Optical conductivity: phase fluctuations and vortices

BCS theory accounts for supercurrent, but doesn't explain it. Clearly "phase coherence" and symmetry breaking underlie the phenomena.

Seen clearly in the Anderson pseudospin picture

$$\mathcal{H}_{BCS} = \sum_k \varepsilon_k \hat{n}_k - V \sum_{k,k'} c^\dagger_{k'} c^\dagger_{-k'} c_k c_{-k}$$

To take advantage of interaction we form coherent states

$$|BCS\rangle \text{ contains } \alpha_k |00\rangle_k + \beta_k |11\rangle_{k\uparrow,-k\downarrow}$$

Superposition of states with time reversed pair occupied and unoccupied.

Anderson pseudospin picture

BCS paired state. Consider time-reversed

pair $|k\uparrow, -k\downarrow\rangle$

BCS $\sim \alpha_k |00\rangle + \beta_k |11\rangle$

No probability for $|01\rangle$ or $|10\rangle$

α, β are the same as the u, v discussed previously

Pseudospins Define $|\uparrow\rangle_k = |\underset{k\uparrow}{1}\ \underset{-k\downarrow}{1}\rangle_k$

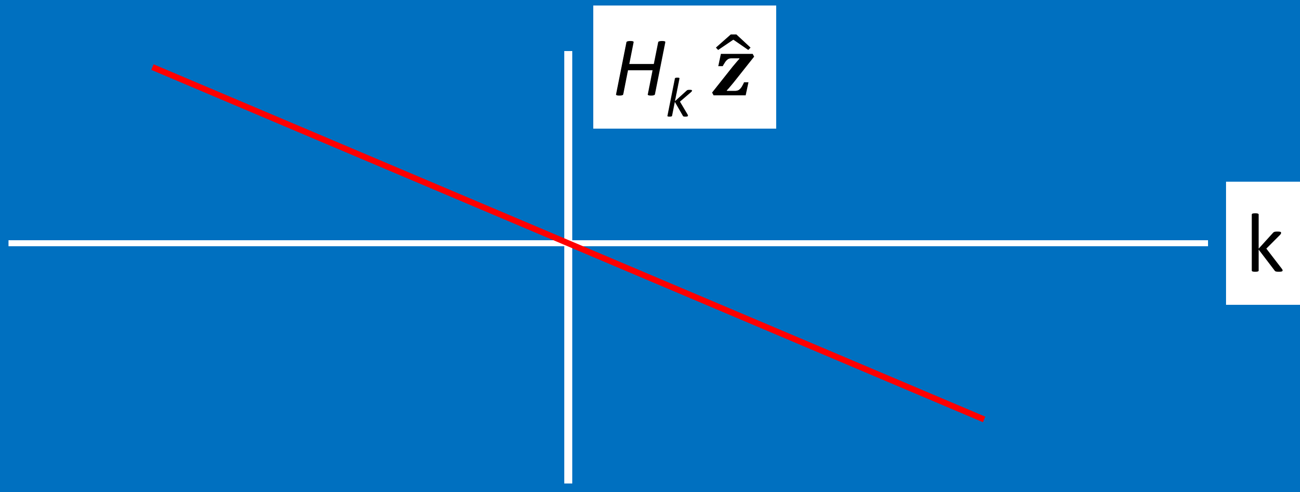
$$|\downarrow\rangle_k = |0\,0\rangle_k$$

We can write the states as spinors

$$\Psi_k = \begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix}$$

Rewriting $\mathcal{H}_{BCS}$ in spin language

$$\sum_k \varepsilon_k \hat{n}_k \rightarrow \sum_k \frac{\epsilon_k}{2} \sigma_z \quad \text{[shifting zero of energy]}$$



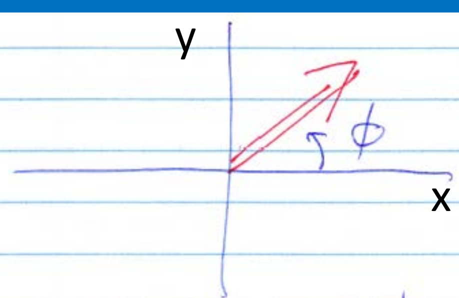
k

Second term scatters one spin state to another

$$-V\sum_{kk'} c^\dagger_{k'}\, c^\dagger_{-k'}\, c_k\, c_{-k}$$

$$\rightarrow \frac{V}{4}\sum_{kk'} \underbrace{\sigma_{xk'}\sigma_{xk} + \sigma_{yk'}\sigma_{yk}}_{\vec{\sigma}_{k'}\cdot\vec{\sigma}_k}$$

