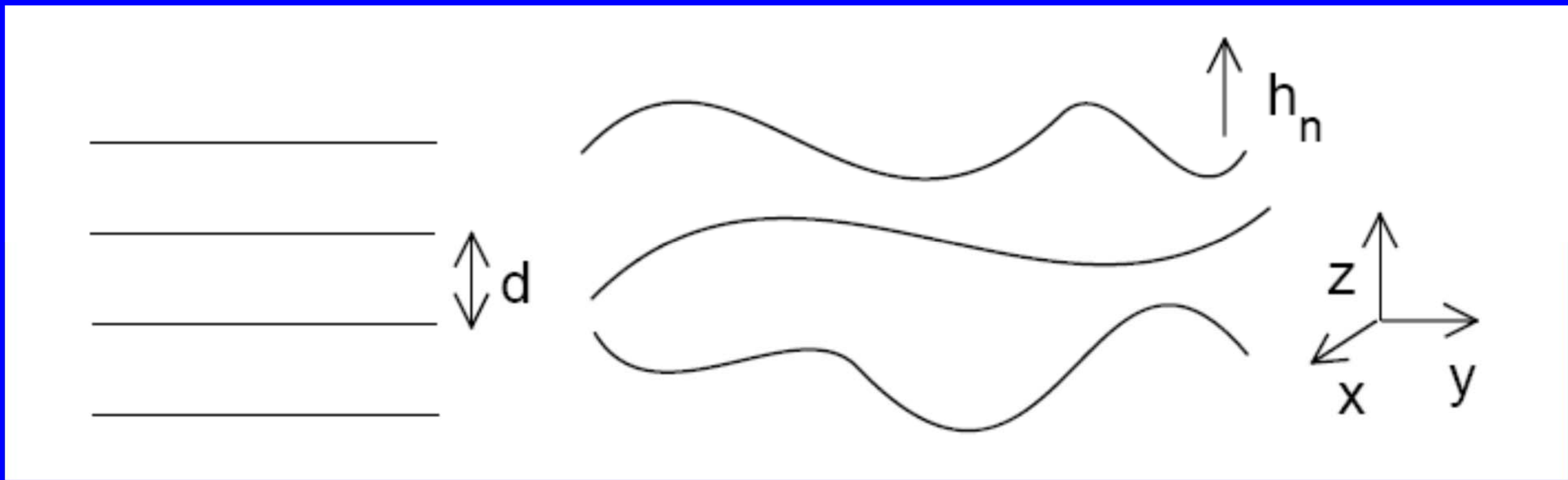
Physics of membranes IV

- Excluded volume – Helfrich interactions
- Attractive interactions and fluctuations
- Unbinding transition
- Pinched membranes

S. A. Safran, Statistical thermodynamics of surfaces, interfaces, membranes (Westview Press)
S. A. Safran, Safran SA. (**1999**) Curvature Elasticity of Thin Films. *Advances in Physics*, **48**:395-448.

Fluctuating, interacting membranes



$$Z_n = nd + h_n(x, y)$$

$$Z_{n+1} - Z_n - d = h_{n+1} - h_n$$

Effective Hamiltonian

$$\mathcal{H} = \int dx\, dy\, u(x,y)$$

$$u = \tfrac{1}{2} B \sum_n (h_n - h_{n+1})^2 + \tfrac{1}{2} k \sum_n \left(h_{n_{xx}} + h_{n_{yy}}\right)^2$$

$$\mathcal{H} = \sum_{\vec{q},Q} |h(\vec{q},Q)|^2 \left[B(1-\cos Qd) + \tfrac{1}{2} k q^4\right]$$

Free energy

$$F = -T \log \left[\prod_{\vec{q},Q} \int dh(\vec{q},Q)\, e^{-\mathcal{H}/T} \right]$$

$$\Delta f = \frac{T}{2(2\pi)^3} \int d\vec{q}\, dQ\ \log \left[\frac{B(Q) + kq^4}{kq^4} \right]$$

$$B(Q) = 2B(1 - \cos Qd)$$

$$\Delta f = \frac{T}{16\pi} \int_{-\pi/d}^{\pi/d} dQ\, \sqrt{\frac{2B(1 - \cos Qd)}{k}}$$

free energy per unit volume !

