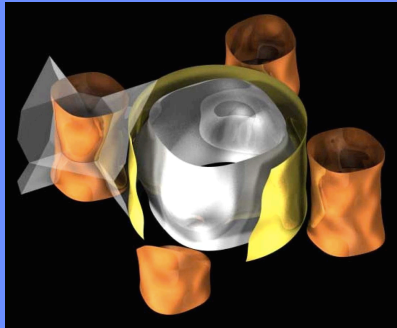
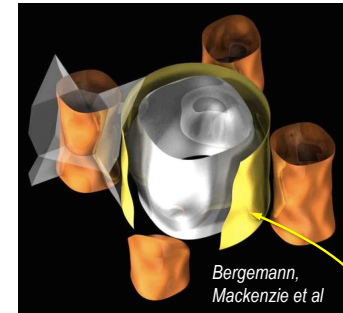
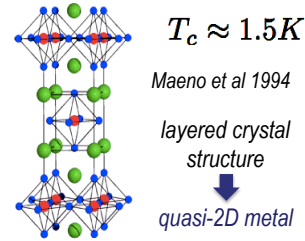


# Sr<sub>2</sub>RuO<sub>4</sub> - quasi-2D-superconductor

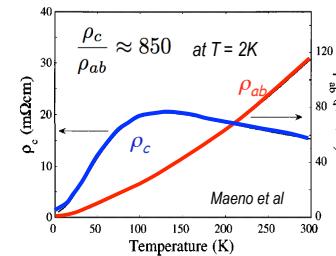


Example 1:  
The spin-triplet superconductor  
Sr<sub>2</sub>RuO<sub>4</sub>



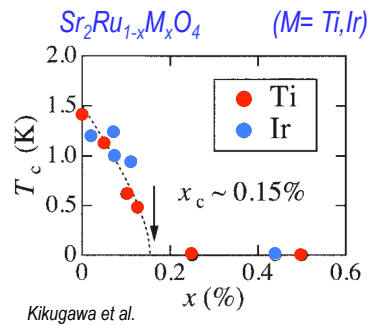
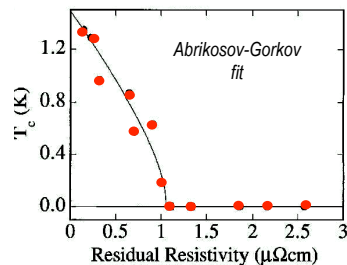
3 quasi-2D bands  
derived from 4d-t<sub>2g</sub> orbitals of Ru

most likely  $\gamma$ -band strongly dominant



## Unconventional pairing - disorder effect

sensitivity to non-magnetic impurities



textbook like  
suppression of  $T_c$

mean free path  
 $\ell \leq \xi \approx 660 \text{ \AA}$   
 $T_c = 0$

## superconducting phases - analogy to <sup>3</sup>He

basic symmetries:

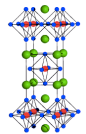
$$\mathcal{G} = G \times S \times \mathcal{K} \times U(1)$$

p-wave pairing states:

point group:  $D_{4h}$  tetragonal

spin-orbit coupling strong

inplane pairing ( $\vec{k} = (k_x, k_y, 0)$ )



<sup>3</sup>He phases

B-phase

$$\vec{d}(\vec{k}) = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$$

A-phase

$$\vec{d}(\vec{k}) = \hat{z}(k_x \pm ik_y)$$

$$\hat{\Psi}_{\vec{k}} = (\vec{d}(\vec{k}) \cdot \hat{\sigma})i\hat{\sigma}^y$$



$\Gamma$	$\vec{d}(\vec{k})$	
$A_{1u}$	$\vec{d} = \hat{x}k_x + \hat{y}k_y$	} B-phase
$A_{2u}$	$\vec{d} = \hat{x}k_y - \hat{y}k_x$	
$B_{1u}$	$\vec{d} = \hat{x}k_x - \hat{y}k_y$	
$B_{2u}$	$\vec{d} = \hat{x}k_y + \hat{y}k_x$	
$E_u$	$\vec{d} = \hat{z}(k_x \pm ik_y)$	} A-phase

## Experimental evidence for pairing symmetry

**spin-polarizability**

NMR-Knight shift

Ishida et al (1998)

inplane equal-spin pairing  $\vec{d} \parallel \hat{z}$

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**magnetic moment**

$\mu$ SR zero-field relaxation

Luke et al (1998)

intrinsic magnetism

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**field distribution**

$\mu$ SR field distribution in vortex phase

Luke et al (2000)

2-component order parameter

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**spin-polarizability**

NMR-Knight shift

Ishida et al (1998)

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Luke et al (1998)

intrinsic magnetism

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$\mu$ SR field distribution in vortex phase

Luke et al (2000)

2-component order parameter

**Ultrasound absorption**

Sound velocity renormalization for transversal mode

Lupien, Taillefer et al.

2-component order parameter

## superconducting phases - analogy to $^3\text{He}$

basic symmetries:

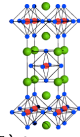
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$^3\text{He}$  phases

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$$\hat{\Psi}_{\vec{k}} = (\vec{d}(\vec{k}) \cdot \hat{\sigma}) i\hat{\sigma}^y$$



$\Gamma$	$\vec{d}(\vec{k})$	
$A_{1u}$	$\vec{d} = \hat{x}k_x + \hat{y}k_y$	} <b>B-phase</b>
$A_{2u}$	$\vec{d} = \hat{x}k_y - \hat{y}k_x$	
$B_{1u}$	$\vec{d} = \hat{x}k_x - \hat{y}k_y$	
$B_{2u}$	$\vec{d} = \hat{x}k_y + \hat{y}k_x$	
$E_u$	$\vec{d} = \hat{z}(k_x \pm ik_y)$	} <b>A-phase</b>

## $\text{Sr}_2\text{RuO}_4$ - chiral p-wave superconductor

$$\vec{d}(\vec{k}) = \hat{z}(k_x \pm ik_y) \Rightarrow \hat{\Psi}_{\vec{k}} = \begin{pmatrix} 0 & k_x \pm ik_y \\ k_x \pm ik_y & 0 \end{pmatrix}$$

degeneracy: 2

topological phase

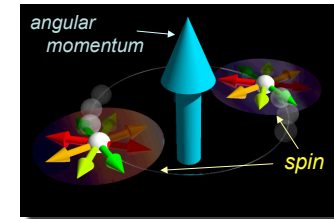
$$k_x + ik_y \xleftrightarrow{\hat{K}} k_x - ik_y$$