for xy components of pseudospin

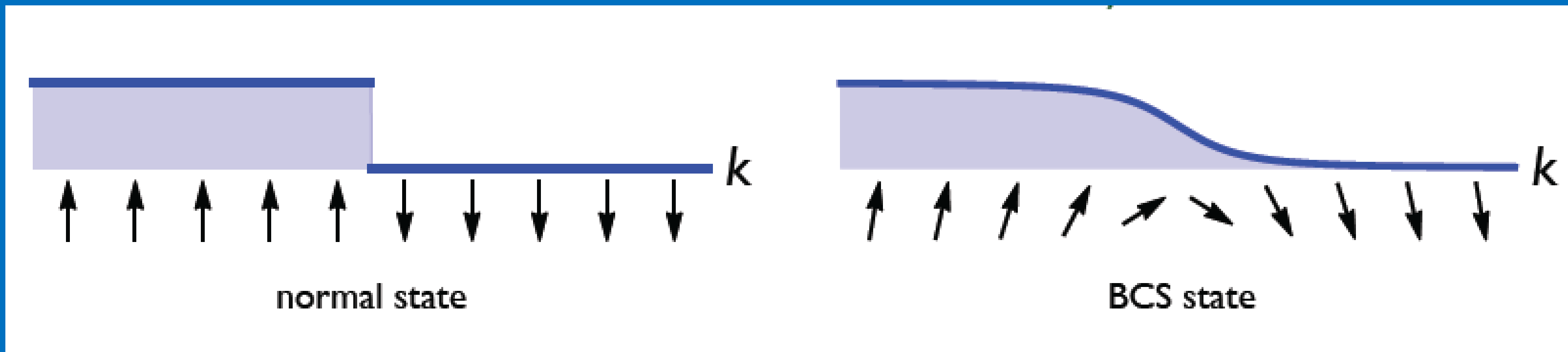


$$\sum_k \langle c_{k\uparrow}\, c_{-k\downarrow}\rangle = \Delta e^{i\phi}$$

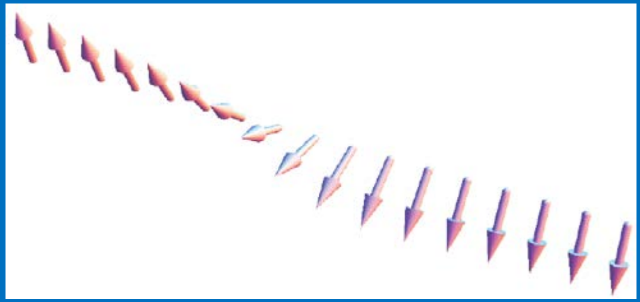
mean field solution

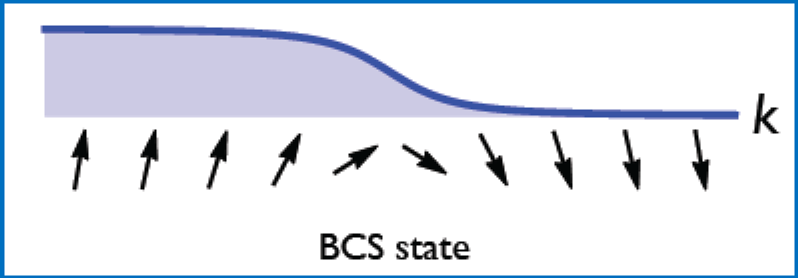
Spins experience a mean field in the xy plane arising from spin-spin interactions – could choose any direction in xy plane: U(1)

Visualizing mean field solution



This is the breaking of U(1) symmetry. The original $\mathcal{H}$ was ~~symmetry~~ symmetric w.r.t ϕ but the mean-field solution breaks this symmetry

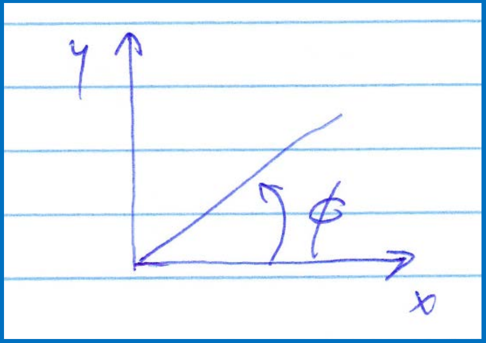




Number and phase are conjugate

Define number operator $\hat{n}_k = 1 + \sigma_{zk}$

Rotation about $\hat{z}$ axis is $e^{i\hat{n}_k \phi_k}$ to within a phase factor. Thus the $\hat{n}$ operator generates rotation in ϕ. Thus $\hat{n}$ and ϕ are conjugate operators



Rotation about z-axis

The energy is minimized when ϕ is constant in space.

Thus the effective Hamilton must contain a term

like $\mathcal{H}_{eff} = \frac{\rho_s}{2}|\nabla\phi|^2$ where ρ_s is the

phase stiffness. In Fourier space $\mathcal{H}_q = \frac{\rho_s}{2}\sum_q q^2\phi_q^2$.

As $\hat{n}_q$ and ϕ_q are conjugate

$$i[\mathcal{H}(\phi_q), \hat{n}_q] = \frac{\partial\mathcal{H}}{\partial\phi_q}\text{; also } \dot{n}_q = \frac{i}{\hbar}[\mathcal{H}(\phi_q), \hat{n}]$$

Josephson relations

Therefore $\frac{\partial n_q}{\partial t} = \frac{i}{\hbar} \frac{\partial \mathcal{H}(\phi_q)}{\partial \phi_q}$

by charge conservation $\frac{\partial n_q}{\partial t} = q J_q$

or $J_q = \rho_s q \phi_q \Rightarrow \boxed{J_s = \rho_s \nabla \phi}$

Also leads to Josephson relation

$$\frac{\partial \phi}{\partial t} = \frac{1}{i\hbar}\frac{\partial H}{\partial n} = \frac{\mu}{i\hbar}$$

Josephson relations: contrast with CDW

Superconductor $J_s \sim \nabla\phi$ $\qquad \dfrac{\partial\phi}{\partial t} \sim \dfrac{\mu}{i\hbar}$

Charge density wave $J_s \sim \dfrac{\partial\phi}{\partial t}$ $\qquad \dfrac{\partial\phi}{\partial x} \sim \rho(x)$

Phase fluctuations and optical conductivity.

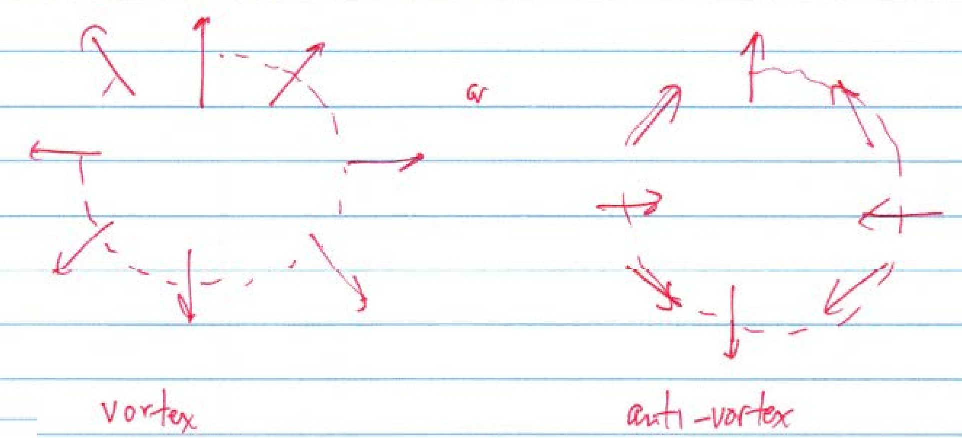
Starting point

$$J_s(r) = \frac{n_s e \hbar}{m} \nabla \varphi(\vec{r}) \quad \text{in static case}$$

where $\nabla \varphi(\vec{r}) = \nabla \theta(\vec{r}) - \frac{e}{\hbar} \vec{A}(\vec{r})$

$$\nabla \times \vec{J}_s = \frac{n_s e \hbar}{m}\left(\nabla \times \nabla\theta(\vec{r}) - \frac{e}{\hbar}\vec{B}\right)$$

which yields a gauge invariant result. For typical scalar field $\nabla \times \nabla\theta(\vec{r}) \equiv 0$. However θ is compact in the sense that $\theta + 2\pi \equiv \theta$. Therefore we can have a topological defect

