Lecture 2

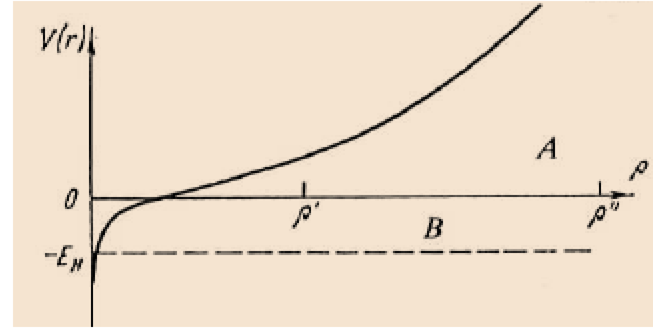
Hopping magnetoresistance

Topics:

1. *Asymptotic behavior of the impurity wave function in magnetic field*
2. *Anomalous tunneling in magnetic field*
3. *Interference effects in hopping magnetoresistance*
4. *Spin-orbit effects in hopping magnetoresistance*
5. *Interplay of interference and orbital effects in hopping magnetoresistance*

Schrödinger equation in cylindrical coordinates

$$-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial F}{\partial\rho}\right)+\frac{\partial^2 F}{\partial z^2}\right]+\frac{\hbar^2\rho^2}{8m\lambda^4}F-\frac{e^2}{\kappa r}F=EF$$



$$\lambda=\left(\frac{c\hbar}{eH}\right)^{1/2}$$ magnetic length

cyclotron barrier $$E\frac{a^2\rho^2}{4\lambda^4}$$

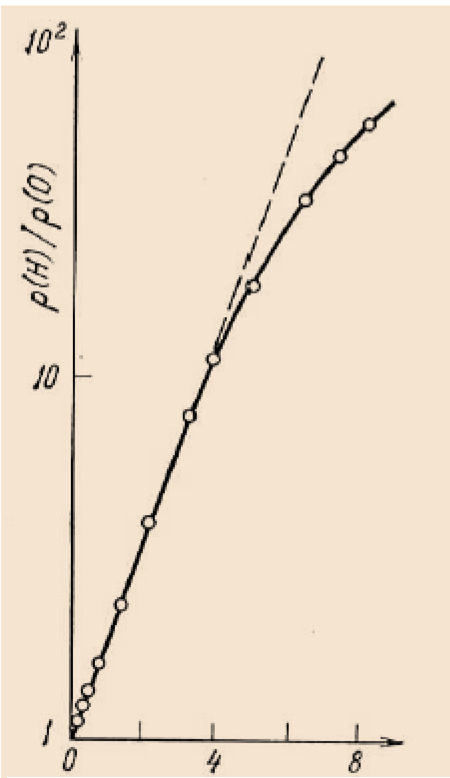


Fig. 7.5. Magnetoresistance of a lightly doped n-GaAs sample at $T = 2.6$ K [7.23]

weak field

$$\lambda \gg a$$

$$a \leqslant \rho \leqslant \lambda^2/a$$

orbital shrinkage

$$F(\mathbf{r}) \propto \exp\left[-\left(\frac{r}{a}+\frac{\rho^2 ra}{24\lambda^4}\right)\right]$$

NNH regime

$$r \sim \rho \sim N^{-1/3}$$

$$t \equiv B_c/24\pi = 0.036$$

$$\frac{\rho_3(H)}{\rho_3(0)}=\exp\left(t\frac{ae^2}{Nc^2\hbar^2}H^2\right)$$

VRH regime

$$r \sim \rho \sim a\left(\frac{T_0}{T}\right)^{1/4}$$

$$\ln\frac{\rho(H)}{\rho(0)}=t_1\frac{e^2a^4H^2}{c^2\hbar^2}\left(\frac{T_0}{T}\right)^{3/4}$$

$$t_1 = 5/2016$$

strong field

$$F(\mathbf{r}) \propto \exp\left[-\left(\frac{\rho^2}{4\lambda^2} + \frac{|z|}{a_H}\right)\right]$$

cigar-shaped wave function

$$a_H = \frac{\hbar}{\sqrt{2mE_H}}$$

"activation energy" grows with field

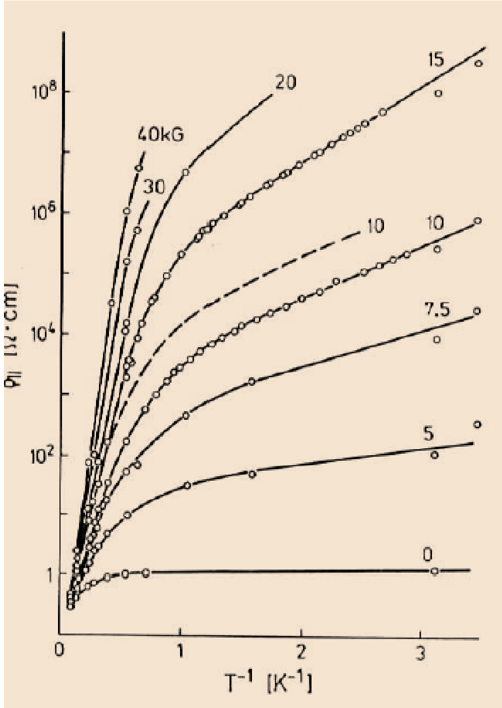


Fig. 7.1. Solid lines represent the inverse temperature dependences of the longitudinal resistivity $\rho_{\parallel}$ in an n-InSb sample with $N_D \approx 6\times10^{14}\,\mathrm{cm}^{-3}$ ([7.6], sample A9) for different values of the magnetic field. The transverse resistivity $\rho_{\perp}$ for $H = 10$ kOe is shown by a dashed line

VRH regime

$$\frac{2|z_{ij}|}{a_H} + \frac{x_{ij}^2 + y_{ij}^2}{2\lambda^2} + \frac{\epsilon_{ij}}{kT} \leqslant \xi$$

$$n(\xi) = 2g(\mu)\epsilon_{\max}x_{\max}y_{\max}z_{\max} = 2g(\mu)\lambda^2 a_H kT\xi^3$$

$$\rho(H) = \rho_0 \exp\{[T_0(H)/T]^{1/3}\}$$

$$T_0(H) = \frac{2.1\, eH}{g(\mu)c\hbar a_H k}$$

$$G_E(\mathbf{r},0) = \sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\,\frac{\psi_{np_z}(\rho,z)\,\psi_{np_z}(0,0)}{\hbar\Omega(n+\frac{1}{2}) + \frac{p_z^2}{2m} - E}$$

$$\psi_{np_z} = \frac{1}{\lambda\sqrt{2\pi}}\,L_n\left(\frac{\rho^2}{2\lambda^2}\right)\mathrm{e}^{-\frac{\rho^2}{4\lambda^2}}\,\mathrm{e}^{-\frac{i}{\hbar}p_z z}$$

$$F(\vec{r}) \propto G_E(\mathbf{r},0) \propto \mathrm{e}^{-\frac{\rho^2}{4\lambda^2}}\int_{-\infty}^{\infty}dp_z\int_0^{\infty}dt\;\mathrm{e}^{-\frac{i}{\hbar}p_z z - \frac{p_z^2}{2m}t - E_H t}\sum_{n=0}^{\infty}\mathrm{e}^{-n\hbar\Omega t}\,L_n\left(\frac{\rho^2}{2\lambda^2}\right)$$

Anomalous tunneling: Tunnel decay of the donor wave function across the magnetic field slows down due to the under-barrier scattering

Variable-range hopping conductivity in a strong magnetic field

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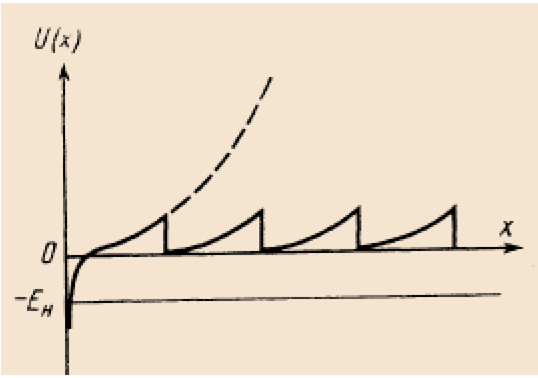
(Submitted 7 June 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **36**, No. 2, 43–46 (20 July 1982)



$$\mathcal{H} = \sum_i \epsilon_i a_i^+ a_i + \sum_{i \neq j} V_{ij} a_i^+ a_j$$

overlap integral between under-barrier scatterers

$$V_{ij} \propto \exp(-r_{ij}^2/4\lambda^2)$$

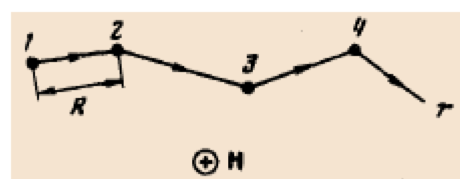
$$\psi_1(\mathbf{r}) \simeq \psi_1^0(\mathbf{r}) + \sum_i \frac{V_{1i}\psi_i^0(\mathbf{r})}{\epsilon_1 - \epsilon_i} + \sum_{i \neq j} \frac{V_{1i} V_{ij} \psi_j^0(\mathbf{r})}{(\epsilon_1 - \epsilon_i)(\epsilon_1 - \epsilon_j)} + \dots,$$

$$\propto \exp\left[-\frac{\vec{r}_i^{\,2} + (\vec{r} - \vec{r}_i)^2}{4\lambda^2}\right] >> \exp\left[-\frac{\vec{r}^{\,2}}{4\lambda^2}\right]$$

$$\exp\left[-\frac{\vec{r}^{\,2}}{n\lambda^2}\right] \cdot C^n$$

and the most probable optimum tunneling path to the point $\mathbf{r}$ in the plane of the film will consist of $n \simeq \mathbf{r}/R$ "steps," each with a length of order R (Fig. 2). For each step we have $V_{ij} \propto \exp(-R^2/4\lambda^2)$ and thus

$$\psi_1 \propto \exp(-nR^2/4\lambda^2) = \exp(-r/b),$$



where $b = s\lambda^2 R^{-1}$, and s is a numerical factor. In the case $\mathbf{r} \gg R$ we obviously have $\psi_1(r) \gg \psi_1^0(r)$.

Tunnel transparency of disordered systems in a magnetic field

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(Submitted 19 August 1982)
Zh. Eksp. Teor. Fiz. **84**, 811–822 (February 1983)

It is shown that multiple nonresonant scattering in a magnetic field alters substantially the character of the decay (the argument of the exponential) of the wave function of a tunneling electron. For example, the wave function in a strong magnetic field is proportional to $\exp(-x^2/2\lambda^2)$ without allowance for scattering, but when scattering is taken into account it takes the form $\exp(-|x|/b)$, where x is the coordinate in the direction perpendicular to the magnetic field, λ is the magnetic length, $b=\lambda/|\ln B|$, and B is a parameter that describes the scattering. The mean square modulus of the Green's function with negative energy in a magnetic field is calculated for scattering by a random Gaussian potential. It is shown that in semiconductor solid solutions this quantity can be used to describe the tunnel transparency of films in a magnetic field parallel to the surface, as well as the magnetoresistance of bulk samples in the region of hopping conduction.

$$\psi(x) \propto \exp\left(-\frac{x}{a}-\frac{x^3a}{6\lambda^4}\right), \qquad x \ll \frac{\lambda^2}{a}, \tag{1a}$$

$$\psi(x) \propto \exp\left(-\frac{x^2}{2\lambda^2}\right), \qquad x \gg \frac{\lambda^2}{a}, \tag{1b}$$

$$G_\varepsilon(\mathbf{r},\mathbf{r}')=G_\varepsilon^0(\mathbf{r},\mathbf{r}')+\int d\mathbf{r}_1 G_\varepsilon^0(\mathbf{r},\mathbf{r}_1)V(\mathbf{r}_1)G_\varepsilon^0(\mathbf{r}_1,\mathbf{r}')$$

$$+\int d\mathbf{r}_1\, d\mathbf{r}_2 G_\varepsilon^0(\mathbf{r},\mathbf{r}_1)V(\mathbf{r}_1)G_\varepsilon^0(\mathbf{r}_1,\mathbf{r}_2)V(\mathbf{r}_2)G_\varepsilon^0(\mathbf{r}_2,\mathbf{r}'),+\dots,$$

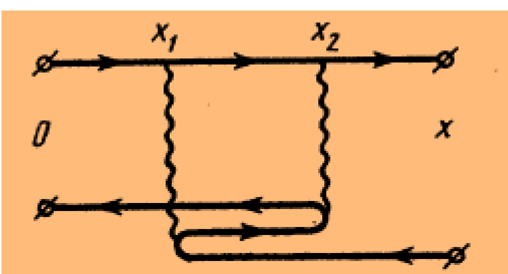
$$G_\varepsilon^0(\mathbf{r},\mathbf{r}')=\left(\frac{m}{2\pi\hbar^2}\right)\frac{1}{R}\exp\left\{-\frac{R}{a}-\frac{R^3a}{24\lambda^4}\sin^2\vartheta\right\}e^{-i\Phi(\mathbf{r},\mathbf{r}')}$$

at $\lambda > a$, $R^2\sin^2\vartheta < \lambda^4/a^2$, and

$$G_\varepsilon^0(\mathbf{r},\mathbf{r}')=\left(\frac{m}{2\pi\hbar^2}\right)\frac{a}{\lambda^2}\exp\left(-\frac{R|\cos\vartheta|}{a}-\frac{R^2\sin^2\vartheta}{4\lambda^2}\right)e^{-i\Phi(\mathbf{r},\mathbf{r}')}$$

at $\lambda < a$.

returns of tunneling electron are exponentially "costly"



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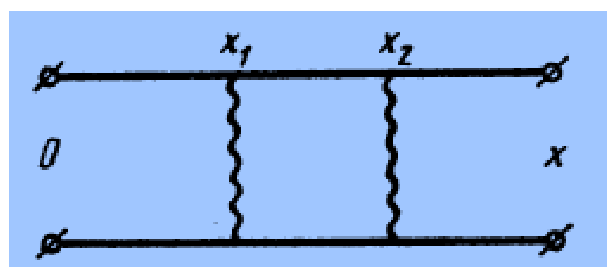
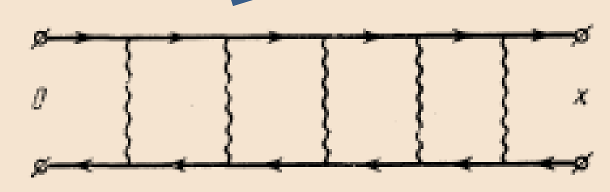


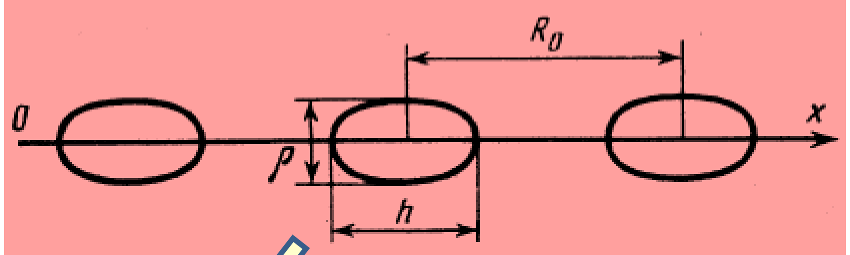
in calculating $\langle|G_\varepsilon(\mathbf{r},\mathbf{r}')|^2\rangle$ *only this sequence should be retained* *in calculating* $\langle|G_\varepsilon(\mathbf{r},\mathbf{r}')|^2\rangle$ *only this sequence should be retained*



n-th term

$$\langle|G_n|^2\rangle = \gamma^n \int |G^0(0,\mathbf{r}_1)|^2 |G^0(\mathbf{r}_1,\mathbf{r}_2)|^2 \ldots |G^0(\mathbf{r}_n,\mathbf{r}_{n+1})|^2 \, d\mathbf{r}_1 \ldots d\mathbf{r}_n.$$

relevant domains of integration do not overlap



$$B^n \exp\left[-(n+1)\left(\frac{x}{n+1}\right)^2 \frac{1}{4\lambda^2}\right]$$

$$n = n_{max} \equiv \frac{x}{2\lambda} |\ln B|^{-1/2} - 1.$$

$$G_\varepsilon(0,x) \propto \exp\left(-\frac{x}{\lambda}|\ln B|^{1/2}\right)$$

localization length $a = \dfrac{\lambda}{(\ln B)^{1/2}}$

Localization and hopping conductivity in the quantum Hall regime

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The long-range asymptotic behavior of the two-particle Green function of a two-dimensional electronic system in the presence of a strong magnetic field and a Gaussian white-noise potential is studied. Away from the center of the Landau level we can show that weak disorder leads to an exponential tail of the Green function, i.e., $G \sim e^{-\alpha|\mathbf{r}-\mathbf{r}'|}$. The rate of the exponential decay is found when α is large to be $\alpha \approx (|\ln W|/2)^{1/2}$, where W parametrizes the strength of the disorder. At shorter distances the Gaussian behavior of the unperturbed system predominates, and so there is a crossover between the two. The hopping conductivity in the quantum Hall devices is also discussed, and it is shown that the temperature dependence of the exponent in the conductance is not a simple power law, although it approaches the usual Mott $T^{-1/3}$ law as $T \rightarrow 0$.

$$G = G_0 + G_0 V G$$

$$|G|^2 = |G_0|^2 + G_0^* G_0 V G + G_0^* V G^* G_0 + G_0^* V G^* G_0 V G$$

$$\langle \delta(V)(\mathbf{r}) \delta V(\mathbf{r}') \rangle = \delta W \, \delta(\mathbf{r} - \mathbf{r}')$$

$$\Delta \langle |G|^2 \rangle = W^2 \int d\mathbf{r}_1 \int d\mathbf{r}_2 \, G_0^*(\mathbf{r},\mathbf{r}_1) G_0^*(\mathbf{r}_1,\mathbf{r}_2) G_0(\mathbf{r},\mathbf{r}_2) G_0(\mathbf{r}_2,\mathbf{r}_1) \langle G^*(\mathbf{r}_2,\mathbf{r}') G(\mathbf{r}_1,\mathbf{r}') \rangle$$