broken time reversal symmetry  $\mathcal{K}$

$$D_{4h} \times SU(2) \times \mathcal{K} \times U(1)_\phi$$

analog to  $^3\text{He}$  A-phase

$$U(1)_{S_z} \times U(1)_{L_z + \phi}$$



Deguchi & Maeno

## $\text{Sr}_2\text{RuO}_4$ - chiral p-wave superconductor

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degeneracy: 2

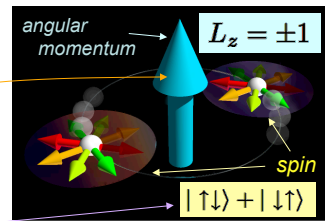
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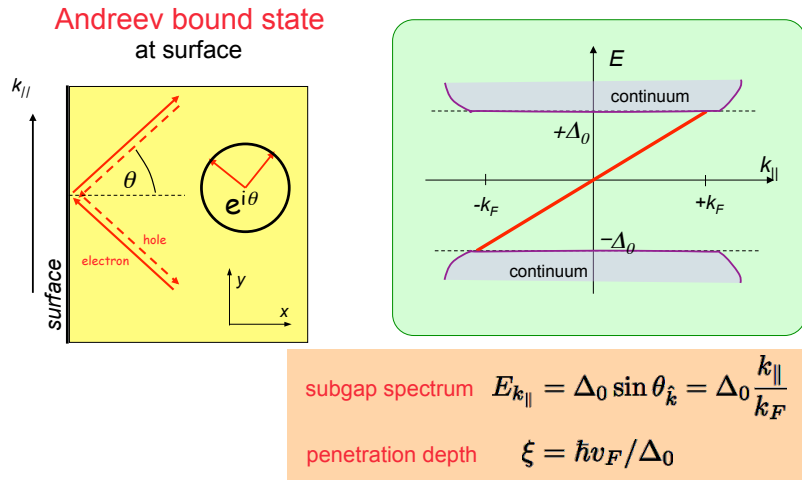


