Outline of the series

- Overview/Review "Equilibrium SM vs. Nonequilibrium SM"
- An Ising-like model in DDS "Shattered expectations"
  - What differences do the drive induce?
  - Surprises about T_c , above it, near it, and below it!
  - What we understand so far and what challenges remain.
- DDS in one-dimension "Bare bones NESM"
- Systems with more than one driven species "American football, Barber poles, and Clouds"
- Summary and Outlook "Come and join in the fun!"

Driven Diffusive Systems

What are DDS? Why study them?

conservation laws crucial

…out of equilibrium !

using

direct drives, e.g., *E, g* fields

or random drives, e.g., multiple *T*'s

Driven Diffusive Systems

What are DDS? Why study them?

- Fundamental issues in NESM
- Physics of many systems "all around us"
  - fast ionic conductors (which started this industry)
  - micro/macro biological systems
  - vehicular/pedestrian traffic, granular flow
  - social/economic networks

    ⋮

Driven Diffusive Systems

Perhaps we can gain some insight through SIMPLE systems, like the Ising model

- Physics of many systems "all around us"
  - fast ionic condu…

is

…EX!

Yet, we learned a great deal about EQUILIBRIUM SM through it.

Driven Diffusive Systems

Simple models and Potential applications

- Single species of particles (Ising lattice gas)
- Two or more species
- ***n*** dimensional objects embedded in ***d*** dim. space
- Variety of drives ("two temperature" models)
- (effects of) quenched disorder

  ⋮

- What's **not** DDS? ***Many, many*** other NE systems:
  - reaction-diffusion (birth-death) processes, with "serious" and/or "ordinary" interactions
  - percolation, directed percolation, BARW, persistence, …
  - epidemics, population dynamics, network dynamics, …
  - aging, glass, …

Simple Models and Applications

- Single species (n,d) DDS

protein synthesis (0,1)
advection of passive particles (0,2)
fast ionic conductors (0,3)
transport through polymeric membranes (0,3)
steps on vicinal surfaces (1,2)
flux lines in superconductors (1,3)
polymer sedimentation (1,3)
surface growth in MBE (2,3)

- Multi-species DDS

bio-molecular motors on microtubules
electrophoresis
vacancy mediated diffusion in alloys
vehicular/pedestrian traffic dynamics

An Ising-like Driven Diffusive System

Shattered expectations

100^{th} Statistical Mechanics Meeting (Rutgers)

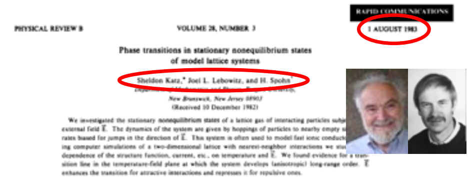
December, 2008

Twenty five years after KLS

a celebration of

non-equilibrium statistical mechanics

What's **KLS** ? and 25 years after?


RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 28, NUMBER 3 1 AUGUST 1983

Phase transitions in stationary nonequilibrium states of model lattice systems

Sheldon Katz,* Joel L. Lebowitz, and H. Spohn†

New Brunswick, New Jersey 08903

(Received 10 December 1982)

We investigated the stationary nonequilibrium states of a lattice gas of interacting particles subje[ct to an] external field $\vec{E}$. The dynamics of the system are given by hoppings of particles to nearby empty s[ites with] rates biased for jumps in the direction of $\vec{E}$. This system is often used to model fast ionic conduct[ors. Us]ing computer simulations of a two-dimensional lattice with nearest-neighbor interactions we stu[died the] dependence of the structure function, current, etc., on temperature and $\vec{E}$. We found evidence for a transition line in the temperature-field plane at which the system develops (anisotropic) long-range order. $\vec{E}$ enhances the transition for attractive interactions and represses it for repulsive ones.

Nonequilibrium Steady States of Stochastic Lattice Gas Models of Fast Ionic Conductors

Journal of Statistical Physics, **34**, 497 (1984)

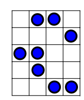
What's the original KLS?

- Take a simple interacting many-particle system…
  (Ising model – lattice gas version, for the ions)
- Drive it far from thermal equilibrium…
  (by an external DC "electric" field)
- Does anything "new" show up ?