Exponential decay of the wave functions away from the center of the Landau level

Anomalous tunneling in a strong magnetic field and a smooth potential

$V(x)=V_0\cos(qx+\theta)$

$V_0 \ll |E| \ll \hbar\omega.$

$R_c = 2\pi/q \gg l.$

magnetic length much smaller than the correlation energy

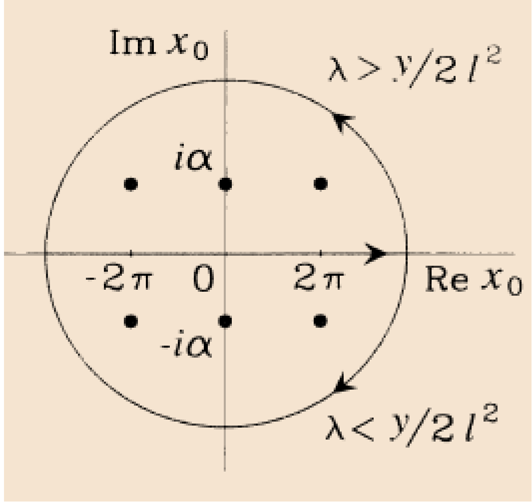
$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{m\omega^2}{2}(x-x_0)^2+V(x)-\epsilon\right]\varphi(x)=0$$

$$\psi_{x_0}(x,y)=\frac{e^{-ix_0y/l^2}}{\sqrt{2\pi l^2}}\frac{e^{-(x-x_0)^2/2l^2}}{\sqrt{\pi^{1/2}l}}.$$

$\epsilon_0(x_0)=\hbar\omega/2+\Delta\epsilon(x_0)$

$\Delta\epsilon(x_0)=e^{-q^2l^2/4}V_0\cos(qx_0+\theta)$

$$G(y)=\frac{1}{2\pi l^2}\int_{-\infty}^{\infty}\frac{dx_0}{\sqrt{\pi l^2}}\frac{e^{-x_0^2/l^2-ix_0y/l^2}}{E-V_0\cos(qx_0+\theta)}.$$



$$G(y)=\frac{1}{2\pi^2 l^2}\int_{-\infty}^{\infty}d\lambda e^{-\lambda^2l^2}\int_{-\infty}^{\infty}dx_0\frac{e^{ix_0(2\lambda-y/l^2)}}{E-V_0\cos(qx_0+\theta)}$$

poles of denominator

$qx_p=2\pi p-\theta\pm i\alpha,$

$\alpha=\operatorname{arcosh}(|E|/V_0)\simeq\ln(2|E|/V_0).$

$$G(y) \propto \exp\left[-\frac{y^2}{4l^2}\right], \quad y < \frac{2\alpha}{q} = \frac{\alpha R_c}{\pi}$$

$$G(y) \propto \exp\left[-\frac{y\alpha}{ql^2} + \frac{\alpha^2}{q^2l^2} - \frac{\theta^2}{l^2q^2} + i\theta\left[\frac{y}{ql^2} - \frac{2\alpha}{q^2l^2}\right]\right] \quad y > \frac{2\alpha}{q} = \frac{\alpha R_c}{\pi}$$

$$\langle G(y)\rangle_\theta \equiv \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} G(y) \propto \exp\left[-\frac{y^2}{4l^2}\right]$$

but

$$\langle |G(y)|^2\rangle_\theta \propto \exp\left[-\frac{2y\alpha}{ql^2} + \frac{2\alpha^2}{q^2l^2}\right]$$

decay length

$$a = \frac{ql^2}{\alpha} = \frac{2\pi l^2}{R_c \ln(2|E|/V_0)} \quad << l$$

$$y_c = 2\alpha/q = R_c \ln(2|E|/V_0)/\pi$$

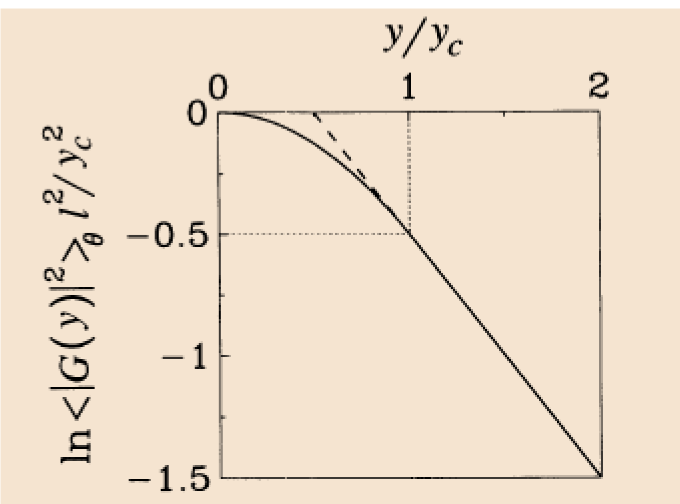


FIG. 2. The plot of $\ln\langle|G(y)|^2\rangle_\theta$ vs y (solid curve). The Gaussian decay is changed to the exponential decay at $y = y_c$.

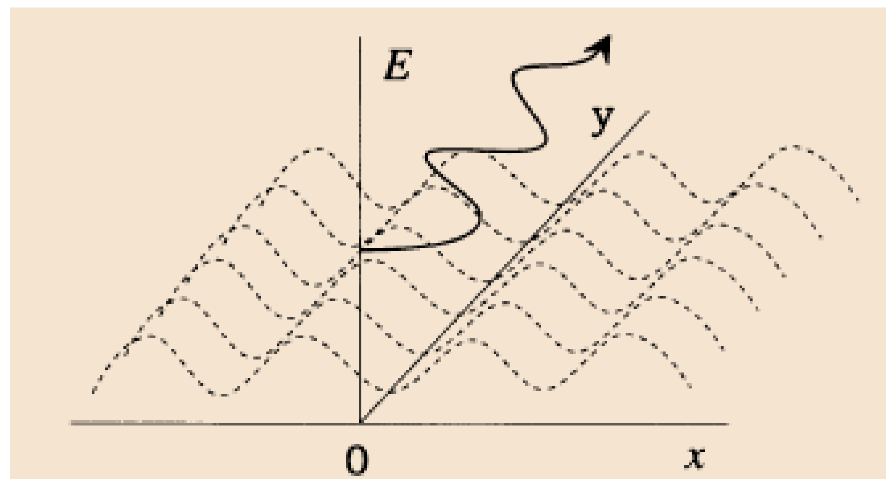
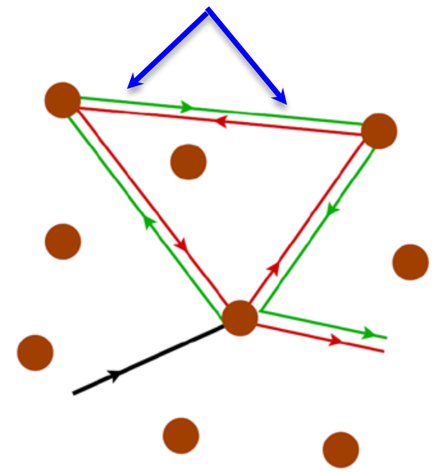


FIG. 3. A schematic representation of the electron trajectory with imaginary "time" (solid curve). The electron energy is much larger than the amplitude of periodic potential (dashed curves). The periodic scattering by potential results in a snake-like shape of the imaginary-time electron trajectory.

Interference of real directed amplitudes + log-averaging in the hopping regime cause negative magnetoresistance

In disordered metals negative magnetoresistance is due to interference of two counterpropagating paths:



Aaronov-Bohm oscillations with normal and superconducting flux quanta in hopping conductivity

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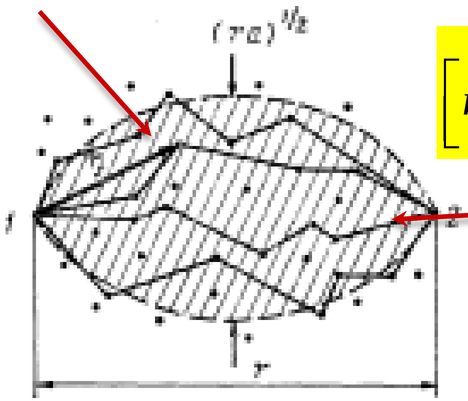
no coherent backscattering in the hopping regime

Scatterers with energies outside the Mott energy strip



$$\left[r^2+\left(\sqrt{ra}\right)^2\right]^{\frac{1}{2}}-r\approx\frac{a}{2}$$

Contributions of different tunneling paths to the net amplitude have random signs

VOLUME 60, NUMBER 15 PHYSICAL REVIEW LETTERS 11 APRIL 1988

Orbital Magnetoconductance in the Variable-Range–Hopping Regime

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Tunnel hopping in disordered systems

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(Submitted 21 May 1985)

Zh. Eksp. Teor. Fiz. **89**, 1770–1784 (November 1985)

All multiple-scattering paths have the same length

$$g(\varepsilon_i)=(1-x)\delta(\varepsilon_i-W)+x\delta(\varepsilon_i-W/A)$$

Random energy denominators take two values

$$I=V\sum_{\{\Gamma\}}\prod_{\{i\Gamma\}}\left(\frac{V}{\varepsilon-\varepsilon_i}\right)\Bigg|_{\varepsilon=0}=V\left(\frac{|V|}{W}\right)^{kn-1}J$$

Aharonov-Bohm phases

$$J=\sum_{\{\Gamma\}}\prod_{\{i\Gamma\}}\alpha_i$$

$$V_{ij}=V\exp(i\varphi_{ij})$$

$$\varphi_{ij}=(e/2\hbar c)\mathbf{H}[\mathbf{r}_i,\ \mathbf{r}_j]$$

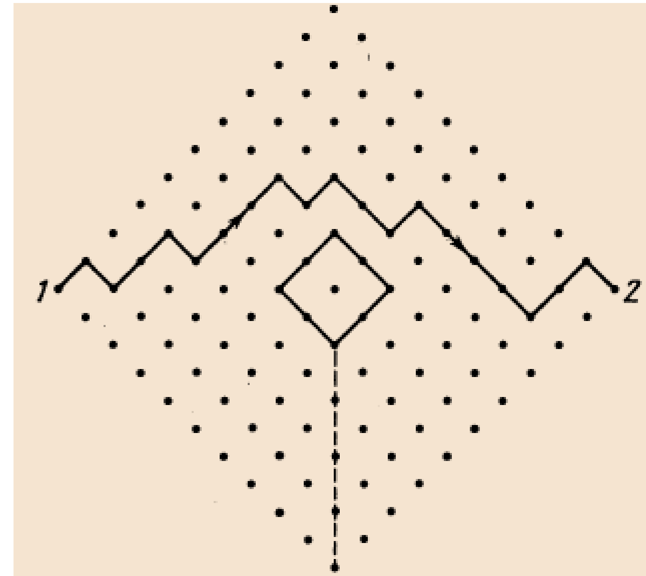


FIG. 1. Lattice used in the simulation of the quantity J. Sites 1 and 2 are the source and observation point. The arrow shows one of the oriented paths between these sites. The square at the center of the lattice is used in Section 3 to simulate an aperture in which a solenoid is placed. The dashed line is a "cut" on which the phase changes discontinuously.

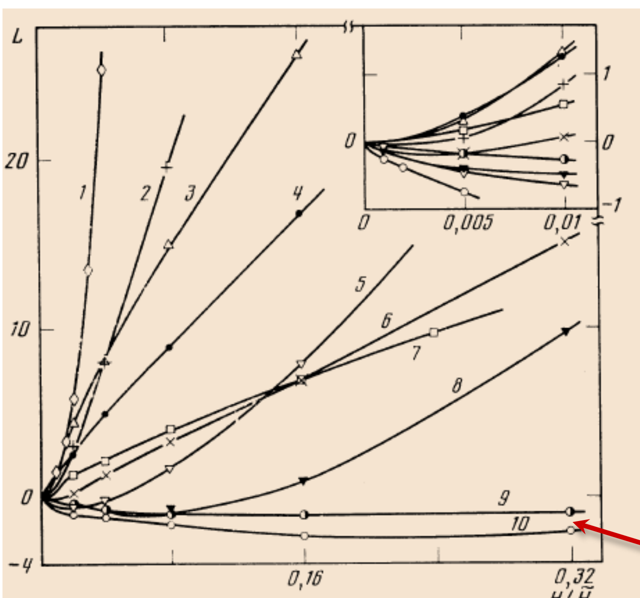


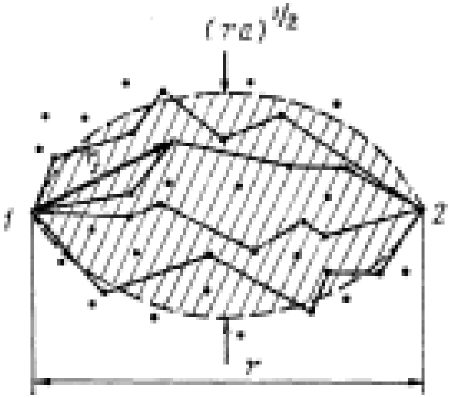
FIG. 7. The quantity $L_T+\langle\ln|J(H)/J(0)|^2\rangle$ as a function of the dimensionless magnetic field $H/\bar{H}$ for the three-dimensional case and various values of A and x: 1—$x=0$; 2—$A=8$, $x=0.001$; 3—$A=2$, $x=0.5$; 4—$A=20$, $x=0.01$; 5—$A=-8$, $x=0.01$; 6—$A=-1$, $x=0.1$; 7—$A=20$, $x=0.1$; 8—$A=-8$, $x=0.02$; 9—$A=8$, $x=0.1$; 10—$A=-1$, $x=0.5$. Shown separately in the inset is the region of very weak fields, $H\leqslant 0.01\bar{H}$.

negative magnetoresistance

Contributions to hopping conductivity from disorder configurations

where virtual amplitudes almost cancel each other

are most sensitive to a weak magnetic field



$$|A_1 + A_2| << |A_1|, |A_2|$$

Aharonov-Bohm flux $2\pi \frac{\Phi}{\Phi_0}$

$$\frac{\sigma(B)-\sigma(0)}{\sigma(0)} = \ln\left[\frac{A_1^2 + 2A_1A_2\cos\varphi + A_2^2}{A_1^2 + 2A_1A_2 + A_2^2}\right] \approx \ln\left[1 + \frac{|A_1A_2|\varphi^2}{(A_1+A_2)^2}\right]$$

"Phase volume" of configurations sensitive to magnetic field B

is proportional to B ⇨ ***linear NMR***

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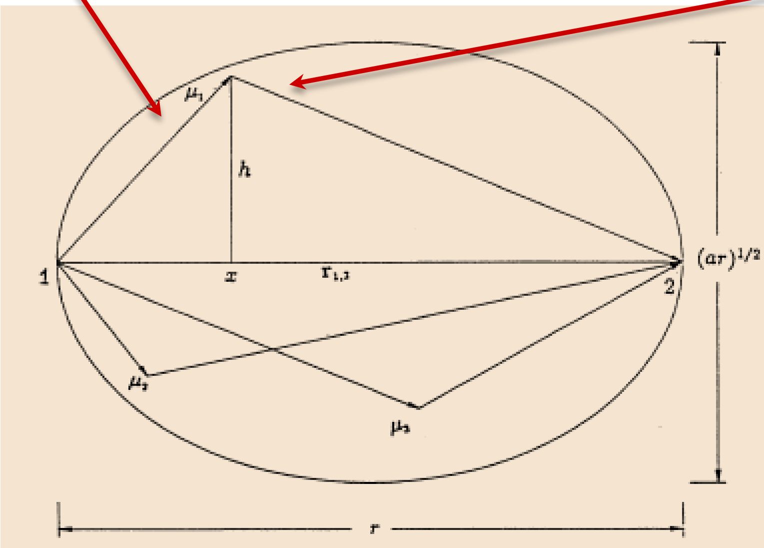
Single-scattering-path approach to the negative magnetoresistance in the variable-range-hopping regime for two-dimensional electron systems

$$\psi_\mu = 2\pi \frac{(\mathbf{B}\cdot\mathbf{S}_\mu)}{\phi_0}$$

Single-scattering paths



$$\frac{\mathcal{R}(B)-\mathcal{R}(0)}{\mathcal{R}(0)} = -K(A)\frac{B}{B_0}$$

Linear NMR at small fields

slope is $\propto T^{-1}$

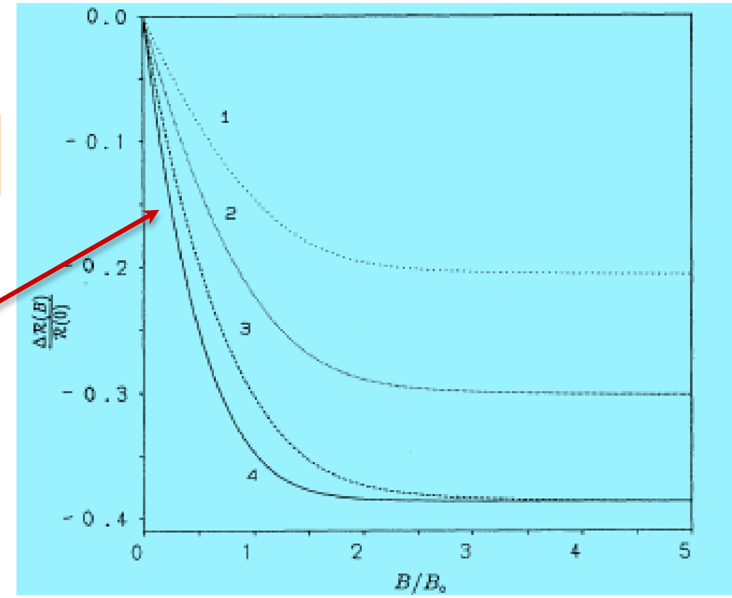


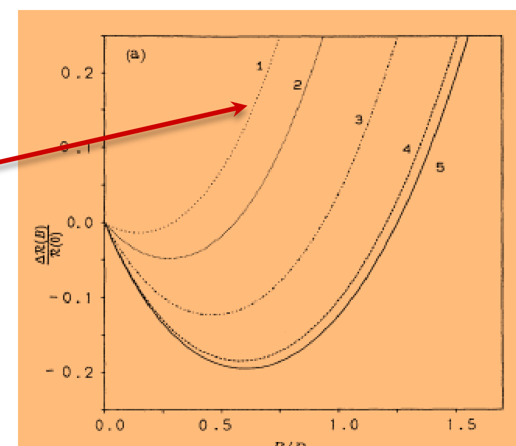
FIG. 3. The behavior of the relative magnetoresistance as a function of the normalized magnetic field for various values of normalized scattering strength A [Eq. (3.18)]. (1) $A=0.07$, (2) $A=0.13$, (3) $A=0.26$, and (4) the strong-scattering limit.