Deguchi & Maeno

Edge states  
&  
Spontaneous currents

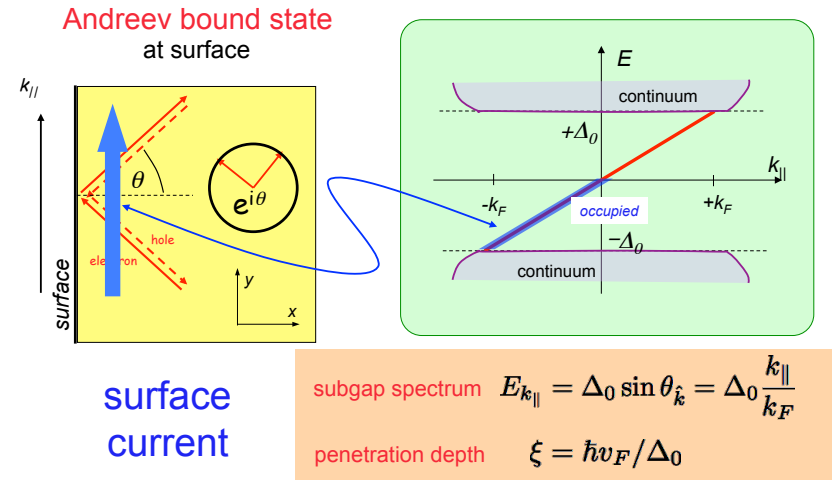
## Spontaneous currents

at inhomogeneities, surface and domain wall



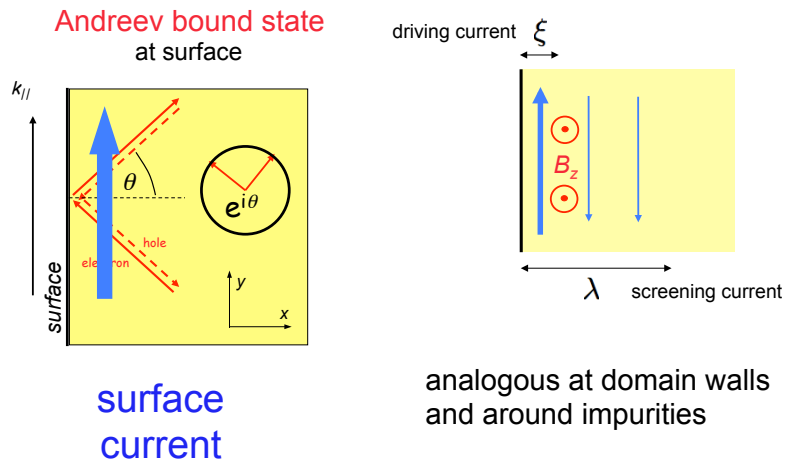
## Spontaneous currents

at inhomogeneities, surface and domain wall



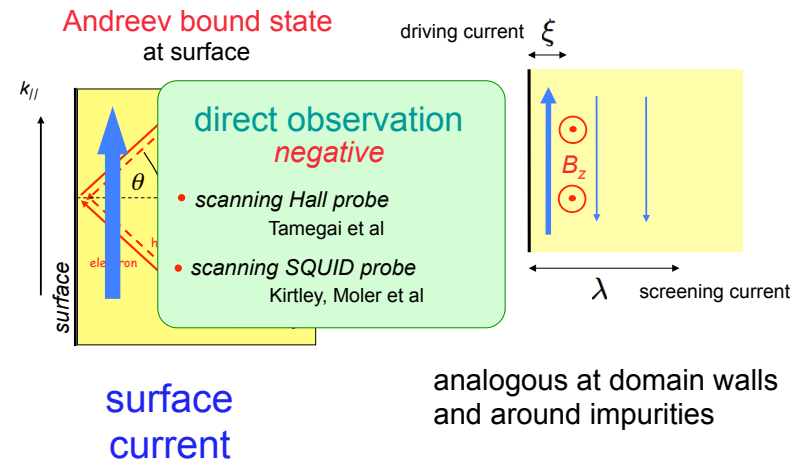
## Spontaneous currents

at inhomogeneities, surface and domain wall



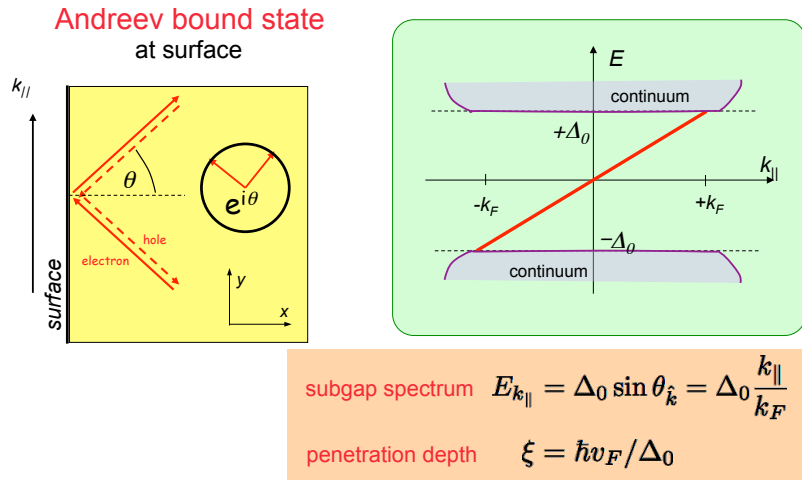
## Spontaneous currents

at inhomogeneities, surface and domain wall



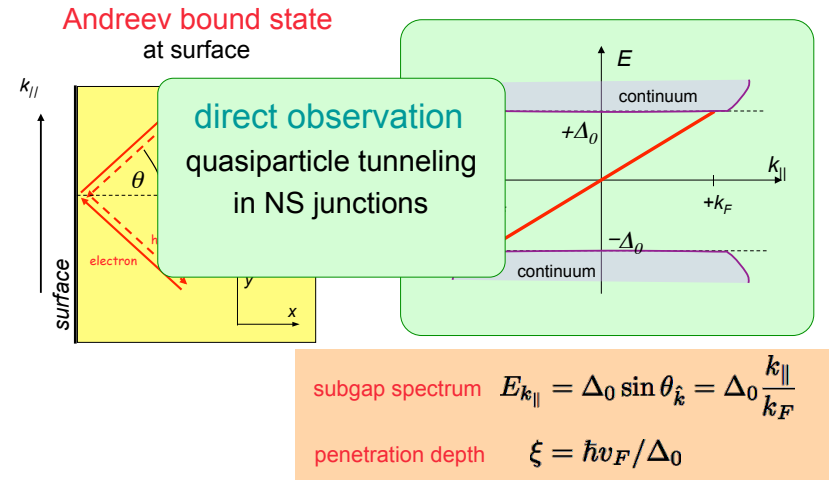
## Quasiparticle states

at inhomogeneities, surface and domain wall



## Quasiparticle states

at inhomogeneities, surface and domain wall

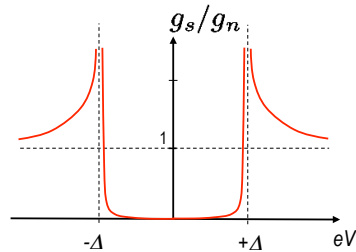


## Quasiparticle tunneling in NS junctions

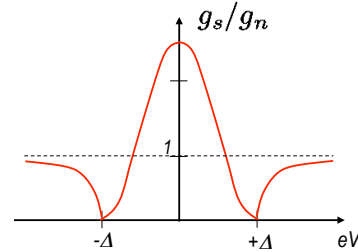
Tunneling conductance

$$g(eV) = \frac{dI}{dV}(eV)$$

conventional superconductor



chiral p-wave superconductor

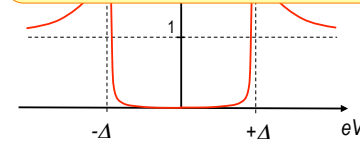
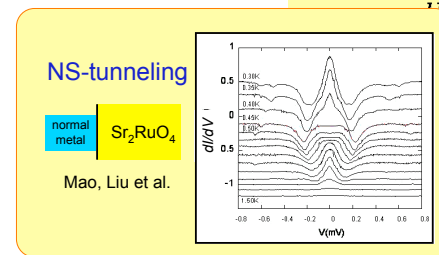


Honerkamp, Matsumoto & MS Yamashiro, Tanaka et al.

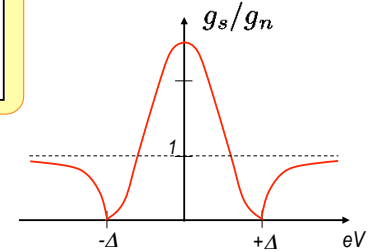
## Quasiparticle tunneling in NS junctions

Tunneling conductance

(eV)

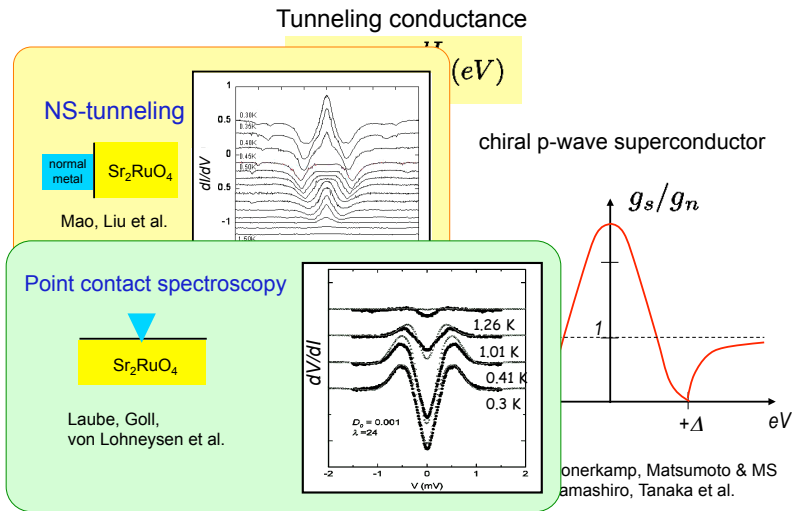


chiral p-wave superconductor



Honerkamp, Matsumoto & MS Yamashiro, Tanaka et al.

# Quasiparticle tunneling in NS junctions

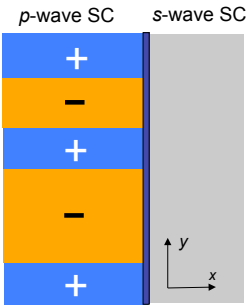


# Further evidence for chiral p-wave pairing

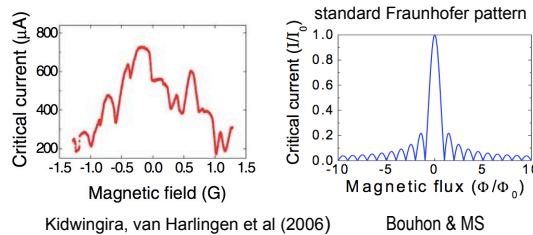
# Phase sensitive experiments

Josephson effect and domain walls

two domains  $\vec{d}_{\pm}(\vec{k}) = \eta_0 \hat{z}(k_x \pm ik_y)$



interference effect in magnetic field



$$I = \max_{\alpha} \int dy I_c(y) \sin[kx + \phi(y) + \alpha]$$

$$k = \frac{2\pi}{\Phi_0} \tilde{d}H_{ex}$$

$$\Phi_0 = \frac{hc}{2e}$$

critical current

intrinsic phase shift associated with domain

# Phase sensitive experiments

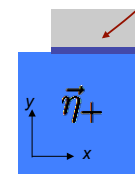
SQUID type of measurements

corner „SQUID“ geometry

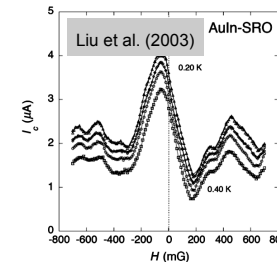
$$J_y \propto i\{\psi^* \eta_x - \psi_s \eta_x^*\} \propto \sin(\alpha)$$

phase difference

$$\pm \pi/2$$



$$J_x \propto i\{\psi^* \eta_y - \psi_s \eta_y^*\} \propto \sin(\alpha + \pi/2)$$

