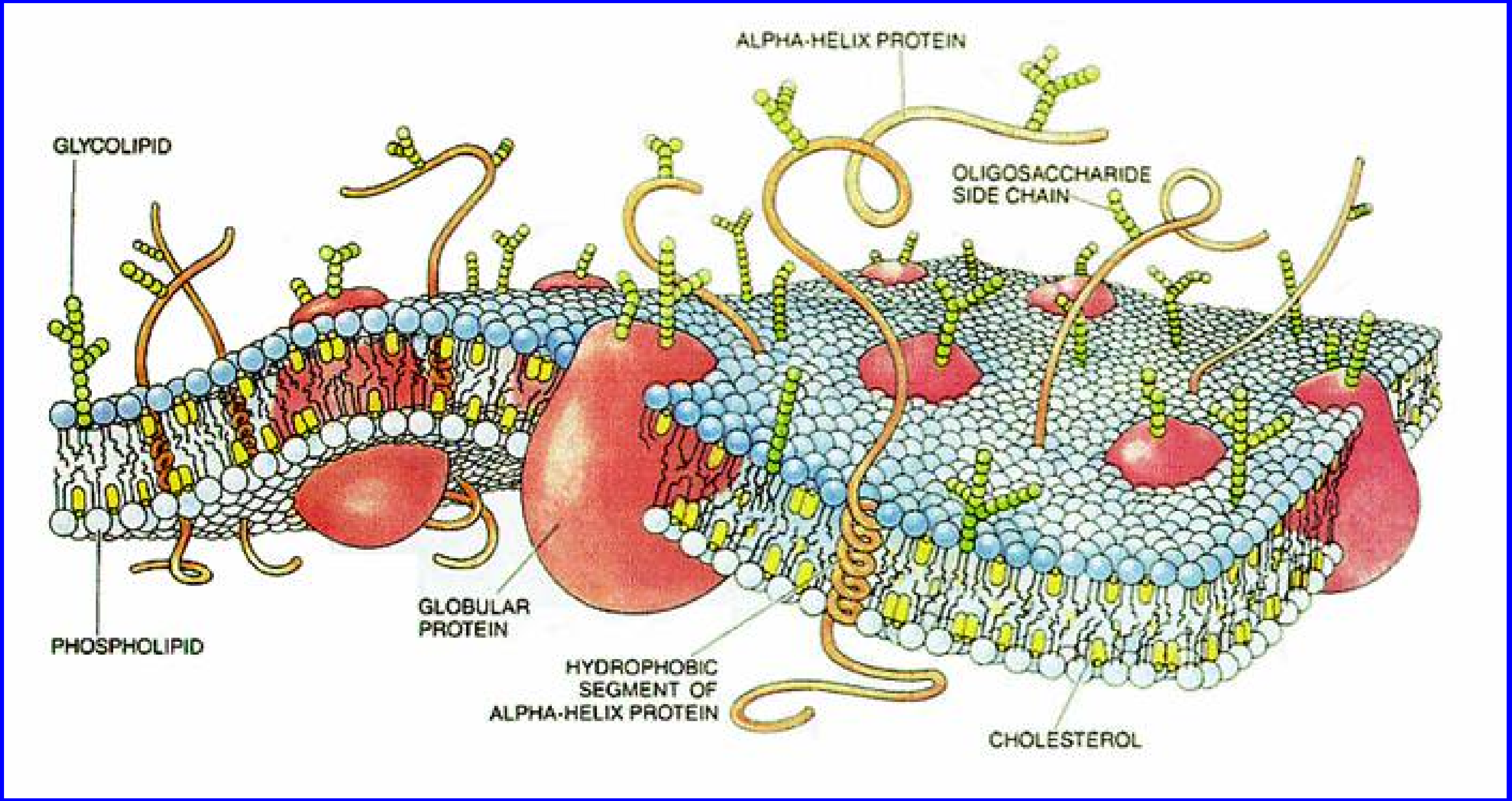
Physics of membranes V

- Mixed membranes w/ internal freedom
- Curvature-composition coupling
- Membrane rafts
- Inclusions in membranes: proteins