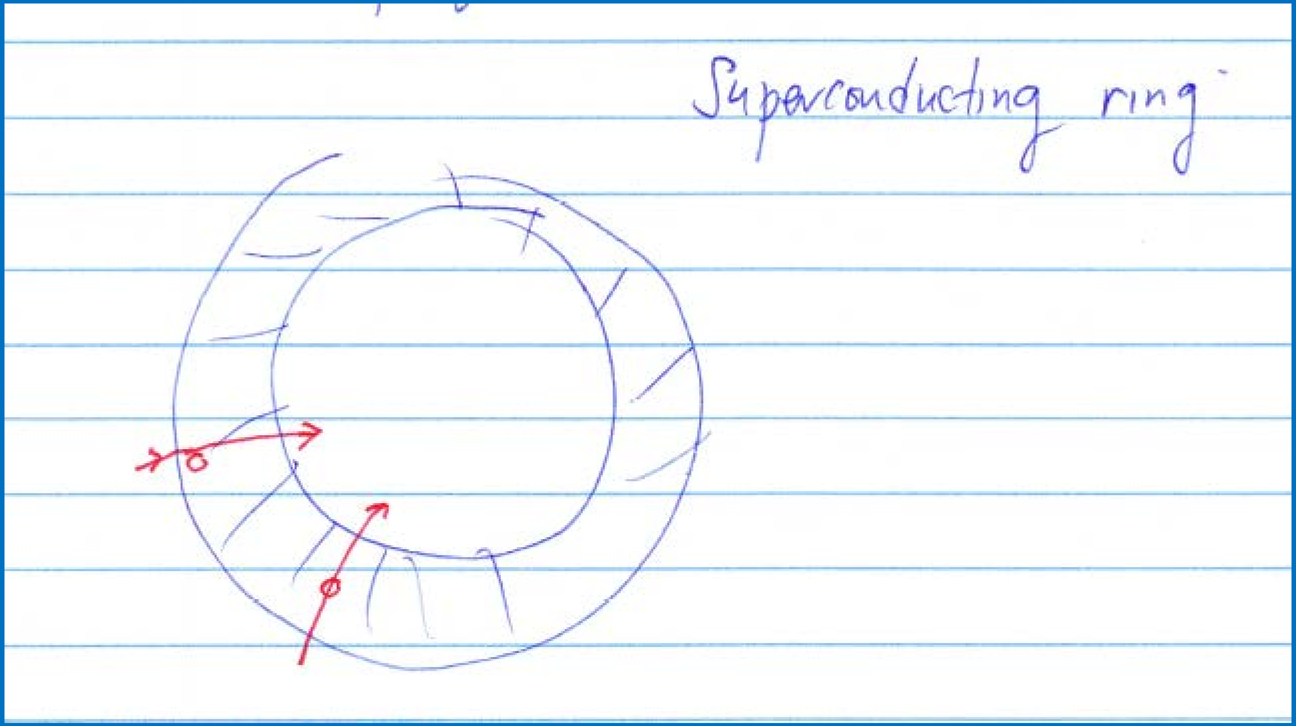


$$\nabla \times \nabla\theta \rightarrow \delta(R)$$

For an ensemble of vortices at position R_i

$$\nabla \times \vec{J}_s = \frac{n_s e \hbar}{m}\left[\sum_i (\pm)\,\delta(\vec{R}_i) - \frac{e}{\hbar}\vec{B}\right]$$

Optical conductivity with vortices



Consider flow of vortices from outside to inside ring.

Each vortex carries a magnetic flux Φ_0

$$\Delta V = -\frac{1}{c}\frac{d\Phi}{dt} = -\frac{\Phi_0}{c}\frac{dN_v}{dt}$$

$$\frac{dN_v}{dt} = n_v v_D \cdot 2\pi R$$

N_v is number that enter ring

n_v is areal density

v_D is ~~dif~~ drift velocity

$$\boxed{\vec{E} = -n_v \vec{v}_D \frac{\Phi_0}{c} \times \hat{z}}$$

Φ_0 is the superconducting flux quantum

Motion of vortices leads to a non-vanishing E-field

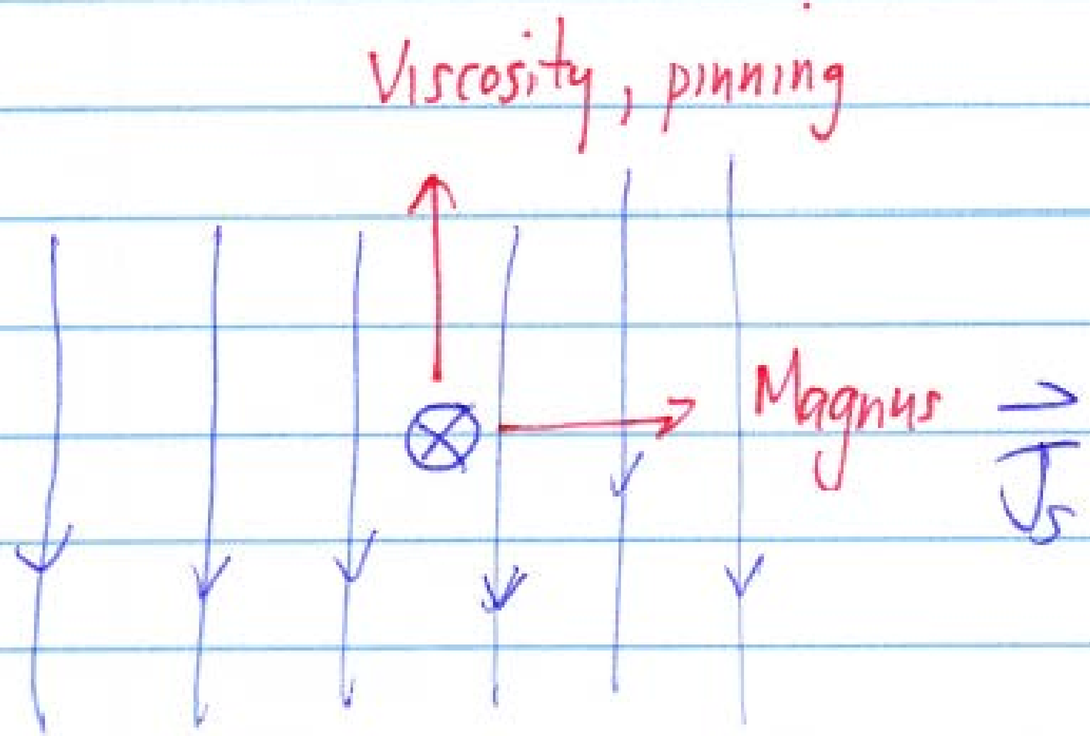
Phenomenological of vortex dynamics in the presence of superfluid.