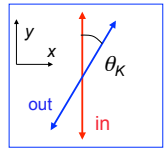


Interference pattern off-centered

broken time reversal chiral p-wave

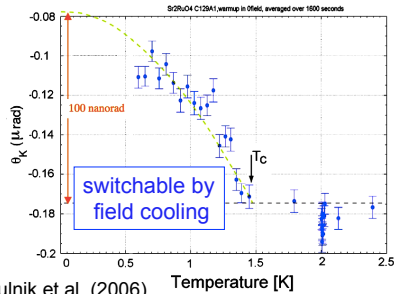
# Kerr effect

rotation of polarization axis for reflected light



$$\theta_K \approx -Im \frac{(n_+ + i\kappa_+) - (n_- + i\kappa_-)}{(n_+ + i\kappa_+)(n_- + i\kappa_-) - 1}$$

$$\theta_K \approx \frac{4\pi\sigma''_{xy}}{n(n^2 - 1)\omega} = \frac{2\pi e^2}{n(n^2 - 1)d} \frac{\Delta^2}{(\hbar\omega)^3} \quad ?$$



Kapitulnik et al. (2006)

strongly reduced for  $\hbar\omega \gg \Delta$

debate:

effects of gauge-invariance

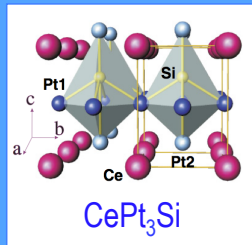
Yakovenko, Kallin, Mineev, ...  
Buhmann & MS

in 90s: Joynt; Yip & Sauls, ...

# Status of evidence for chiral p-wave

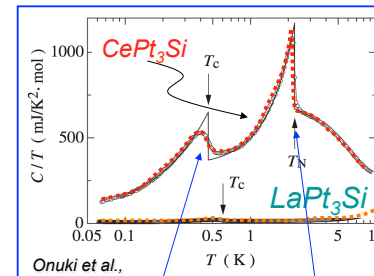
NMR Knight shift $\vec{H} \perp z$	consistent
NMR Knight shift $\vec{H} \parallel z$	unexplained
$\mu$ SR ZF relaxation rate	consistent
flux distribution/ultrasound (2-comp OP)	consistent
phase sensitive test / Josephson effect	consistent
edge states in quasiparticle tunneling	consistent
direct observation of spontaneous currents	negative
phenomenology of 3K-phase	consistent
limiting behavior for inplane field	unexplained
Kerr effect	consistent
disorder effects non-magnetic impurities	consistent

Case 2:  
Superconductors  
without inversion center  
Key symmetries for Cooper pairing



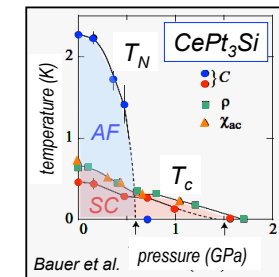
# Heavy Fermion superconductor CePt<sub>3</sub>Si

Superconductivity and magnetism



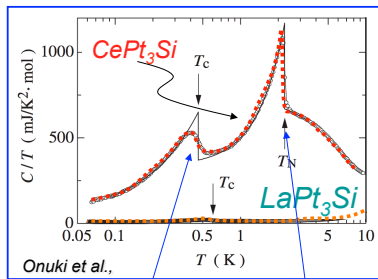
$T_c = 0.45 \text{ K}$      $T_N = 2.2 \text{ K}$

discovered by Ernst Bauer et al (2003)



# Heavy Fermion superconductor $CePt_3Si$

## Superconductivity and magnetism

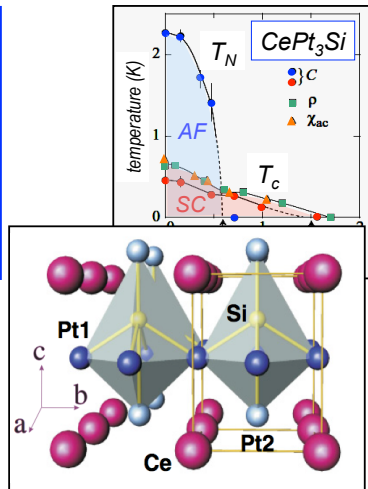


$T_c = 0.45 K$     $T_N = 2.2 K$

non-centrosymmetric crystal

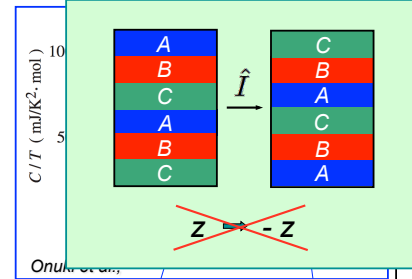
$P4mm \rightarrow C_{4v}$

tetragonal



# Heavy Fermion superconductor $CePt_3Si$

## Superconductivity and magnetism

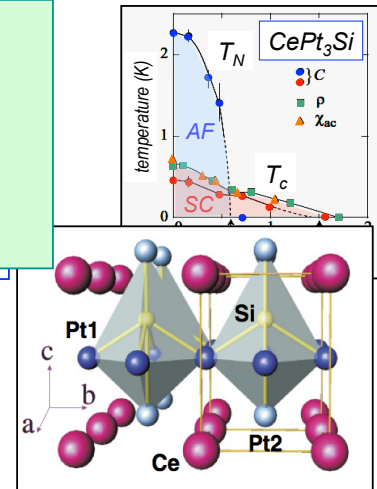


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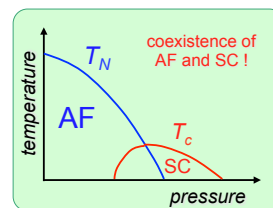


## Other Ce-based compounds

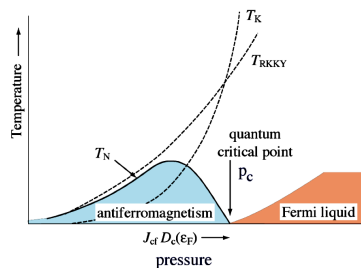
$CeRhSi_3$     $CeIrSi_3$     $CeCoGe_3$

antiferromagnets with superconductivity under pressure at QCP

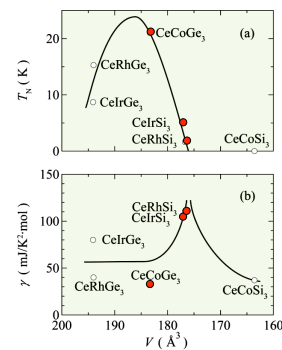
non-centrosymmetric point group  $C_{4v}$



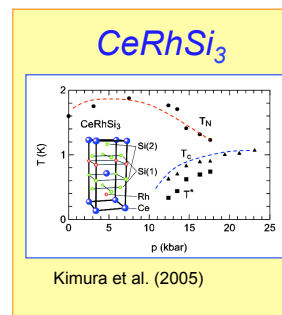
Doniach's phase diagram



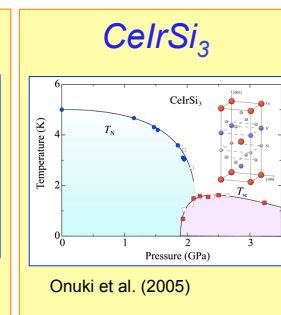
Onuki et al



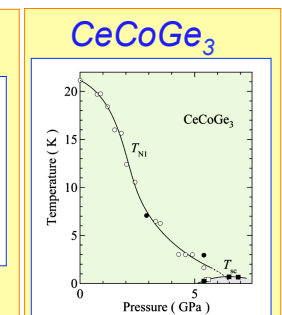
## Other Ce-based compounds



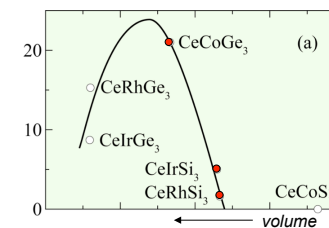
Kimura et al. (2005)



