)



- (2D) MIT as a Mott transition with disorder (**DMFT picture**)
- glass as a **precursor** of MIT
- **melting of glass** even at $T=0$ (by quantum fluctuations)
- **Metallic glass phase:**
 $\sigma(T) - \sigma(0) \sim T^{3/2}$
- hierarchical, **correlated** dynamics

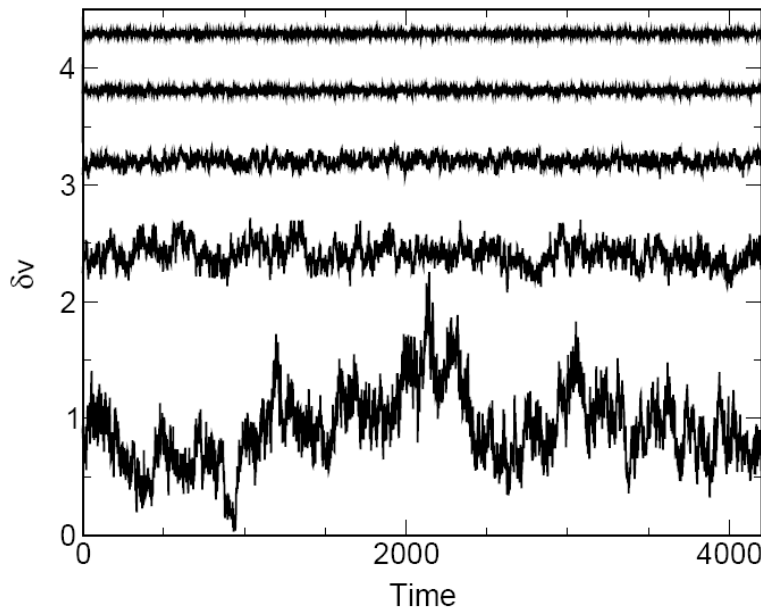
Physical trajectory: $E_F \sim n_s$; $U \sim n_s^{1/2}$; $W \sim \text{const.}$ $\Rightarrow (E_F/U) \sim (W/U)^{-1}$

① High-mobility samples, ② Low-mobility samples

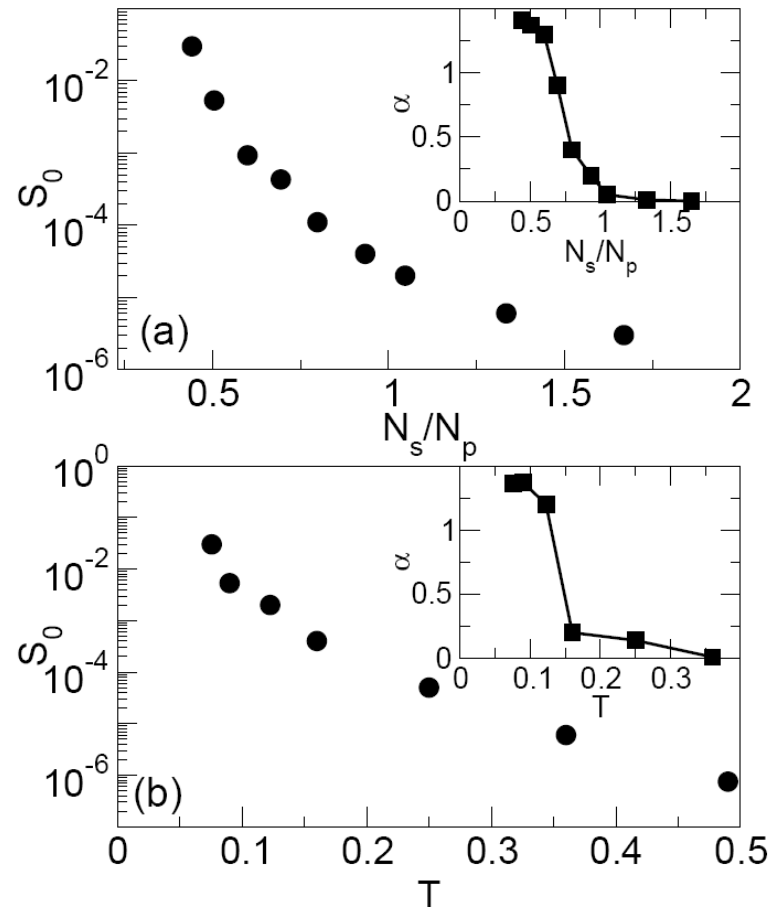
Simulations



- **Molecular Dynamics** [C. Reichardt and C. J. Olson Reichardt, PRL 93, 176405 (2004): a classical model of interacting electrons in 2D with disorder
- **increase of noise power and α with decreasing density and T**
- **non-Gaussianity at low n_s and T**



Similar to experiments in 2DES in Si



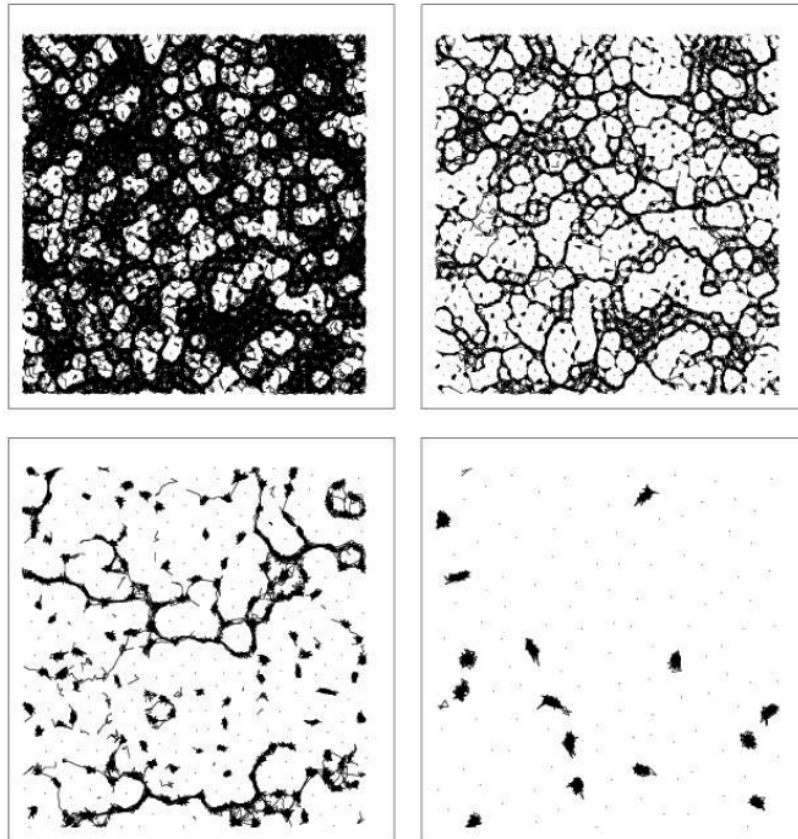
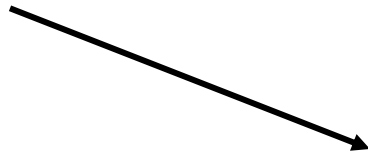


FIG. 5. Electron trajectories for a fixed period of time for fixed $T = 0.09$ at (a) $N_s/N_p = 1.67$, (b) 1.37, (c) 0.5, and (d) 0.3.