Self consistent interaction

$$B = \frac{\partial^2(\Delta f d)}{\partial d^2}$$

B is defined from free energy per unit area

$$B = \frac{9T^2}{\pi^2 k}\frac{1}{d^4}$$

$$\Delta f_a = \Delta f\ d.$$

$$\Delta f_a = \frac{3T^2}{2\pi^2 k}\frac{1}{d^2}$$

Free energy per unit area: repulsive interactions
Stabilize lamellar phase when spacing gets too small

Scattering experiments

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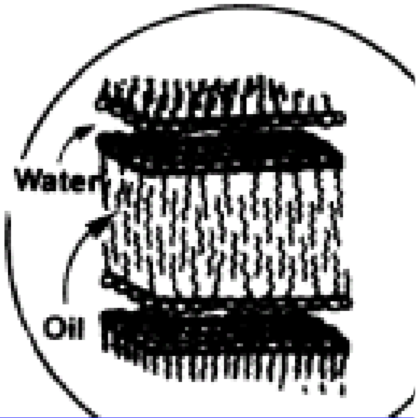
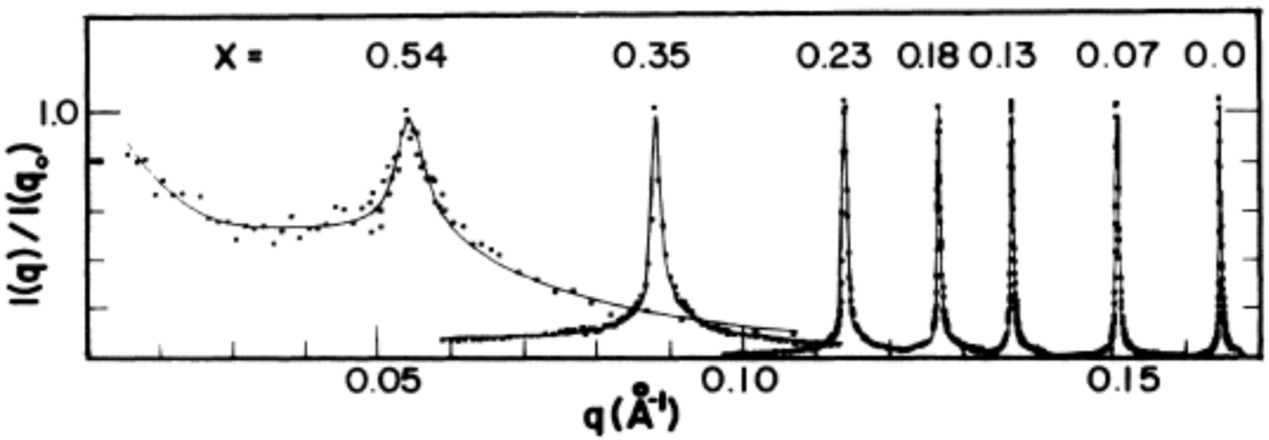
Steric Interactions in a Model Multimembrane System: A Synchrotron X-Ray Study

C. R. Safinya,[(1)] D. Roux,[(1),(3)] G. S. Smith,[(1),(2)] S. K. Sinha,[(1)] P. Dimon,[(1)] N. A. Clark,[(2)]
and A. M. Bellocq[(3)]