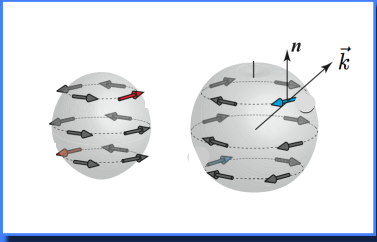
Onuki et al. (2005)



Onuki et al. (2007)



# Electronic states of non-centrosymmetric metal



## Non-centrosymmetric superconductors

**CePt<sub>3</sub>Si**

no inversion center

$$\mathcal{H} = \sum_{\vec{k},s} \epsilon_{\vec{k}} c_{\vec{k}s}^\dagger c_{\vec{k}s} + \sum_{\vec{k},s,s'} \vec{g}_{\vec{k}} \cdot \vec{\sigma}_{ss'} c_{\vec{k}s}^\dagger c_{\vec{k}s'}$$

spin-orbit coupling

$$\vec{g}_{\vec{k}} = -\vec{g}_{-\vec{k}}$$

Rashba spin-orbit coupling

$$\vec{g}_{\vec{k}} = \alpha(\hat{z} \times \vec{k})$$

spin-splitting

tetragonal  $D_{4h} \rightarrow C_{4v}$

## Non-centrosymmetric superconductors

## pairing symmetry

gap function

$$\hat{\Delta}_{\vec{k}} = \begin{pmatrix} \Delta_{\vec{k},\uparrow\uparrow} & \Delta_{\vec{k},\uparrow\downarrow} \\ \Delta_{\vec{k},\downarrow\uparrow} & \Delta_{\vec{k},\downarrow\downarrow} \end{pmatrix}$$

$$\hat{\Delta}_{\vec{k}} = i \sum_{\mu=0}^3 d_{\mu}(\vec{k}) \hat{\sigma}^{\mu} \hat{\sigma}^y$$

parity

$$[d_0(\vec{k}), \vec{d}(\vec{k})] = [d_0(-\vec{k}), -\vec{d}(-\vec{k})]$$

even spin-singlet      odd spin-triplet

symmetry operation

$$g[d_0(\vec{k}), \vec{d}(\vec{k})] = [d_0(R_g \vec{k}), R_g^s \vec{d}(R_g \vec{k})]$$

$$\mathcal{H} = \sum_{\vec{k},s} \epsilon_{\vec{k}} c_{\vec{k}s}^\dagger c_{\vec{k}s} + \sum_{\vec{k},s,s'} \vec{g}_{\vec{k}} \cdot \vec{\sigma}_{ss'} c_{\vec{k}s}^\dagger c_{\vec{k}s'}$$

symmetry classification

inversion is not a symmetry

→ mixed-parity pairing

full-symmetry pairing state

$$d_0(\vec{k}) = \Delta_e \quad \vec{d}(\vec{k}) = \Delta_o \vec{g}_{\vec{k}}$$

$$\hat{\Delta}_{\vec{k}} = (\Delta_e + \Delta_o \vec{g}_{\vec{k}} \cdot \hat{\sigma}) i \hat{\sigma}^y$$

non-unitary  $\hat{\Delta}^\dagger \hat{\Delta} \neq \hat{\sigma}^0$

## Non-centrosymmetric superconductors

## pairing symmetry

**CePt<sub>3</sub>Si**

$D_{4h}$   $\xrightarrow{\text{Inversion } z \rightarrow -z}$   $C_{4v}$

full-symmetry pairing state

$$d_0(\vec{k}) = \Delta_e \quad \vec{d}(\vec{k}) = \Delta_o \vec{g}_{\vec{k}}$$

$\Gamma_1^+$   $\Gamma_2^-$   
 $\Gamma_1$   $\Gamma_1$

$D_{4h}$   $C_{4v}$

even-parity component  $\Gamma^+$

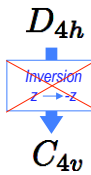
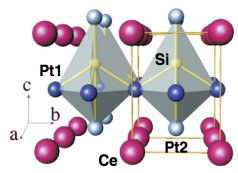
odd-parity component  $\Gamma^+ \otimes \Gamma_2^-$

$\Gamma$

$\Gamma$	$d_0(\vec{k})$	$\vec{d}(\vec{k})$
$\Gamma_1$	1	$\vec{g}_{\vec{k}}$
$\Gamma_2$	$k_x k_y (k_x^2 - k_y^2)$	$k_x k_y (k_x^2 - k_y^2) \vec{g}_{\vec{k}}$
$\Gamma_3$	$k_x^2 - k_y^2$	$(k_x^2 - k_y^2) \vec{g}_{\vec{k}}$
$\Gamma_4$	$k_x k_y$	$k_x k_y \vec{g}_{\vec{k}}$
$\Gamma_5$	$\{k_x k_z, k_y k_z\}$	$\{k_x k_z \vec{g}_{\vec{k}}, k_y k_z \vec{g}_{\vec{k}}\}$

# Non-centrosymmetric superconductors

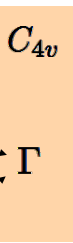
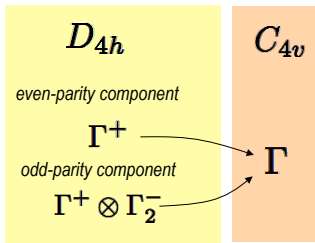
# pairing symmetry



full-symmetry pairing state

$$d_0(\vec{k}) = \Delta_e \quad \vec{d}(\vec{k}) = \Delta_o \vec{g}_{\vec{k}}$$

$\Gamma_1^+$                        $\Gamma_2^-$   
 $\Gamma_1$                                $\Gamma_1$



$\Gamma$	$d_0(\vec{k})$	$\vec{d}(\vec{k})$
$\Gamma_1$	1	$\vec{g}_{\vec{k}}$
$\Gamma_2$	$k_x k_y (k_x^2 - k_y^2)$	$k_x k_y (k_x^2 - k_y^2) \vec{g}_{\vec{k}}$
$\Gamma_3$	$k_x^2 - k_y^2$	$(k_x^2 - k_y^2) \vec{g}_{\vec{k}}$
$\Gamma_4$	$k_x k_y$	$k_x k_y \vec{g}_{\vec{k}}$
$\Gamma_5$	$\{k_x k_z, k_y k_z\}$	$\{k_x k_z \vec{g}_{\vec{k}}, k_y k_z \vec{g}_{\vec{k}}\}$

$$\hat{\Delta}_{\vec{k}} = (\Delta_e + \Delta_o \vec{g}_{\vec{k}} \cdot \hat{\sigma}) i \hat{\sigma}_y$$

mixed-parity pairing

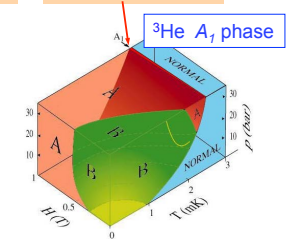
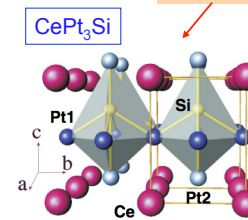
# Mixed parity states are non-unitary

unitary superconducting states:  $\hat{\Delta} \hat{\Delta}^\dagger = |\Delta|^2 \hat{\sigma}_0 \propto 2 \times 2$  unit matrix

