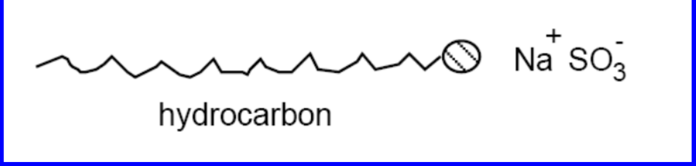
Physics of membranes II

- Membranes in materials and in biology
- Molecular interactions
- Curvature energy

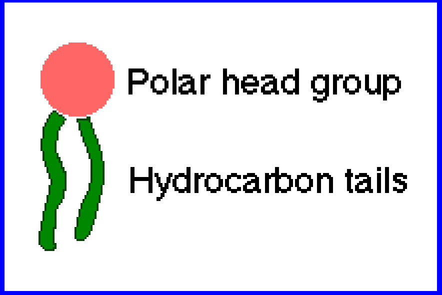
S. A. Safran, Statistical thermodynamics of surfaces, interfaces, membranes (Westview Press)
S. A. Safran, Safran SA. (**1999**) Curvature Elasticity of Thin Films. *Advances in Physics*, **48**:395-448.

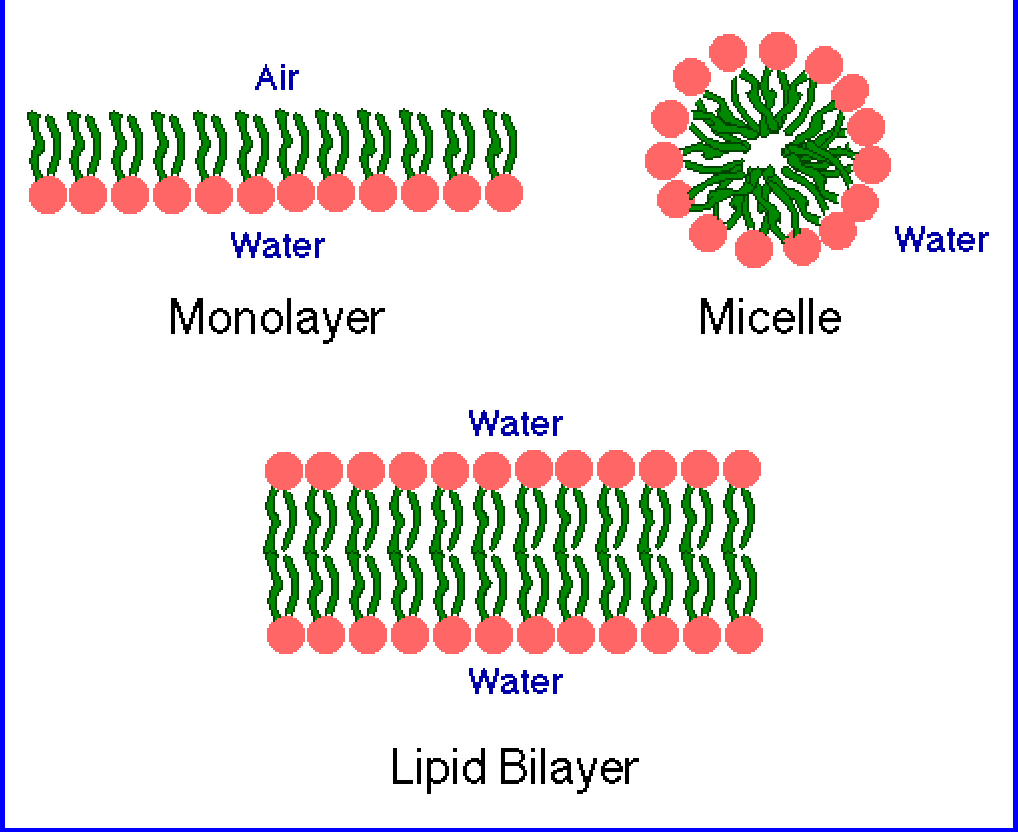
Amphiphiles and membranes

- Amphiphiles: soaps, lipids



- polar head (water)
- hydrocarbon tail (oil)