Noise power and α maximum



Trajectories change with time:
dynamical inhomogeneities

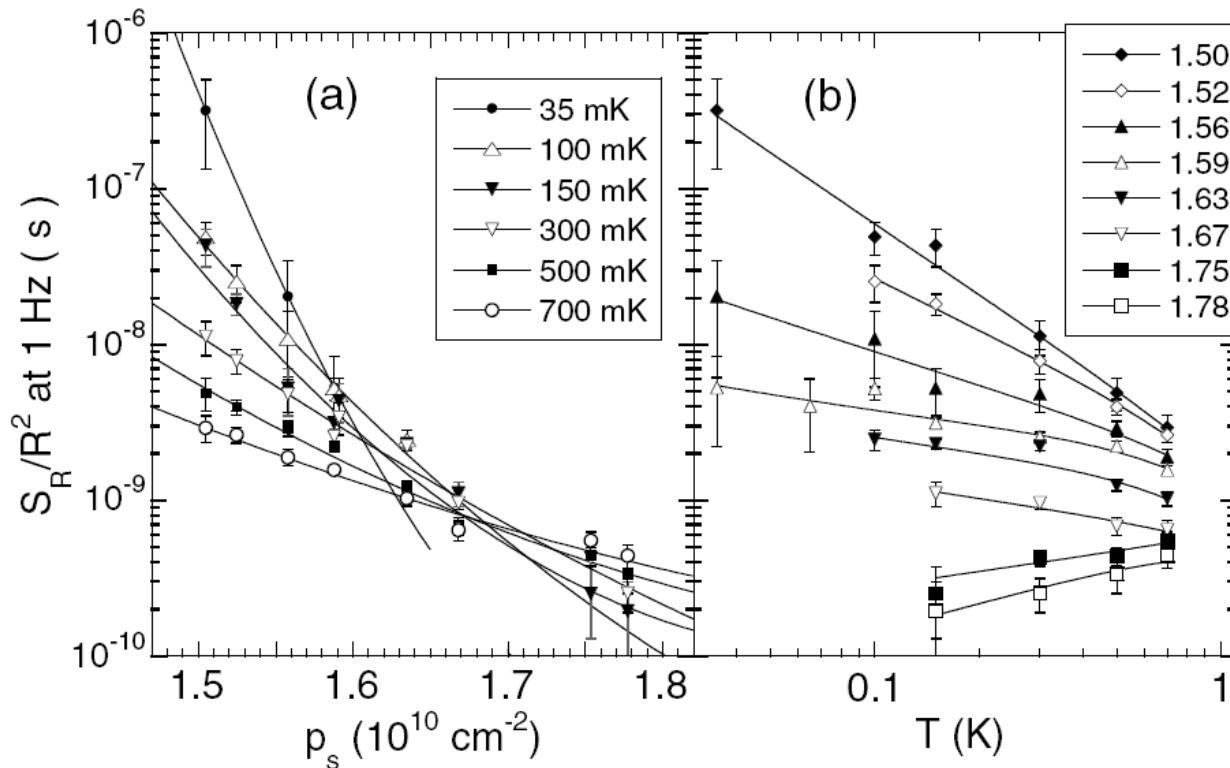
- Monte Carlo [Kolton, Grempel, Dominguez, PRB 71, 024206 (2005)]:
3D Coulomb glass - **heterogeneous dynamics**

Experiments in other systems



- $1/f^\alpha$ noise in 2D holes in GaAs (much less disorder than in Si) – some similarities, but no changes in α (≈ 1) and $(1-\beta)$ (≈ 0)
- not low enough p_s , T , f , or disorder?

[Leturcq, L'Hôte, Tourbot, Mellor, Henini, PRL 90, 076402 (2003)]