Forces on vortex

viscous damping $\vec{f}_\eta = -\eta \vec{v}_D$

pinning $\vec{f}_p = -K(\vec{r} - \vec{r}_{pin})$

Magnus force $\frac{n_s \hbar}{2}(\vec{v}_s - \vec{v}_D) \times \hat{z}$ depends on relative motion



Assume $J_s = J_s e^{-i\omega t}$. In steady state $\sum \vec{f} = 0$

$$\boxed{v_{Dy} \simeq \frac{J_s \hbar / e}{\eta + K/i\omega}}$$

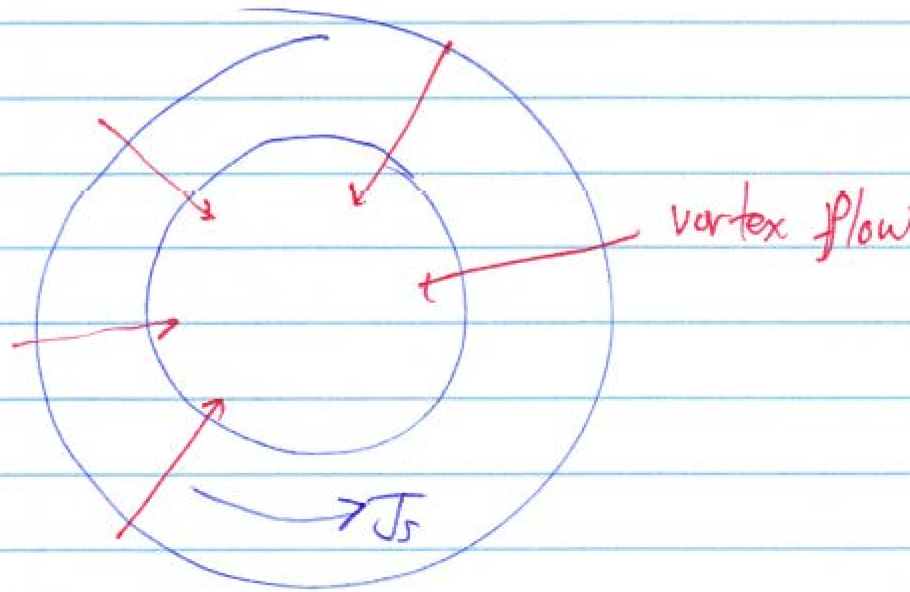
Motion to perpendicular to J_s

$$\boxed{v_{Dx} \simeq \frac{- J_s \frac{\hbar}{e}\left(\frac{n_s \hbar}{2}\right)}{(\eta + K/i\omega)^2}}$$

Motion parallel to J_s

Because of Magnus force v_D is $\perp$ to J_s

and yet its magnitude is governed by K, η



Recall $\vec{E} = - n_V \vec{v}_D \frac{\Phi_0}{c} \times \hat{z}$

We can write as flux flow resistivity

$$\rho_{ff} = \frac{n_v (\Phi_0)^2}{\cancel{\pi}\ \eta + k/i\omega}$$

Note that this looks like the conductivity of electrons in Drude model

particle-vortex duality

in that $\rho_v \sim n_v (\Phi_0)^2$

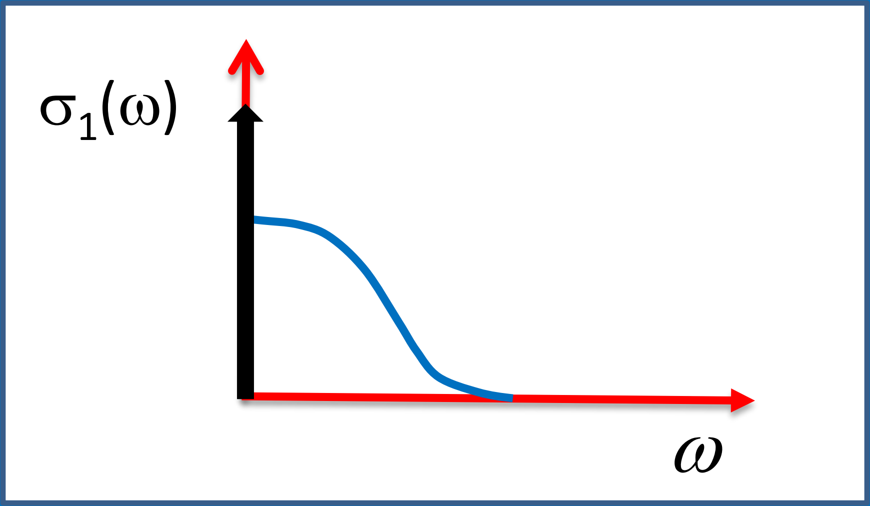
$\sigma_e \sim n_e e^2$

Predictions.

① For pinned vortices $K \neq 0$

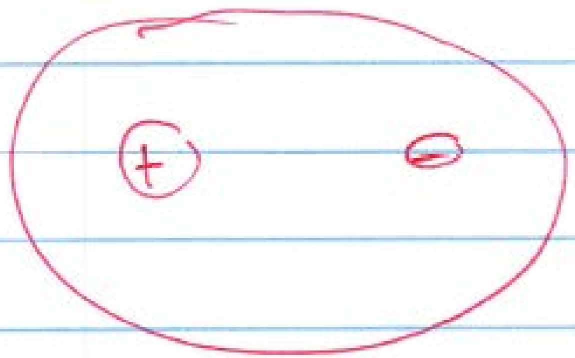
$$\rho_{ff} \to 0 \quad \text{as } \omega \to 0$$

Therefore pinned vortices do not destroy the superconducting state.



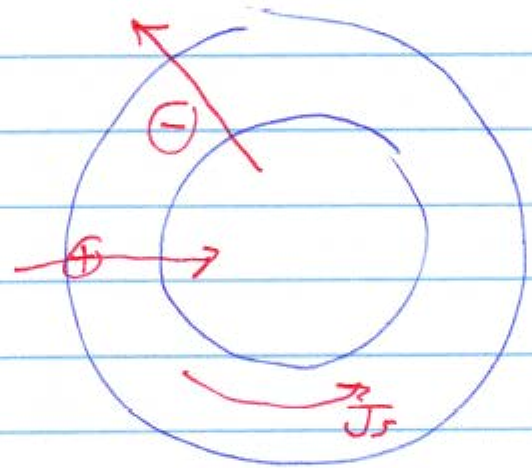
② Bound vortex-anti vortex pairs have no net vorticity. Do not experience Magnus force and will not drift under influence of J_s.

They do not destroy the SC state



Bound vortex-anti vortex pair.

(3) Unbound, free vortices do destroy zero resistance state



Same net change in flux inside ring.

