

Outline

1. Antiferromagnetism and quantum criticality in insulators
2. Onset of antiferromagnetism in metals, and d-wave superconductivity
3. Competing density wave order, and the pseudogap of the cuprate superconductors
4. Non-Fermi liquids

Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i < j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i < j} J_{ij} \left(\Psi_{i\alpha a}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta a} \right) \cdot \left(\Psi_{j\gamma b}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{j\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha a} \rightarrow U_{i,ab} \Psi_{i\alpha b}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J .

- I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
- E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
- P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

Pseudospin symmetry of the exchange interaction

$$H_{tJ} = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

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This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J .

The pseudospin symmetry is fully broken by t_{ij} , V_{ij} , and the chemical potential, and so expected to be unimportant away from half-filling.

$$\mathcal{L}_f = \psi_{1\alpha}^\dagger (\partial_\tau - i\mathbf{v}_1 \cdot \nabla_r) \psi_{1\alpha} + \psi_{2\alpha}^\dagger (\partial_\tau - i\mathbf{v}_2 \cdot \nabla_r) \psi_{2\alpha}$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$

“Yukawa” coupling: $\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right)$

The “hot-spot” theory for the onset of antiferromagnetism in metals is exactly invariant under the pseudospin symmetry!

$$\mathcal{L}_f = \psi_{1\alpha}^\dagger (\partial_\tau - i\mathbf{v}_1 \cdot \nabla_r) \psi_{1\alpha} + \psi_{2\alpha}^\dagger (\partial_\tau - i\mathbf{v}_2 \cdot \nabla_r) \psi_{2\alpha}$$

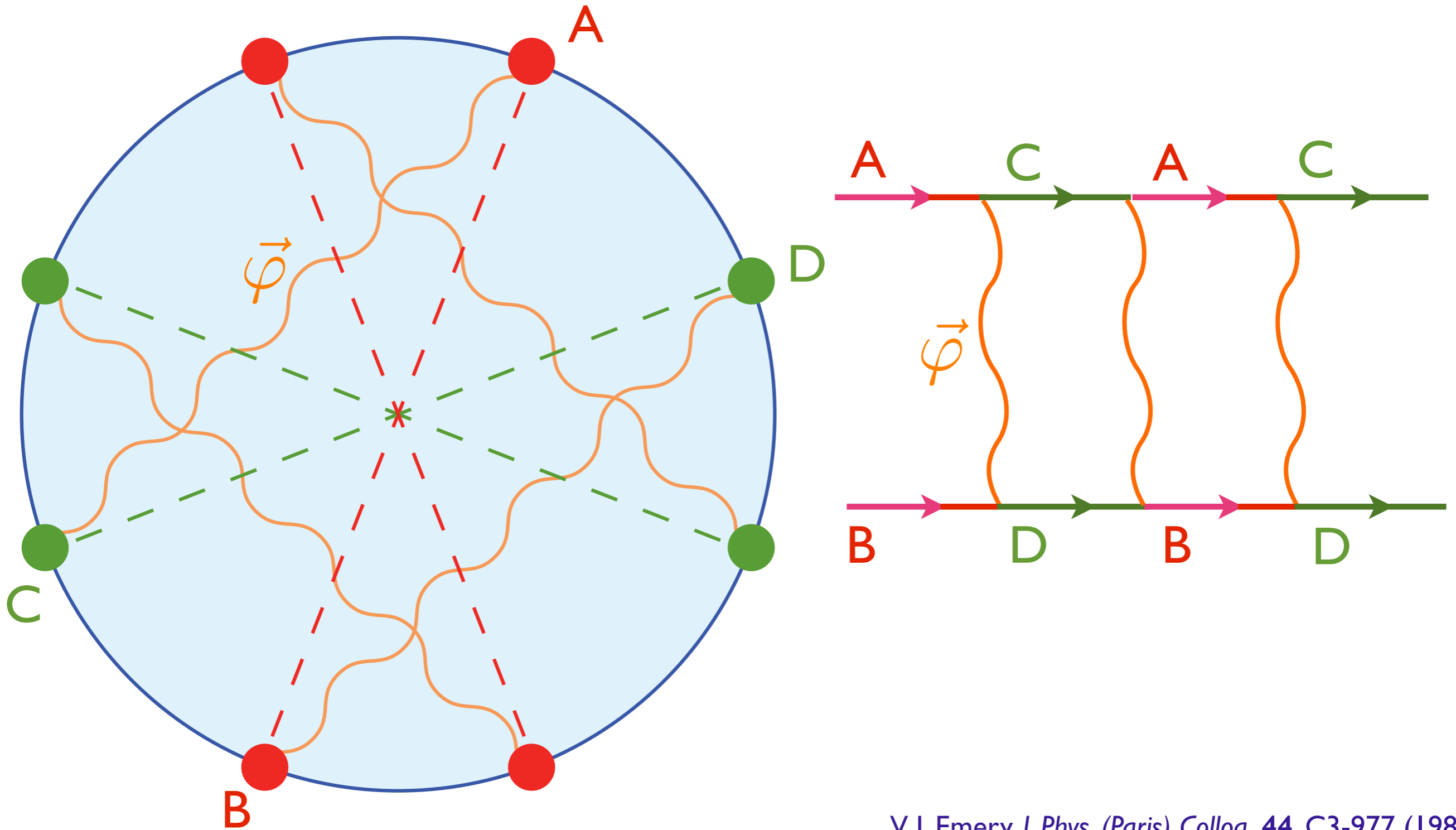
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We will now move away from the antiferromagnetic critical point (and the hot-spot theory) and investigate if there are any remnant consequences of this unexpected symmetry.

Pairing “glue” from antiferromagnetic fluctuations



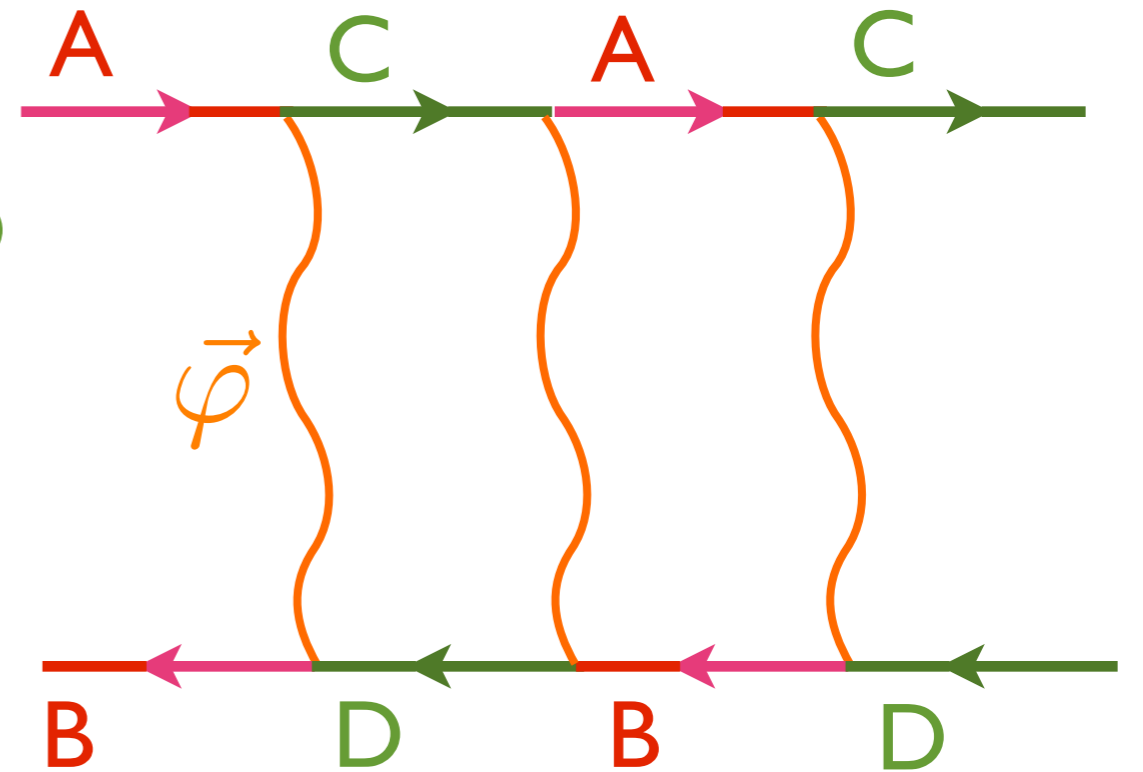
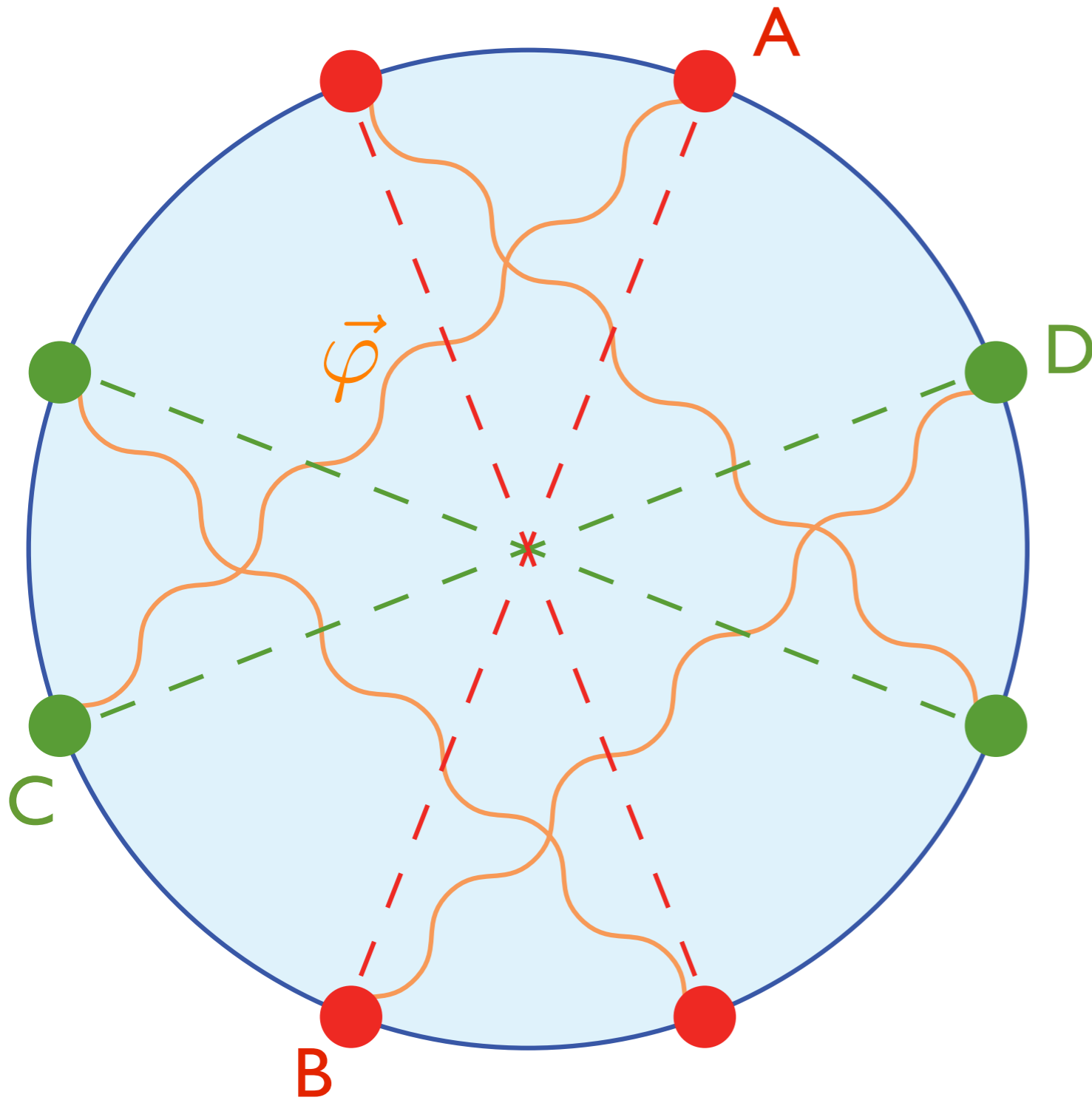
V. J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)

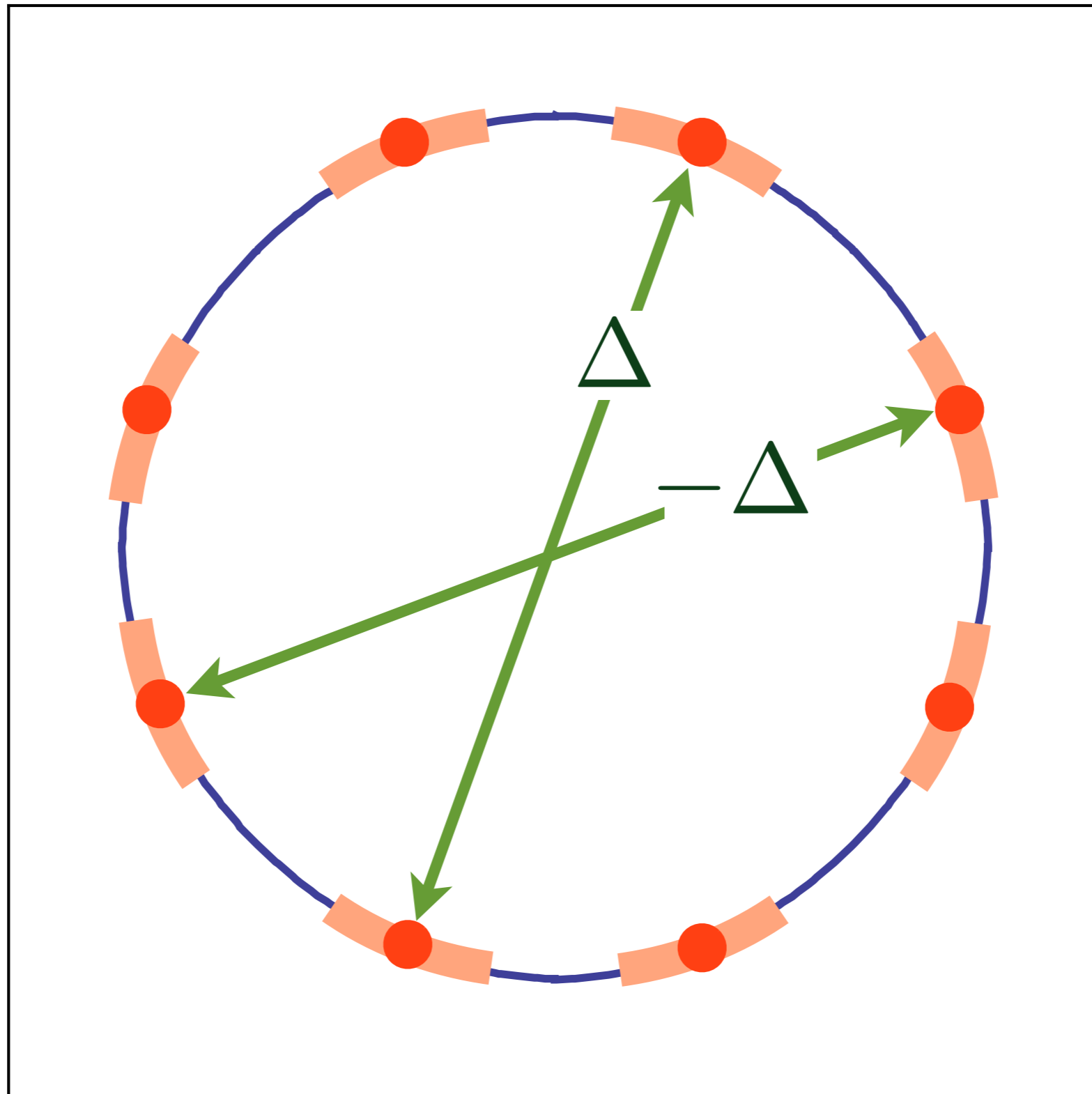
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

S. Raghu, S. A. Kivelson, and D. J. Scalapino, *Phys. Rev. B* 81, 224505 (2010)

Same “glue” leads to particle-hole pairing



$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$

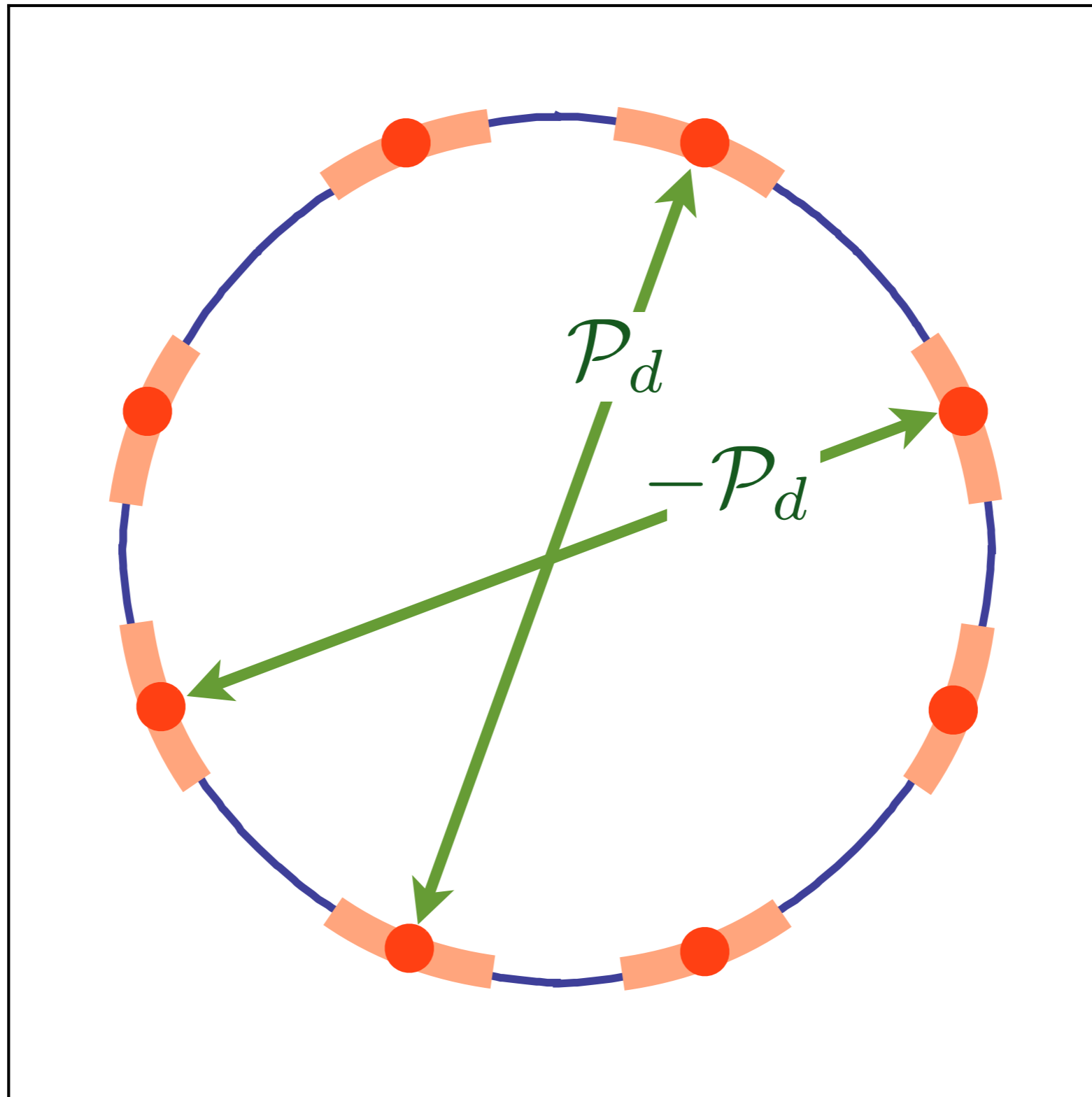


Unconventional pairing at and near hot spots

$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \mathcal{P}_d (\cos k_x - \cos k_y)$$

\mathbf{Q} is ' $2k_F$ '
wavevector

After pseudospin
rotation on *half*
the hot-spots

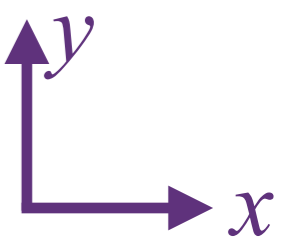


“*d*-form factor” density wave

Charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r})c_{\alpha}(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

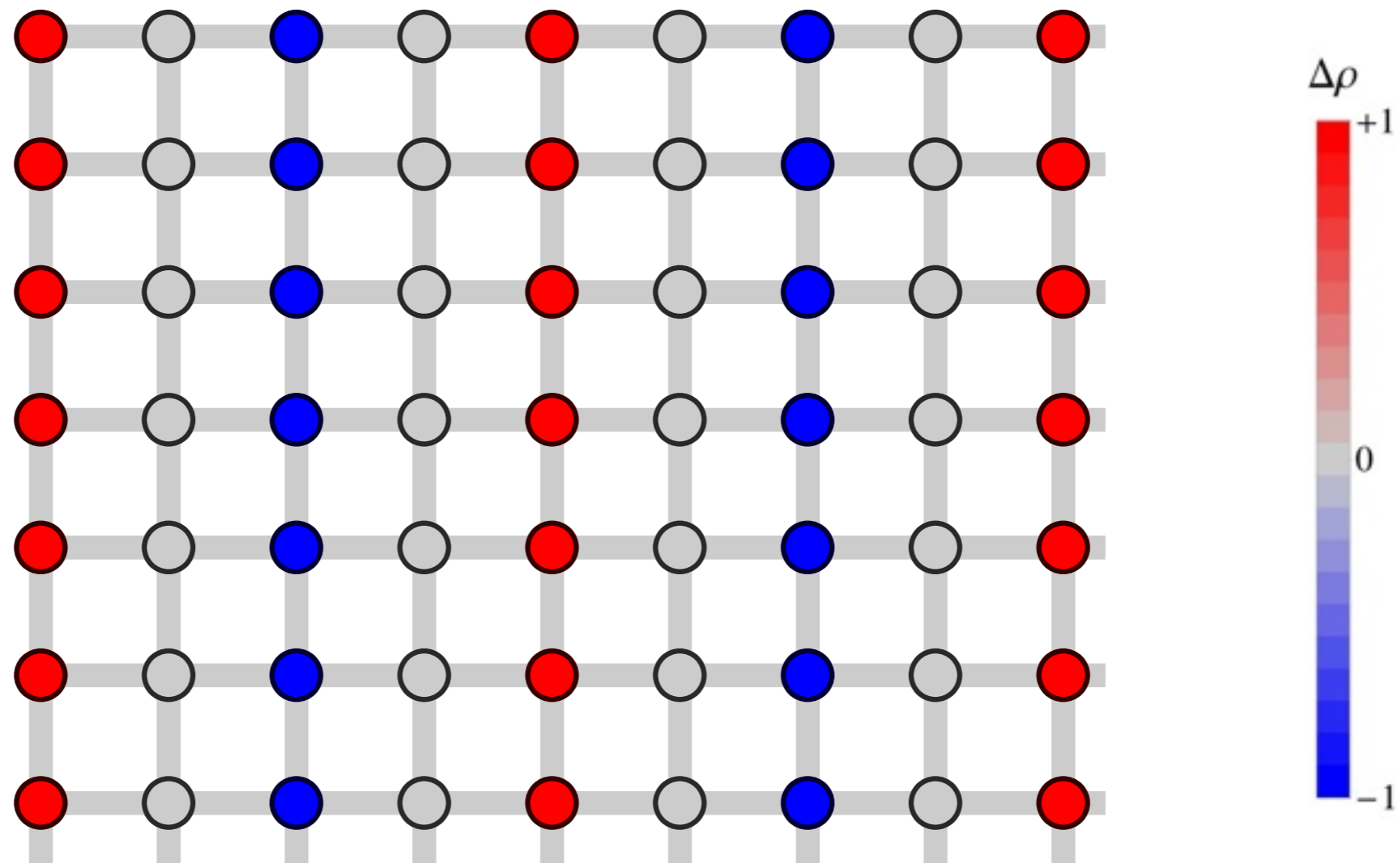
CDW order.



Plot of $P_{ii} = \langle c_{i\alpha}^\dagger c_{i\alpha} \rangle$ with

$$P_{ii} = e^{i\mathbf{Q}\cdot\mathbf{r}_i} + \text{c.c.}$$

with $\mathbf{Q} = 2\pi(1/4, 0)$

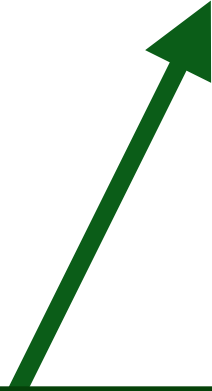


Unconventional density wave (DW) :
Bose condensation of particle-hole pairs

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$

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Crucial “center-of-mass” co-ordinate.
(Not used in previous work)
Simplifies action of time-reversal

Unconventional density wave (DW) :
Bose condensation of particle-hole pairs

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle = \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$

Density wave form factor (internal particle-hole pair wavefunction)

$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand (using reflection symmetry for \mathbf{Q} along axes or diagonals)

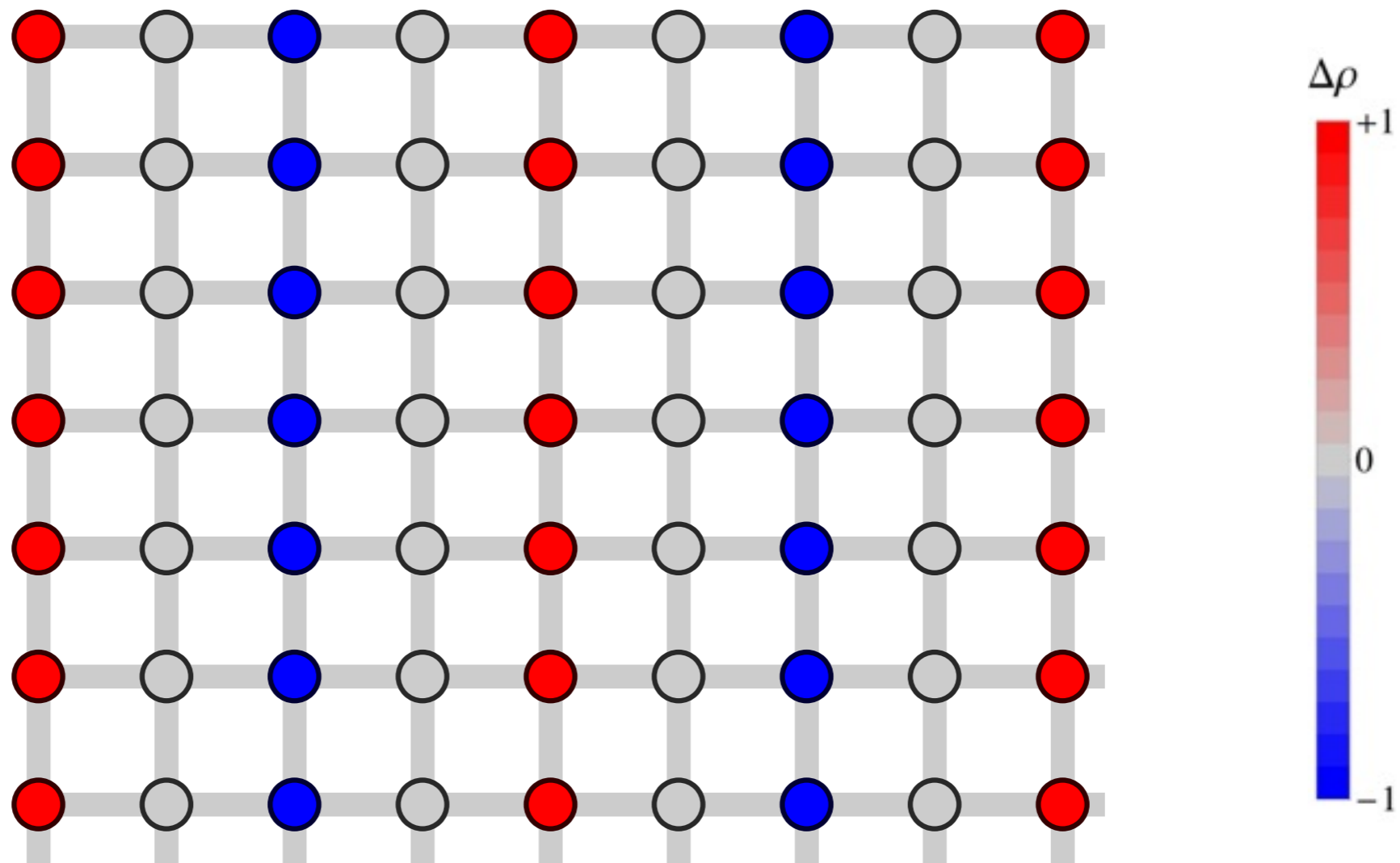
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

Conventional CDW order: s -form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

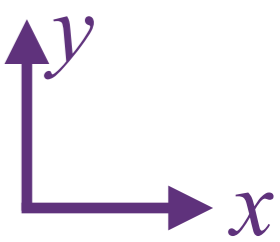
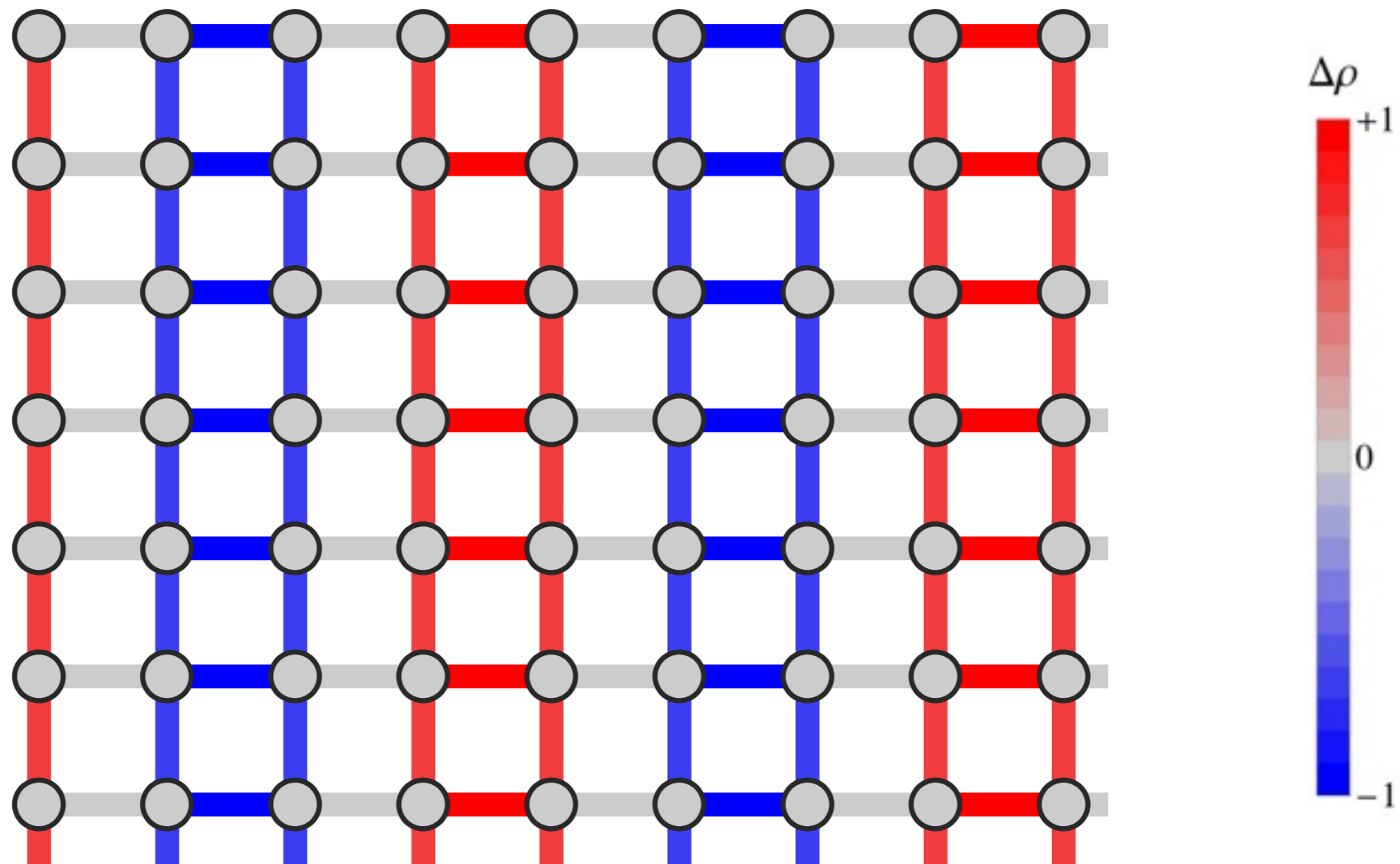


Unconventional DW order: s' -form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

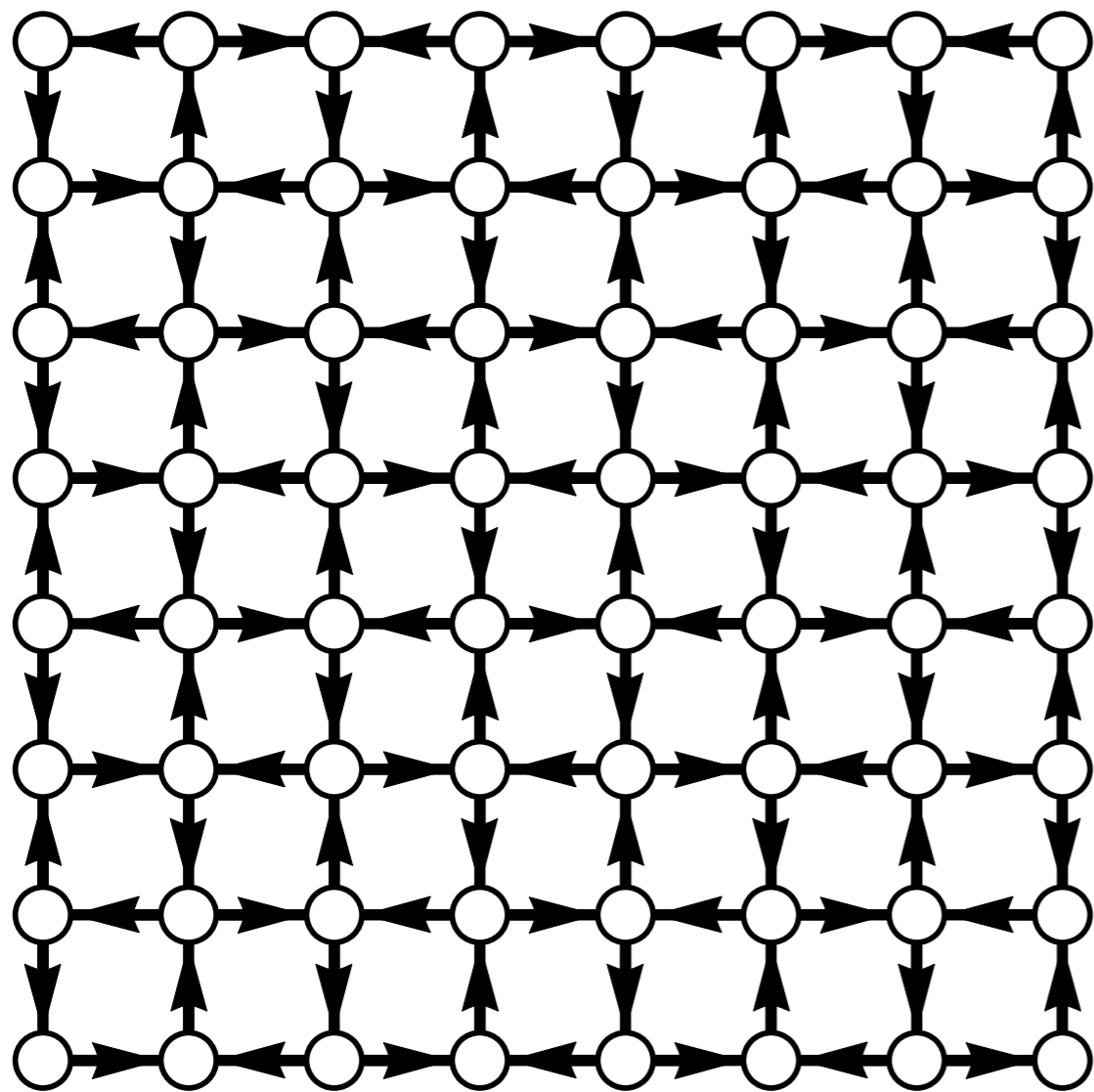


Current order: p -form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\sum_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

$$\mathcal{P}(\mathbf{k}) = \sin(k_x) - \sin(k_y) \quad \text{and} \quad \mathbf{Q} = (\pi, \pi)$$



This state breaks time-reversal and is also known as “ d -density wave” (but is p -form factor in our notation), and “staggered-flux (SF)”. (Similar comments apply to “loop” orders of Varma and others.)

