Granular Matter II

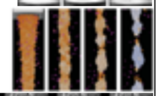
Heinrich Jaeger

University of Chicago

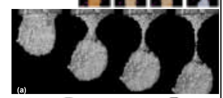
http://jfi.uchicago.edu/~jaeger/group/index.html

Outline

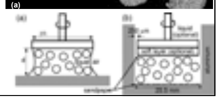
- Brief intro to granular matter



- Granular jets & freely falling streams



- Dense suspension droplet break-up



- Shear thickening in suspensions

Break-up = catastrophic deformation, topological transition

In pure simple liquids: distinct, smooth structures during break-up; signature of liquid type (viscous, inviscid, etc.)

Self-similarity and scaling near break-up at time $t = t_0$

Ingredients: surface tension, viscosity, density

Cut-off: molecular scale

- Model system to test scaling ideas about topological transitions (fluids, condensed matter physics, materials science, …, black holes)
- Technological relevance (spraying, fuel injection, …)

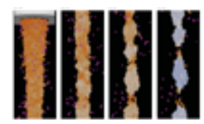
Break-up in dry granular systems (jets, streams):

distinct, much more ragged structures in neck region; signature of granularity

Scaling near break-up at time $t = t_0$; but: where does 2/3 power law come from?

Ingredients = ? (material density, eff. surface tension due to cohesion, coeff. of restitution, plus??)

Cut-off: grain scale

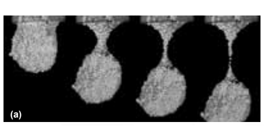


- Model system to test for tiny (nm, nN) cohesive forces between macroscopic grains

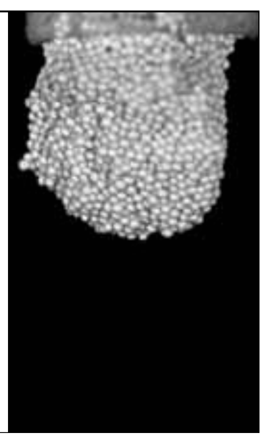
Suspension Break-Up

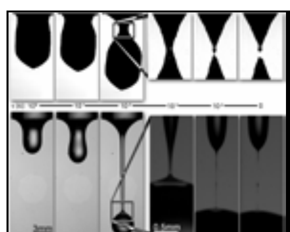
Large particles, > 10µm
Non-Brownian

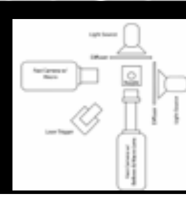
Cut off bottom



850µm zirconium dioxide in water

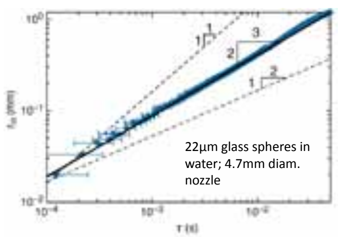






145µm zirconium dioxide in water
50cst silicon oil; 4.7mm nozzle

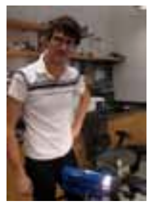
Surprise: dense suspension break-up has scaling like water!



Inviscid: $r_m \sim \tau^{2/3}$

Bubble pinch-off: $r_m \sim \tau^{1/2}$

Viscous: $r_m \sim \tau$

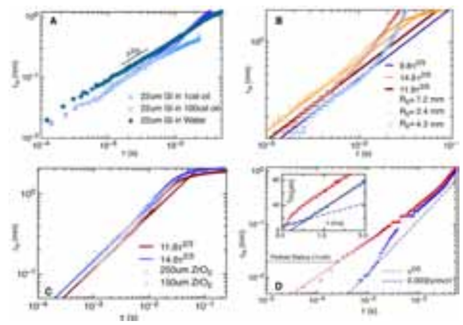


Marc Miskin

$\tau = t_0 - t$ = time remaining until break-up

r_m = neck radius at thinnest point

2/3 power law robust; independent of viscosity!