$$\delta \ln \mathcal{R} = -\frac{2}{\pi}\int_0^{\pi/2} d\varphi \ln\left[1+\frac{G^2(\varphi)}{\sin^2\varphi}\right]$$

$$G(\varphi) = \pi g \int d^2\mathbf{r} \frac{V_{1,\mathbf{r}}V_{2,\mathbf{r}}}{V_{1,2}} |\sin[\psi(\mathbf{r})-\varphi]|$$

With orbital effect taken into account

$$\tilde{V}_{\mu,\nu} = V_{\mu,\nu}\exp\left[-\frac{r^3_{\mu,\nu}a}{24\lambda^4}\right]$$



linear NMR is due to the paths for which direct and scattered amplitudes cancel each other

$$(\delta \ln \mathcal{R})_{\mathrm{orb}} = \frac{r^3_{1,2}a}{12\lambda^4} = \frac{2}{3}\frac{B^2}{B_0^2}$$

Negative magnetoresistance and oscillations of the hopping conductanc of a short n-type channel in a GaAs field-effect transistor

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(Submitted 6 May 1987)
Zh. Eksp. Teor. Fiz. **93**, 2204–2218 (December 1987)

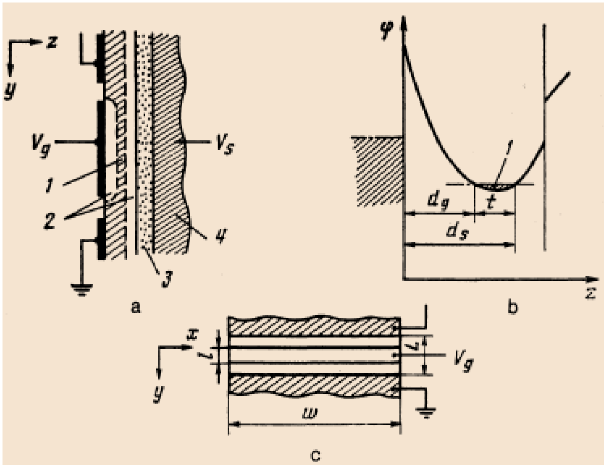


FIG. 2. a) Section through a structure: 1) conducting channel in GaAs; 2) depletion regions; 3) semiinsulating GaAlAs layer; 4) conducting substrate. b) Energy diagram of the channel. c) View of the structure from above.

$R(H=0)$ increases

r increases ⇒ NMR increases

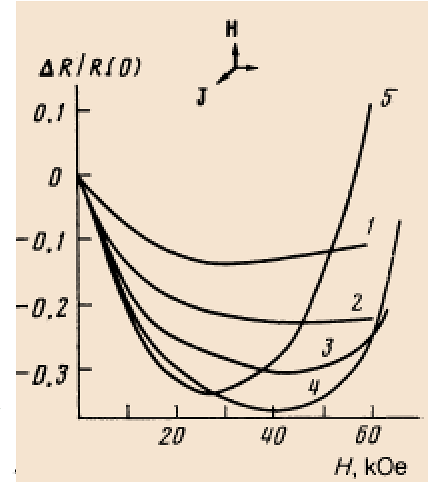
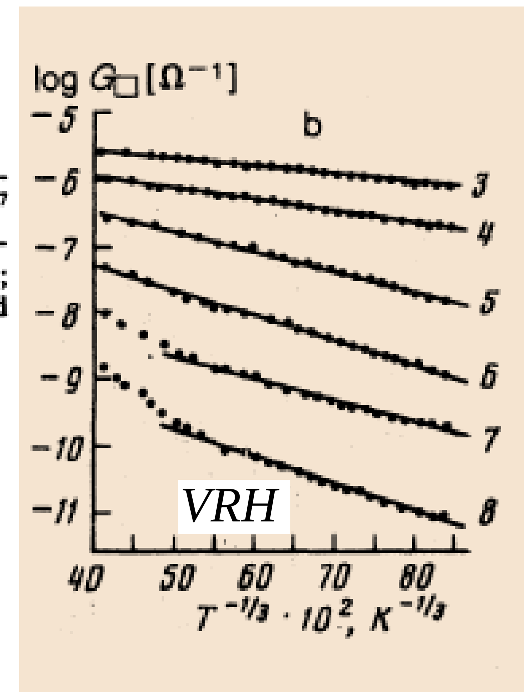


FIG. 5. Transverse magnetoresistance of a channel of sample H6 with $N_d = 1.8\times10^{17}\,\mathrm{cm}^{-3}$ at $T = 4.2$ K recorded for different values of V_g and $R_\square$ $(H=0)$: 1) 2.24 V, 160 kΩ; 2) 2.38 V, 270 kΩ; 3) 2.57 V, 1.7 MΩ; 4) 2.59 V, 2.7 MΩ; 5) 2.70 V, 28 MΩ.

FIG. 3. a) Temperature dependences of the channel conductance (sample H1, $N_d = 1.0\times10^{17}\,\mathrm{cm}^{-3}$) obtained for different values of the gate voltage $-V_g$ (V): 1) 0; 2) 1; 3) 1.95; 4) 2.04; 5) 2.17; 6) 2.27; 7) 2.34; 8) 2.395. b) Curves 3–5 replotted using the coordinates $\log G$ and $T^{-1/3}$.



$R(H=0)$ increases

anisotropy increases

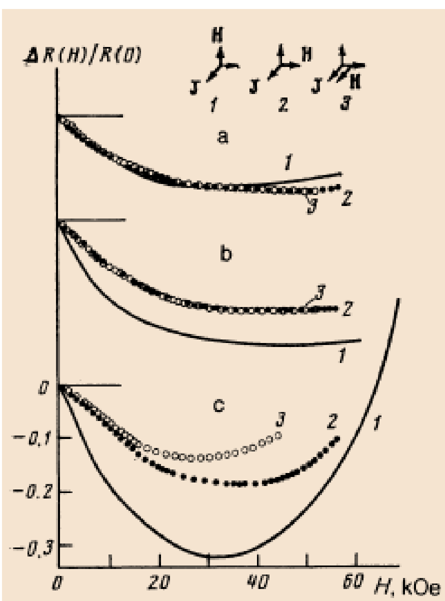


FIG. 6. Channel magnetoresistance (of sample H6) obtained for three orientations of the magnetic field: the continuous curves correspond to $H_\perp$; the black dots (●) correspond to $H_{\parallel\parallel}$, and the open circles (○) correspond to $H_\parallel$. The curves were obtained for different values of V_g and $R_\square$ $(H=0)$: a) 2.24 V, 160 kΩ; b) 2.38 V, 270 kΩ; c) 2.61 V, 7.0 MΩ (the curves are shifted along the y axis).

PHYSICAL REVIEW B VOLUME 38, NUMBER 8 15 SEPTEMBER 1988-I

Magnetoconductance in the variable-range-hopping regime due to a quantum-interference mechanism

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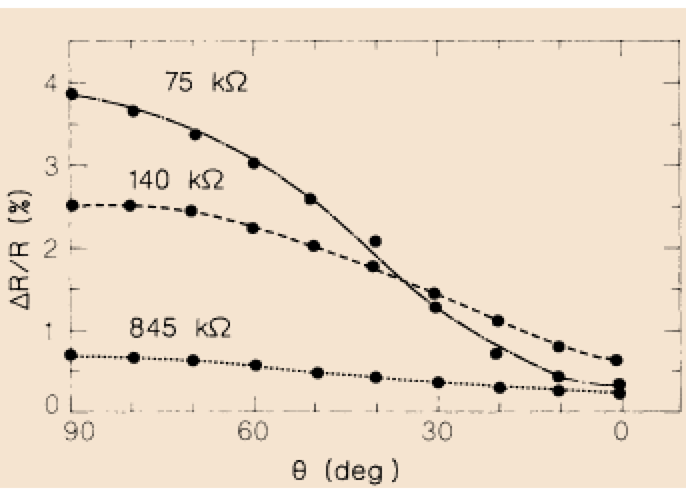
(Received 31 December 1987)

Results of systematic magnetoconductance measurements on highly disordered In_2O_{3-x} films are described. Measurements were performed as a function of magnetic field, electric field, temperature, system dimensionality, and amount of static disorder. It is shown that in the hopping regime, the low-field magnetoconductance is always positive, and anisotropic in sufficiently thin films. The latter feature is suggestive of a nonlocal (orbital) mechanism. We demonstrate that the spatial range of phase coherence, involved in the phenomenon, scales with the hopping length. This length may be controlled by either the temperature or the electric field. It is further shown that several aspects of the experimental results support the basic ideas of a newly proposed quantum-interference mechanism. An intuitive physical description of the reason for the positive magnetoconductance is discussed based on the percolation model for the hopping transport.



strongly localized

FIG. 2. MR for three SL films (with indicated sheet-resistances), measured at 4.2 K as a function of the angle between the field direction and the sample plane ($d = 100$ Å).

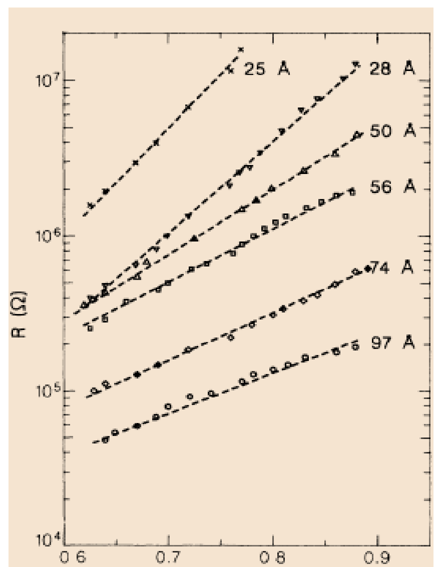


FIG. 5. Resistance as a function of temperature for several 100-Å-thick samples. The curves are labeled by the respective ξ values derived through Eq. (1) using $N(0)=10^{32}$ $erg^{-1}cm^{-3}$ (cf. Sec. II) and T_* was taken from the logarithmic slope of the $R(T)$ data.

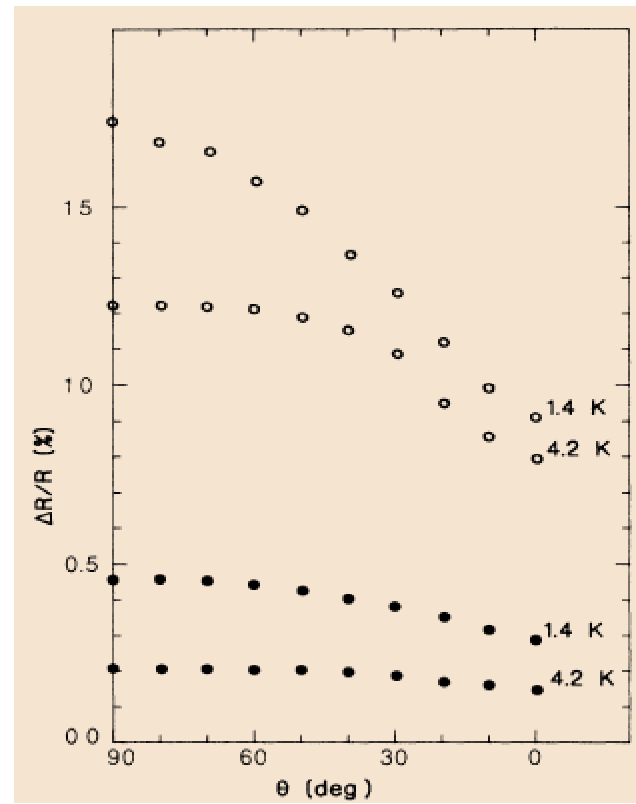


FIG. 3. MR of two 250-Å-thick films as function of the angle between a 2.4-kOe field direction ans samples plane: solid circles, sample with $\xi = 35$ Å; open circles, sample with $\xi = 85$ Å. (Note that anisotropy is stronger at the lower temperature.)

RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 45, NUMBER 16 15 APRIL 1992-II

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(Received 3 February 1992)

A magnetoconductance is observed for weakly insulating *n*-type CdSe which, depending on the temperature of the measurement, is quadratic or approximately linear with field in small magnetic fields, and exhibits saturation as the field is increased. The crossovers from quadratic to linear behavior and to saturation occur at magnetic fields which are consistent with theoretical expectations for the effect of quantum interference in the hopping regime.

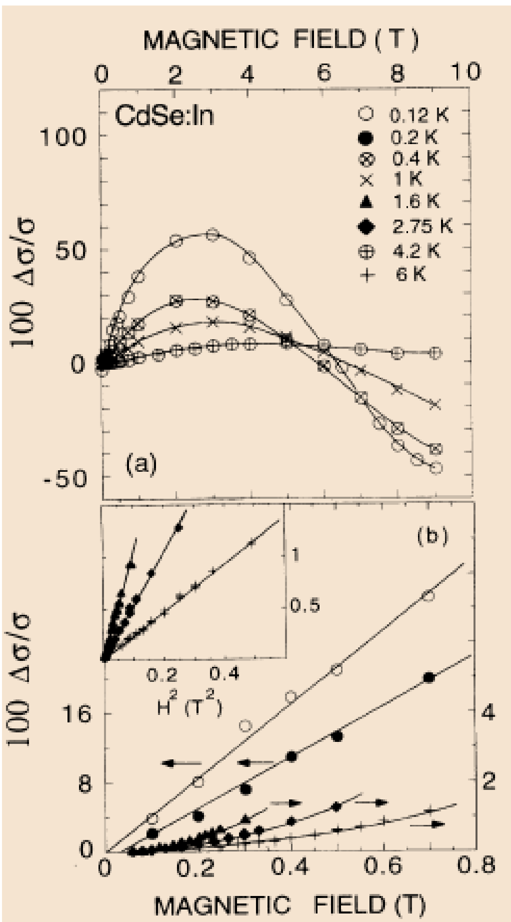


FIG. 1. The magnetoconductance of compensated *n*-type CdSe with net In dopant concentration $N=(N_D-N_A)=2.25\times10^{17}$ cm^{-3} (sample 4), where N_D and N_A are the donor and acceptor concentrations, respectively: (a) magnetic fields to 10 T; (b) magnetic fields to 0.8 T. To demonstrate the quadratic behavior at higher temperatures, the inset in (b) shows the same data plotted as a function of H^2. The list of symbols and corresponding temperatures refers to both (a) and (b). The straight lines represent least-mean-square fits to the data and the curves are drawn to guide the eye.

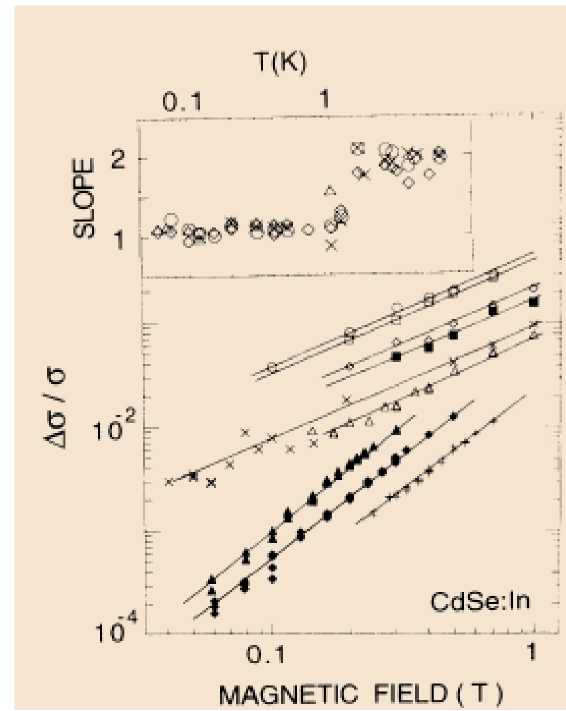


FIG. 2. The magnetoconductance $\Delta\sigma/\sigma$ of sample 4 vs magnetic field plotted on a log-log scale. The symbols denote the following: □, 0.1 K; ○, 0.12 K; ◇, 0.3 K; ■, 0.5 K; ×, 1.0 K; △, 1.2 K; ▲, 1.6 K; ◆, 2.75 K; +, 6.0 K. The straight lines are least-mean-square fits to the data, and their slopes give the exponent s of $\Delta\sigma/\sigma\propto H^s$. The inset shows the slope s as a function of the logarithm of the temperature for samples 1 (◇), 2 (△), 4 (○), and 5 (×).