S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B **63**, 094503 (2001).

P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006).

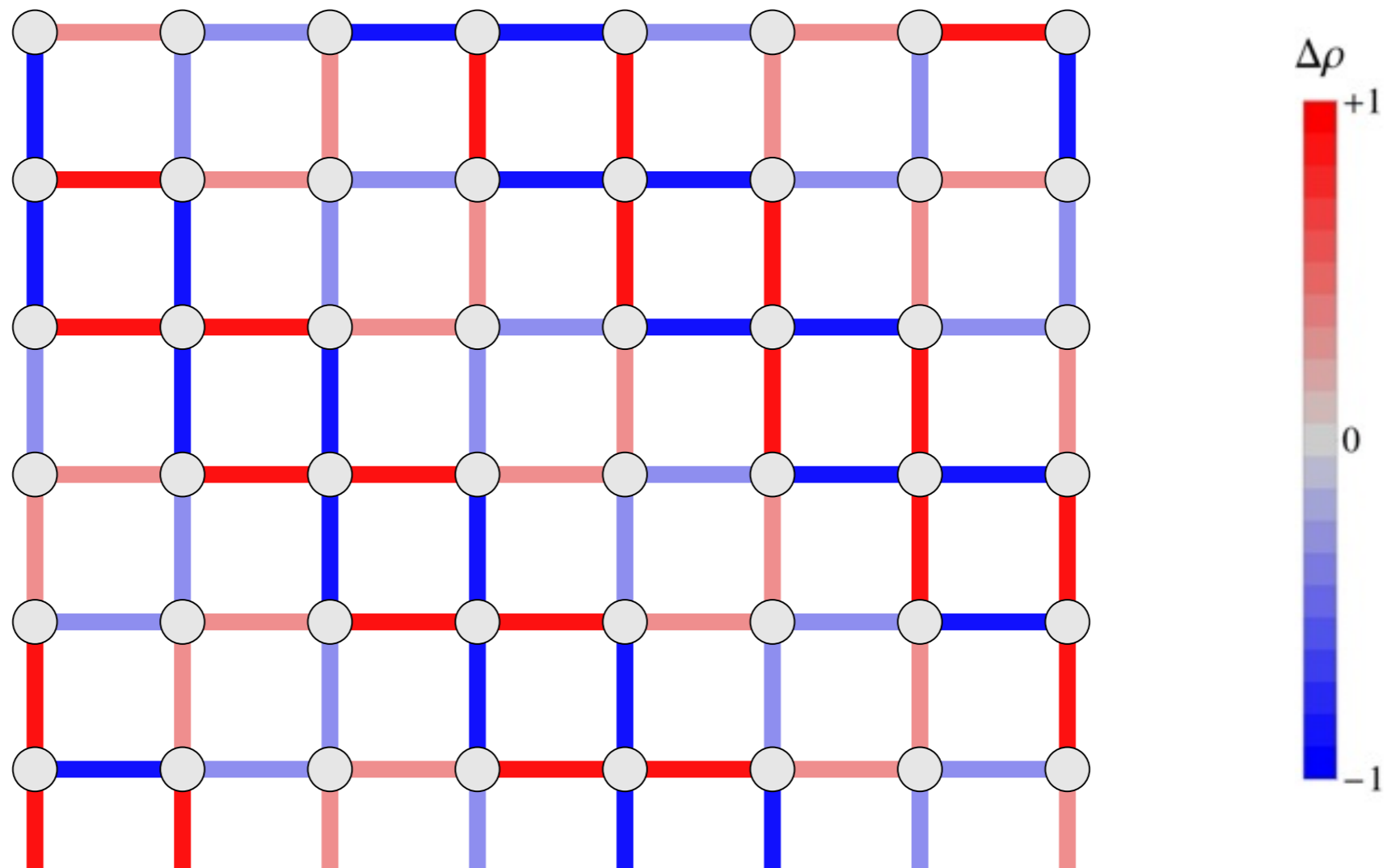
R. B. Laughlin, Phys. Rev. B **89**, 035134 (2014).

Unconventional DW order: d -form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 1/4)$$



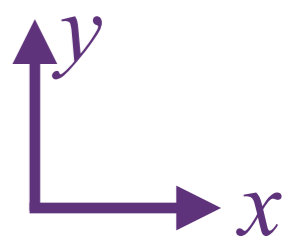
Density wave predicted by hot-spot theory

Unconventional DW order: d -form factor

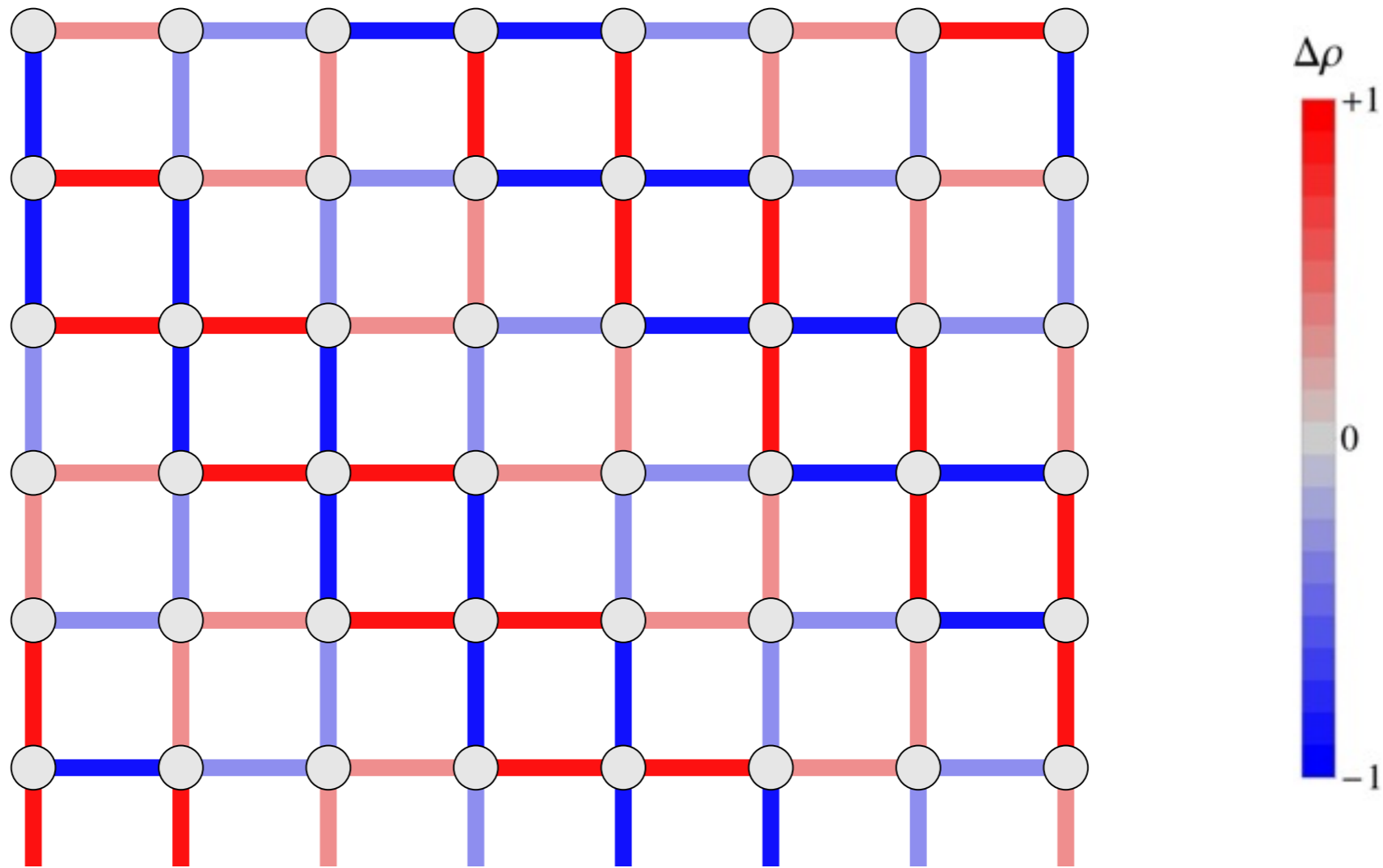
Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 1/4)$$



Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



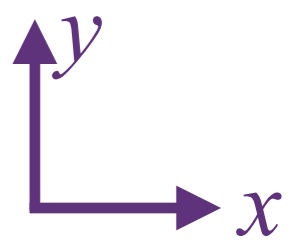
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Unconventional DW order: d -form factor

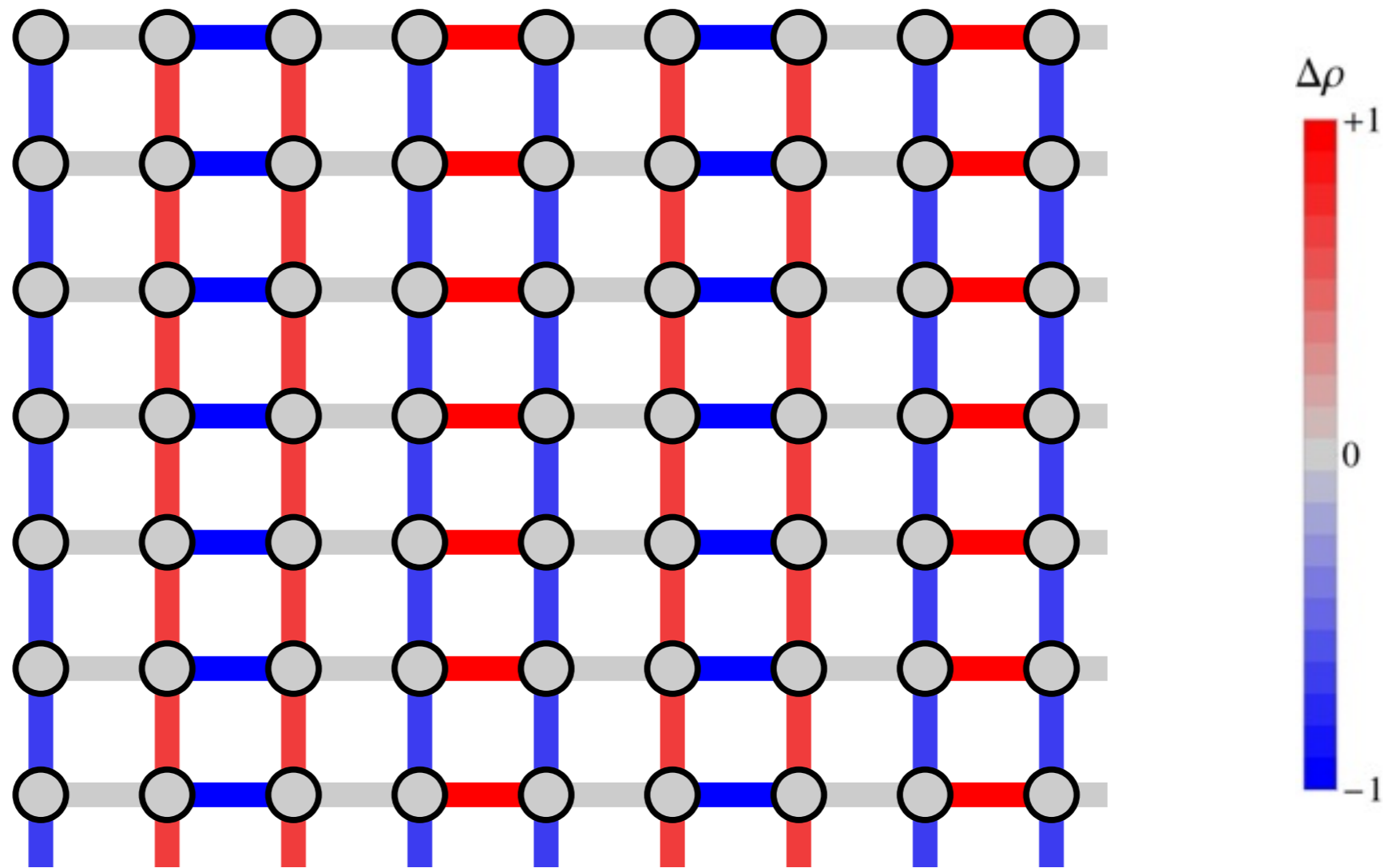
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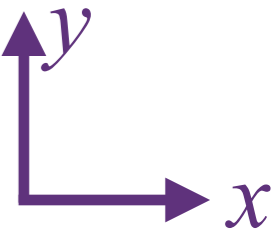
Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



This specific d -form factor density wave order (with \mathbf{Q} along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Unconventional DW order: d -form factor

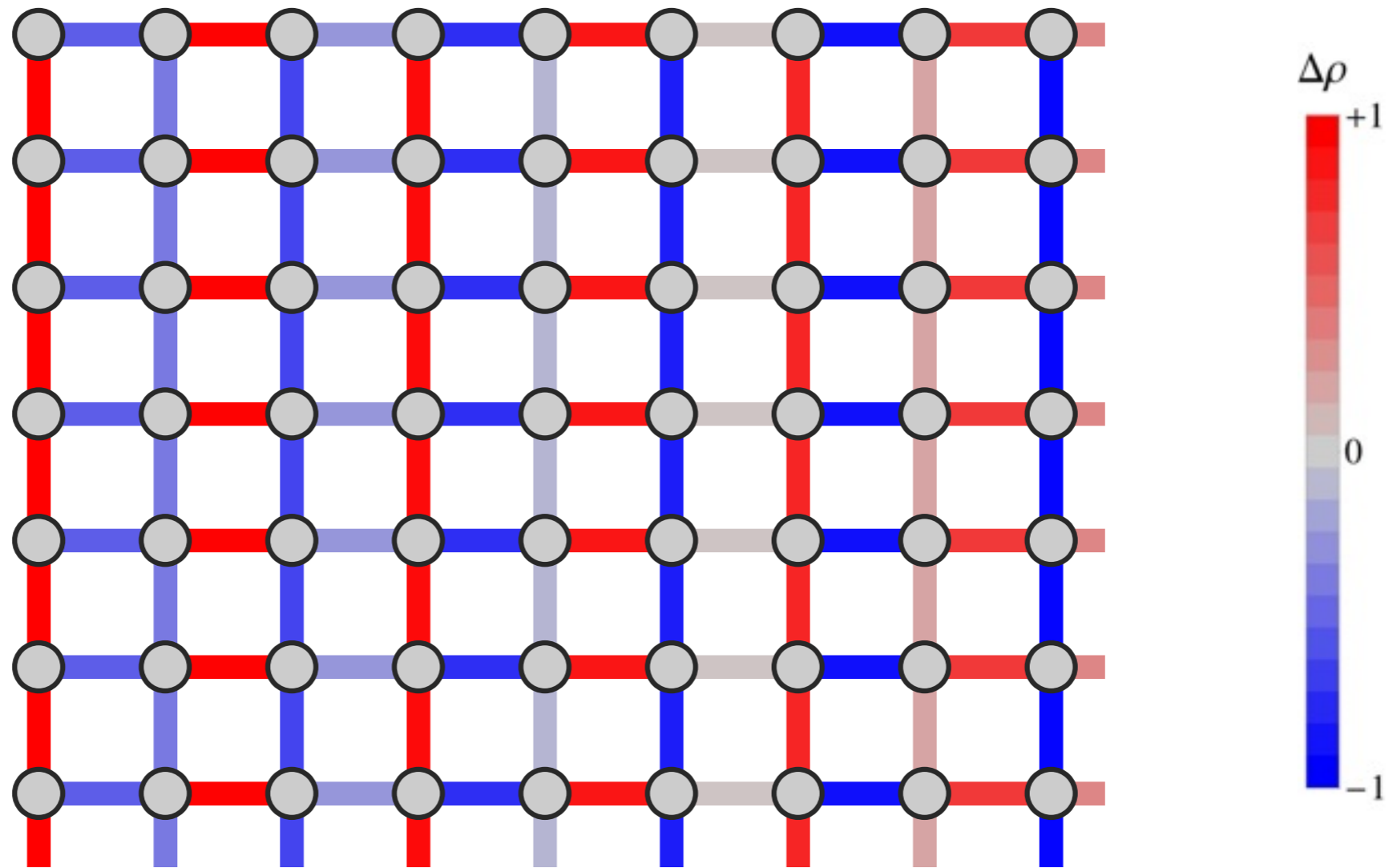
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$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

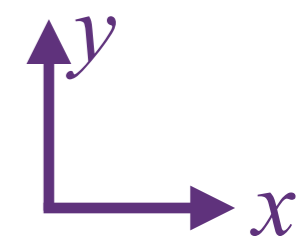
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



This specific d -form factor density wave order (with \mathbf{Q} along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Unconventional DW order: $(d + s)$ -form factor

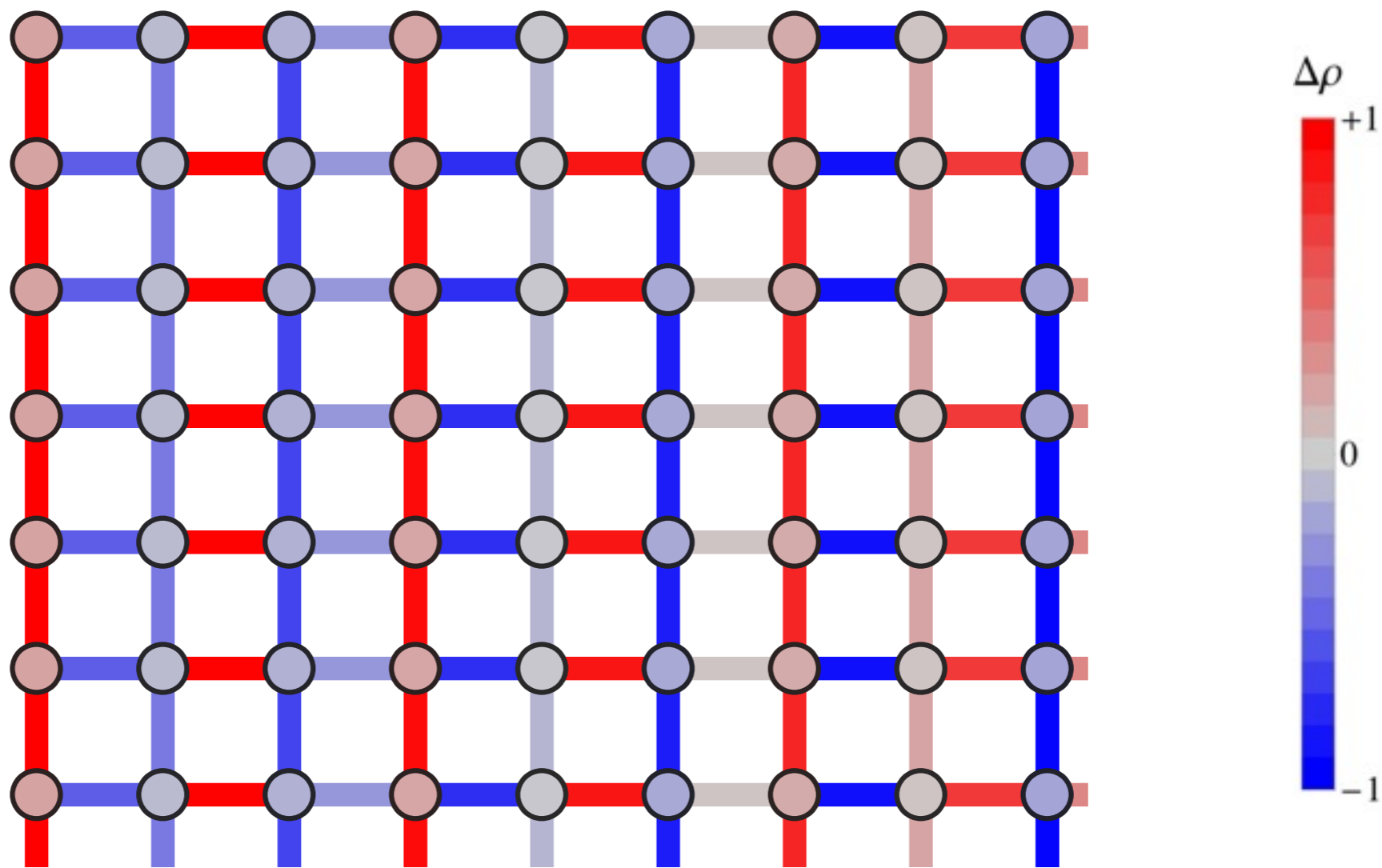


Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [0.2 + \cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

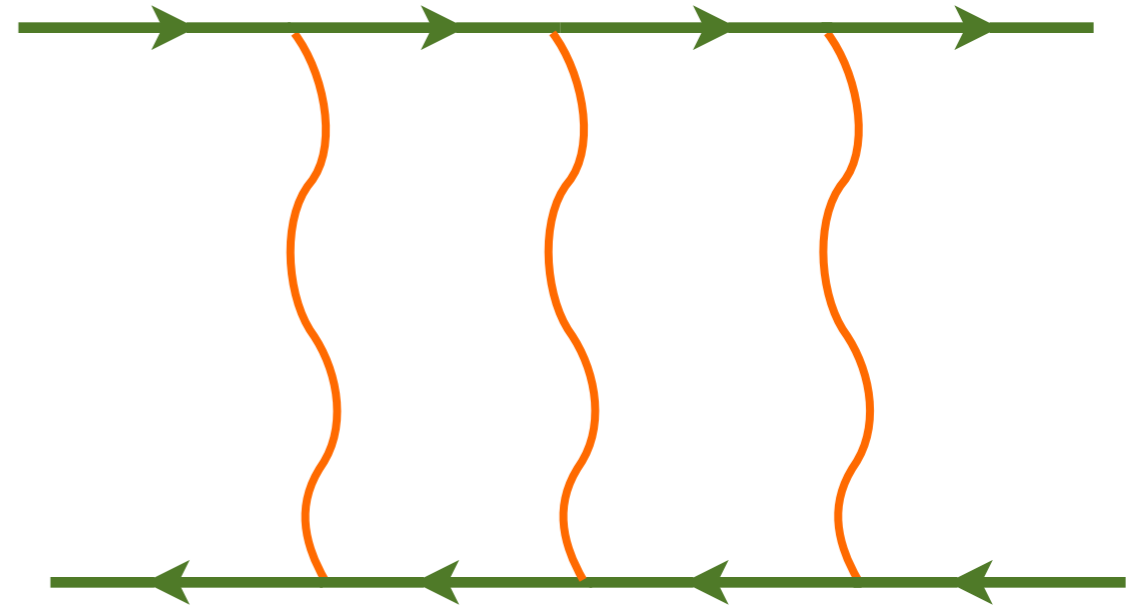
Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



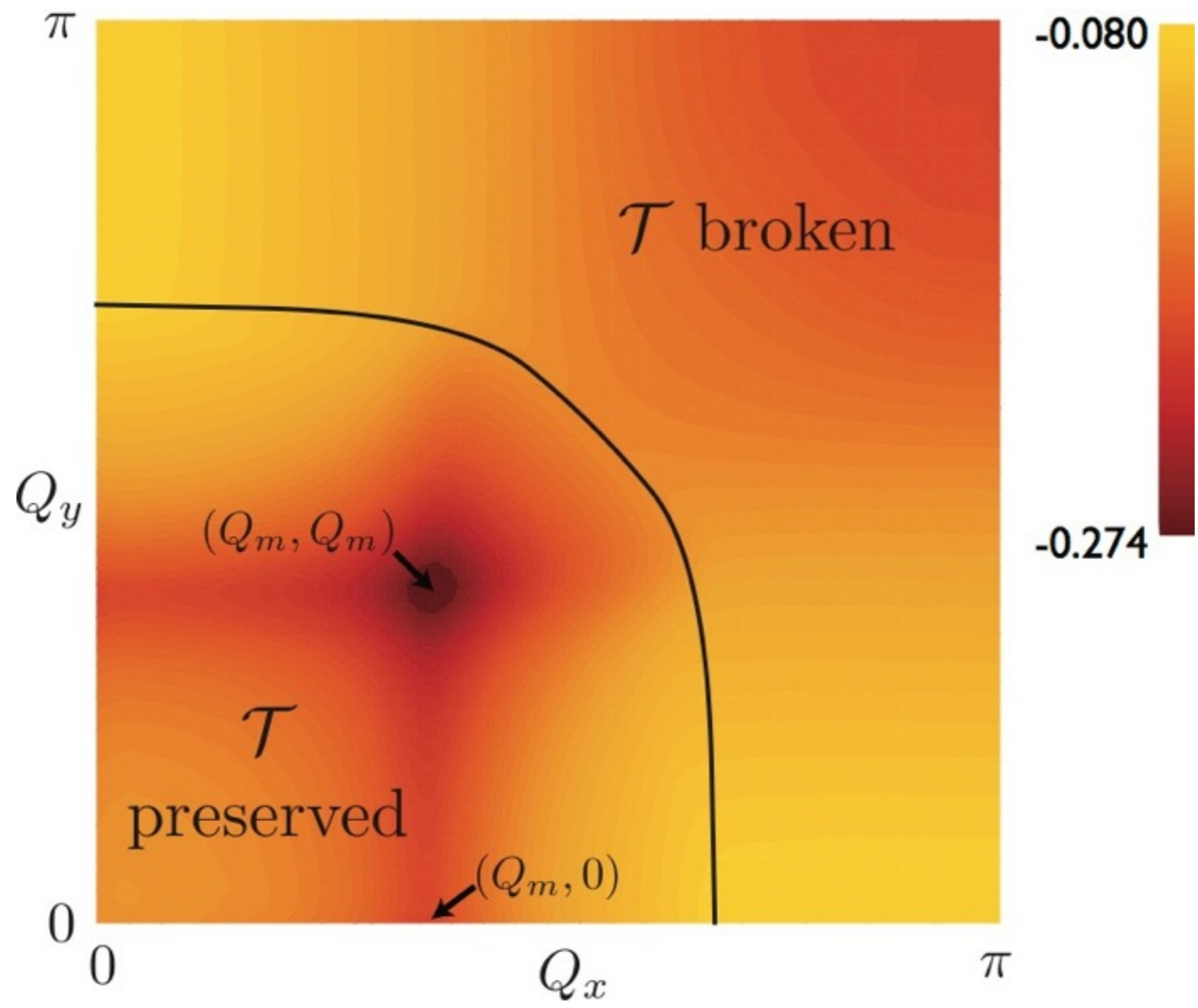
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Determination of form factor of density wave by solution of Bethe-Salpeter equation for the particle-hole pair about the entire Fermi surface

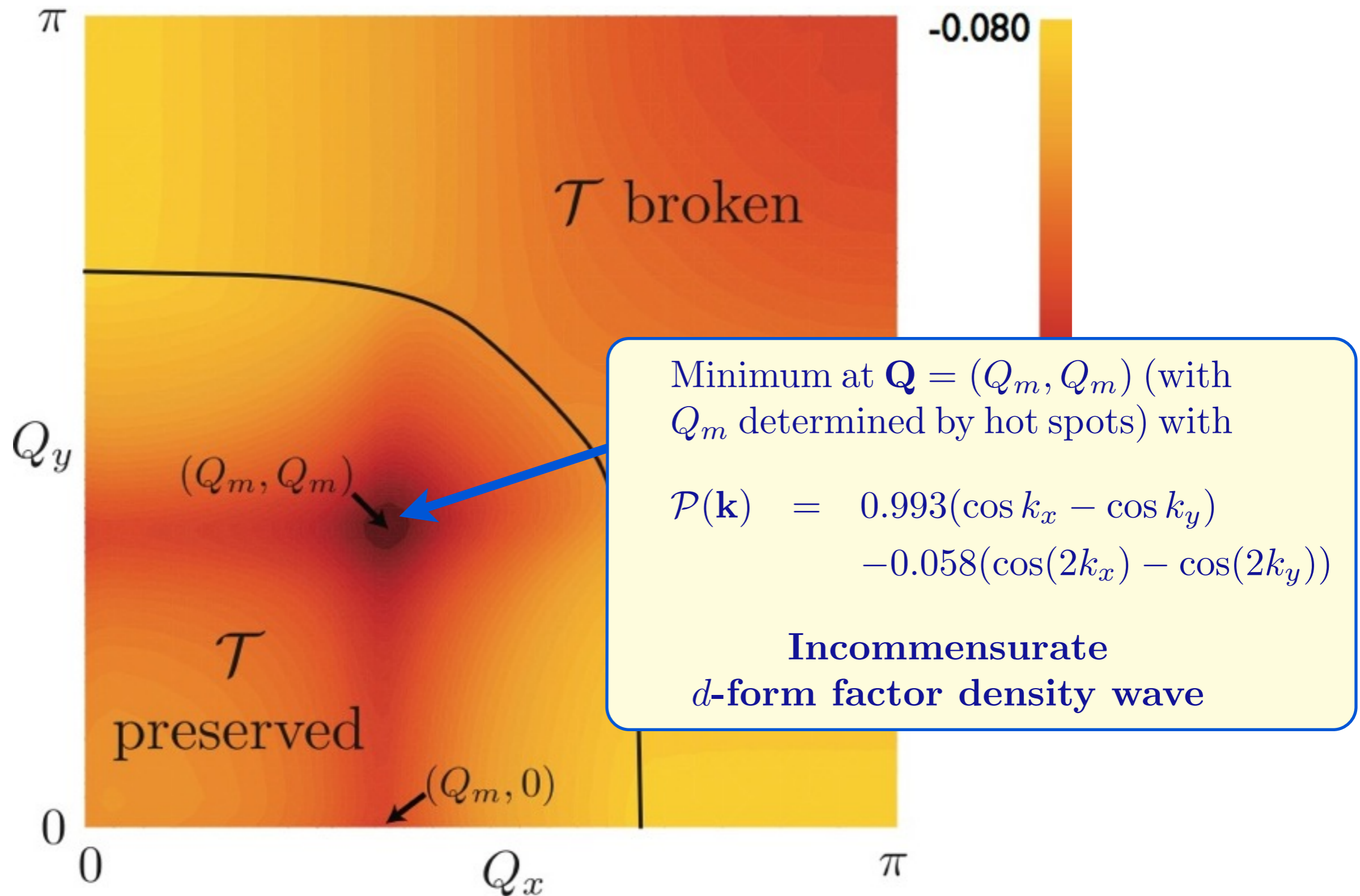
$$H = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \dots$$



