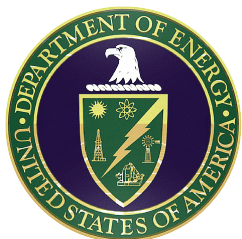


# Four Lectures at the 2014 Boulder School: "Modern Aspects of Superconductivity"

1

## Outline of Lectures:

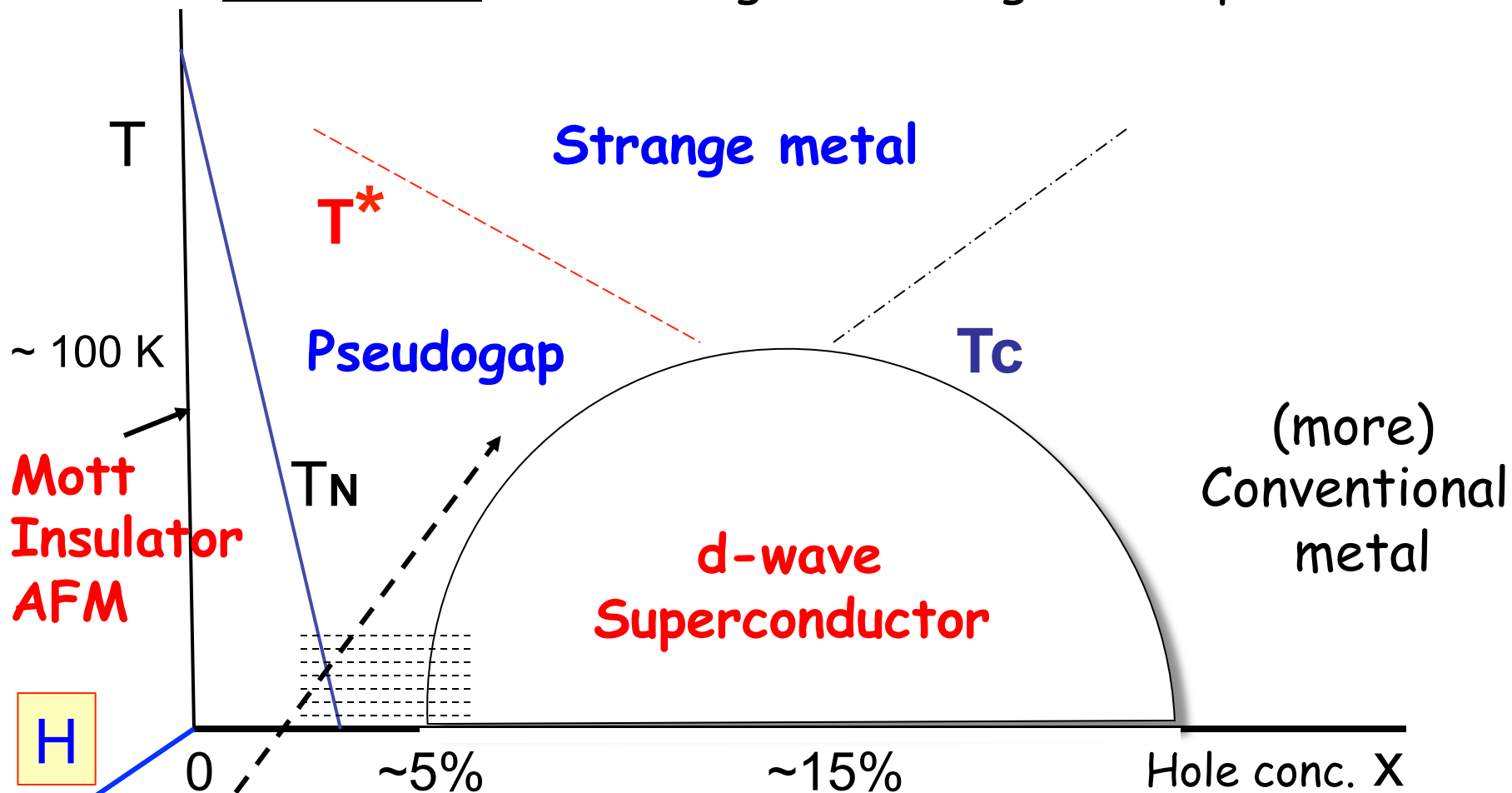
- Phenomenology of High T<sub>c</sub> Cuprates I
  - Phenomenology of High T<sub>c</sub> Cuprates II
  - Superconductivity in Doped Mott Insulators
- 
- BCS-BEC crossover & unitary Fermi gas



**Mohit Randeria**  
Ohio State University



# Schematic Phase Diagram of High Tc Cuprates

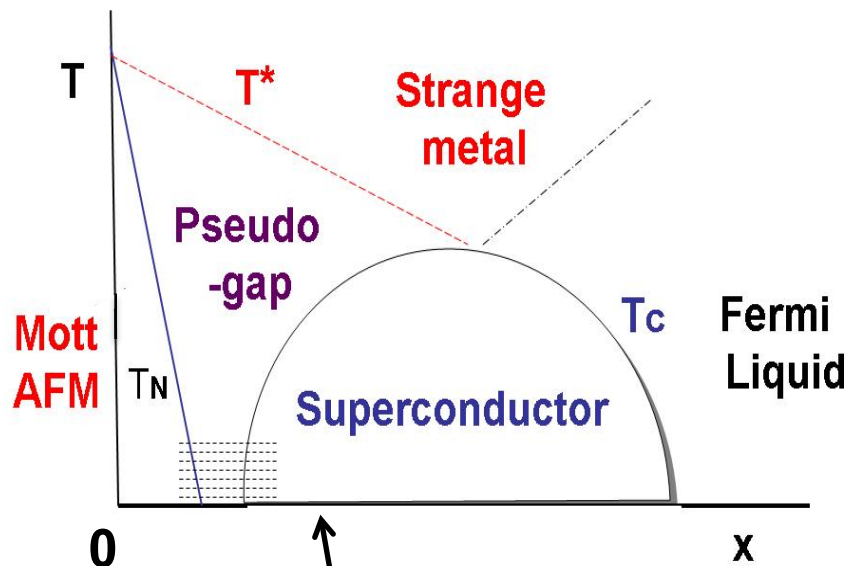


**H**  
Competing Orders:  
CDW...

Goal of Lectures # 1 & 2:  
to help you understand this  
phase diagram

# Failure of central paradigms of 20<sup>th</sup> Century Solid State Physics

Band theory fails for parent insulator



BCS MF theory fails for underdoping

Landau Fermi liquid theory fails for strange metal and pseudogap regimes

Competing orders:  
Antiferromagnetism  
CDW; nematicity; T-reversal...

Hidden Quantum Critical Point under the dome?

# Phenomenology of High Tc Cuprates I

- Standard theory of metals, insulators & superconductors
- High Tc cuprate materials
- Parent insulators: Mott AFM
- Problem of doped Mott insulator
- ARPES & quantum oscillations
  - Fermi liquid state at overdoping
  - Strange metal at optimal doping
  - d-wave superconductor

# Phenomenology of High Tc Cuprates II

- underdoped materials
  - pseudogap
  - SC fluctuations
  - competing order parameters
  - high field experiments

## Lightening quick (superficial) review of the central paradigms:

- I) band theory of solids [1]
- II) Landau's Fermi liquid theory [2]
- III) BCS theory of superconductivity [3]

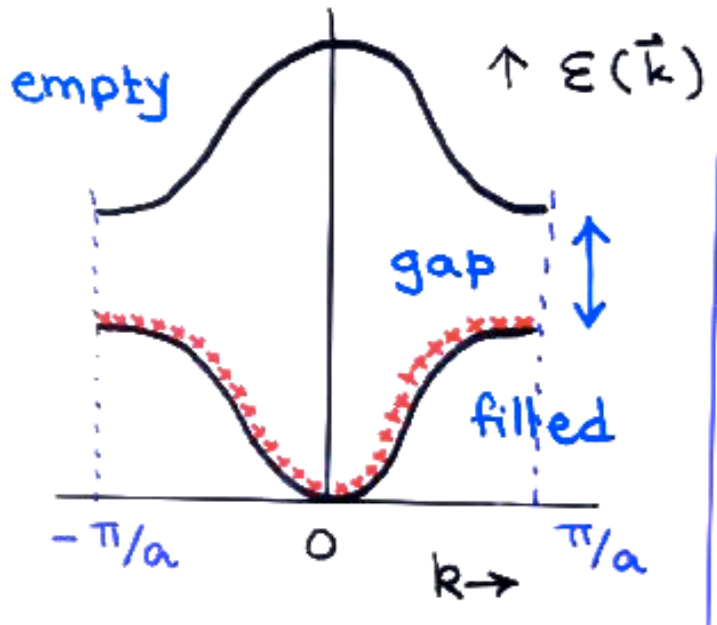
[1] Ashcroft & Mermin; Kittel

[2] Pines & Nozieres; Lifshitz & Pitaevski (L&L Stat. Phys. II)

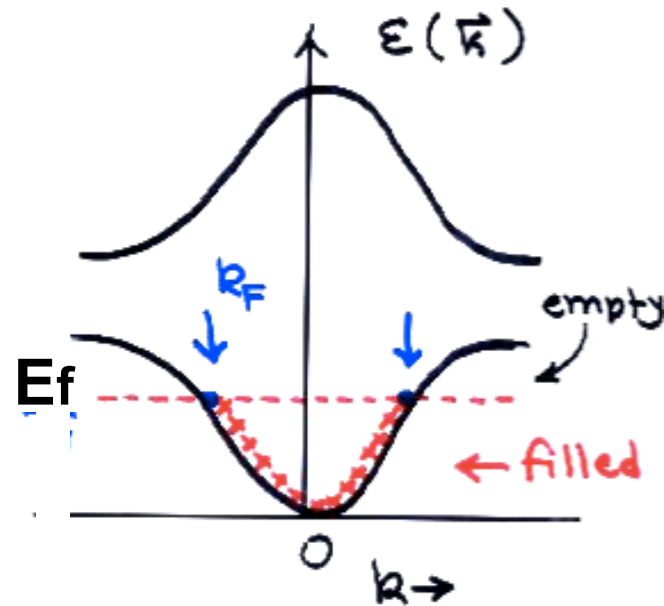
[3] Tinkham

# I) Band Theory: Insulators vs. Metals

- solve Schrodinger equation for a single electron in a periodic potential
- obtain “bands” of energy levels and “gaps”
- fill up states consistent with Pauli exclusion



Excitation gap  
 → Insulator



Fermi surface of  
Gapless excitations  
 → Metal

## Band Theory:

Spectacular success in understanding

- conventional metals
- band insulators/semiconductors
- topological insulators

Why can one (more or less) ignore Coulomb repulsion between electrons in a metal?

Landau's Theory  
of Fermi Liquids



## II) Landau's Theory of Fermi Liquids

Interactions between electrons lead to only quantitative changes (not qualitative ones) in low energy, long wavelength properties

- specific heat  $C \sim T$
- spin susceptibility  $\chi \sim \text{const.}$  etc.
- Boltzmann eqn. for transport

Low energy excitations:

Quasiparticles ... "look like" electrons

\* live on the Fermi surface (Luttinger theorem)

\*  $Q = 1, S = \frac{1}{2}$

\* sharp  $\Gamma \ll \omega$       \* weakly interacting

Why does  
It work?

\* Screening      \* Pauli exclusion

\* all orders in perturbation theory ( $d \geq 2$ )  
[for RG approach: see Shankar RMP (1993)]

# Spectral Function for Electrons:

$$A(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im}G(\mathbf{k}, \omega + i0^+)$$

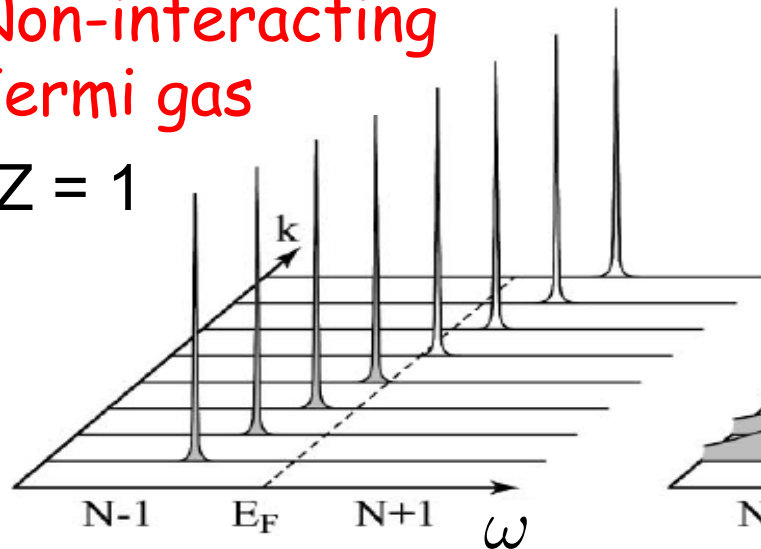
= Probability density of making a single-particle excitation with a momentum  $\mathbf{k}$  within a small energy interval around  $\omega$

[Relation to ARPES discussed later]

**Quasiparticle** = pole in  $G \rightarrow \delta$ -function in  $A$   
Quasiparticle weight  $Z$

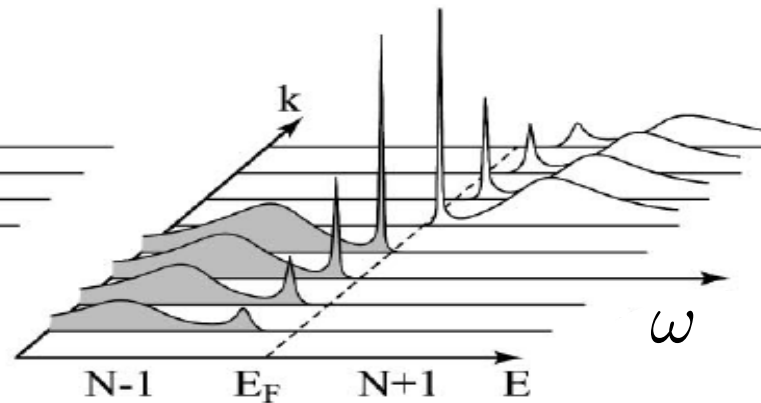
**Non-interacting  
Fermi gas**

$$Z = 1$$



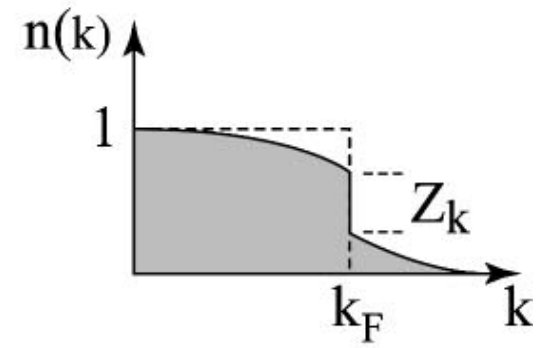
**Landau Fermi liquid**

$$0 < Z < 1$$



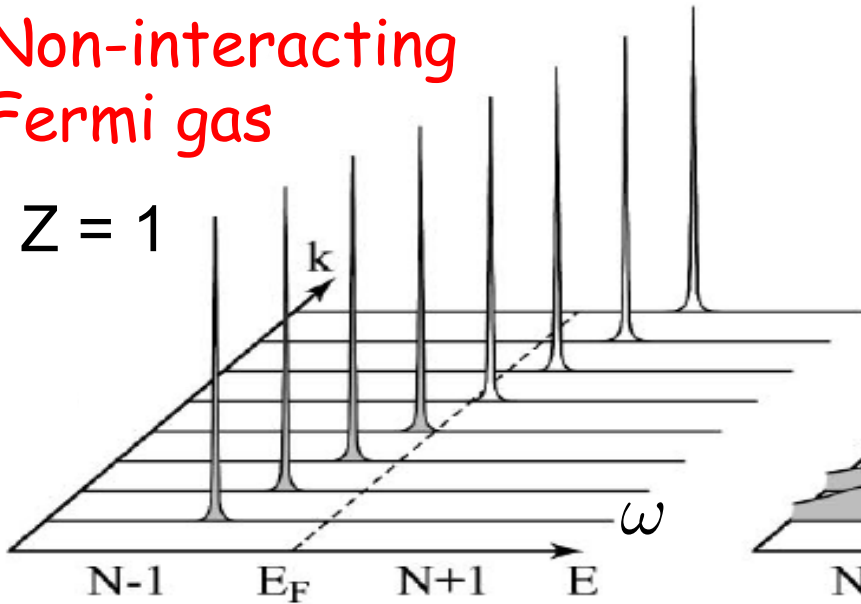
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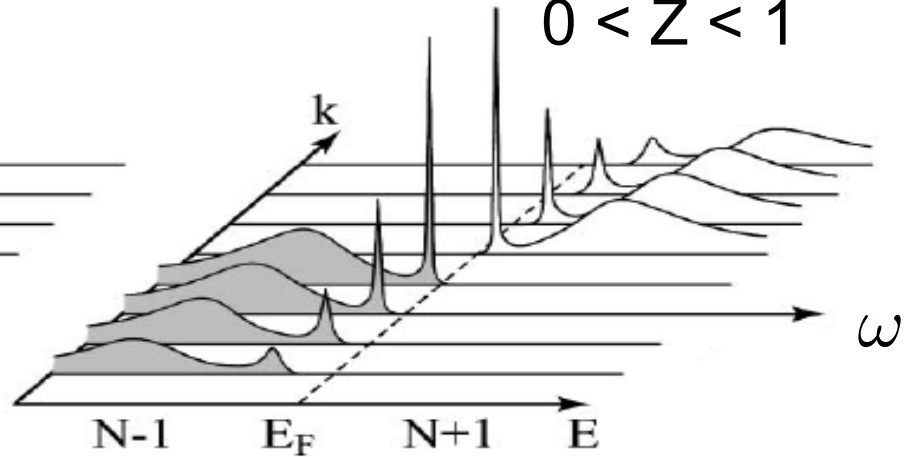
Non-interacting  
Fermi gas

$$Z = 1$$



Landau Fermi-liquid

$$0 < Z < 1$$



$$A(k, \omega) = \delta(\omega - \epsilon_k^0)$$

$$\epsilon_k^0 \simeq \frac{k_F}{m} (k - k_F)$$

$$A(k, \omega) = Z\delta(\omega - \epsilon_k) + \text{incoherent weight}$$

$$\epsilon_k \simeq \frac{k_F}{m^*} (k - k_F)$$

# III) BCS Theory of Superconductivity

## Pairing and Condensation



### \* Pairing:

Weak attraction “g” between electrons  
(electron-phonon int. in conventional SCs)