In the presence of spin-orbit scattering

VOLUME 73, NUMBER 10 PHYSICAL REVIEW LETTERS 5 SEPTEMBER 1994

Low-Field Anomaly in 2D Hopping Magnetoresistance Caused by Spin-Orbit Term in the Energy Spectrum

With SO scattering the hopping amplitude is an operator

$$A = G_0(r_{12}) + G_0(r_{13}) Q G_0(r_{32}) \exp\left(2\pi i \frac{\mathbf{B}\cdot\mathbf{S}}{\Phi_0}\right)$$

$$A^{\uparrow\uparrow} = A_d^{\uparrow\uparrow} + A_s^{\uparrow\uparrow} \qquad A^{\uparrow\downarrow} = A_s^{\uparrow\downarrow}$$

tunneling probability

$$W = |A^{\uparrow\uparrow}|^2 + |A^{\uparrow\downarrow}|^2$$

in a weak magnetic field:

$$\ln[R(B)/R(0)] \simeq \delta R(B)/R(0)$$

distribution function

$$\frac{\delta R(B)}{R(0)} = -\int d\tau\, F_S(\tau) \ln\left(1 + \frac{4\pi^2 B^2 S^2}{\Phi_0^2} \frac{1}{\tau^2 + \lambda^2}\right)$$

$$\tau = A^{\uparrow\uparrow}(0)/A_s^{\uparrow\uparrow}$$ ← *interference parameter*

$$\lambda = Q^{\uparrow\downarrow}/Q^{\uparrow\uparrow}$$ ← *strength of SO scattering*

Major contribution to the integral comes from small zero-field amplitudes

$$\frac{\delta R(B)}{R(0)} = -2\pi F_S(0)\left(\sqrt{\lambda^2 + \frac{4\pi^2 B^2 S^2}{\Phi_0^2}} - |\lambda|\right)$$

SO scattering turns linear NMR into quadratic

SO coupling in the bare Hamiltonian instead of SO scattering

$$\hat{H}_{SO} = \alpha n \cdot (\sigma \times k)$$

normal to the 2D plane

$$\hat{G}(\mathbf{r}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{E - \hbar^2\mathbf{k}^2/2m - \alpha\mathbf{n}\cdot(\boldsymbol{\sigma}\times\mathbf{k})} = \exp[-ik_0\mathbf{n}\cdot(\boldsymbol{\sigma}\times\mathbf{r})]G_0(r).$$

$$k_0 = \frac{\alpha m}{\hbar^2}$$

with the help of identity

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{A}+\hat{B}+\frac{1}{2}[\hat{A},\hat{B}]+\dots}$$

$$\hat{G}(\mathbf{r}_{13})\hat{G}(\mathbf{r}_{32}) = G_0(r_{13})G_0(r_{32})\exp\left\{-ik_0\mathbf{n}\cdot(\boldsymbol{\sigma} - \frac{k_0^2}{2}[\mathbf{n}\cdot(\boldsymbol{\sigma}\times\mathbf{r}_{13}),\mathbf{n}\cdot(\boldsymbol{\sigma}\times\mathbf{r}_{32})]\right\}.$$

the commutator $[\mathbf{n}\cdot(\boldsymbol{\sigma}\times\mathbf{r}_{13}),\mathbf{n}\cdot(\boldsymbol{\sigma}\times\mathbf{r}_{32})]$ *is equal to* $4i(\mathbf{S}\cdot\mathbf{n})(\boldsymbol{\sigma}\cdot\mathbf{n})$

$$\hat{A} = t\hat{G}(\mathbf{r}_{12})\left[\tau - 1 + \exp\left(2\pi i\frac{(\mathbf{B}-\hat{\mathbf{B}}_0)\cdot\mathbf{S}}{\Phi_0}\right)\right]$$

$$t = QG_0(r_{13})G_0(r_{32})/G_0(r_{12}).$$

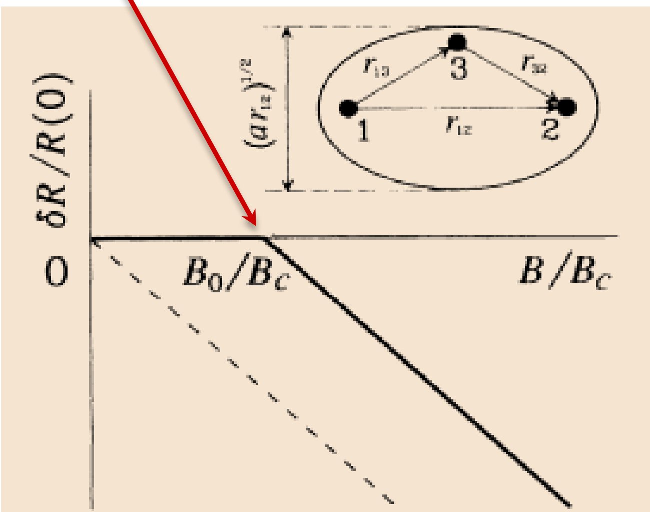
$$\hat{\mathbf{B}}_0 = B_0(\boldsymbol{\sigma}\cdot\mathbf{n})\mathbf{n},$$

SO magnetic field

$$B_0 = \frac{k_0^2\Phi_0}{\pi} = \frac{\alpha^2 m^2}{\pi\hbar^4}\Phi_0$$

$$\frac{\delta R(B)}{R(0)} = -\frac{1}{2}\sum_{s=\pm 1}\int d\tau\, F_S(\tau) \times \ln\left[1 + \frac{(B + sB_0)^2 - B_0^2}{\tau^2\Phi_0^2/4\pi^2 S^2 + B_0^2}\right]$$

SO anomaly



$$\frac{\delta R(B)}{R(0)} = -\frac{1}{2B_c}(|B + B_0| + |B - B_0| - 2B_0)$$

$$B_c = \Phi_0/4\pi^2 S F_S(0)$$

Position of anomaly $B_0 = \frac{k_0^2\Phi_0}{\pi} = \frac{\alpha^2 m^2}{\pi\hbar^4}\Phi_0$ ***is independent of disorder***

for Dresselhaus mechanism $\alpha = 2a_{42}\langle\hat{k}_z^2\rangle$

For AlGaAs/GaAs heterostructure with width $d = 50A^\circ$ ***we get*** $\alpha = 2\times 10^{-9}\, eV \bullet cm$

$$B_o = 0.18T$$

Phys. Status Solidi C 9, No. 2, 183–186 (2012)

Electrical transport properties in a single-walled carbon nanotube network

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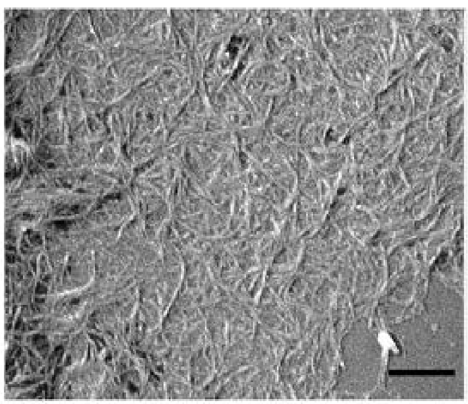

Figure 1 Scanning electron microscopy image of the SWCNT network. The black scale bar represents 1 µm.

$$k_B T_0 = 3 / N(E_F)\, \xi^2 d \quad (1)$$

where k_B is Boltzmann constant, T_0 is the degree of disorder [6] equal to the slope determined in Fig. 2(b), $N(E_F)$ is the density of states at the Fermi level E_F and d is the thickness of the SWCNT network, we can extract the localisation length ξ. $N(E_F)$ has been reported to be ~ 10^{18} - 10^{19} eV^{-1} cm^{-3} [10]. With our sample thickness d equal to 200 nm, ξ is evaluated to be between 3.6 nm and 11 nm, indicating that our SWCNT sample forms a 3D network.

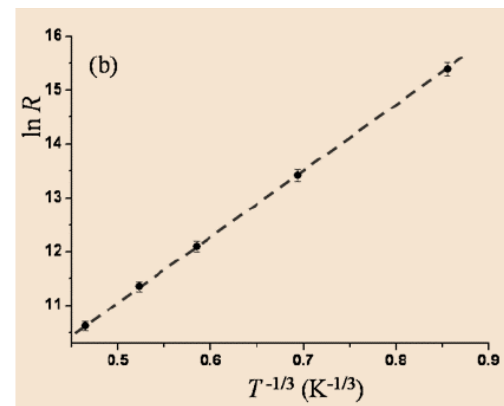

Figure 2 (a) The resistance of the SWCNT network within the temperature T range 0.5 K-249 K depends strongly on the temperature. The inset shows the linear I-V characteristics of the SWCNT network at T = 0.5, 0.6, 0.7, 0.8 and 0.9 K. The corresponding values of the resistance are also displayed. (b) The plot shows the linear relationship between ln R and $T^{1/3}$.

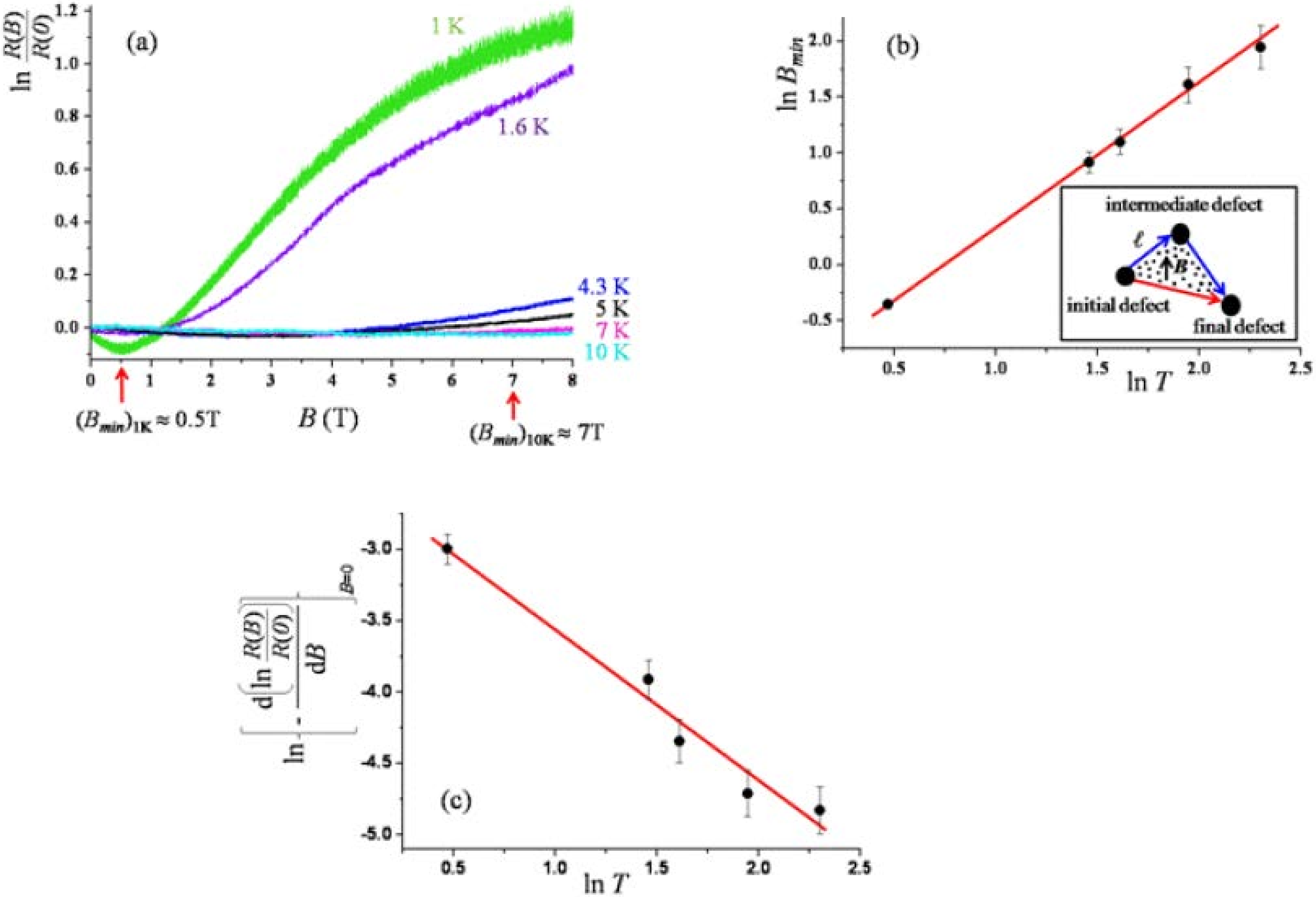

Figure 3 (a) The natural logarithm of ratio $R(B)/R(0)$ versus B at T = 1, 1.6, 4.3, 5, 7 and 10 K. B_{min} values are indicated for T = 1 K and 10 K. (b) B_{min} is proportional to the temperature. The inset shows a schematic drawing of alternative tunnelling paths for strongly localised hopping electrons. Applying a $\boldsymbol{B}$-field destroys the quantum interference process between the paths. (c) Temperature dependence of the negative slope of the magnetoresistance curves when B is close to 0 T.

Negative Hopping Magnetoresistance of Two-Dimensional Electron Gas in a Smooth Random Potential

electron gas breaks up into lakes

each lake accommodating many electrons

saddle-point potential between two lakes

$$U(x,y) = U_0 - \frac{m\omega_x^2 x^2}{2} + \frac{m\omega_y^2 y^2}{2}$$

$$\exp\left[-\pi\left(U_0 - E + \frac{m\omega_y^2 y^2}{2}\right)\Big/\hbar\omega_x\right]$$

transmission of barrier as a function of y

tunnel matrix element between two lakes

$$\psi_p^1\left(-\frac{d}{2}, y\right) \propto \sin(py - \chi_1),$$

$$\psi_k^r\left(-\frac{d}{2}, y\right) \propto \sin(ky - \chi_r),$$

$$\lambda = (2\hbar\omega_x/\pi m\omega_y^2)^{1/2}$$

$$t_{pk} \sim \exp[-\pi(U_0 - E)/\hbar\omega_x] \int_{-\infty}^{\infty} dy \times \exp(-y^2/\lambda^2)\frac{\partial}{\partial x}\psi_p^1\left(-\frac{d}{2}, y\right) \times \left[\frac{\partial}{\partial x}\psi_k^r\left(\frac{d}{2}, y\right)\right]^*.$$

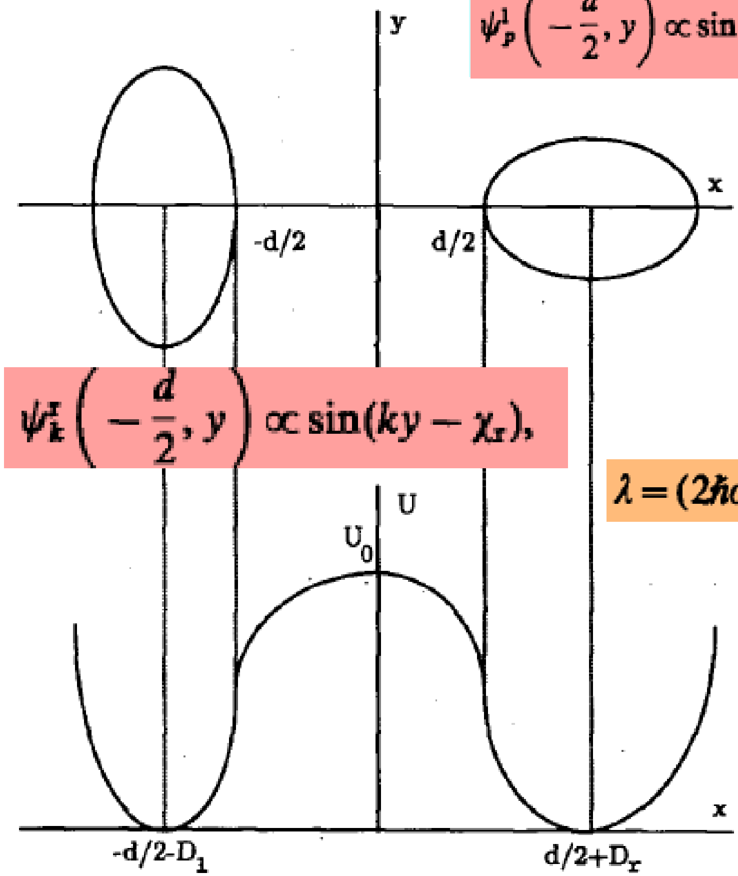

Fig. 1. (a) Two electronic lakes separated by a saddle point. (b) The schematic potential profile in the cross-section $y = 0$.

In a zero magnetic field

suppression due to mismatch of momenta

$$t_{pk} \sim \exp[-\pi(U_0 - E)/\hbar\omega_x] \times \{\cos(\chi_l - \chi_r)\exp[-(p-k)^2\lambda^2/4] - \cos(\chi_l + \chi_r)\exp[-(p+k)^2\lambda^2/4]\}.$$

$$B_0 = \frac{\Phi_0}{\sqrt{2}\pi(D_l + D_r)\lambda}$$

In a weak magnetic field

$$\psi_p^l\left(-\frac{d}{2}, y\right) \text{ and } \psi_k^r\left(-\frac{d}{2}, y\right)$$

acquire the gauge phase factors

lake sizes

$$\exp\left[\frac{2\pi i y B D_1}{\Phi_0}\right] \text{ and } \exp\left[-\frac{2\pi i y B D_2}{\Phi_0}\right]$$

$$t_{kp}(B) = t_{kp}(0)\exp\left(-\frac{B^2}{2B_0^2}\right) \times \left[\cosh\frac{B}{B_1} + i\tan(\chi_l + \chi_r)\sinh\frac{B}{B_1}\right]$$

$$B_1 = \frac{\Phi_0}{\pi(D_l + D_r)|k - p|\lambda^2}.$$

exponential growth

$$\frac{R(B)}{R(0)} = \left(\frac{t_{kp}(B)}{t_{kp}(0)}\right)^{-2} = \exp\left(\frac{B^2}{B_0^2}\right) \times \frac{1}{\cosh^2 B/B_1 + \tan^2(\chi_l - \chi_r)\sinh^2 B/B_1}.$$

exponential fall off

The physical meaning of the negative magnetoresistance is that the phase factors acquired by the wavefunctions in a magnetic field effectively compensate the difference in their wave numbers in the y-direction and, therefore, lead to an increase of the coupling.