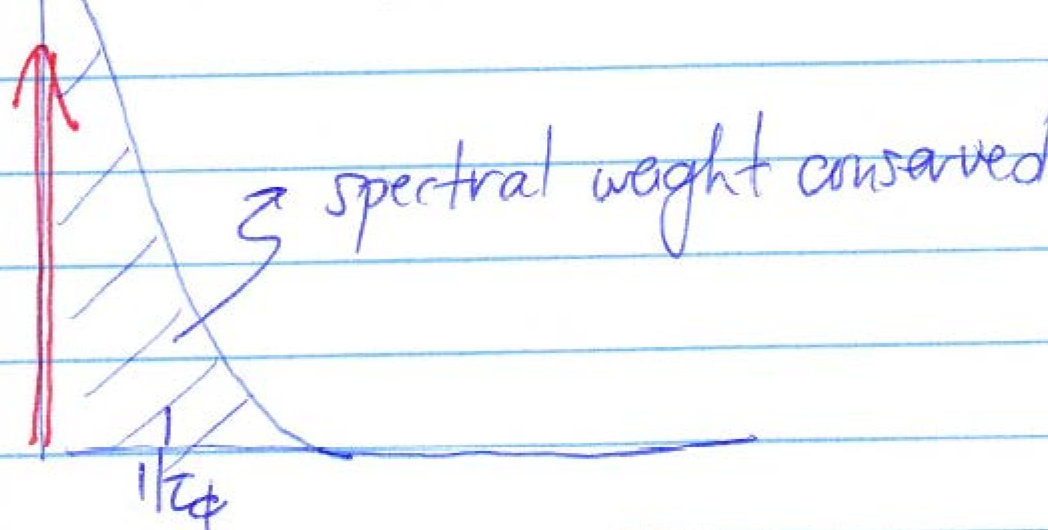
For free vortices we find

$$\rho_{ff} = \frac{n_v \Phi_0^2}{\pi \eta}$$

or $$\sigma_{ff} = \frac{\pi \eta}{n_v \Phi_0^2}$$

This leads to a "pseudo-δ function" for vortex plasma

$$\sigma_{ff} = \frac{\pi\eta}{n_v \Phi_0^2}$$



To find the width which is phase correlation time

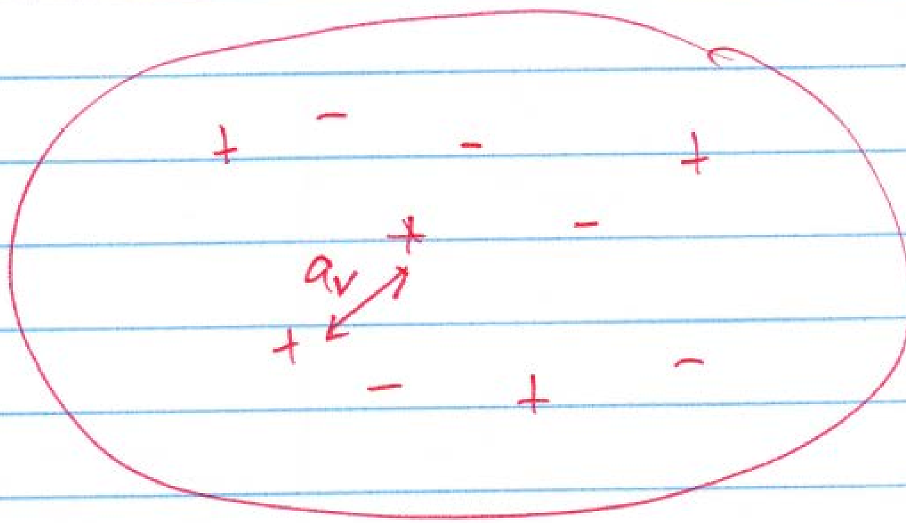
Use conservation of spectral weight

$$\sigma_{ff} \cdot \frac{1}{\tau_\phi} \simeq \frac{n_s e^2}{m}$$

$$\frac{1}{\tau_\phi} \simeq \pi n_v \left(\frac{\rho_s}{kT} \right) D$$

Note that $\rho_s \simeq k_B T$ near vortex unbinding transition

τ_ϕ is the time required ~~to~~ for vortices to diffuse the intervortex separation



