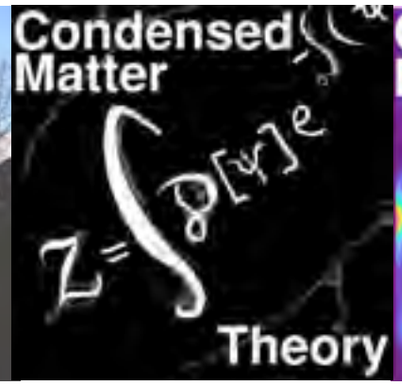
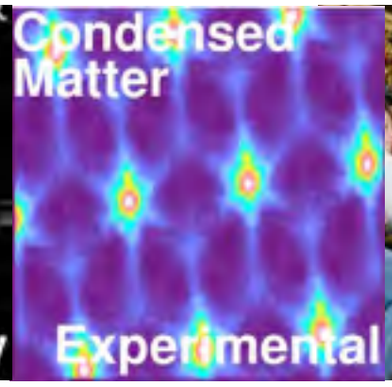
The Physics of Heavy Fermion Superconductivity

Lecture IV. Composite Pairing Hypothesis. Hastatic Order.





Piers Coleman Center for Materials Theory, Rutgers.

Boulder School 2014: Modern Aspects of Superconductivity

June 30-July 25, 2014


14-17 July 2014

Royal Holloway University of London

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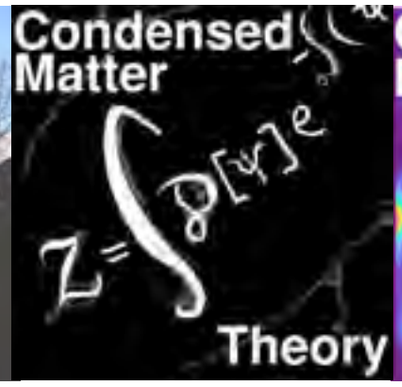

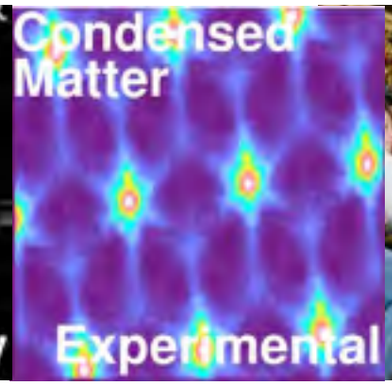

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A solvable model of composite pairing.

PC, Tsvelik, Kee, Andrei PRB 60, 3605 (1999).

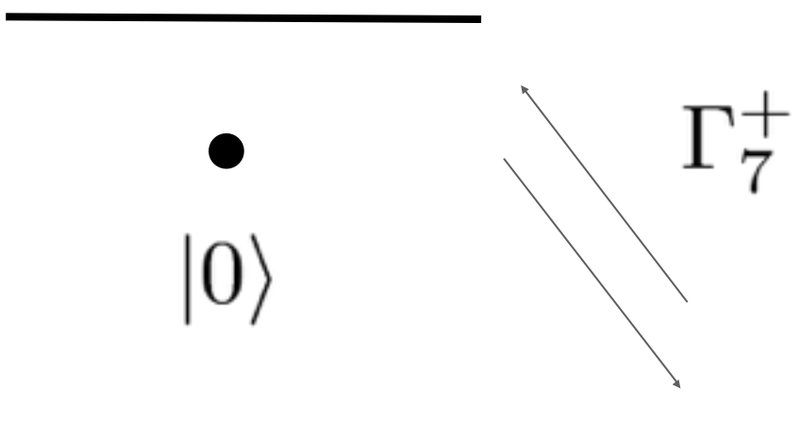
Flint, Dzero, PC, Nature Physics 4, 643 (2008).

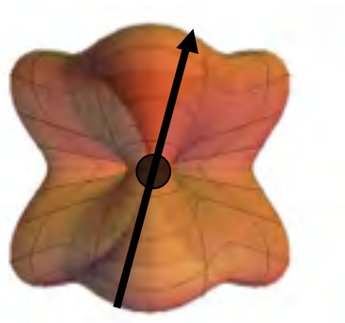
Flint, PC, PRL, 105, 246404 (2010).

Flint, Nevidomskyy, PC, PRB 84, 064514 (2011).

The Two Channel Kondo Model

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi_{1j\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_j$$





$|\Gamma_7^+\rangle$

Wannier functions at site j:

$$\psi_{\Gamma j}^\dagger = \sum_k \Phi_{\Gamma k} \mathrm{e}^{i\vec{k}\cdot\vec{R}_j} c_k$$

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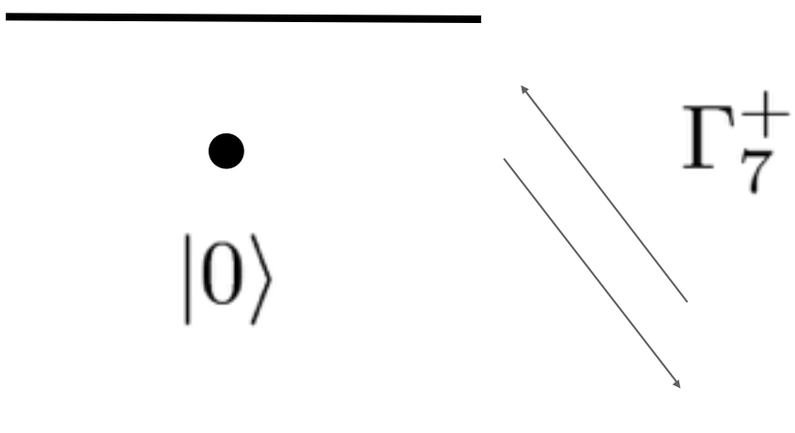