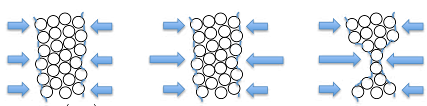


Different solvent viscosities (A), nozzle diameters (B), particle diameters (C).
(D) = comparison with oil of same effective viscosity of 50 cst (blue trace)

Particles in interior: fully submerged
Particles at interface: linked by menisci

Viscosity irrelevant near break-up

Inertial force ~ force exerted by menisci at bounding surface



$P = \alpha(\gamma/a)$

$$\rho\frac{d\vec{u}}{dt} = -\nabla P \quad \Longrightarrow \quad \rho\frac{r_m}{\tau^2} \propto \frac{P}{r_m} \qquad P = ?$$

Flat surface: particle packing uniform ➔ P = αγ/a = const. ➔ no flow

Curved surface: particle packing depends on curvature ➔ P ≠ const. ➔ flow

For sphere packing on surface with Gaussian curvature $K < 0$:

$$P = \alpha\gamma a K / \tfrac{\pi}{3}(6 - \bar{Z})$$

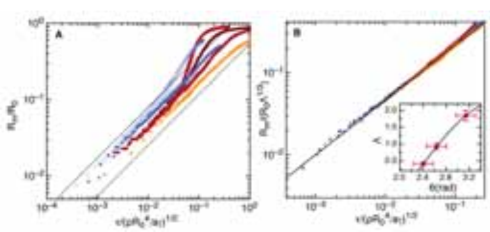
$= 1/(r_m R_0)$ Avg. coordination #

Marc Miskin: find avg $Z(K)$ for configuration of spheres puncturing the 2D surface with Gaussian curvature $K < 0$ that bounds a disordered 3D packing:

$$P = -\alpha\gamma\left[\frac{1}{2\sqrt{3}a} + \frac{(6-\sqrt{3}\pi)}{12\pi}Ka + \mathcal{O}(\sqrt{-Ka})^3\right].$$

$$\rho\frac{r_m}{\tau^2} \propto \frac{P}{r_m} \rightarrow \rho\frac{r_m}{\tau^2} \propto \frac{\alpha\gamma}{r_m^2}\frac{a}{R_0} \rightarrow r_m \propto \left(\frac{\alpha\gamma}{\rho}\frac{a}{R_0}\right)^{1/3}\tau^{2/3}$$

Scaling Collapse



Residual variation due to wetting angles   Corrected for wetting

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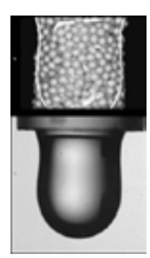
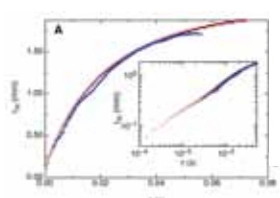
Pure inviscid liquid limit reached by reducing particle radius a toward zero?

$$r_m \propto \left(\frac{\alpha\gamma}{\rho}\frac{a}{R_0}\right)^{1/3}\tau^{2/3}$$





No, set $a \sim R_0$ instead ➔ dense suspension behaves most like water when particle size approaches fraction of nozzle diameter and individual particles become clearly discernable by naked eye!!

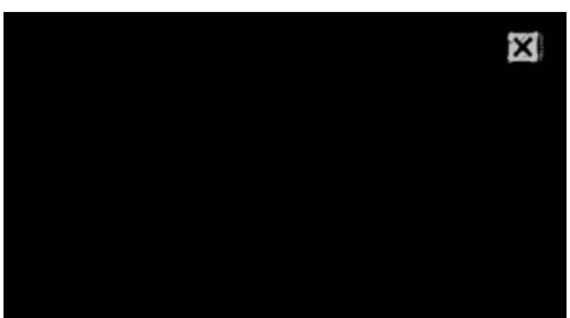
Dense Suspension Break-Up

- New class of break-up phenomena, intermediate between simple liquids and dry grains
- Interplay between break-up dynamics and particle arrangement on curved bounding surface
- Scaling but no self-similar structure; instead memory of initial conditions (nozzle size)
- Viscosity unimportant ➔ dense suspension not simply a very viscous liquid as often assumed
- Beyond single particle scale: break-up proceeds like pure liquid

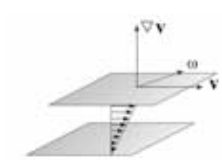
When and how to walk on water:
A new perspective on shear thickening in suspensions

National Geographic
http://www.youtube.com/watch?v=45yabrnryXk&feature=fvw

BBC 1
http://www.youtube.com/watch?v=AQ6OtKywyD8&feature=related



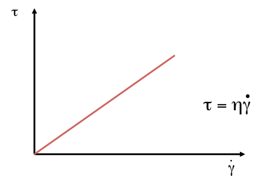
Liquid Mountaineering
http://www.youtube.com/watch?v=Oe3St1GgoHQ