$$\frac{1}{\tau_\phi} \simeq n_v D$$

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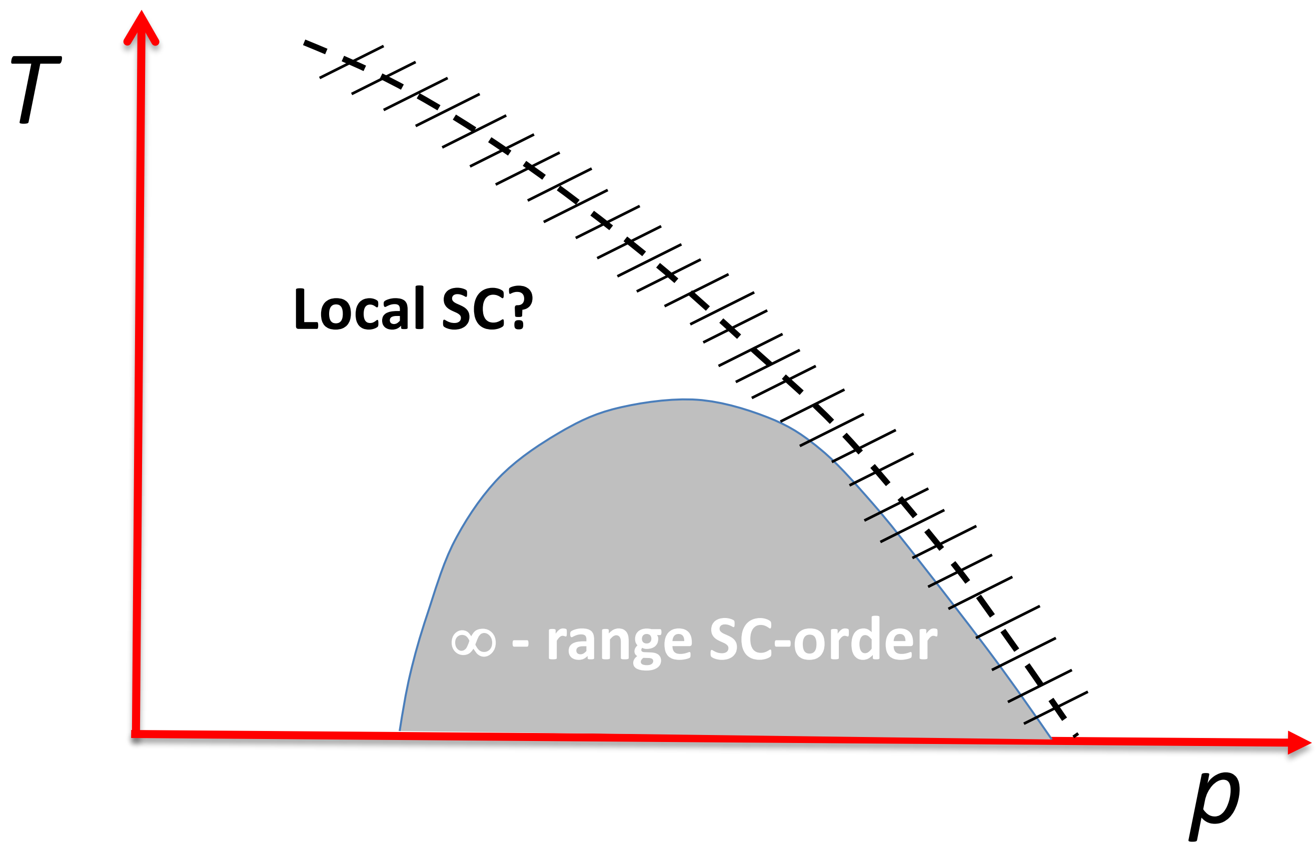
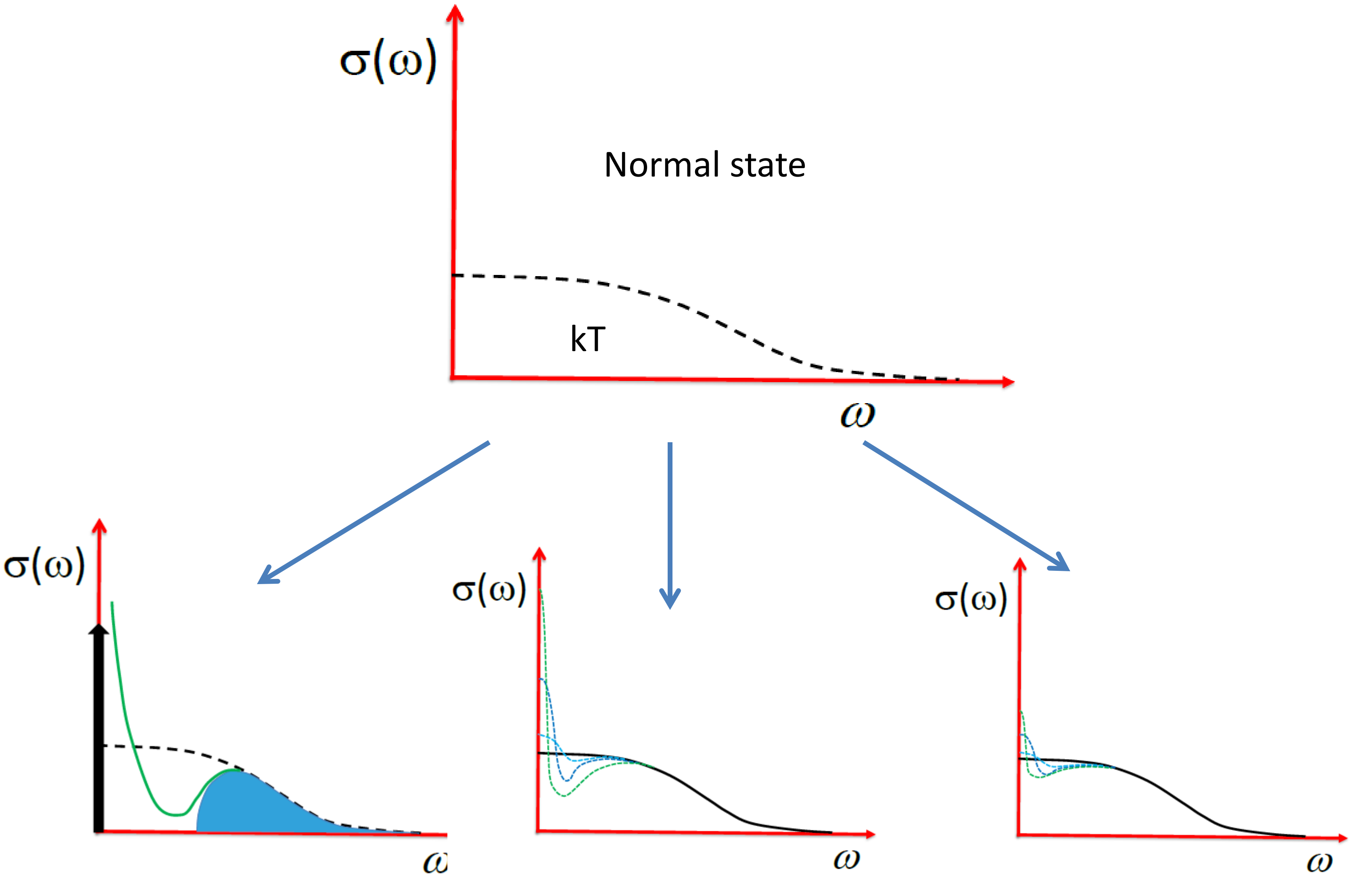
Importance of phase fluctuations in superconductors with small superfluid density