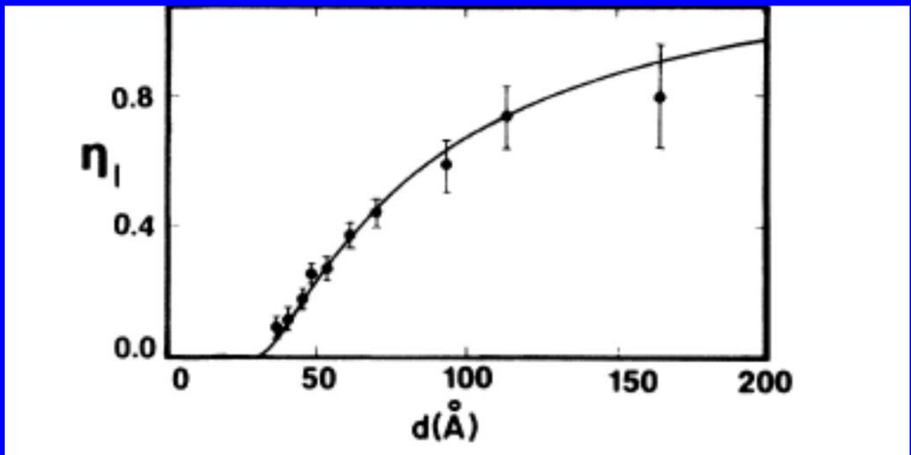
Lineshape exponent

$$S(0,0,q_z) \sim |q_z - q_m|^{-2+\eta_m}$$

$$\eta_m = \frac{m^2 q_0^{\,2} k_B T}{8\pi\sqrt{Bk}}$$

$$q_0 = 2\pi / d$$

$$B \sim \frac{T^2}{kd^4}$$



Supplementary material

Membrane interactions

- Long range attraction
  - Van der waals $1/r^6$ (fluctuations of dipoles)

The interaction energy per unit area between two membranes of thickness, $2d$, separated by a distance, $2D$ is proportional to d^2/D^4 when $d \ll D$ and is attractive. For small separations relative to the membrane thickness, $D \ll d$, the interaction decays as $1/D^2$. In this case,

- Short range repulsion: hydration $V_h = A_h e^{-d/\lambda}$

$$V_{tot} = A_h e^{-d/\lambda} - \frac{A\delta^2}{2\pi d^4}$$

Virial Expansion

- Two body virial term: ϕ^2, patch volume v

$$\chi = -\frac{1}{2v^2}\int d^3r\left[1-\exp(-V_{tot}/k_BT)\right]$$

- Helrich repulsion – cubic in density, ϕ^3
- Layer thickness δ, separation d, $\phi=\delta/d$

per unit volume

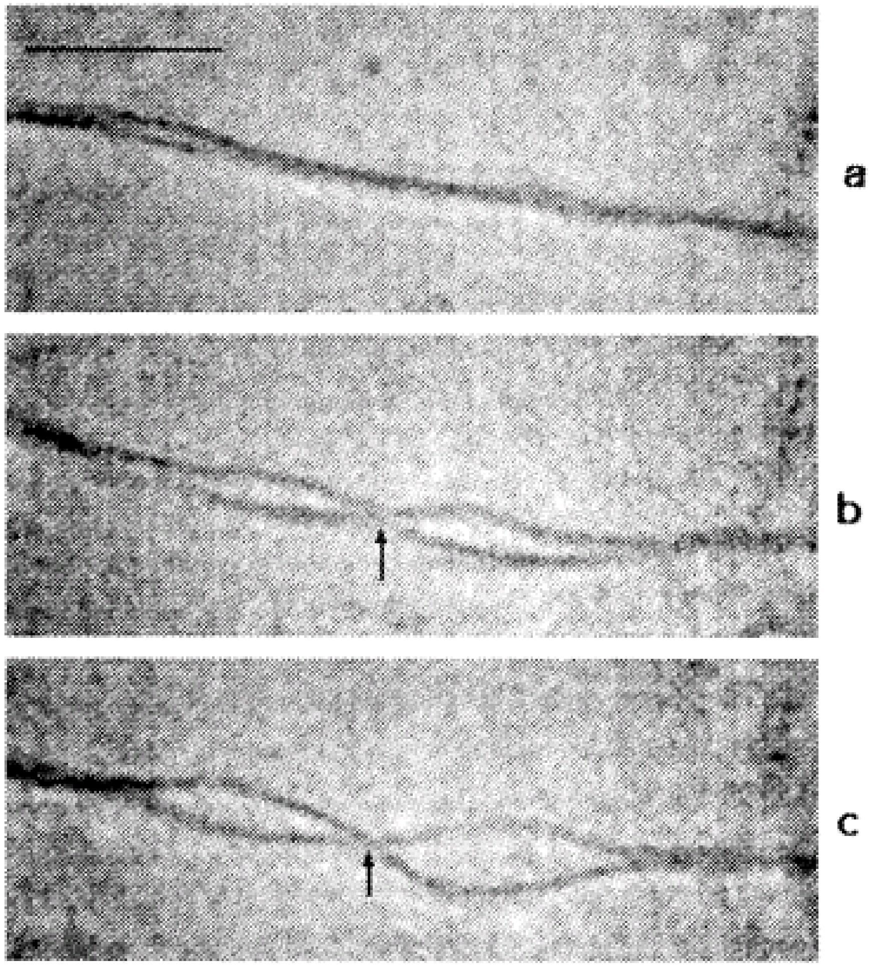
$$f(\phi) = \frac{3\pi^2T^2}{128K\delta^3}\phi^3 - T\chi\phi^2$$