→ instability of the Fermi liquid:

$S = 0$ , ‘ $L = 0$ ’ Pairs

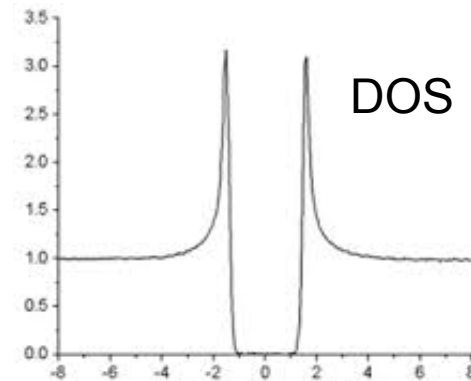
Binding energy of pairs  
= Energy gap

$$\Delta = \omega_0 \exp(-1/g)$$

Experimental observation of gap  $\Delta$ :

Thermodynamics:  $C(T)$ ;  $\chi(T)$

Spectroscopy: tunneling, optics



# BCS Theory of Superconductivity: Pairing and Condensation

Condensation: 2<sup>nd</sup> order phase transition at  $T_c \sim \Delta$   
Macroscopic number ( $\sim 10^{20}$ ) of pairs  
in a single quantum state (cf. BEC)

Order parameter  
broken U(1)

$$\Delta = |\Delta| \exp(i\theta)$$

gap      phase

Experimental manifestation  
of **the phase  $\theta$**

- superconductivity
- Meissner effect
- Josephson effects

**Condensation:**  
Off Diagonal Long Range Order  
ODLRO (or algebraic order)

$$\langle c_{\mathbf{k},\alpha}^\dagger c_{-\mathbf{k},\beta}^\dagger \rangle$$

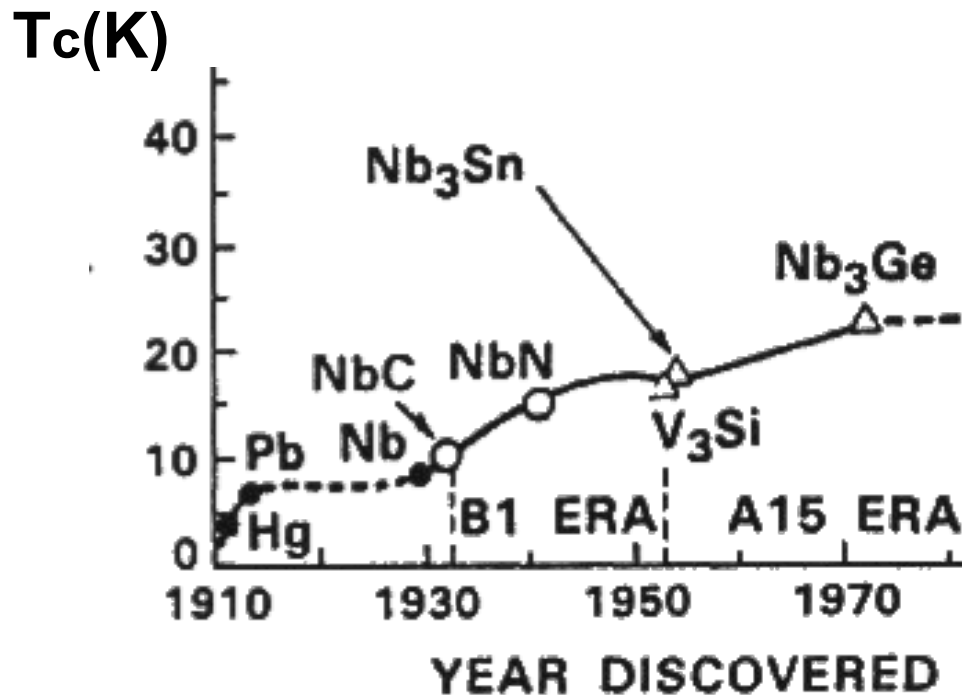
**Phase Coherence  
& rigidity**  
Superfluid stiffness

$$\rho_s > 0$$

By the late 1970's ...

**BCS and Ginzburg-Landau Theories** solved all existing physics problems in superconductors

... and even superfluid Helium-3 which is a triplet, p-wave paired superfluid!



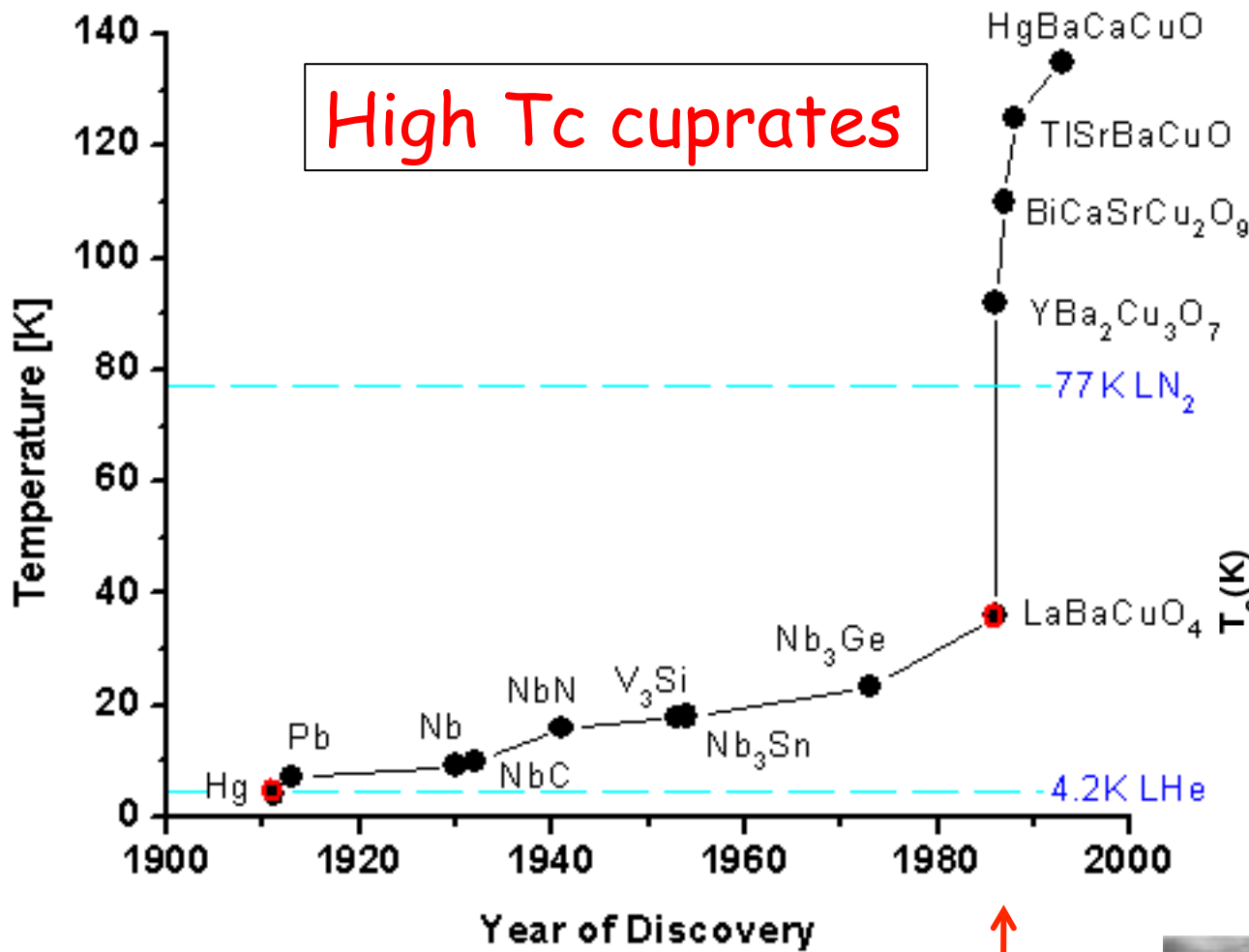
Plateau in the search for higher T<sub>c</sub> materials

# Phenomenology of High Tc Cuprates I

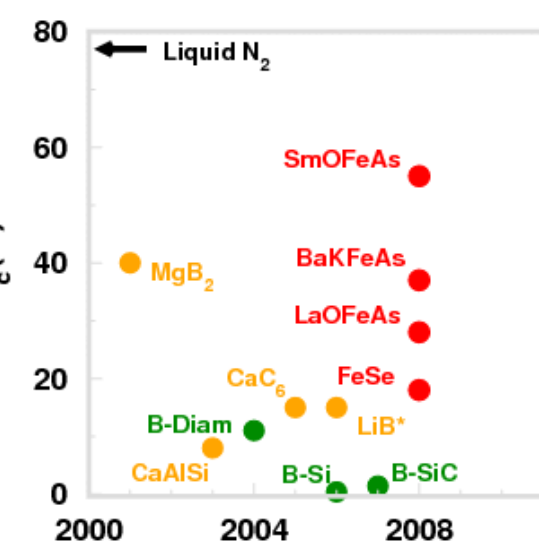
- Standard theory of metals, insulators & superconductors
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- Parent insulators: Mott AFM
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## Phenomenology of High Tc Cuprates II

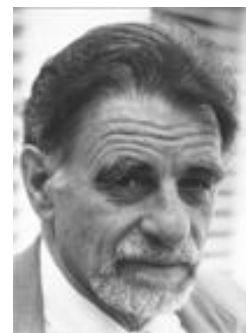
- underdoped materials
  - pseudogap
  - SC fluctuations
  - competing order parameters
  - high field experiments



- $MgB_2$
- Fullerenes
- Pnictides



“Possible high  $T_c$  superconductivity in the Ba-La-Cu-O system”, J. G. Bednorz & K. A. Müller  
Z. Phys. B 64, 189 (1986)



# High $T_c$ Cuprates

$\text{La}_{(2-x)}\text{Sr}_x\text{CuO}_4$  (LSCO) max  $T_c = 40 \text{ K}$

$\text{YBa}_2\text{Cu}_3\text{O}_{(6+x)}$  (YBCO)  $T_c = 90 \text{ K}$

$\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{(8+x)}$  (Bi2212)  $T_c = 90 \text{ K}$

$\text{Tl}_2\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{(10+x)}$  (Tl2223)  $T_c = 125 \text{ K}$

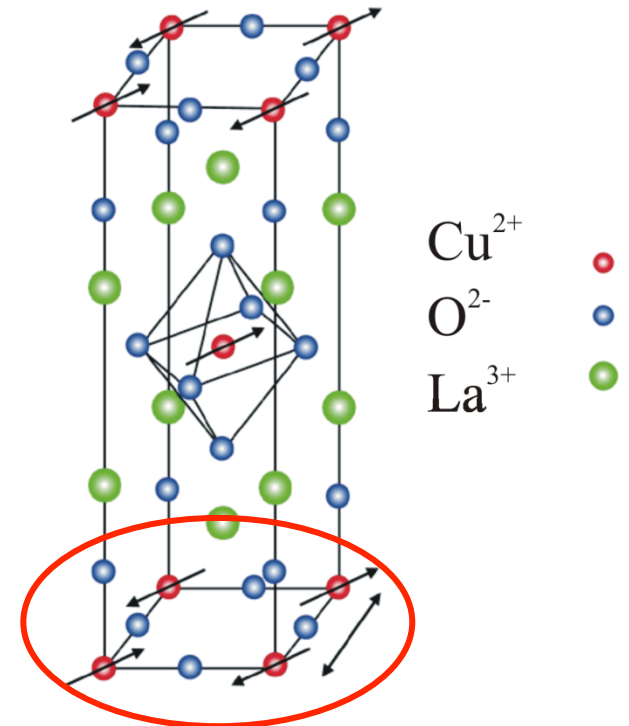
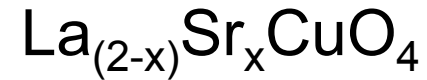
Highest  $T_c \sim 160\text{K}$  under pressure

- 4-5 elements
- Off-stoichiometric
- \* Yet very similar properties

\* Common feature:

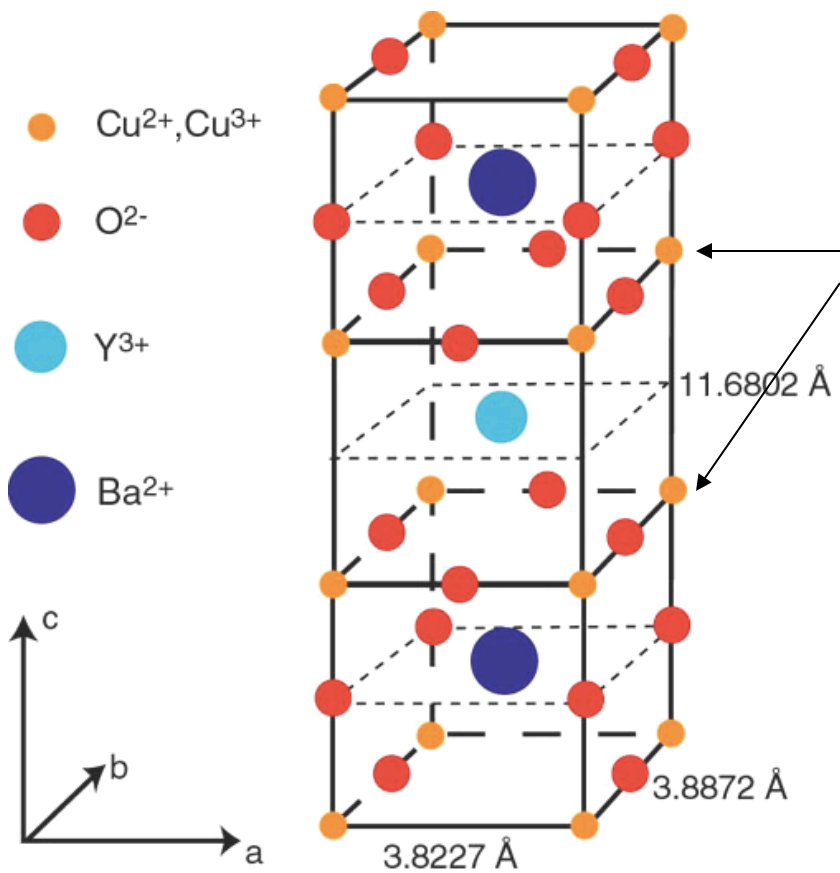
**$\text{CuO}_2$  plane**

1, 2 or 3 planes/unit cell



# High Tc Cuprates:

Layered structure  
→ Quasi-2D  
electronic structure



CuO<sub>2</sub> planes

All the action -  
superconductivity,  
magnetism -  
is on the  
2D CuO<sub>2</sub> planes



Stoichiometry of “spacer layers” (between planes)  
→ controls the carrier concentration of CuO<sub>2</sub> layers

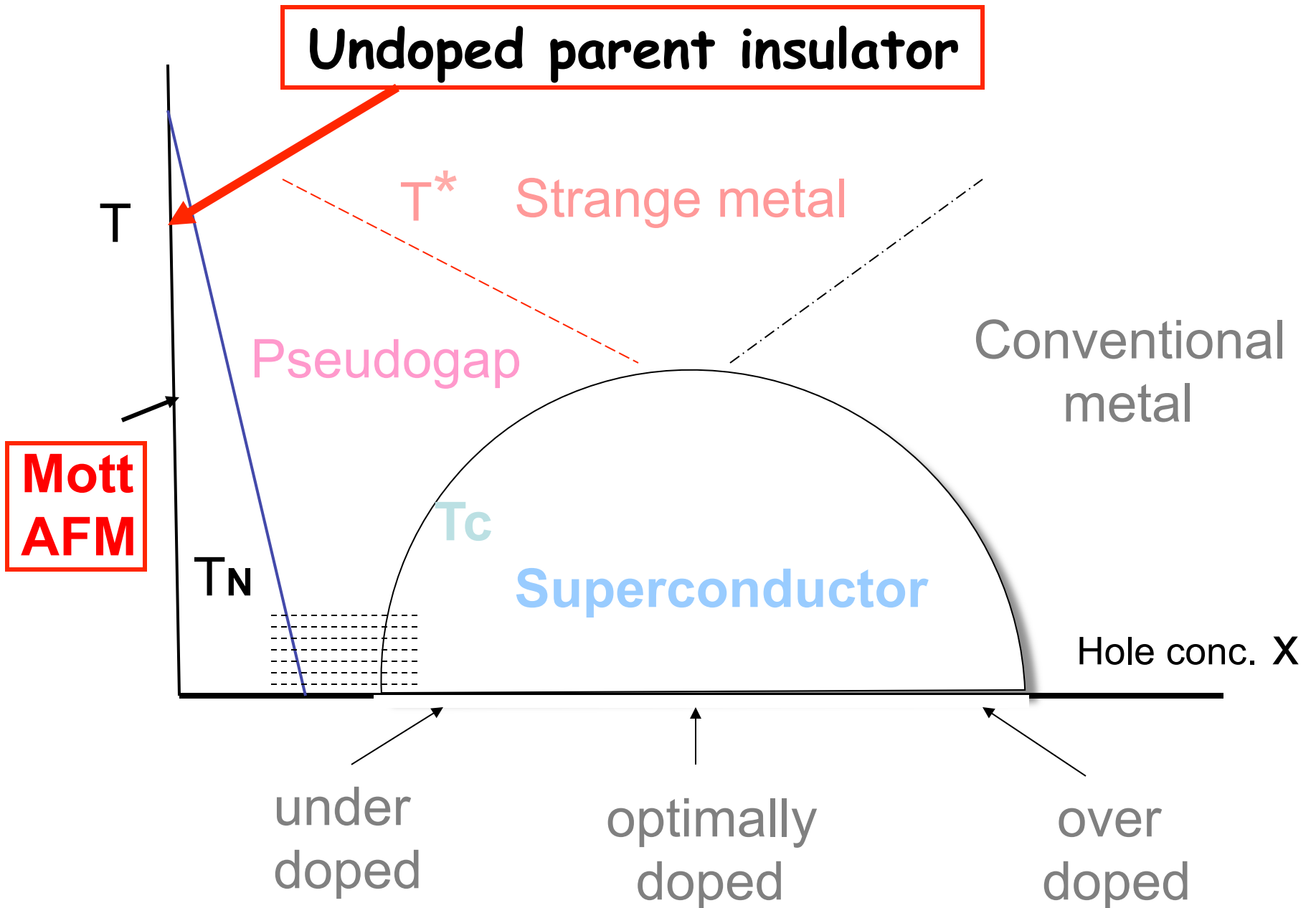
## “Matthias’ Rules” for finding new superconductors (1960’s)

1. Transition metals are better than simple metals
2. Peaks of density of states at the Fermi level are good
3. High crystallographic symmetry is good: Cubic is best
4. Stay away from oxygen
5. Stay away from magnetism
6. Stay away from insulating phases
7. Stay away from theorists

Strongly  
Violated  
by  
Bednorz  
& Muller

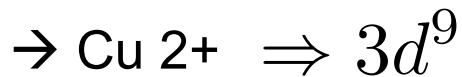
T. H. Geballe & J.K. Hulm,  
“Bernd Theodor Matthias”  
(National Academies Press, 1996).