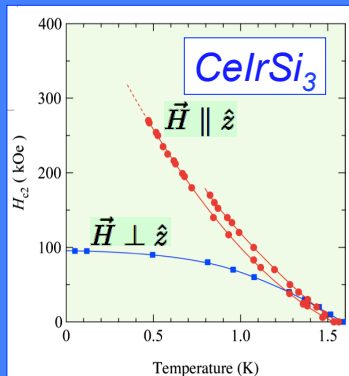
$$\hat{\Delta} = \{ \psi(\vec{k}) \hat{\sigma}_0 + \vec{d}(\vec{k}) \cdot \hat{\sigma} \} i \hat{\sigma}_y$$

$$\hat{\Delta} \hat{\Delta}^\dagger = (|\psi|^2 + |\vec{d}|^2) \hat{\sigma}_0 + \underbrace{\{ \psi^* \vec{d} + \psi \vec{d}^* \}}_{\propto \vec{\lambda}_{\vec{k}}} \cdot \hat{\sigma} + i \underbrace{\{ \vec{d} \times \vec{d}^* \}}_{\propto \vec{H}} \cdot \hat{\sigma}$$

inversion symmetry violated      time reversal symmetry violated



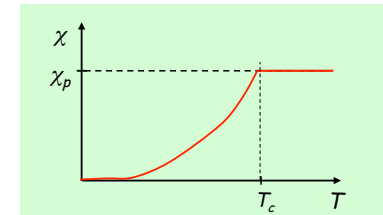
# Upper critical field



# Spin susceptibility

- spin singlet pairing  $\rightarrow$  Yosida behavior of spin susceptibility

pair breaking by spin polarization

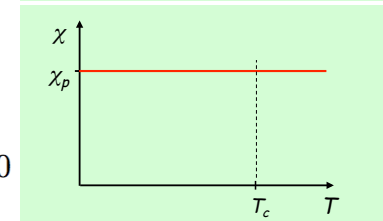


- spin triplet pairing

no pair breaking for equal-spin pairing

$$\chi = \text{const. for } \vec{d}(\vec{k}) \cdot \vec{H} = 0$$

note:  $\vec{S} \parallel \vec{H}$  with  $\vec{d} \perp \vec{S}$

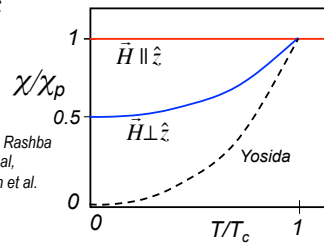


## Spin susceptibility & Rashba spin-orbit coupling

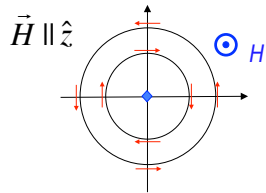
spin susceptibility of non-centrosymmetric SC

$$\vec{g}_{\vec{k}} = \hat{z} \times \vec{k} = \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}$$

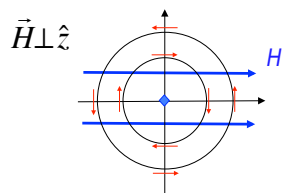
→  
Gorkov & Rashba  
Frigeri et al.  
Samokhin et al.



as in CePt<sub>3</sub>Si, CeRhSi<sub>3</sub>, CeIrSi<sub>3</sub>



"transverse field"  
interband spin polarization



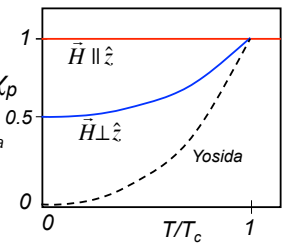
partially "transverse field"  
interband spin polarization limited

## Spin susceptibility & Rashba spin-orbit coupling

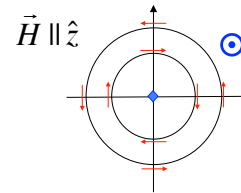
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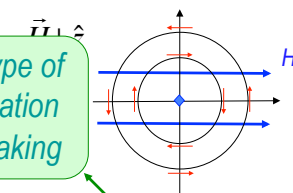


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"transverse field"  
interband spin polarization

van Vleck type of  
spin polarization  
not pair breaking



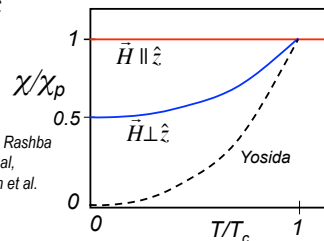
partially "transverse field"  
interband spin polarization limited

## Spin susceptibility & Rashba spin-orbit coupling

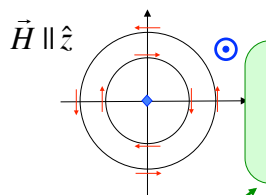
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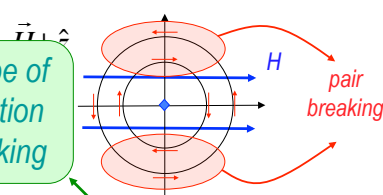


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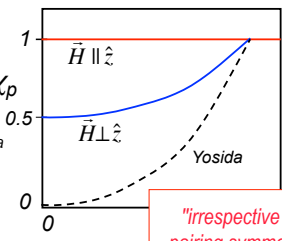
partially "transverse field"  
interband spin polarization limited

## Spin susceptibility & Rashba spin-orbit coupling

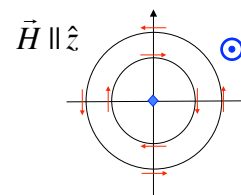
spin susceptibility of non-centrosymmetric SC

$$\chi_{\mu\nu}(T=0) \approx \chi_p \left\{ \delta_{\mu\nu} - \left\langle \frac{g_{\vec{k}}^\mu g_{\vec{k}}^\nu}{|\vec{g}_{\vec{k}}|^2} \right\rangle_{\vec{k}} \right\}$$

→  
Gorkov & Rashba  
Frigeri et al.  
Samokhin et al.