- M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)
S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)
J. C. Davis and Dung-Hai Lee, *Proc. Natl. Acad. Sci.* **110**, 17623 (2013)
J. D. Sau and S. Sachdev, *Phys. Rev. B* **89**, 075129 (2014)
A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311



Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order $\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = [\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$

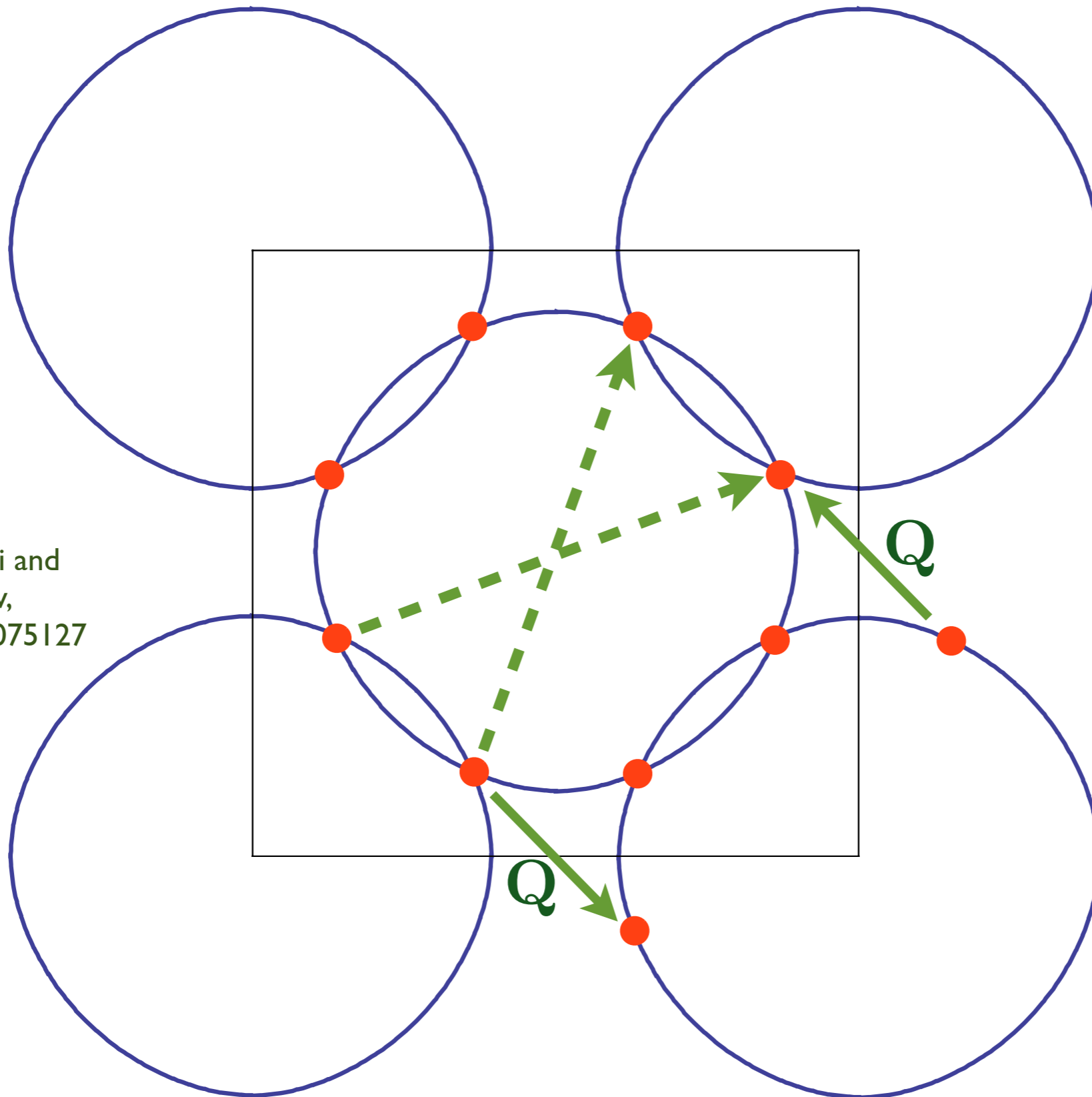


Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

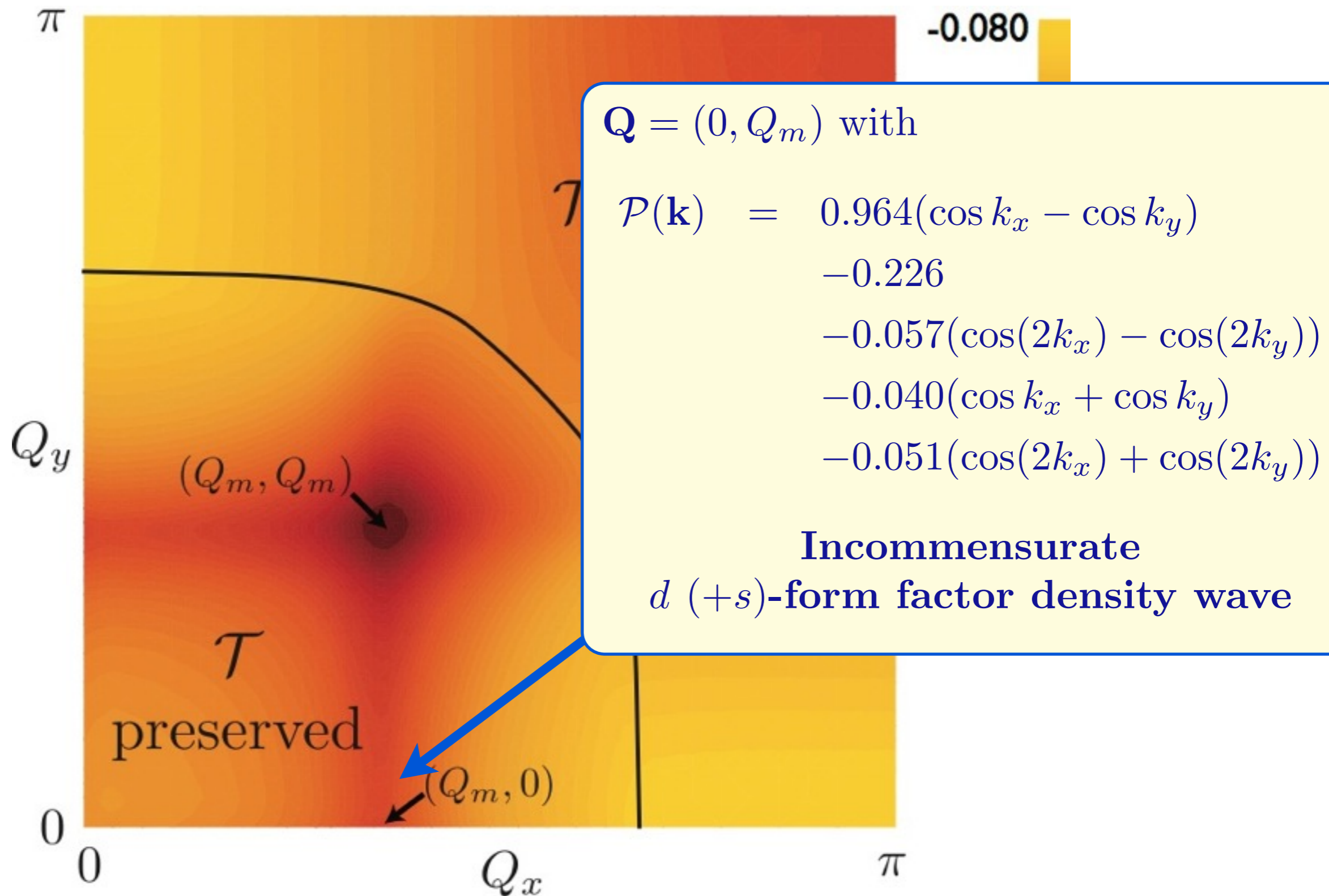
$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

Incommensurate d -form factor density wave

M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)



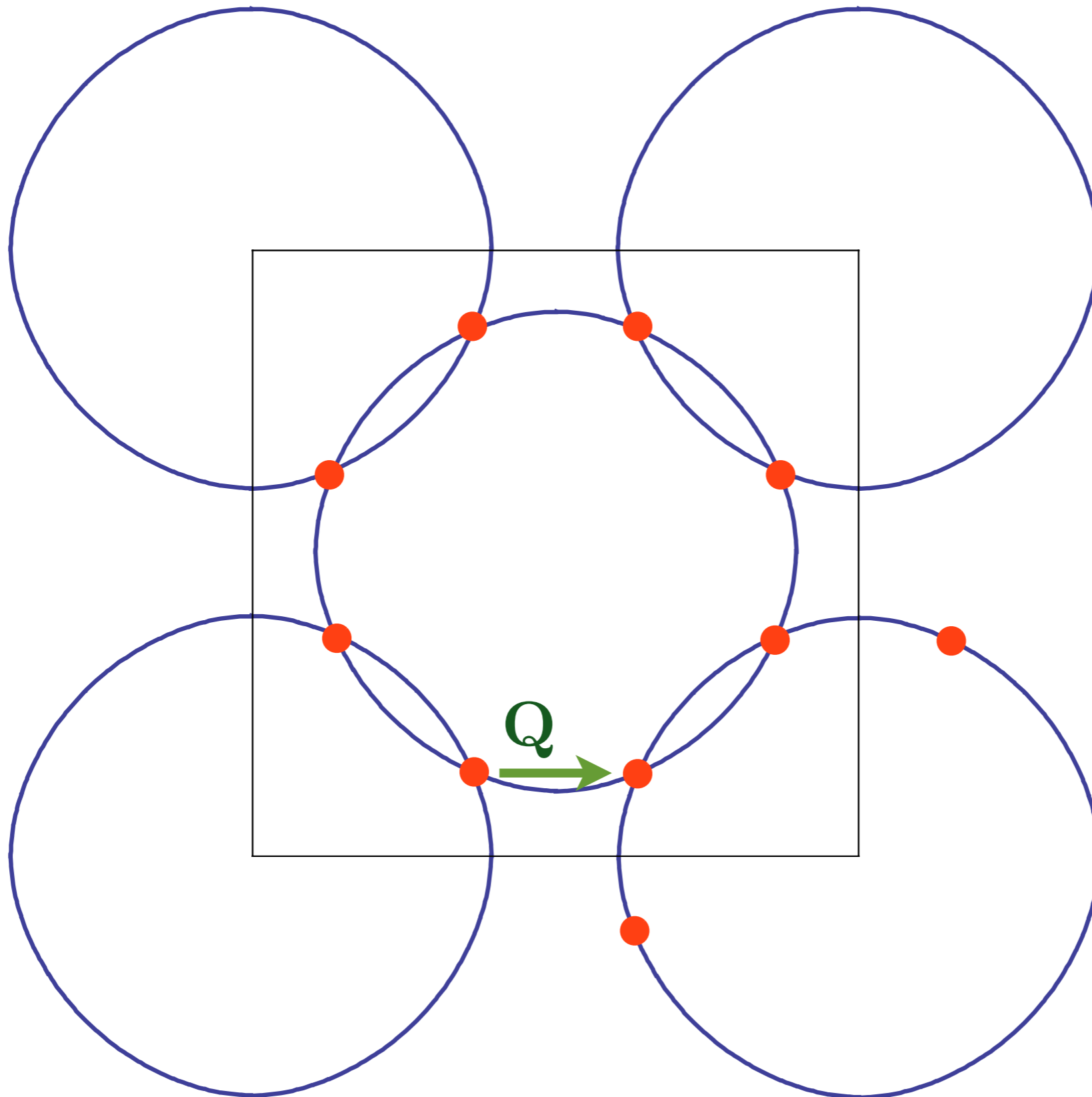
$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \mathcal{P}_d(\cos k_x - \cos k_y)$$



Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

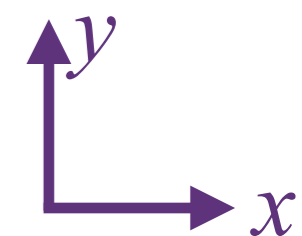
$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

Incommensurate d -form factor density wave



$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle \sim -0.226 + 0.964(\cos k_x - \cos k_y)$$

Unconventional DW order: $(d + s)$ -form factor

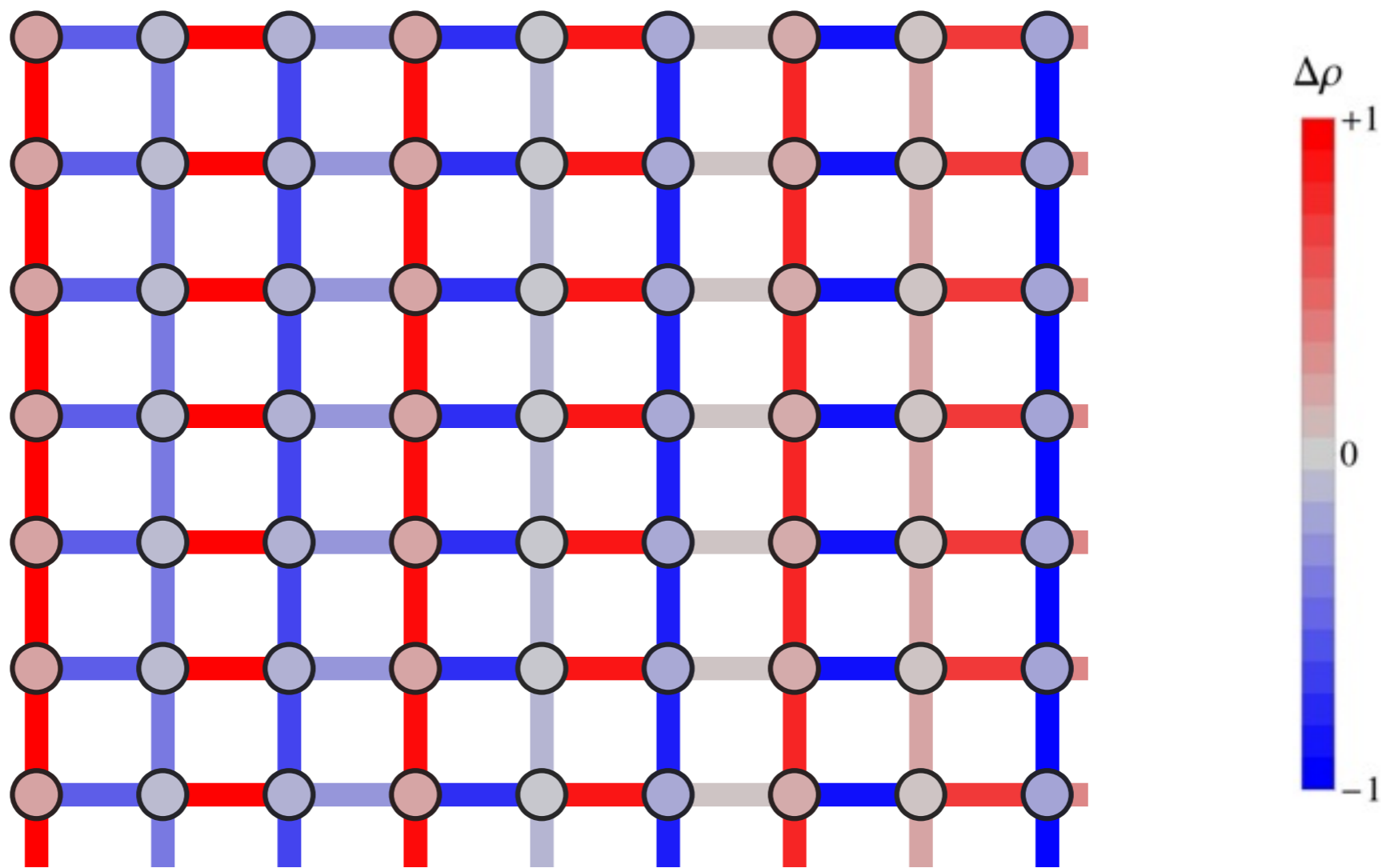


Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

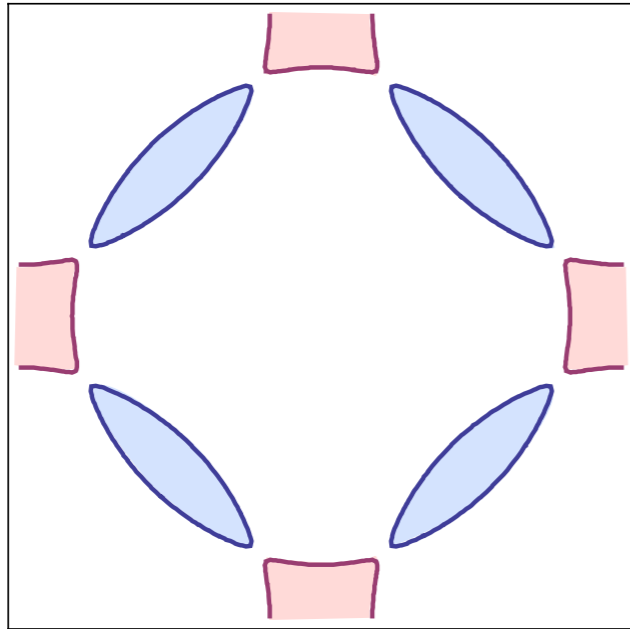
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [0.2 + \cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



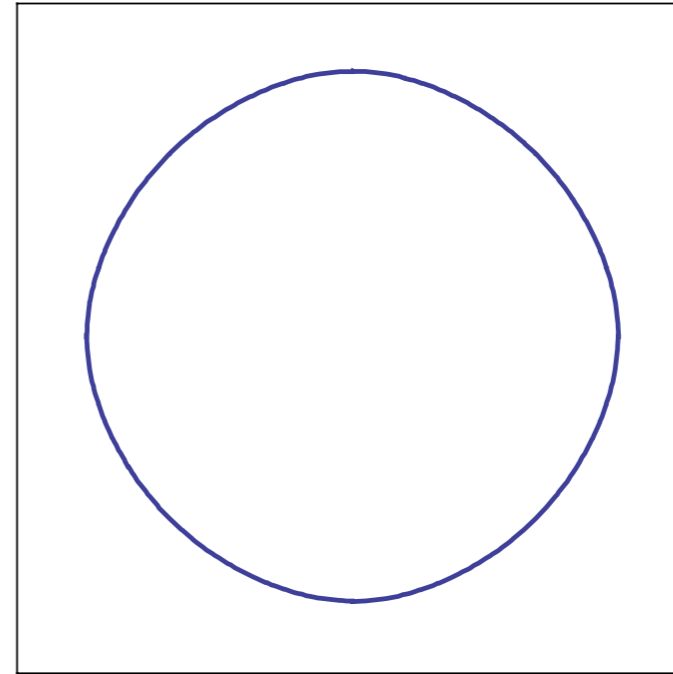
This specific d -form factor density wave order (with \mathbf{Q} along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

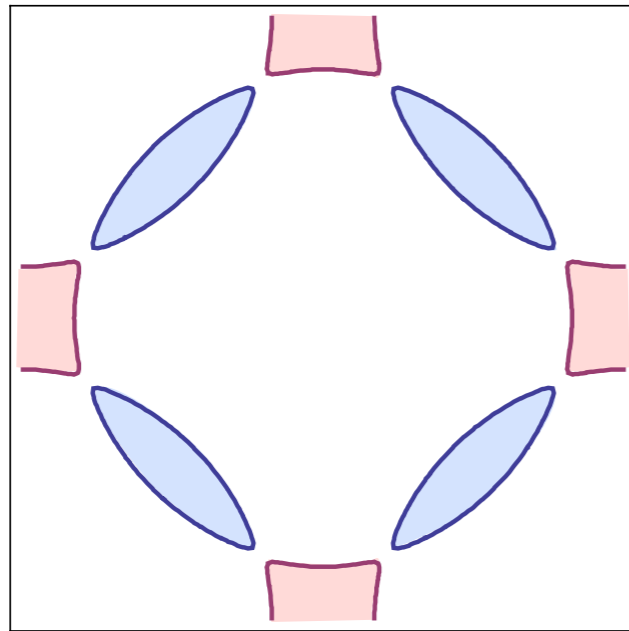


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

r

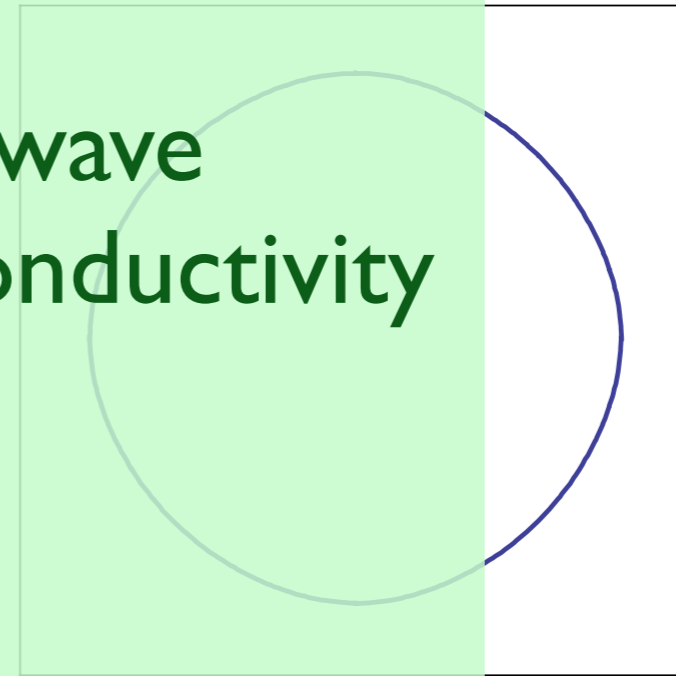
Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

d-wave
superconductivity

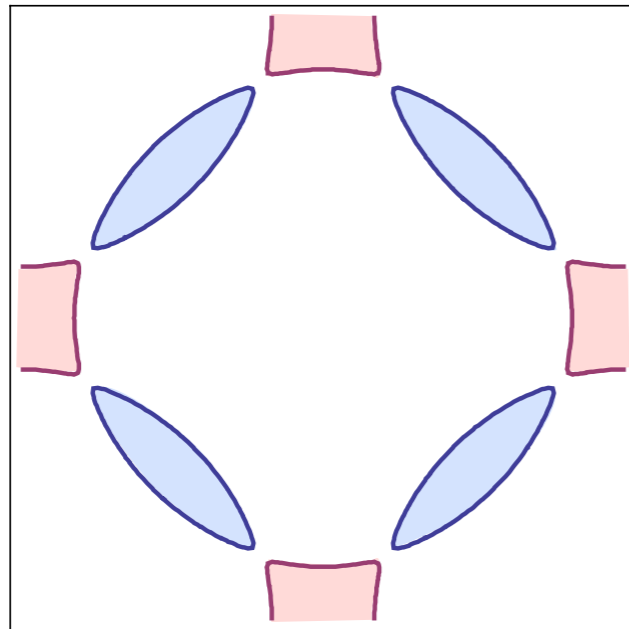


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

r

Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

d-wave
superconductivity
and
an unconventional
charge density wave
with a
d-wave form factor

$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

r

Direct phase-sensitive identification of a d -form factor density wave in underdoped cuprates

Kazuhiro Fujita^{a,b,c,1}, Mohammad H. Hamidian^{a,b,1}, Stephen D. Edkins^{b,d}, Chung Koo Kim^a, Yuhki Kohsaka^e, Masaki Azuma^f, Mikio Takano^g, Hidenori Takagi^{c,h,i}, Hiroshi Eisaki^j, Shin-ichi Uchida^c, Andrea Allais^k, Michael J. Lawler^{b,l}, Eun-Ah Kim^b, Subir Sachdev^{k,m}, and J. C. Séamus Davis^{a,b,d,2}

The identity of the fundamental broken symmetry (if any) in the underdoped cuprates is unresolved. However, evidence has been accumulating that this state may be an unconventional density wave. Here we carry out site-specific measurements within each CuO_2 unit cell, segregating the results into three separate electronic structure images containing only the Cu sites [$\text{Cu}(r)$] and only the x/y axis O sites [$O_x(r)$ and $O_y(r)$]. Phase-resolved Fourier analysis reveals directly that the modulations in the $O_x(r)$ and $O_y(r)$ sublattice images consistently exhibit a relative phase of π . We confirm this discovery on two highly distinct cuprate compounds, ruling out tunnel matrix-element and materials-specific systematics. These observations demonstrate by direct sublattice phase-resolved visualization that the density wave found in underdoped cuprates consists of modulations of the intraunit-cell states that exhibit a predominantly d -symmetry form factor.

**Direct phase-sensitive visualization of the
d-form factor density wave in underdoped cuprates
arXiv:1404.0362, PNAS to appear.**



J. C. Seamus Davis
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Kazuhiro Fujita
Cornell/ BNL



Mohammad Hamidian
Cornell / BNL



Stephen Edkins
St Andrews/Cornell



Yuhki Kohsaka
RIKEN - Tokyo



Michael Lawler
Cornell / Binghamton



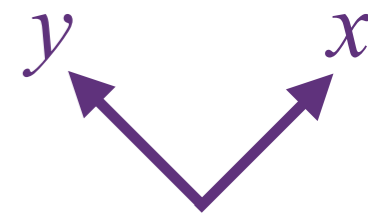
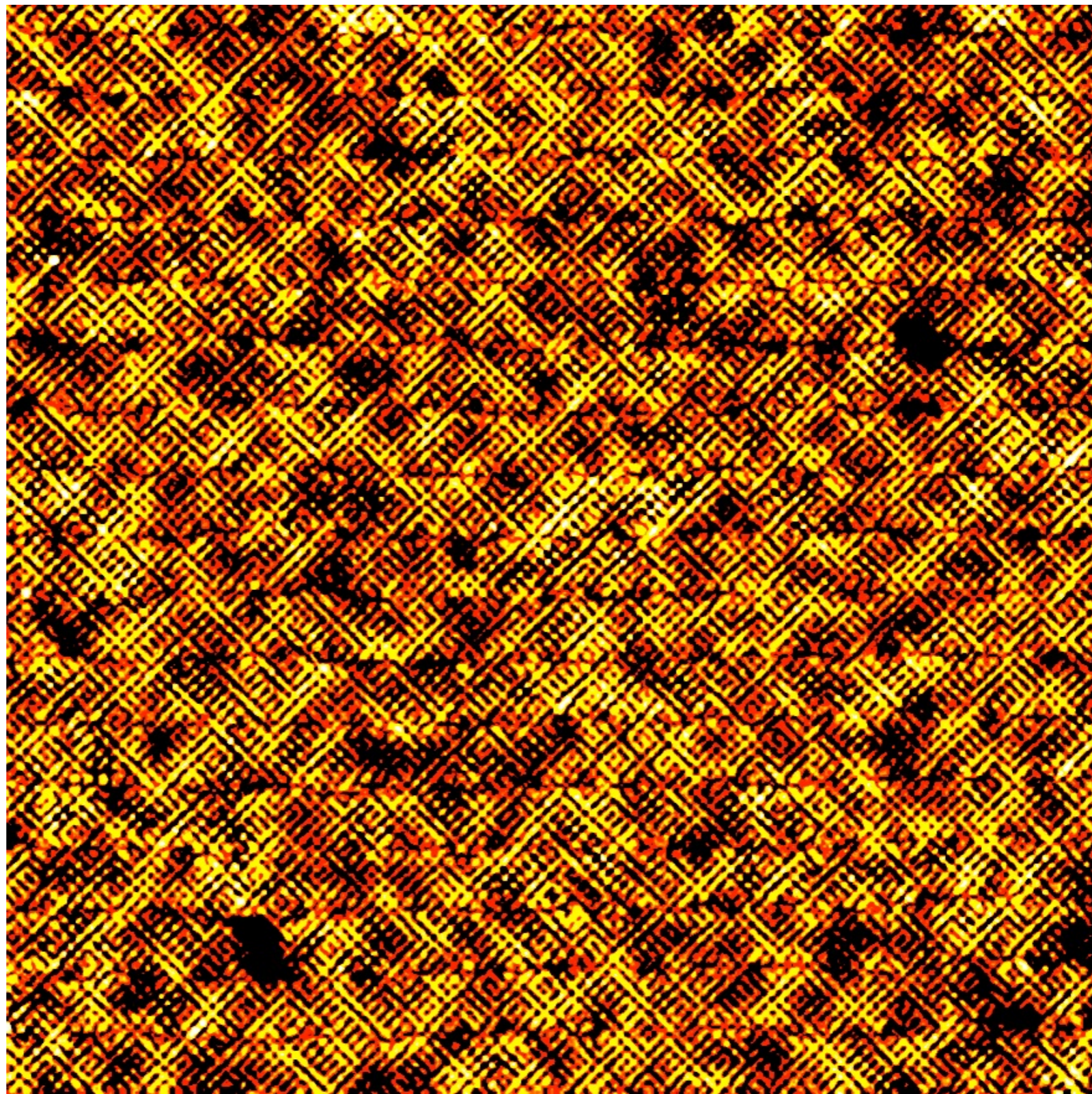
Eun-Ah Kim
Cornell

See also

C. Howald, H. Eisaki,
N. Kaneko, M. Greven,
and A. Kapitulnik,
Phys. Rev. B **67**,
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M. Vershinin, S. Misra,
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Nature Phys. **4**, 696
(2008).