**Undoped parent insulator**

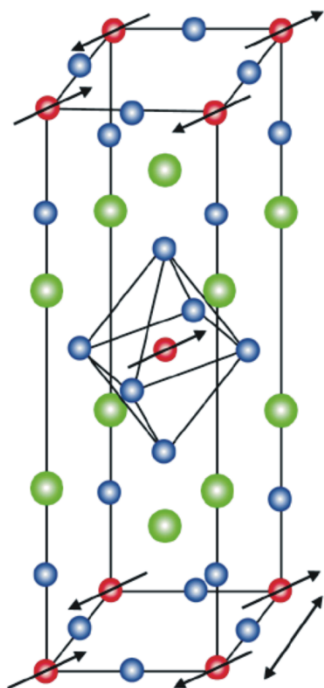


# La<sub>2</sub>CuO<sub>4</sub>

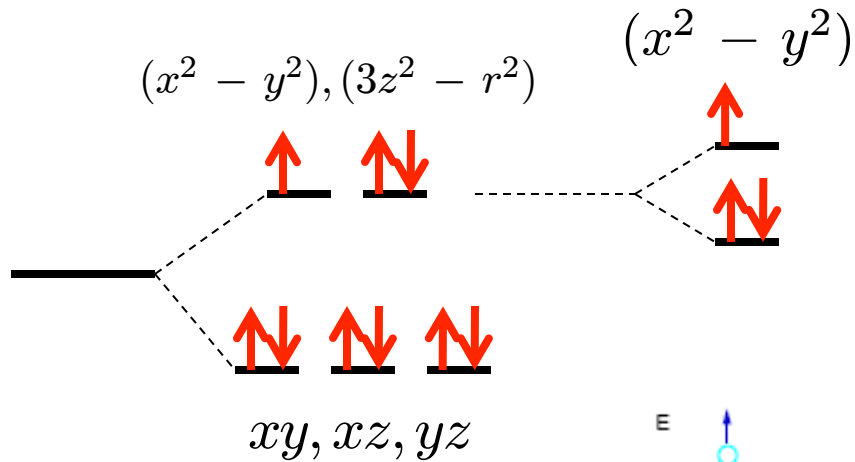
$$2(+3) + \text{Cu} + 4(-2) = 0$$



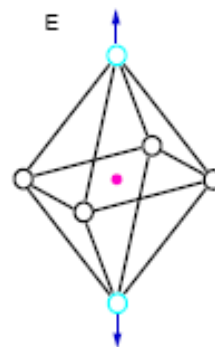
Cu<sup>2+</sup>  
O<sup>2-</sup>  
La<sup>3+</sup>



corner sharing octahedra

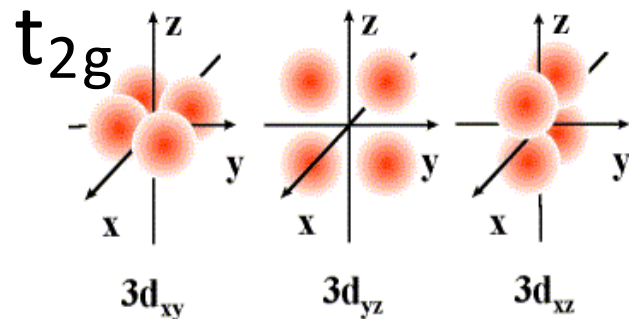
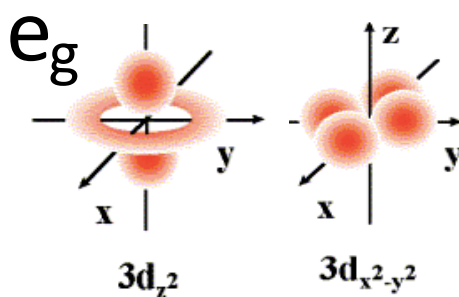


Octahedral/  
Cubic crystal  
field



1 elect.  
(or 1 hole)  
per u.c.  
= Spin 1/2

Hybridize  
with O  
pσ orbitals

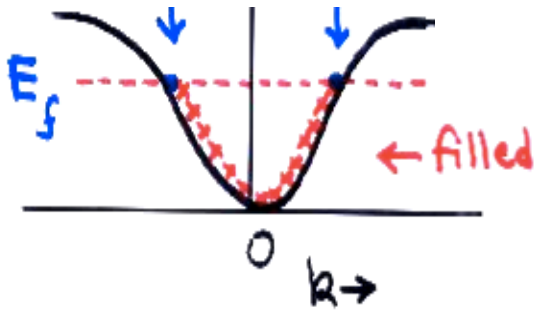


# Band theory

La2CuO4

1 el/unit cell

→ Half-filled band (k-space)



Treating Interactions perturbatively → ~~metal~~

# Experiments → La2CuO4 Insulator!

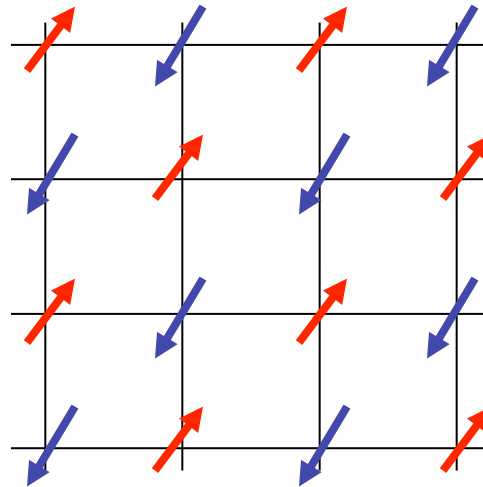
## Mott Insulator\*

driven by strong Coulomb correlations in a half-filled band

On-site Coulomb = Hubbard U

$$U n_{i\uparrow} n_{i\downarrow} \quad U \gg t$$

Half-filled in r-space: one el./site



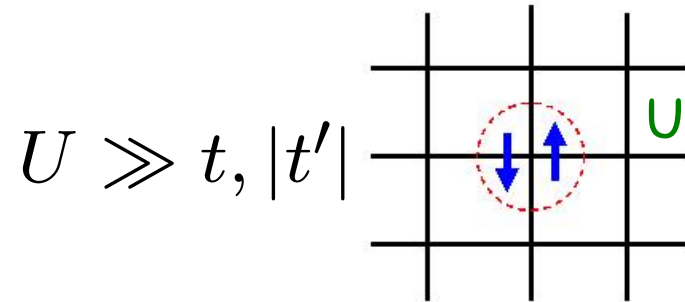
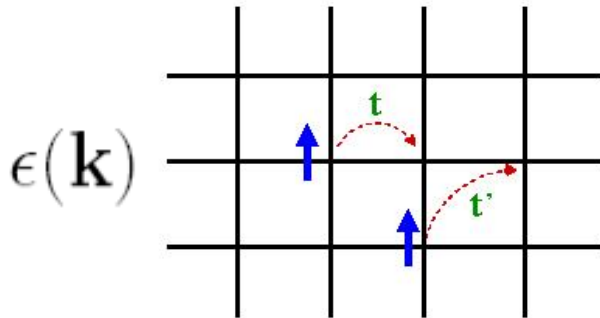
Charge →  
Mott Insulator

Spin →  
Antiferromagnet

(\*will not discuss charge-transfer Insulator)

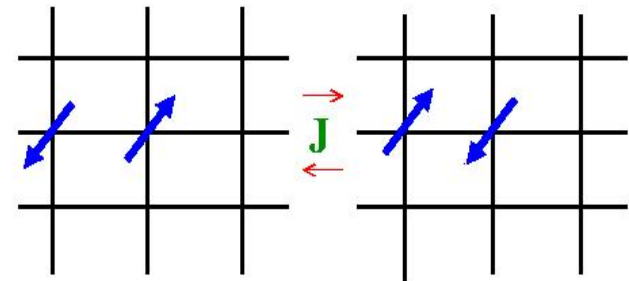
# Hubbard model: minimal model for CuO<sub>2</sub> planes

$$\mathcal{H} = \sum_{\mathbf{k}, \sigma} \epsilon(\mathbf{k}) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + U \sum_{\mathbf{r}} n_{\mathbf{r}\uparrow} n_{\mathbf{r}\downarrow}$$



## AF superexchange

$$J \sum \mathbf{S}_i \cdot \mathbf{S}_j \quad J = 4t^2/U$$



- neutron
- Raman

**~ 100-150 meV**

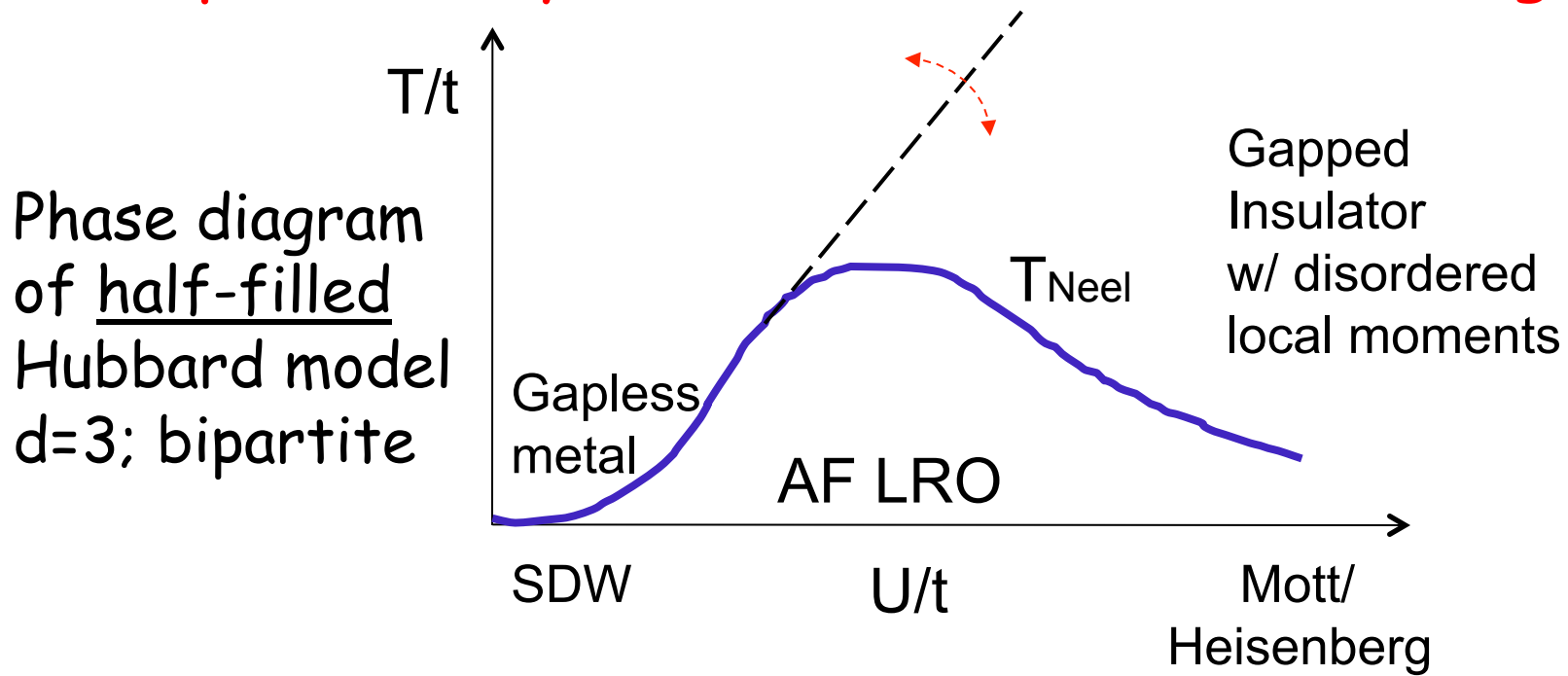
$$J \leq |t'| \leq t \ll U \rightarrow 3.6 \text{ eV}$$

**300 meV**  
**t' ~ t/4**

- photoemission
- electronic structure calc.

Insulator →  
2D S=1/2 Heisenberg AFM

# Spin Density Wave (SDW) vs. Heisenberg AF



**Do Mott insulators always show AFM order at  $T=0$  ?**

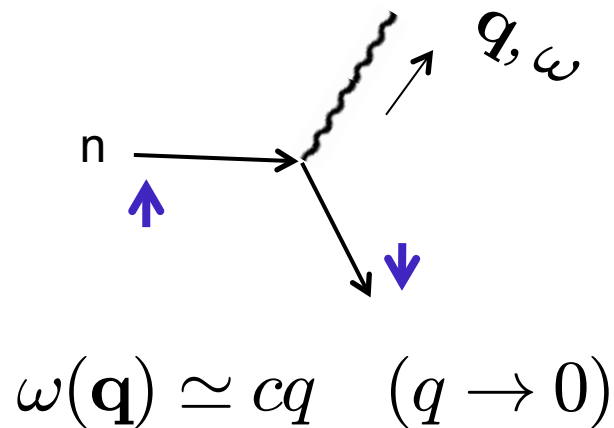
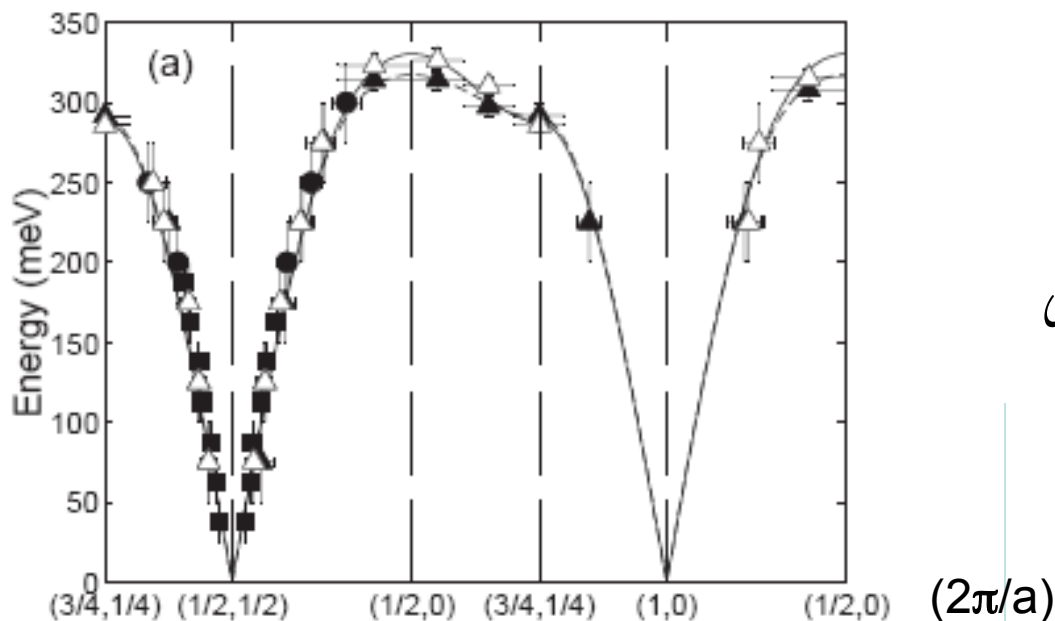
- **Yes**, for bipartite lattices, no frustration &  $d > 1$   
→ parent insulator in cuprates has a **Neel Long Range Order**
- **No**, for non-bipartite & frustrated systems  
→ **quantum spin liquids** (triangular lattice organics, kagome lattice materials,...)