Ising Lattice Gas

- Take a well-known equilibrium system…

e.g., Ising lattice gas (2-d, Onsager)

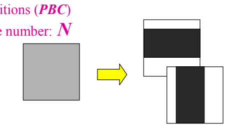


$\mathcal{C}$: { $n(x,y)$ } with $n = 0,1$

$\mathcal{H}(\mathcal{C}) = -J \sum_{x,a} n(x)\, n(x+a)$

+ periodic boundary conditions (***PBC***)

+ ***FIXED*** total particle number: N

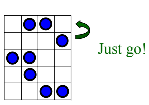


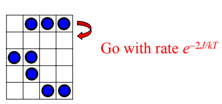
Ising Lattice Gas

- Take a well-known equilibrium system,
- evolving with a simple (DB obeying) dynamics…

$R(\mathcal{C} \to \mathcal{C}') \,/\, R(\mathcal{C}' \to \mathcal{C}) = exp[\{\mathcal{H}(\mathcal{C}') - \mathcal{H}(\mathcal{C})\}/kT\,]$

…one favorite R is Metropolis, e.g.,

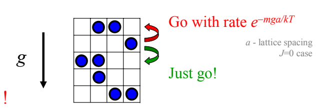




Driven Ising Lattice Gas

- Take a well-known equilibrium system
- Drive it far from thermal equilibrium
  (by some additional external force, so particles suffer *biased* diffusion.)

e.g., effects of gravity (uniform field)

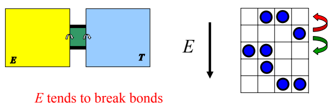


- Can't have PBC !!
- Get to equilibrium with extra potential term… NOTHING new, or NESM!

Driven Ising Lattice Gas

- Take a well-known equilibrium system
- Drive it far from thermal equilibrium
  (by some additional external force, so particles suffer *biased* diffusion.)

PBC possible with "electric" field, E
(non-potential, rely on $\partial_t B$)



Go with rate $e^{-(E-2J)/kT}$

unit "charge" and a with $E > 2J$

Just go!

E tends to break bonds

T tends to satisfy bonds

Driven Ising Lattice Gas

How does this differ from the equilibrium case?

- Dynamics violates detailed balance.
- System goes into non-equilibrium steady state:
  non-trivial particle current *and* energy through-flux.

In most cases, this is not easy to see!

In this case, it has to do with the PBC.

Irreversible w-loops are **global**!

Driven Ising Lattice Gas

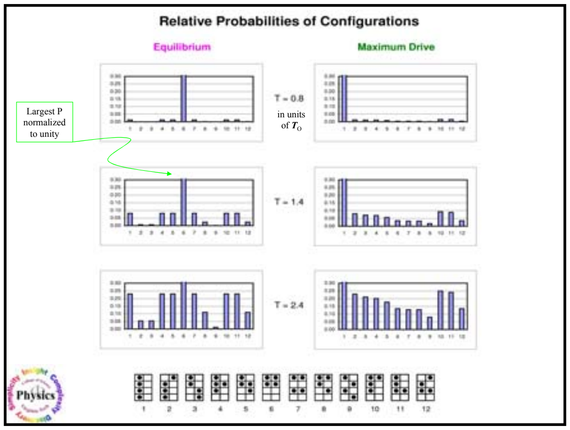
How does this differ from the equilibrium case?

- Dynamics violates detailed balance.
- System goes into non-equilibrium steady state
- Stationary distribution, $P^*(\mathscr{C})$, exists…
  ...but very different from Boltzmann.

Another simple, exactly solvable, case:

half filled, 2×4 lattice

70 configurations, 12 unrelated by symmetries



Driven Ising Lattice Gas

How does this differ from the equilibrium case?

- Dynamics violates detailed balance.
- System goes into non-equilibrium steady state
- Stationary distribution, $P^*(\mathscr{C})$, exists…
  ...but very different from Boltzmann.
- Usual fluctuation-response relation violated.

Even in simplest case: 2×3 (E=∞)

- "specific heat" $-\partial_\beta\langle U\rangle$ has a *peak* at $\beta = \ell n3\,/4J$
- energy fluctuations $\langle \Delta U^2\rangle$ *monotonic* in β

Driven Ising Lattice Gas

How does this differ from the equilibrium case?

- Dynamics v…
- System …
- Stationar…
  …but v… …ltzmann.
- Usual fluctuation… …pation theorem violated.
- The many *surprises* they bring!!

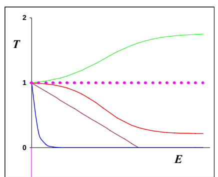
BTW, presence of ***anisotropy*** should not be a surprise…
The drive ***E*** is imposed in only one axis.

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition

for example, consider phase diagram:



Lenz-Ising, Onsager

What's your bet?

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses (E "adds" noise ~ higher T ; *but …*)

"Freezing by heating"
H.E. Stanley, *Nature* **404**, 718 (2000)

" Getting more by pushing less "
RKPZ, E.L. Praestgaard, and O.G. Mouritsen
American Journal of Physics **70,** 384 (2002)

Driven Ising Lattice Gas

The surprises they bring!!

- bre… …nded intuition
- neg…

For all $T > T_c$!!