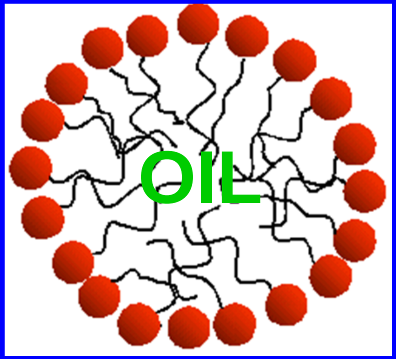


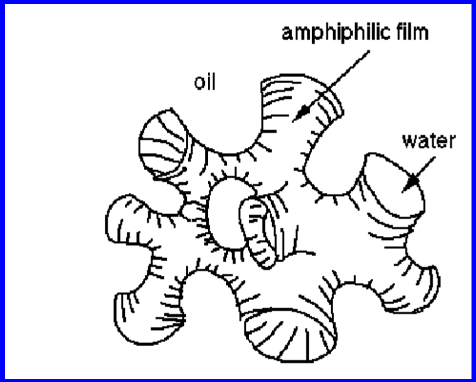
http://138.192.68.68/bio/Courses/biochem2/Membranes/Membranes.html

Microemulsions

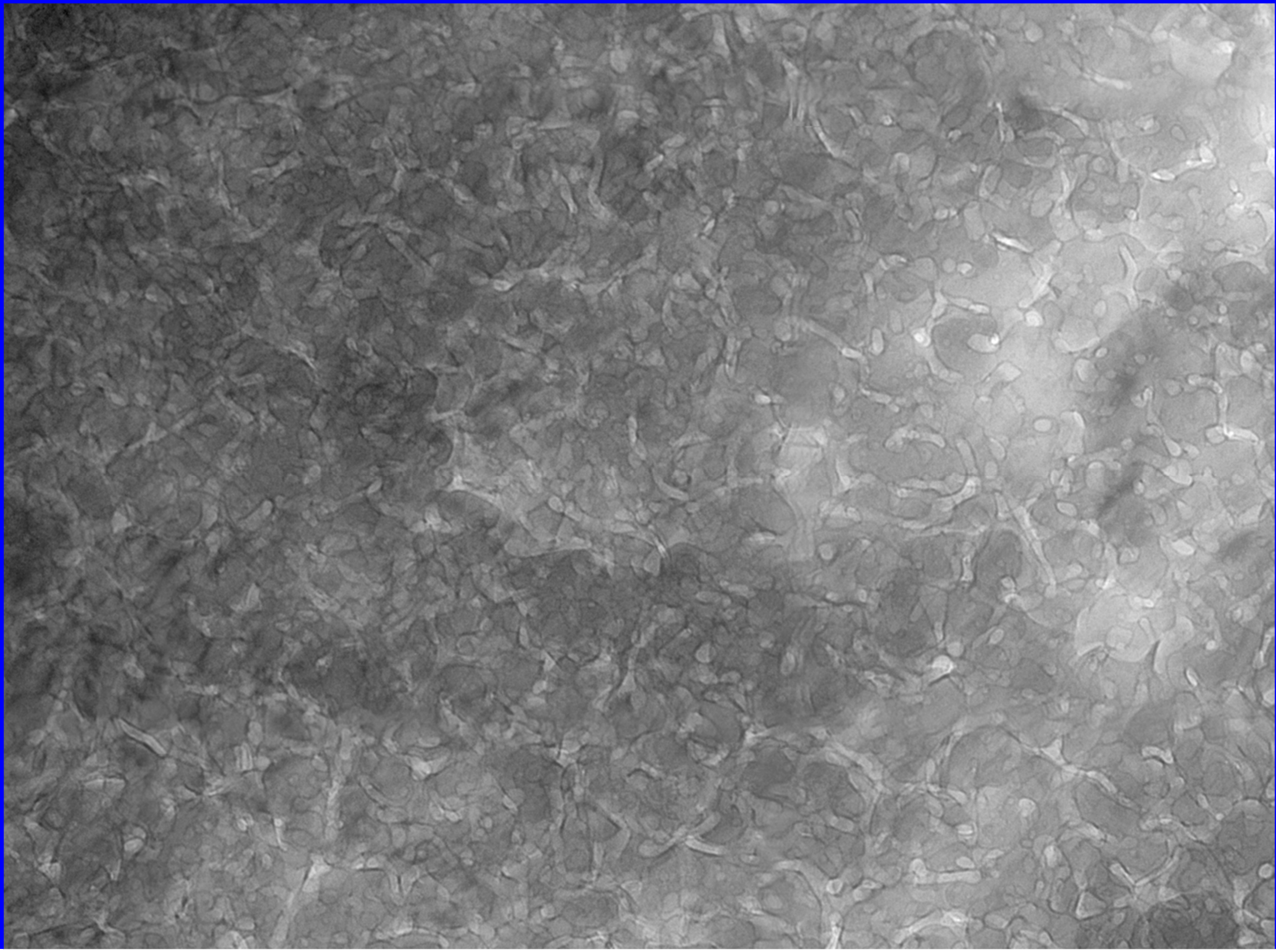
- Dispersion of oil in water or water in oil
  - amphiphile at interface
  - domain size >> molecular