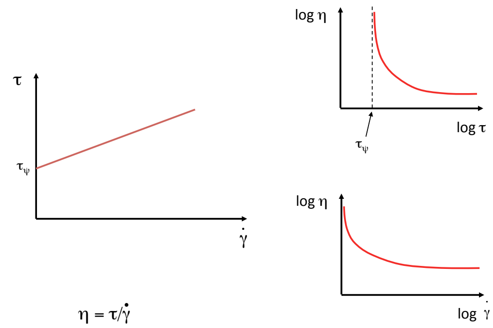
shear stress — shear rate

$$\tau = \eta(dv/dz) = \eta\dot{\gamma}$$

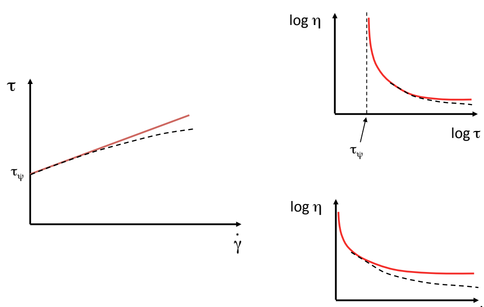
viscosity



Newtonian: viscosity = const.



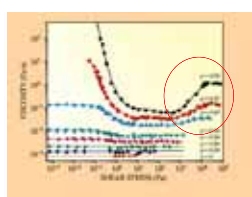
$\eta = \tau/\dot{\gamma}$



$\eta = \tau/\dot{\gamma}$

- - - $\tau = \tau_\psi + \tau_1\dot{\gamma}^{1/2}$

Norman J. Wagner & John F. Brady, Physics Today 2009



Viscosity of colloidal latex dispersions, as a function of applied shear stress.

Shear thickening in colloidal dispersions

"Shampoos, paints, cements, and soft body armor that stiffens under impact are just a few of the materials whose rheology is due to the change in viscosity that occurs when colloidal fluids experience shear stress."

BARNES 1989

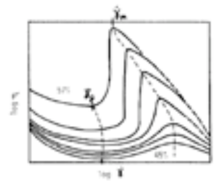
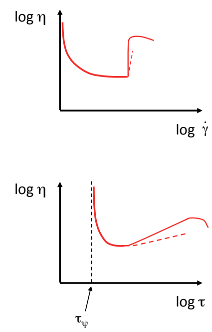


Fig. 1. Schematic representation of viscosity versus shear rate for shear-thickening systems, with approximate phase volume as parameter. Also shown are the loci of $\dot{\gamma}_c$ and $\dot{\gamma}_m$, the shear rates at the beginning and end of the shear-thickening region.



$\eta = \tau/\dot{\gamma}$

—— discontinuous

- - - - continuous

"the harder you push the more it resists"

http://www.youtube.com/watch?v=f2XQ97XHjVw

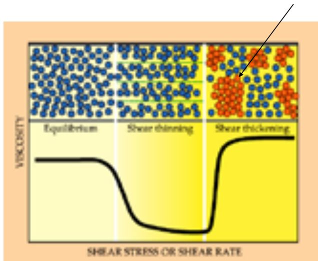
Shear-Thickening ("Dilatancy") in Suspensions of Nonaggregating Solid Particles Dispersed in Newtonian Liquids

H. A. BARNES, *Unilever Research, Port Sunlight Laboratory, Bebington, Wirral, Merseyside, England L63 3JW*

> We shall find that so many kinds of suspensions show shear thickening that one is soon forced to the conclusion that given the right circumstances, *all* suspensions of solid particles will show the phenomenon.

Journal of Rheology, 33(2), 329–366 (1989) CCC 0148-6055/89/020329-38$04.00

hydroclusters



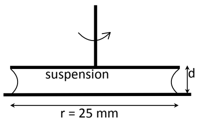
Norman J. Wagner & John F. Brady, Physics Today 2009

Eric Brown

- What determines the range of observable shear thickening?
  OR: Why don't all suspensions shear thicken?
- What is the mechanism for discontinuous shear thickening?