V. J. Emery* & S. A. Kivelson†



Overview of optical conductivity in a superconductor

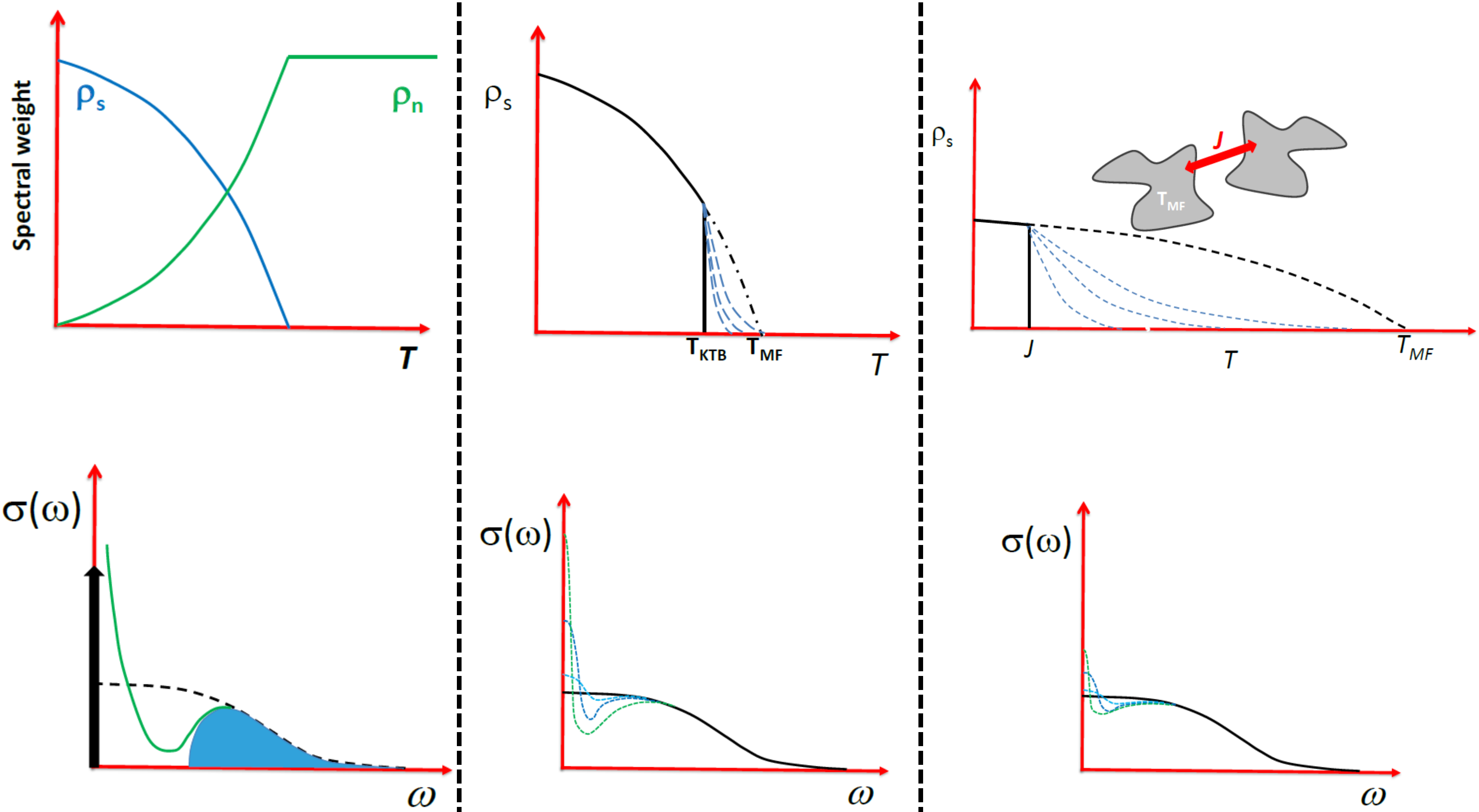


Mean-field theory (Mattis-Bardeen)

Fluctuations/homogeneous (KTB); Halperin-Nelson

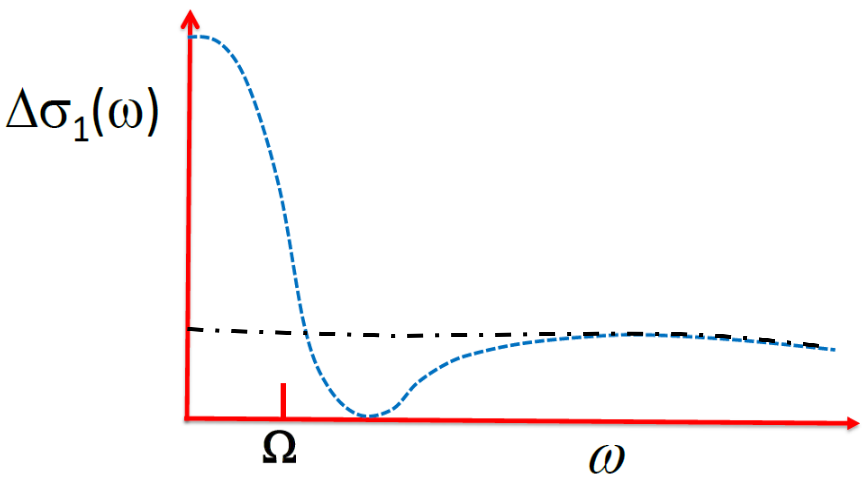
Fluctuations/"granular"

Mean-field theory (Mattis-Bardeen) | Fluctuations/homogeneous (KTB); Halperin-Nelson | Fluctuations/"granular"



Terahertz spectroscopy probes pseudo-δ conductivity

Pseudo-δ conductivity



Bare superfluid density

$$\rho_{s0}(T) \equiv \int \Delta\sigma_1(\omega)d\omega = \lim_{\omega \gg \Omega} \omega\sigma_2(\omega)$$