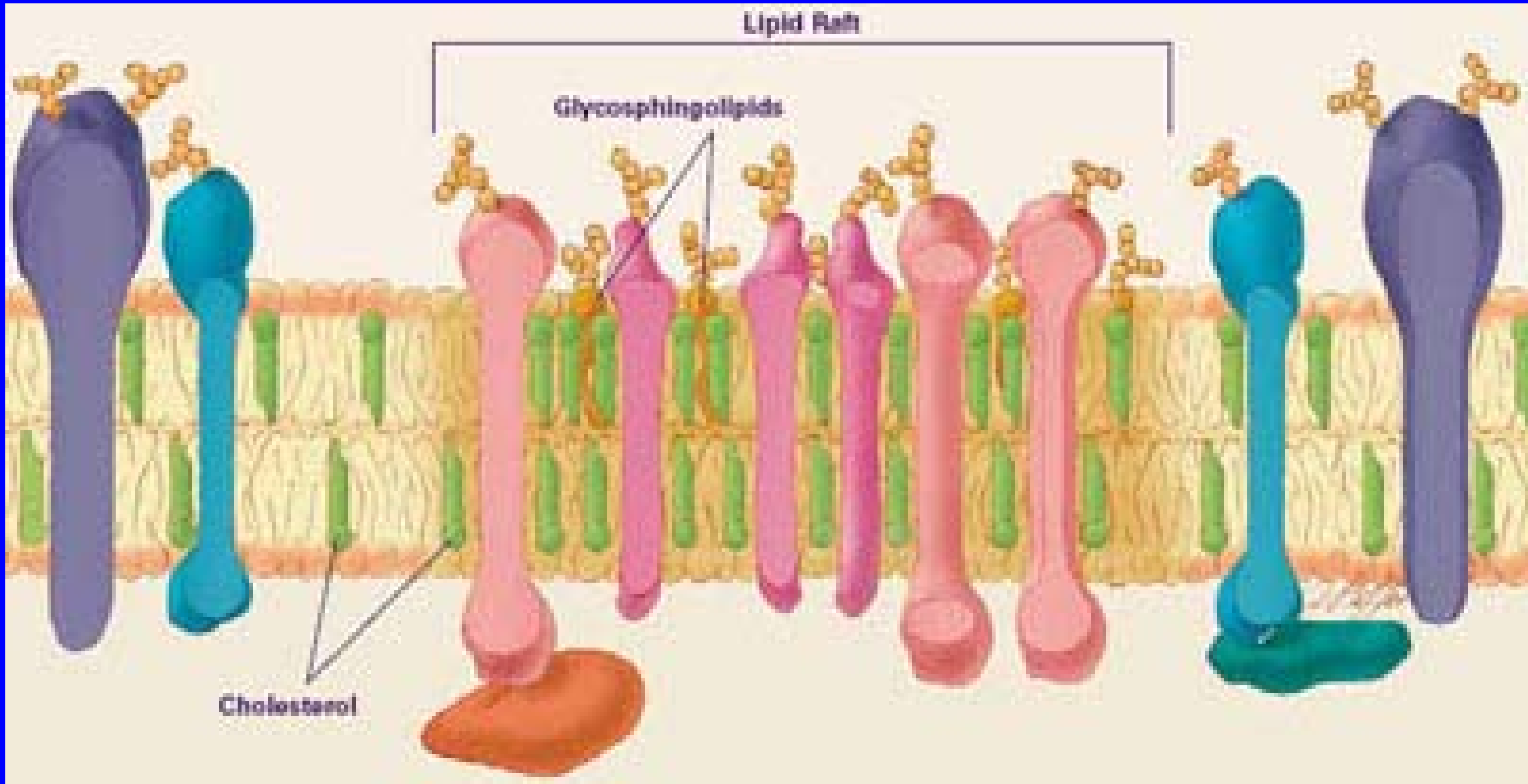
Refs. Papers listed on slides.