See http://arxiv.org/abs/1010.4921

Rheometry



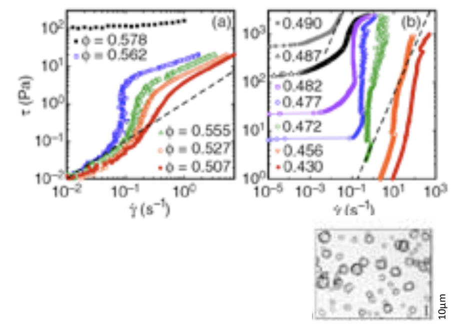
constant applied torque T
measured rotation rate ω

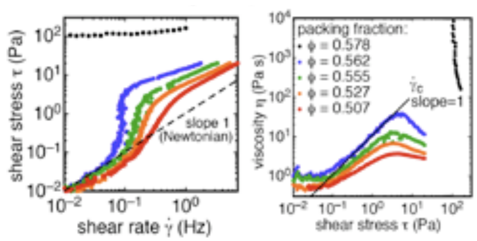
shear stress $\tau = T/\pi r^3$
shear rate $\dot{\gamma} = \omega r/d$
viscosity $\eta = \tau/\dot{\gamma}$ in steady state

suspensions:
6μm - 1000μm glass spheres in various liquids
14 μm cornstarch in water w/ CsCl (density matched)
125 μm polyethylene spheres in silicone oil (density matched)
150 μm ZrO_2 spheres in mineral oil
PEG (various shapes) in liquid PEG

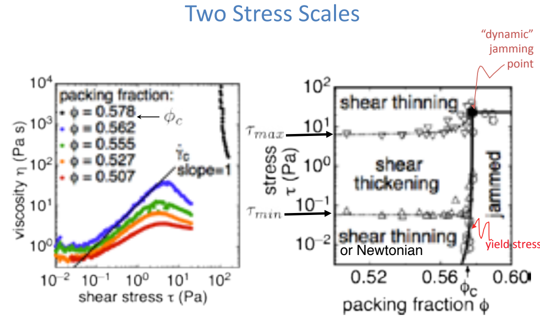
100 μm glass spheres in mineral oil

Cornstarch in water





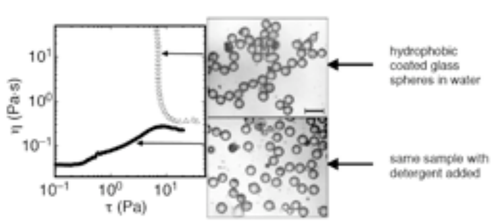
Two Stress Scales



100 μm glass spheres in mineral oil

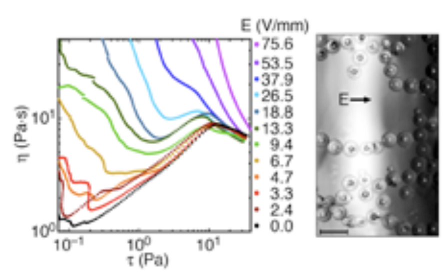
Brown & Jaeger, PRL 2009

Onset Stress set by Particle Attractions



attractions → clustering → yield stress → no shear thickening

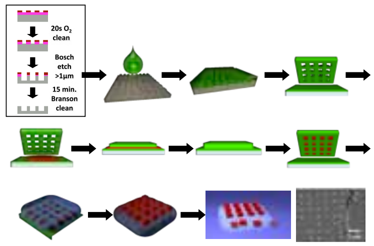
Tunable Yield Stress: E or B Field



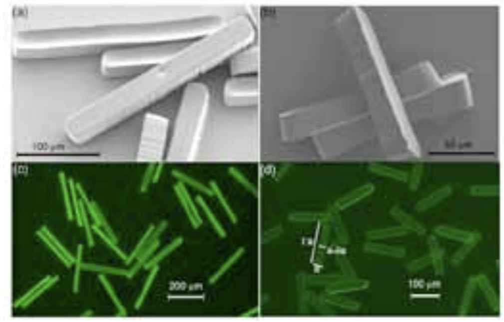
Thinning stress (from yielding) simply additive to thickening stress

Joe DeSimone (UNC), Liquidia

Particle Replication in Non-wetting Templates (PRINT Platform)

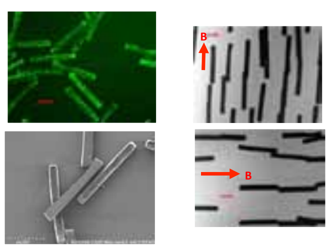


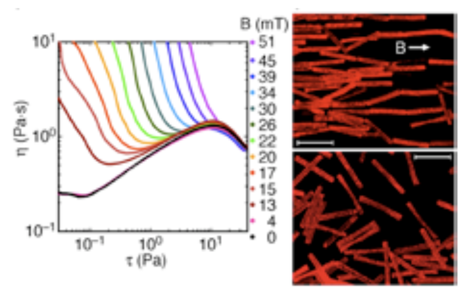
"Direct Fabrication and Harvesting of Monodisperse, Shape Specific Nano-Biomaterials"; Rolland, J. P.; Maynor, B. W.; Euliss, L. E.; Exner, A. E.; Denison, G. M.; DeSimone, J. M *J. Am. Chem. Soc.* 2005, 127, 10096