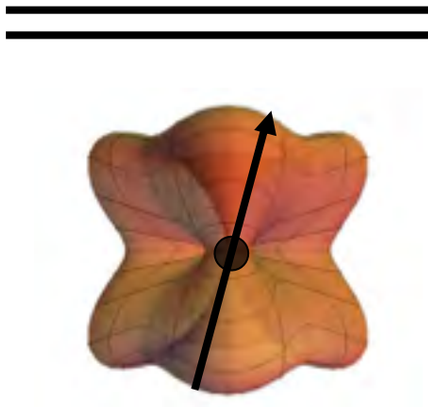

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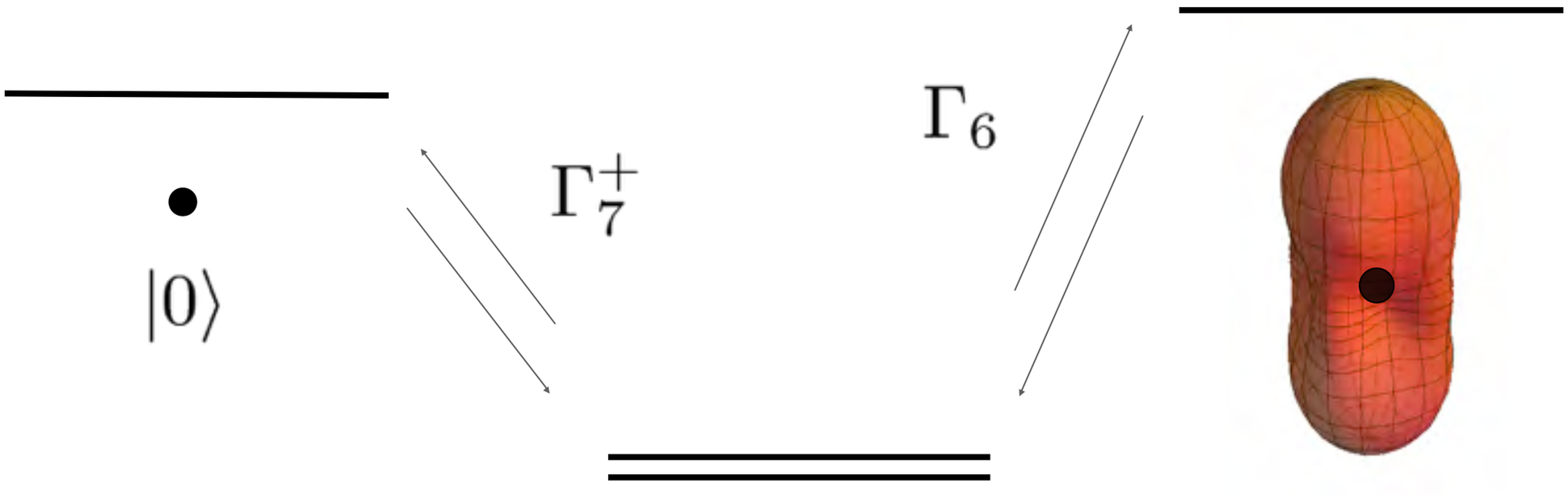

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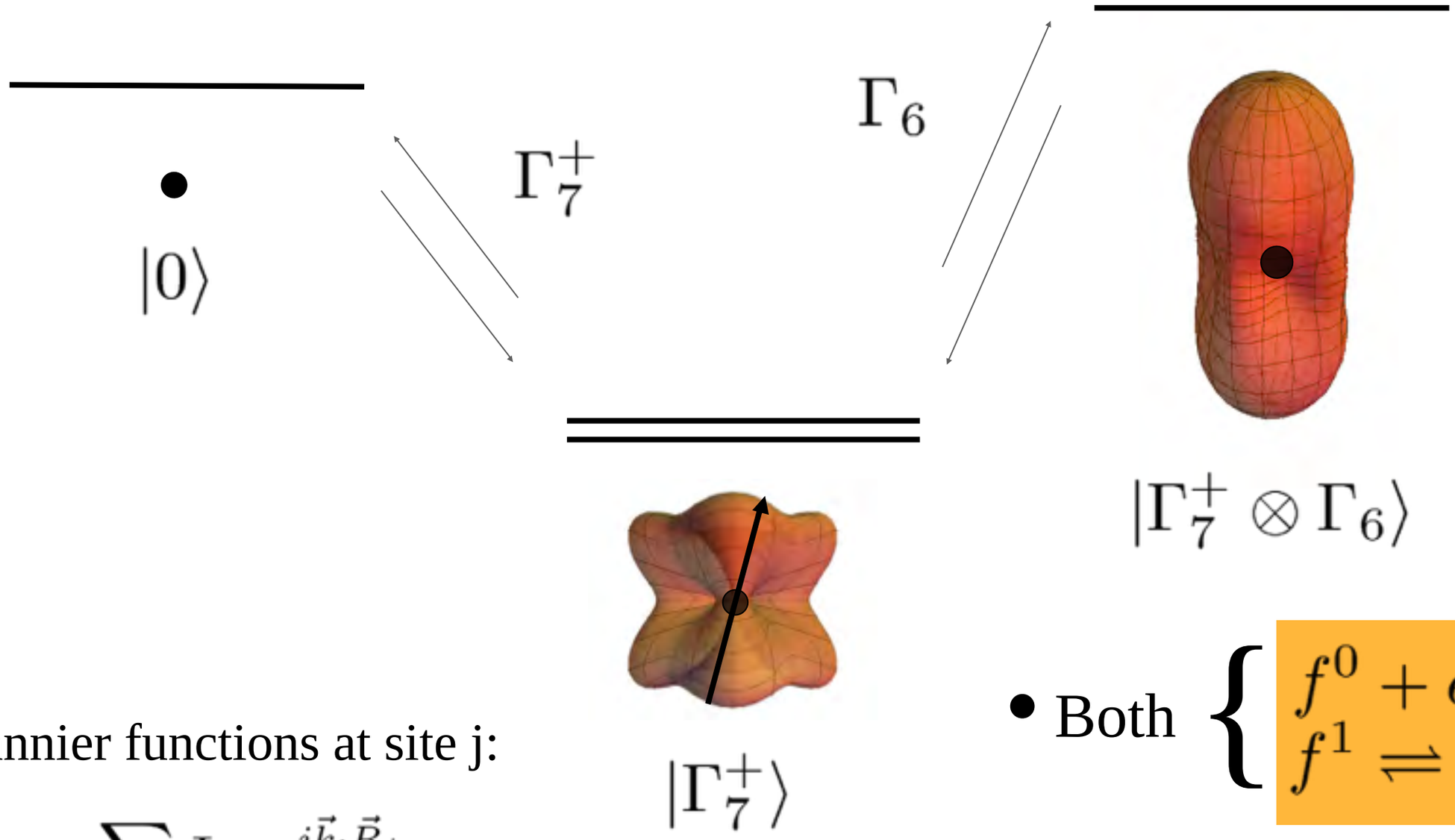