Realistic Membranes



http://science.nasa.gov/headlines/y2004/images/microbetrap/fig1_portal.jpg

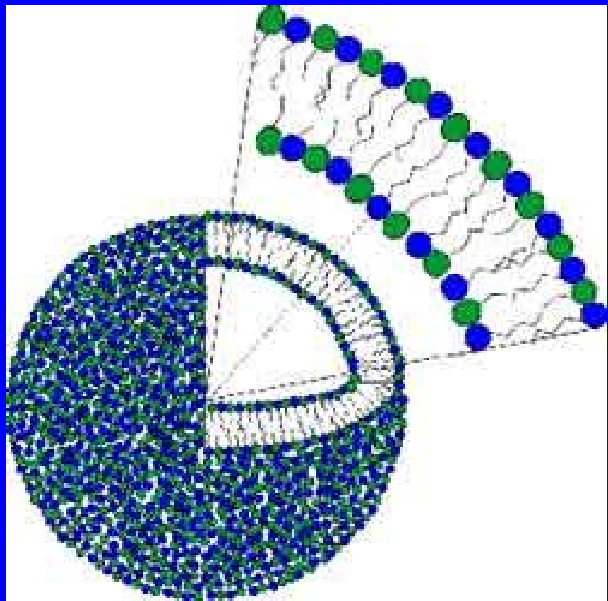
Lipid rafts in cells



- Rafts are often sites for membrane activity and interactions

http://publications.nigms.nih.gov/insidethecell/chapter2.html

Mixed Vesicles



Kaler group, U. Del.

Composition and curvature

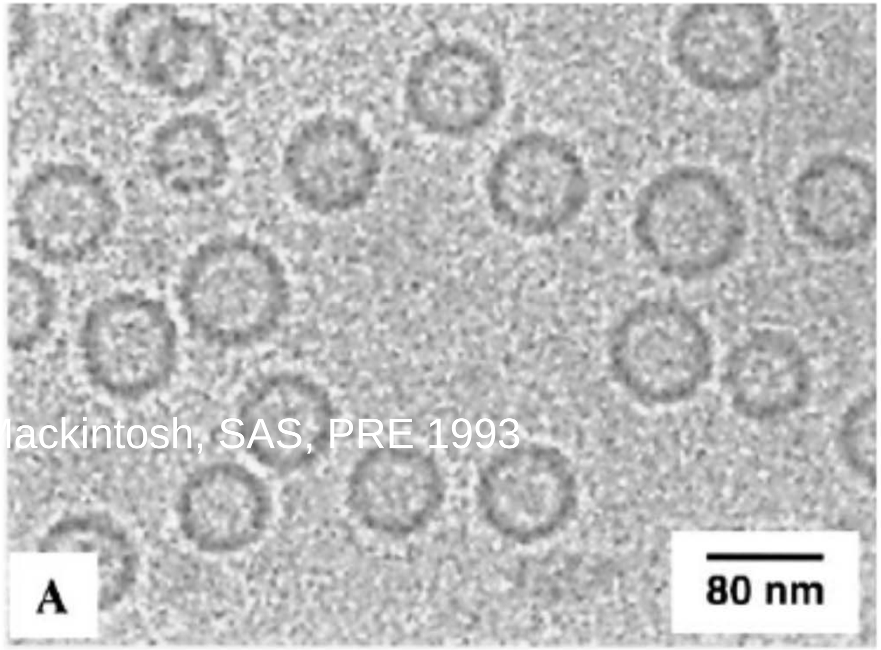
- Mixed amphiphilic layers
- Two species phase separate below critical temperature
- Flat: lamellar phase with layers with different compositions
- Curved: vesicles – inner and outer layers different compositions
- Phase separation can stabilize vesicles
  - Amphiphile "A" has positive spontaneous curvature
  - Amphiphile "B" has negative spontaneous curvature
  - Separation can be promoted by chemical interactions
  - Separation can also be promoted by dependence of spontaneous curvature of mixture on AB interactions