Magnetic High Aspect Ratio Rods

- 20 x 20 x 240 μm composite PEG rods loaded with 30 wt.% magnetite

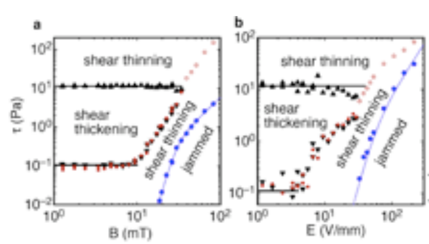




B = 30 mT

B = 0 mT

- Aspect ratio 9 rods
- Magnetite-doped (30% by weight), PEG rods made by PRINT suspended in PEG at a packing fraction ϕ = 0.20
- Shear thickening region shrinks and is eventually eliminated as it is encroached on by the increasing yield stress



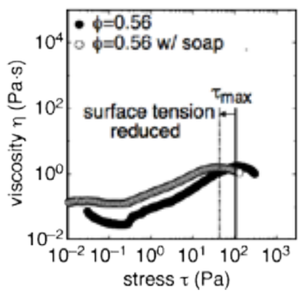
Brown et al., Nature Materials 2010

Shear thickening onset

- Controlled by yield stress, which in turn can arise from a variety of sources: local confinement @ high concentrations, attractive interactions (chemistry, E-field, B-field)
- Predicted by simple addition of thinning and thickening stresses (at least in limit of discontinuous thickening), independent of microstructural details

Where does upper stress scale come from?

100 µm glass spheres in water



Suspensions that shear-thicken tend to dilate

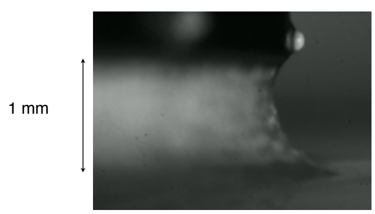
Metzner & Whitlock, Trans. Soc. Rheol. 1958
Lootens et al. PRL 2003
Fall et al. PRL 2008

layer of cornstarch (14µm dia.) in water, $\phi = \phi_c - 0.02$
sheared from upper right side (movie = real time)

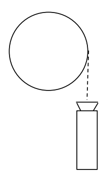
Dilation causes change in surface texture

150 µm ZrO_2 in mineral oil

side view



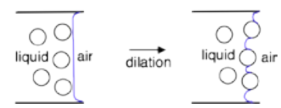
top view



3Hz rotation, movie playback at 0.33x

ZrO₂ videos taken with help from Marc Miskin

Mechanism for dynamic stress increase: dilation against liquid-air interface

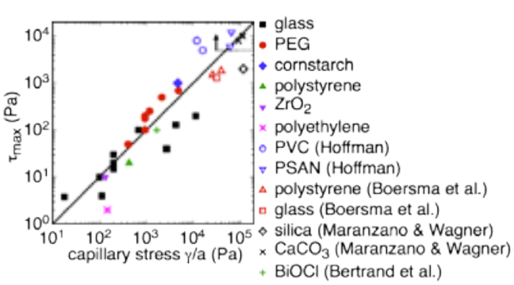


confining stress $\tau \approx \frac{\gamma}{r} \sim \frac{\gamma}{a}$
γ = surface tension
a = particle diameter

confining stress converted to shear, normal stress via friction

See also Holmes et al., J. Rheol. (2005)

Maximum stress of shear thickening regime scales as confining stress from surface tension γ/a

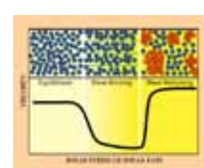


Granular Picture

Particles-particle interactions give yield stress
Thinning from yielding can mask ST
ST from "frustrated dilation" by confinement
Upper limit when confinement is overcome