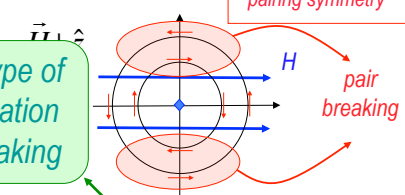


"irrespective of  
pairing symmetry"



"transverse field"  
interband spin polarization

van Vleck type of  
spin polarization  
not pair breaking



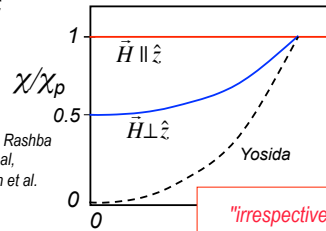
partially "transverse field"  
interband spin polarization limited

## Paramagnetic limiting

spin susceptibility of non-centrosymmetric SC

$$\chi_{\mu\nu}(T=0) \approx \chi_p \left\{ \delta_{\mu\nu} - \left\langle \frac{g_{\vec{k}}^{\mu} g_{\vec{k}}^{\nu}}{|\vec{g}_{\vec{k}}|^2} \right\rangle_{\vec{k}} \right\}$$

Gorkov & Rashba  
Frigeri et al.  
Samokhin et al.



"irrespective of pairing symmetry"

paramagnetic limiting field

$$H_p = \frac{H_c(0)}{\sqrt{4\pi(\chi_p - \chi(0))}}$$

$\vec{H} \perp \hat{z}$  paramagnetic limiting

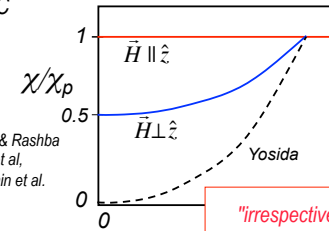
$\vec{H} \parallel \hat{z}$  no paramagn. limiting

## Paramagnetic limiting

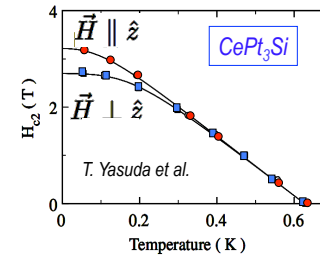
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Gorkov & Rashba  
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"irrespective of pairing symmetry"



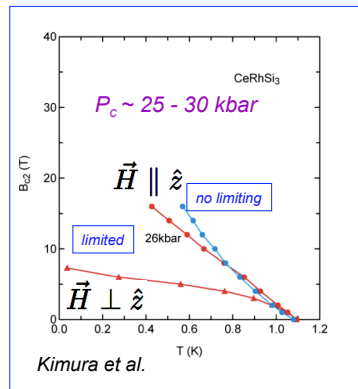
does not follow the expectations !

$\vec{H} \perp \hat{z}$  paramagnetic limiting

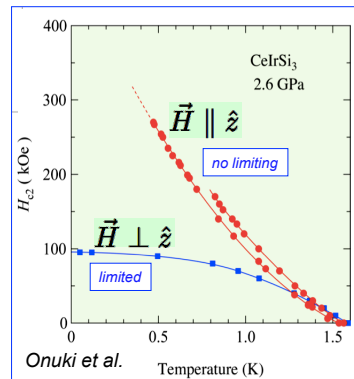
$\vec{H} \parallel \hat{z}$  no paramagn. limiting

## Upper critical field and paramagnetic limiting

CeRhSi<sub>3</sub>



CeIrSi<sub>3</sub>



fits very well to theoretical expectations of paramagnetic limiting

## Comparison of different heavy Fermion superconductors

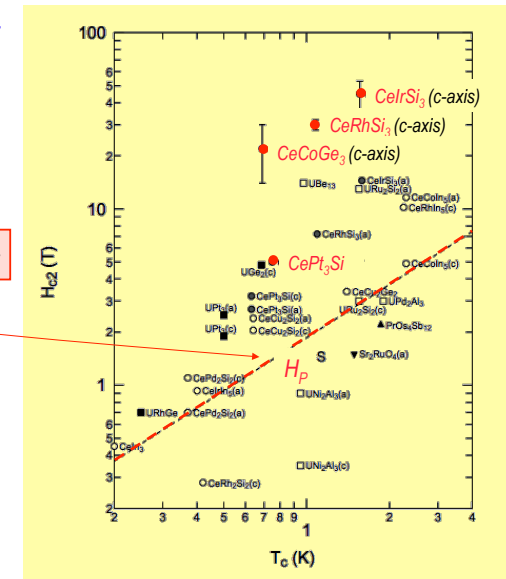
paramagnetic limit (BCS weak coupling)