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cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Single FS, two channels.

$$\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma\mathbf{k}} \;\; c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_j}$$

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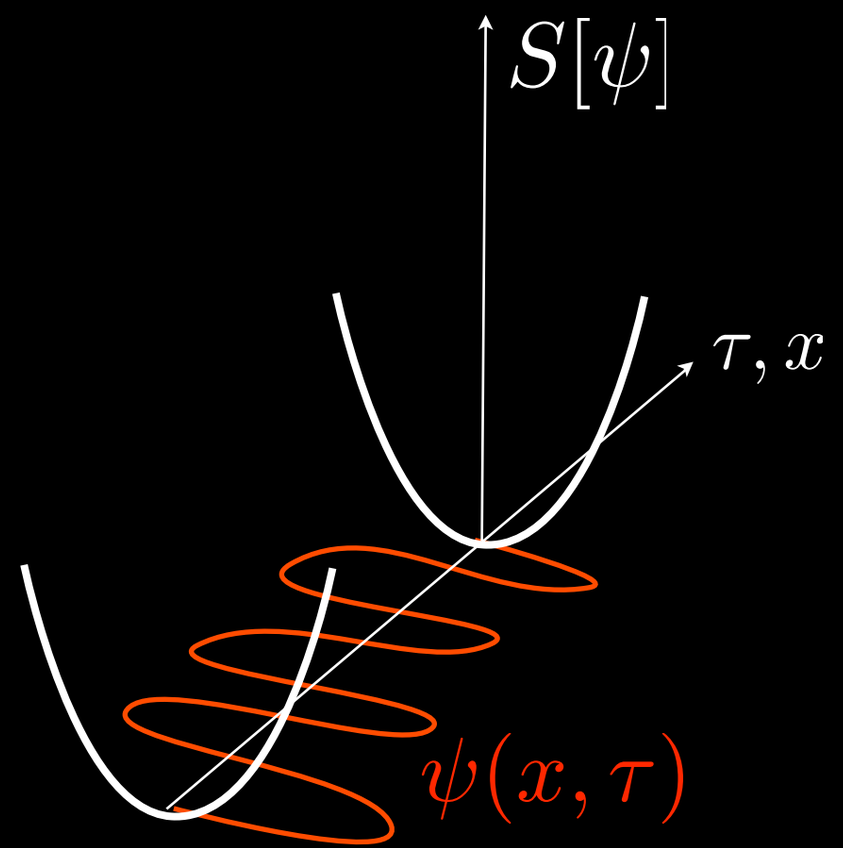

Wild quantum fluctuations!

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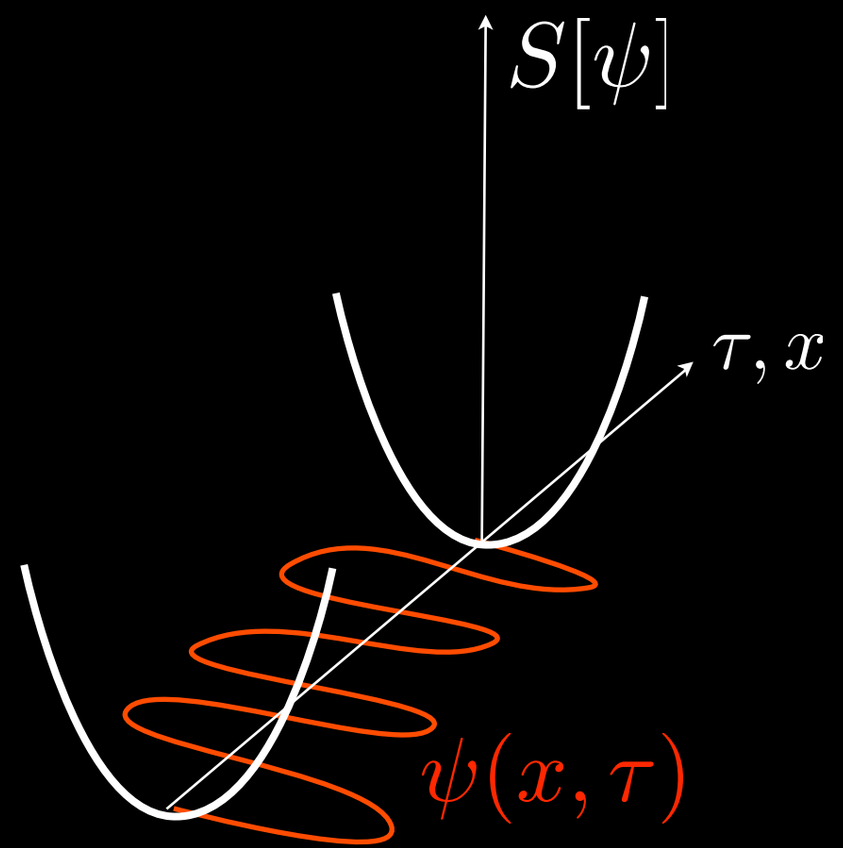

So how can we solve this model?

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Method: large N

Flint, Dzero Coleman Nature Physics '08
PRB 79, 014424(2009)