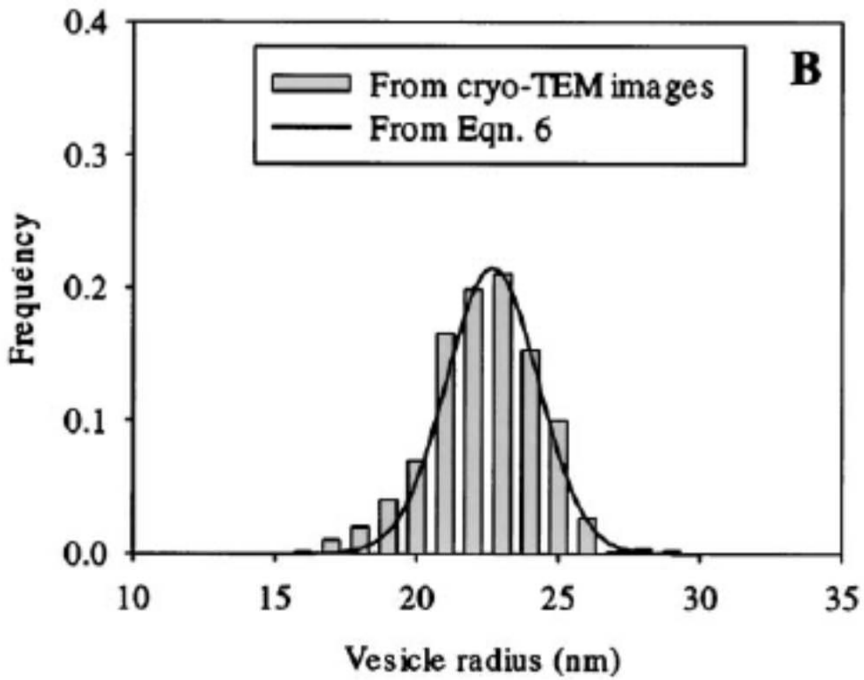
F. Mackintosh et al. PRE 1991, 1993; SAS et at. Science, 1990; Kaler /Zaszadisnky Science 1989

Spontaneous vesicles in surfactant mixtures

The origins of stability of spontaneous vesicles
H. T. Jung*, B. Coldren*, J. A. Zasadzinski*†, D. J. Iampietro‡, and E. W. Kaler; PNAS (2001)

Theory: Mackintosh, SAS, PRE 1993; SAS, Andelman, Pincus, Sicence 1990





Curvature-composition coupling in modulated membranes

$$H\{\ell, \phi\} = \int \mathrm{d}^2x \left[\frac{\sigma}{2}(\nabla \ell)^2 + \frac{\kappa}{2}(\nabla^2 \ell)^2 + \frac{b}{2}(\nabla \phi)^2 + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 - \mu\phi + \Lambda(\nabla^2 \ell)\phi\right]$$

Membrane local height: ℓ Local composition difference: ϕ

Surface tension: σ Line tension; b Bending modulus: κ

Integrate out membrane degrees of freedom in Fourier space, keeping up to 4th order in q

Leibler, Andelman:J. Phys. 1987;
Komura, Andelman, Langmuir 2006.

Concentration-height instabilities

$$H_{\text{eff}}\{\phi\} \approx \int d^2x \left[\frac{B}{2}(\nabla\phi)^2 + \frac{C}{2}(\nabla^2\phi)^2 + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 - \mu\phi\right]$$

$$B \equiv b - \frac{\Lambda^2}{\sigma} \qquad C \equiv \frac{\Lambda^2\kappa}{\sigma^2}$$

Can be negative

Most unstable wavevector: $q^* = (-B/2C)^{1/2}$

Stripe phase: $\phi_S = \phi_0 + \phi_{q^*}\cos(q^*x)$

Hexagonal phase: $\phi_H = \phi_0 + \sum_{i=1}^{3}\phi_{q^*}\cos(\vec{q}_i\cdot\vec{r})$