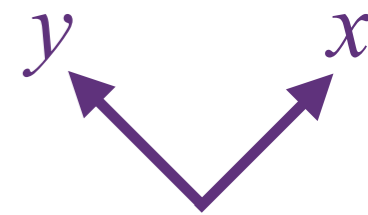
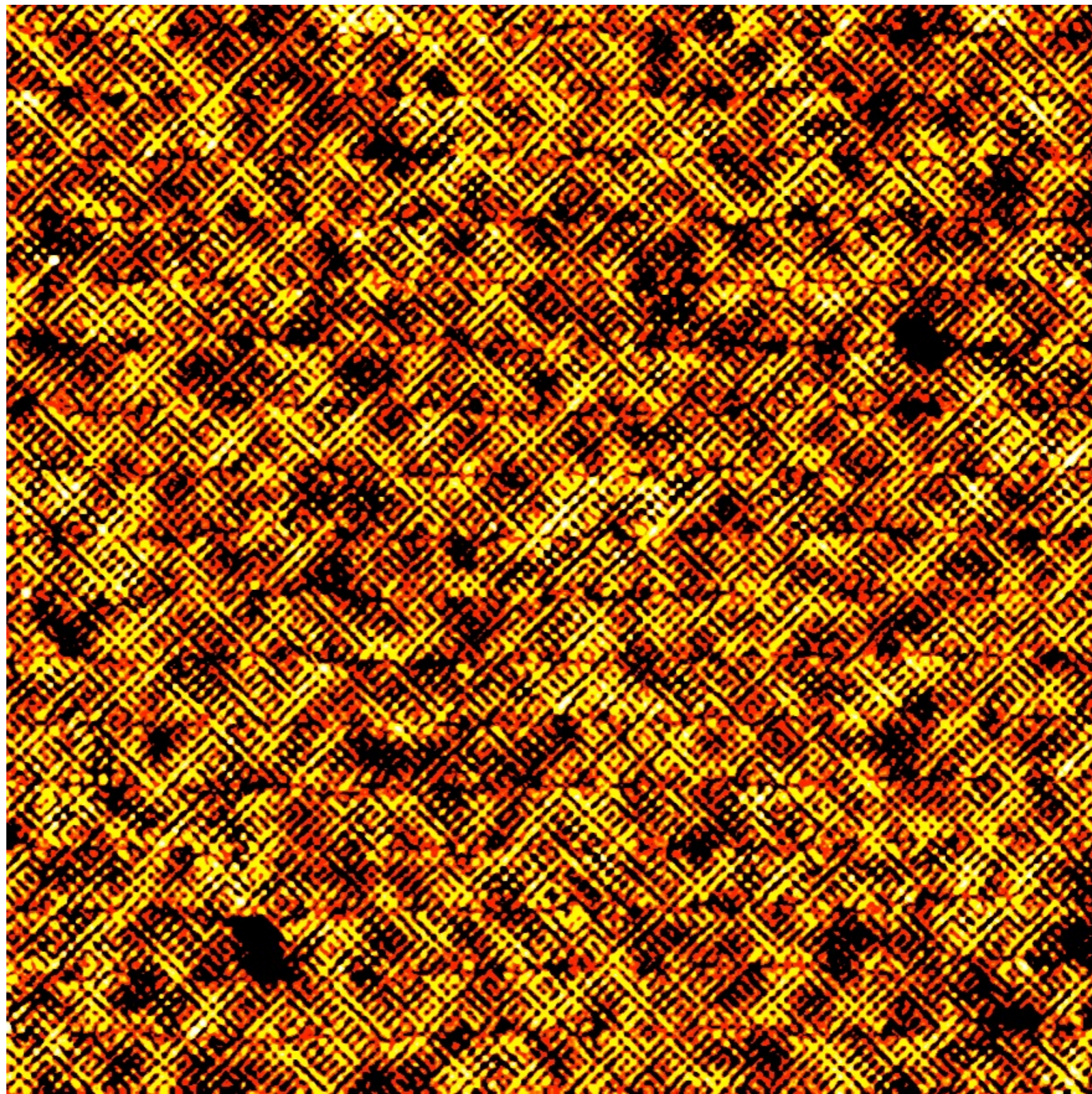
“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). **Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.**

See also

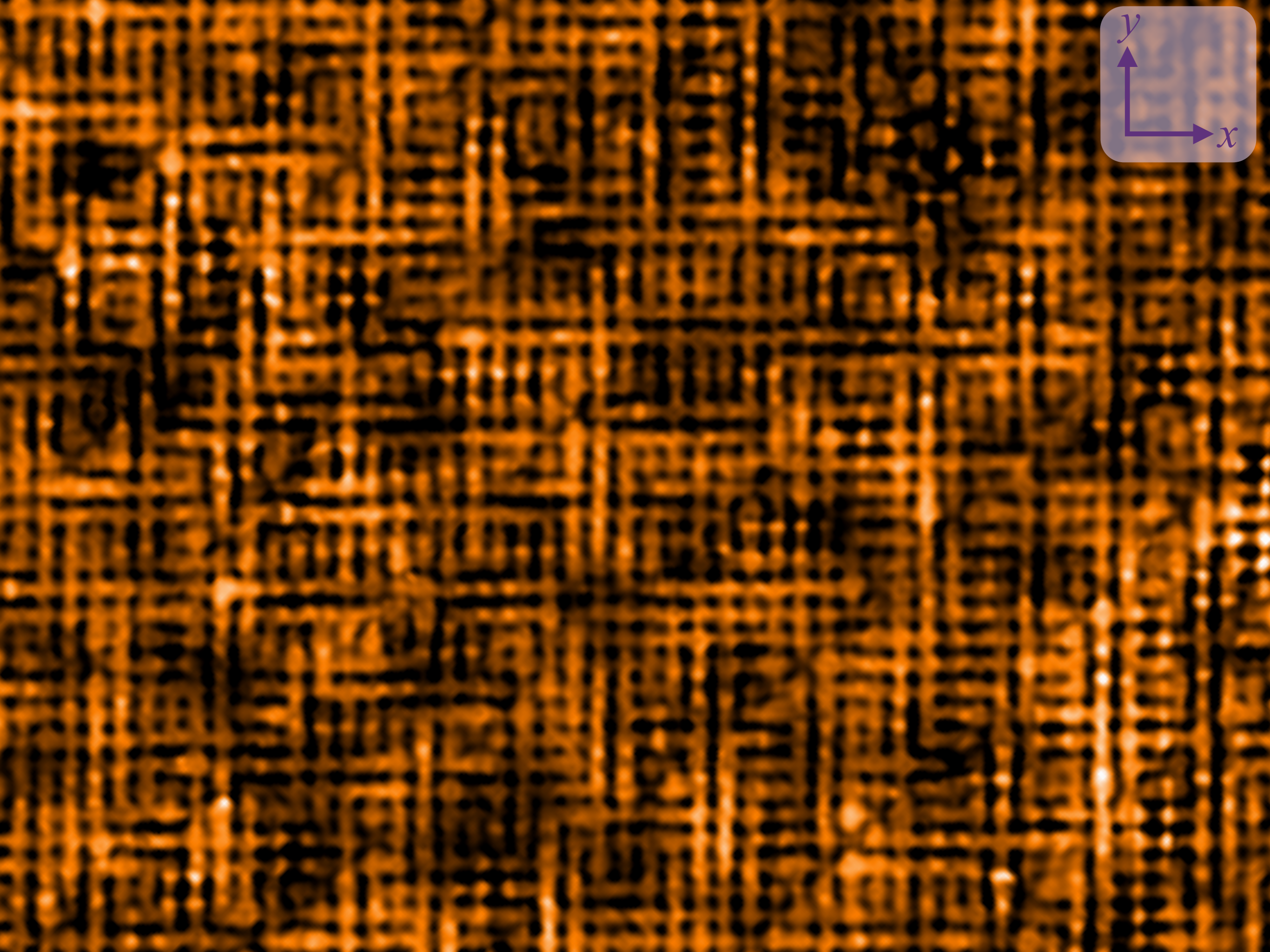
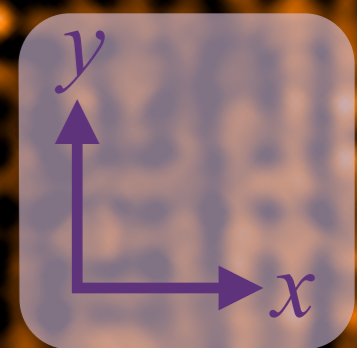
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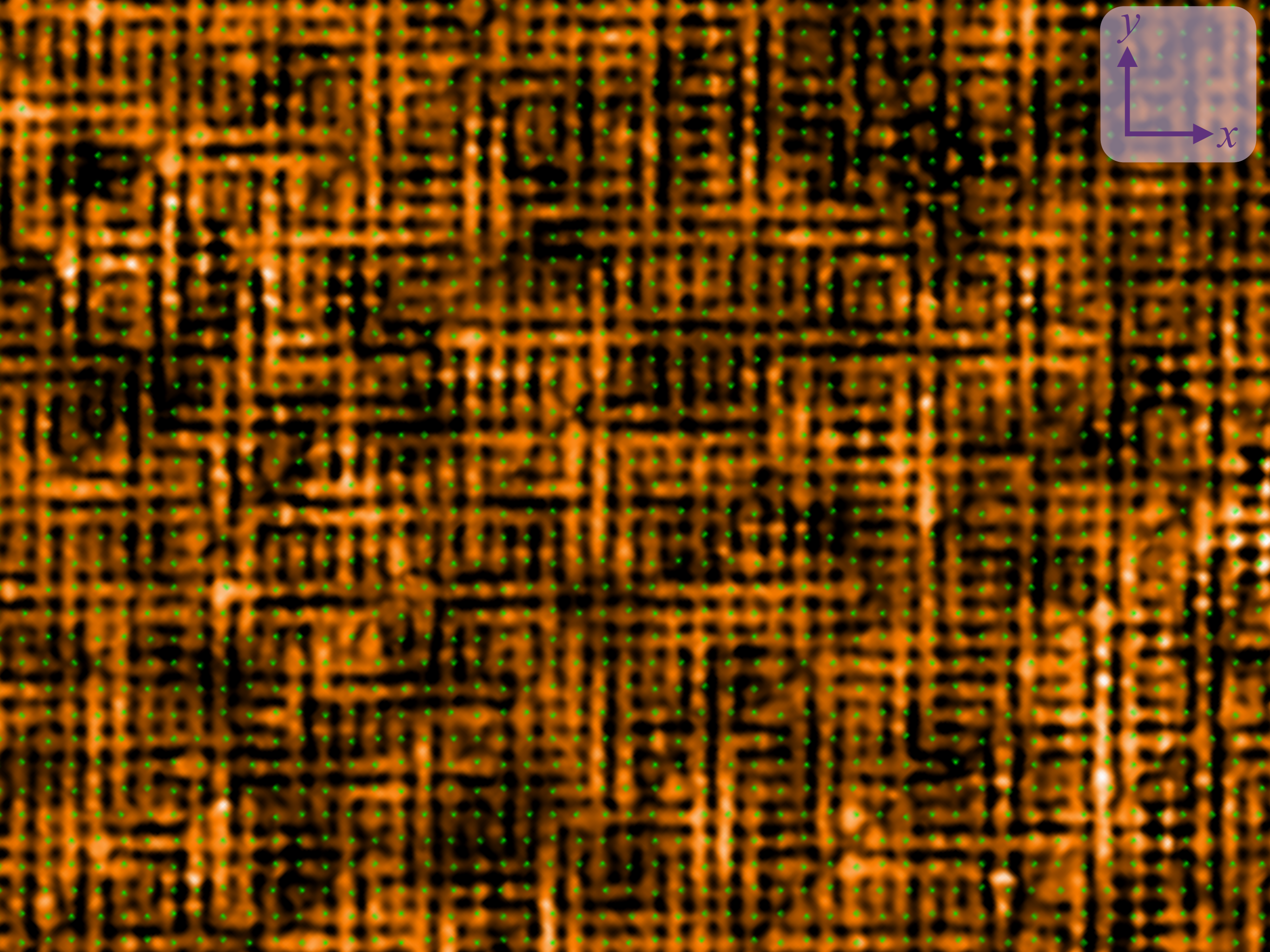
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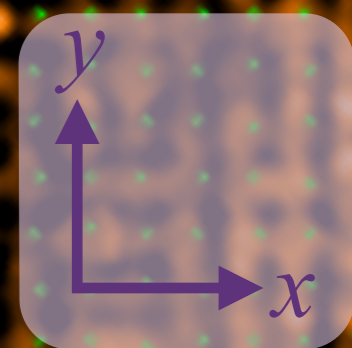


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A density wave with
wavelength ≈ 4 lattice sites ?

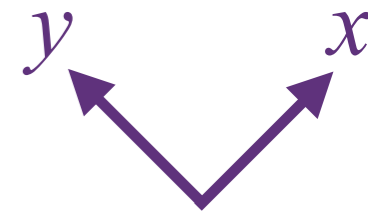
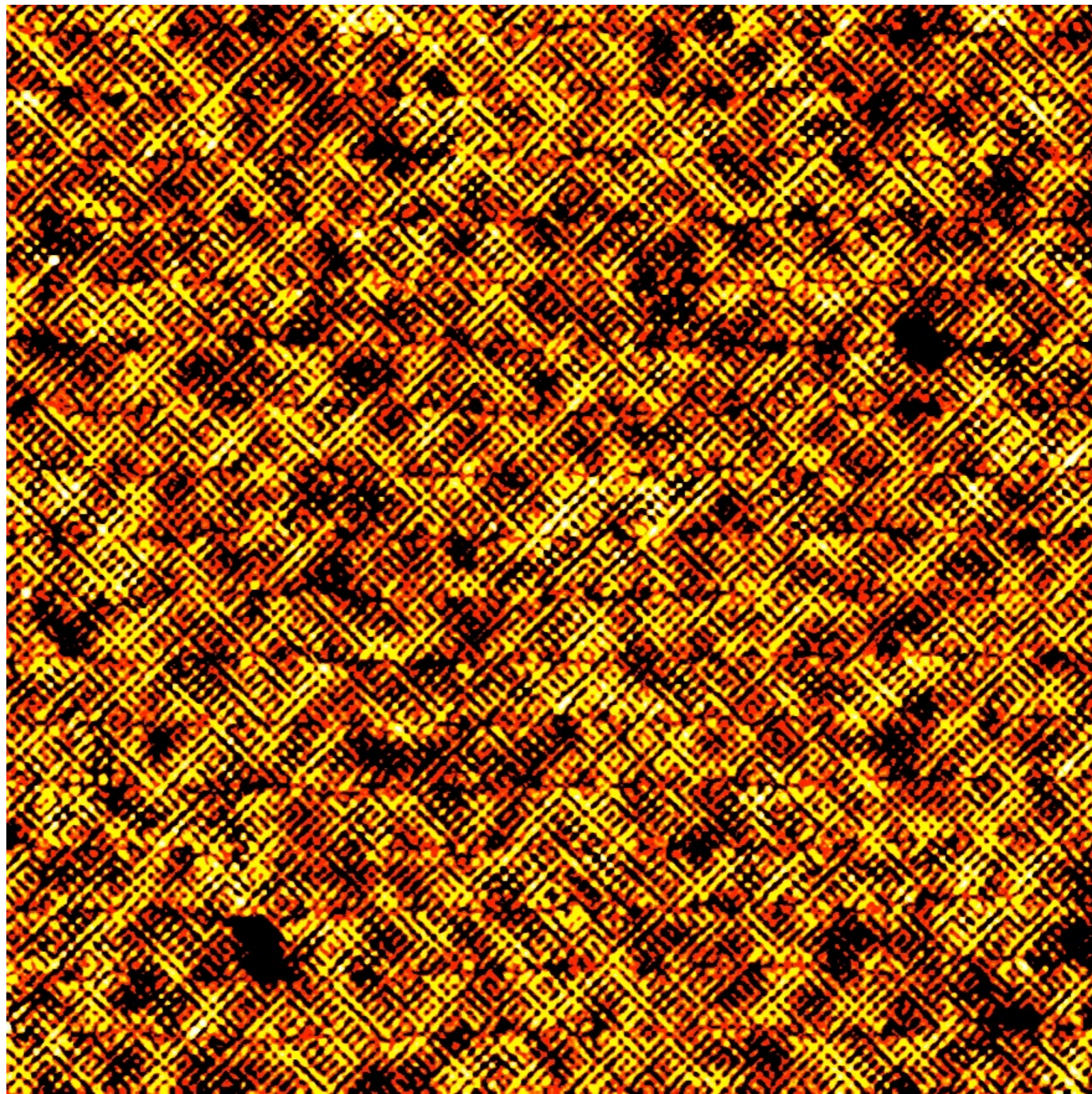


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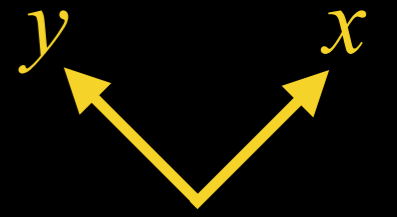
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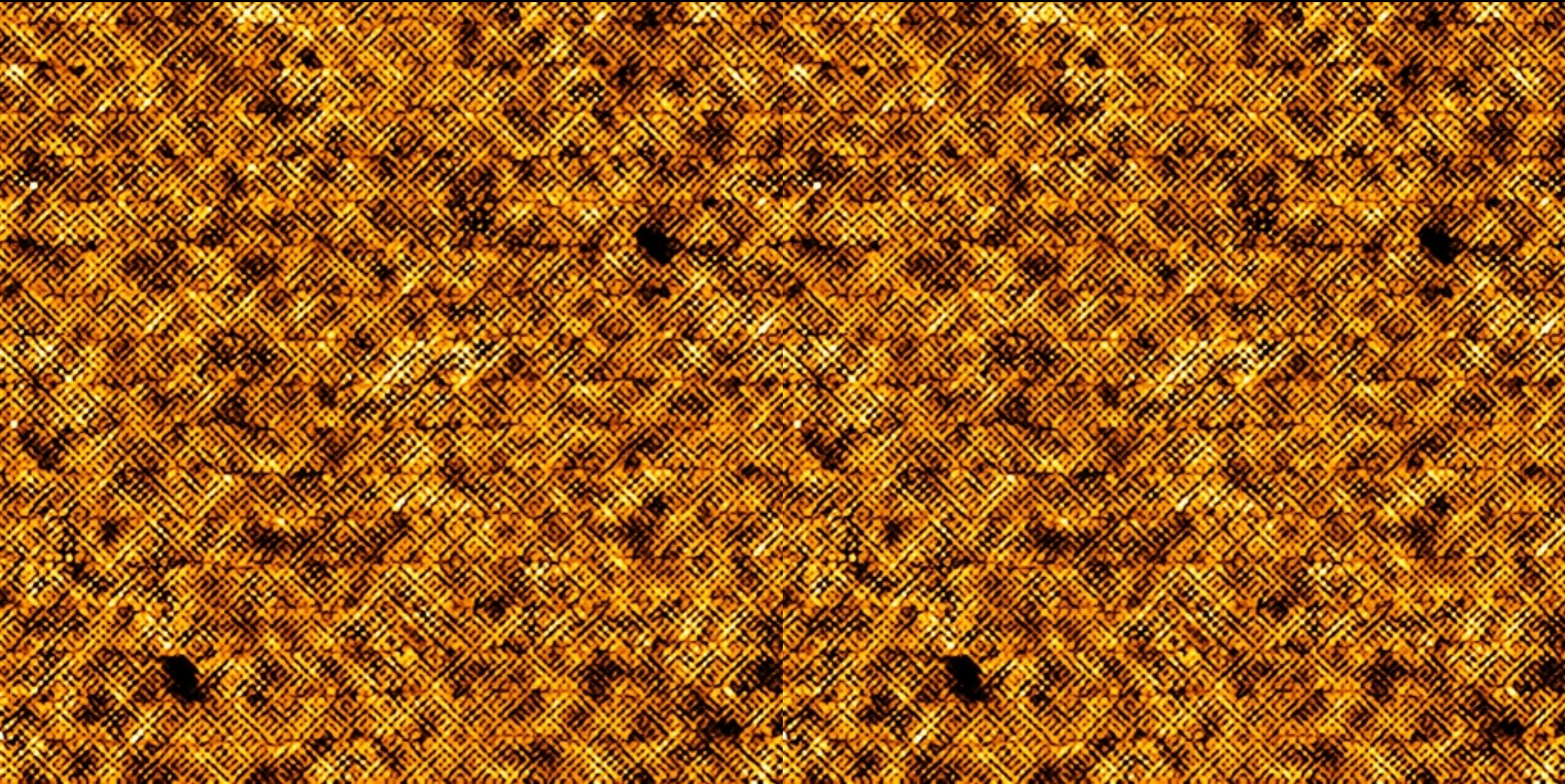
UD45K
BSCCO

$R(r, 150\text{mV})$



$R(r, 150\text{mV})$

$R(r, 150\text{mV})$



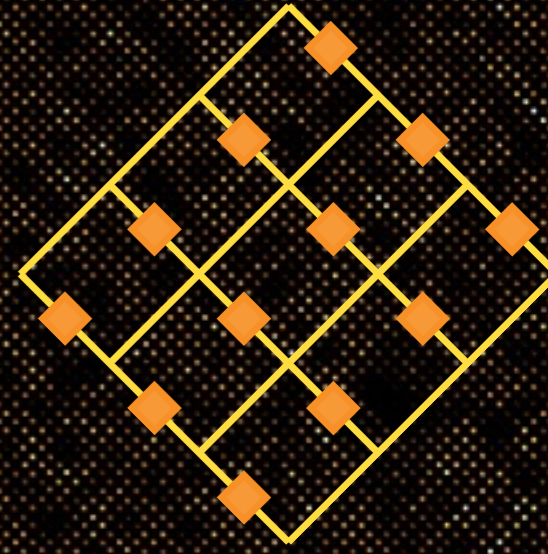
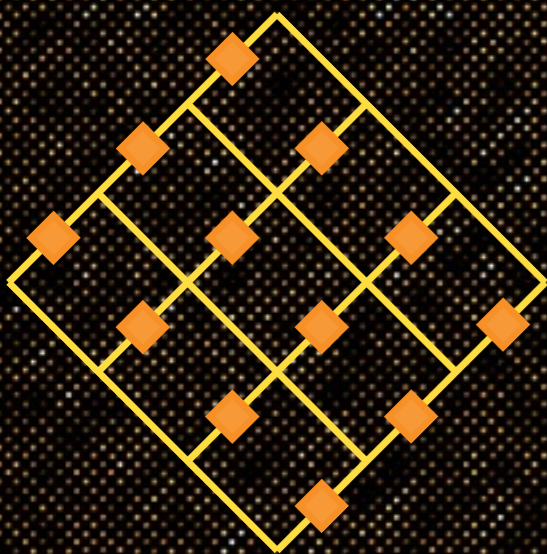
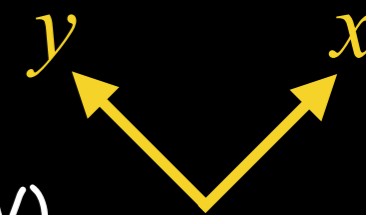
Note that these are identical images.

UD45K

$R(r=0, 150\text{mV})$

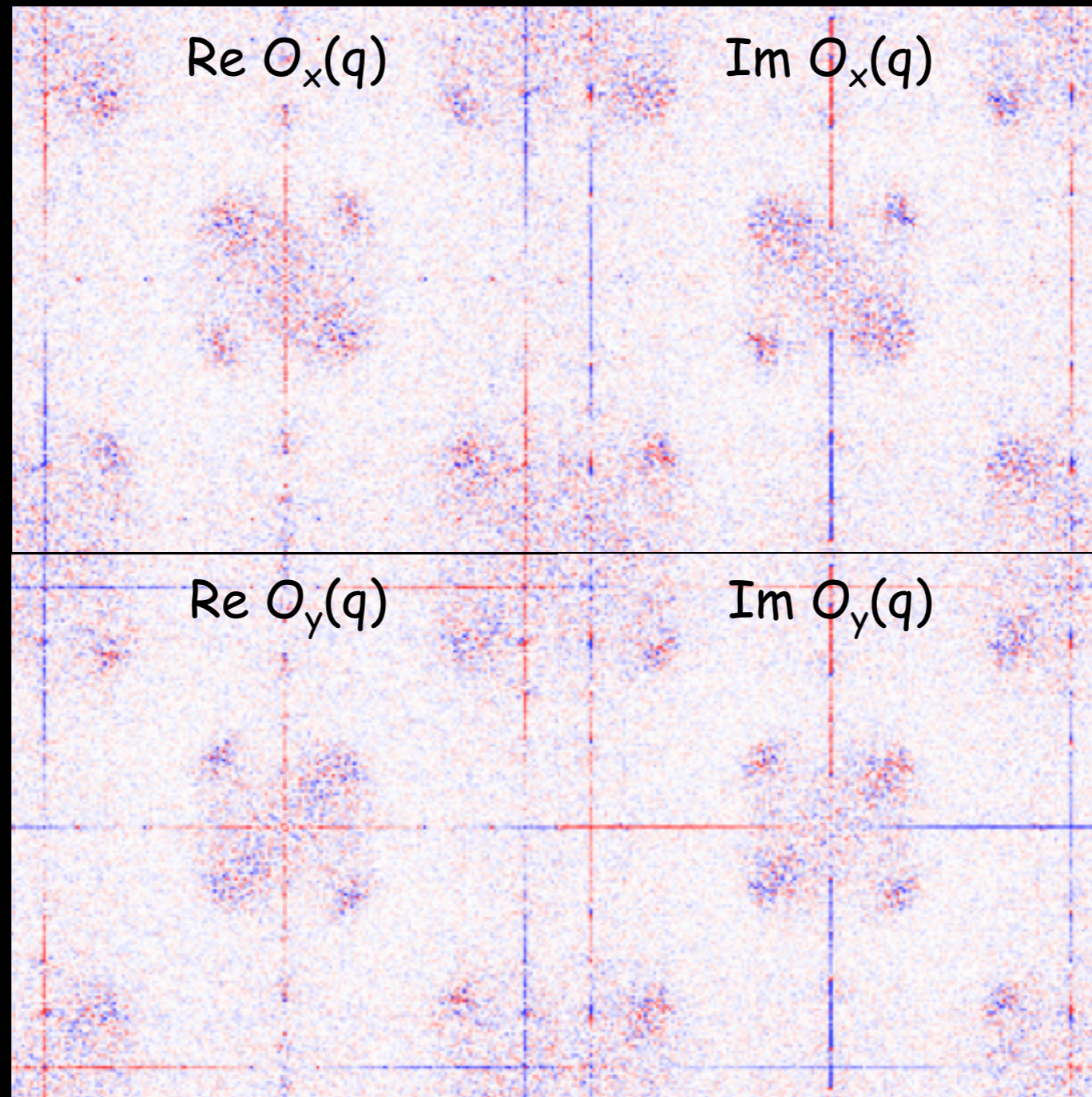
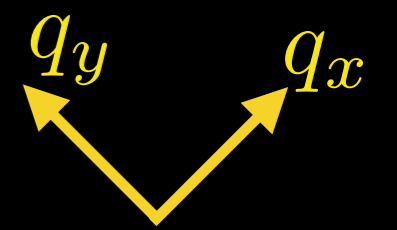
$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$

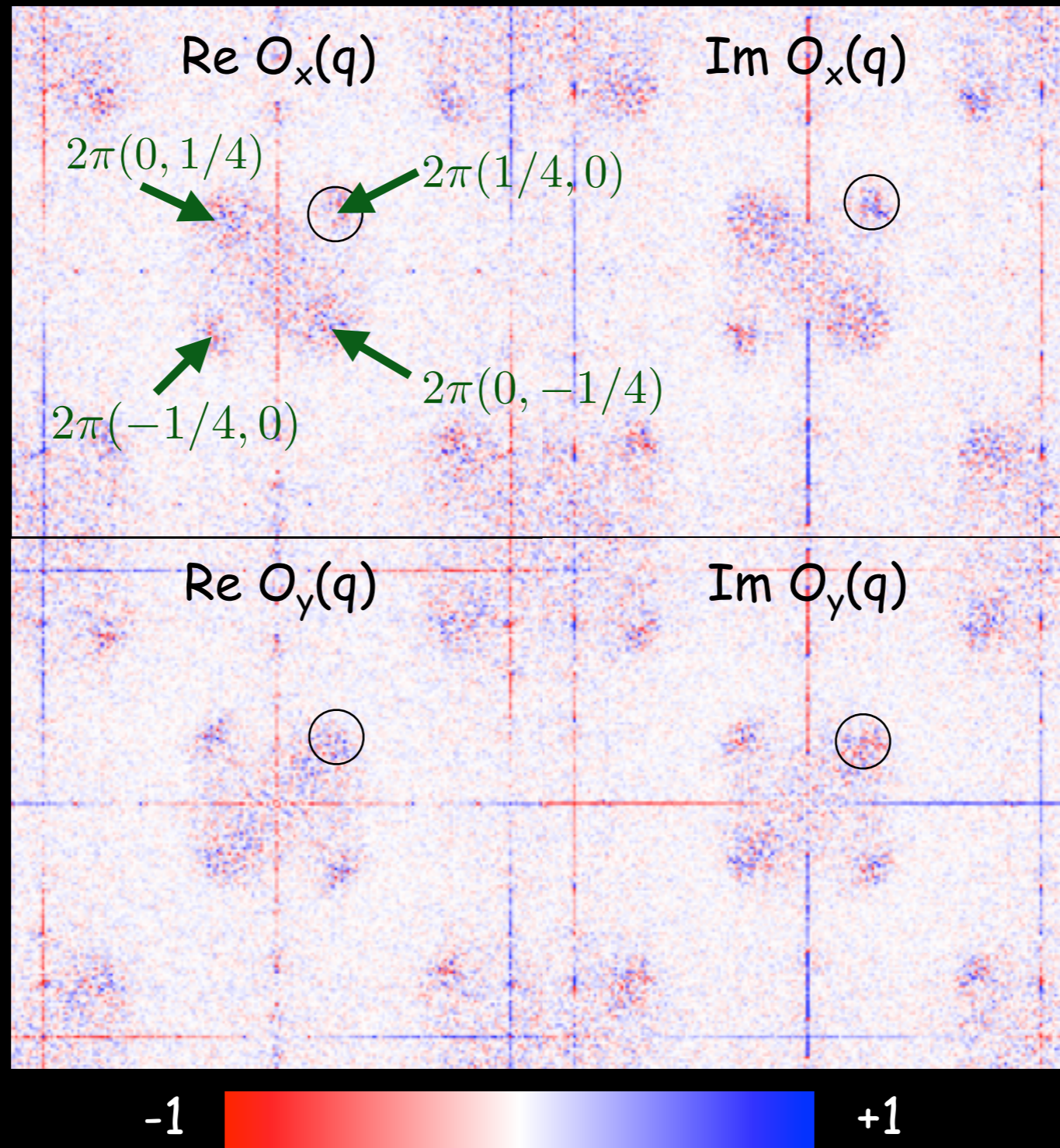
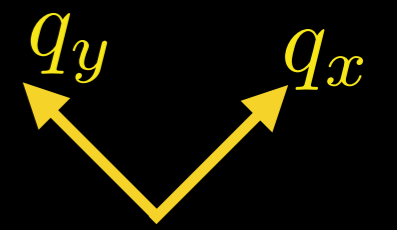