$$p_c = 1.44 \times 10^{10} \text{ cm}^{-2}$$

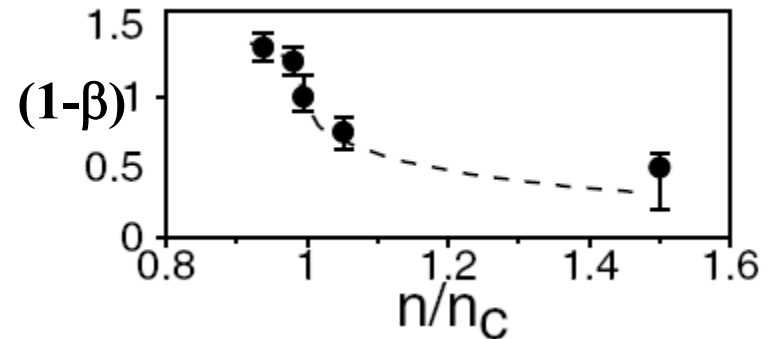
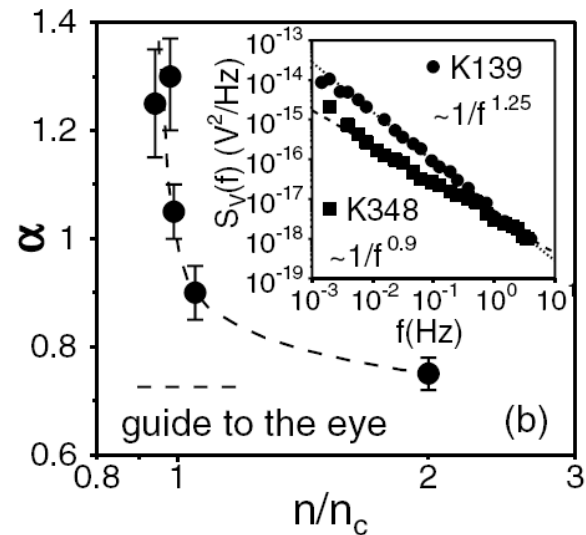
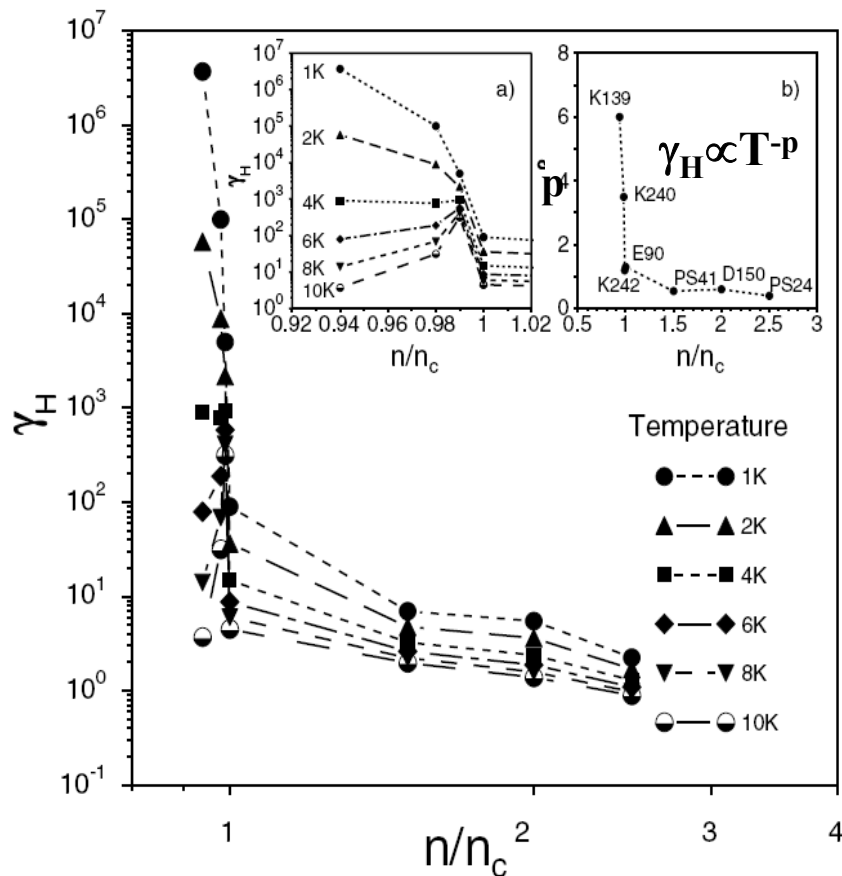
$$p_s^* = 1.57 \times 10^{10} \text{ cm}^{-2}$$

Bulk Si:P(B)

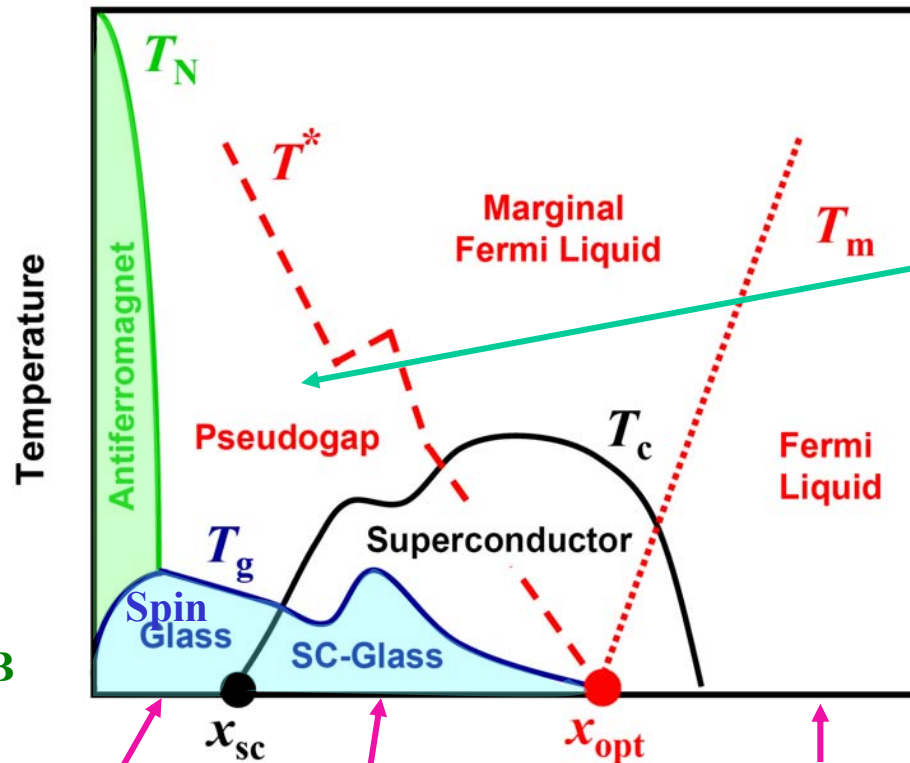


- $1/f^\alpha$ noise in bulk Si:P(B) - a huge increase of noise and the onset of non-Gaussianity at the MIT!!! Similar to 2DES in Si!

[Kar, Raychaudhuri, Ghosh, v. Löhneysen, Weiss, PRL 91, 216603 (2003)]



High temperature superconductors



**Emergence of
nanoscale
inhomogeneities!**

[C.Panagopoulos and
V.Dobrosavljević, PRB
72, 014536 (2005)]

Glassy insulator?