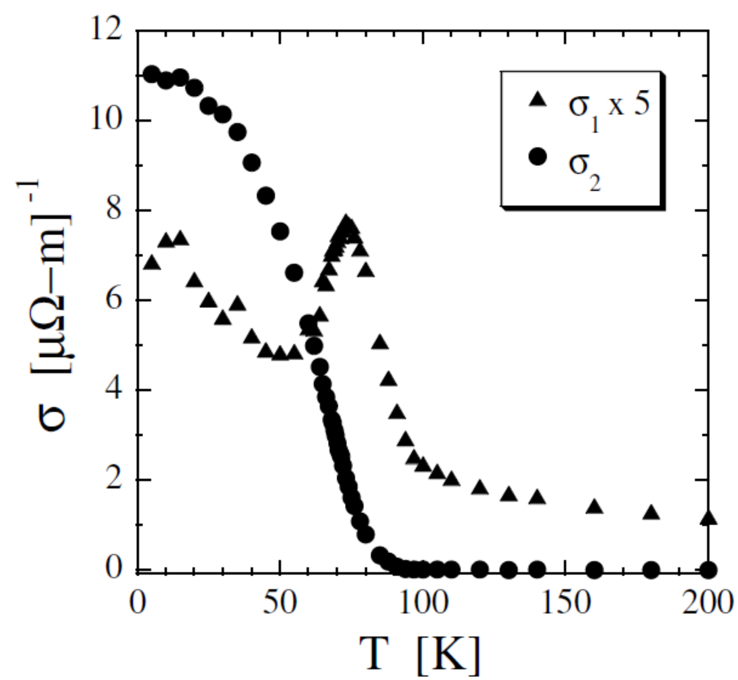
$$\chi \sim \rho_{s0}\xi^2$$

Renormalized superfluid density

$$\rho_s(\omega, T) = \omega\sigma_2(\omega, T)$$

Phase fluctuation frequency $\Omega(T)$

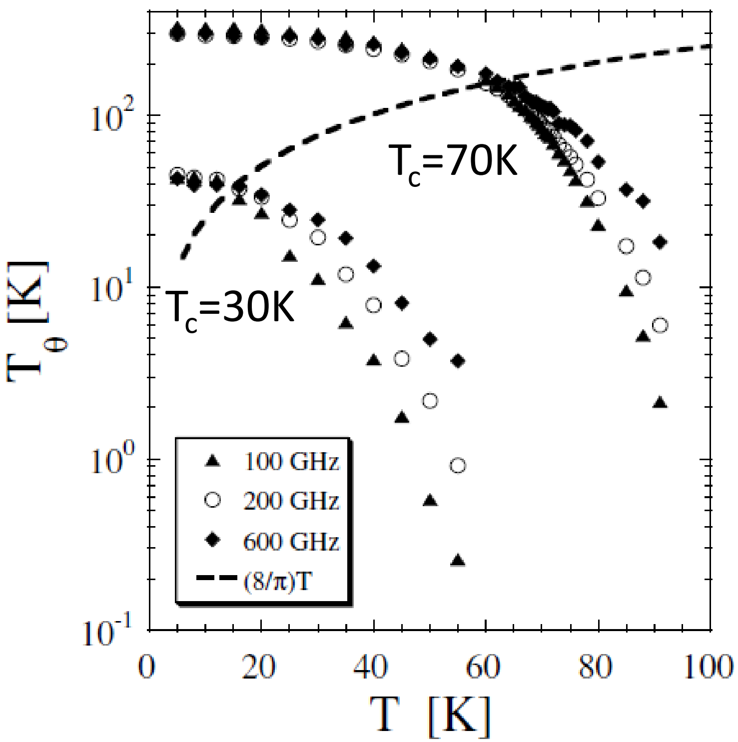
THz conductivity measured on BSCCO T_c = 70 K
Frequency is 100 GHz = Δ/100



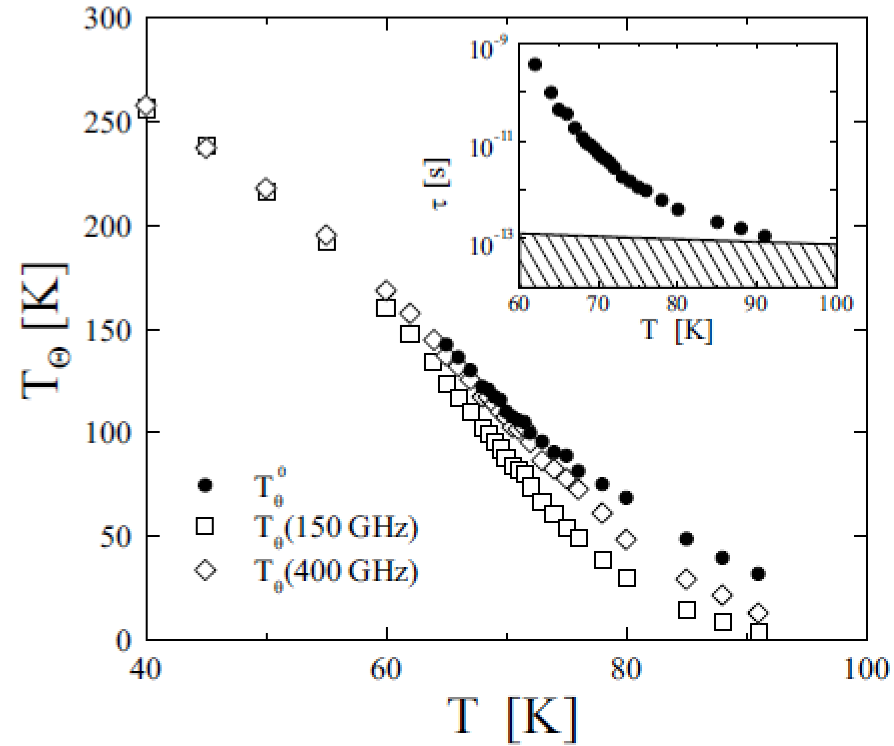
Can see renormalized superfluid density and fluctuation frequency directly in the data

Superfluid density becomes frequency dependent when it decreases it equals T

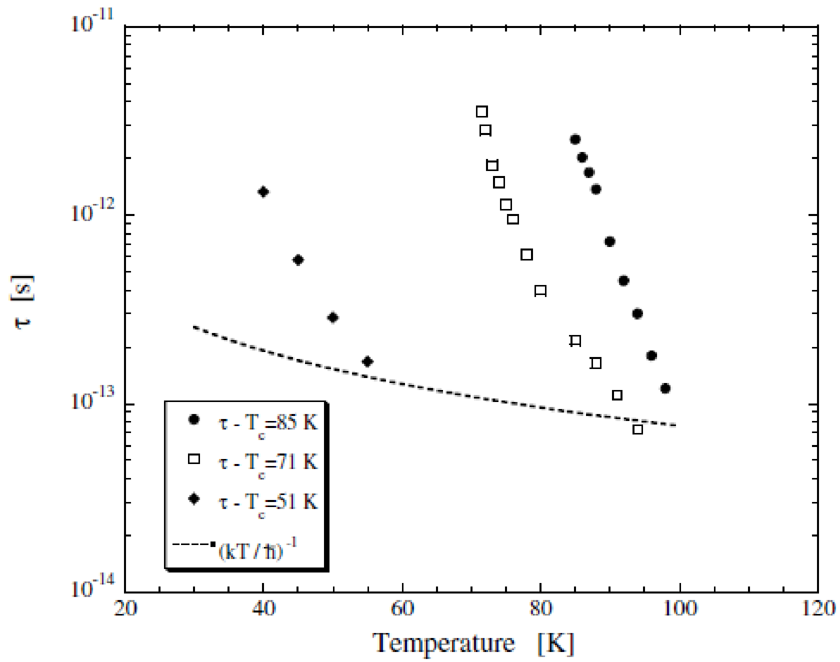
Relevance of KTB phase fluctuations



Phase fluctuation time reaches inverse of k_BT at 15 K above T_c

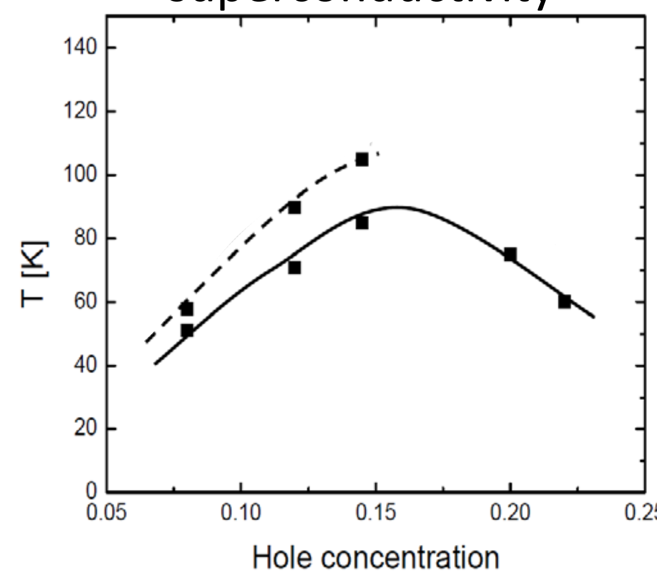


Phase fluctuation time for underdoped samples



T_{MF} tracks T_c

Single energy scale describes onset of superconductivity



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The materials under consideration are rather complex, and there are many subtleties involved