UD45K

Broad (0,Q) and (Q,0) DW Features

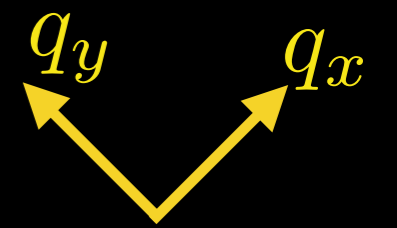


Broad (0,Q) and (Q,0) DW Features

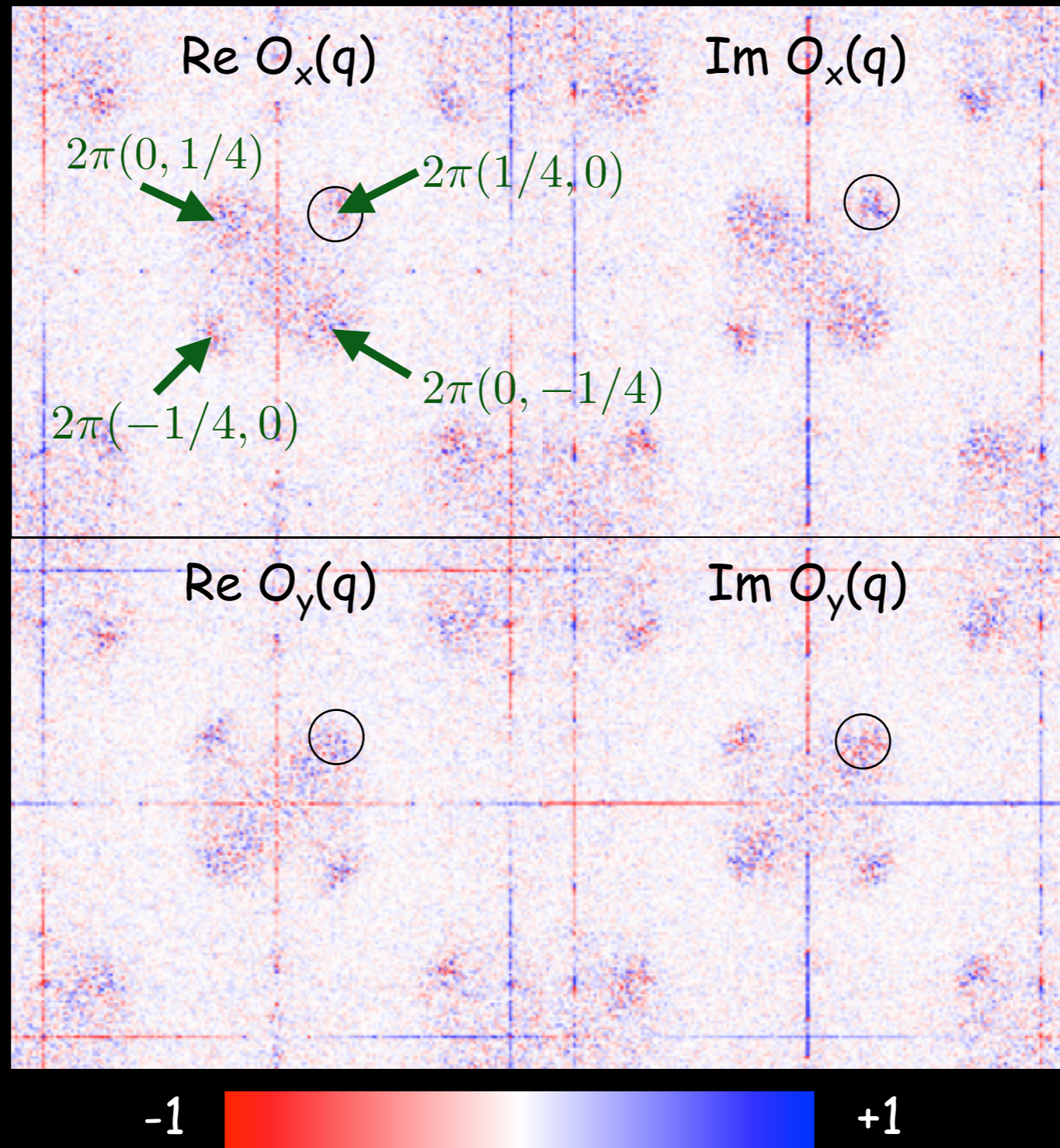


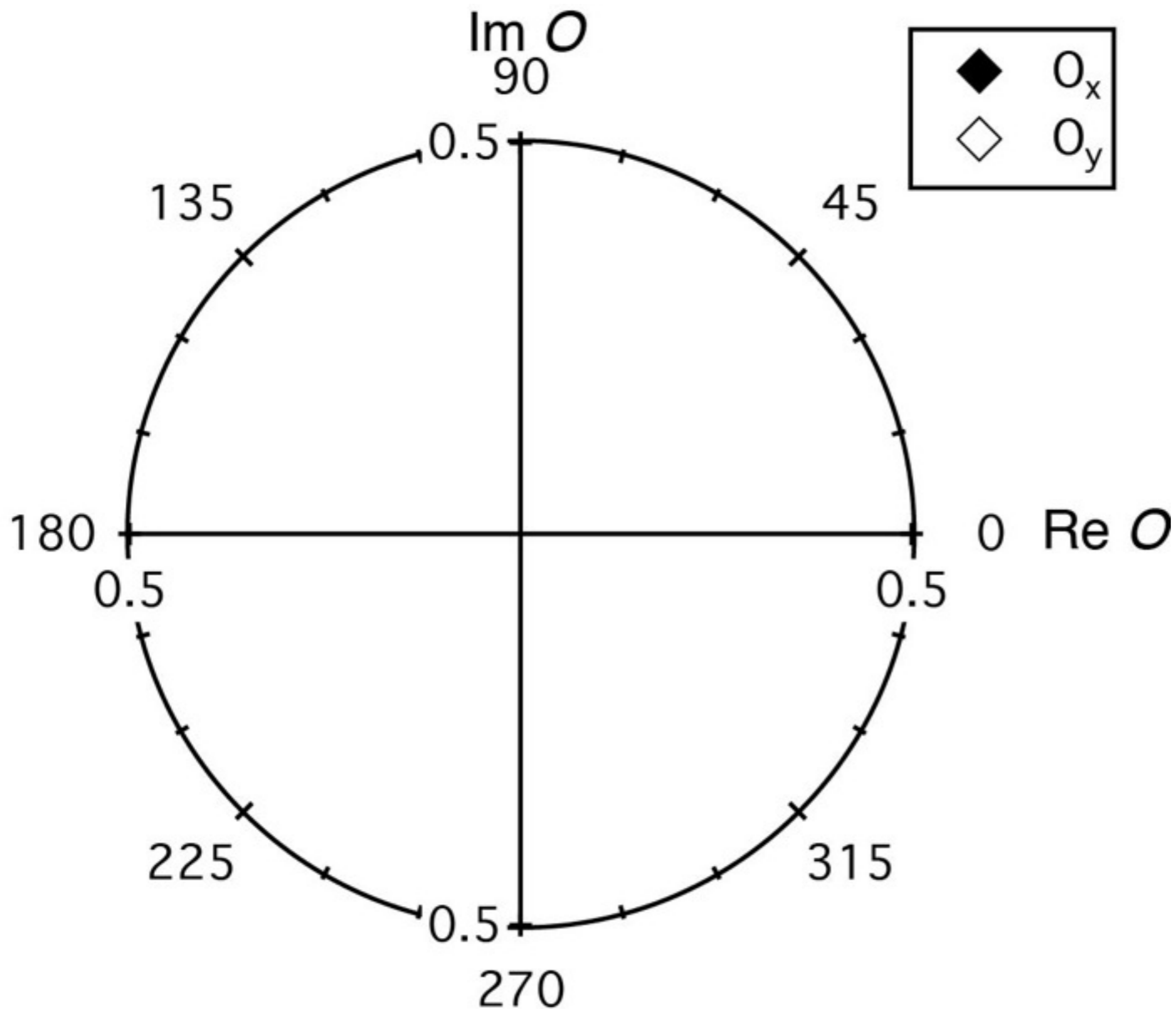
UD45K

Broad (0,Q) and (Q,0) DW Features

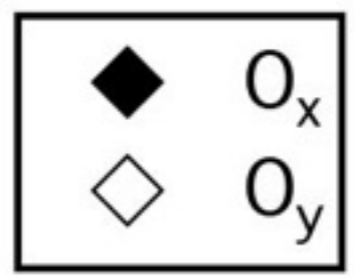
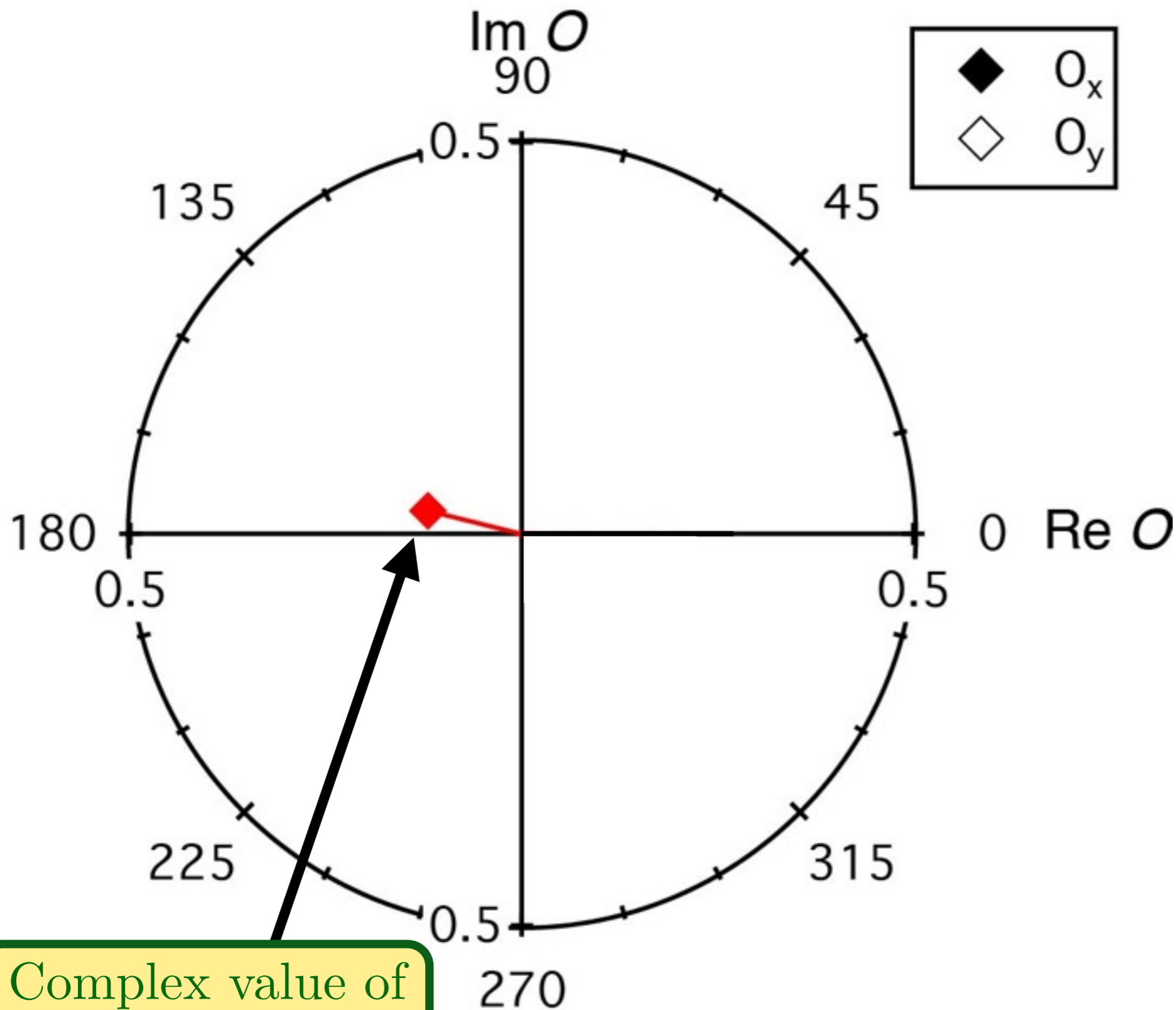


For each pixel in the circles, we obtain 2 complex numbers, $O_x(q)$ and $O_y(q)$.



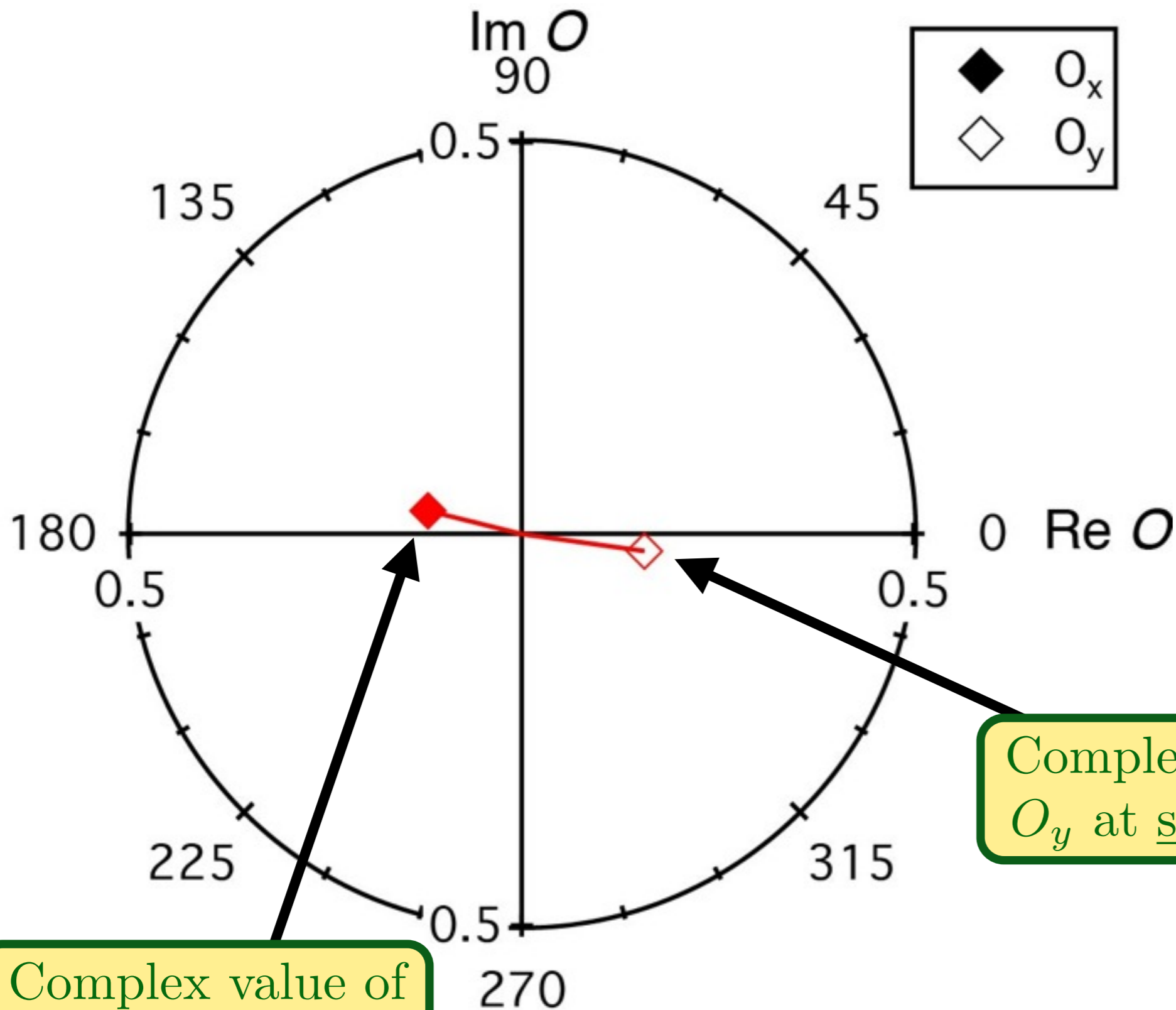


Phase-sensitive measurement of the d -form factor of density wave order



Phase-sensitive measurement of the d -form factor of density wave order

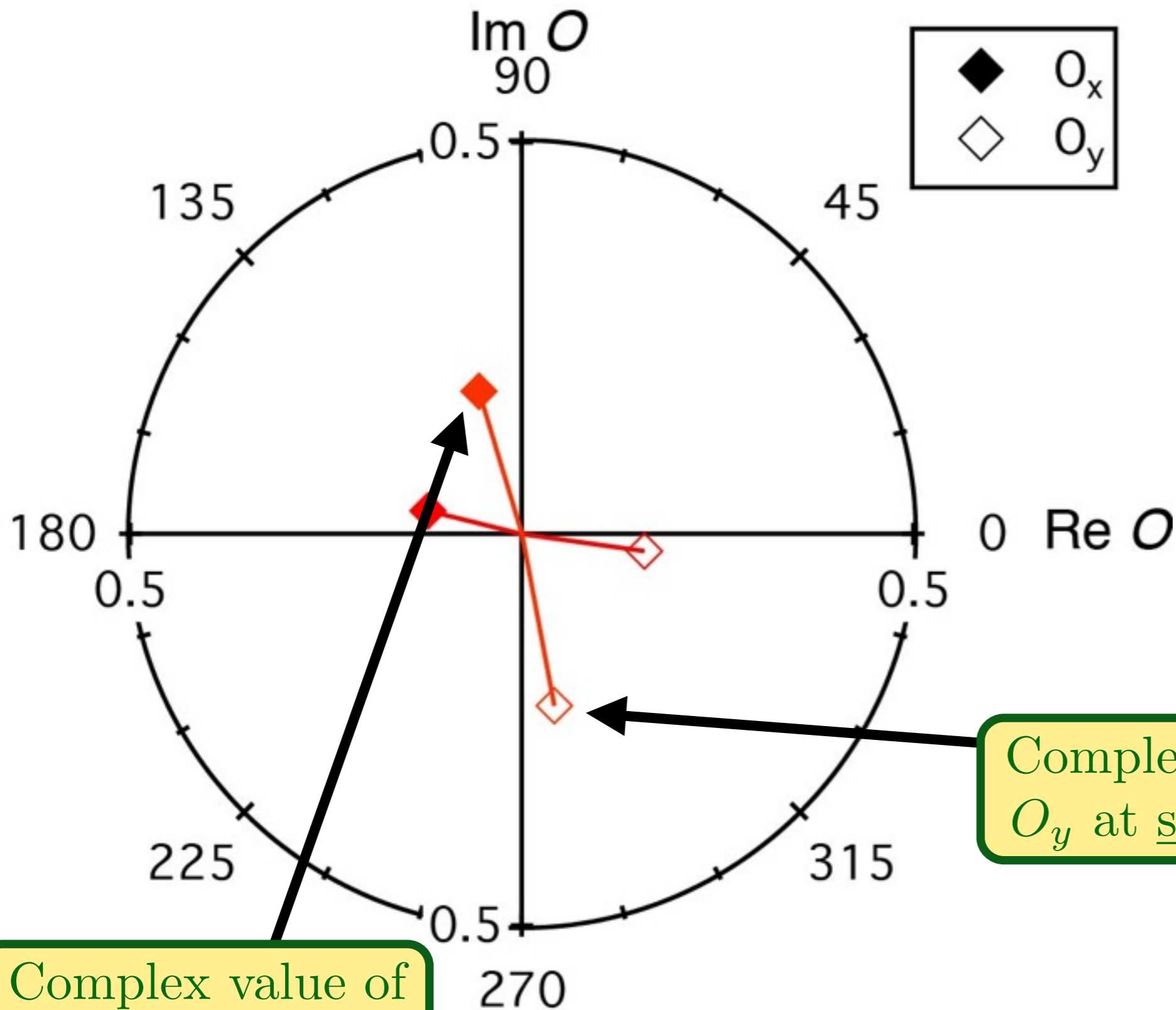
Complex value of O_x at a pixel



Phase-sensitive measurement of the d -form factor of density wave order

Complex value of O_x at a pixel

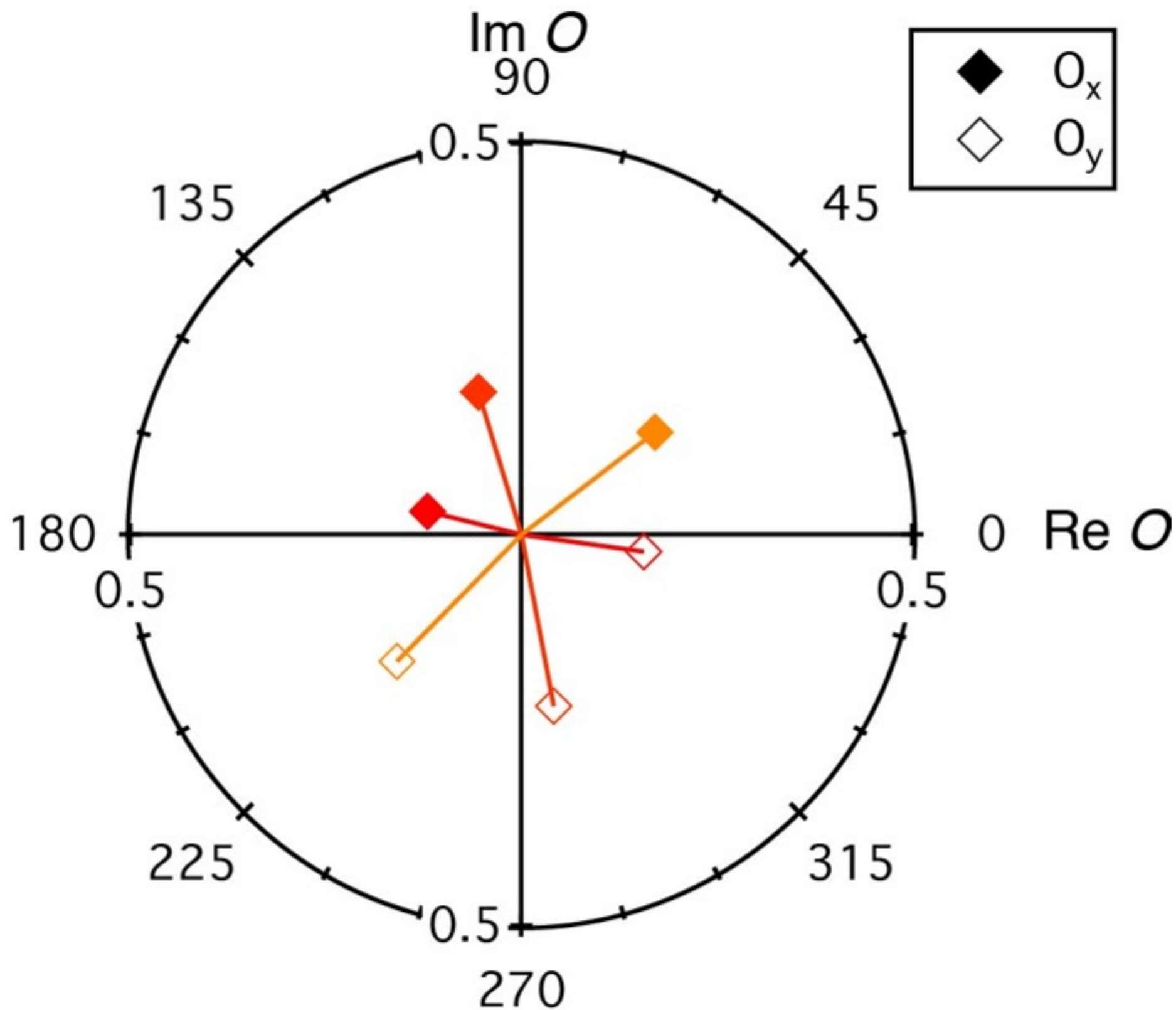
Complex value of O_y at same pixel



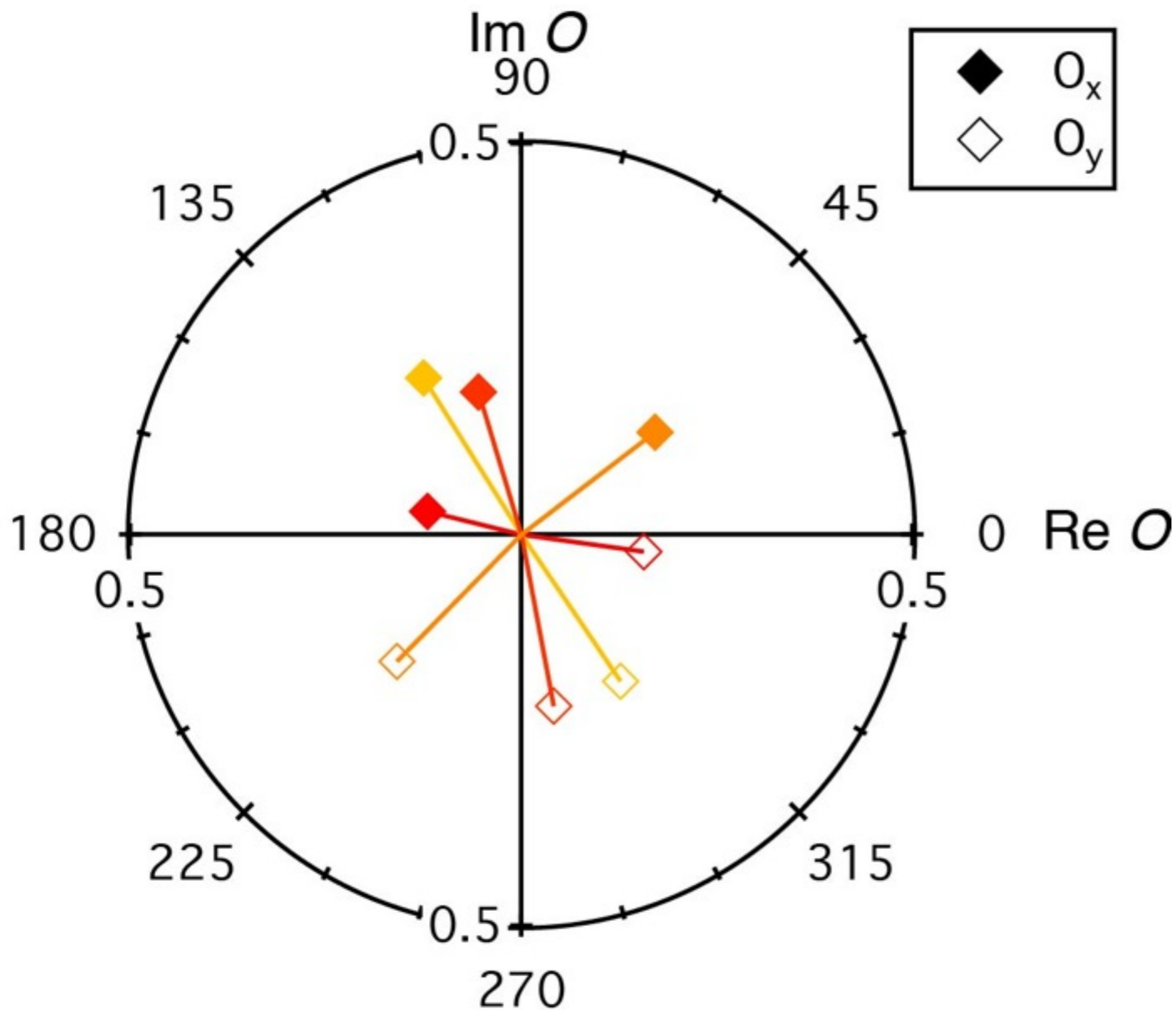
Phase-sensitive measurement of the d -form factor of density wave order

Complex value of O_x at a pixel

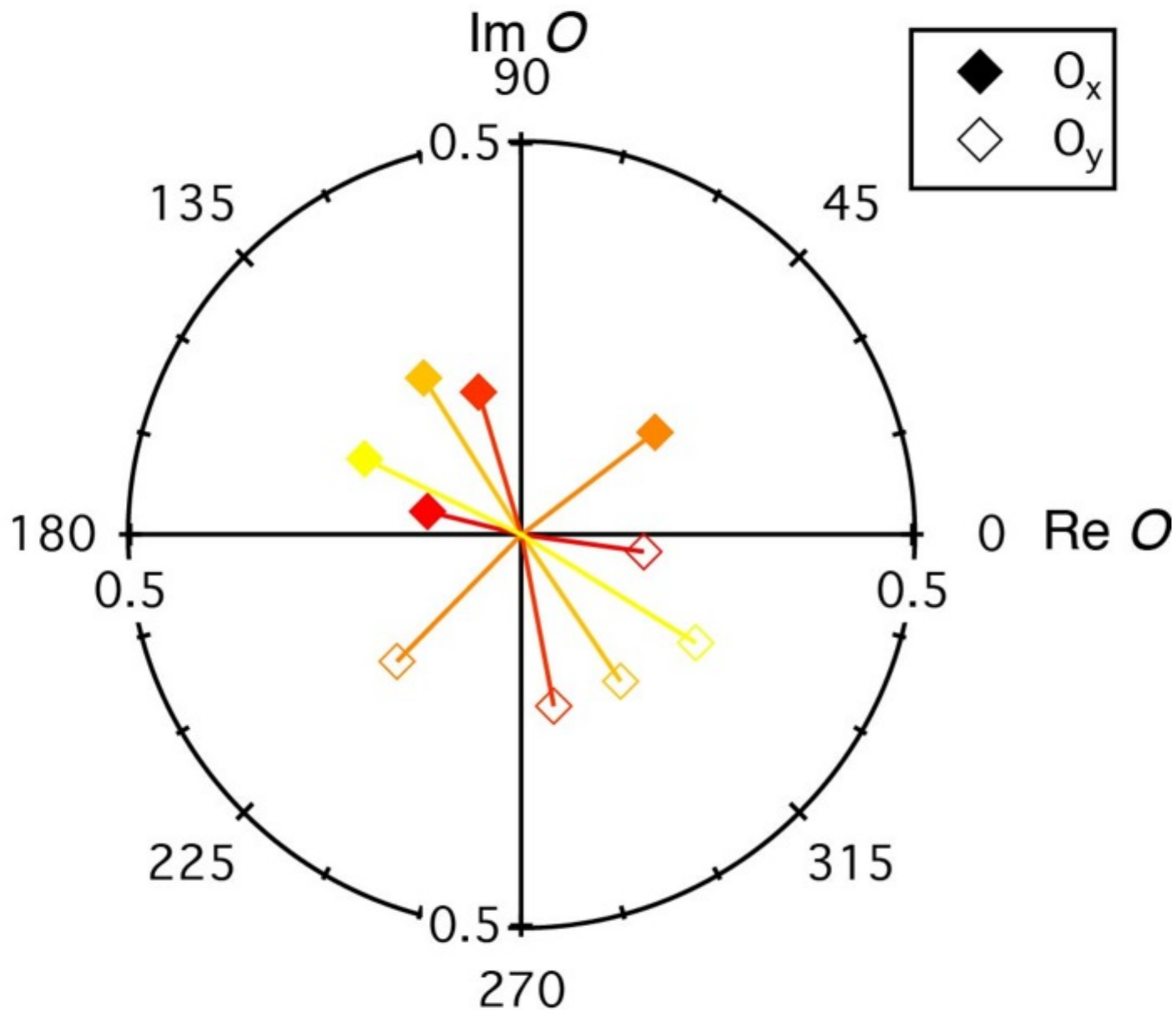
Complex value of O_y at same pixel



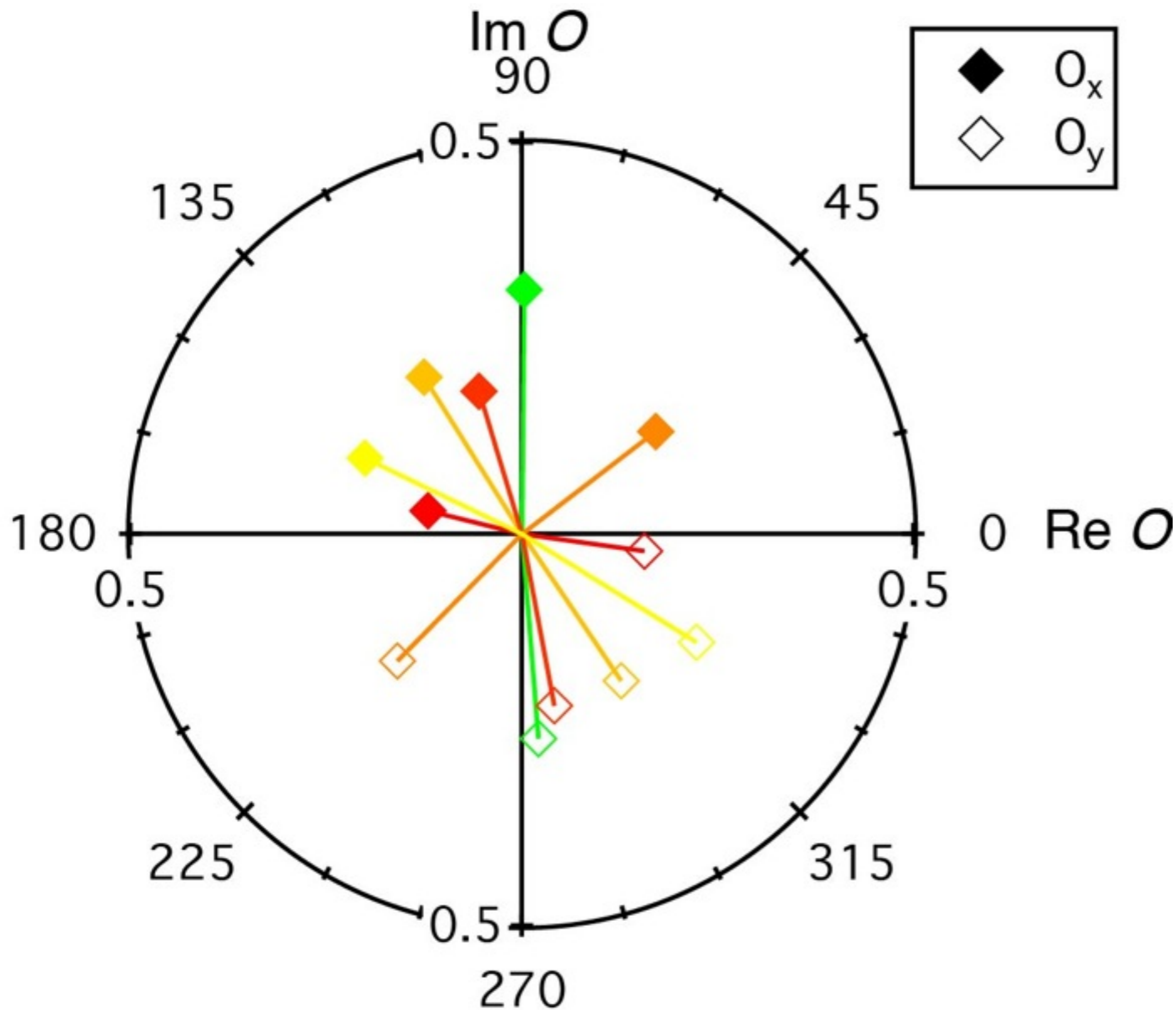
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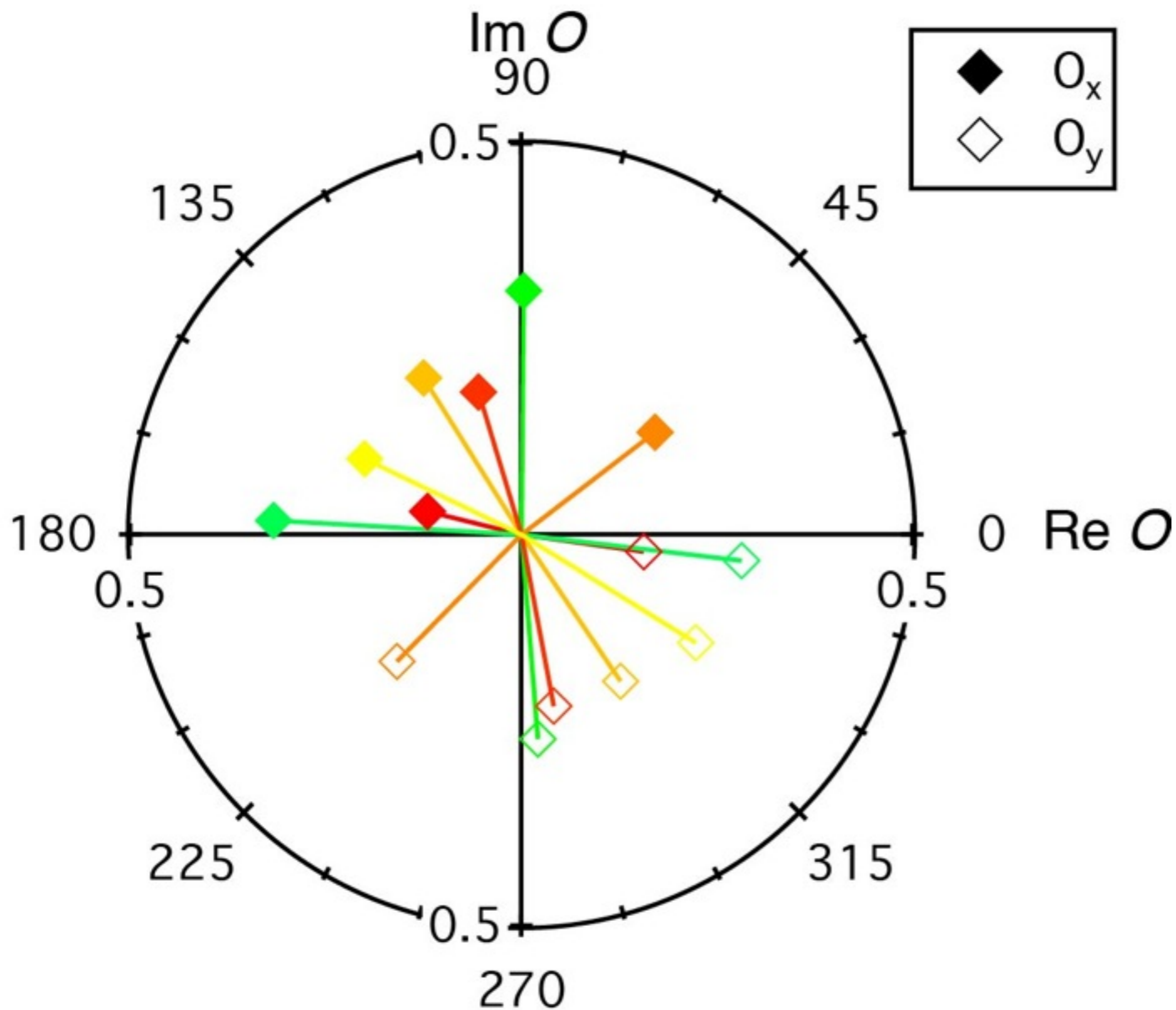
Phase-sensitive measurement of the d -form factor of density wave order