Small- field expansion:

$$\frac{\delta R(B)}{R(0)} = \left(\frac{1}{B_0^2} - \frac{1}{B_1^2 \cos^2(\chi_l - \chi_r)}\right) B^2$$

in a smooth potential

$$R >> \frac{e^2}{h}$$

while the temperature dependence is weak

and it is negative for any χ_l and χ_r if $B_0 > B_1$. The ratio $B_0/B_1 = |k-p|\lambda/\sqrt{2}$ can be estimated as $(E_F/\hbar\Omega)^{1/2}$, where E_F is the Fermi energy of an electron in a lake and $\Omega = \omega_y^2/\omega_x$. This ratio is large in the semiclassical regime. Hence, in the small field region the resistance decreases as $\delta R(B)/R(0) \simeq -B^2/B_1^2$, the characteristic value B_1 being of the order of Φ_0/S, where $S \sim D_l^2 \sim D_r^2$ is of the order of the area of the lake. The resistance falls off exponentially with B in the region $B_c \gg B \gg B_1$, where the crossover field is $B_c = B_0^2/B_1$. After reaching a minimum at $B \approx B_c$, the resistance rises sharply at higher fields. Note however that at $B = B_c$ the cyclotron frequency becomes of the order of $\hbar\Omega$ so that the effect of magnetic field on the eigenstates in the lakes cannot be reduced to the phase factors only.

PHYSICAL REVIEW B 70, 205306 (2004)

Electronic transport through a quantum dot network

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(Received 7 May 2004; published 5 November 2004)

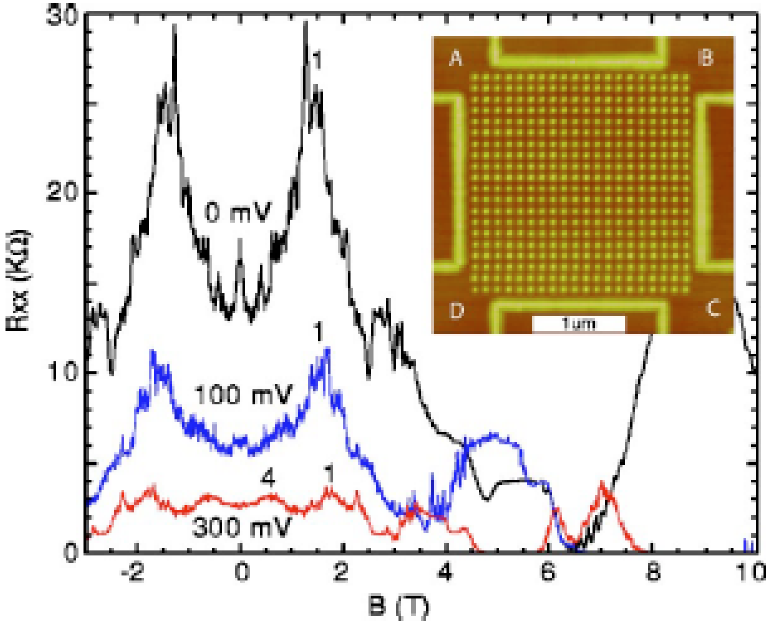


FIG. 1. (Color online) Magnetoresistance measured from A to C across diagonal 1 at different top gate voltages at T=90 mK, commensurability peaks around 1 and 4 antidots are marked. Inset: AFM-micrograph of the antidot lattice and the enclosing cavity. Bright regions are oxidized and correspond to depletion in the underlying two-dimensional electron system.

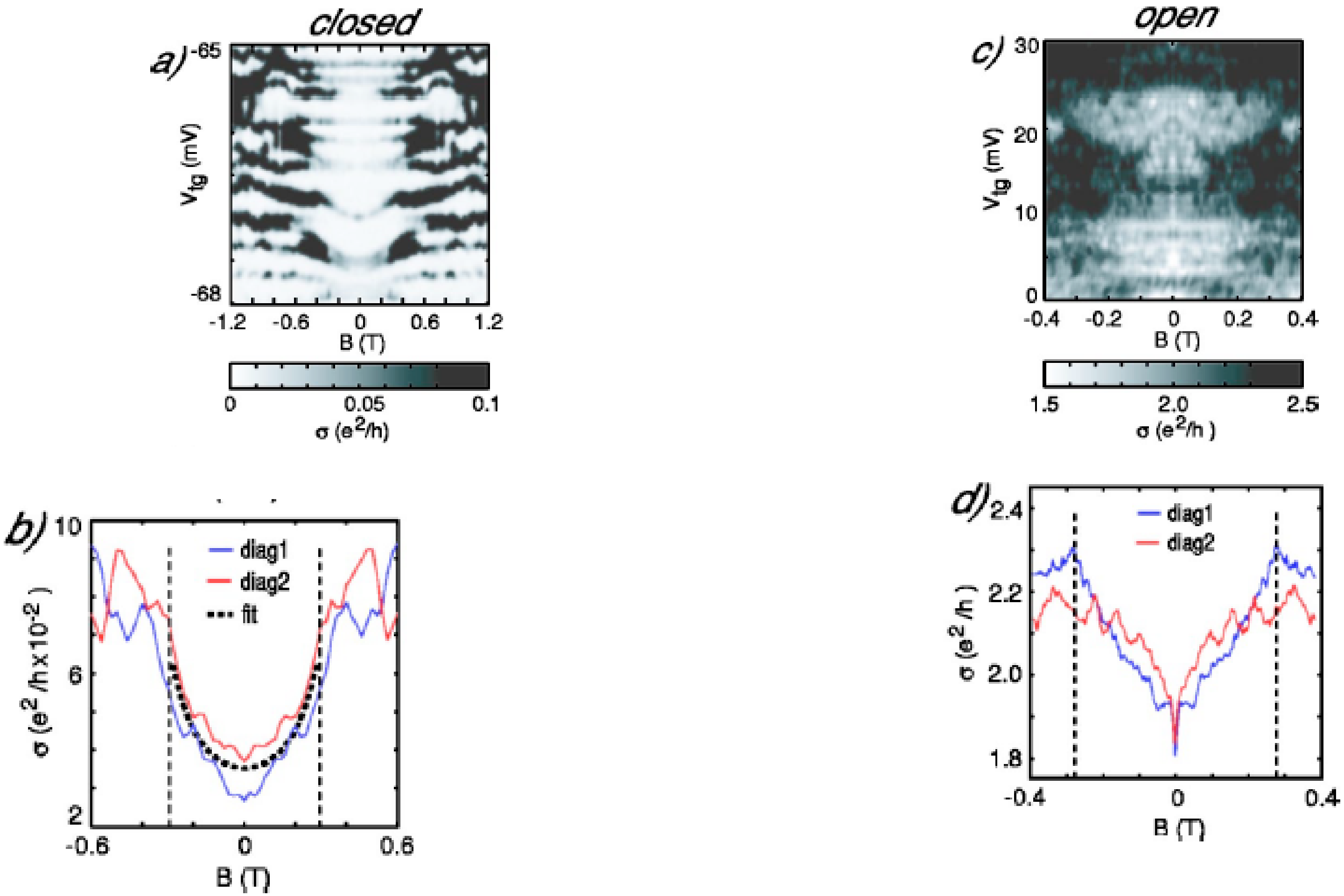
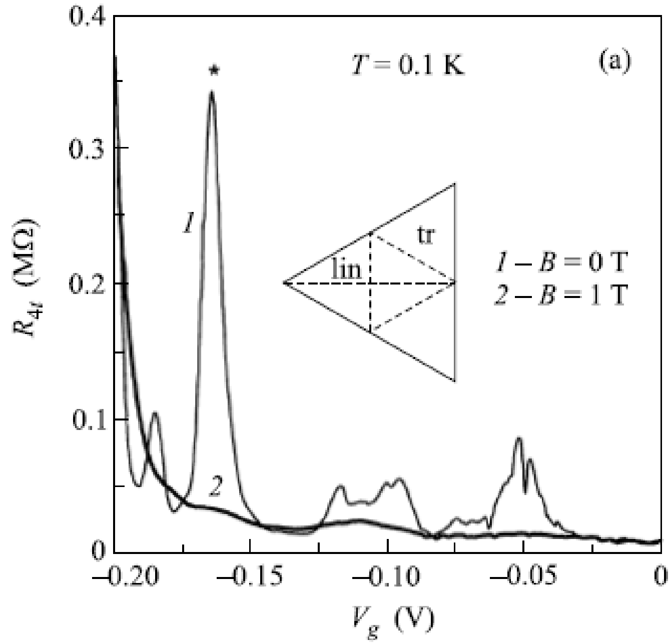


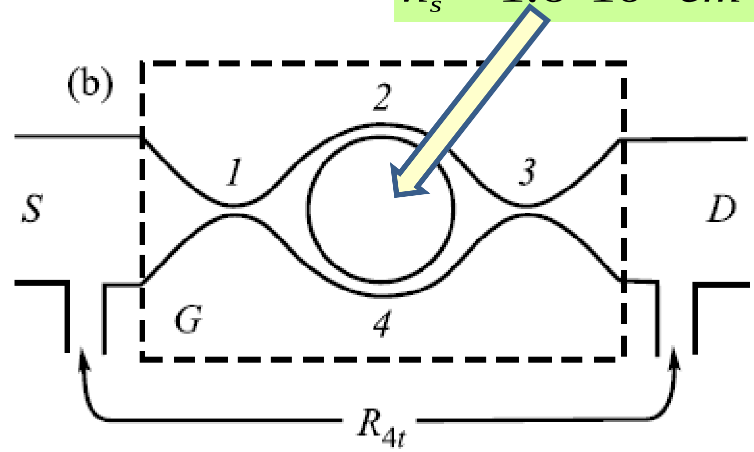
FIG. 5. (Color online) (a) Conductance as a function of top gate voltage and magnetic field in the Coulomb blockade regime at $T = 90$ mK. A clear positive magnetoconductance for individual Coulomb peaks is observed. (b) Averaged magnetoconductance between −54 and −70 mV in steps of 0.02 mV. Dashed lines mark a flux quantum through the unit cell (290 mT) and a fit using Eq. (4). (c) Conductance as a function of top gate voltage and magnetic field in the open regime. (d) Averaged magnetoconductance between 30 and 0 mV in steps of 0.25 mV. Dashed lines mark a flux quantum through the unit cell (290 mT).

A.A. Bykov, D.V. Nomokonov, A.K. Bakarov, O. Estibals, J.C. Portal

Pis'ma JETP 78, 36 (2003).



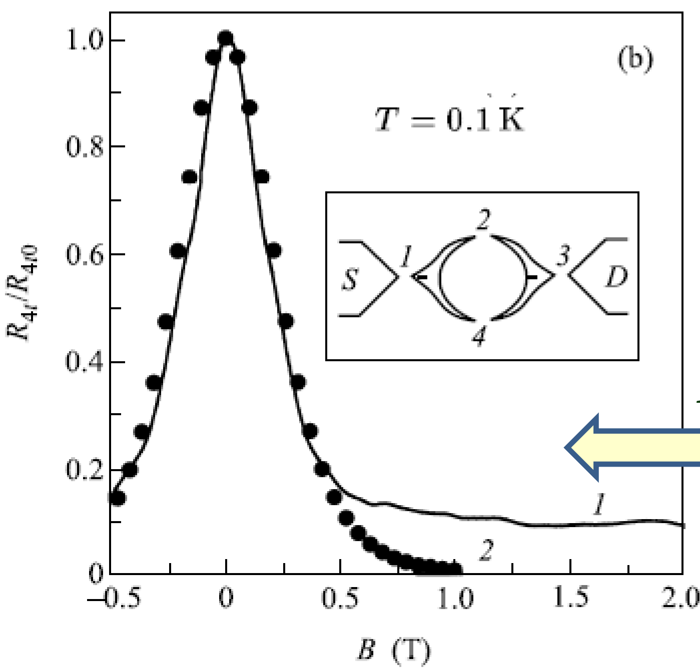
$n_s = 1.6 \cdot 10^{12} cm^{-2}$



Transport through a closed ring

$r_{eff} = 0.13\,\mu m$

$\mu = 4 \cdot 10^5 cm^2/V \cdot s$



Fit to

$$R(B)/R(0) = \exp(B^2/B_0^2)/\mathrm{ch}^2(B/B_1)$$

$B_0 = 2.0\,T$

$B_1 = 0.294\,T$

PHYSICAL REVIEW B VOLUME 41, NUMBER 12 15 APRIL 1990-II

Hopping transport in δ-doping layers in GaAs

δ -doped GaAs

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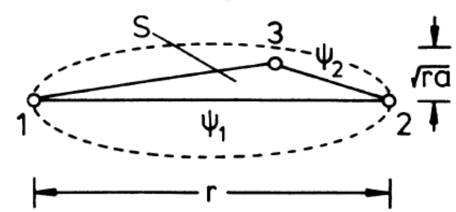
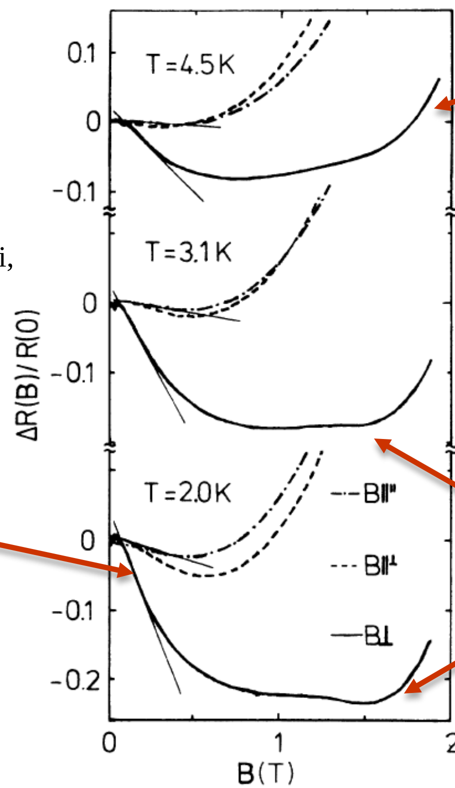
(Received 9 June 1989; revised manuscript received 4 December 1989)

weak-field negative-hopping magnetoresistance due to interference of tunneling paths

V. L. Nguen, B. Z. Spivak, and B. I. Shklovskii, JETP Lett. 43, 44 (1986).



positive magnetoresistance due to orbital shrinkage of the donor wave functions



feature

FIG. 8. Relative magnetoresistance $\Delta R(B)/R(0)$ as function of the magnetic field for the three configurations at various temperatures. The thin solid lines at low field indicate that the $\Delta R/R$ is linear in the B field.

VRH

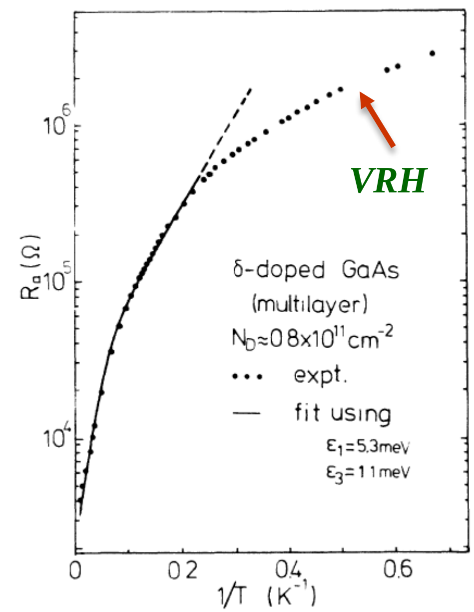


FIG. 2. Sheet resistance ($R_\square$) vs reciprocal temperature for layers contributing to the transport. The solid line models the transport on the basis of excitation to the band edge and nearest-neighbor hopping (NNH). Variable-range hopping (VRH) dominates above 0.2 K^{-1}.