$|\vec{q}_i| = q^*$ and $\sum_{i=1}^{3}\vec{q}_i = 0$

Phase Diagram

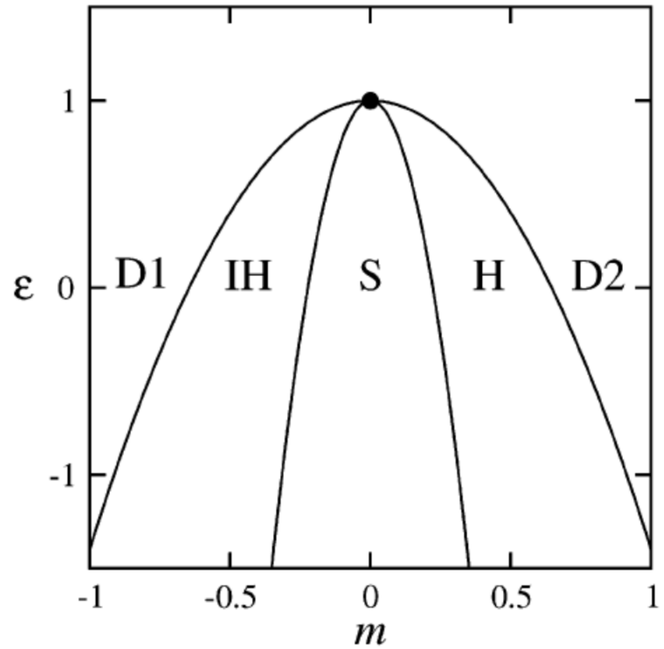


Figure 2. Mean-field phase diagram in the (m, ϵ) plane. m represents the reduced composition, and ϵ represents the reduced temperature as defined in eq 6. There are four different phases: the disordered phase (denoted D1 and D2 on the two sides of the transition lines), the striped phase (S), the hexagonal phase (H), and the inverted hexagonal phase (IH). These phases are separated by the first-order transition lines. For simplicity, we show here by solid lines only the crossover in the free energies while avoiding plotting two-phase

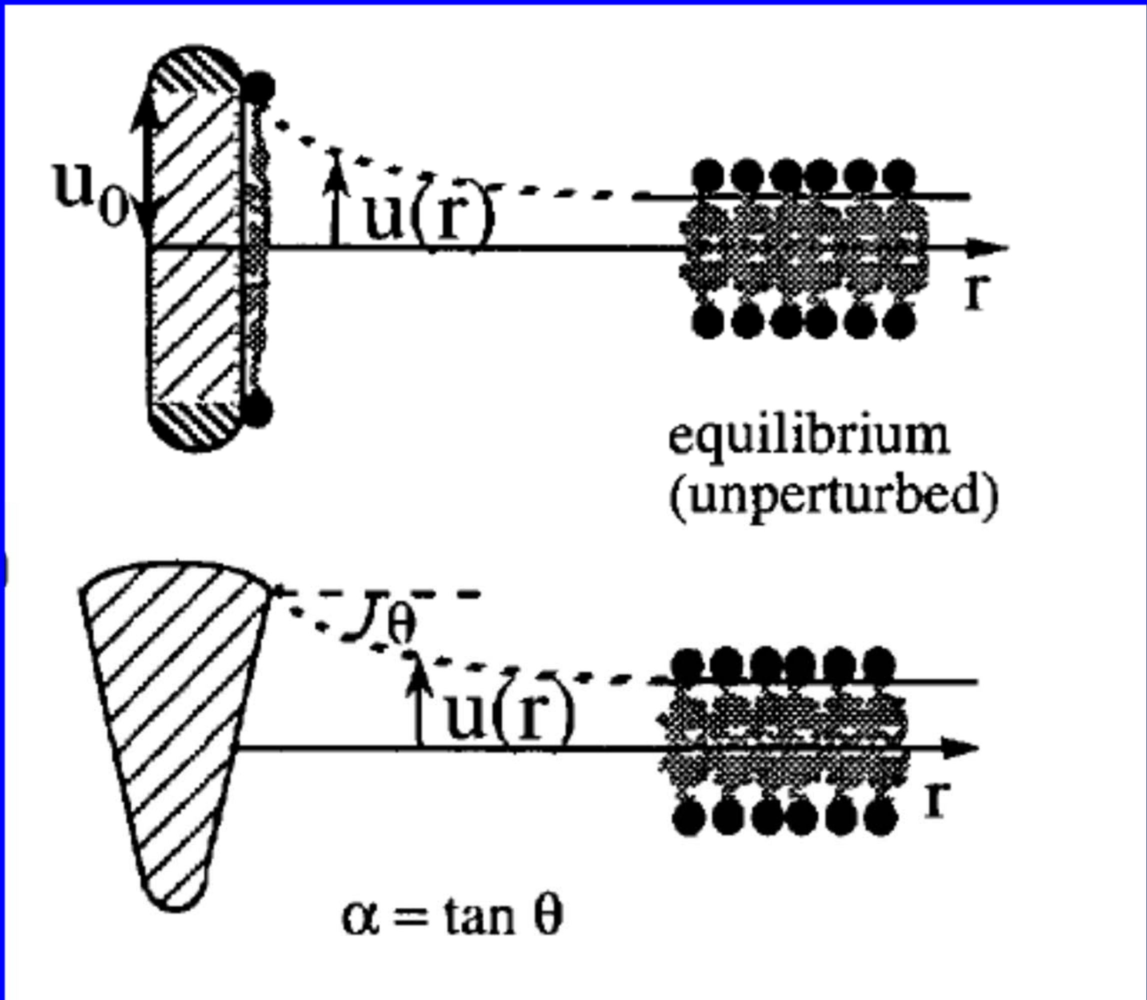
Leibler, Andelman:J. Phys. 1987;
Expts: Keller and McConnel, PRE 1995.