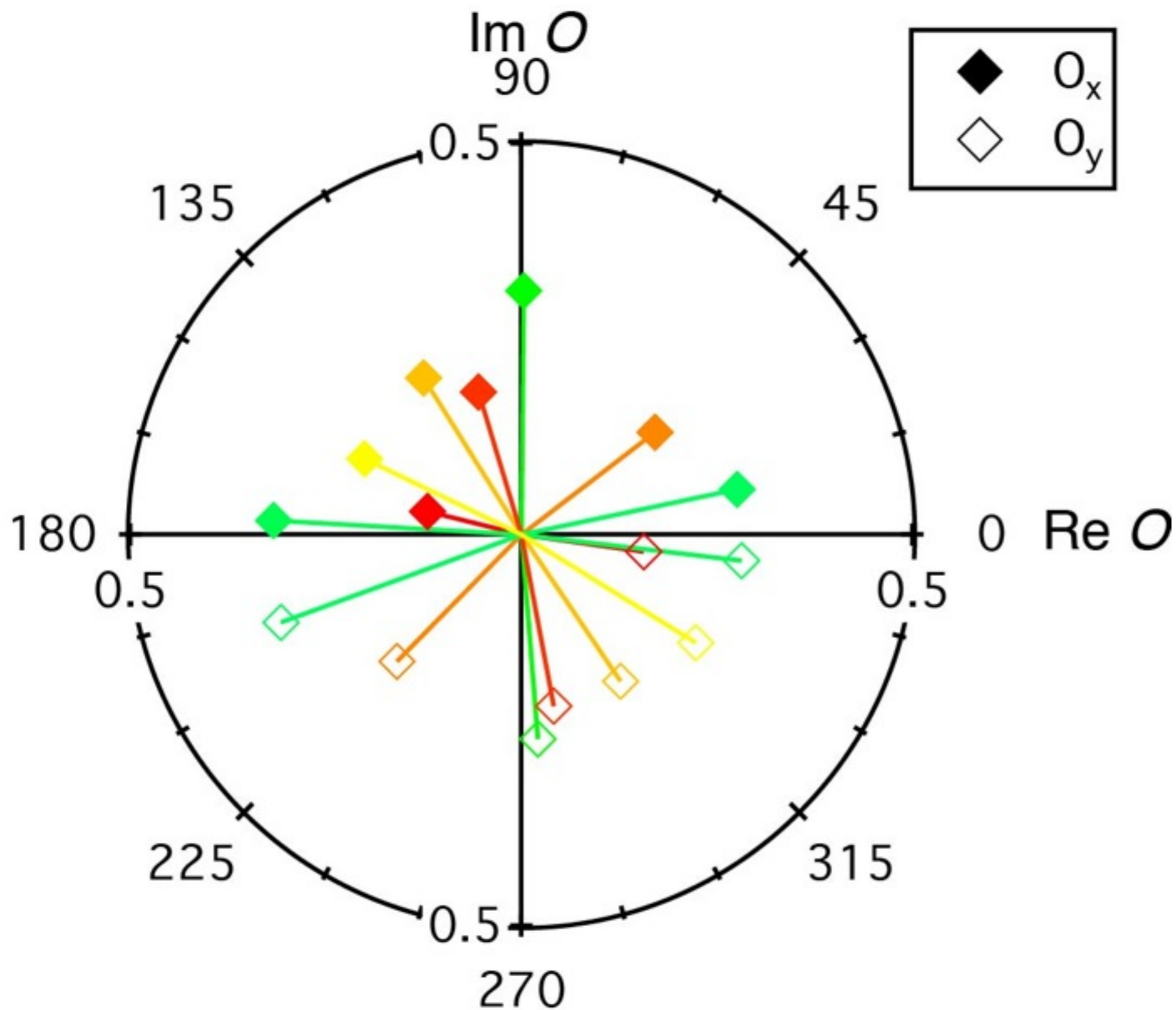
**Phase-sensitive
measurement of
the d -form factor
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order**



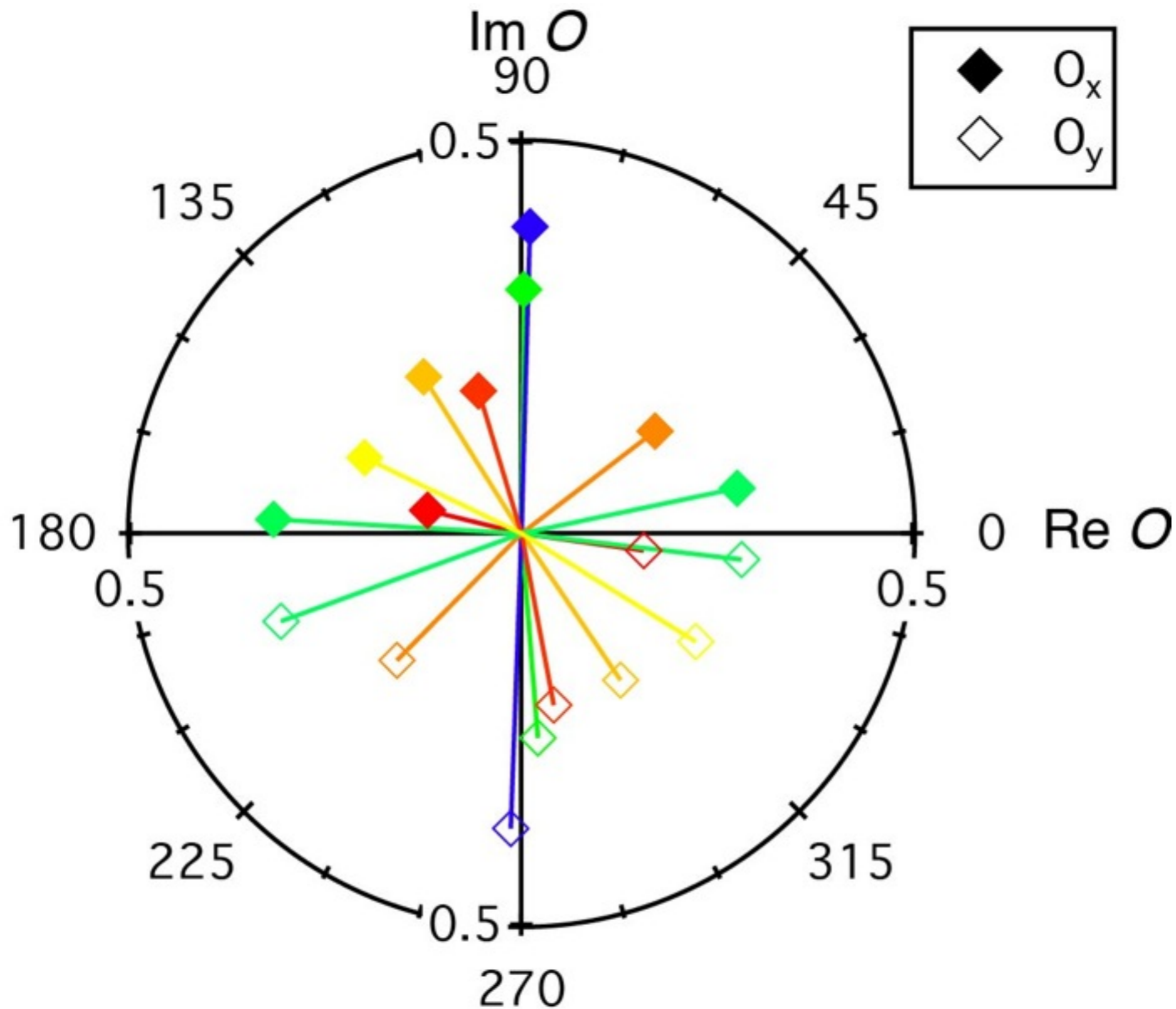
Phase-sensitive measurement of the *d*-form factor of density wave order



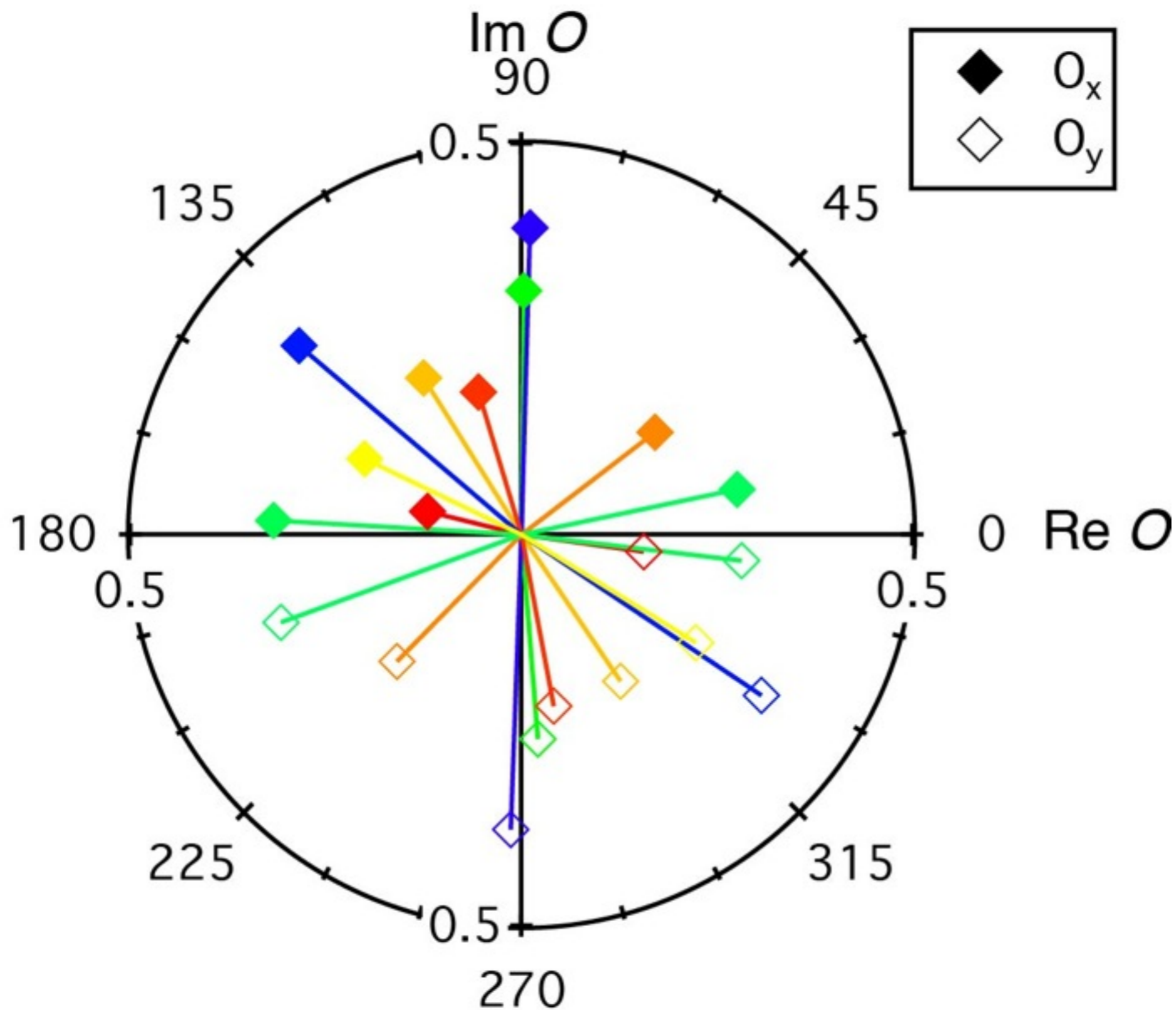
Phase-sensitive measurement of the *d*-form factor of density wave order



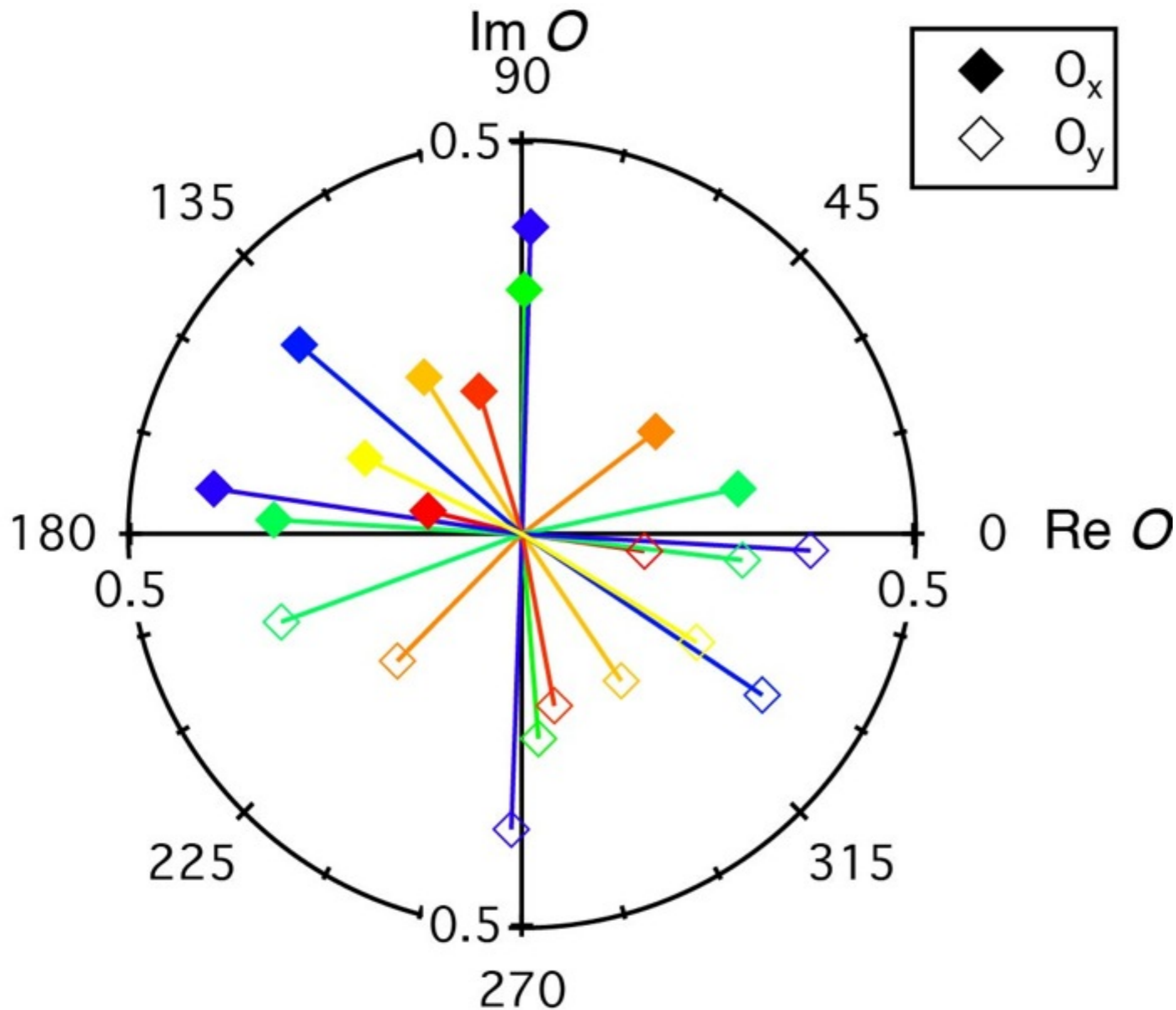
Phase-sensitive measurement of the d -form factor of density wave order



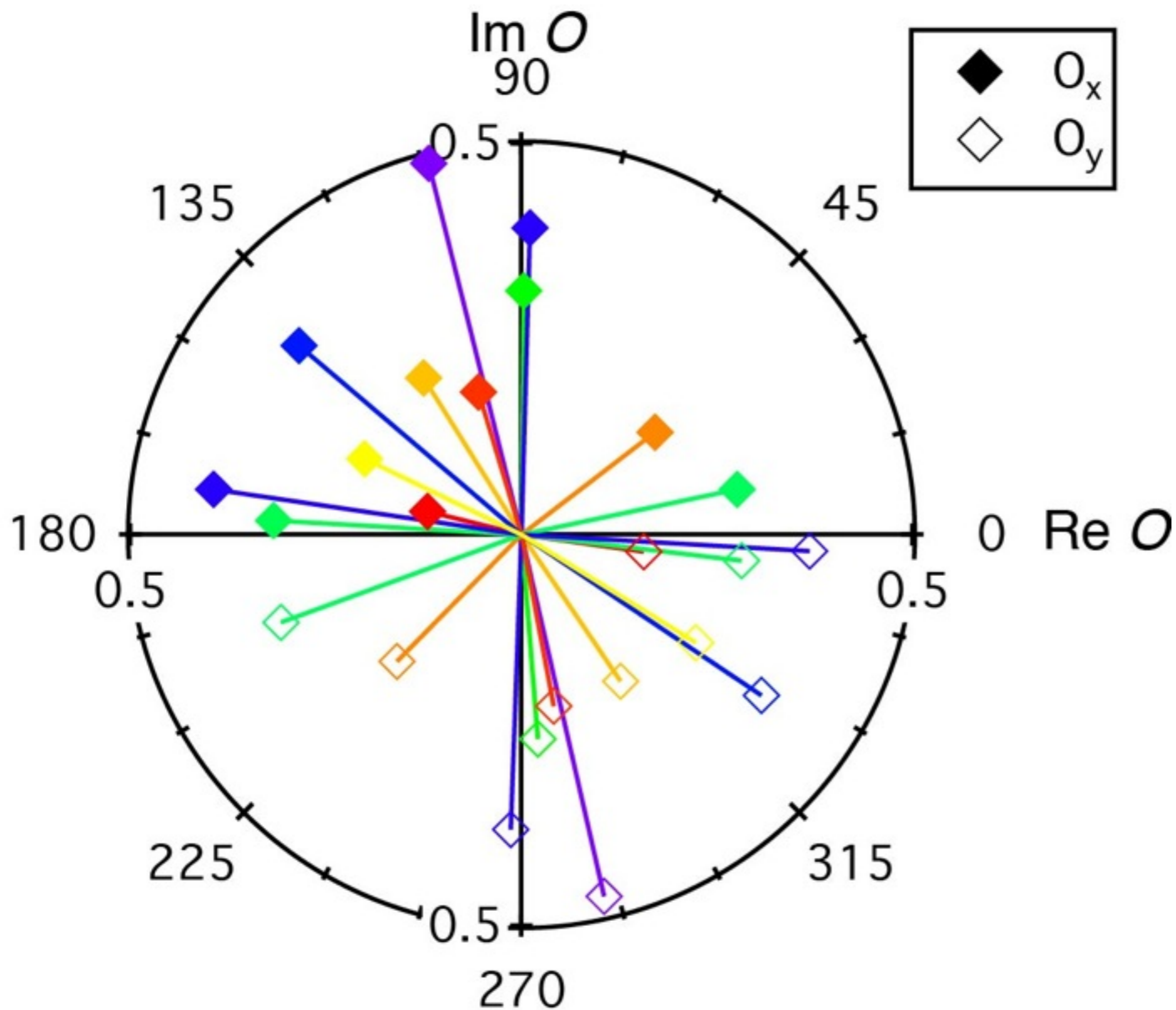
**Phase-sensitive
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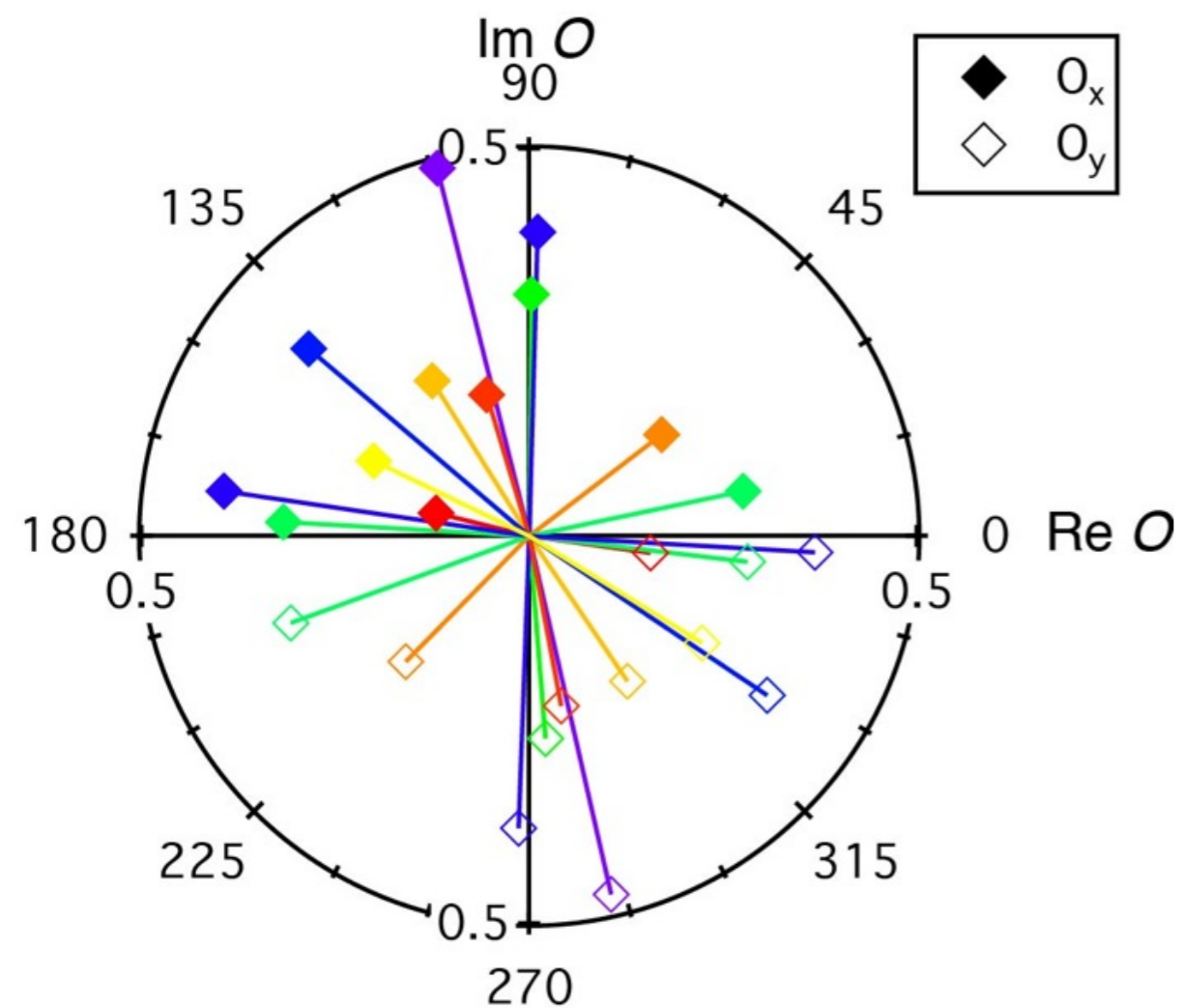
Phase-sensitive measurement of the d -form factor of density wave order



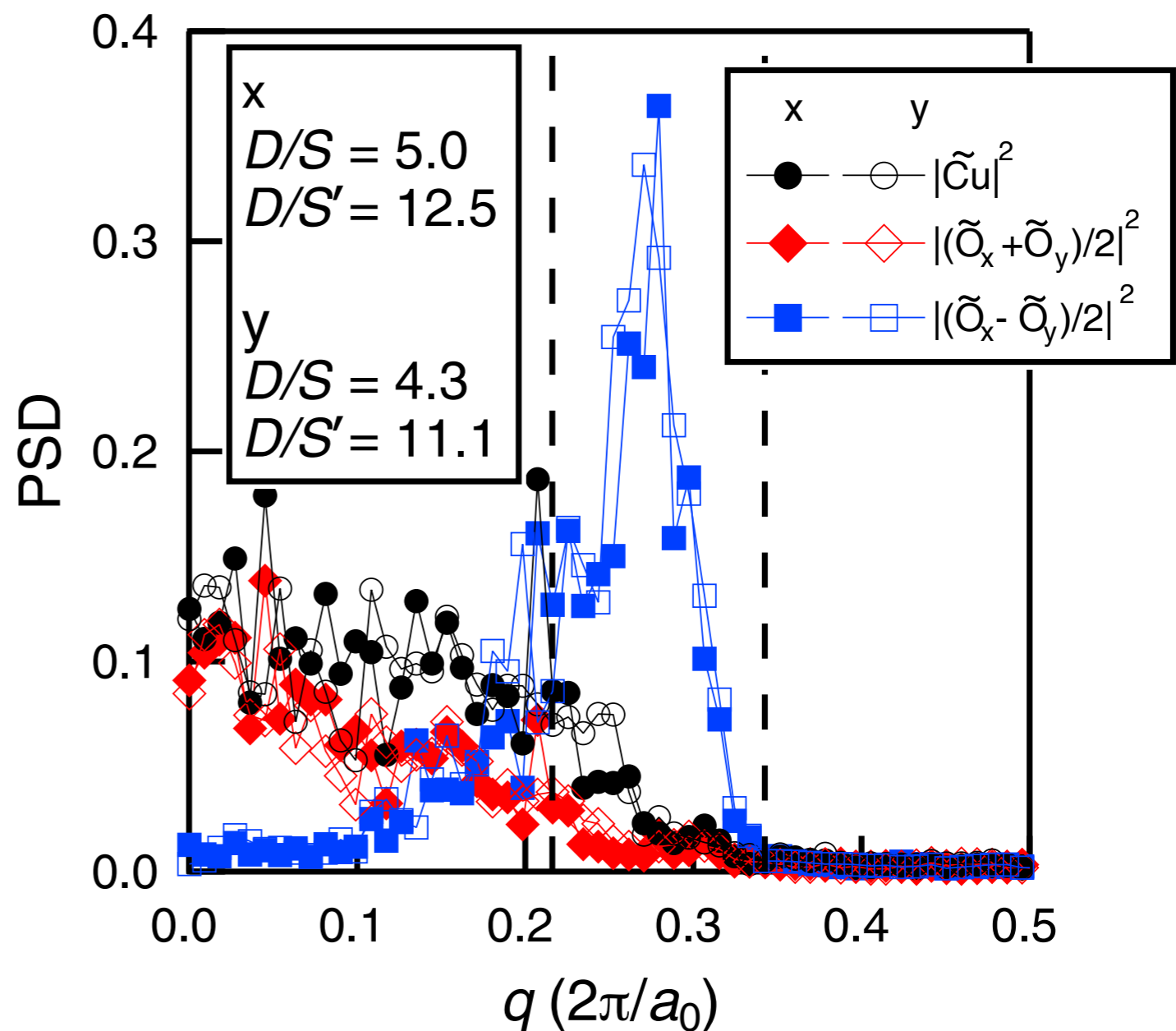
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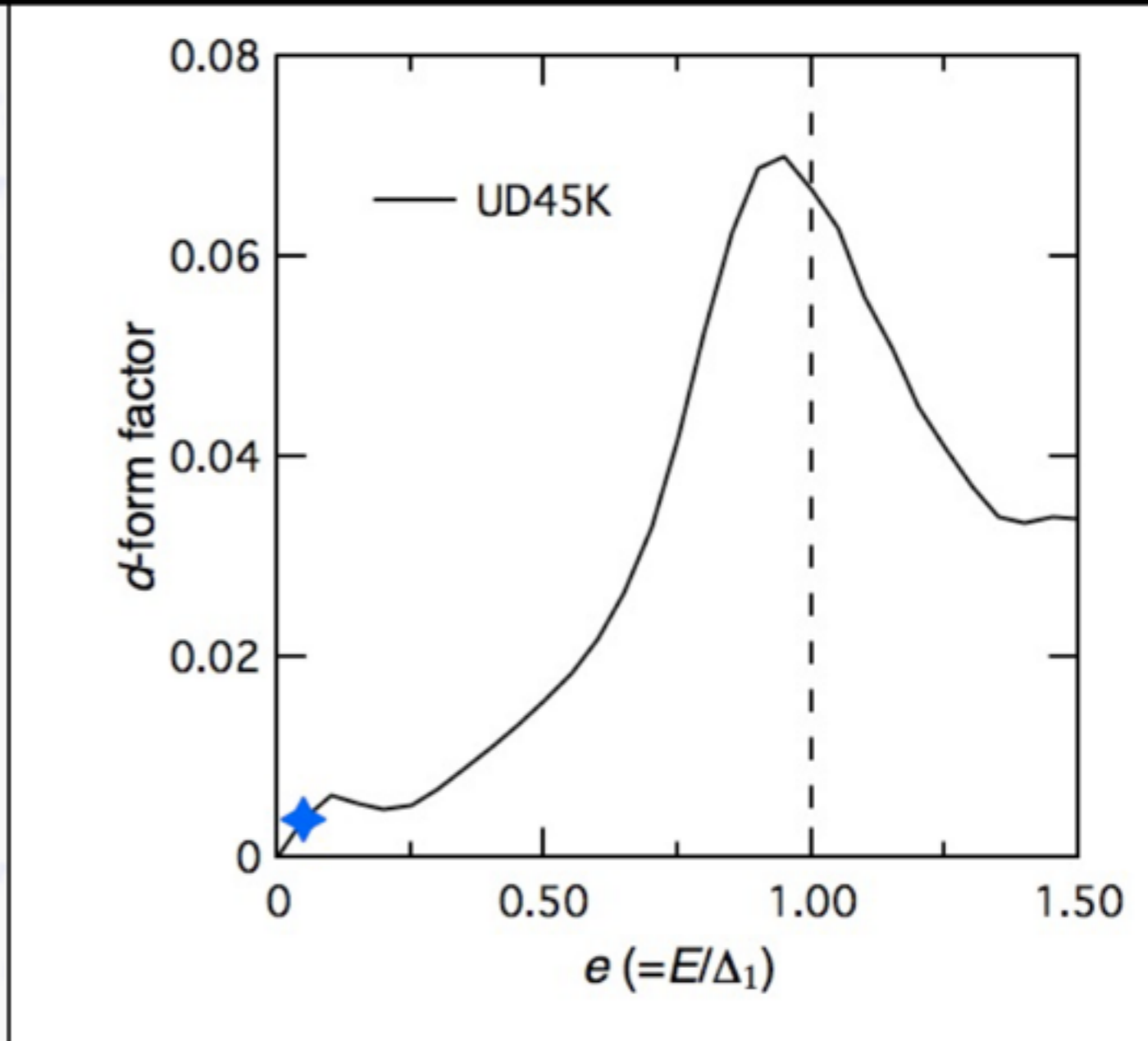
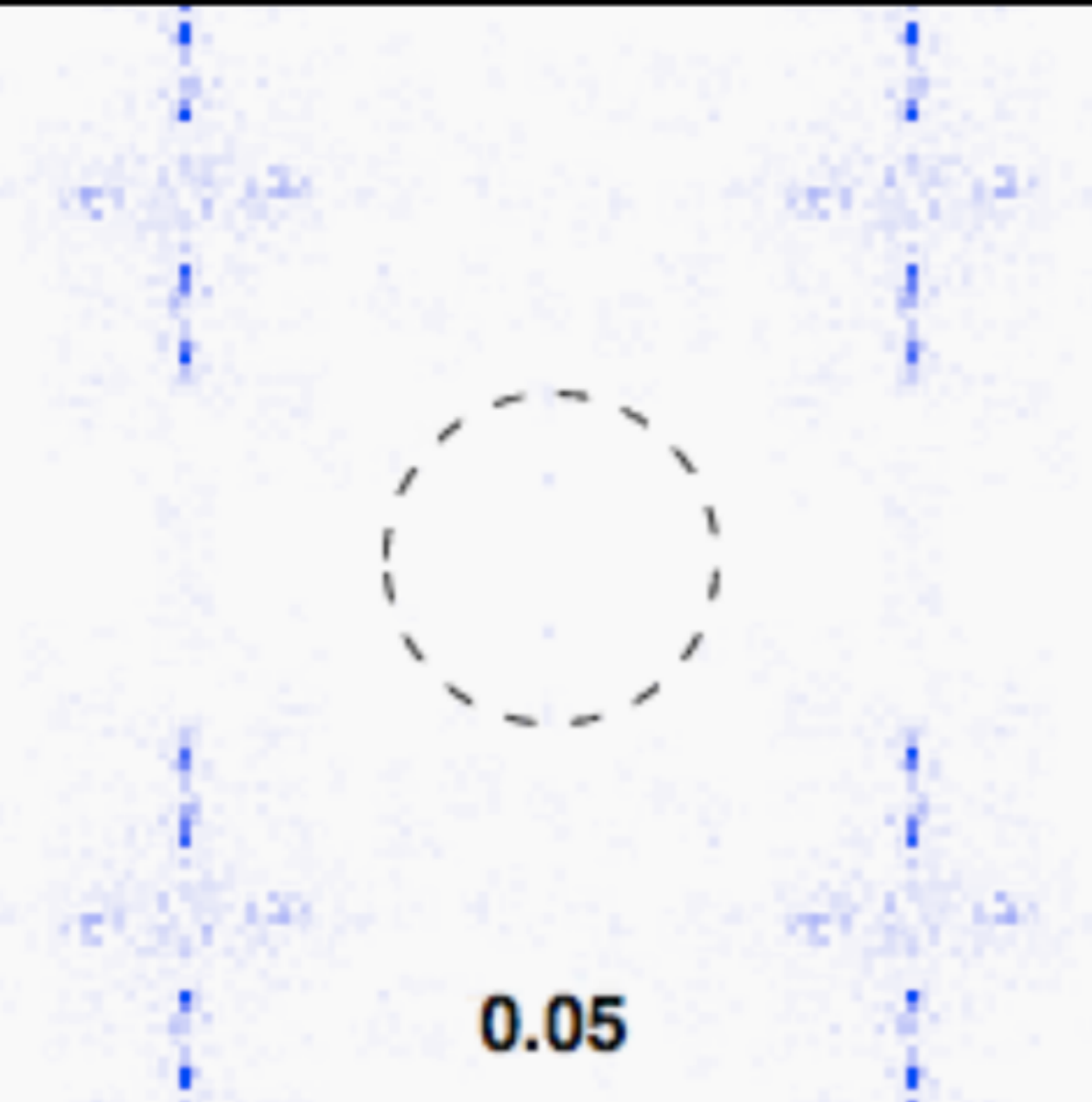
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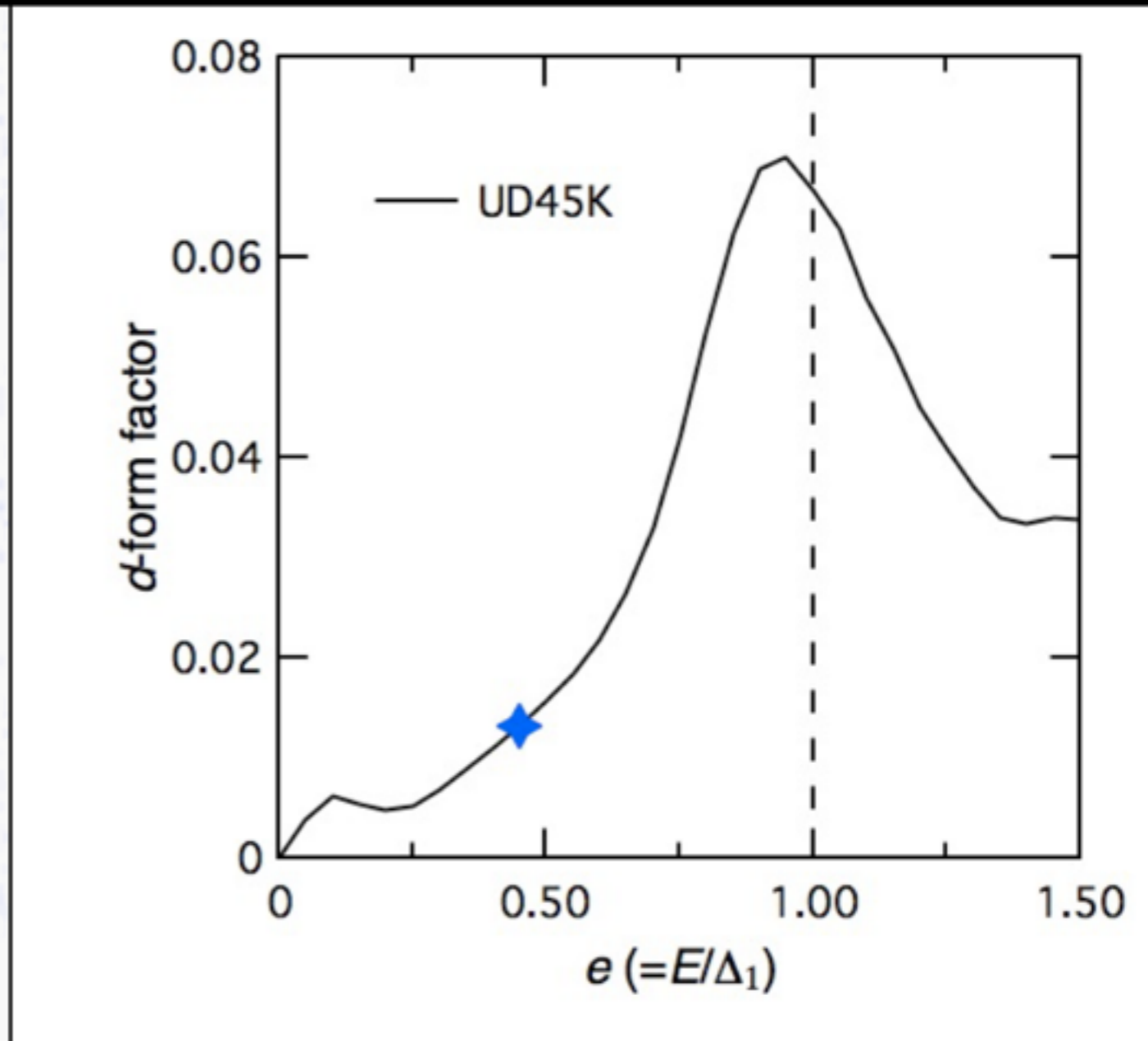
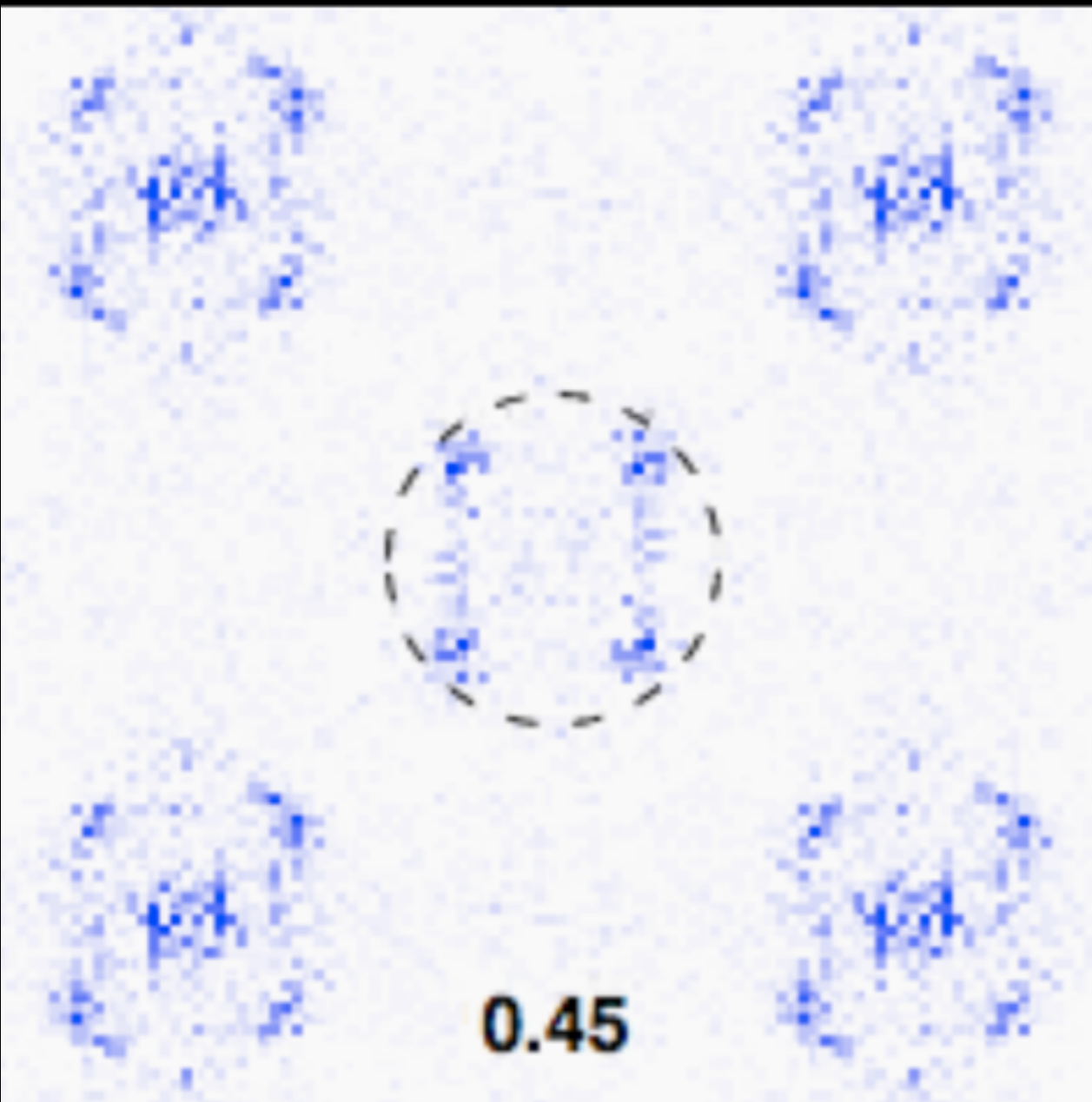
d-form Factor Predominant at Pseudogap Energy