Mechanisms of magnetoresistance in variable-range-hopping transport for two-dimensional electron systems

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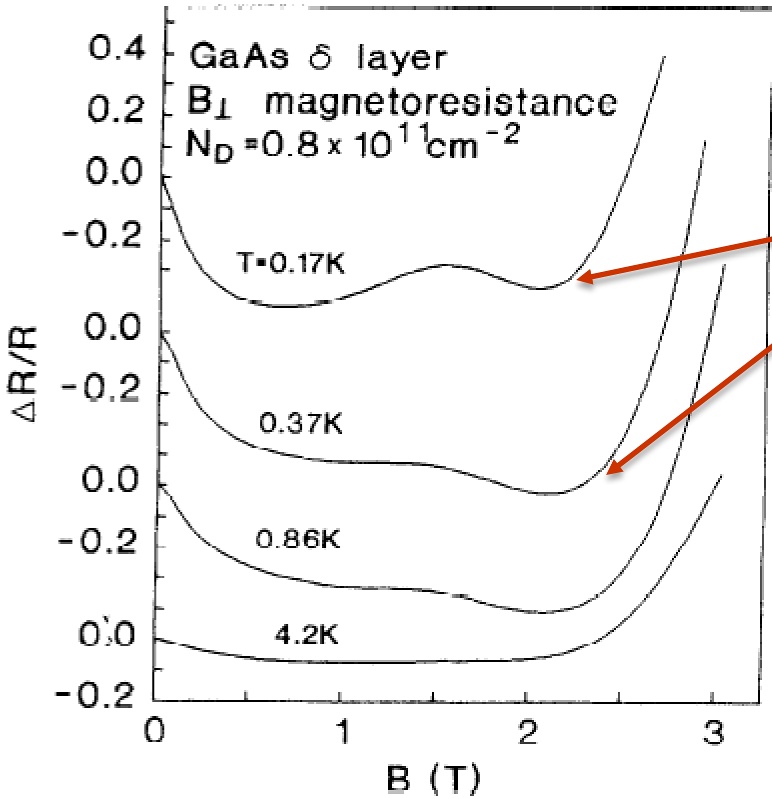
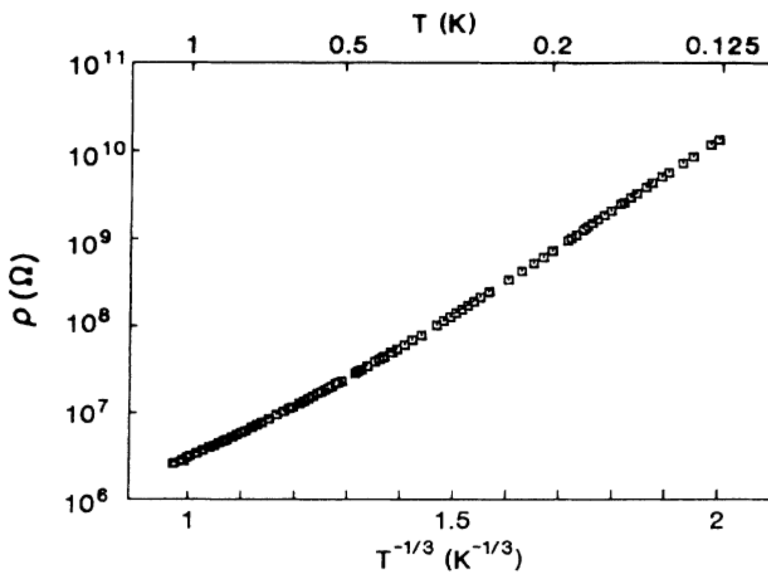


FIG. 1. Evolution of the negative magnetoresistance from the NNH regime (4.2 K in Ref. 8) to low temperatures where VRH applies. Note that $\Delta R(B)/R(0)$ exceeds 50% and is non-monotonic.

feature becomes more pronounced with decreasing temperature



Fitting to Mott's law

PHYSICAL REVIEW B VOLUME 46, NUMBER 19 15 NOVEMBER 1992-I

Giant negative magnetoresistance of a degenerate two-dimensional electron gas in the variable-range-hopping regime

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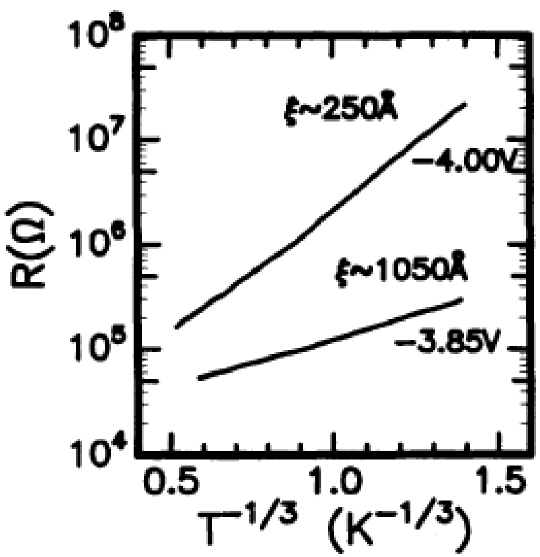
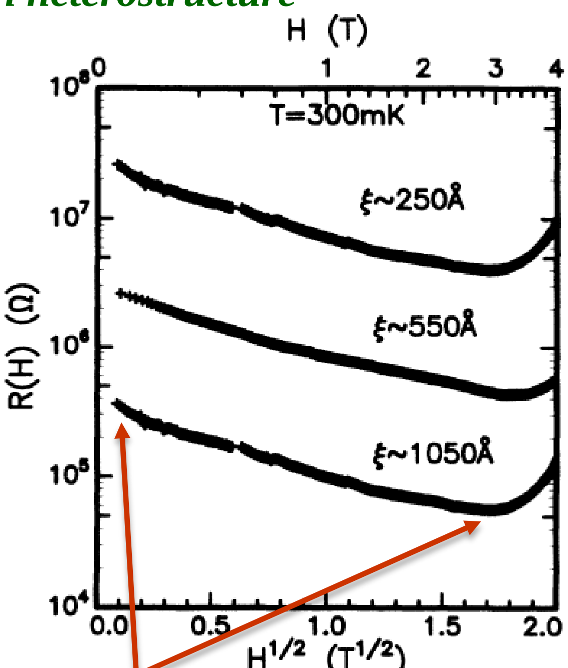
(Received 10 July 1992)

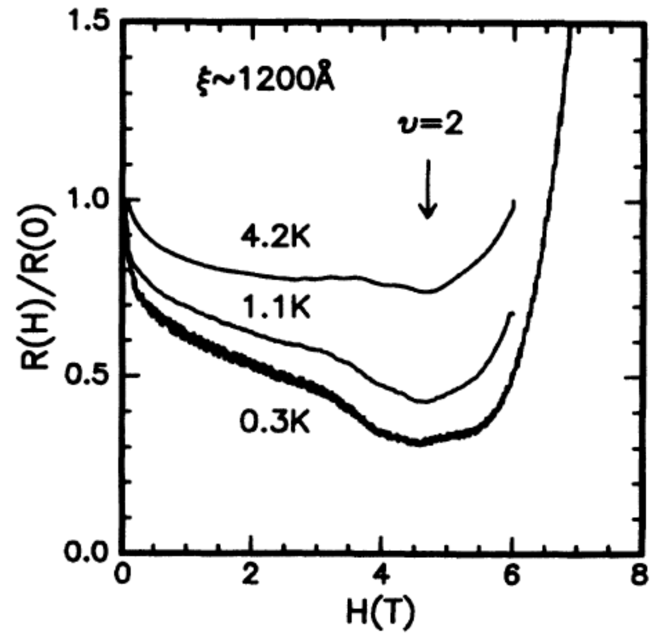
$GaAs/Al_xGa_{1-x}As$ ***MBE-grown heterostructure***



Localization radius is inferred from fitting to Mott's law



3 ***times net drop of resistance***



feature evolved into a deep minimum

VOLUME 71, NUMBER 9 PHYSICAL REVIEW LETTERS 30 AUGUST 1993

Observation of Magnetic-Field-Induced Delocalization: Transition from Anderson Insulator to Quantum Hall Conductor

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Massachusetts Institute of Technology, Francis Bitter National Magnet Laboratory, Cambridge, Massachusetts 02139
(Received 27 April 1993)

$GaAs/Al_xGa_{1-x}As$ *MBE-grown heterostructure*

measurements at **T=80mK**

two delocalization transitions are resolved within resistance minimum

in field domain betweeen two transitions Hall resistivity is quantized

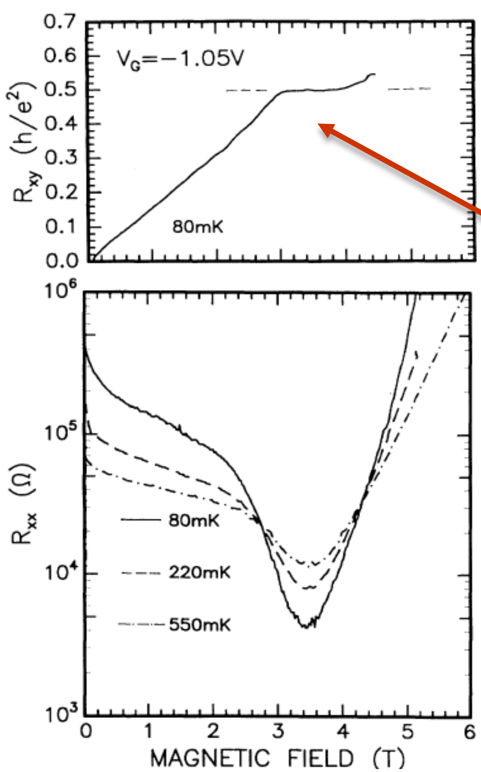
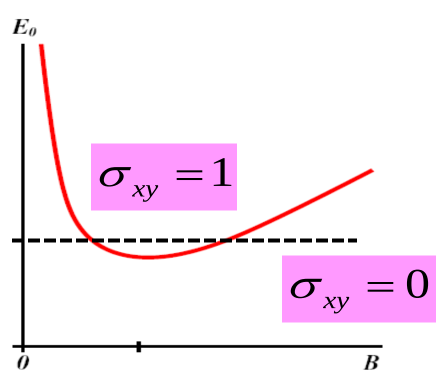


FIG. 2. Hall resistance and the longitudinal resistance as a function of the magnetic field (sample A).



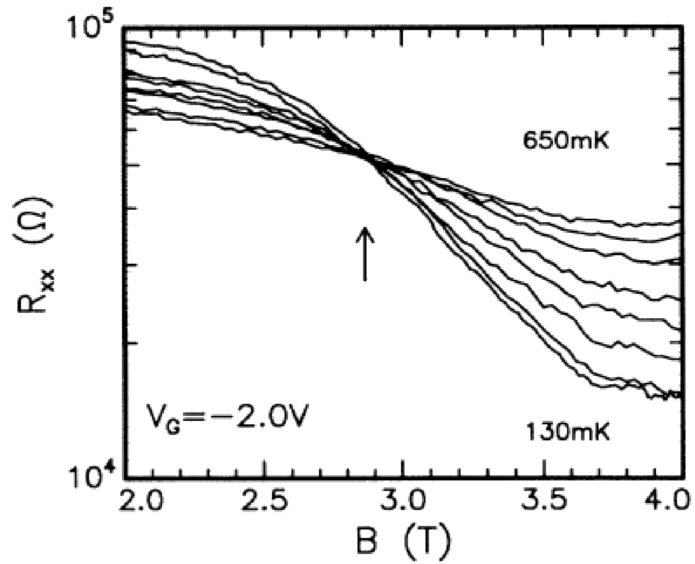


FIG. 4. R_{xx} vs B at $V_G = -2.0$ V (sample B) in a temperature range from 130 to 650 mK (160, 180, 250, 300, 400, and 550 mK in between the top and bottom curves).

resistivity insensitive to temperature-signature of criticality

J. Phys.: Condens. Matter 6 (1994) 4763–4770. Printed in the UK

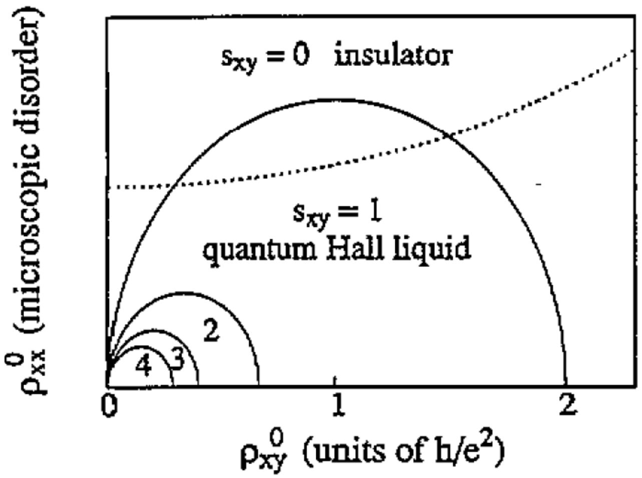
Magnetic-field-induced insulator–quantum Hall–insulator transition in a disordered two-dimensional electron gas

R J F Hughes†, J T Nicholls‡, J E F Frost†, E H Linfield†, M Pepper†, C J B Ford†, D A Ritchie†, G A C Jones†, Eugene Kogan†‡ and Moshe Kaveh†‡

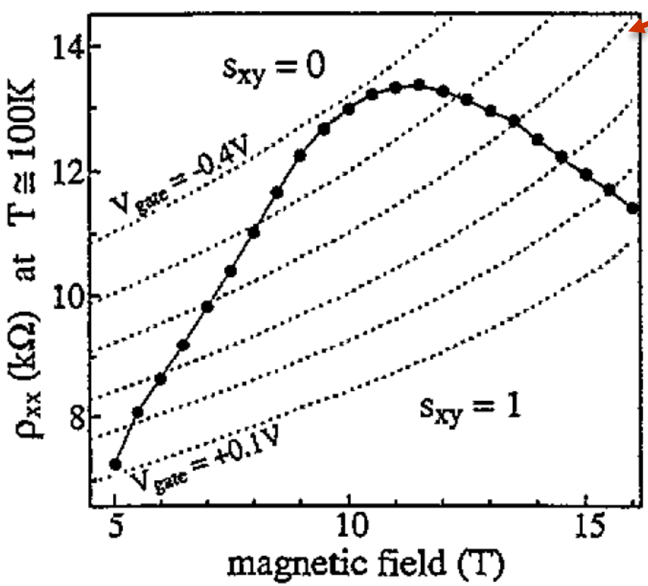
† Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, UK

‡ The Jack and Pearl Resnick Institute of Advanced Technology, Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

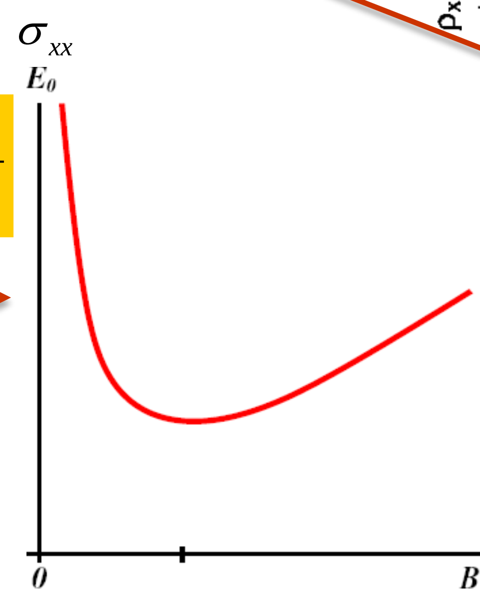
δ -doped GaAs gated structure

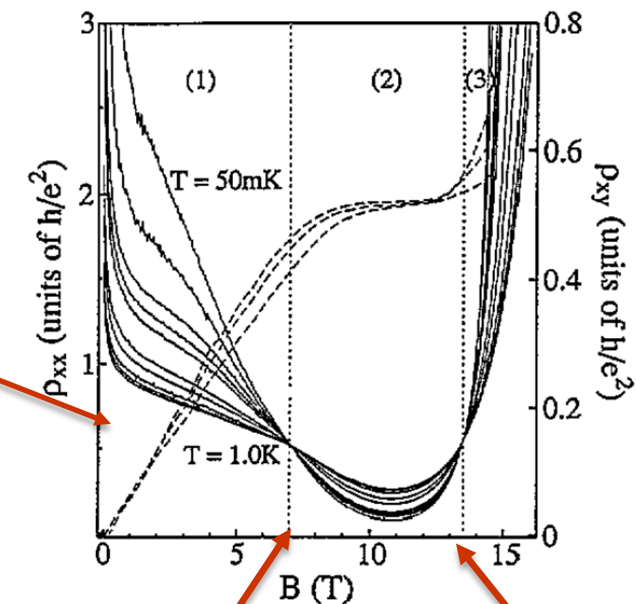


$V_{gate} = -0.2V$



$$\rho_{xx} \to \frac{1}{\sigma_{xx}}$$





B_L

B_H

$B_H \approx 2B_L$

Magnetic-Field-Induced Metal-Insulator Transition in Two Dimensions

T. Wang,[1] K. P. Clark,[1] G. F. Spencer,[1] A. M. Mack,[2] and W. P. Kirk[1]
[1]*Center for Nanostructure Materials and Quantum Device Fabrication, Engineering-Physics Building, Texas A&M University, College Station, Texas 77843-4242*
[2]*Department of Physics, University of Minnesota, Minneapolis, Minnesota 55455*

$GaAs/Al_{0.3}Ga_{0.7}As$ ***heterostructure***

$B_H \approx 4B_L$

two delocalization transitions survive at high temperatures

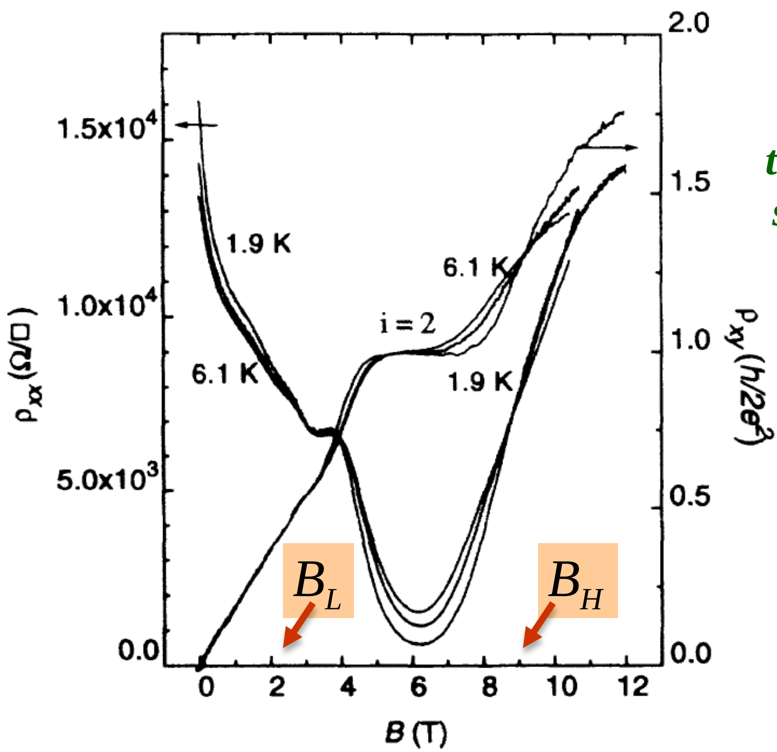


FIG. 1. Both ρ_{xx} (left axis) and ρ_{xy} (right axis) as a function of magnetic field B for sample A. Plotted are three temperatures: 6.1, 4.2, and 1.9 K.

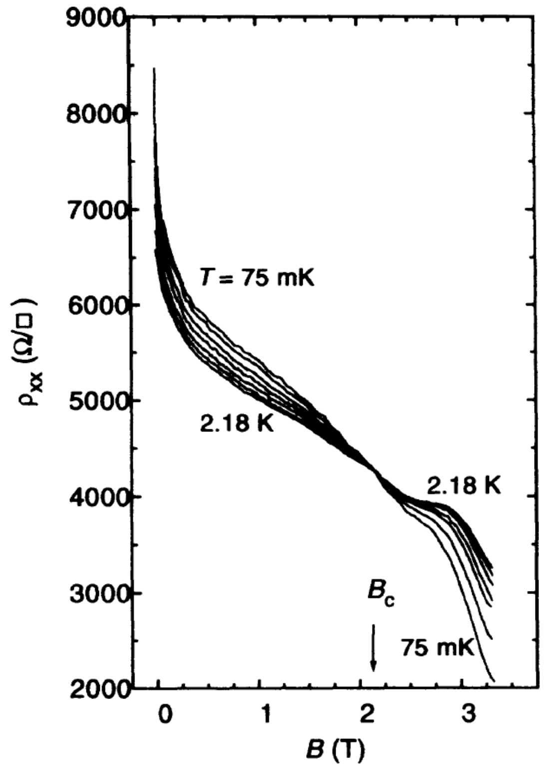


FIG. 3. The magnetoresistance curves $\rho_{xy}(B)$ for sample B. From the upper insulating curve to the lower insulating curve (i.e., $B < B_c$), the temperatures are 75 mK, 200 mK, 400 mK, 600 mK, 1.0 K, 1.5 K, and 2.18 K.