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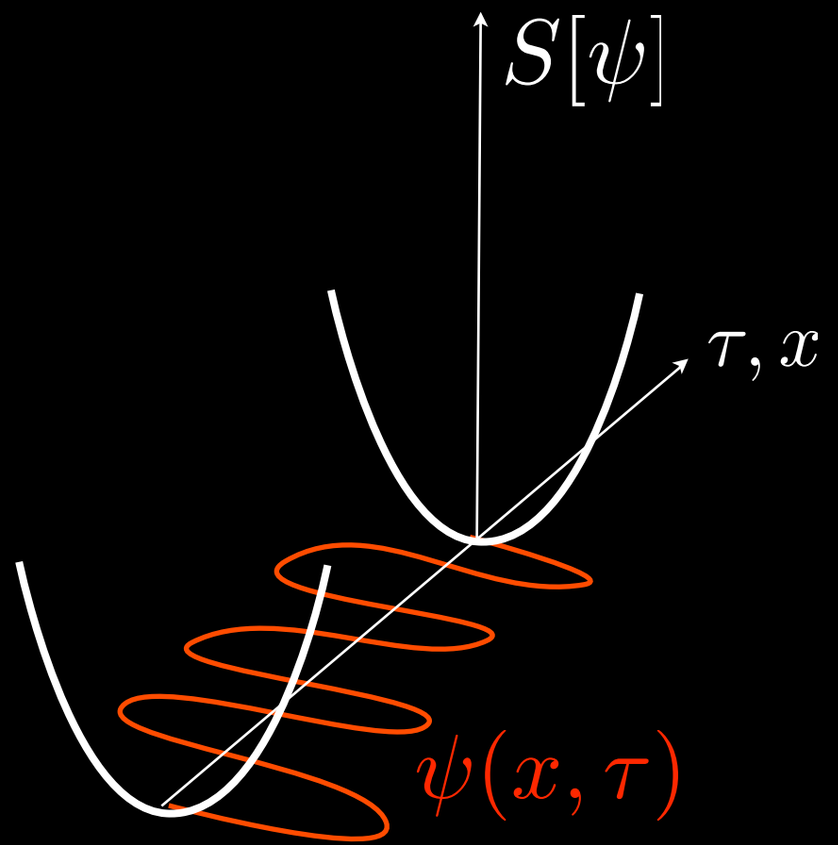

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$\sigma \in (-\frac{1}{2}, \frac{1}{2}) \longrightarrow$

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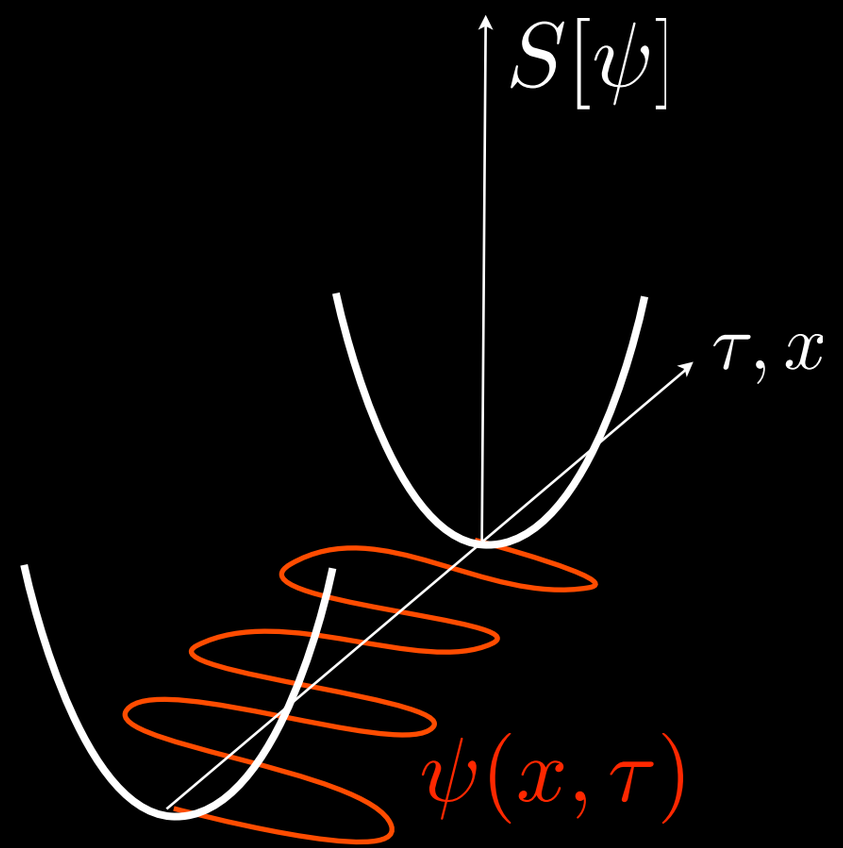

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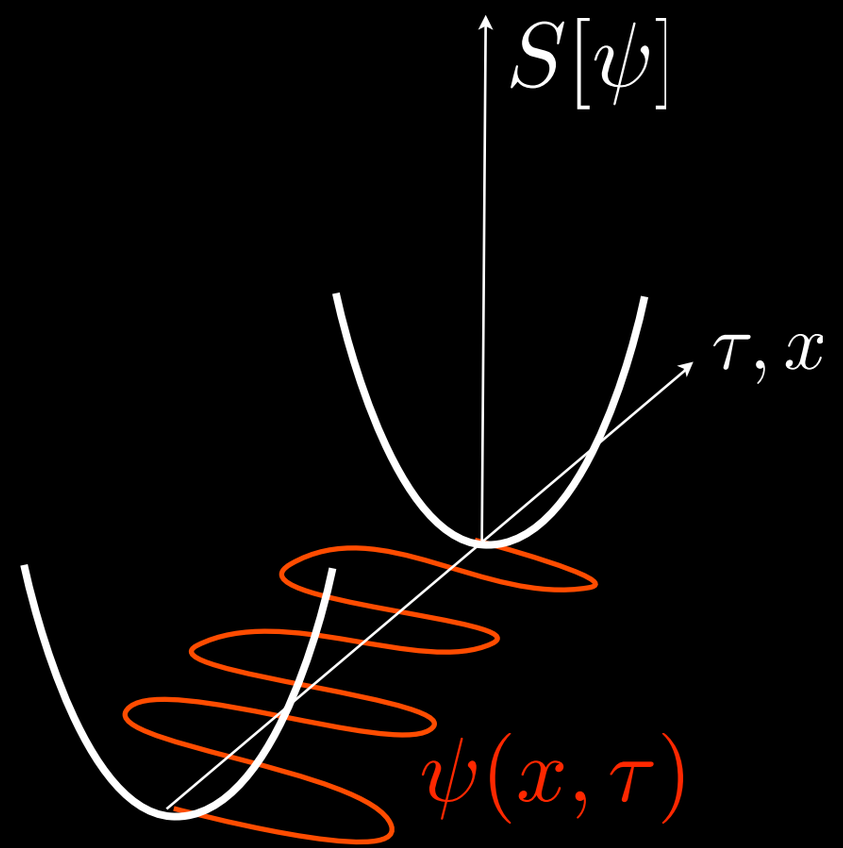