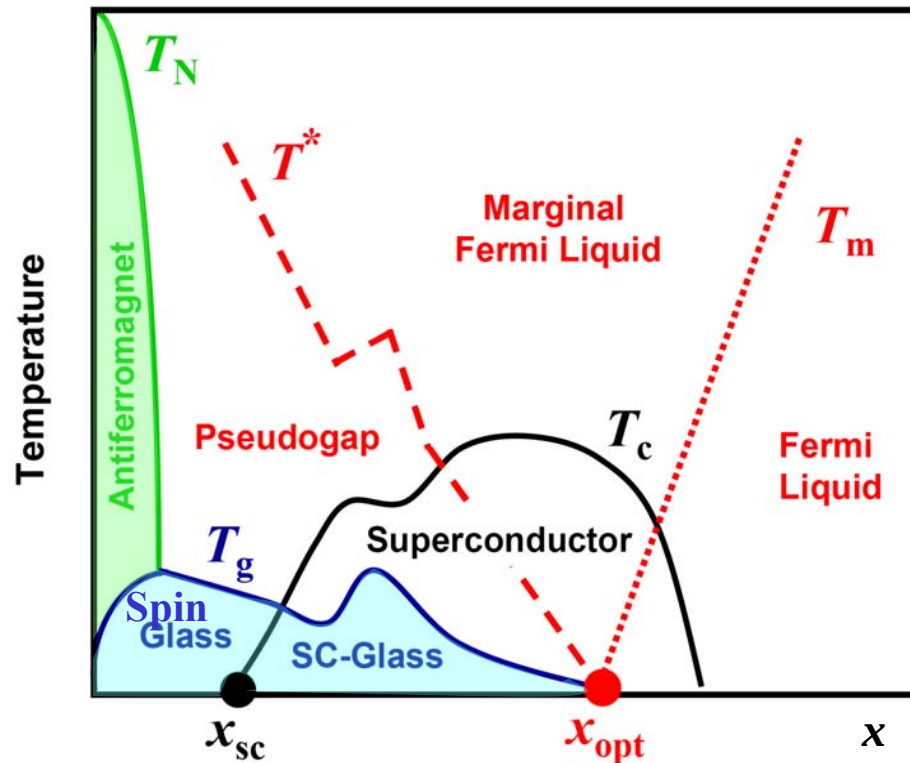
Metal-insulator transition?

**Inhomogeneous,
conducting glassy
state?**

**Homogeneous
metal**

**Quantum
glass transition?**

High temperature superconductors



MIT in a 2DES in Si
at $T=0$
[Coulomb glass; $T_g=0$]

Glassy insulator

Metallic glassy phase
($k_F l < 1$; “bad” metal)

Metal

n_c

n_g

carrier density

metal-insulator transition

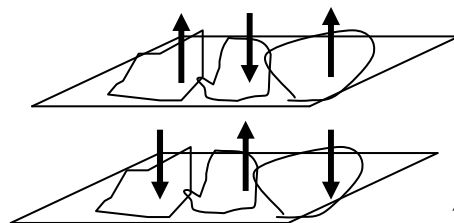
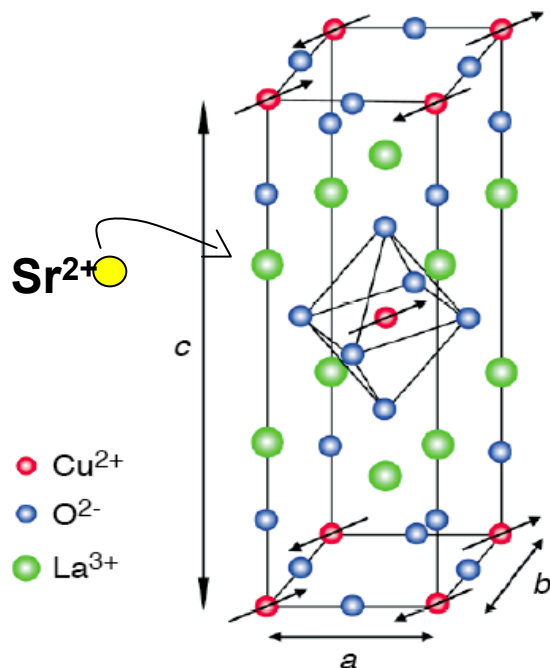
glass transition



Lightly doped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

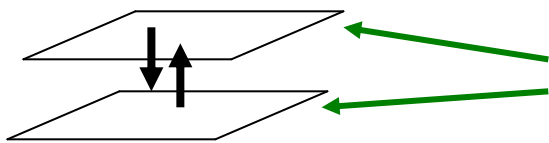


$x \geq 0.02$: short-range magnetic order for $T > T_{\text{sg}}$
(T_{sg} - spin glass transition)

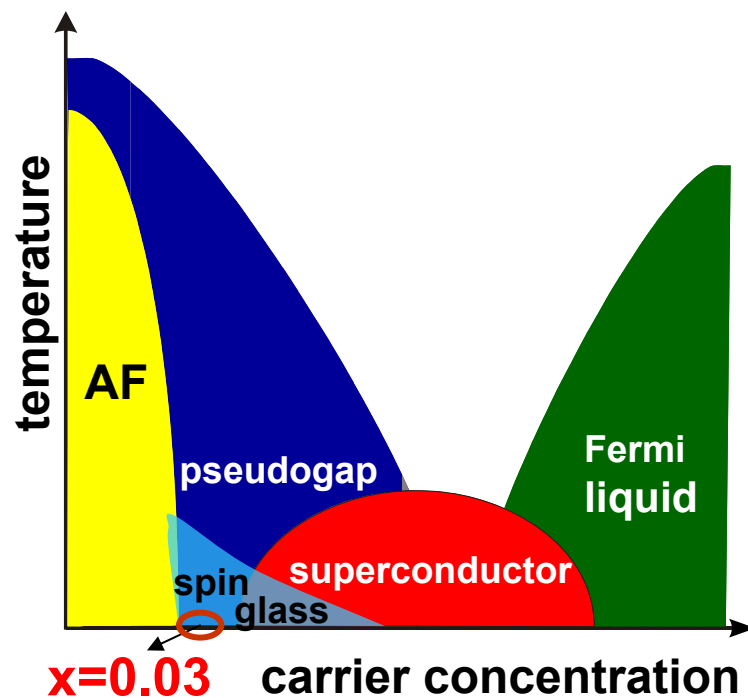


• AF domains; holes along domain walls

3D long-range antiferromagnetic (AF) order:

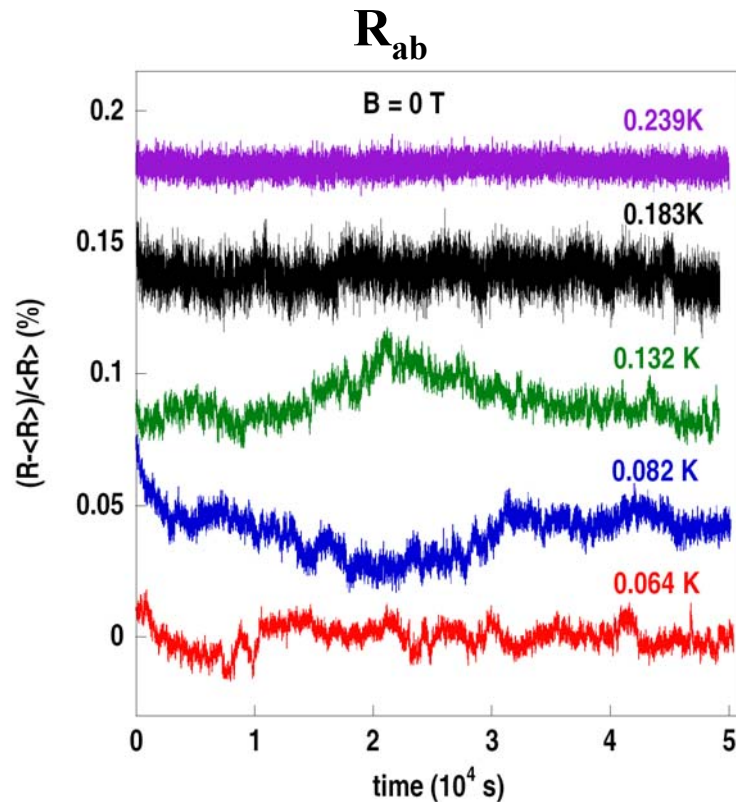
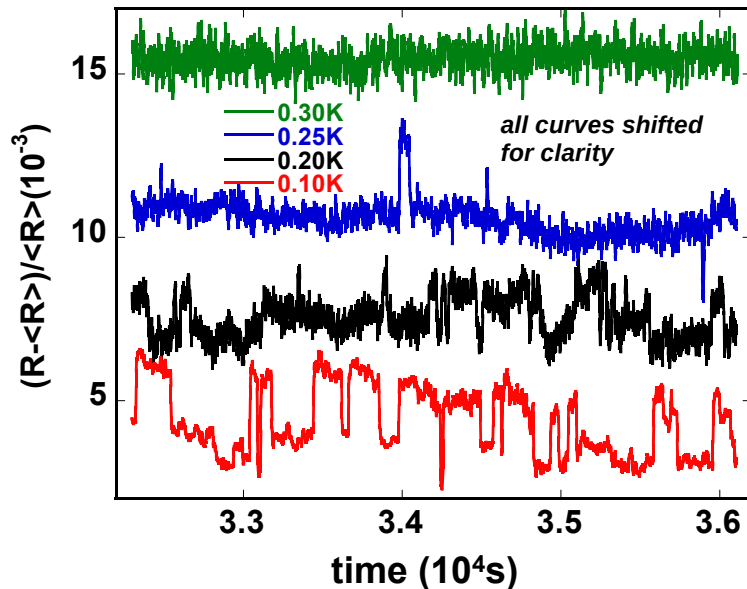
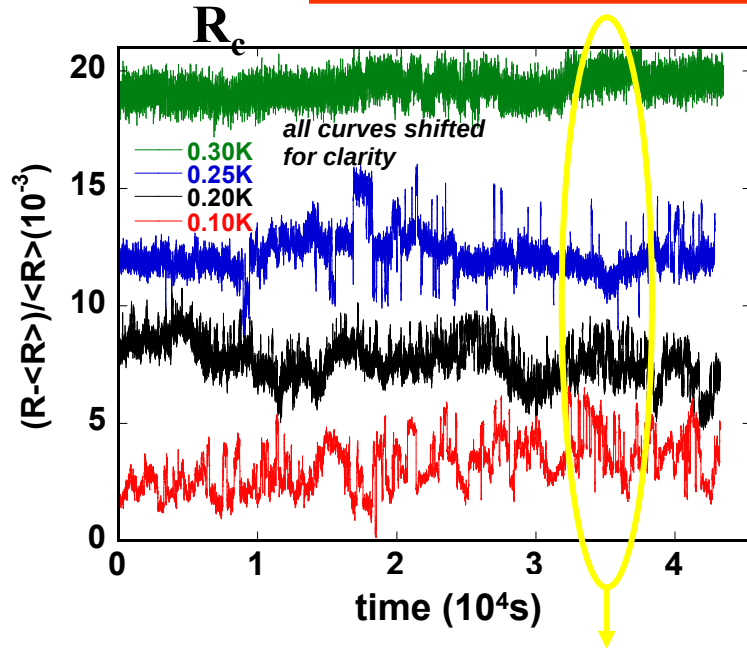