Nuclear Physics B235[FS11] (1984) 277–298

ON LOCALIZATION IN THE THEORY OF THE QUANTIZED HALL EFFECT: A TWO-DIMENSIONAL REALIZATION OF THE θ-VACUUM

A.M.M. PRUISKEN

Schlumberger-Doll Research, PO Box 307, Ridgefield, CT 06877, USA

It is shown that the localization problem in the theory of the quantized Hall effect is governed by the zero-component grassmannian U(2m) non-linear σ-model with a θ-term, a two-dimensional analogue of the θ-vacuum in Yang-Mills theory. In this case, θ is to be interpreted as the "bare" value for the Hall conductivity, determined by an underlying non-critical theory. A detailed derivation is presented starting from the replica method and a delta function distribution for the impurities.

A. M. M. Pruisken and I. S. Burmistrov, Ann. Phys. (N.Y.) **316**, 285 (2005).

Topological term-Shubnikov oscillations $\cos\left(2\pi E_F/\hbar\omega_c\right)$

$$\frac{\partial\sigma_{xx}}{\partial\ln L} = -\frac{1}{2\pi^2\sigma_{xx}} - \sigma_{xx}^2 D e^{-2\pi\sigma_{xx}}\cos\left(2\pi\sigma_{xy}\right)$$

$$-\frac{2\pi}{\omega_c\tau}\frac{E_F}{\hbar\omega_c}$$ ***instead of the Dingle factor*** $$-\frac{2\pi}{\omega_c\tau} = -2\pi\frac{\sigma_{xx}}{\sigma_{xy}}$$

$$\frac{\partial\sigma_{xy}}{\partial\ln L} = -\sigma_{xx}^2 D e^{-2\pi\sigma_{xx}}\sin\left(2\pi\sigma_{xy}\right)$$

Fate of the Delocalized States in a Vanishing Magnetic Field

I. Glozman, C. E. Johnson, and H. W. Jiang
Department of Physics, University of California at Los Angeles, Los Angeles, California 90024
(Received 2 June 1994)

GaAs/Al_xGa_{1-x}As ***MBE-grown heterostructure***

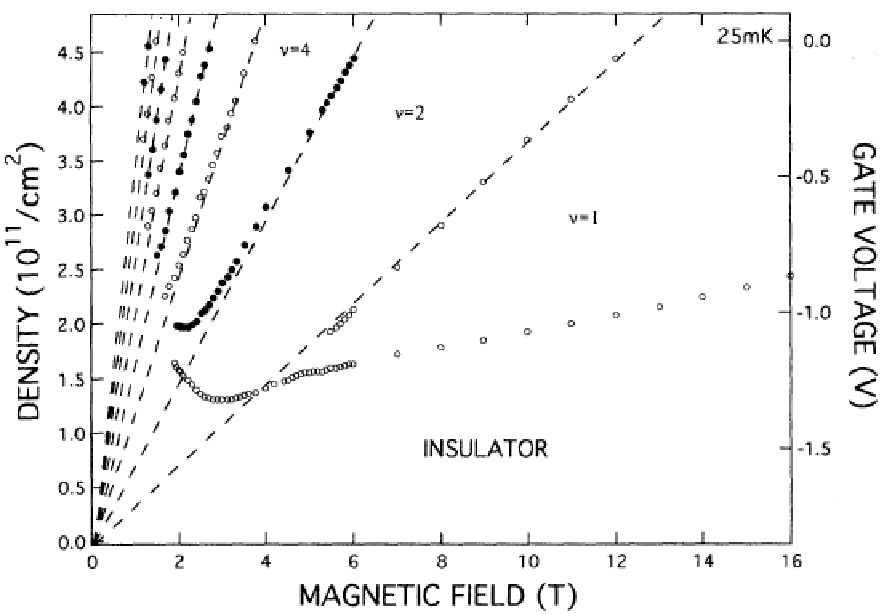


FIG. 2. Floating of the lowest delocalized state is shown here in the context of the delocalized states in the n-B plane. Quantum Hall conductor regions are labeled with appropriate filling factors or values of $\sigma_{xy}/(e^2/h)$, and are separated by metallic delocalized states. The energy of these delocalized states can be considered directly proportional to n in the data region. Dotted lines represent the traditional Landau levels, with both spin states plotted for the lowest level.

density vs. magnetic field phase diagram is inferred from the peaks in diagonal conductance at temperature
T=25mK

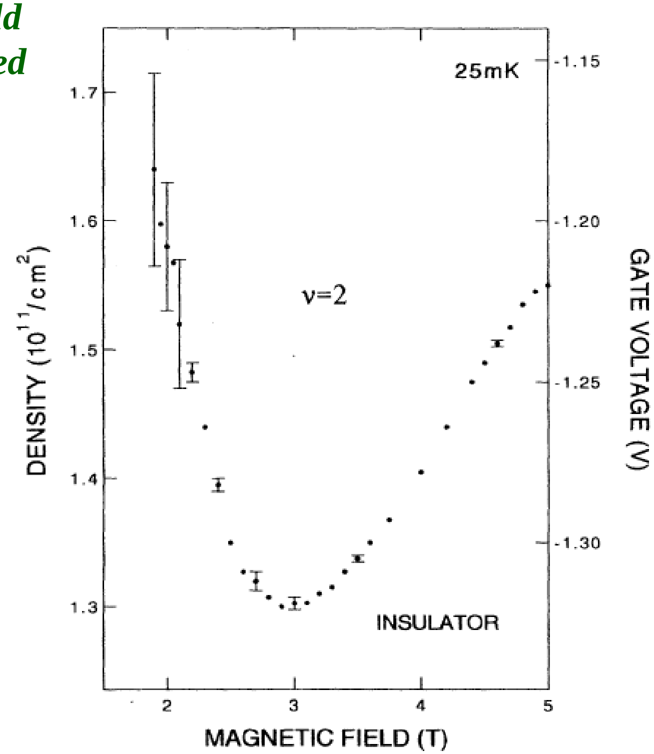


FIG. 4. Phase diagram for the lowest delocalized state, derived from the peaks in σ_{xx}, as in Fig. 3, unequivocally demonstrates floating. Error bars suggest the diminishing resolvability of the peak as $B \rightarrow 0$ and represent a reasonable uncertainty in the peak position.

Symmetry in the insulator–quantum-Hall–insulator transitions observed in a Ge/SiGe quantum well

M. Hilke, D. Shahar, S. H. Song, and D. C. Tsui
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544

Y. H. Xie and Don Monroe
Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974

(Received 29 August 1997)

hole gas in Ge/SiGe ***strained quantum well***

$B_H \approx 2B_L$

$\rho_c^L = \rho_c^H$

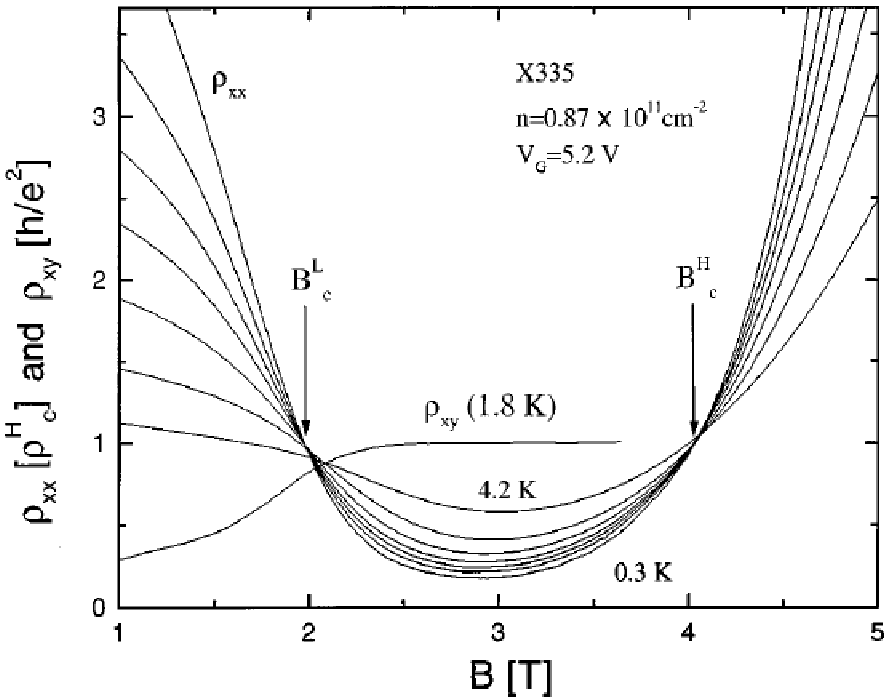
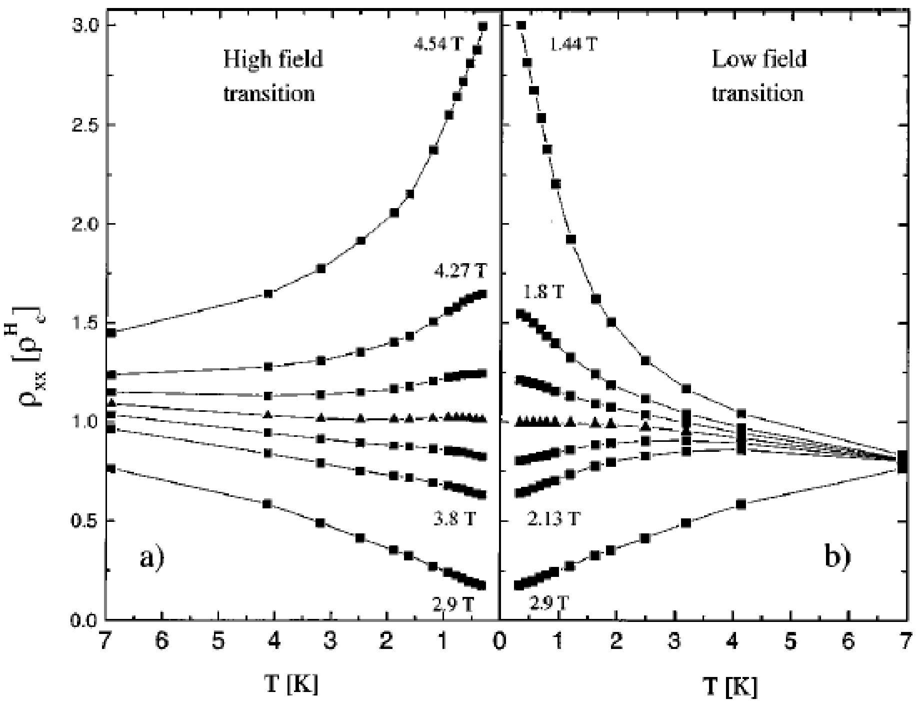


FIG. 1. Diagonal and Hall resistivity as a function of magnetic field. $\rho_c^H = 2.2h/e^2$. The temperatures are 0.3, 0.55, 0.75, 0.9, 1.2, 2.4, and 4.2 K.



energy scale for high-field transition is larger

Nuclear Physics B235[FS11] (1984) 277–298

ON LOCALIZATION IN THE THEORY OF THE QUANTIZED HALL EFFECT: A TWO-DIMENSIONAL REALIZATION OF THE θ-VACUUM

A.M.M. PRUISKEN

Schlumberger-Doll Research, PO Box 307, Ridgefield, CT 06877, USA

It is shown that the localization problem in the theory of the quantized Hall effect is governed by the zero-component grassmannian U(2m) non-linear σ-model with a θ-term, a two-dimensional analogue of the θ-vacuum in Yang-Mills theory. In this case, θ is to be interpreted as the "bare" value for the Hall conductivity, determined by an underlying non-critical theory. A detailed derivation is presented starting from the replica method and a delta function distribution for the impurities.

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$$-\frac{2\pi}{\omega_c\tau}\frac{E_F}{\hbar\omega_c}$$ ***instead of the Dingle factor*** $$-\frac{2\pi}{\omega_c\tau}=-2\pi\frac{\sigma_{xx}}{\sigma_{xy}}$$

$$\frac{\partial\sigma_{xy}}{\partial\ln L}=-\sigma_{xx}^2 De^{-2\pi\sigma_{xx}}\sin\left(2\pi\sigma_{xy}\right)$$

New Universality at the Magnetic Field Driven Insulator to Integer Quantum Hall Effect Transitions

S.-H. Song, D. Shahar, and D. C. Tsui
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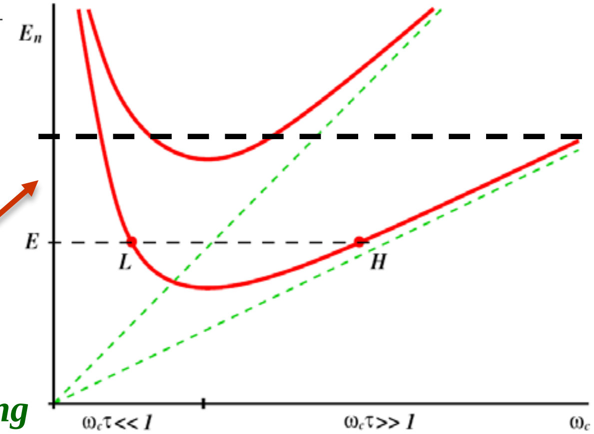
Y. H. Xie and Don Monroe
Bell Laboratories, Lucent Technologies, 700 Mountain Ave., Murray Hill, New Jersey 07974
(Received 3 July 1996)



experimentally, levitation is not strong

hole gas in Ge/SiGe ***strained quantum well***

transition sequence

$0 \to 3 \to 2 \to 1 \to 0$

is resolved

$B_H \approx 4B_L$

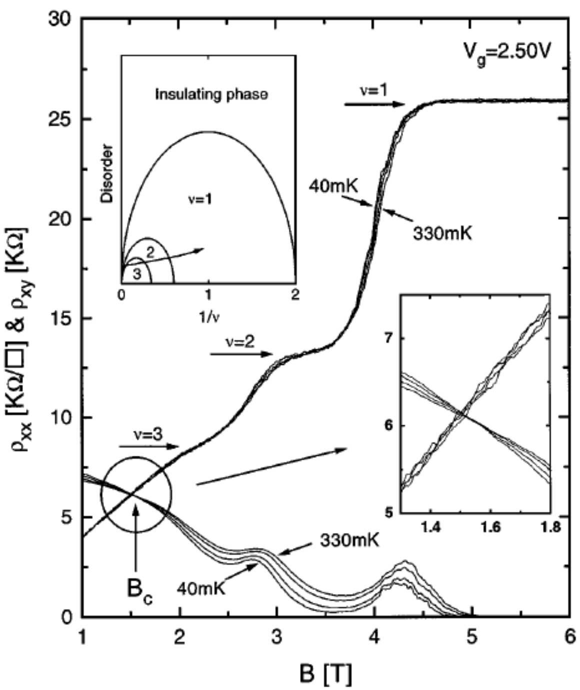


FIG. 1. ρ_{xx} and ρ_{xy} versus magnetic field (B) for $V_g = 2.5$ V at $T = 40$, 130, 230, and 330 mK. B_c (1.56 T) denotes the critical magnetic field at which the transition from an insulator to QHE occurs. The top-left inset shows the IQHE portion of the theoretical phase diagram suggested by KLZ. The arrow indicates a possible trajectory for a sample. The bottom-right inset shows a magnified view around the transition.

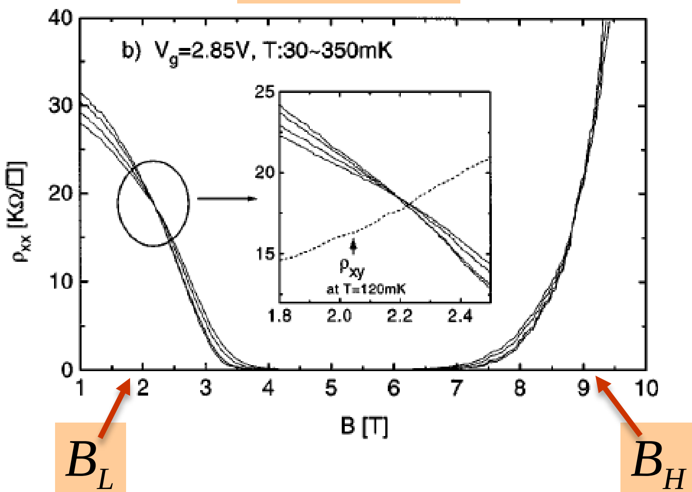


FIG. 2. ρ_{xx} vs B traces for two V_g values. The insets show traces of ρ_{xy} at 120 mK and magnified views of the crossing points. (a) $V_g = 2.72$ V and $T = 30$, 120, 240, and 320 mK. (b) $V_g = 2.85$ V and $T = 30$, 120, 250, and 350 mK.

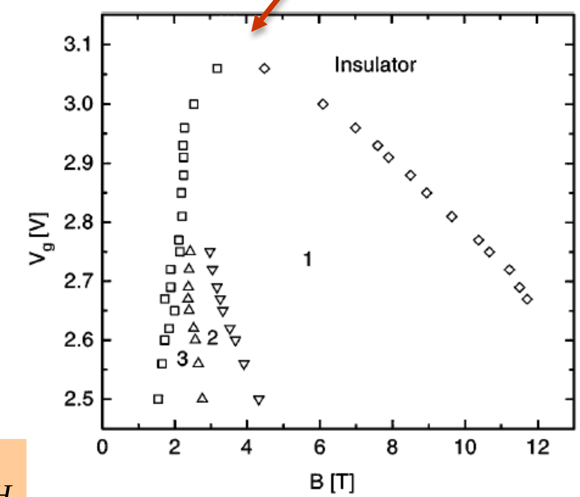


FIG. 4. The experimental phase diagram for 2DHG in our strained Ge quantum well. Different symbols denote different transitions; (□) low-B insulator to QHE transition, (△) $\nu = 3$ to $\nu = 2$ QHE transition, (▽) $\nu = 2$ to $\nu = 1$ QHE transition, and (◇) $\nu = 1$ to high-B insulator transition deduced from B values where $\rho_{xx} = h/e^2$.