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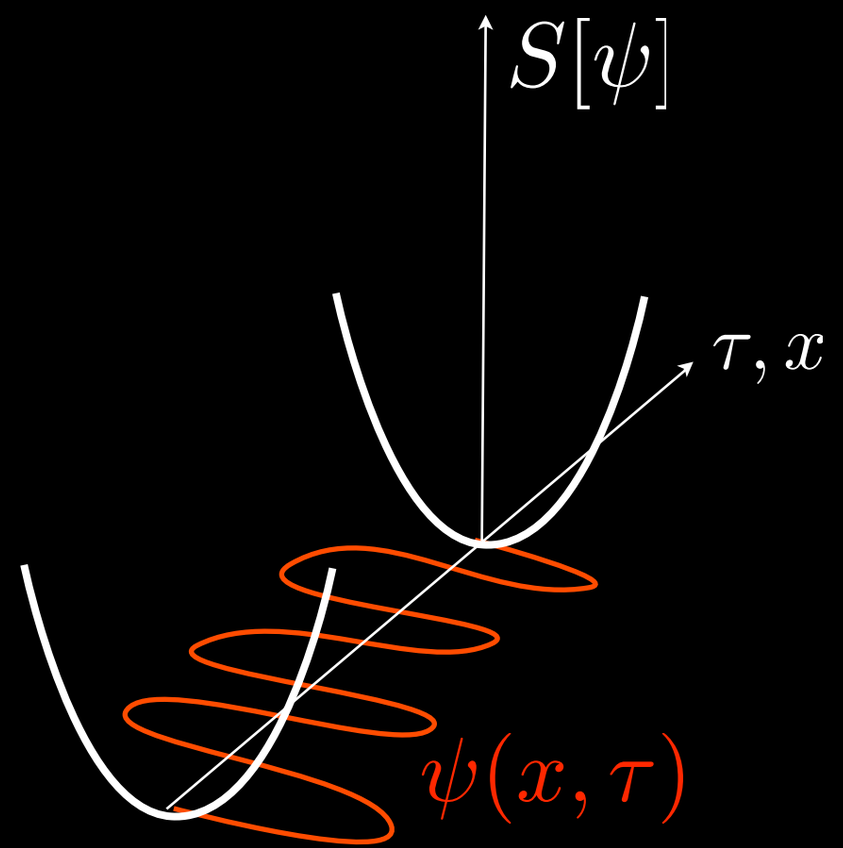

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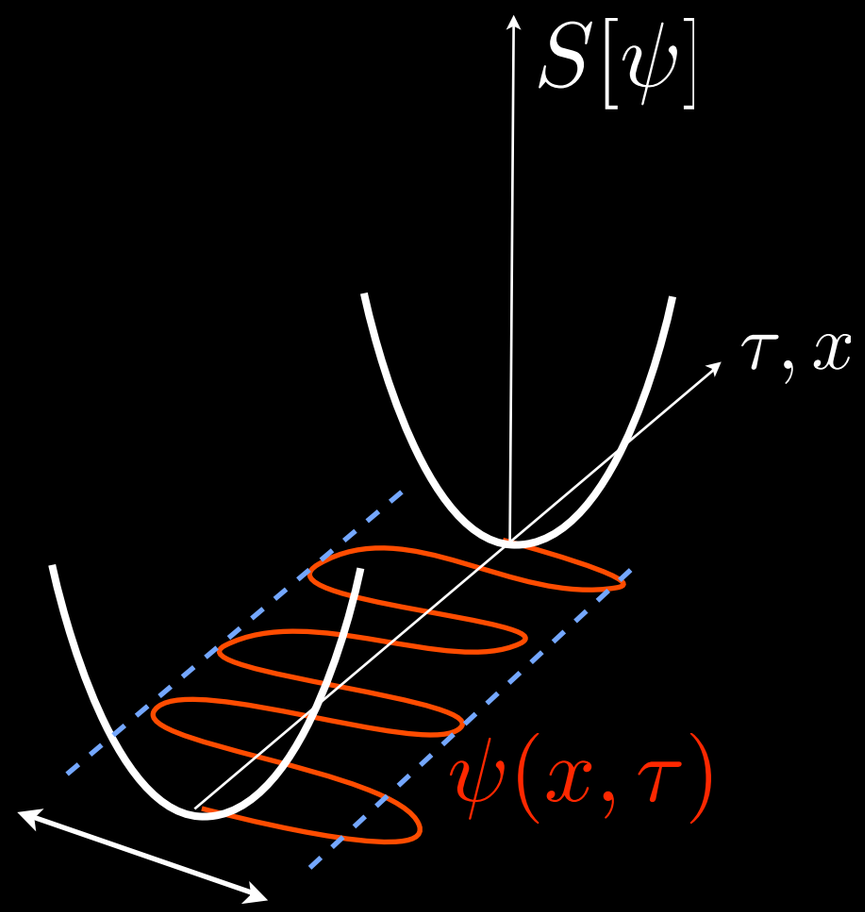