$$H_p = 1.85 k_B T_c / g \mu_B$$

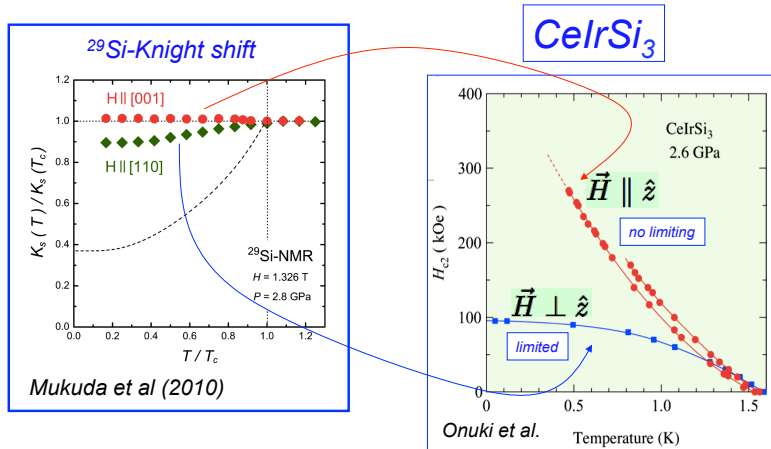
Non-centrosymmetric superconductors

CeIrSi<sub>3</sub>  
CeRhSi<sub>3</sub>  
CeCoGe<sub>3</sub>

highest H<sub>c2</sub> among heavy Fermion materials

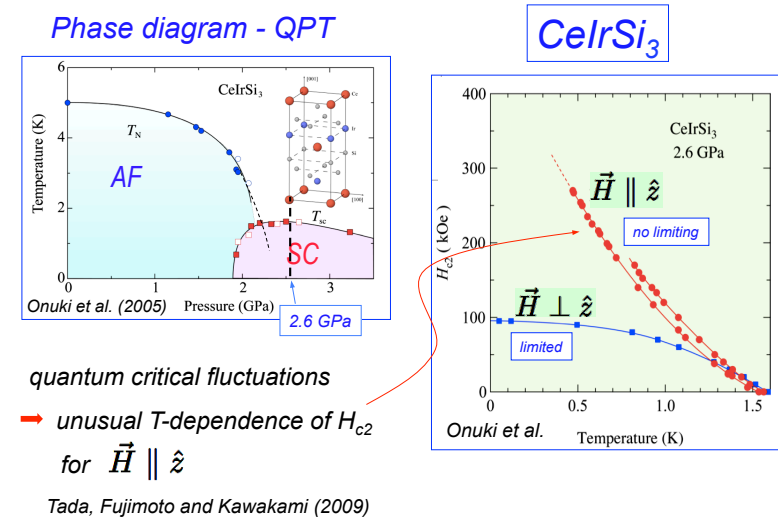


## Upper critical field and paramagnetic limiting

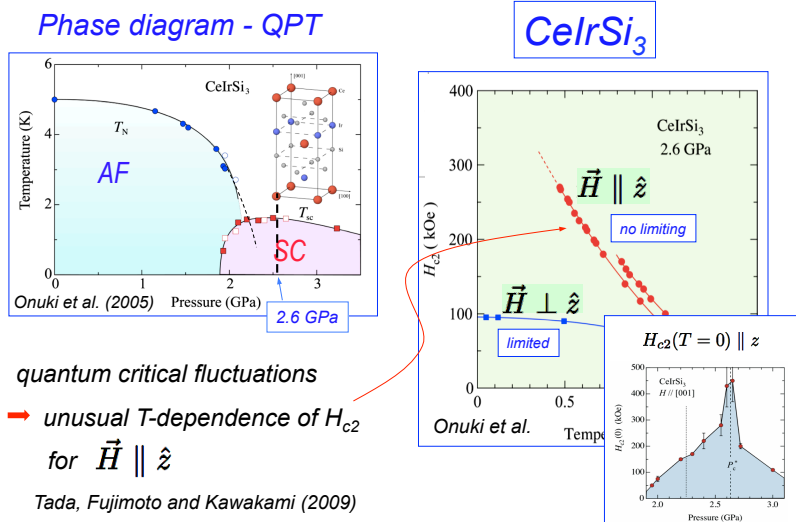


fits very well to theoretical expectations of paramagnetic limiting

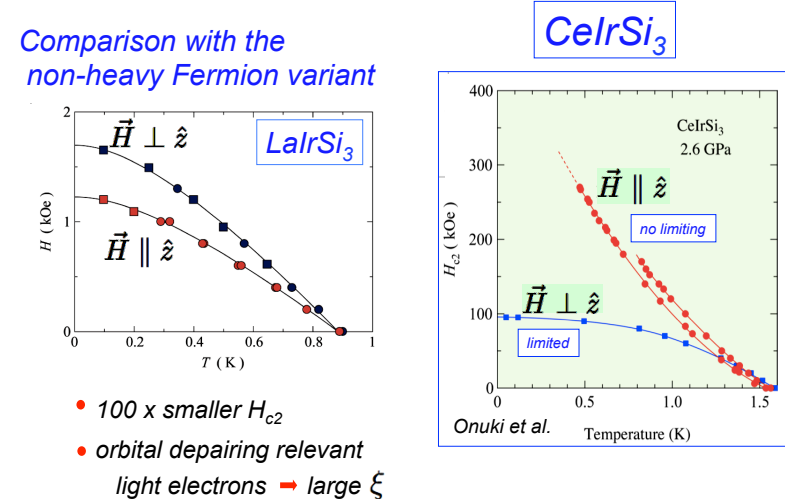
## Upper critical field and paramagnetic limiting



## Upper critical field and paramagnetic limiting



## Upper critical field and paramagnetic limiting



# Conclusion & Remarks

inversion key symmetry for Cooper pairing

without inversion symmetry → mixed-parity pairing

dominant unconventional component

rich phenomena and complex phenomenology

Intriguing novel features:

magnetolectric phenomena connection to spintronics and multiferroics Edelman, Mineev, Samokhin, Eschrig, Aoyama, ...

Josephson effect phase sensitive probes Hayashi, Linder, Subdo, Borkje, Klam, ...

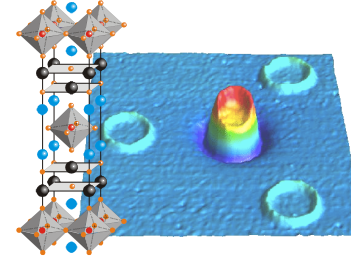
Coexistence of magnetism and superconductivity at quantum critical points Yanase, Fujimoto, ...

# Order parameter symmetry of unconventional superconductors

## Identified order parameters

• High- $T_c$  superconductors

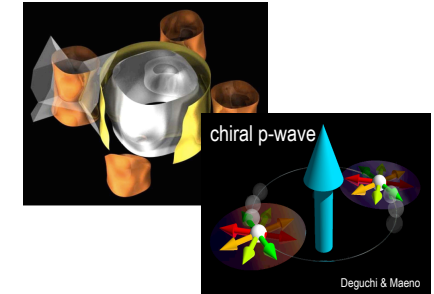
$La_{2-x}Sr_xCuO_4, YBa_2Cu_3O_7, \dots$



spin singlet,  $d$ -wave pairing

$$\psi(\vec{k}) = k_x^2 - k_y^2 \quad 1D \text{ rep.}$$

• Ruthenate  $Sr_2RuO_4$



2D metallic analog to  $^3He$  spin triplet chiral  $p$ -wave

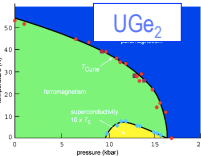
$$\vec{d}(\vec{k}) = \hat{z}(k_x \pm ik_y) \quad 2D \text{ rep.}$$

# Order parameter symmetry of unconventional superconductors

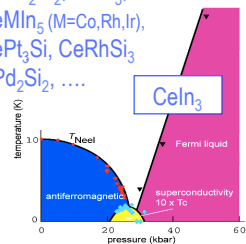
## open order parameters

• Heavy Fermion superconductors

likely spin triplet  
 $UBe_{13}, UPt_3, UGe_2, ZnZr_2, PrOs_4Sb_{12}$

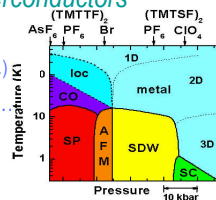


likely spin singlet  
 $CeCu_2Si_2, CeIn_3, CeMn_5 (M=Co, Rh, Ir), CePt_3Si, CeRhSi_3, UPd_2Si_2, \dots$



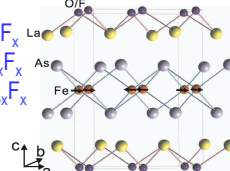
• Organic superconductors

$(TMTSF)_2M$  ( $M=PF_6, SbF_6, \dots$ )  
 $(BEDT-TTF)_2M$  ...  
 singlet or triplet



• Fe-oxypnictides

$LaFeAsO_{1-x}F_x, NdFeAsO_{1-x}F_x, SmFeAsO_{1-x}F_x, CaFe_2As_2, BaFe_2As_2$



unconv. multi-band spin singlet ?

# Conclusions and final remarks

- Superconductivity in strongly correlated electron systems likely unconventional
  - ◆ strong Coulomb repulsion favors angular momentum  $l > 0$
  - ◆ exotic pairing mechanisms in particular close to quantum critical points

- Unconventional order parameters give rise to new phenomena
  - ◆ quasi-particle properties, tunneling and Josephson effect
  - ◆ mixed phase, vortex matter, flux dynamics
  - ◆ superconducting multi-phase diagrams
  - ◆ magnetism and connection to competing phases
  - ◆ disorder effects

→ higher dimensional order parameters ( $Sr_2RuO_4, (U,Th)Be_{13}, UPt_3, \dots$ ) are more interesting than one-dimensional ones (high- $T_c$  superconductors, ...)

Many chapters on unconventional superconductivity are still unwritten and new materials are discovered at an accelerating pace (sample purity is mandatory ! ?)