Volume 106A, number 4 PHYSICS LETTERS 3 December 1984

QUANTUM HALL EFFECT AND ADDITIONAL OSCILLATIONS OF CONDUCTIVITY IN WEAK MAGNETIC FIELDS

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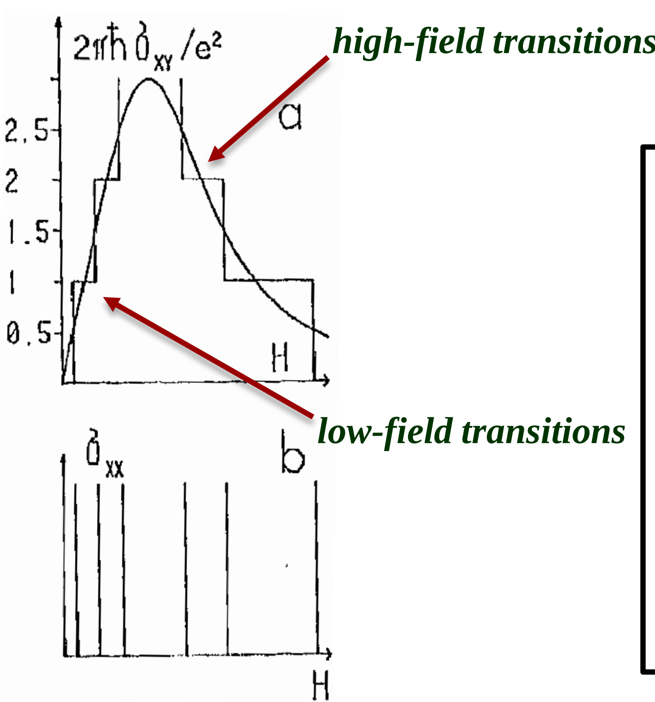
Received 23 July 1984

In the framework of the scaling hypothesis for localization of 2D electrons in a magnetic field B it is shown that at $T = 0$ the conductivity σ_{xx} has maxima at $B_n^{(1,2)} = (mc/e\hbar)\{E_F/(2n+1) \pm \{[E_F/(2n+1)]^2 - (\hbar/\tau)^2\}^{1/2}\}$, i.e. besides oscillations of the Shubnikov type with maxima at $B_n^{(1)} \approx (mc/e\hbar)E_F/(n+1/2)$ there is the same number of additional oscillations at $B_n^{(2)} \approx (mc/e)(\hbar/E_F\tau^2)(n+1/2)$. The Hall conductivity $\sigma_{xy} = (e^2/2\pi\hbar)n$ at $B_n^{(1)} < B < B_{n-1}^{(1)}$ and at $B_{n-1}^{(2)} < B < B_n^{(2)}$ gives the number of delocalized states at $E < E_F$ with energies E_n described by the interpolating relation $E_n = \hbar\Omega(n+1/2)[1 + (\Omega\tau)^{-2}]$. At $B < B_0$, when $r_H > l$ (r_H is the magnetic length, l is the mean free path), all states with $E < E_F$ are localized.



VOLUME 52, NUMBER 25 PHYSICAL REVIEW LETTERS 18 JUNE 1984

Levitation of Extended-State Bands in a Strong Magnetic Field

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Livermore, California 94550

$$\sigma_{xy}^0 = (\omega_c\tau)\sigma_{xx}^0$$

$$E_n = (n + \tfrac{1}{2})\hbar\omega_c\left[\frac{1 + (\omega_c\tau)^2}{(\omega_c\tau)^2}\right]$$

with initial conditions

$\sigma_0 = k_F l$

$$\sigma_{xx}\big|_{L\sim l} = \frac{\sigma_0}{1+(\omega_c\tau)^2}, \qquad \sigma_{xy}\big|_{L\sim l} = \frac{\sigma_0(\omega_c\tau)}{1+(\omega_c\tau)^2}$$

fixed point

$$\sin 2\pi\sigma_{xy} = 0, \quad \cos 2\pi\sigma_{xy} = -1$$

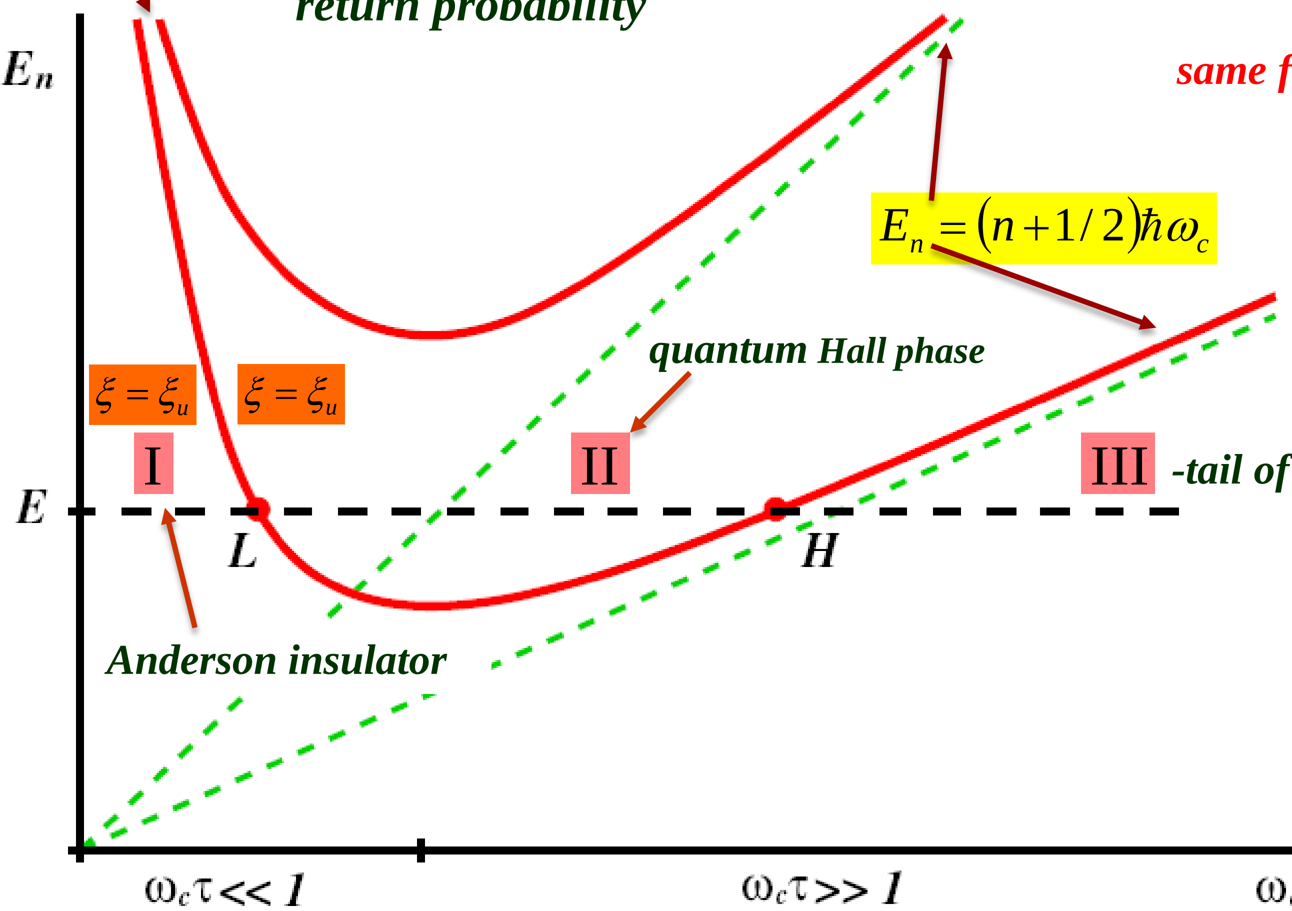
$$E_n = (n+1/2)\frac{\hbar}{\omega_c\tau^2} \;\Rightarrow\; \frac{1}{k_F l} \sim \omega_c\tau$$

return probability ***rotation angle***

$$E_n = \hbar\omega_c\left(n+\frac{1}{2}\right)\left[1+\frac{1}{(\omega_c\tau)^2}\right]$$

same for unitary and orthogonal classes

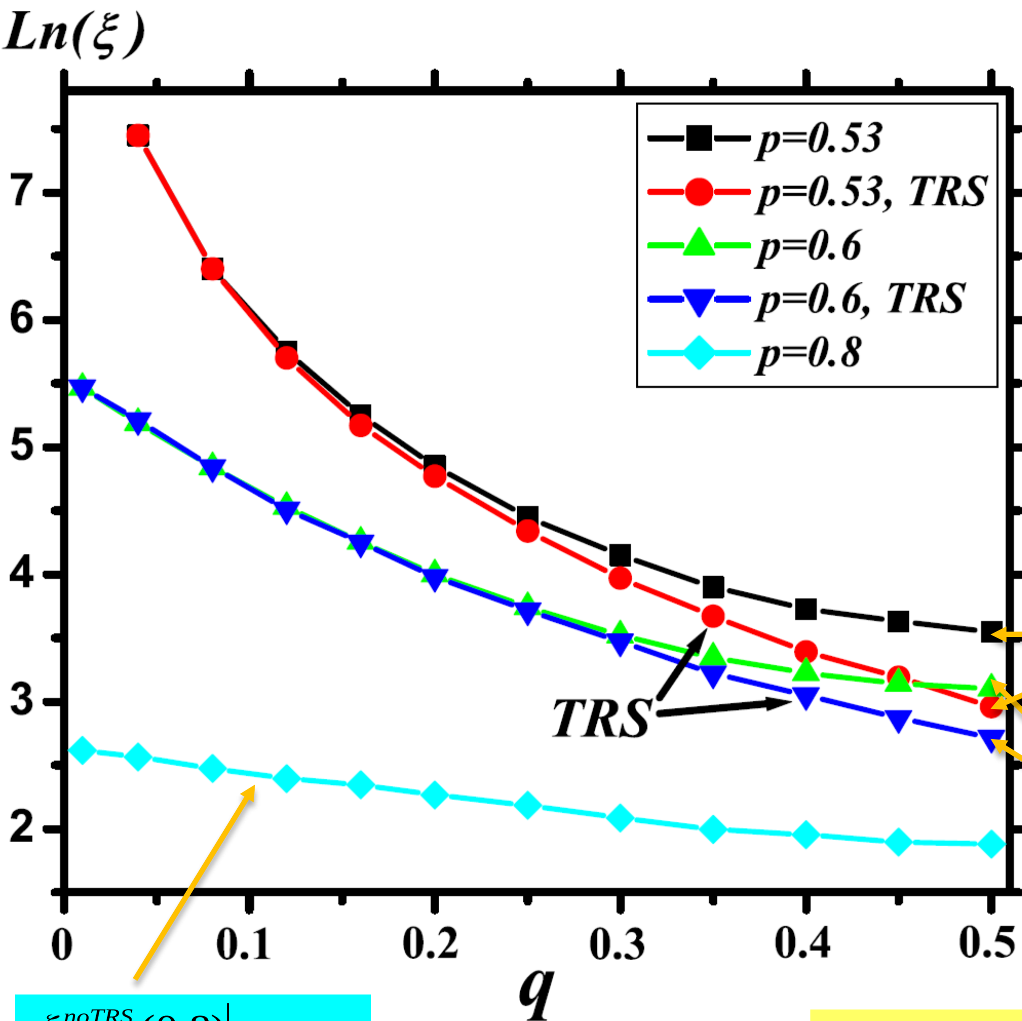
at any field when classical non-diagonal conductivity assumes the value $\sigma_{xy} = n+1/2$ ***quantum effects lead to discrete delocalized state***

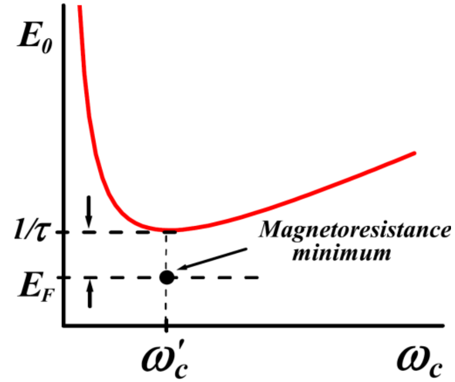


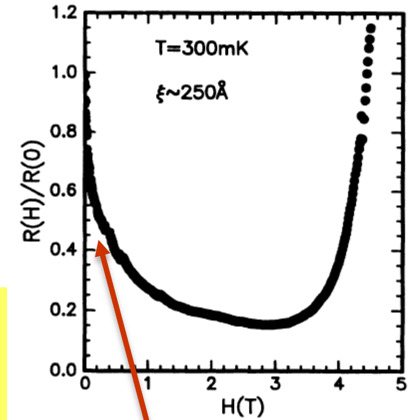
-tail of the lowest Landau level

Deeply insulating regime: ξ determines ρ_{xx} via Mott's law

$$\ln \rho_{xx} \propto \frac{1}{\left(\xi^2 T\right)^{1/3}}$$







$$\left.\frac{\xi^{noTRS}(0.53)}{\xi^{TRS}(0.53)}\right|_{q=1/2} = 1.82$$

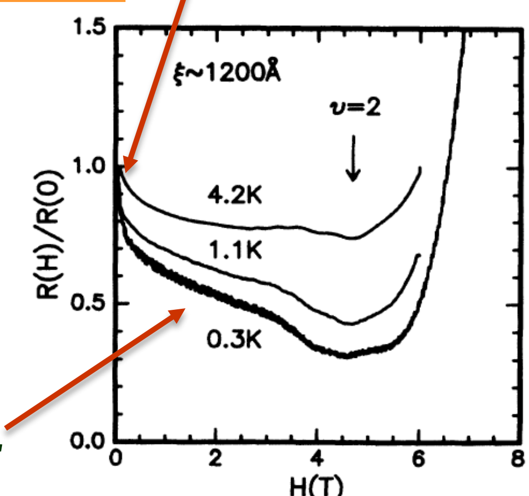
$$\left.\frac{\xi^{noTRS}(0.6)}{\xi^{TRS}(0.6)}\right|_{q=1/2} = 1.47$$

interference-induced NMR

$$\frac{\xi^{noTRS}(0.8)\big|_{q=0}}{\xi^{noTRS}(0.8)\big|_{q=1/2}} = 2$$

$$\frac{\xi^{noTRS}(0.6)\big|_{q=0}}{\xi^{noTRS}(0.6)\big|_{q=1/2}} = 11$$

$$\frac{\xi^{noTRS}(0.53)\big|_{q=0}}{\xi^{noTRS}(0.53)\big|_{q=1/2}} = 60$$



with orbital action of magnetic field, NMR is much stronger

PHYSICAL REVIEW B VOLUME 53, NUMBER 15 15 APRIL 1996-I

Magnetic-field-induced crossover from Mott variable-range hopping to weakly insulating behavior

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(Received 1 November 1995)

In three-dimensional n-CdSe samples that obey Mott variable-range hopping, $\rho=\rho_0\exp(T_0/T)^{1/4}$, in the absence of a magnetic field, we report a field-induced crossover at low temperatures to a resistivity that exhibits a weak power-law divergence, $\rho=\rho_0 T^{-\alpha}$, with an exponent α that decreases slowly with increasing field.

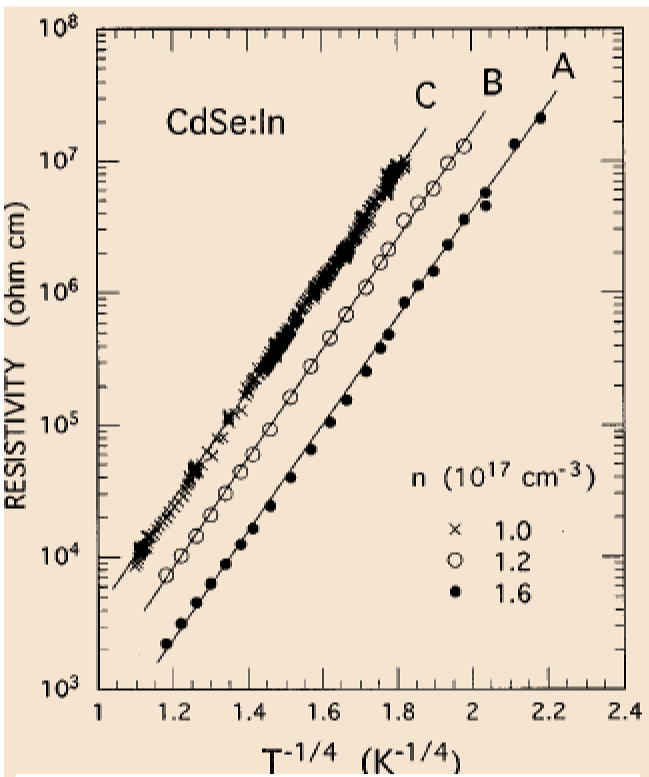

FIG. 1. Resistivity in zero field vs $T^{-1/4}$ on a semilogarithmic scale for three insulating, n-type CdSe:In samples with dopant densities $0.57n_c$ (sample A), $0.43n_c$ (sample B), and $0.36n_c$ (sample C) (based on a critical concentration $n_c=2.8\times10^{17}$ cm^{-3}). The resistivity obeys Mott variable-range hopping $\rho=\rho_0\exp[(T_0/T)^{1/4}]$ with $T_0\approx7400$, 8400, and 8400 K, and $\rho_0=0.037$, 0.083, and 0.30Ω cm for samples A, B, and C, respectively.