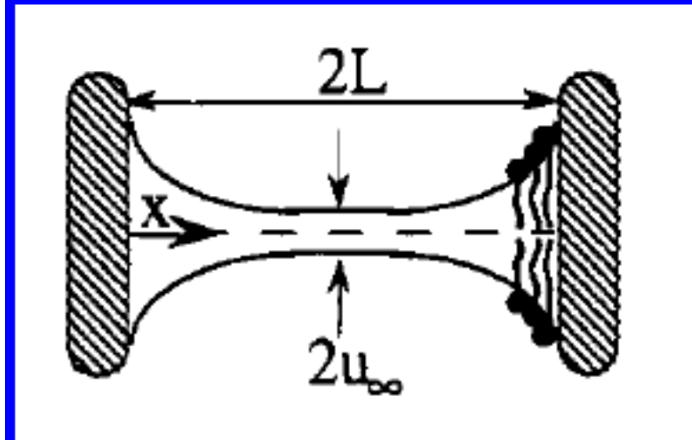
Inclusions in membranes

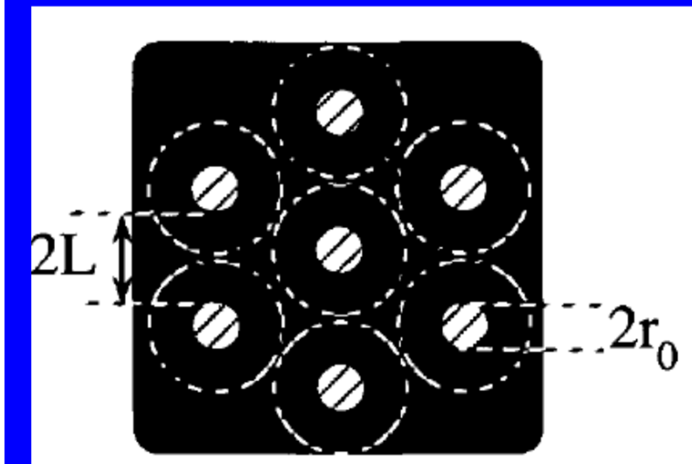
- Proteins, colloidal particles
- Membrane deformations
- (Dan et al. Langmuir 1993, J. Phys. 1994)
- Reduce fluctuation entropy

  (Goulian et al. Europhys Lett. 1993)
- Attractive interactions – exponential, power law
- Membrane pressure on protein
  - Affects protein function
  - (Cantor, Biochemistry 1997; Dan, SAS, Biophys. J. 1998)