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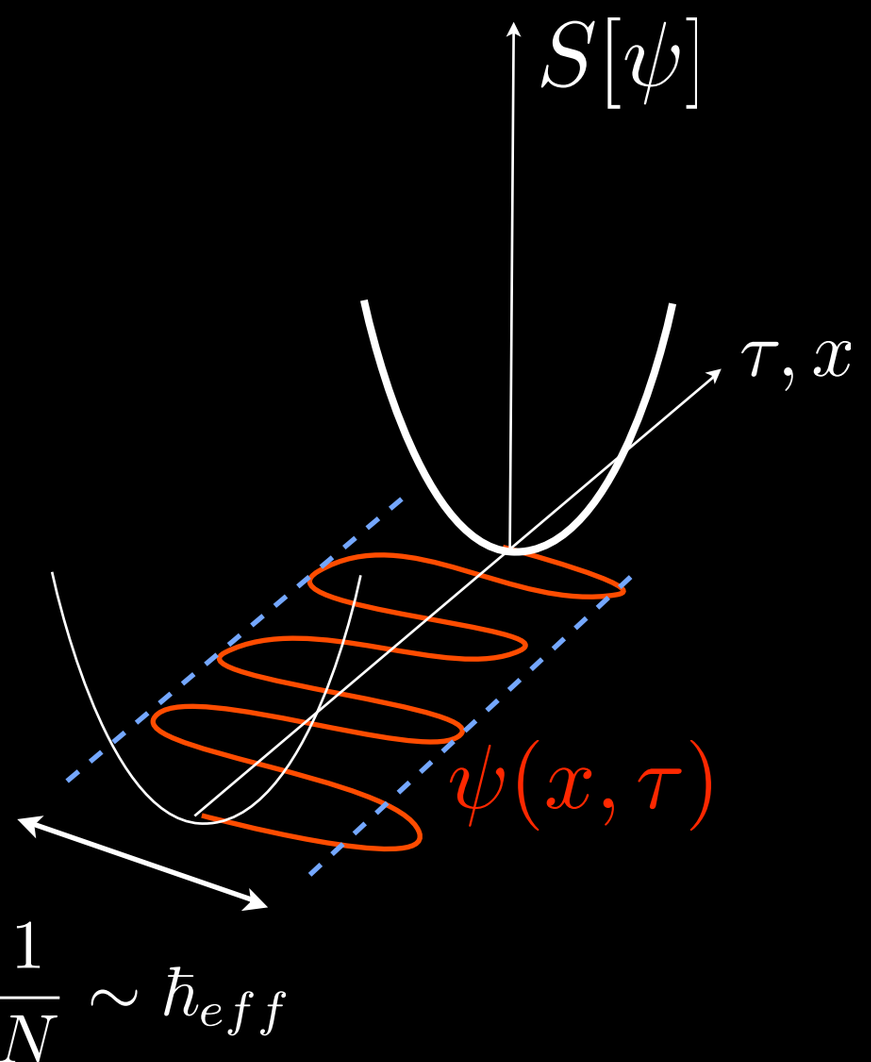

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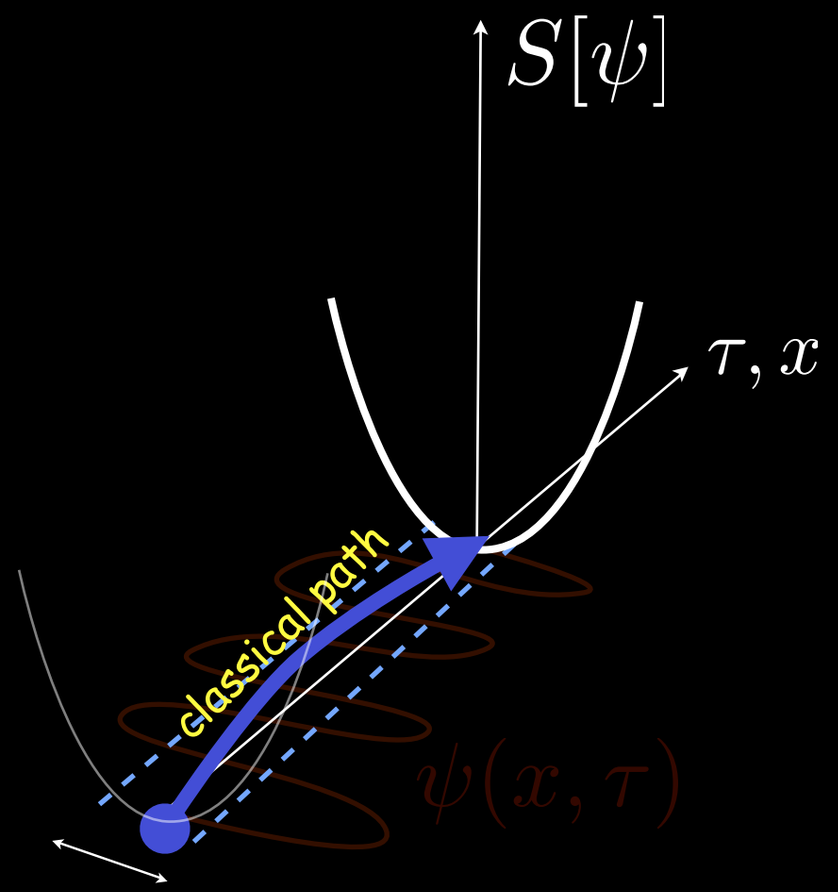

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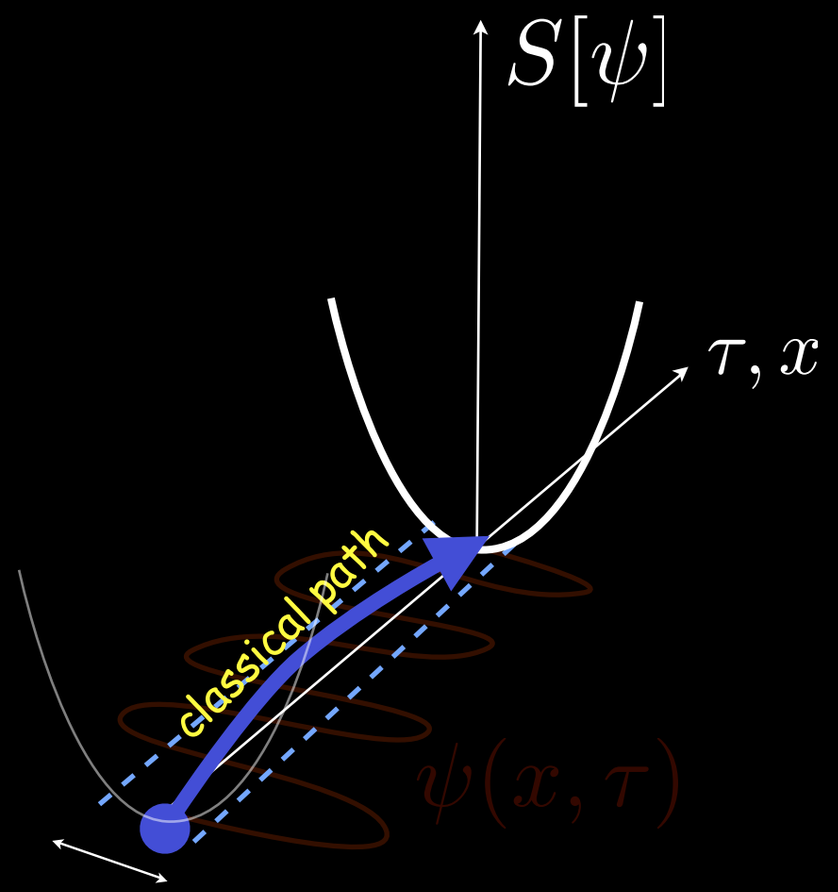