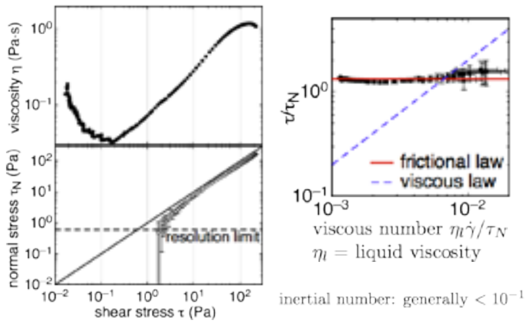
Hydrocluster Picture

Liquid-mediated hydrodynamic interactions
Peclet number determines ST onset



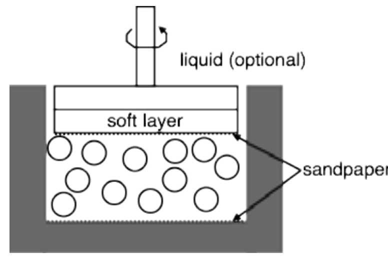
Stresses scale according to frictional law

100 μm glass spheres in water $\phi = \phi_c - 0.02$



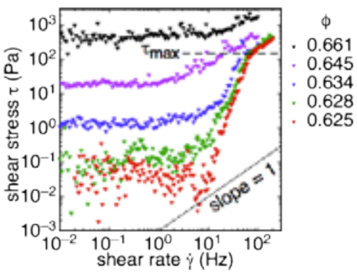
inertial number: generally $< 10^{-1}$

Generalize boundary effect: solid walls



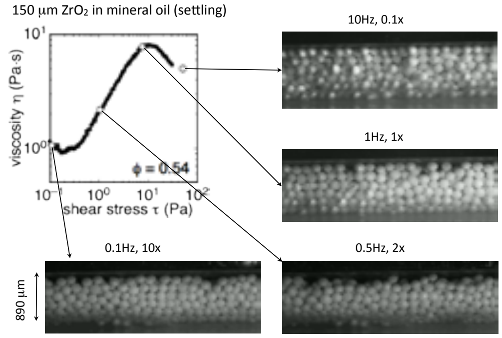
Discontinuous Shear Thickening in dry grains!

600 μm glass spheres (no liquid) with solid walls

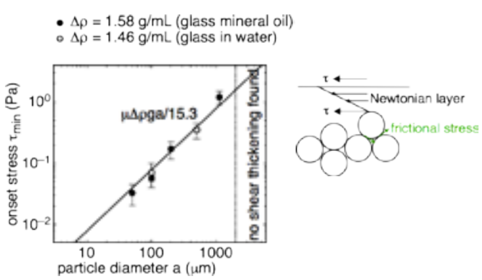


Shear thickening onsets when grains start to shear

150 μm ZrO_2 in mineral oil (settling)



Onset of shear thickening for large particles: when gravitational stress is exceeded



calculation: Goldman et al. Chem. Eng. Sci. (1967)

When to walk on water…

Use suspension with particle size that gives largest shear thickening range

Smaller size gives better confinement γ/a as well as lower onset for particle diam. > 10μm

Use colloids?

Shear thickening starts when shear stress exceeds stress scale of particle interactions

- Brownian colloids (osmotic pressure):
  (Farr et al. 1997, Bergenholtz et al. 2002, Maranzano & Wagner 2002)

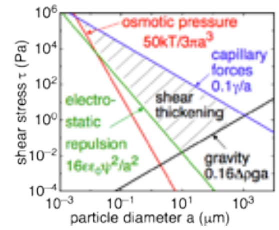
$$\tau_{min} = \frac{50kT}{3\pi a^3}$$

- Charge-stabilized colloids (electrostatic potential ψ):
  (Hoffman 1982, Maranzano & Wagner 2001)

$$\tau_{min} = \frac{16\epsilon\epsilon_0\psi^2}{a^2}$$

- Induced dipole attractions from applied electric field E
  (Brown et al. 2010)

$$\tau_{min} = 12\pi\epsilon_0\epsilon\beta^2 E^2$$



cornstarch

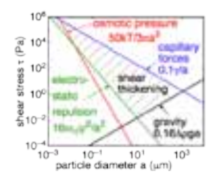
See paper by Eric Brown on www.arXiv.org

How and when to walk on water…

How: fast … to generate large stresses

When: after adding 40+ percent of ~10μm particles

To sum up shear thickening:



- Conditions for Discontinuous Shear Thickening:
  - dilation under shear
  - confining stress in response to dilation
  - dominance of confining stress
  - particle interactions small so that yield stress well below upper limit due to confinement
- Granular picture can explain phase diagram & many of the details (frictional law for stresses, dilation, shear banding,…)
- Outstanding issues: Transient response? Compressive vs. shear vs. tensile stresses?