CuO_2 planes



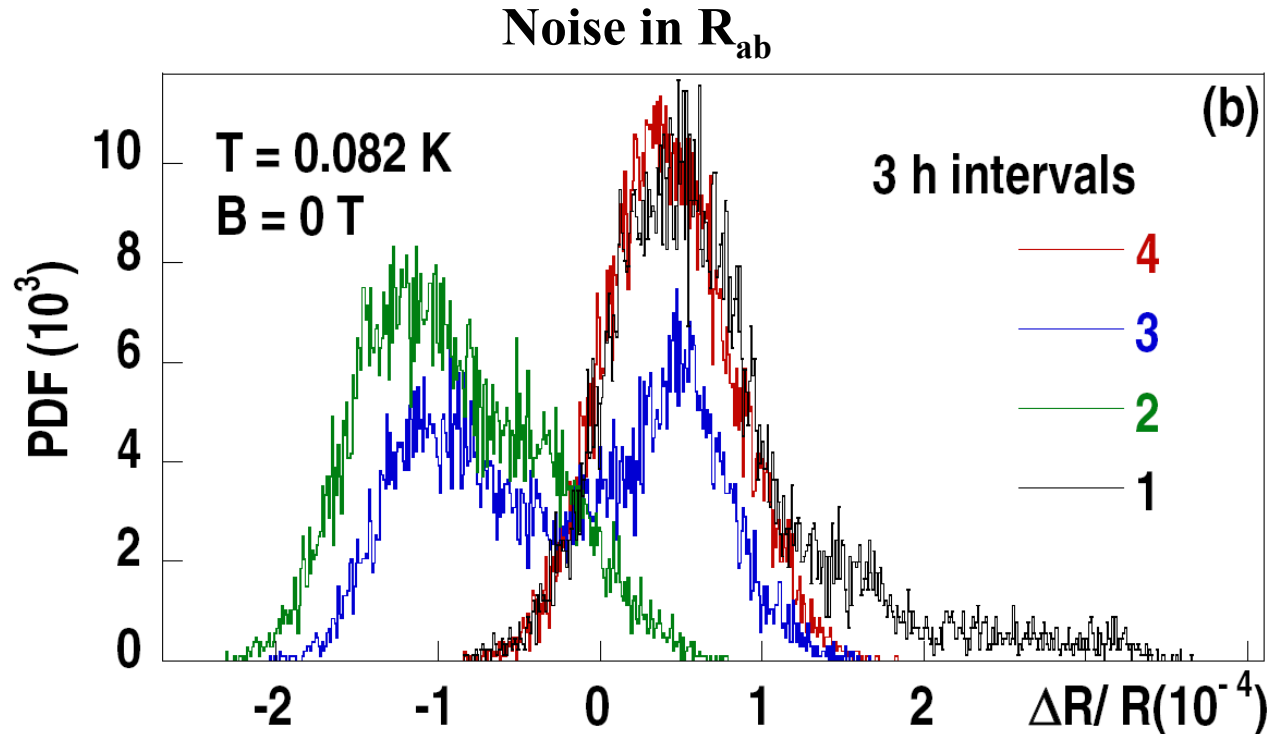
$T_{\text{sg}} \sim 7-8 \text{ K}$

Resistance fluctuations (noise); LSCO

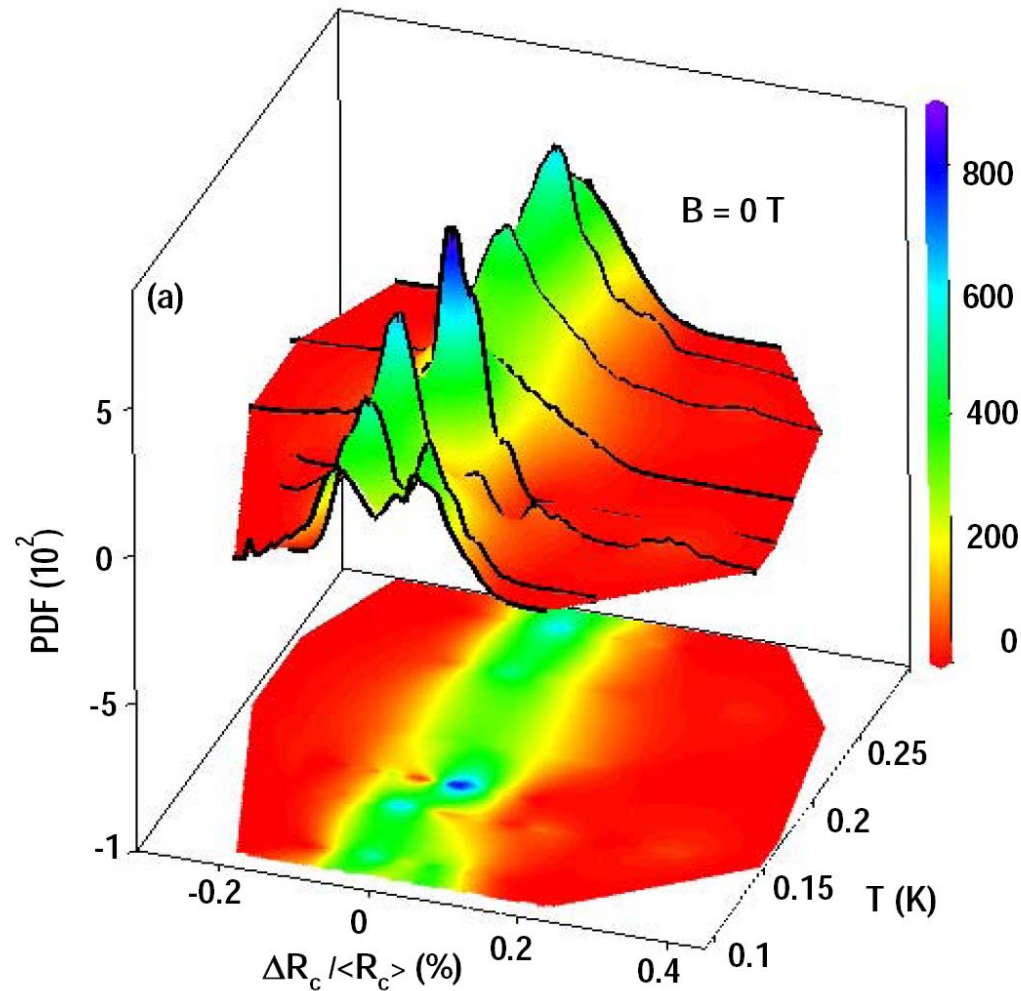


- noise Gaussian at “high” T
(e.g. $T > 0.18$ K for R_{ab} noise)
- at low T, non-Gaussian noise
metastable states (out-of-equilibrium)

Probability density functions (PDF) of fluctuations



- structure depends on the observation time –
different states contribute $\Rightarrow$ nonergodic