$$|(O_x(q,e) - O_y(q,e))/2|^2$$

Spectral weight
inside the broken circle

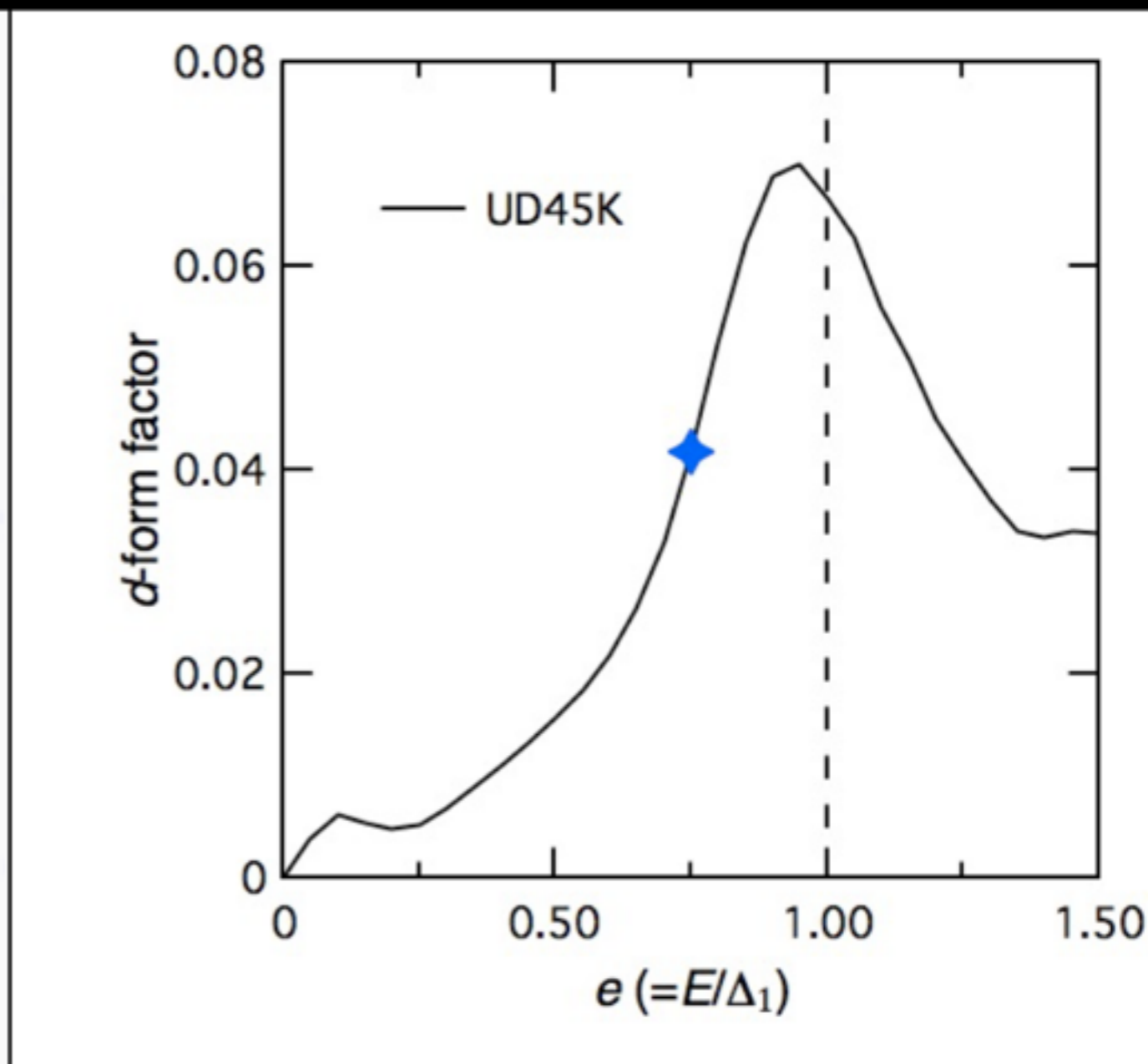
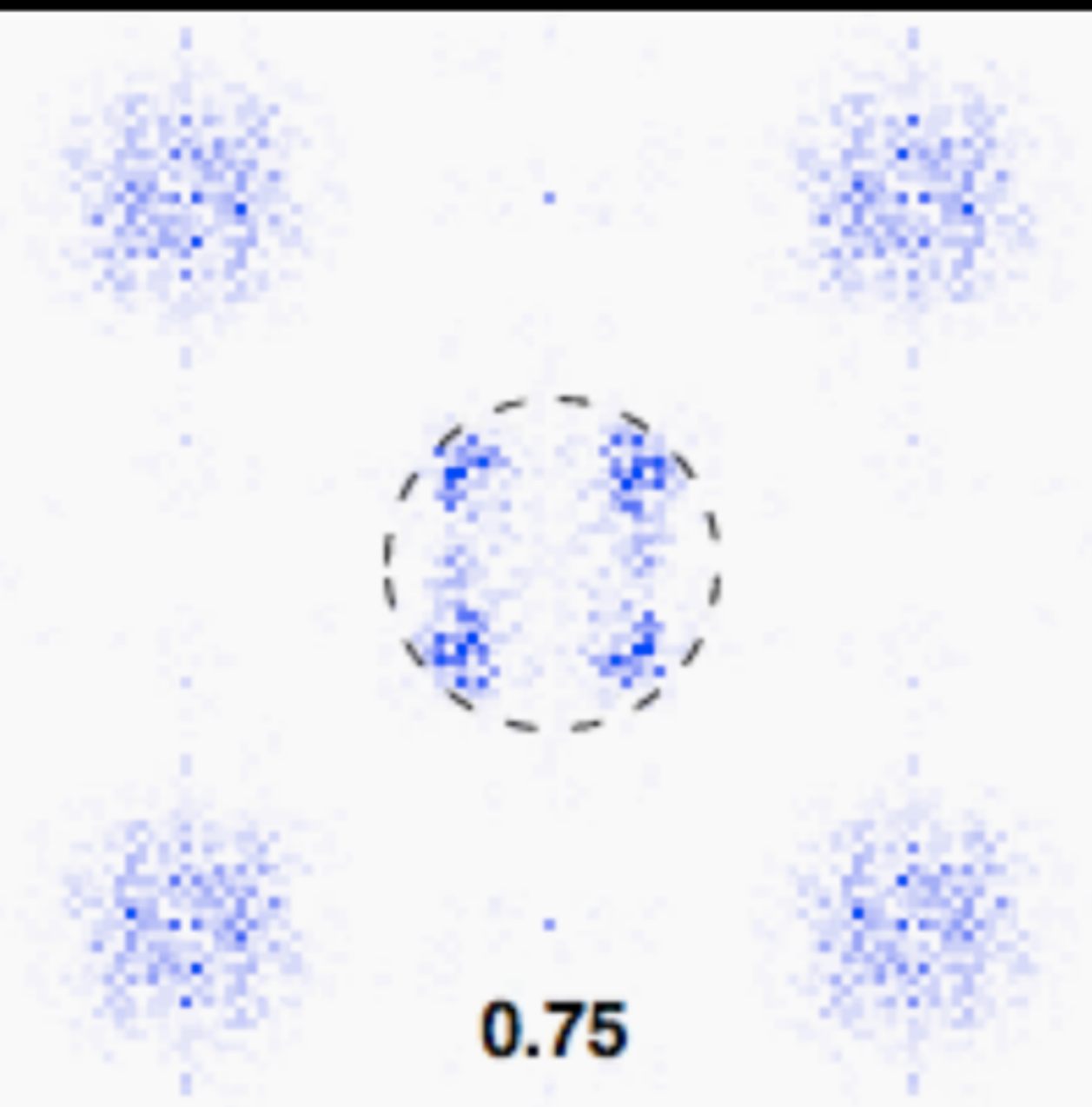
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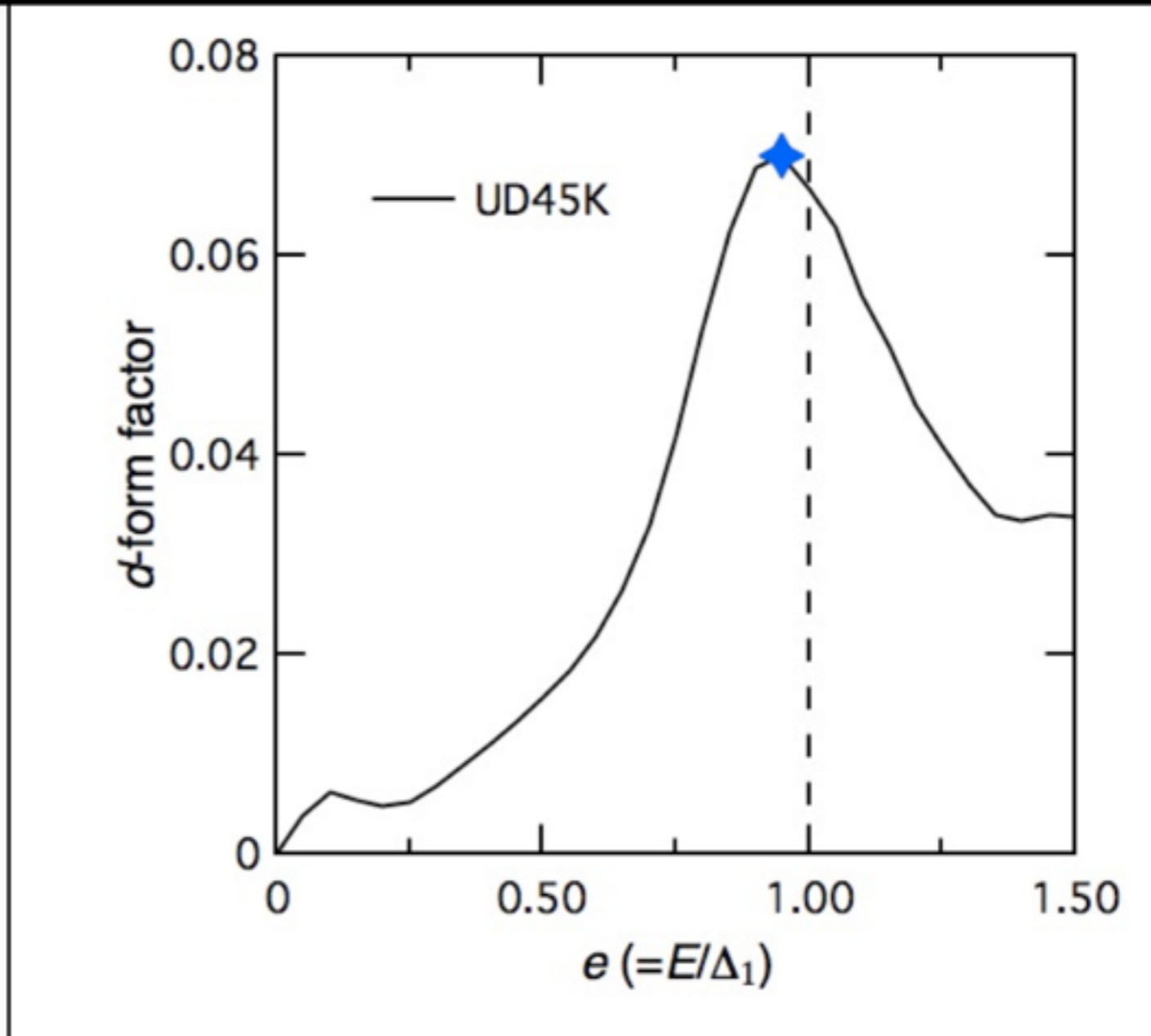
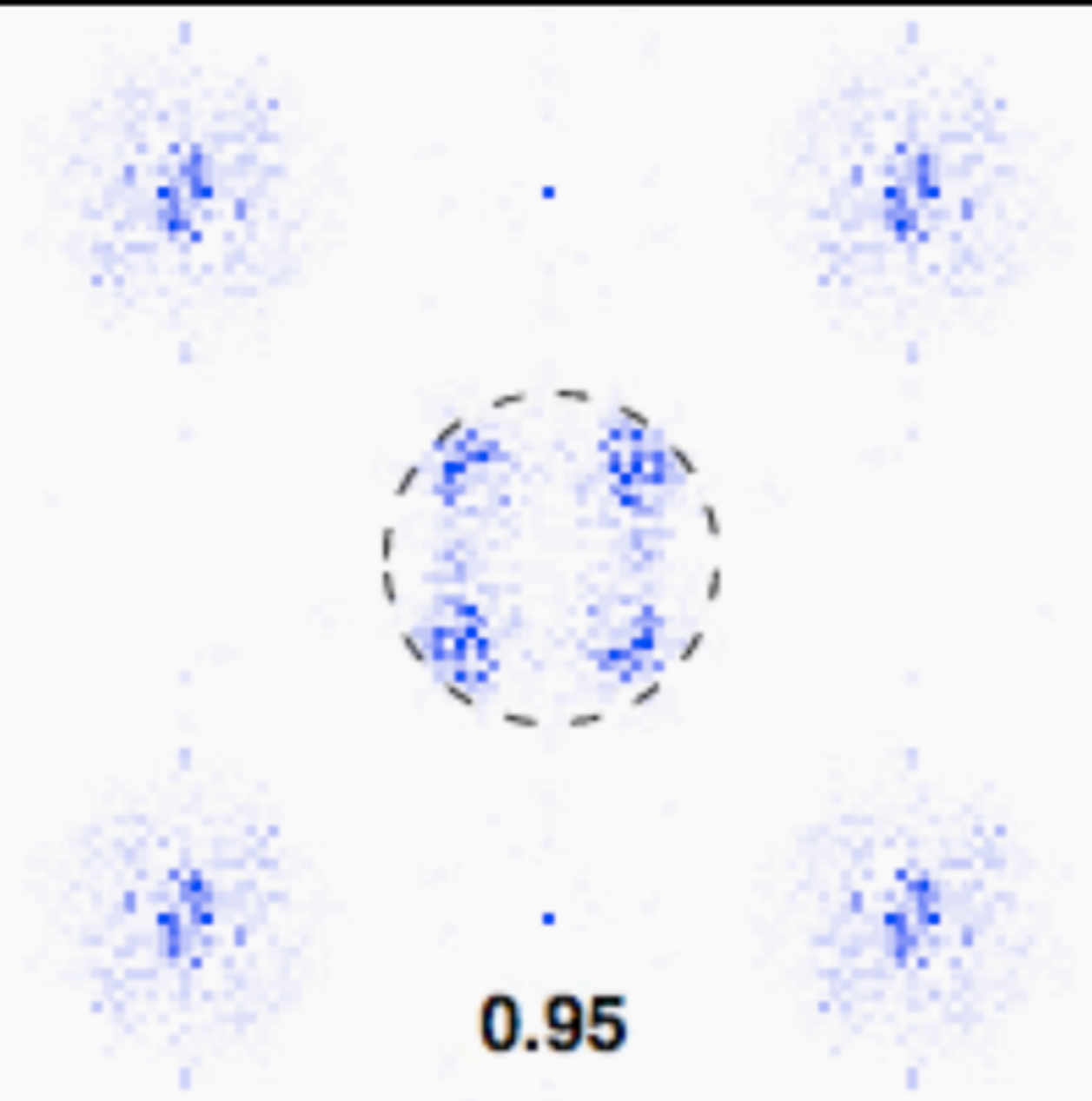
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$$|(O_x(q,e) - O_y(q,e))/2|^2$$