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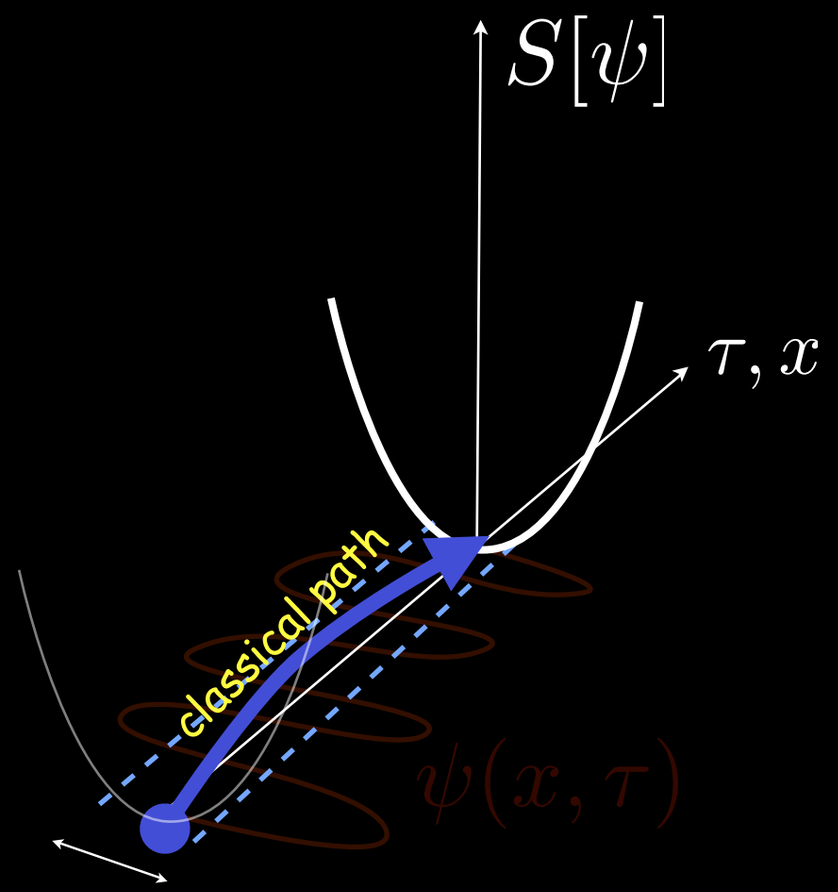

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"Symplectic N" *(large N + time reversal symmetry)*

R. Flint, Dzero and PC '08

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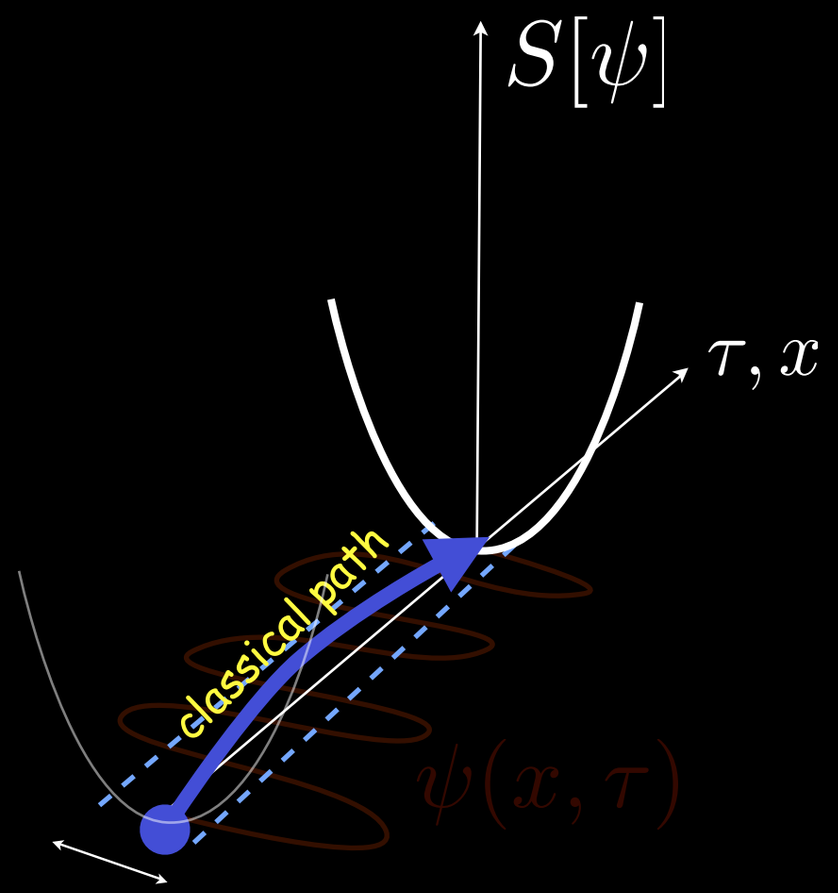

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PC, Kee, Andrei, Tsvelik (98)

$$S_{\alpha\beta} = f^{\dagger}_{\alpha} f_{\beta} + \mathrm{sgn}(\alpha\beta) f_{-\alpha} f^{\dagger}_{-\beta}$$

$$S_{\alpha\beta} = f^{\dagger}_{\alpha} f_{\beta} + \mathrm{sgn}(\alpha\beta) f_{-\alpha} f^{\dagger}_{-\beta}$$

$$H = \sum \epsilon_k c^{+}_{k\alpha} c_{k\alpha} + \sum_{j,r=1,2} J_r c^{+}_{jr\alpha} c_{jr\beta} \; S_{\beta\alpha}(j)$$

$$c^{+}_{jr\alpha} = \frac{1}{\sqrt{N}} \sum c^{+}_{k\alpha} \; \Phi_{\alpha\alpha'}(k) \; e^{-i\vec{k}\cdot\vec{R}_j}$$

T

Form factor

$$S_{\alpha\beta} = f^{\dagger}_{\alpha} f_{\beta} + \mathrm{sgn}(\alpha\beta) f_{-\alpha} f^{\dagger}_{-\beta}$$