See Senthil's lectures

# Undoped cuprates → Probe spins using neutrons

- elastic scattering → magnetic order  $Q = (\pi, \pi)$  or  $(1/2, 1/2)$
- ordered moment reduced by quantum fluctuations
- inelastic → spin wave excitations

R. Coldea et al, PRL (2001)



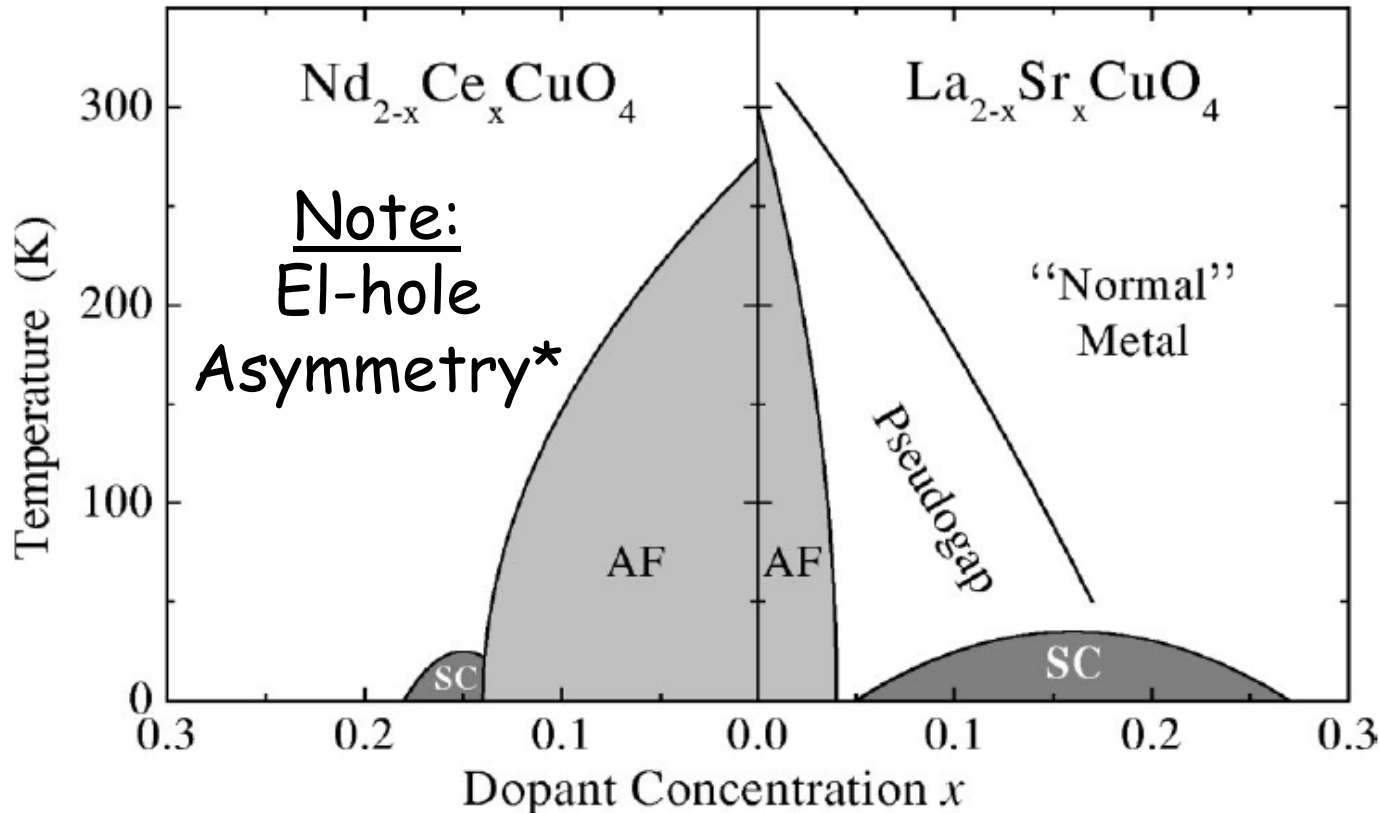
$$\omega(\mathbf{q}) \simeq cq \quad (q \rightarrow 0)$$

highest J among  
2 & 3 dim.  
TM oxides

$\text{La}_2\text{CuO}_4 \rightarrow S=1/2$  Heisenberg AF with n.n.  $J = 150$  meV

+ small ring exchange consistent with  $t^4/U^3$

# What happens when you put mobile carriers in a Mott insulator?



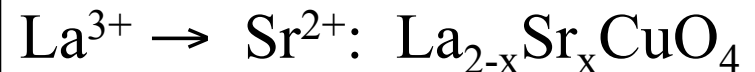
focus  
only  
on hole  
doping  
here

← Electron doping

Hole doping →

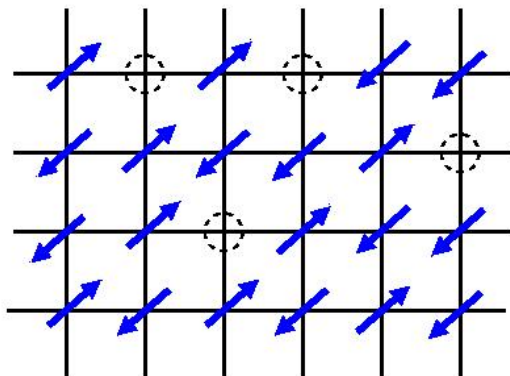
\*see A. Paramekanti's lectures

Mott

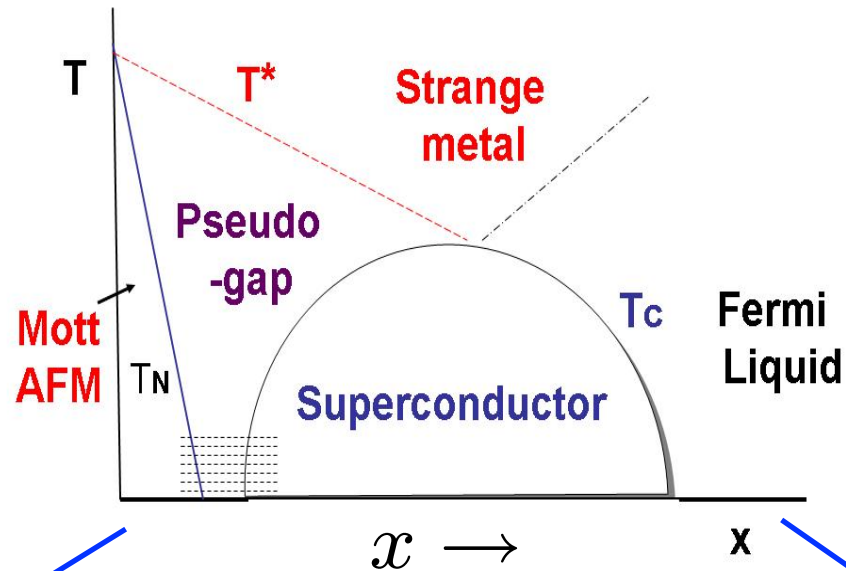
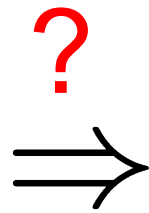


Central problem:

Electronic structure of a doped  
2D  $S=1/2$  Mott insulator?

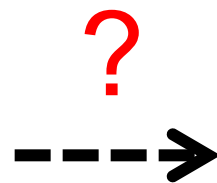
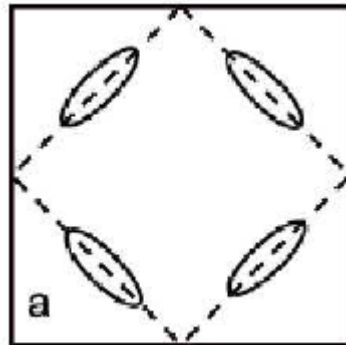


hole conc.  $x$

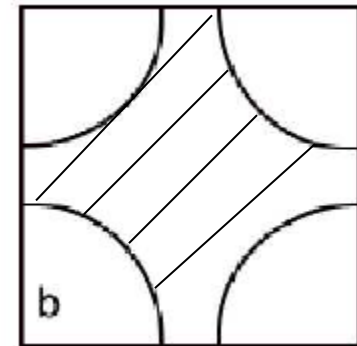


FS Pockets  
with  $x$  holes

(violates Luttinger  
without AFM order  
Or exotic order)



Large FS with  
Luttinger count  
( $1-x$ ) electrons  
= ( $1+x$ ) holes



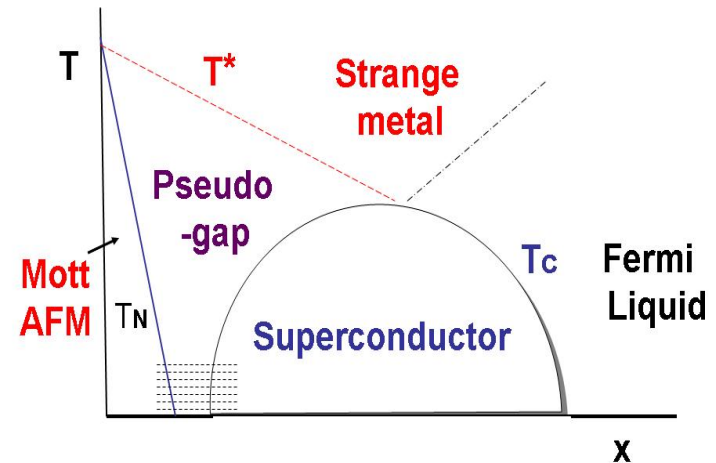
# Phenomenology of High Tc Cuprates I

- Standard theory of metals, insulators & superconductors
- High Tc cuprate materials
- Parent insulators: Mott AFM
- Problem of doped Mott insulator
- **ARPES** & quantum oscillations
  - Fermi liquid state at overdoping
  - Strange metal at optimal doping
  - d-wave superconductor

## Phenomenology of High Tc Cuprates II

- underdoped materials
  - pseudogap
  - SC fluctuations
  - competing order parameters
  - high field experiments

One of the most important probes of the electronic structure of all phases of the High  $T_c$  cuprates



## Angle-Resolved PhotoEmission Spectroscopy (ARPES)

Review articles:

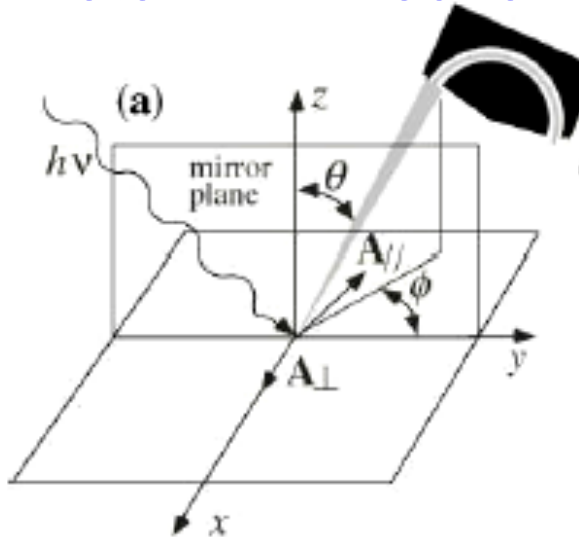
- A. Damascelli, Z. Hussain and Z. X. Shen, Rev. Mod. Phys. **75**, 473 (2003).
- J. C. Campuzano, M. R. Norman and M. Randeria; in *"Physics of Conventional and Unconventional Superconductors"*, eds. K. H. Bennemann and J. B. Ketterson, (Springer 2004); cond-mat/0209476.

See also: D. Dessau's lectures at this School

# ARPES

$$I(\mathbf{k}, \omega) = I_0(\mathbf{k}) f(\omega) A(\mathbf{k}, \omega)$$

Photon-in Electron-out



$\mathbf{k}$  = momentum in 2D zone  
 $\omega$  = - binding energy  
(w.r.t. chemical potential)

Matrix elements:  $I_0(\mathbf{k})$

- polarization selection rules
- incident photon energy  $h\nu$
- No T or  $\omega$  dependence

Fermi fn  $f(\omega) = \frac{1}{\exp(\omega/T) + 1}$

Spectral Function  $A(\mathbf{k}, \omega)$

$f(\omega)A(\mathbf{k}, \omega) =$  occupied part of Spectral Function

## Spectral function:

$$A(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} G(\mathbf{k}, \omega + i0^+)$$

= Probability density of making a single-particle excitation with momentum  $\mathbf{k}$  at an energy  $\omega$

Green's  
function:

$$G(\mathbf{k}, \omega) = \frac{1}{\omega - \epsilon_{\mathbf{k}} - \Sigma(\mathbf{k}, \omega)}$$

Self Energy

$$\Sigma(\mathbf{k}, \omega) = \Sigma'(\mathbf{k}, \omega) + i\Sigma''(\mathbf{k}, \omega)$$

→ Effects of  
Interactions

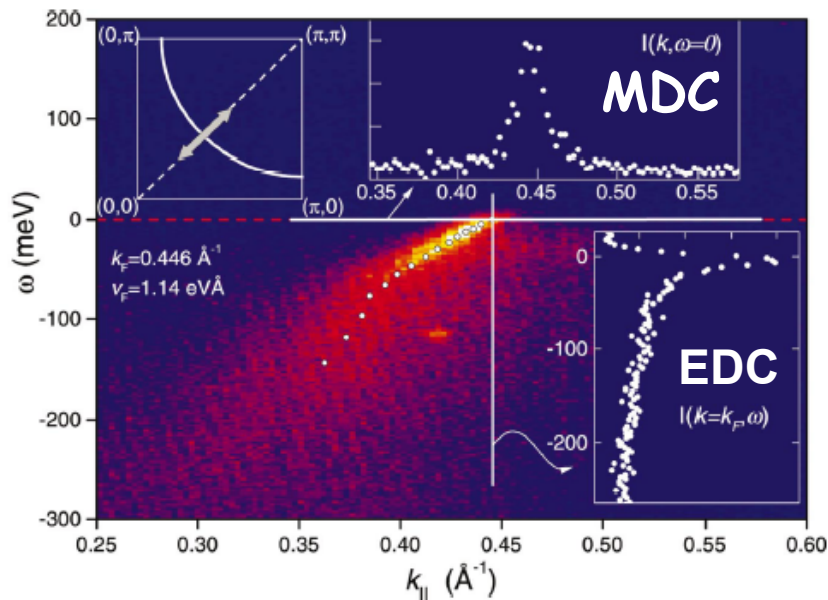
renormalized  
dispersion

scattering rate  
~ 1/(lifetime)

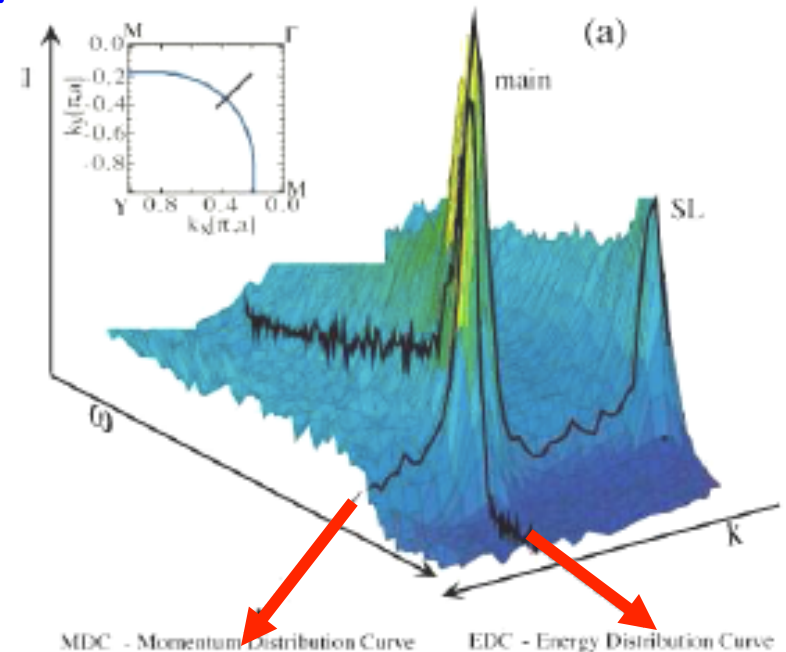
$$A(\mathbf{k}, \omega) = \frac{1}{\pi} \frac{|\Sigma''(\mathbf{k}, \omega)|}{[\omega - \epsilon_k - \Sigma'(\mathbf{k}, \omega)]^2 + [\Sigma''(\mathbf{k}, \omega)]^2}$$

$\Sigma'(\omega) \rightarrow$  Renormalized dispersion

$\Sigma''(\omega) \rightarrow$  Line width



Valla et al, Science (2000)