Spectral weight
inside the broken circle

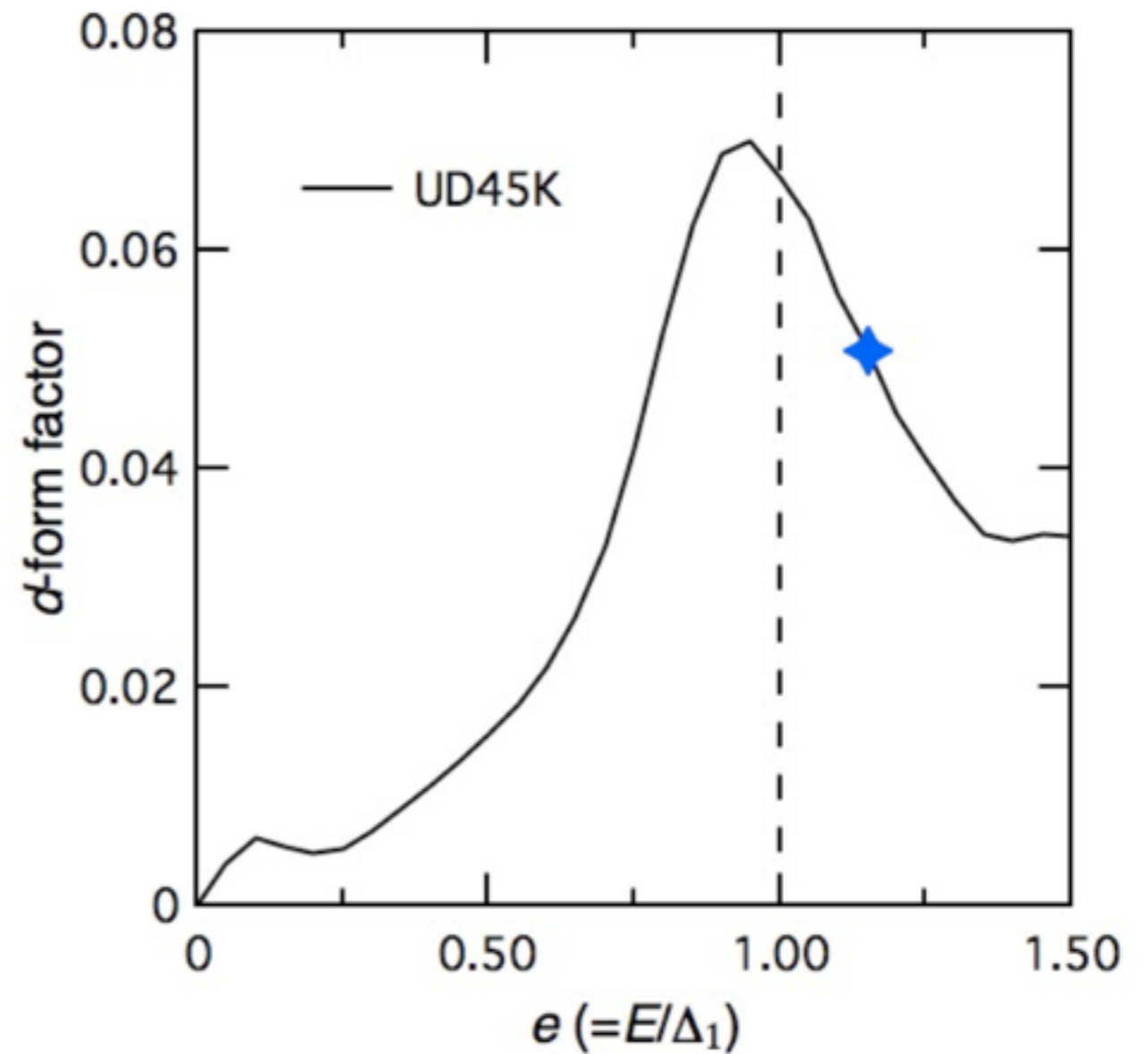
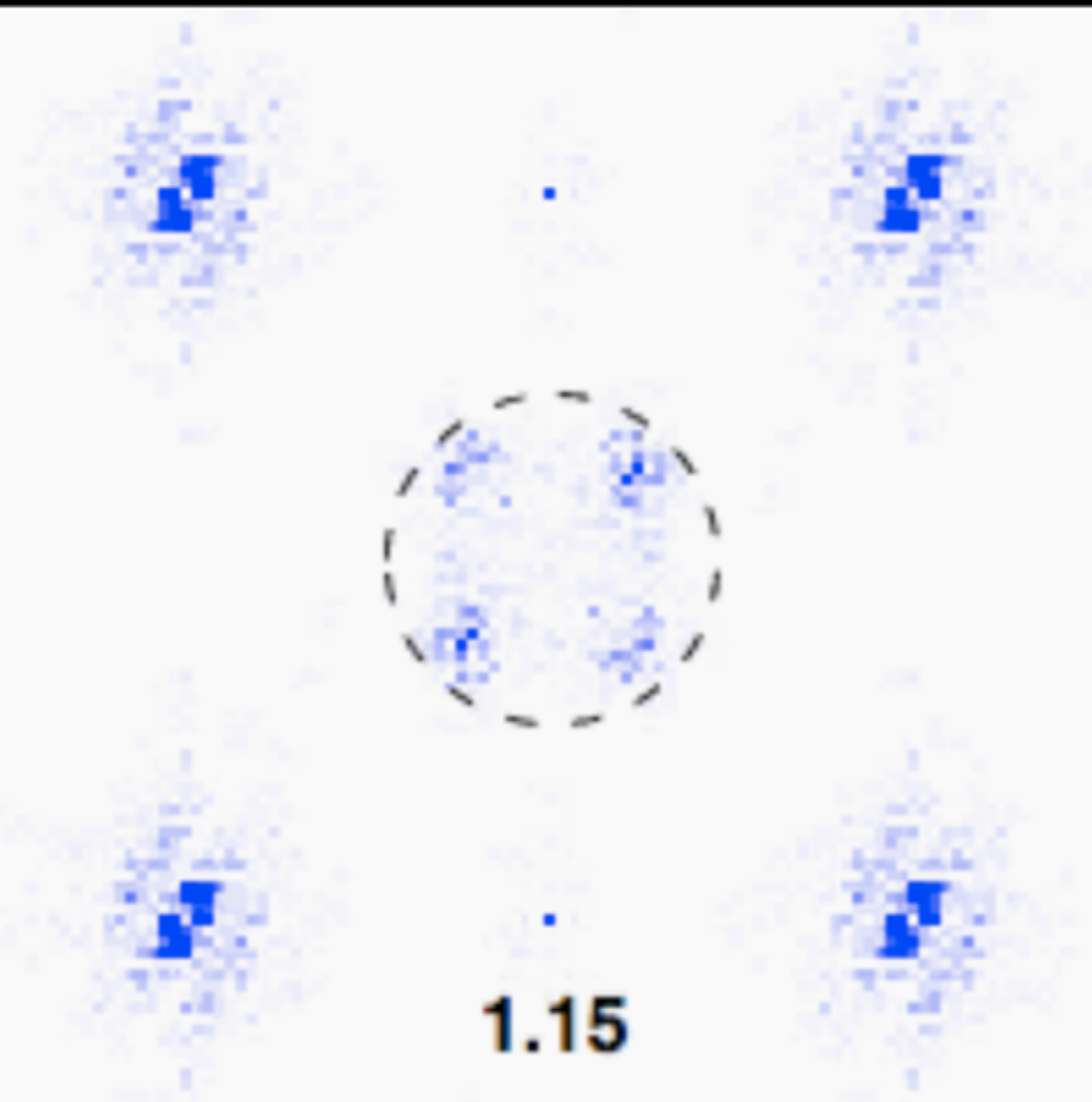
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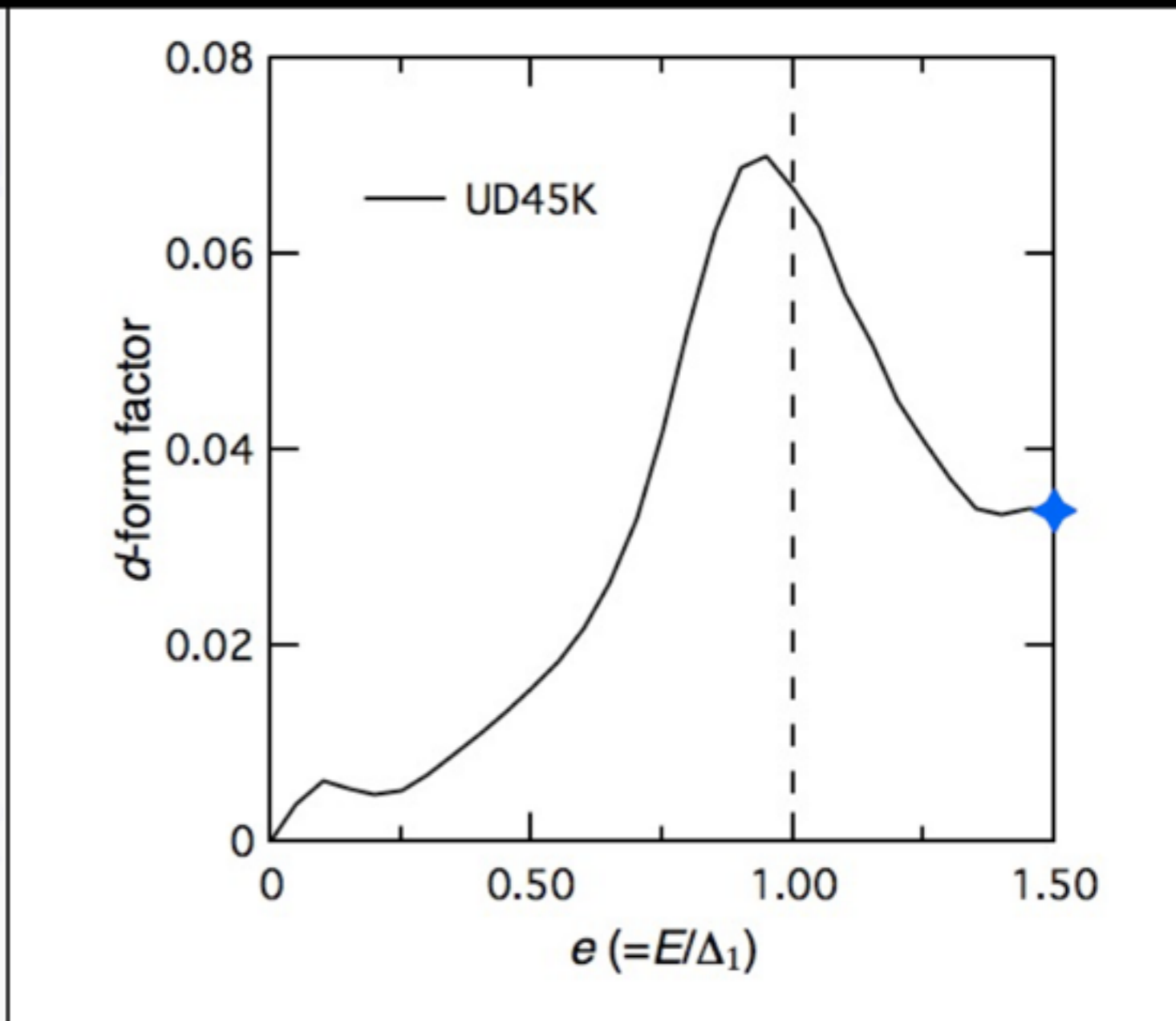
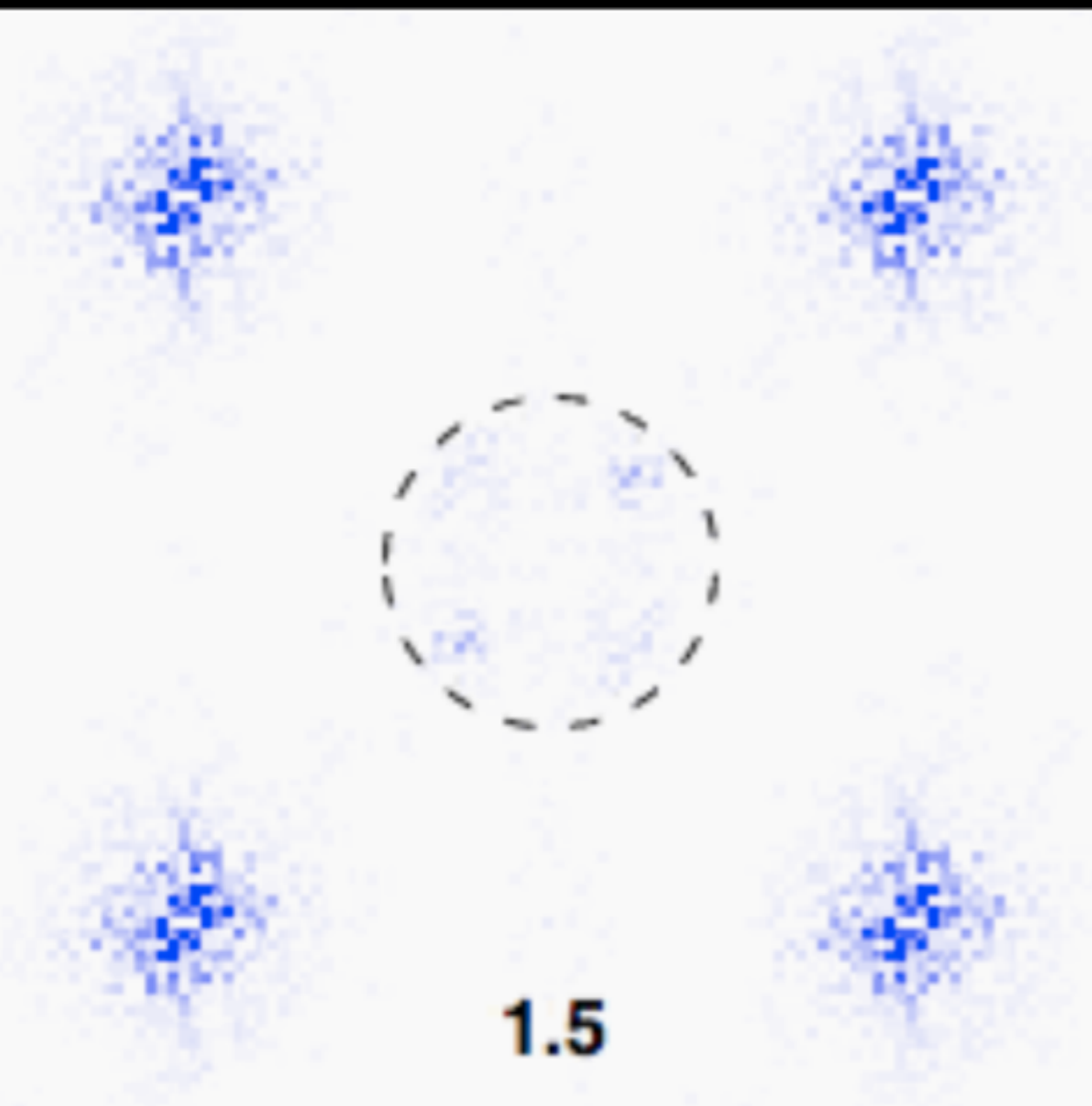
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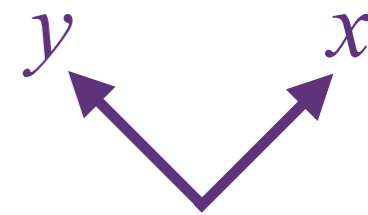
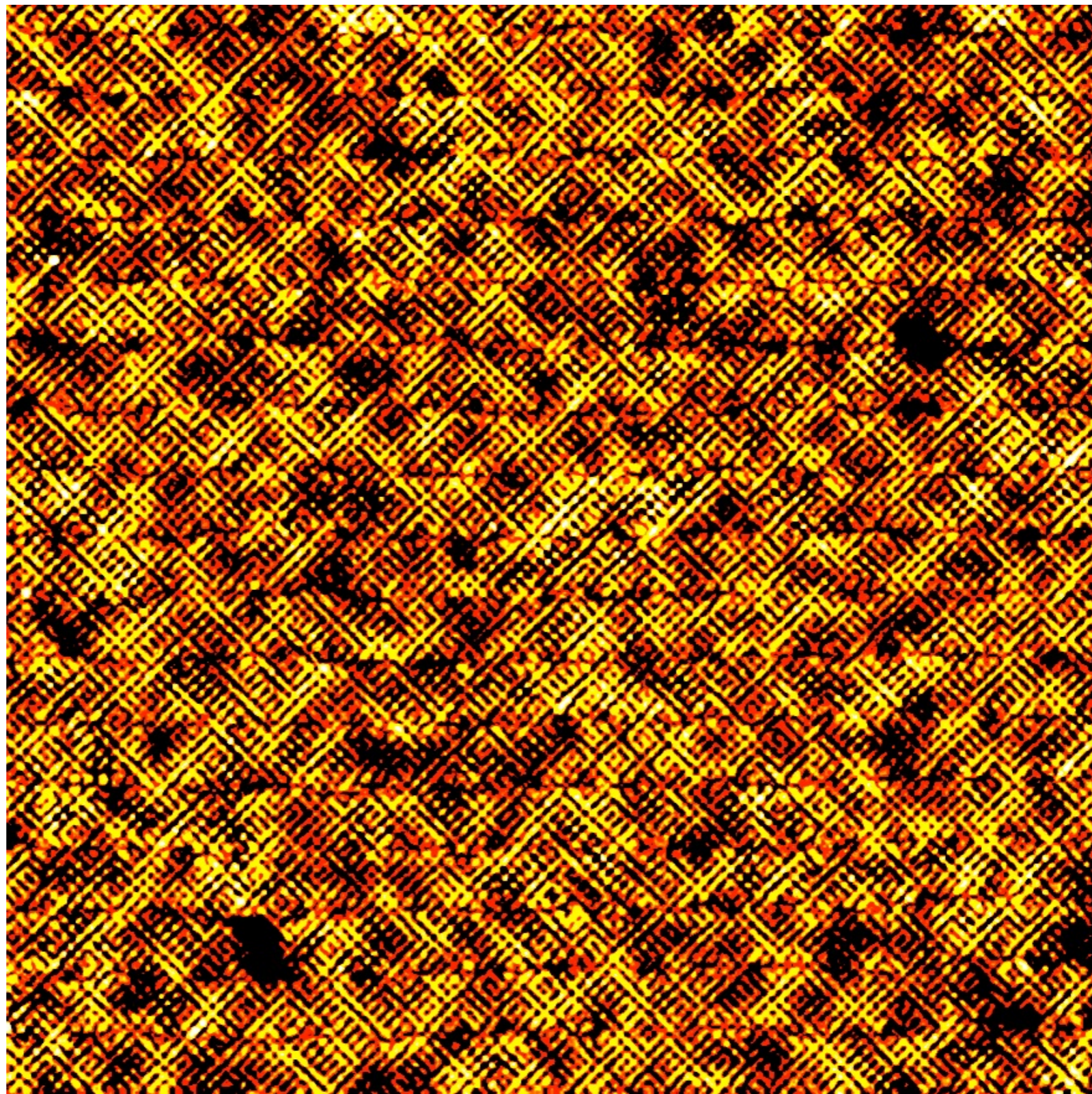
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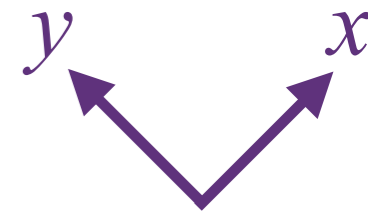
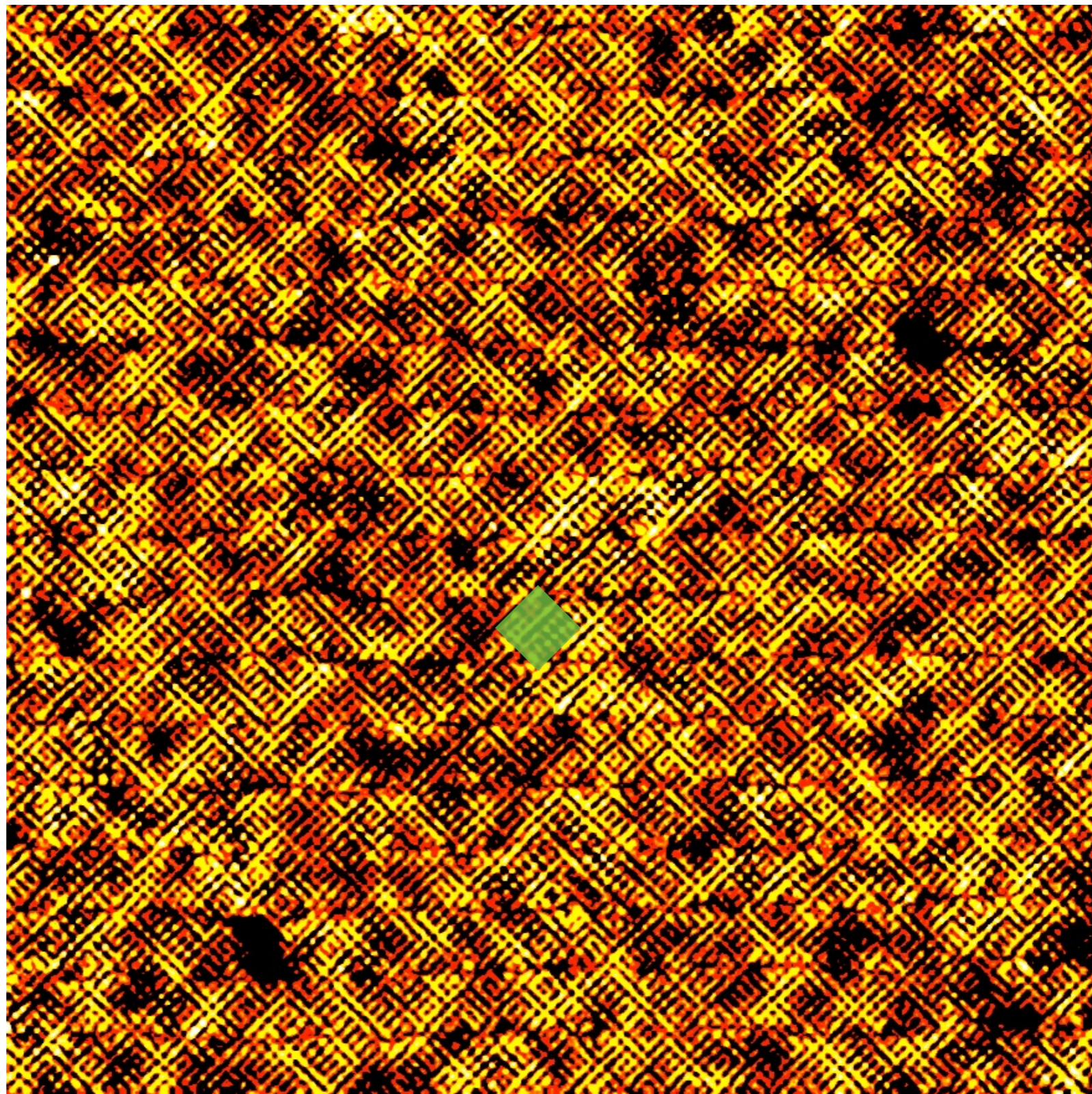
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See also

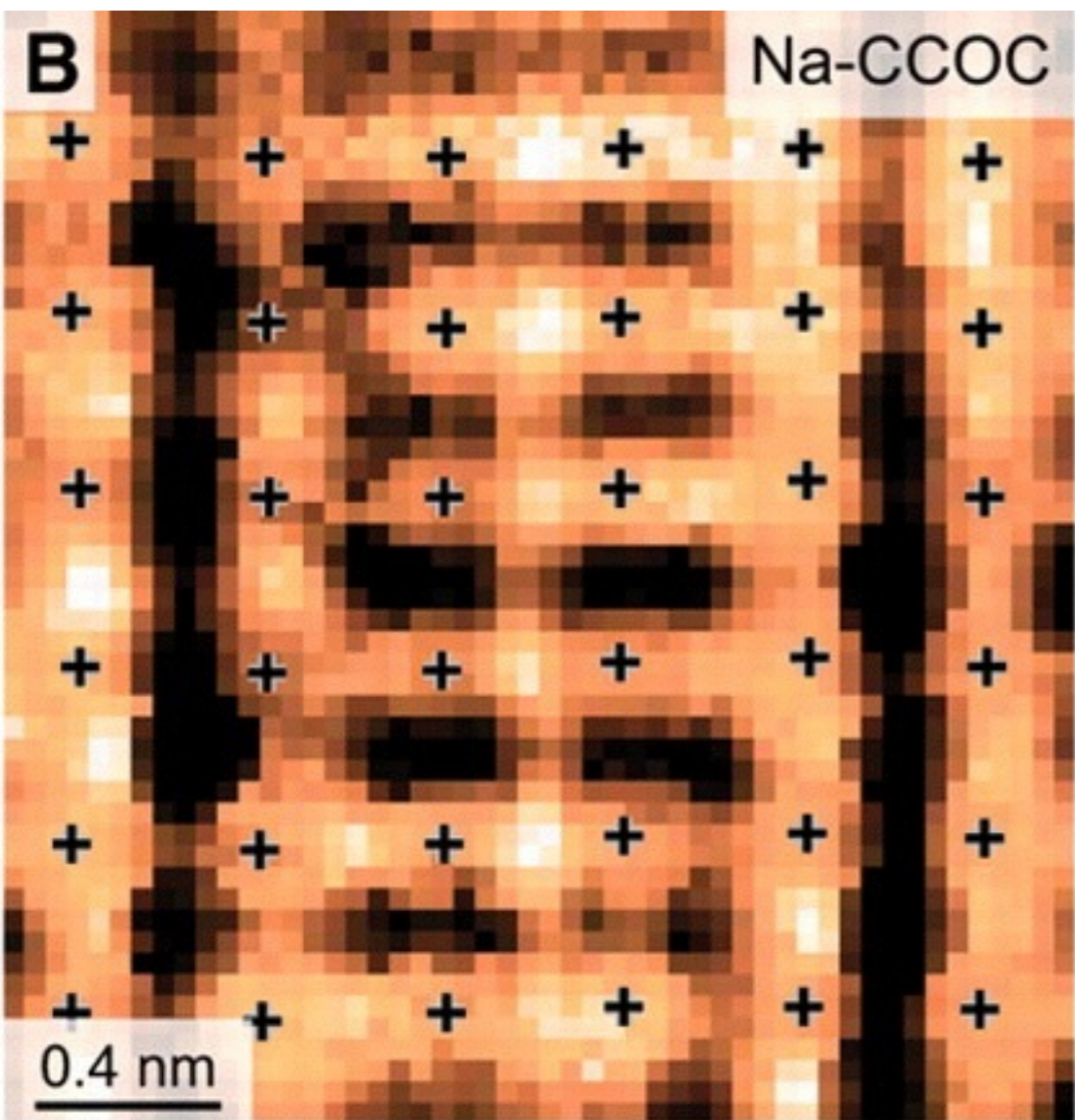
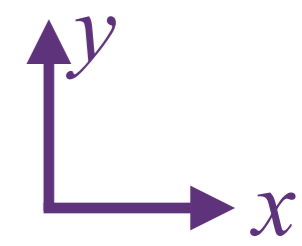
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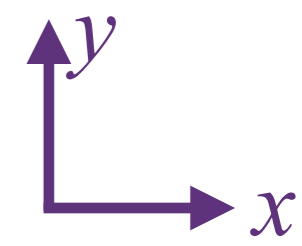
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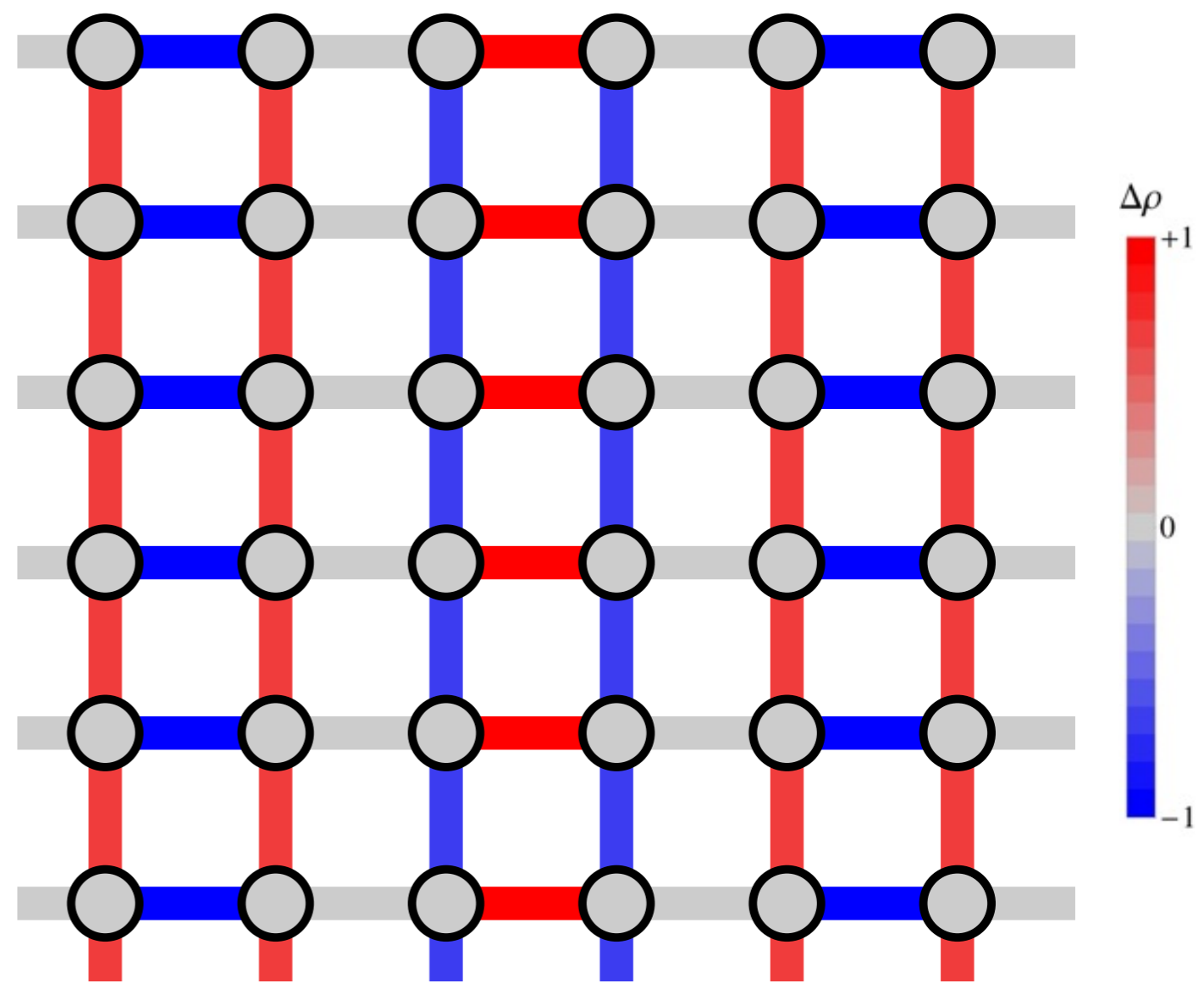
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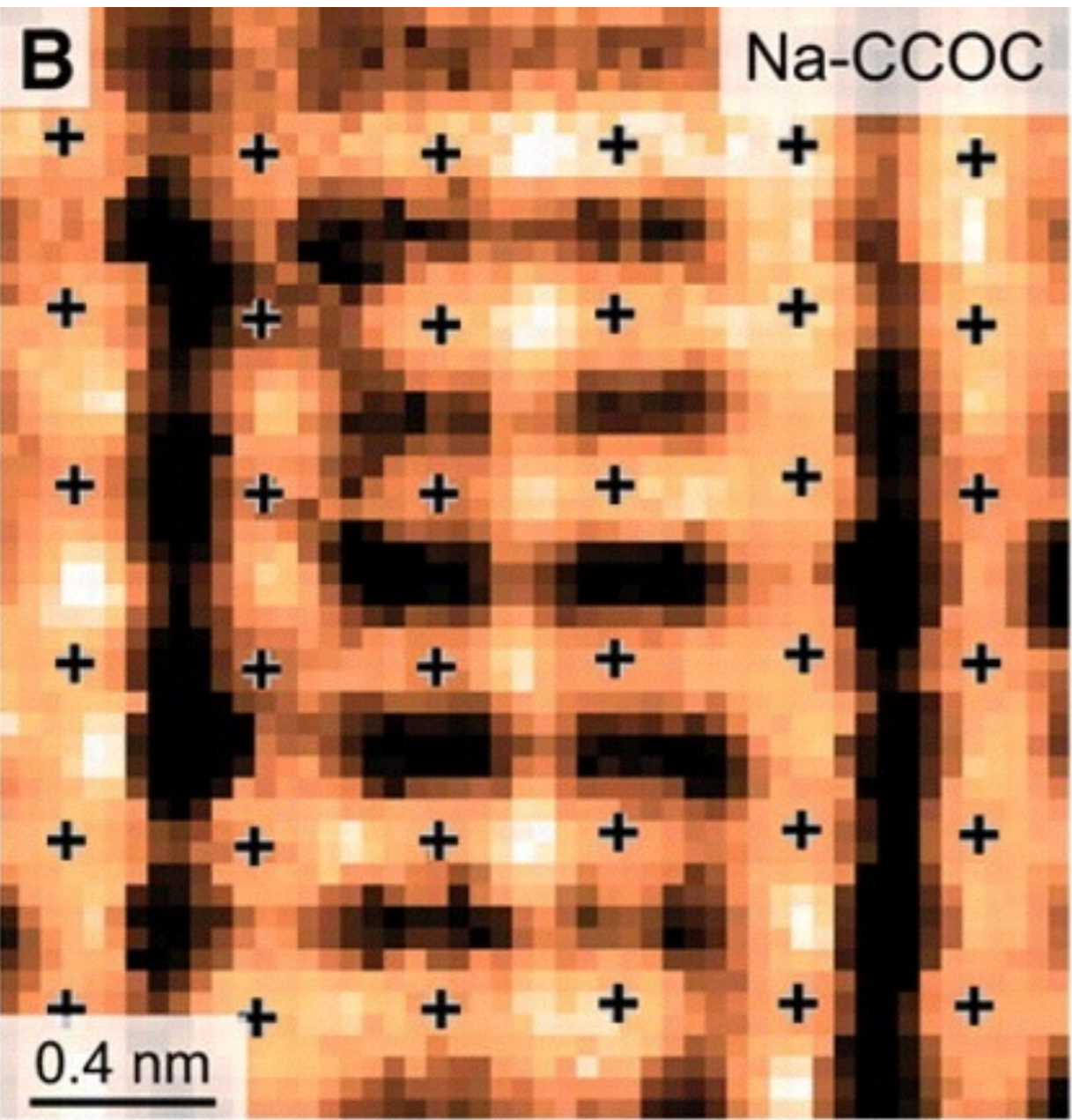
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$\mathbf{Q} = (\pi/2, 0)$

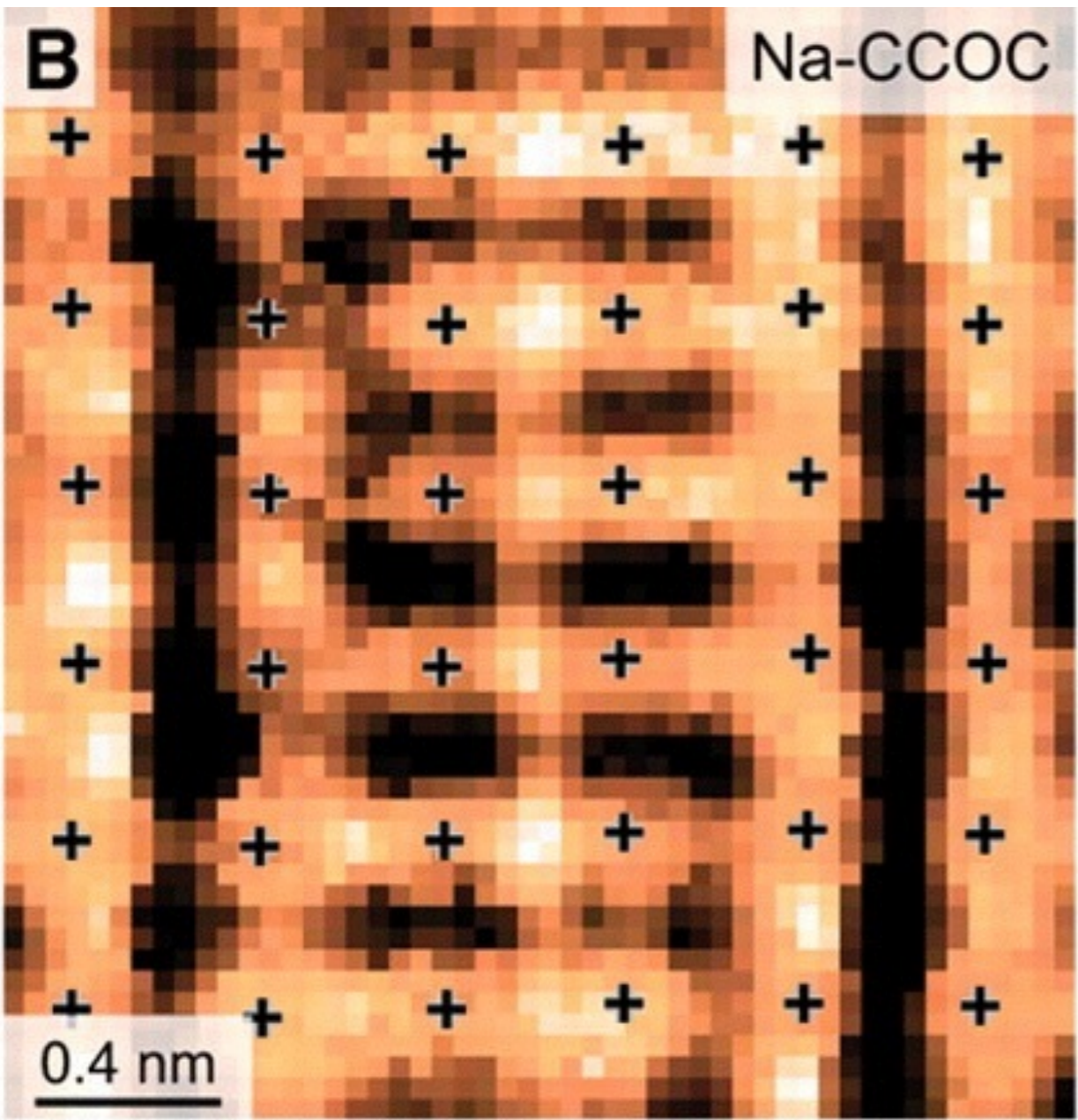
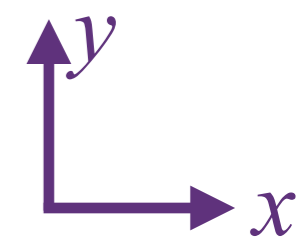


d-form factor density wave order

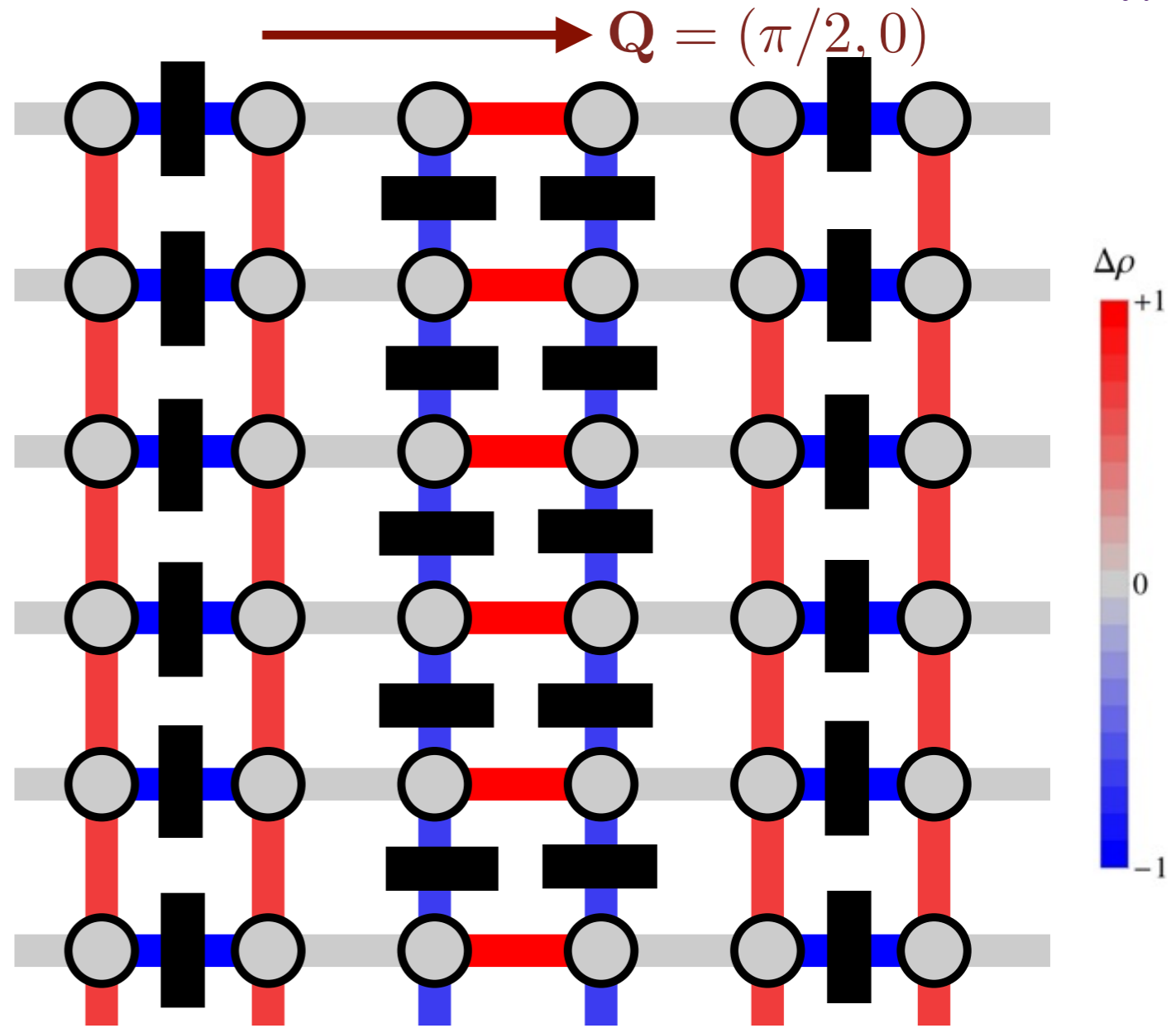


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This specific *d*-form factor density wave order (with \mathbf{Q} along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).



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