- Used in cleaning, drug delivery



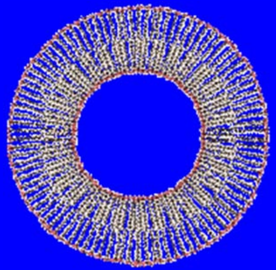


Microemulsion Network

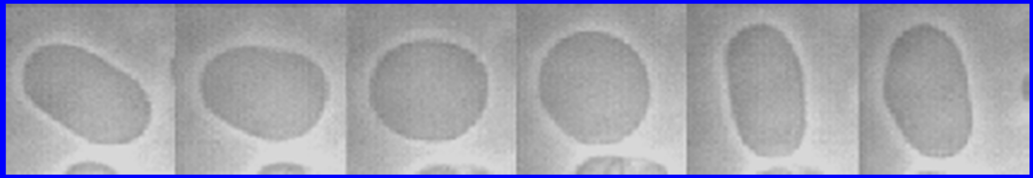


Strey group
(Koln)

Vesicles

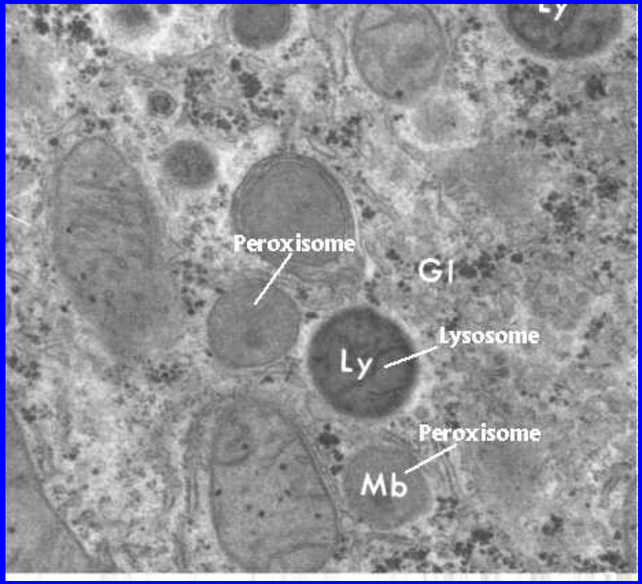


bilayer vesicle

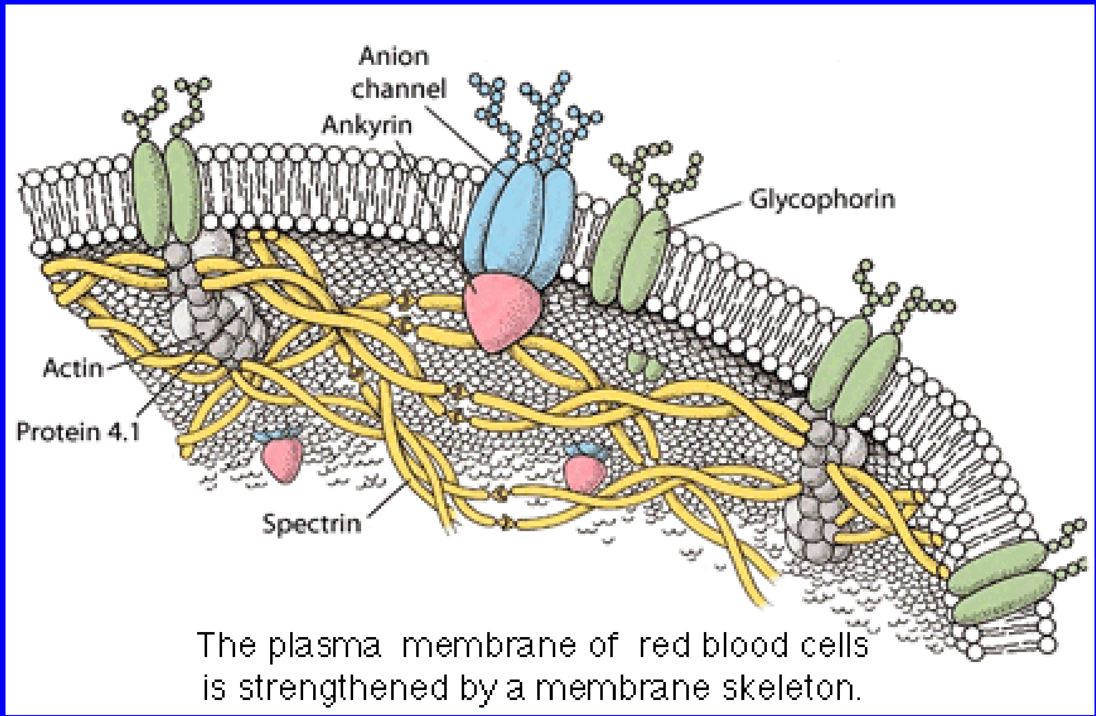


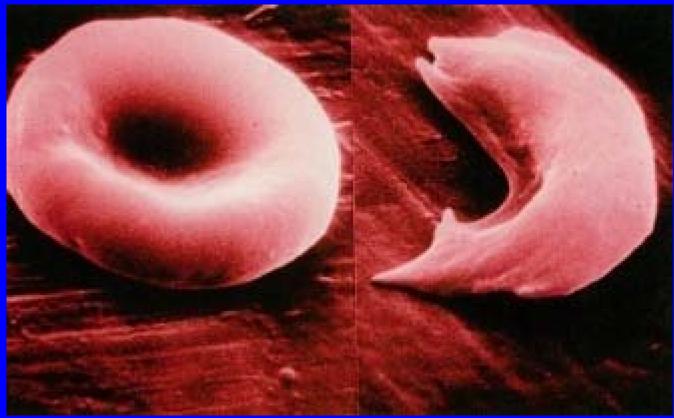
vesicle fluctuations
(mpi-golm)

- Cells: lysosomes
  - digestion
  - transport
- Smart nanocapsules



Red blood cell





http://www.humanillnesses.com/original/Se-Sy/Sickle-cell-Anemia.html

http://138.192.68.68/bio/Courses/biochem2/Membranes/Membranes.html

Curvature