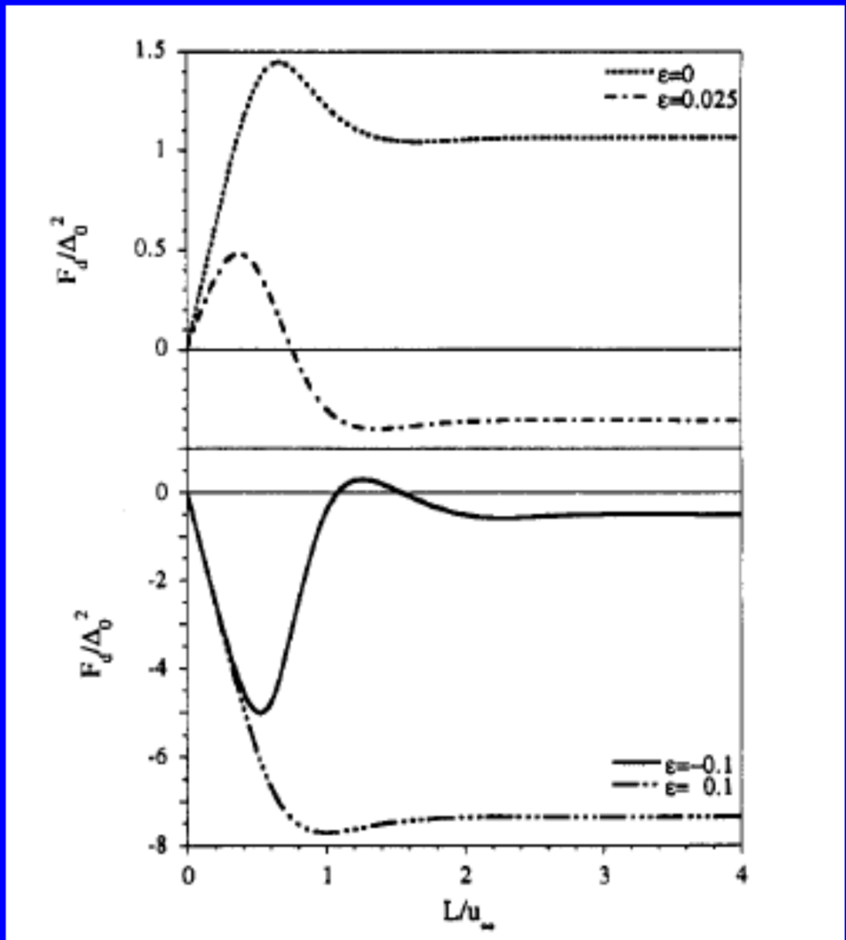
Inclusions induce membrane curvature







Interaction energy



Can be repulsive or attractive depending on inculsion boundary condition and membrane spontaneous curvature

Dan et al. Langmuir 1993, J. Phys. 1994

Experimental implications

- Changing monolayer spontaneous curvature can favor protein aggregation or repulsion

KELLER SL, BEZRUKOV SM, GRUNER SM, et al.

PROBABILITY OF ALAMETHICIN CONDUCTANCE STATES VARIES WITH NONLAMELLAR TENDENCY OF BILAYER PHOSPHOLIPIDS

BIOPHYSICAL JOURNAL 65 (1): 23-27 JUL 1993

Suppression of fluctuations induces attraction between rigid inclusions

GOULIAN M, BRUINSMA R, PINCUS P
Long-Range force in heterogeneous fluid membranes
EUROPHYSICS LETTERS 22 (2): 145-150 APR 10 1993

Park and Lubensky, J. Phys. I , 1996

- Inclusions locally modify curvature moduli (more rigid)
- This can reduce fluctuations and lead to attractions
- Repulsion or attraction depends on details of moduli
- Simple calculation: constraint of zero curvature at location of each inclusion
- Scaling argument: proportional to area, a^2, of each inclusion
  - Free energy per inclusion
  - Suppression of thermal fluctuations: cost $k_B T$

$$-k_B T \left(\frac{a}{L} \right)^4$$

1.1 Interactions between inclusions in a membrane

This problem applies to a membrane whose fluctuations can be described by a height function, $h(x, y)$ in the Monge gauge where the membrane height is a function of its (x,y) position and in which there are no overhangs. Consider the case in which there are two inclusions (as models of proteins in a membrane), separated by a distance $\vec{R}$ that are very rigid so that at the inclusions, the membrane is flat. That is, at the location of each inclusion we have a constraint that the curvatures in the x and y directions of the membrane height function are each (separately) zero. The rest of the membrane is described by a bending energy of the usual type with bending modulus k with no spontaneous curvature; for this problem, you can also ignore the Gaussian curvature. We would like to know by how much the free energy of this membrane is reduced due to the constraints induced by the inclusions, compared to a freely fluctuating membrane with no inclusions. This will tell us the interaction free energy of the two inclusions.

(a) Why is the free energy of the fluctuating membrane increased (as opposed to decreased) by the action of the inclusions?

(b) Why is their effect not just a local effect at the points at which the inclusions are located.

(c) Derive an expression for the partition function of the membrane with the inclusions, by including these point like constraints on h_{xx} and h_{yy} at the location of each inclusion. In any integrations that diverge at $q = 0$, you can use a lower cutoff related to the membrane persistence length, since the Monge gauge is not valid for larger length scales. If integrations diverge at large q use a cutoff of a molecular size, a, (presumably the same size as the inclusion) by inserting a factor of $\exp[-qa]$ in the integrals. Use the relationship

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, e^{i\omega t}$$

to represent all the constraints (2 constraints at each point). Using a Fourier representation and treating the real and imaginary parts carefully, evaluate the partition function.

(d) What is the free energy difference between the membrane with inclusions and one that has two inclusions separated by an infinite distance? What is interesting about this expression and why?