**MDC**

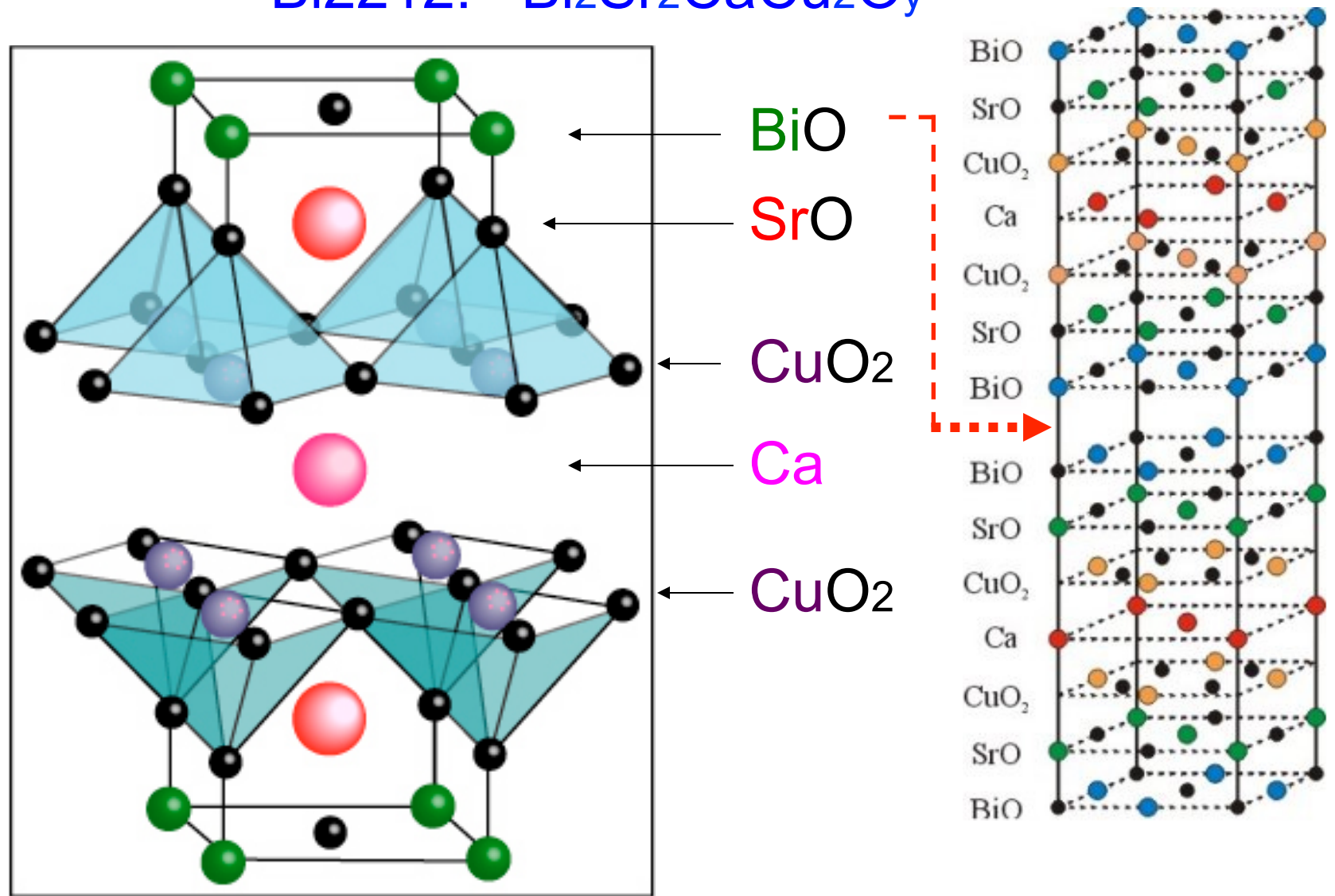
**EDC**

A. Kaminski et al, PRL (2001)

**EDC: energy distribution curve:**  $A(\mathbf{k}, \omega)$  fixed  $\mathbf{k}$ , as a function of  $\omega$

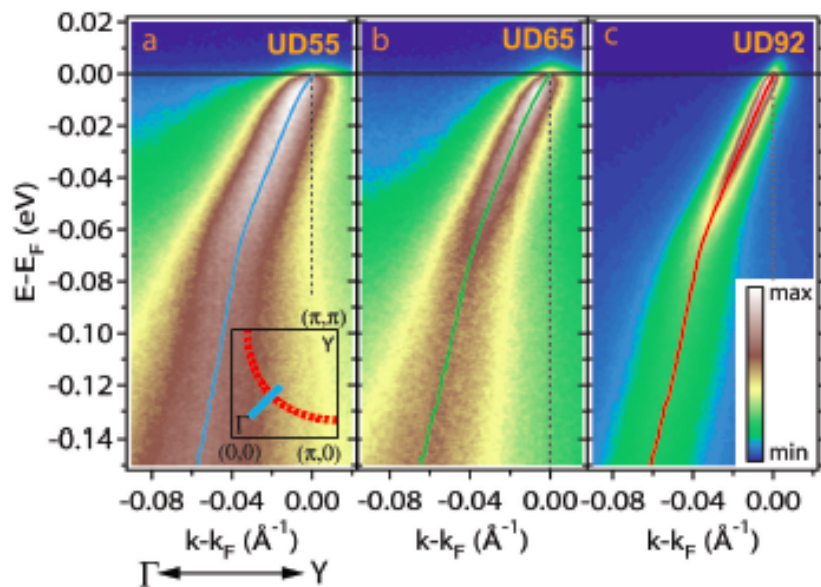
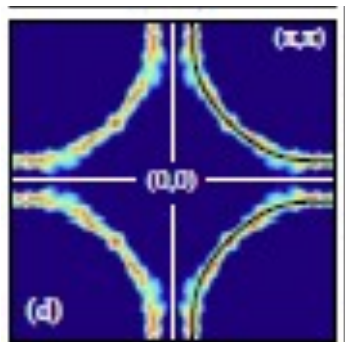
**MDC: momentum distribution curve:**  $A(\mathbf{k}, \omega)$  fixed  $\omega$  as a function of  $\mathbf{k}$

# Bi2212: $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_y$



Both ARPES & STM are surface sensitive probes  
→ exploit cleavage plane in Bi2212 & Bi2201

# What can we learn from ARPES?



Vishik, PRL (2012)

- Fermi surface  $k_f$
- Fermi velocity  $v_f$
- Dispersion &  $\Sigma'$
- Linewidths  $\Sigma''$  and  $Z$   
 → Fermi Liq or not?
- k-resolved Gaps  
 → SC gap  
 → “pseudogap”

# Phenomenology of High Tc Cuprates I

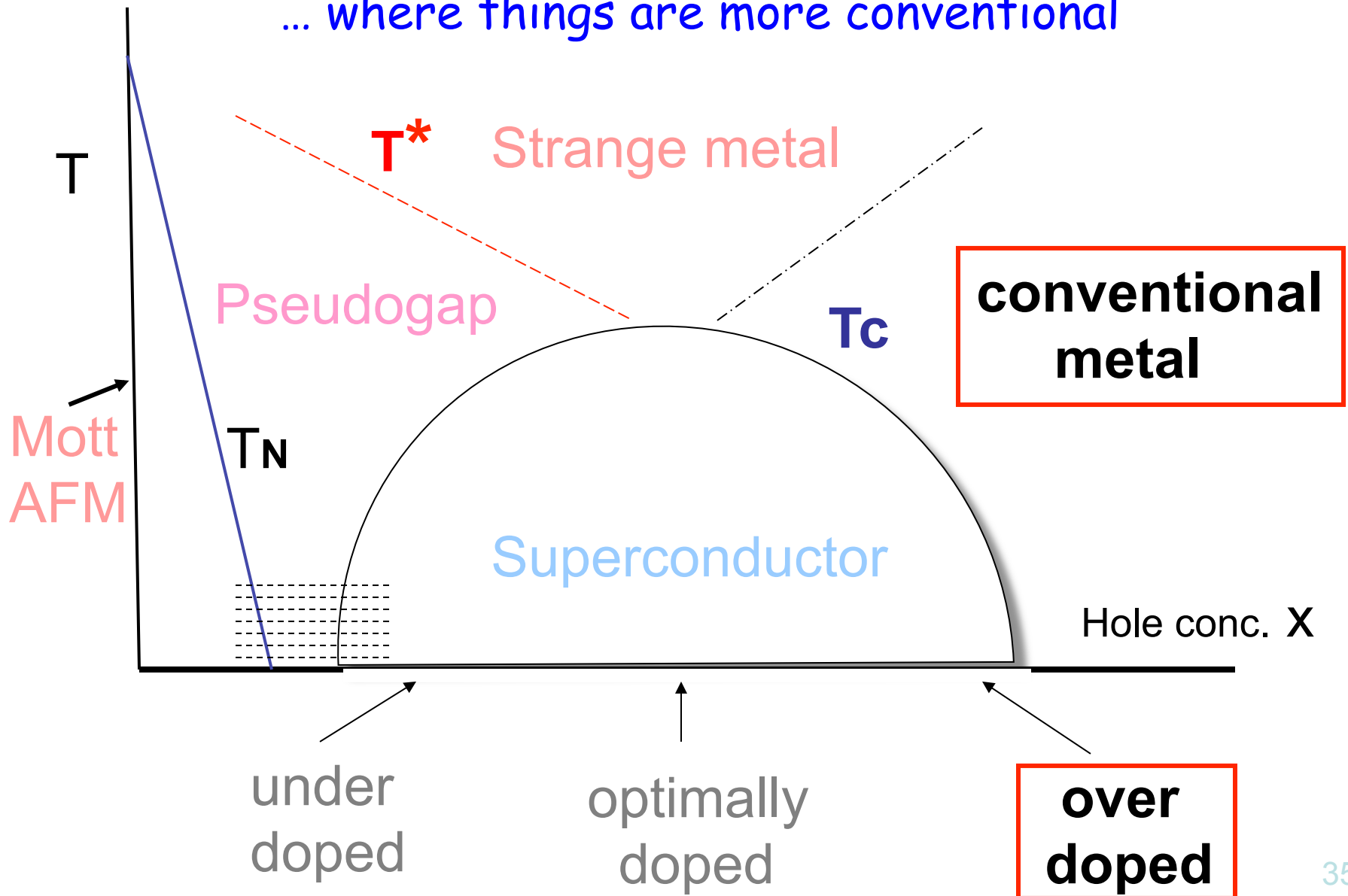
- Standard theory of metals, insulators & superconductors
- High Tc cuprate materials
- Parent insulators: Mott AFM
- Problem of doped Mott insulator
- ARPES & quantum oscillations
  - Fermi liquid state at overdoping
  - Strange metal at optimal doping
  - d-wave superconductor

Conducting  
Phases of  
Doped Mott  
Insulators

# Phenomenology of High Tc Cuprates II

- underdoped materials
  - pseudogap
  - SC fluctuations
  - competing order parameters
  - high field experiments

Start far from the Mott insulator  
... where things are more conventional



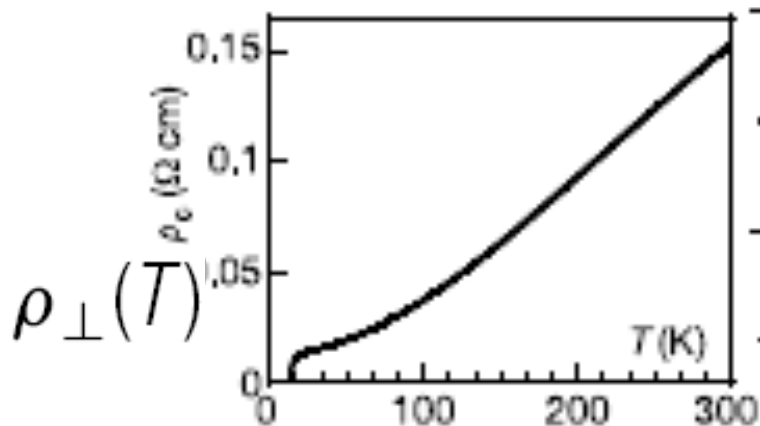
# Overdoped Cuprates look like Fermi liquids

OD TI 2201 ( $T_c = 20$  K)

Resistivity "looks like"

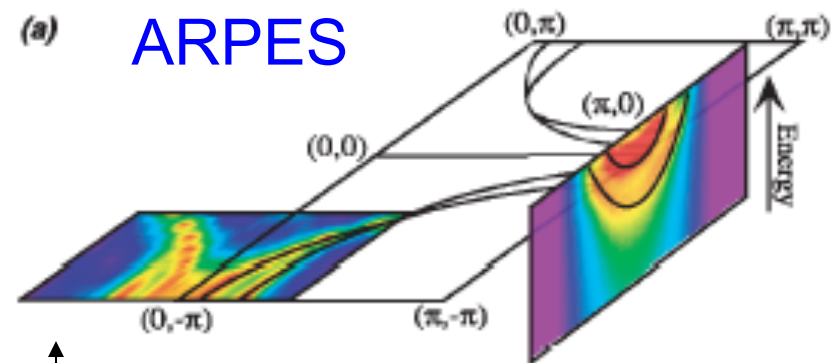
Fermi liquid  $\rho \sim T^2$

$$\rho \sim T^\alpha \quad \alpha > 1$$



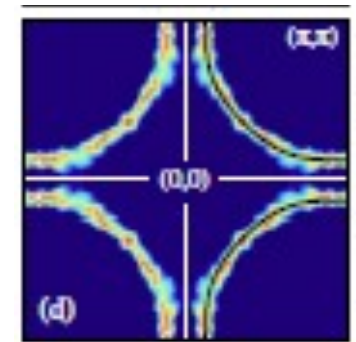
$$\rho_{\perp} / \rho_{ab} \approx 1,000$$

N. Hussey et al, Nature (2003)



OD Bi2212:  
A. Kaminski et al, PRL (2003)

$$A(\mathbf{k}_F, \omega = 0)$$



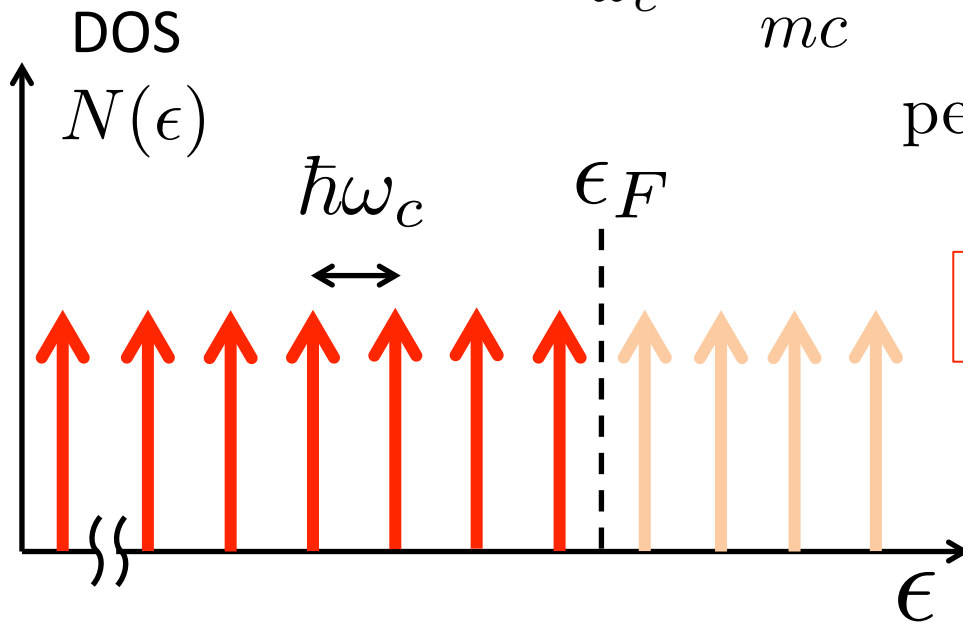
OD TI2201:  
Damascelli group, PRL (2005)

Quantum Oscillations  $\rightarrow$  Large FS with  $(1+x)$  holes in TI2201

# Quantum Oscillations: the simplest example

Exactly solved problem  
of free electrons in 2-dim  
→ Landau levels

$$\omega_c = \frac{eH}{mc}$$



peak in  $N(\epsilon_F)$  when

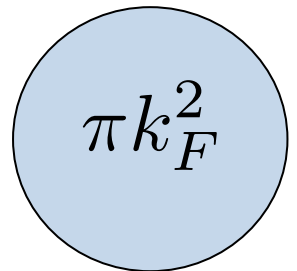
$$\epsilon_F = (n + 1/2) \hbar\omega_c$$

$$\frac{1}{H_n} = \left(n + \frac{1}{2}\right) \frac{\hbar e}{mc} \frac{1}{\epsilon_F}$$

period in  $\frac{1}{H} = 2\pi \left(\frac{e}{\hbar c}\right) \frac{1}{\pi k_F^2}$

Frequency  $\leftrightarrow$  FS area

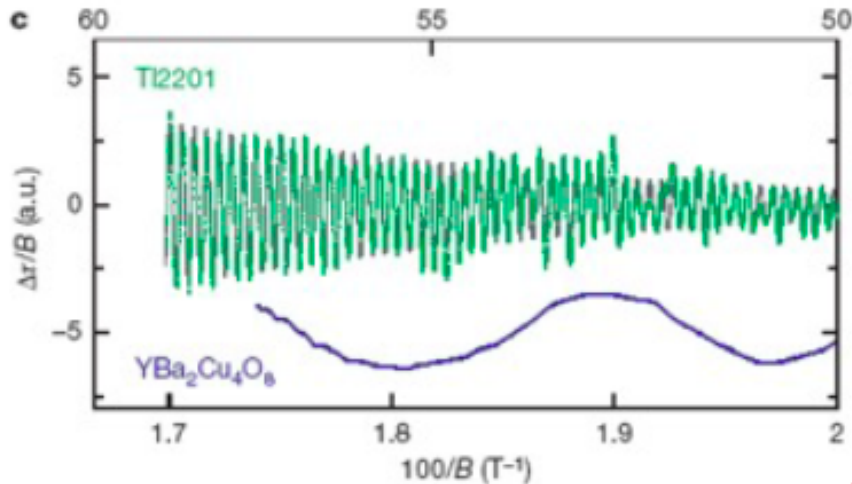
$$f = \frac{\hbar c}{2\pi e} A_{\mathbf{k}}$$



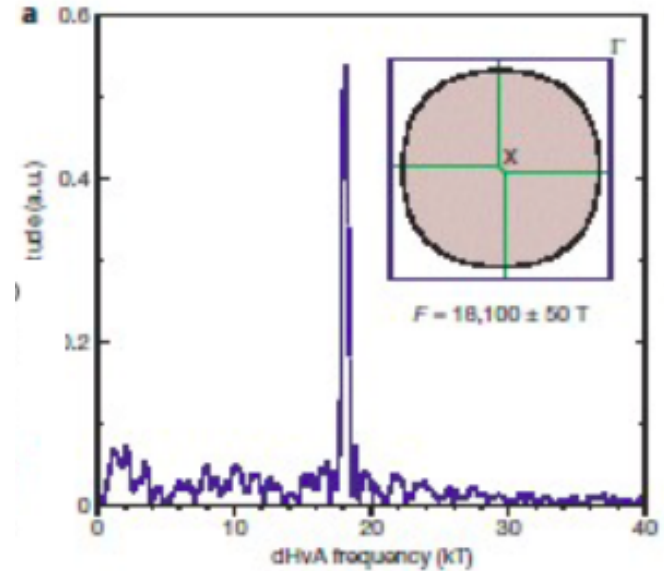
Oscillations in the DOS  
→ Oscillations in "all  
observables"

General Fermi-liquid result  
for 3D, arbitrary dispersion  
[Onsager, Lifshitz-Kosevich]