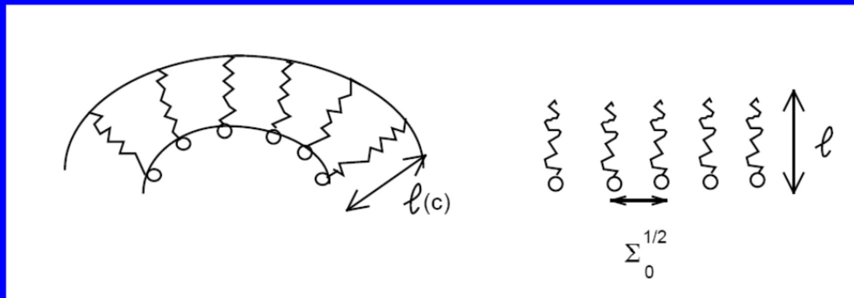
- Two orthogonal directions in which curvature of surface measured: $$\kappa_1 = \frac{1}{R_1}, \kappa_2 = \frac{1}{R_2}$$

- Mean curvature: $$H = \frac{\kappa_1 + \kappa_2}{2}$$

- Gaussian curvature: $$K = \kappa_1 \kappa_2$$

- Can show: these are invariants

Simple spring model I



$$\Sigma_0 \ell = v_0$$

$$f = \tfrac{1}{2} k_s (\ell - \ell_s)^2$$

$$k_s \sim 1/\ell_s \approx 1/\ell_0.$$

f should be extensive with spring length

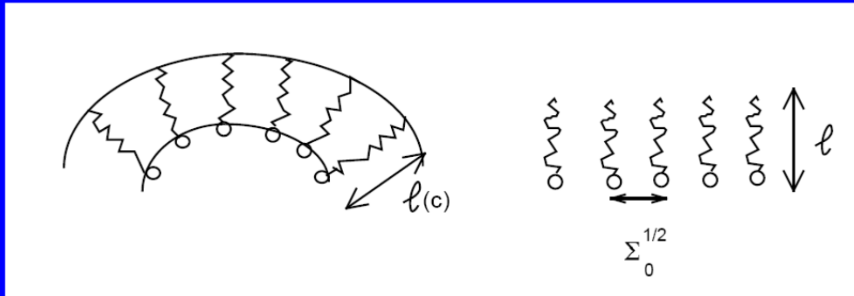
$$v_0 = \Sigma_0 \ell \left(1 + \ell H + \tfrac{1}{3}\ell^2 K\right)$$