- generic long range correlations: $G(\boldsymbol{r}) \sim r^{-d}$
  - *related to* generic discontinuity singularity in $S(\boldsymbol{k})$
  - *related to* number fluctuations in a window is
    …/orienta…

Amplitude depends on direction – ***anisotropy*** !

Reminder: *Away from criticality,*

$G(\boldsymbol{r}) \sim \exp(-r/\xi)$

in the ***equilibrium*** *Ising model*

Driven Ising Lattice Gas

The surprises the…

- bre…
- neg…

For all

Structure factor
FT of $G(\boldsymbol{r})$

- generic long range correlations: $G(\boldsymbol{r}) \sim r^{-d}$
  - *related to* generic discontinuity singularity in $S(\boldsymbol{k})$ at $\mathbf{0}$

Reminder: *In* ***equilibrium*** *Ising,*

$S(\boldsymbol{k}) \sim 1/(k^2 + \xi^{-2})$

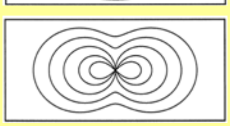
is of Ornstein-Zernike form. Adding anisotropic interactions just splits k^2 *into* $k_\perp^2 + \rho k_\parallel^2$ *, with* **no discontinuity** *at origin:* $\boldsymbol{k} = \mathbf{0}$

Driven Ising L…

Cahn-Hilliard equation, modified to include noise and E, leads to…

$$S(\boldsymbol{k}) = \frac{\sigma_\perp k_x^2 + \sigma_\parallel k_y^2}{\tau_\perp k_x^2 + \tau_\parallel k_y^2 + O(k_x^4, k_x^2 k_y^2, k_y^4)}$$

$(\sigma_\parallel \tau_\perp)/(\sigma_\perp \tau_\parallel)$



$S(\boldsymbol{k})$ in KLS

$$R \equiv \frac{\lim_{k_y \to 0} S(0, k_y)}{\lim_{k_x \to 0} S(k_x, 0)} < 1$$

Driven Ising Lattice Gas

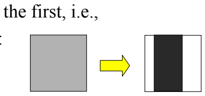
The surprises they bring!!

- bre… …nded intuition
- neg…
- generic long range correlations: r^{-d}

As $T \to T_c$

$R \equiv 1$ *in equilibrium*

- $S(\boldsymbol{k})$ diverges at the orgin, but…
  - Three possible scenarios: $R \to 0, \infty$, nonzero constant.
  - Simulations showed only the first, i.e.,
  - Transverse ordering only:



Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses
- generic long range correlations: r^{-d} (*all T not near* T_c)
- anisotropic scaling

For T near T_c

$k_{||} \sim k_\perp^{1+\Delta}$

$\Delta = 2$ (for KLS)!

$d_c = 4$ for *equilibrium* Ising

$d_c = 5$ [3] for uniformly [randomly] driven case

Fixed point ***violates*** detailed balance: "truly NESM"
Mostly confirmed by simulations, though a controversy lingers!

Fixed point ***satisfies*** detailed balance: Equilibrium "restored under RG"

For all $T < T_c$!!

Co-existence of phases ⇒ interface between them, with interesting properties, e.g., capillary waves, ***anisotropic*** surface tension

No idea how to predict 0.67 !
A theory predicts $1/|q|$,
but ***not*** the c !!

- generic long range correlations: r^{-d} (*all T not near* T_c)
- new universality classes
- anomalous interfacial properties, e.g.,

$S_h(q) \sim q^{-0.67}$ $[1/(|q|+c)]$ for uniformly [randomly] driven case

q^{-2} ⇔ interfacial widths do not diverge with L ! $L^{1/2}$

meaning/existence of surface tension ***unclear***!

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses
- generic long range correlations
- new universality classes
- anomalous interfacial properties
- new ordered states if PBC replaced by

Shifted PBC or Open BC

For all $T < T_c$!!

Reminder: Interesting, new, but understandable phenomena:

anisotropic surface tension

surface boundary layer

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses
- generic long range correlations: r^{-d} (*all T not near* T_c)
- new universality classes
- anomalous interfacial properties
- new ordered states if PBC → SPBC, OBC
- complex phase separation dynamics

For all $T < T_c$!!

Driven Ising Lattice Gas

The surprises they bring!!

- *anisotropic interactions* : $J_{||} = \alpha^2 J_\perp$
- *rectangular geometry* : $L_{||} = L_\perp^{\,p}$
- *bi-layer systems with* $J_\times > 0$ *or* < 0

⋮

- simple variations of "the standard model"

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- …need new intuition/paradigm…

One way forward is

to study many other, similar systems

How about if we follow Ising?
and consider $d = 1$ systems?