# Quantum Oscillations in Overdoped (OD) Tl2201

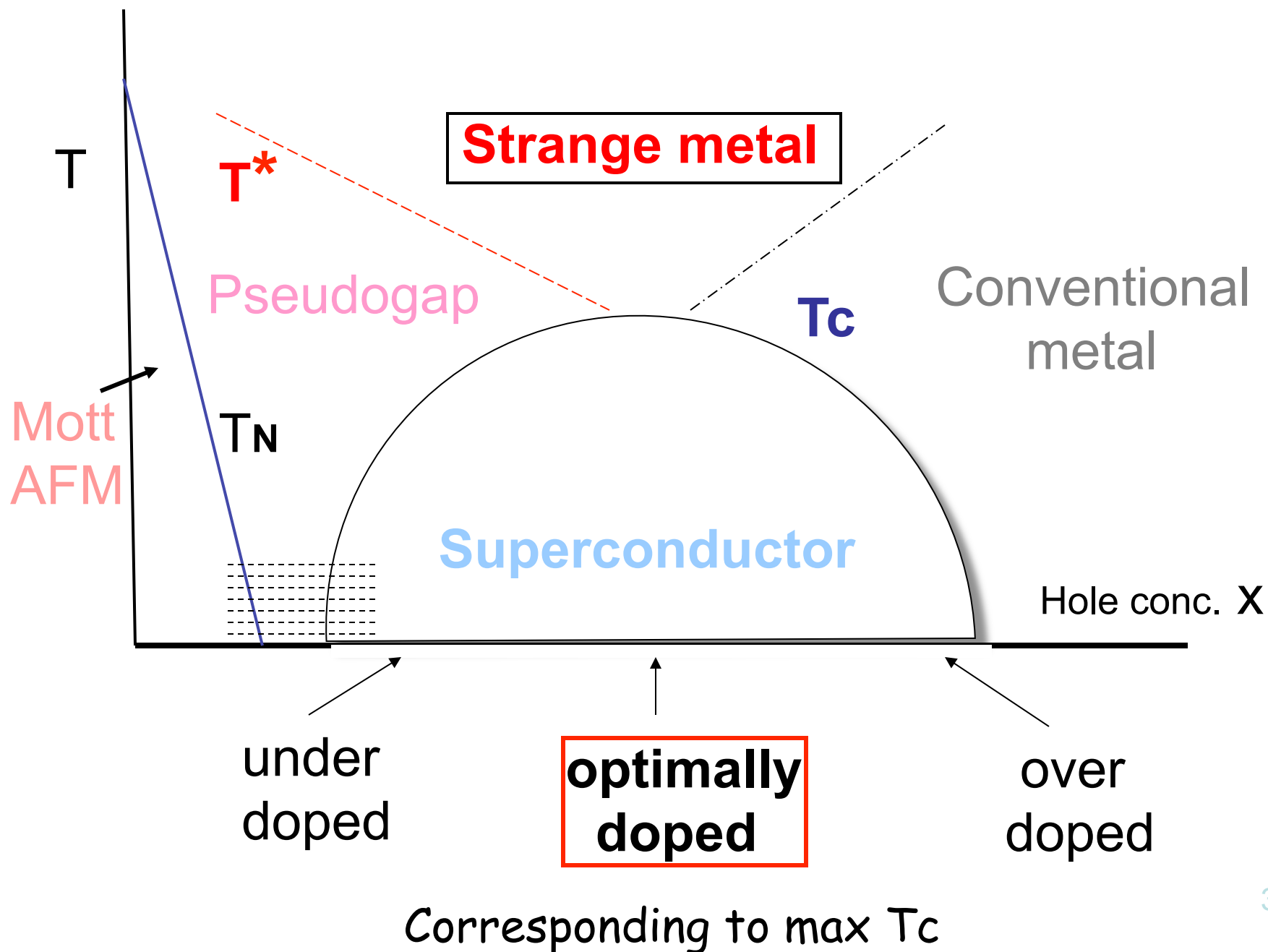


Vignolle, ..., Hussey, Nature (2008)



- \*  $T_c = 10\text{K}$ ,  $B < 60\text{T}$ ; dHvA in M; SdH in c-axis  $\rho$
- \* T and B-dependences fit Lifshitz-Kosevich theory  
 Freq. = 18,100 T  $\rightarrow$  large (Luttinger) FS with  $(1+x)$  holes  
 Sharp quasiparticles with  $m^*/m \sim 4$  (consistent with other probes)

For OD cuprates, no conflict between  
 ( $T > T_c$ ,  $H=0$ ) expts, like ARPES &  
 (low T, high H) expts, like quantum oscillations } ... Unlike  
 underdoped cuprates!

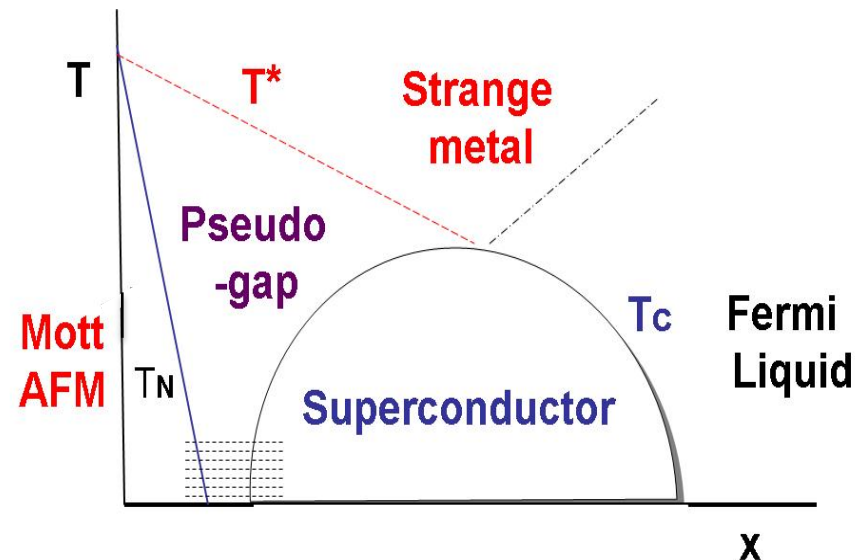


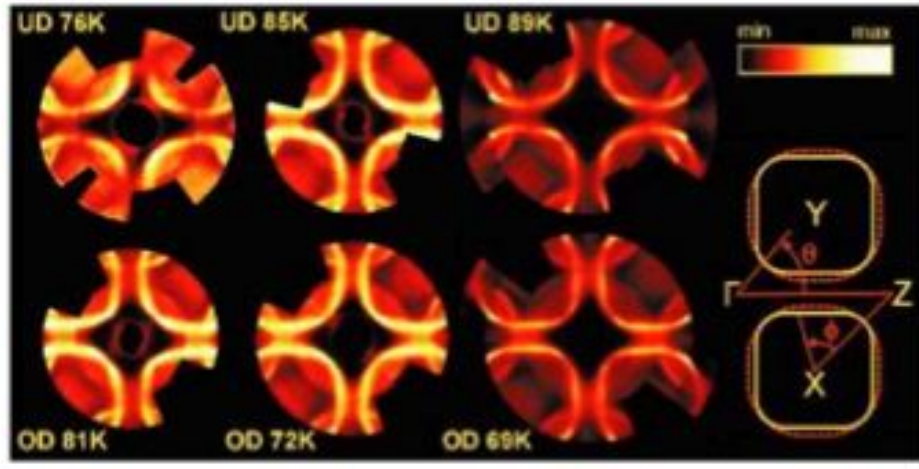
# Summary of experiments in Strange metal regime

- electronic excitations show dispersion similar to band theory with  $m^*/m \sim 3$
- large "Fermi surface" with  $(1+x)$  holes = Luttinger count

but

- anomalous spectral linewidths: quasiparticles ill-defined
- anomalous transport





$A(\mathbf{k}_F, \omega = 0)$  A. Kordyuk et al PRB (2002)

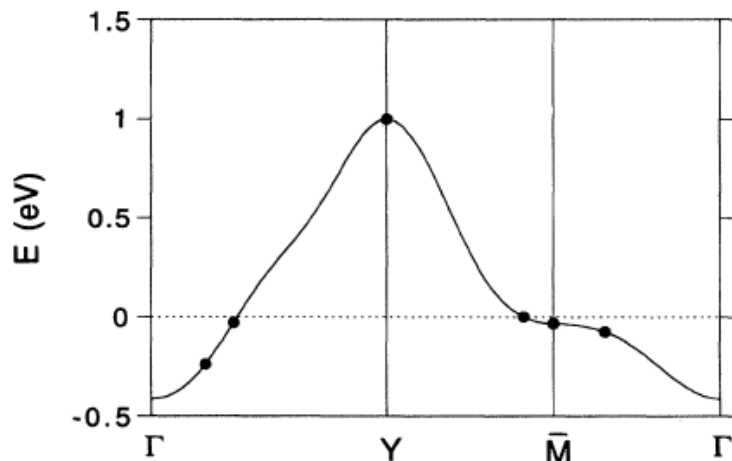
## ARPES

\* large "Fermi surface"  
consistent with  $(1+x)$  holes

\* Dispersing "band"

$$\epsilon(\vec{k}) = \sum c_i \eta_i(\vec{k})$$

Norman, Randeria, Ding & Campuzano, PRB (1995)



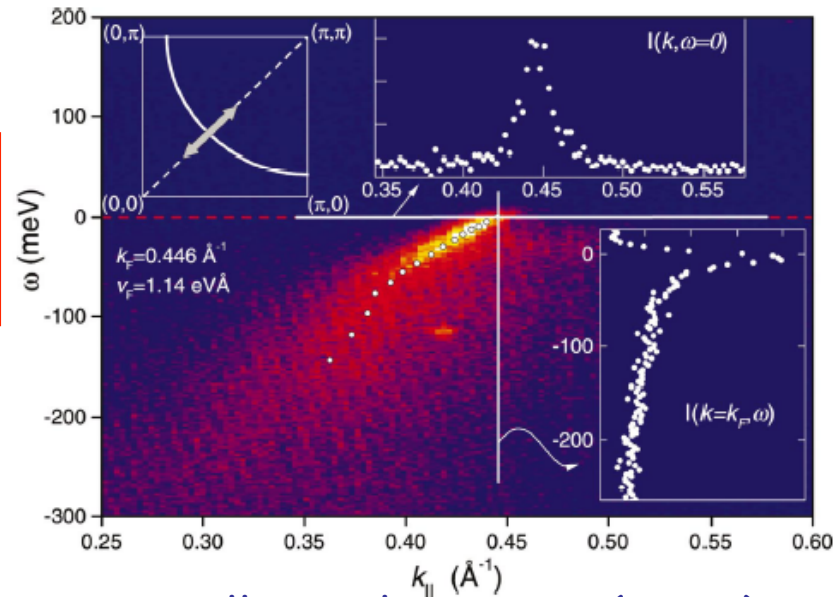
$c_i$	$\eta_i(\vec{k})$
0.1305	1
-0.5951	$\frac{1}{2}(\cos k_x + \cos k_y)$
0.1636	$\cos k_x \cos k_y$
-0.0519	$\frac{1}{2}(\cos 2k_x + \cos 2k_y)$
-0.1117	$\frac{1}{2}(\cos 2k_x \cos k_y + \cos k_x \cos 2k_y)$
0.0510	$\cos 2k_x \cos 2k_y$

# Absence of Quasiparticles in Strange Metal ( $T > T_c$ )

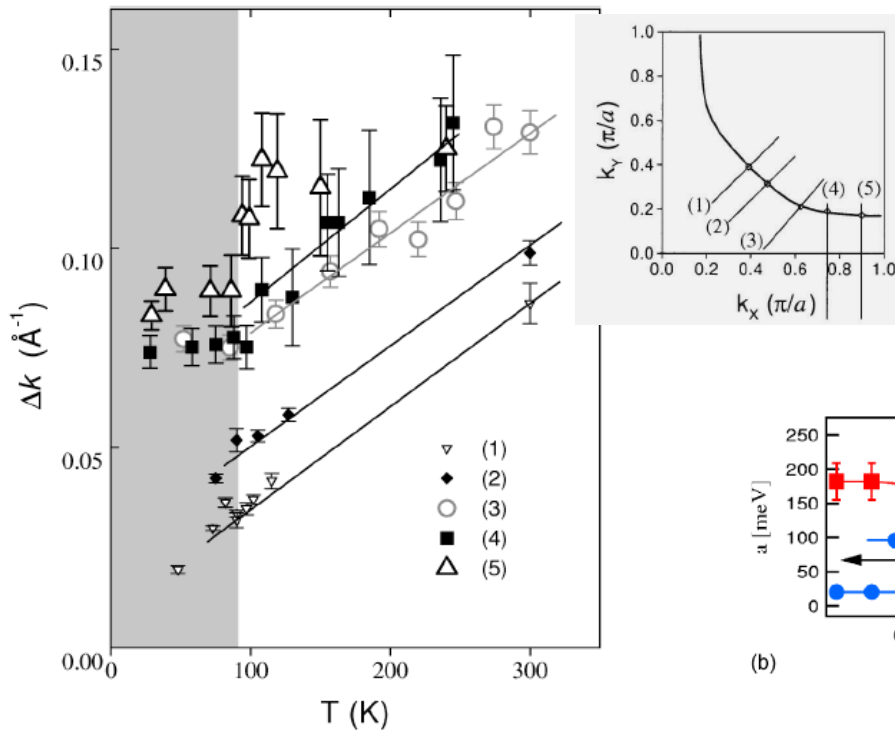
Arpes lineshape

$$\text{Im}\Sigma \sim a + b\omega \quad \text{fixed } T$$

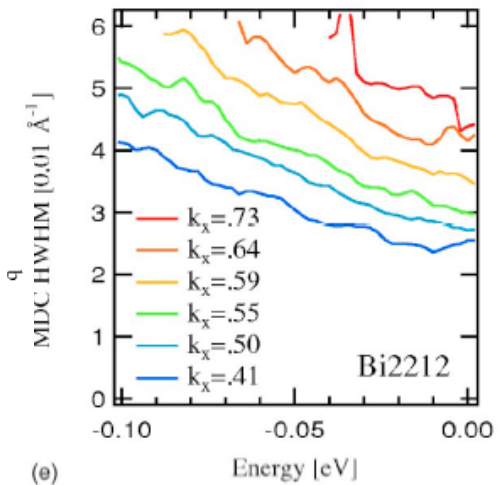
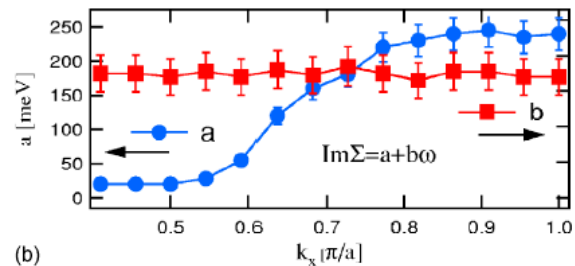
$$a' + b'T \quad \omega = 0$$



Valla et al, Science (2000)



MDC linewidths



Valla et al, (2000)

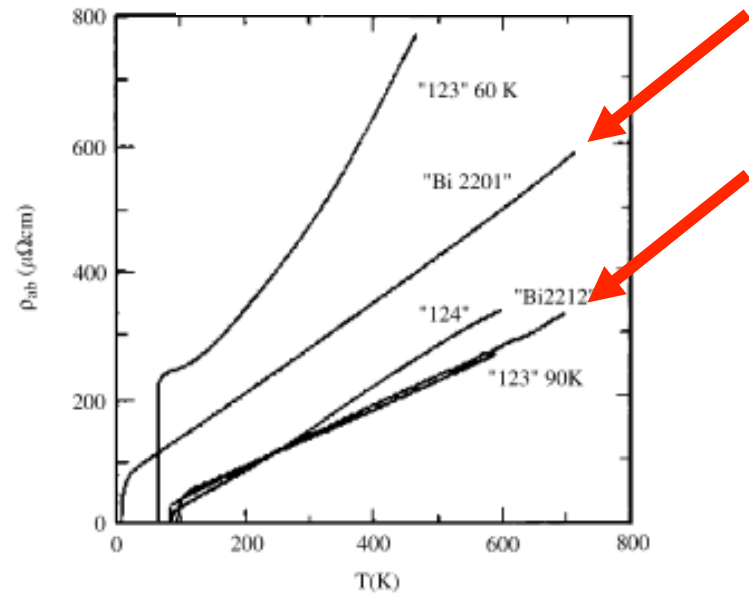
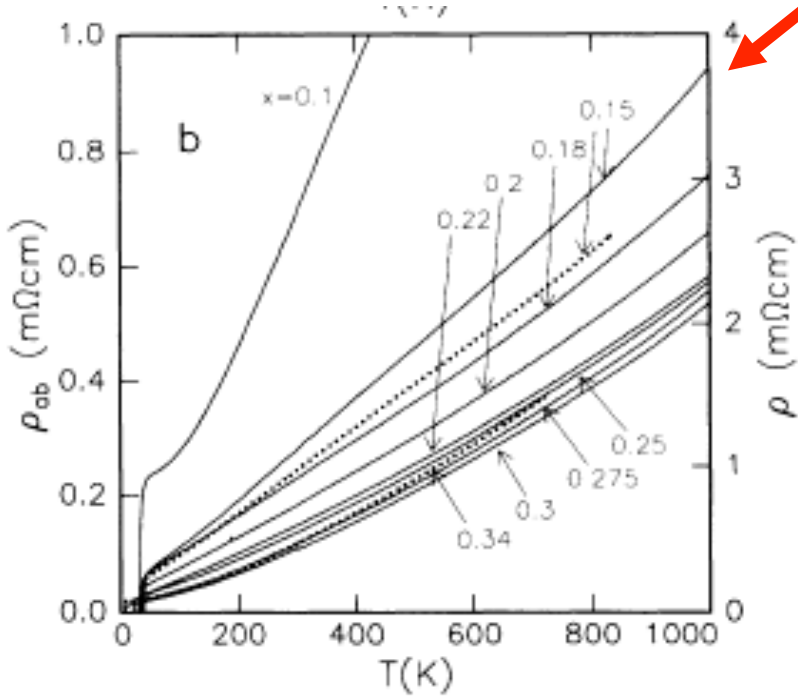
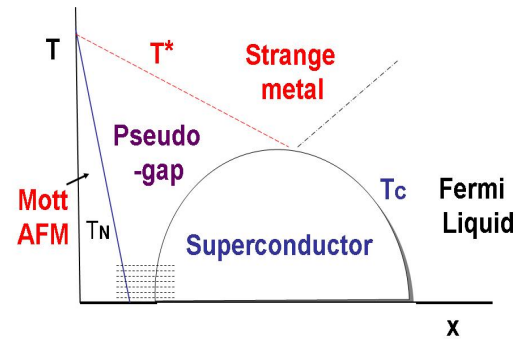
Kaminski et al, (2005)

# The “strange metal” regime

Linear-T resistivity in ab-plane

$$\rho \sim T$$

Why?