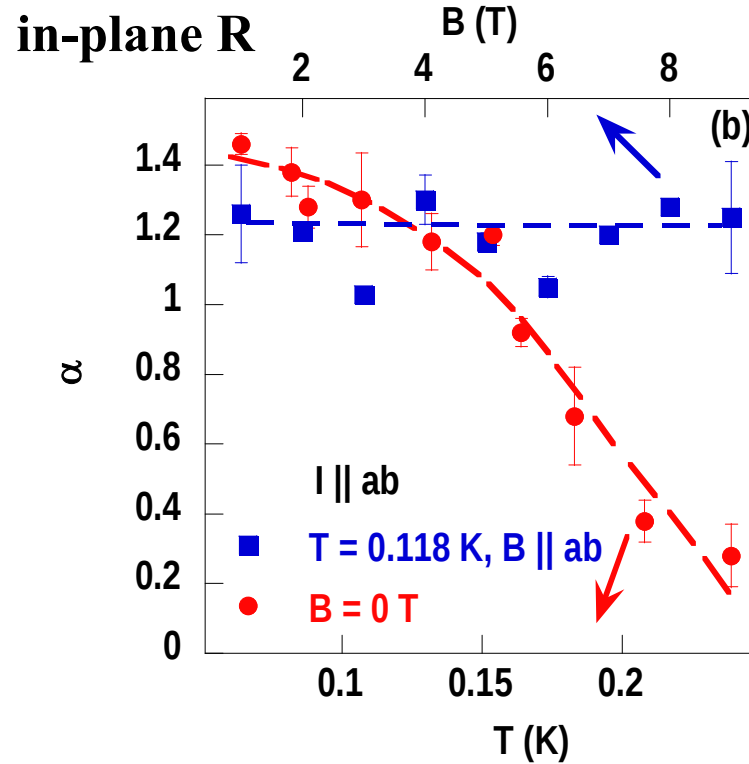
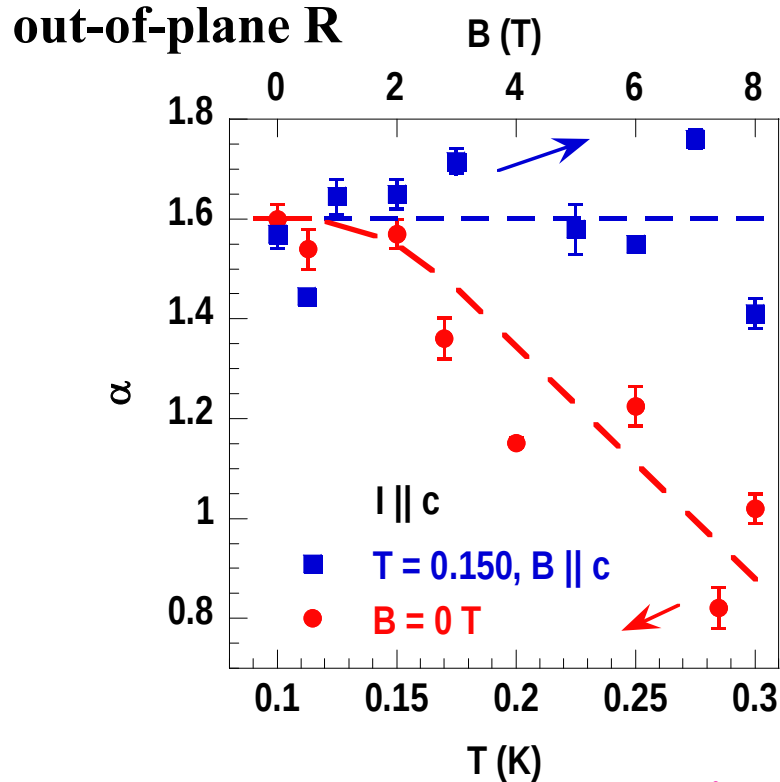


- increase sampling time to 12 hours, but **never becomes Gaussian** at low T
- **nonergodic, does not reach equilibrium** on experimental time scales at low T

Onset of glassiness in transport at $T \ll T_{SG}$: suggests spin and charge glass not directly related

Noise statistics: T and B dependence

- Power spectrum: $S_R \sim 1/f^\alpha$



- α increases as T is reduced; no effect of B !

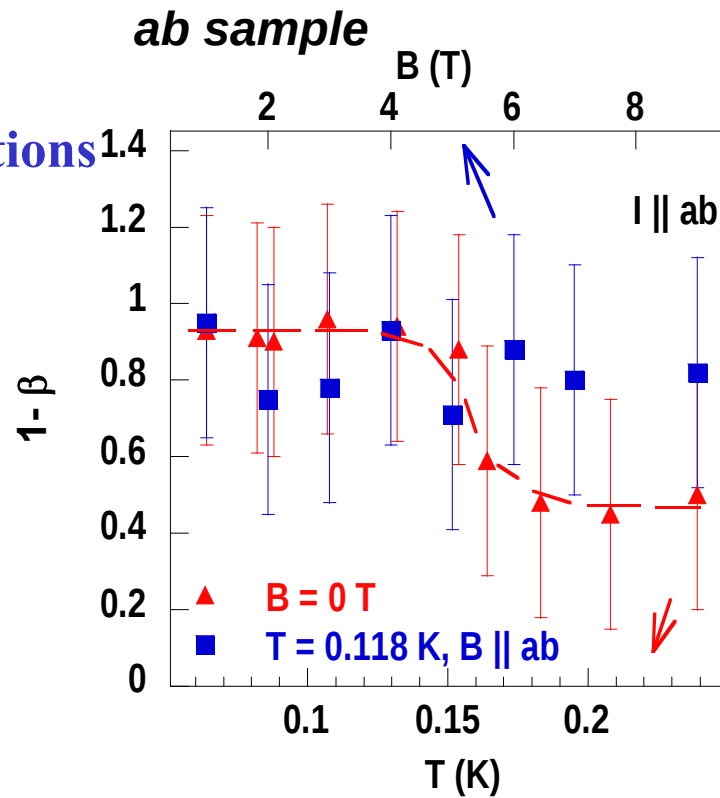
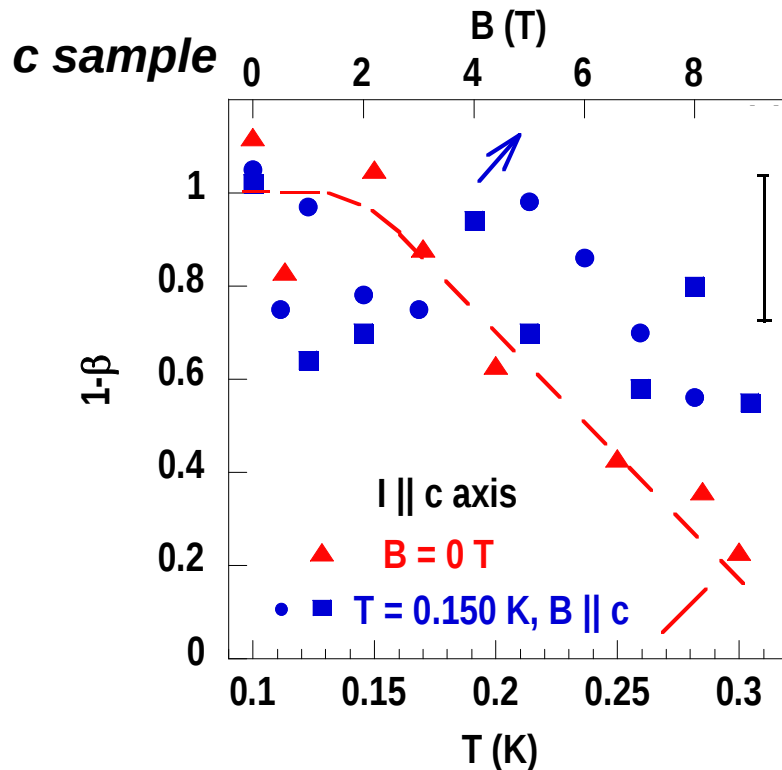
fewer metastable states that dominate
at low T in the exp. time window