Prove: find film volume for sphere and for cylinder

$$\ell = \ell_0 + \ell_1 H + \ell_2 H^2 + \ell_3 K$$

$$\ell_0 = v_0/\Sigma_0,\ \ell_1 = -\ell_0^2,\ \ell_2 = 2\ell_0^3,\ \ell_3 = -\ell_0^3/3.$$

Simple spring model II



$$k_s \sim 1/\ell_s \approx 1/\ell_0.$$

$$f = \frac{k_s \ell_0^4}{2}\left[(H - c_0)^2 - \frac{2c_0\ell_0}{3}K\right]$$

$$c_0 = \frac{(v_0 - \ell_s \Sigma_0)}{\Sigma_0 \ell_0^2}$$

- Bending modulus ~ power law (3rd power) of molecule length
- Spontaneous curvature ~ competition of packing area of heads and of tails

Curvature energy of self assembling monolayers

- Molecules constrained to interface
- Internal degree of freedom: area/molecule
- Area changes: large energies – adjust
- Impose curvature

$$f(\Sigma, H, K) = f_0(\Sigma) + f_1(\Sigma)H + f_2(\Sigma)H^2 + \bar{f}_2(\Sigma)K$$

$$f(\Sigma, H, K) \approx f_0(\Sigma_0) + \tfrac{1}{2} f_0''(\Sigma_0)(\Sigma - \Sigma_0)^2 + f_1(\Sigma_0)H$$

$$+ f_1'(\Sigma_0)(\Sigma - \Sigma_0)H + f_2(\Sigma_0)H^2 + \bar{f}_2(\Sigma_0)K$$

“Zero tension” membranes

- Minimize to find optimal area
- Free energy at optimum: curvature energy

$$\Sigma^* = \Sigma_0 - \left(\frac{f_1'}{f_0''}\right) H$$

$$f(\Sigma^*, H, K) = g_0 + g_1 H + g_2 H^2 + \bar{g}_2 K$$

$$g_0 = f_0(\Sigma_0),\ g_1 = f_1(\Sigma_0),\ \bar{g}_2 = \bar{f}_2(\Sigma_0)$$

$$g_2 = f_2(\Sigma_0) - \frac{1}{2}\frac{{f_1'}^2}{f_0''}$$

Curvature free energy

$$f_c = 2k\,(H - c_0)^2 + \bar{k}\,K$$

mean curvature

Gaussian curvature

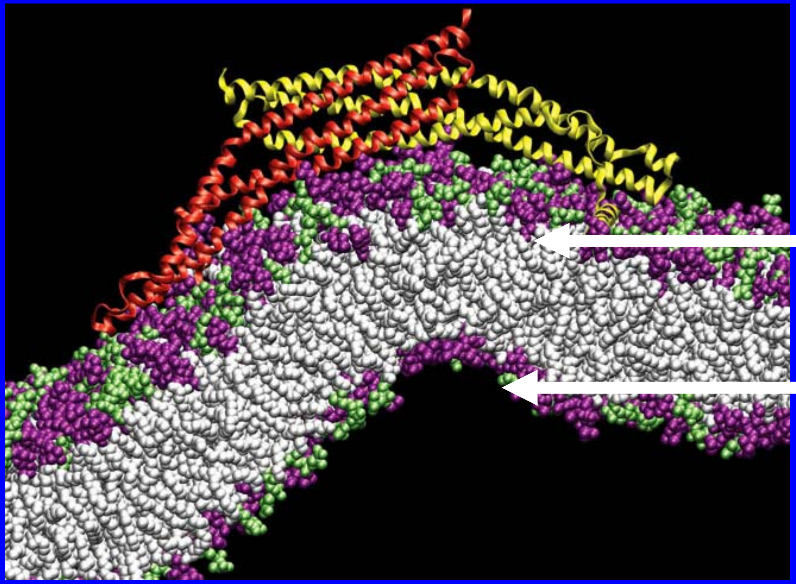
$$f_c = \frac{1}{2}k\,(\kappa_1 + \kappa_2 - 2c_0)^2 + \bar{k}\,\kappa_1\kappa_2$$