- \* Not el-phonon
- linear down to 10 K
- no “saturation” (Ioffe-Regel-Mott)

Slope of resistivity/layer roughly the same (1.5  $\mu\Omega$  cm/K) for all materials.  
 Sheet resistance =  $\rho/d \sim (h/e^2) T/J$

LSCO: Takagi et al, PRL (1992)

# “Marginal Fermi Liquid” Phenomenology for Strange Metal

C. M. Varma et al, PRL (1989)

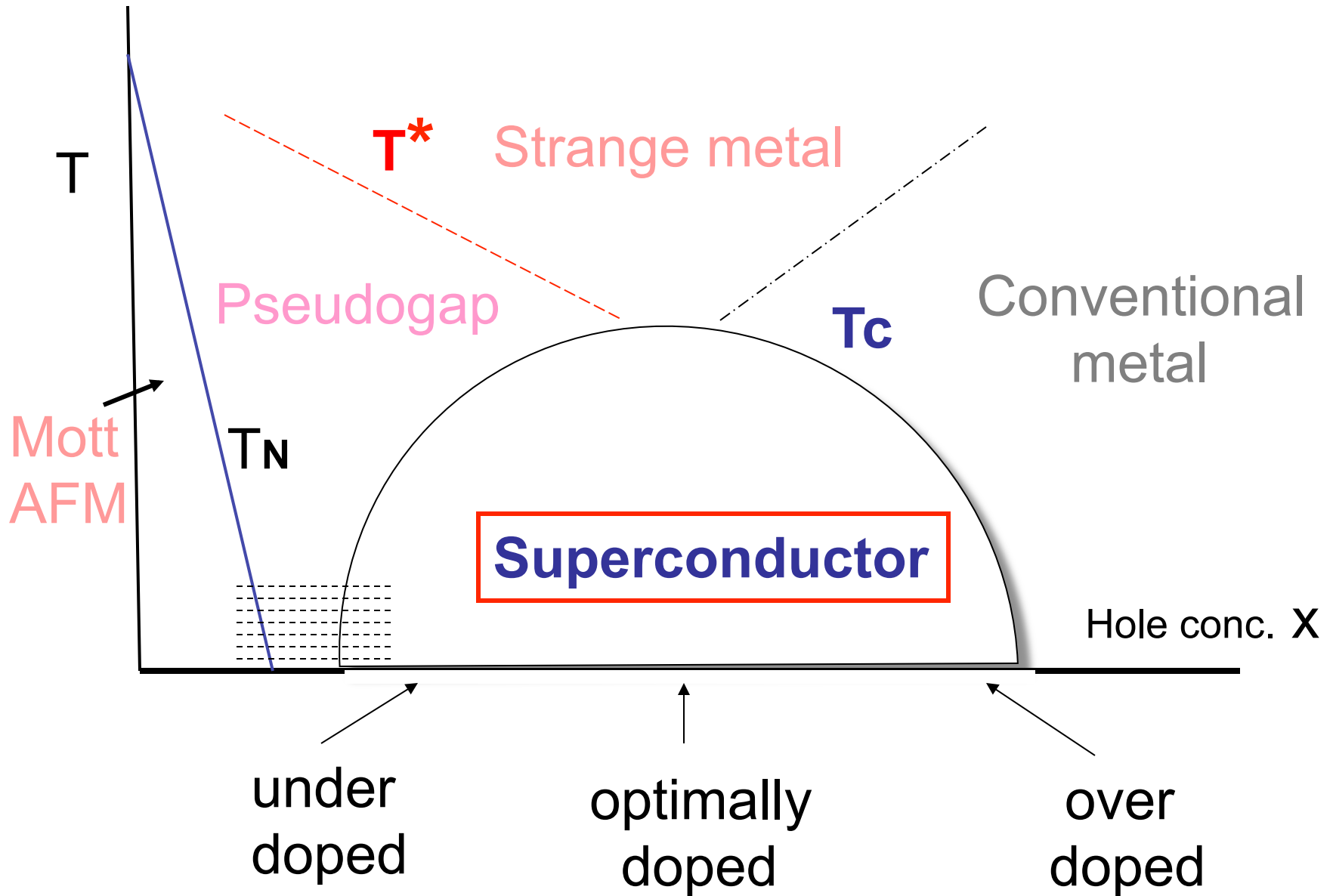
- No energy scale ( $E_f, \Theta_D$ )
- Only energy scale is  $T$
- Both single-particle and transport scattering rates  $1/\tau \simeq \max(\omega, T)$

$$\left. \begin{aligned} \Sigma'' \sim \omega &\Rightarrow \Sigma' \sim \omega \log \omega \\ \Rightarrow Z \sim 1/|\log \omega| &\rightarrow 0 \end{aligned} \right\} \begin{array}{l} \text{Quasiparticles} \\ \text{Not well-defined} \end{array}$$

Open qs:

Microscopic origin?

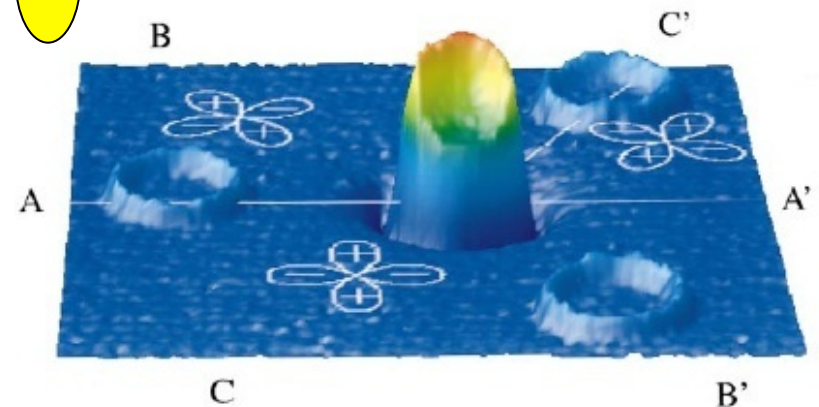
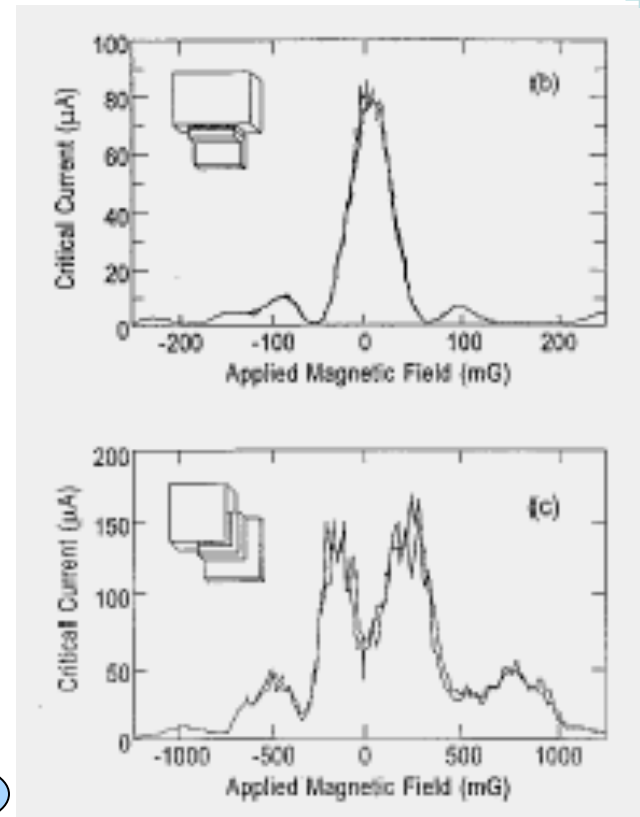
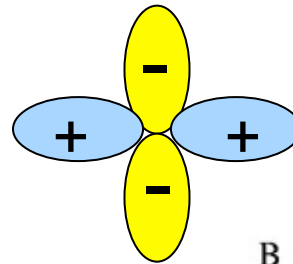
Quantum Critical Point under the SC dome ?



# SC Order Parameter:

- pairing:  
 $(hc/2e)$  flux quantization  
 $2eV/\hbar$  a.c. Josephson
- $S = 0$  singlet pairing  
 NMR  $\chi(T \rightarrow 0) = 0$
- d-wave ( $B_{1g}$ )  
 changes sign under  $\pi/2$  spatial rotation

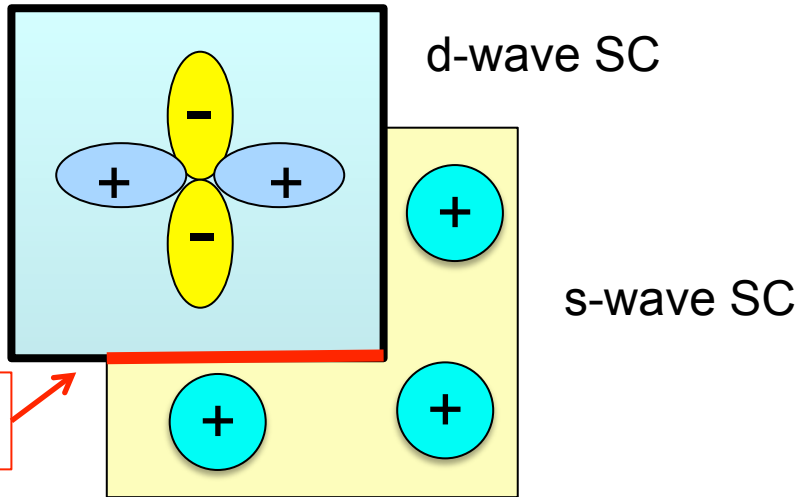
phase sensitive  
 experiments:  
 $\pi$ -junctions



van Harlingen (1993,95); Tsuei & Kirtley (1994,95)

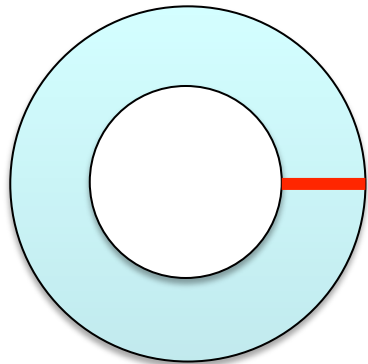
# $d_{x^2-y^2}$ Pairing in High $T_c$ cuprates

Initial skepticism - robustness against disorder - was wrong! Proof from "Josephson interference"



$\pi$ -junction

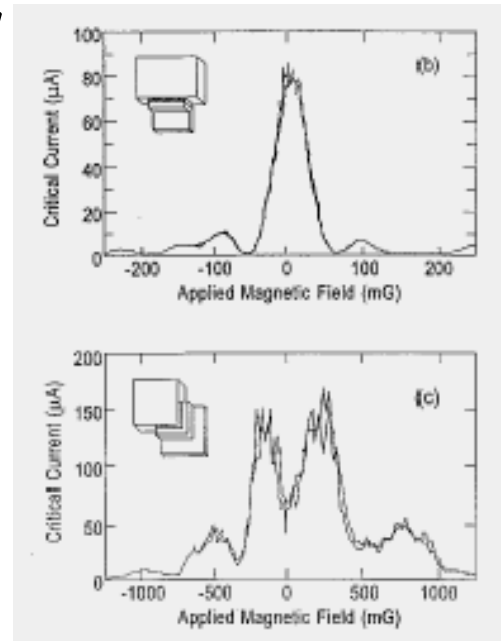
$$I = I_c \sin(\delta\theta + \pi)$$



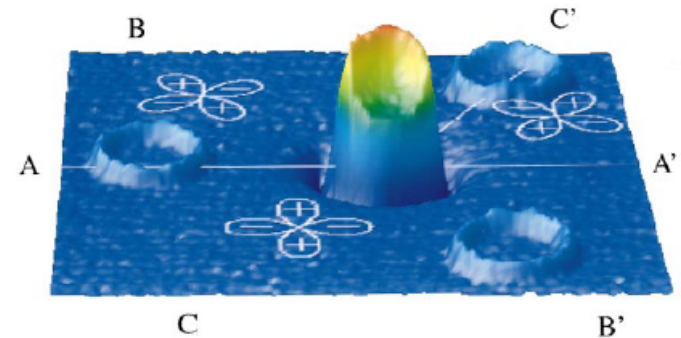
d-wave SC

Frustration across  $\pi$ -junction  $\rightarrow$  half-flux quantum

Sigrist & Rice, RMP 67, 503 (1995)



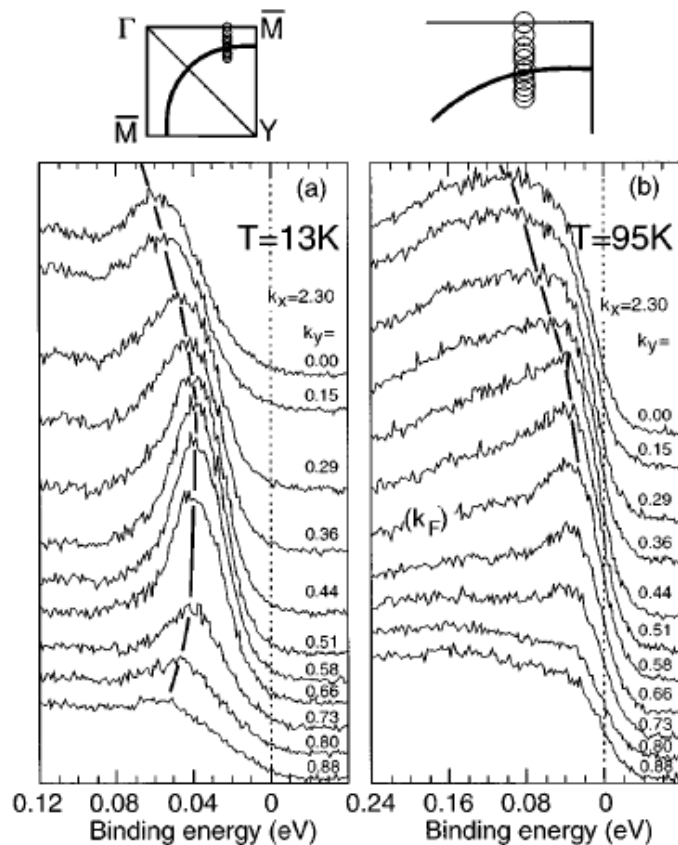
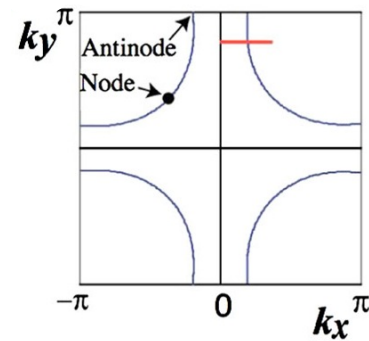
Van Harlingen, RMP 67, 515 (1995)



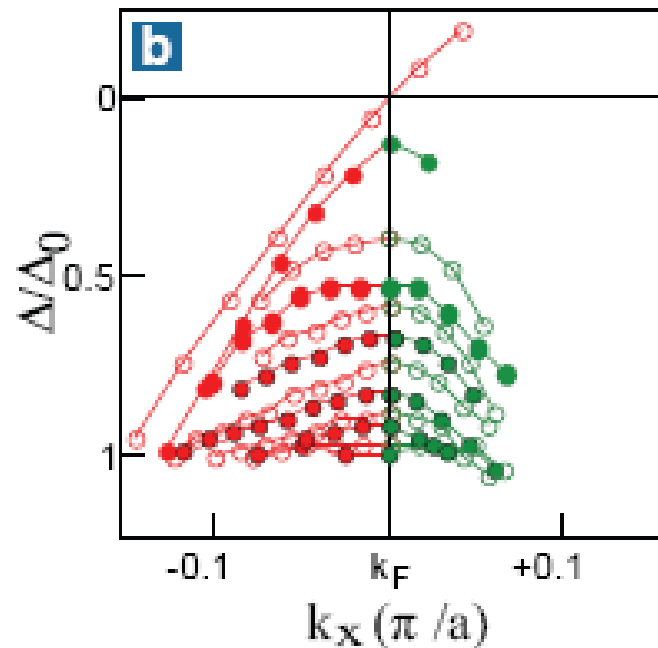
Tsuei & Kirtley, RMP 72, 969 (2000)

# SC gap $\rightarrow$ Change in ARPES dispersion: “p-h mixing”

$$\epsilon_k \rightarrow \sqrt{\epsilon_k^2 + \Delta_k^2}$$



Campuzano et al, PRB (1996)