$$\chi = A - A_c$$

Theory similar to polymer collapse: cubic virial term
Milner, Roux, J. Phys. 1992; Lipowsky, Leibler PRL, 1986

Pinched membranes

- Expt.: overshooting
  - Bar Ziv et al.: PRL, 1995

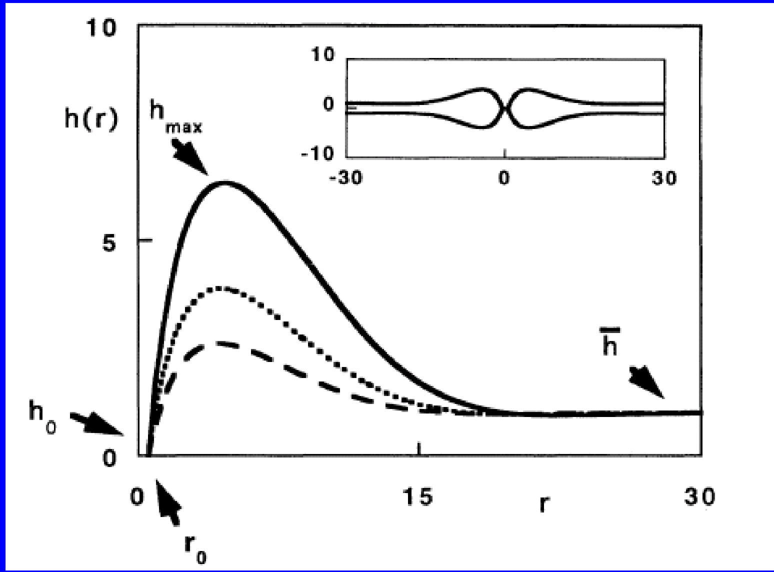


Pinching theory

$$F = \int \left(\frac{\kappa}{2} (\nabla^2 h)^2 + V(h) + Ph \right) dS$$

$$\kappa \nabla^4 h(r) + dV/dh + P = 0$$

$$V(h) = \frac{3\pi^2}{64} \frac{(k_B T)^2}{\kappa} \frac{1}{h^2}.$$



Menes, SAS: PRL 1995; PRE 1997

Pinching and adhesion

- Interactions of pinch sites – attraction
- Models for cell adhesion
  - Menes PRL, PRE 1995,1997; Zuckerman PRL 1995; Lipowsky PRL 1996
- Can Helfrich interaction be coarse grained?
- Cell adhesion experiments
  - Specific ligand binding (integrin)
  - Connection to cytoskeleton
  - Precursors: hyaluronic acid chains