$$k = \frac{g_2}{2\Sigma_0}$$

$$\bar{k} = \frac{\bar{g}_2}{\Sigma_0}$$

$$c_0 = -\frac{g_1}{2g_2}$$

Spontaneous curvature: c_0



expanded

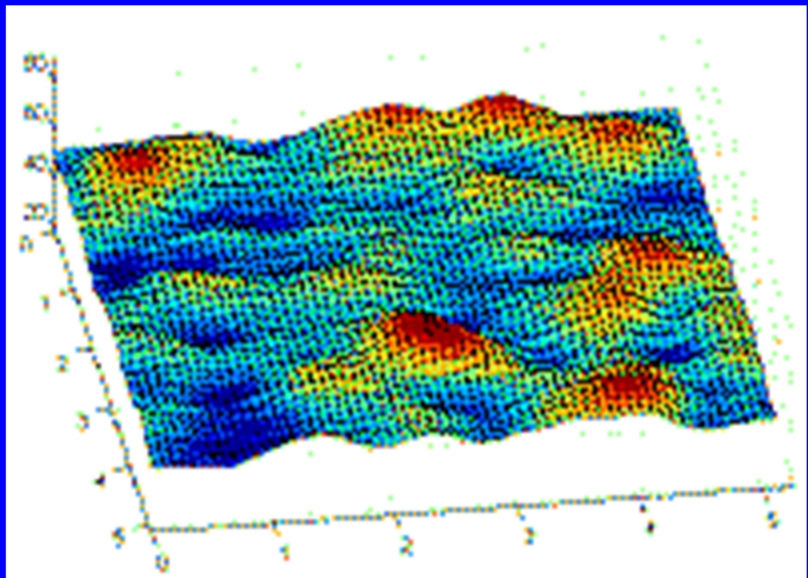
compressed

http://www.psc.edu/science/2006/inprogress/images/blood_tg_fig1b.jpg

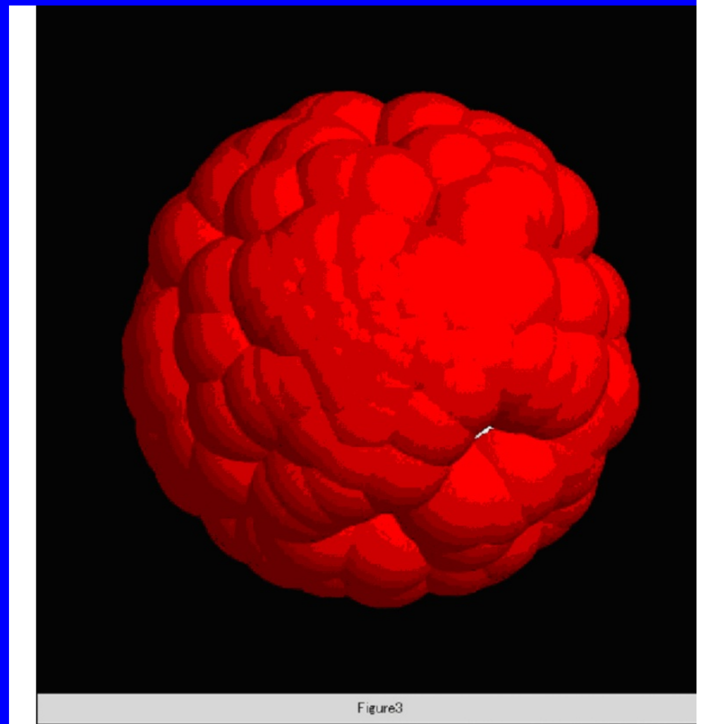
Chains vs. Heads

- Both are entropic contributions to bending “energy”
    - statistical subsystems
- Chain contributions to bending ~ $k_B T\, N^p$
- Polar head electrostatic contribution ~ $k_B T$
- Chains usually dominate bending modulus
- Polarity can control spontaneous curvature
    - Charge dependence
    - Salt dependence
    - Delicate balance of head/chain packings

Fluctuating membrane



http://www.berkeley.edu/news/media/releases/2002/10/24_laser.html



http://www.rgi.tut.fi/ijbem/volume6/number2/003.htm