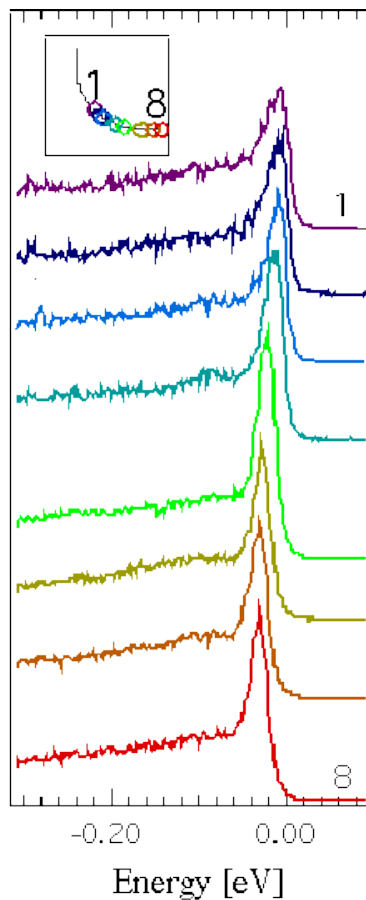
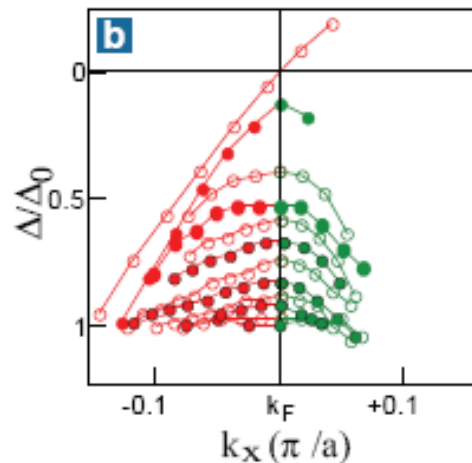


Kanigel et al, PRL (2008)

# SC state gap anisotropy from ARPES

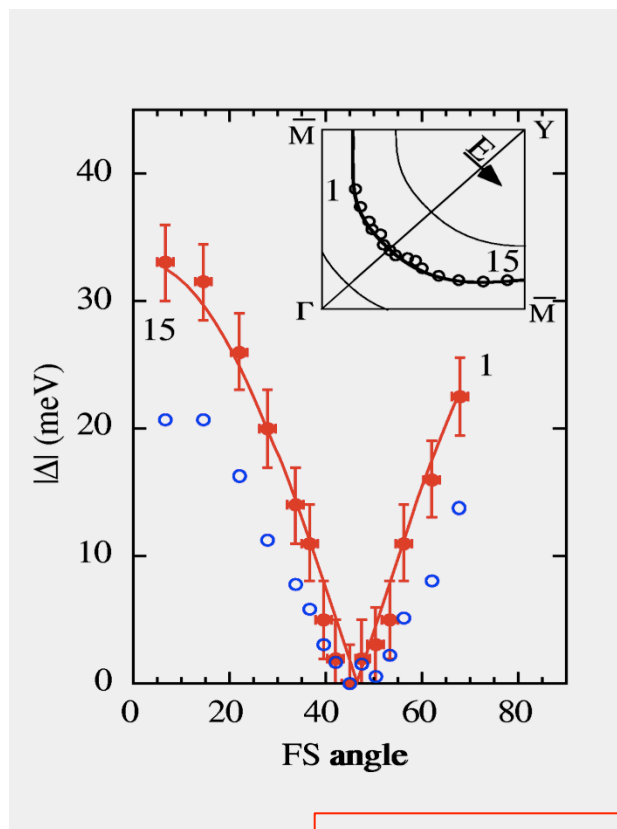
Node/antinode: Shen & Dessau (1993)

Ding et al., PRB (1996)

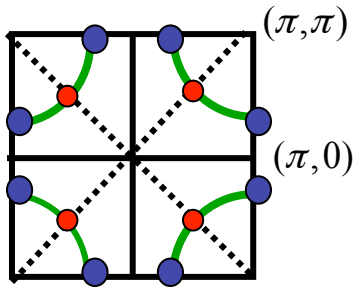


Optimal/Overdoped  
Samples:

$$\Delta(\mathbf{k}) = \Delta_0 |(\cos k_x - \cos k_y)|/2$$



Warning! Can have strong deviations with underdoping!  
→ see lecture#2

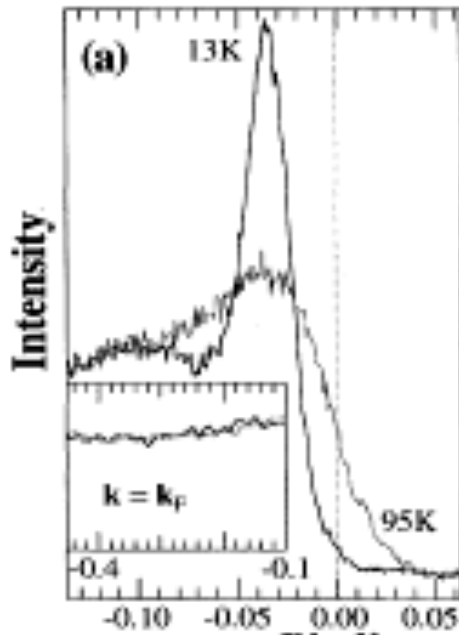


## Sharp Quasiparticles:

- **Absent** above  $T_c$
- **Present** for  $T \ll T_c$

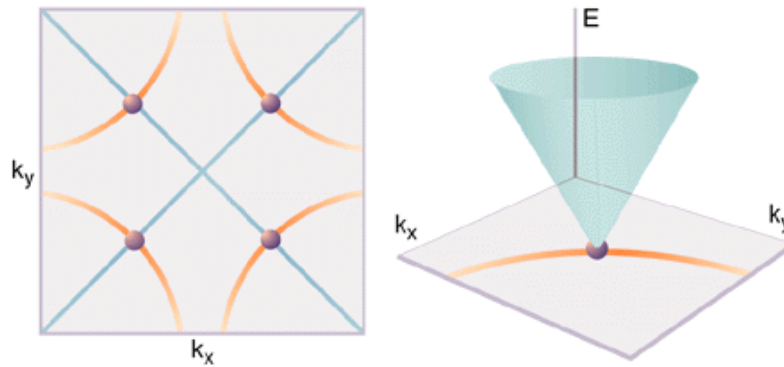
## Dirac cone for d-wave SC Bogoliubov QP excitations

ARPES



Randeria et al  
PRL (1995)

$$E_{\mathbf{k}} \approx \sqrt{(v_F \delta k_{\perp})^2 + (v_{\Delta} \delta k_{\parallel})^2}$$



→ low-energy  
DOS

$$N(\omega) \approx \frac{|\omega|}{\pi v_F v_{\Delta}}$$

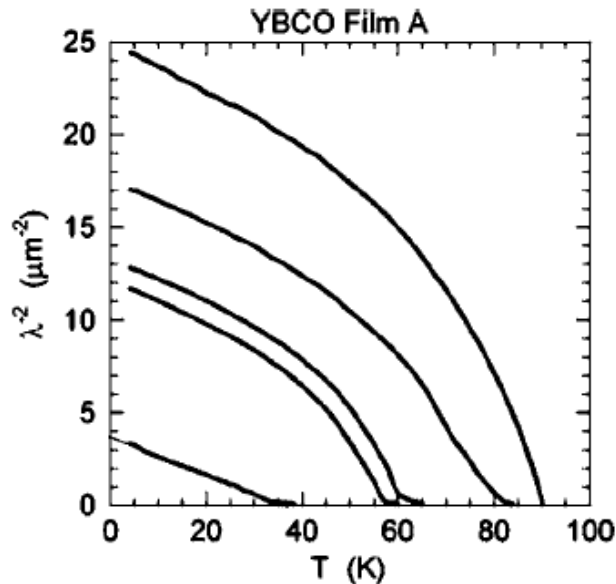
# Nodal QPs & SC state properties

## Linear T sf density

Bonn & Hardy (UBC)

$$\lambda_{\perp}^{-2} = \frac{4\pi n_s^{3d} e^2}{m^* c^2}$$

$$\frac{n_s(T)}{m} = \frac{n_s(0)}{m} - \frac{2 \ln 2}{\pi} \alpha^2 \left( \frac{v_F}{v_{\Delta}} \right) T.$$



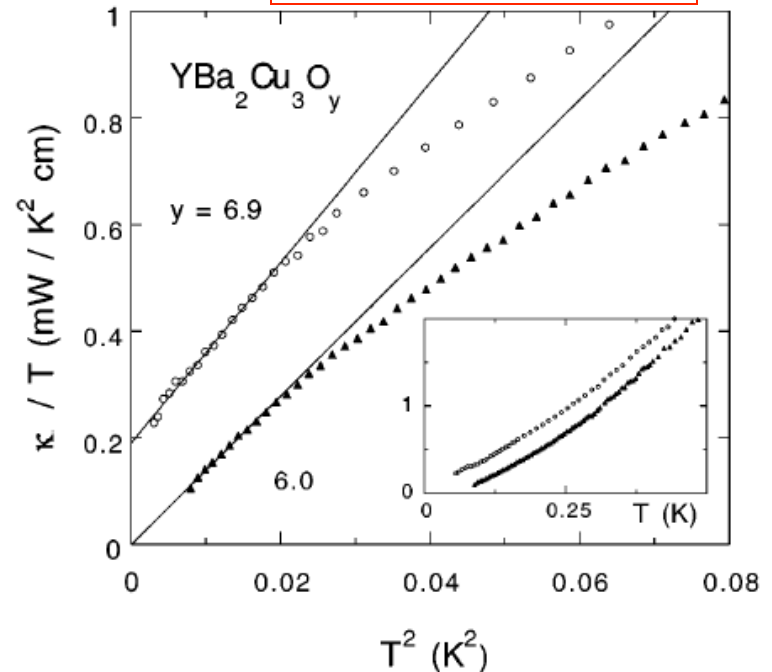
Lemberger group, Physica C 341,561 (2000)

## Universal transport

(independent of impurities)  
thermal conductivity

Durst and Lee PRB (2000)

$$\frac{\kappa}{T} = \frac{k_B^2}{3\hbar} \left( \frac{v_F}{v_{\Delta}} + \frac{v_{\Delta}}{v_F} \right)$$



Taillefer group, PRL 79, 483 (97)

What is the **doping dependence** of Superconducting state parameters as measured by ARPES?

- Gap anisotropy  $\Delta(\mathbf{k})$
- Antinodal gap  $\Delta_{\max}$
- Nodal QP dispersion  $v_F$  &  $v_{\Delta}$

Nontrivial answers as function of underdoping,

Will discuss together with pseudogap physics  
in Lecture #2

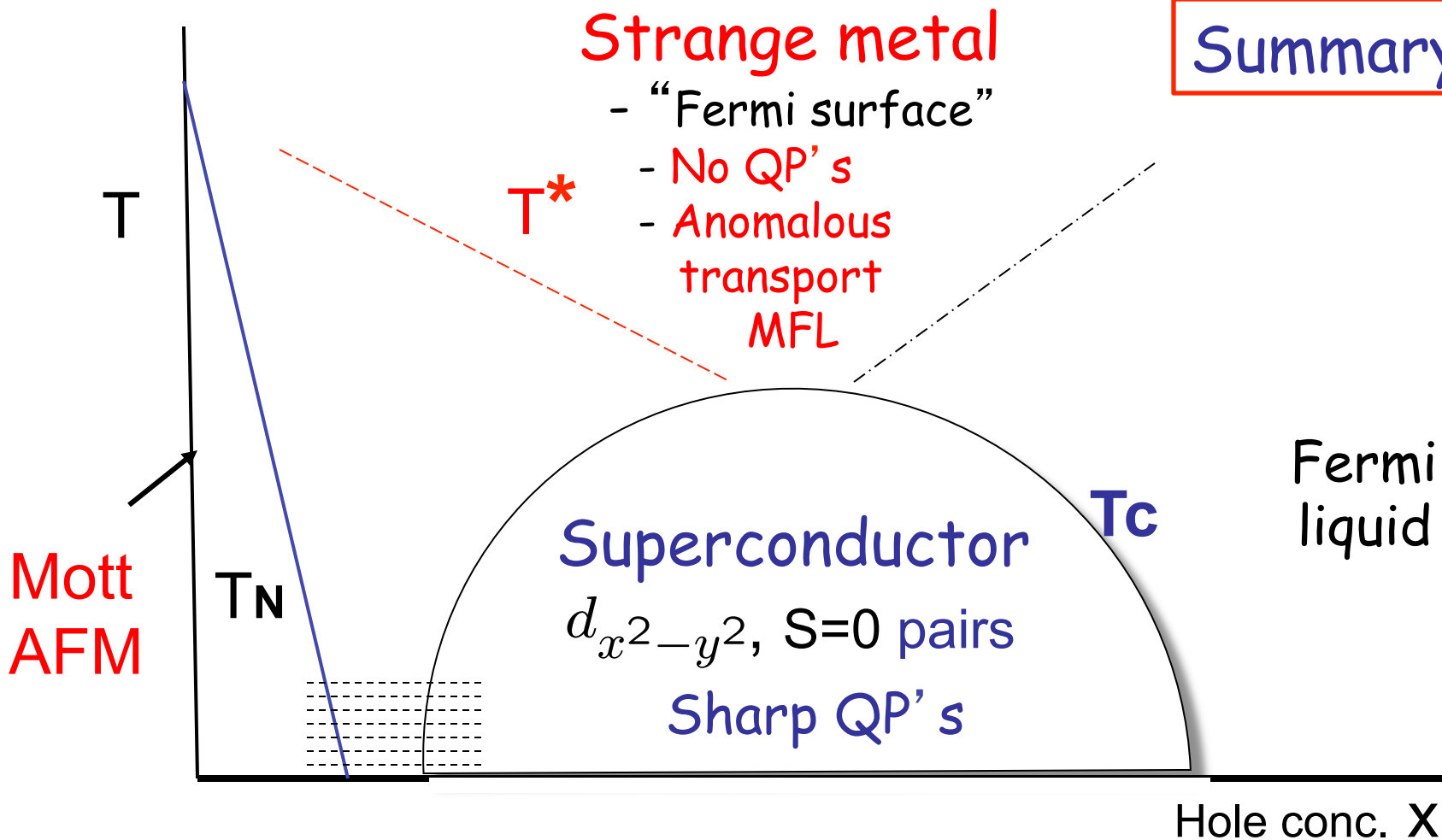
# Phenomenology of High Tc Cuprates I

- Standard theory of metals, insulators & superconductors
- High Tc cuprate materials
- Parent insulators: Mott AFM
- Problem of doped Mott insulator
- ARPES & quantum oscillations
  - Fermi liquid state at overdoping
  - Strange metal at optimal doping
  - d-wave superconductor

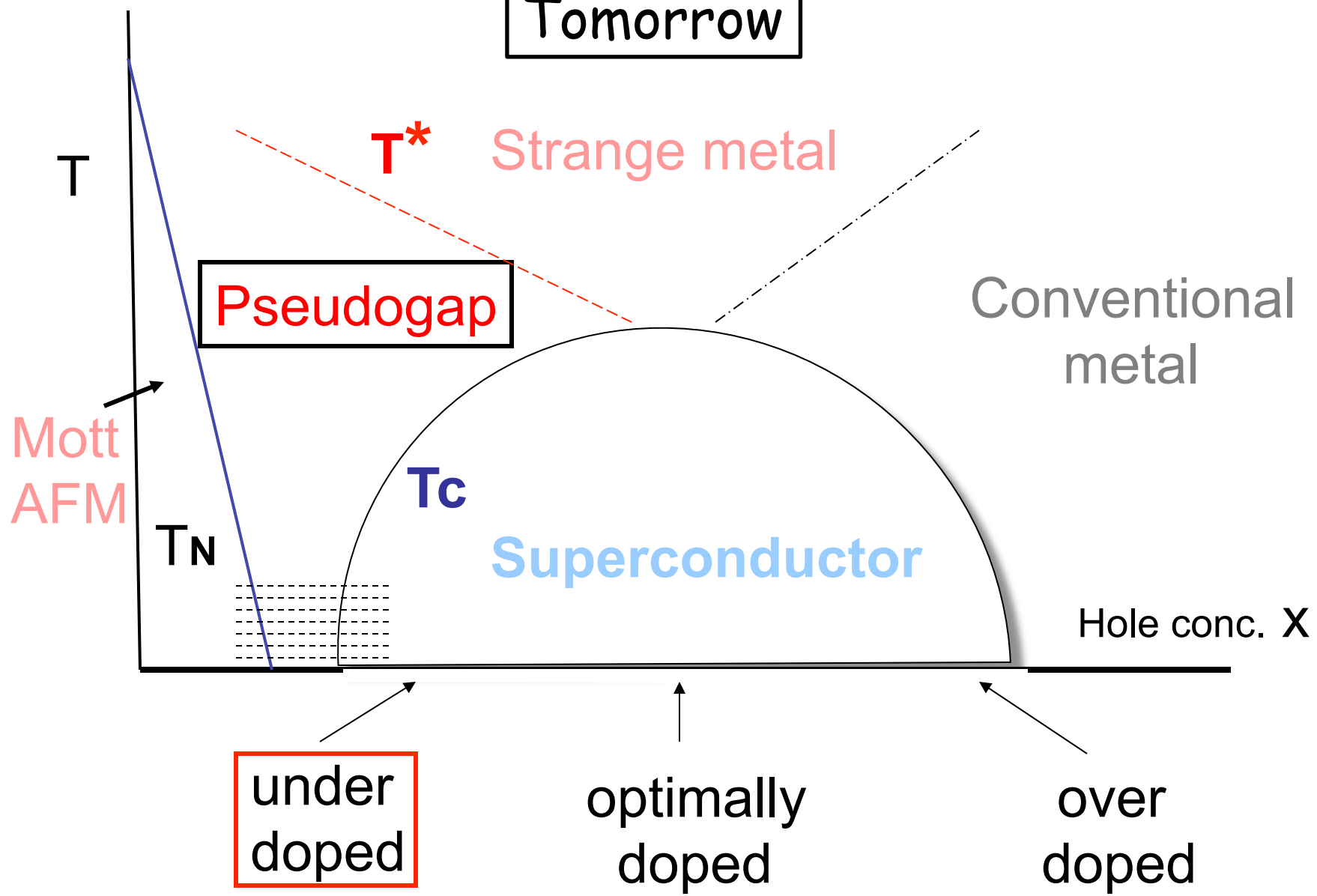
## Phenomenology of High Tc Cuprates II

- underdoped materials
  - pseudogap
  - SC fluctuations
  - competing order parameters
  - high field experiments

Summary



Tomorrow



The end