Slowing down of the dynamics as $T \rightarrow 0$

Second spectrum $S_2(f_2, f)$

- the power spectrum of the **fluctuations** of $S_R(f)$ with time

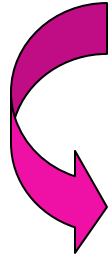
- white** ($1-\beta=0$) for uncorrelated fluctuators (**Gaussian**)
- $S_2(f_2, f) \propto 1/f_2^{1-\beta}$ for interacting fluctuators (**non-Gaussian**)



Increase of correlations as $T \rightarrow 0$

Noise statistics **independent** of both **B** and **magnetic history**

(Partial) summary of noise results in LSCO



- **Slowing down** of the dynamics as $T \rightarrow 0$
- Increase of **correlations** as $T \rightarrow 0$

Glass transition at $T=0$

- Noise statistics independent of both B and magnetic history
(**unlike conventional spin glasses**) $\Rightarrow$ **charge, not spin!**
- Onset of hysteretic and memory effects in magnetoresistance:

$$T_{\text{onset}} \ll T_{\text{sg}}$$

$\Rightarrow$ **Charge glass transition $T_{\text{cg}}=0$**

[I. Raičević *et al.*, PRL101, 177004 (2008)]

Scaling of the second spectra



- can distinguish between different models:
- droplet approach
- hierarchical, tree-like model

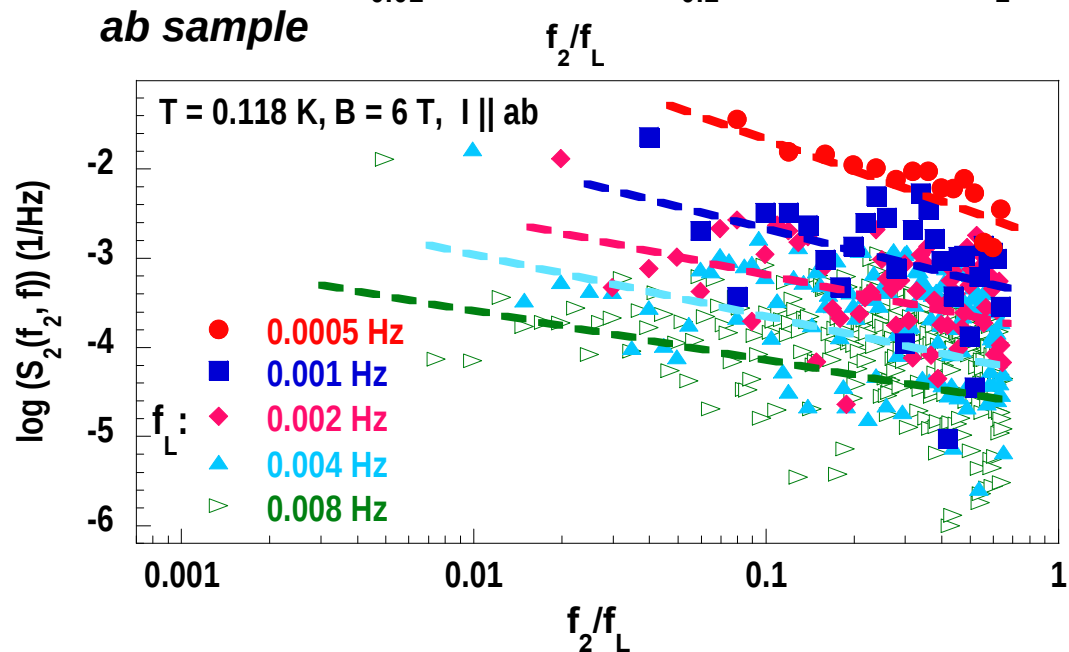
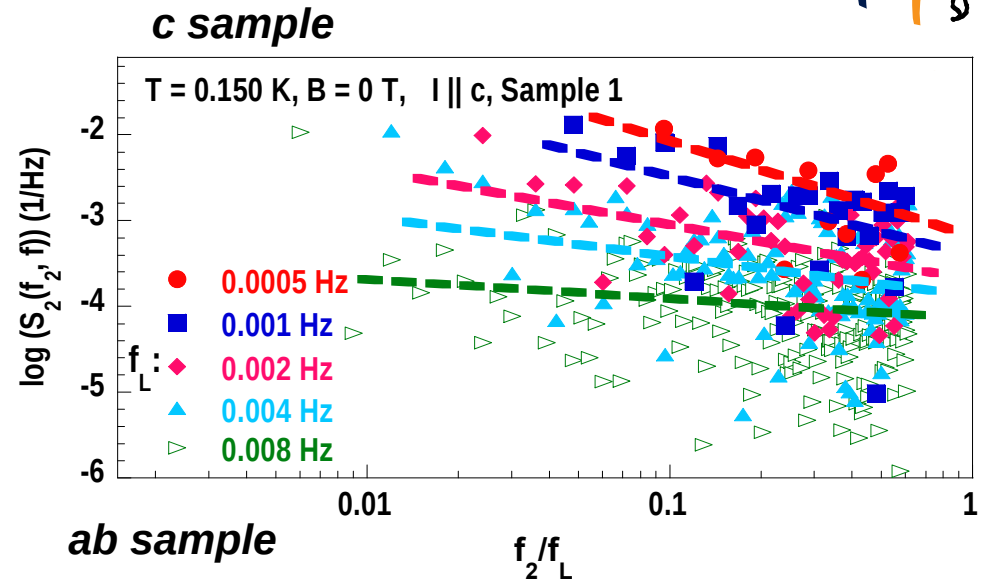
S_2 decreases with f
for a fixed f_2/f ,

consistent with droplet picture
(short-range interactions)



Spatial segregation of holes as a
result of competing interactions
on different length scales

Cluster charge glass



Summary of Lecture III



- Noise study in a 2DES in Si:
 - **sharp transition** to a glassy phase **as a function of density**
 - glass transition at **T=0**
 - **glassy phase: slow, correlated, hierarchical dynamics**
 - glassy ordering due to **charge**, not spin $\Rightarrow$ **Coulomb glass**
- Noise study in lightly doped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$:
 - transition to a glassy phase at **T=0**
 - **glassy phase: slow, correlated dynamics; clusters**
 - glassy ordering due to **charge**, not spin $\Rightarrow$ **Coulomb glass**
- Noise as a probe of glassy dynamics in other systems (**e.g. doped Si**)

Glasses everywhere...