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T

Form factor

e.g without spin orbit

$\Phi_1(\vec{k}) = 1$ s

$\Phi_2(\vec{k}) = \cos k_x - \cos k_y$ d

more realistic with s.o. c.

$$\Phi_{\Gamma}(\vec{k})_{\sigma\alpha} = \sum_{m} Y^{3}_{m-\sigma}(\vec{k}) \langle \ell\, m-\sigma\ \tfrac{1}{2}\,\sigma \,|\, \tfrac{5}{2}\, m\rangle \langle m|\alpha\rangle$$

$\Gamma = \Gamma_6, \Gamma_7$

more realistic with s.o. c.

$$\Phi_{\Gamma}(\vec{k})_{\sigma\alpha} = \sum_{m} Y^{3}_{m-\sigma}(\vec{k}) \langle \ell\, m-\sigma\, \tfrac{1}{2}\sigma \,|\, \tfrac{5}{2}\, m\rangle \langle m|\alpha\rangle$$

$\Gamma = \Gamma_6, \Gamma_8$

$$\Gamma_7^- \quad\quad \beta\,|\tfrac{5}{2}\rangle - \alpha\,|{-\tfrac{3}{2}}\rangle$$

$$\Gamma_6 \quad\quad |\pm\tfrac{1}{2}\rangle$$

$$\Gamma_7^+ \quad\quad \alpha\,|\tfrac{5}{2}\rangle + \beta\,|{-\tfrac{3}{2}}\rangle$$

$$\frac{J}{2N} \sum_{\alpha}^{+} c_{j\alpha}^{\dagger} c_{j\beta} \left(f_{j\beta}^{+} f_{j\alpha} + \tilde{\alpha}\beta f_{j,-\alpha} f_{j,-\alpha} \right)$$

$$\frac{J}{2N} \; c_{j\alpha}^{+} \, c_{j\beta} \left(f_{j\beta}^{+} f_{j\alpha} + \tilde{\alpha}\tilde{\beta} f_{j-\beta} f_{j-\alpha} \right)$$

$$\equiv \quad -\frac{J}{2N} \Big[\left(c_{j\alpha}^{+} f_{j\alpha} \right) \left(f_{j\beta}^{+} c_{j\beta} \right) + \left(\tilde{\alpha} c_{j\alpha}^{+} f_{j-\alpha}^{+} \right) \left(\tilde{\beta} f_{j-\beta} c_{j\beta} \right) \Big]$$

$$\frac{J}{2N}\; c_{j\alpha}^{+}\, c_{j\beta}\left(f_{j\beta}^{+} f_{j\alpha} + \tilde{\alpha}\tilde{\beta} f_{j-\beta} f_{j-\alpha}\right)$$

$$\equiv \quad -\frac{J}{2N}\Big[\left(c_{j\alpha}^{+} f_{j\alpha}\right)\left(f_{j\beta}^{+} c_{j\beta}\right) + \left(\tilde{\alpha} c_{j\alpha}^{+} f_{j-\alpha}^{+}\right)\left(\tilde{\beta} f_{j-\beta} c_{j\beta}\right)\Big]$$

$$\longrightarrow \quad \sum_{q}\left(\vec{V}\, c_{j\alpha}^{+} f_{\alpha} + H.c\right) + \left(\Delta\, c_{j\alpha}^{+} f_{j-\alpha}^{+}\tilde{\alpha} + h.c\right)$$

$$+2N\left(\frac{\bar{V} V}{J} + \frac{\bar{\Delta}\Delta}{J}\right)$$

$$\frac{J}{2N}\; c_{j\alpha}^{+}\, c_{j\beta}\left(f_{j\beta}^{+} f_{j\alpha} + \tilde{\alpha}\tilde{\beta} f_{j-\beta} f_{j-\alpha}\right)$$

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$$+2N\left(\frac{\bar{V}V}{J} + \frac{\bar{\Delta}\Delta}{J}\right)$$

$$= \quad \sum_{\alpha>0}\Big[\left(f_{j\alpha}^{+}, f_{j-\alpha}\right)\begin{pmatrix} V & \Delta \\ \bar{\Delta} & -\bar{V}\end{pmatrix}\begin{pmatrix} c_{j\alpha} \\ c_{j-\alpha}^{+}\end{pmatrix} + 4.c\Big]$$

$$+ \quad Tr\left[\begin{pmatrix}\bar{V} & \bar{\Delta} \\ \Delta & -V\end{pmatrix}\begin{pmatrix} V & \Delta \\ \bar{\Delta} & -\bar{V}\end{pmatrix}\right]\left(\frac{N}{2J}\right)$$

$$\frac{J}{2N}\; c_{j\alpha}^{+}\, c_{j\beta}\left(f_{j\beta}^{+} f_{j\alpha} + \tilde{\alpha}\tilde{\beta} f_{j-\beta} f_{j-\alpha}\right)$$

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$$= \left[\tilde{f}_j^{+} \mathcal{V} c_j + c_j^{+} \bar{\mathcal{V}} f_j \right] + Tr \left(\frac{\bar{V} V}{J} \right) \left(\frac{N}{2} \right)$$

$$\mathcal{V} = \begin{pmatrix} V & \Delta \\ +\bar{\Delta} & -\bar{V} \end{pmatrix}$$

$$= \sum_{\alpha>0} \left[\left(f^{+}_{j\alpha}, f_{j-\alpha} \right) \begin{pmatrix} V & \Delta \\ \bar{\Delta} & -\bar{V} \end{pmatrix} \begin{pmatrix} c_{j\alpha} \\ c^{+}_{j-\alpha} \end{pmatrix} + h.c \right]$$

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$$\mathcal{V} = \begin{pmatrix} V & \Delta \\ +\bar{\Delta} & -\bar{V} \end{pmatrix}$$

$$\Rightarrow \sum_{\Gamma, ij} \left(f^{+}_{j} \, \mathcal{V}_{\Gamma} \, c_{\Gamma j} + h.c + \frac{N}{J_\Gamma} \, Tr \left(\mathcal{V}^{+}_{\Gamma} \mathcal{V}_{\Gamma} \right) \right)$$

$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi^\dagger_{1a}(j) \psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

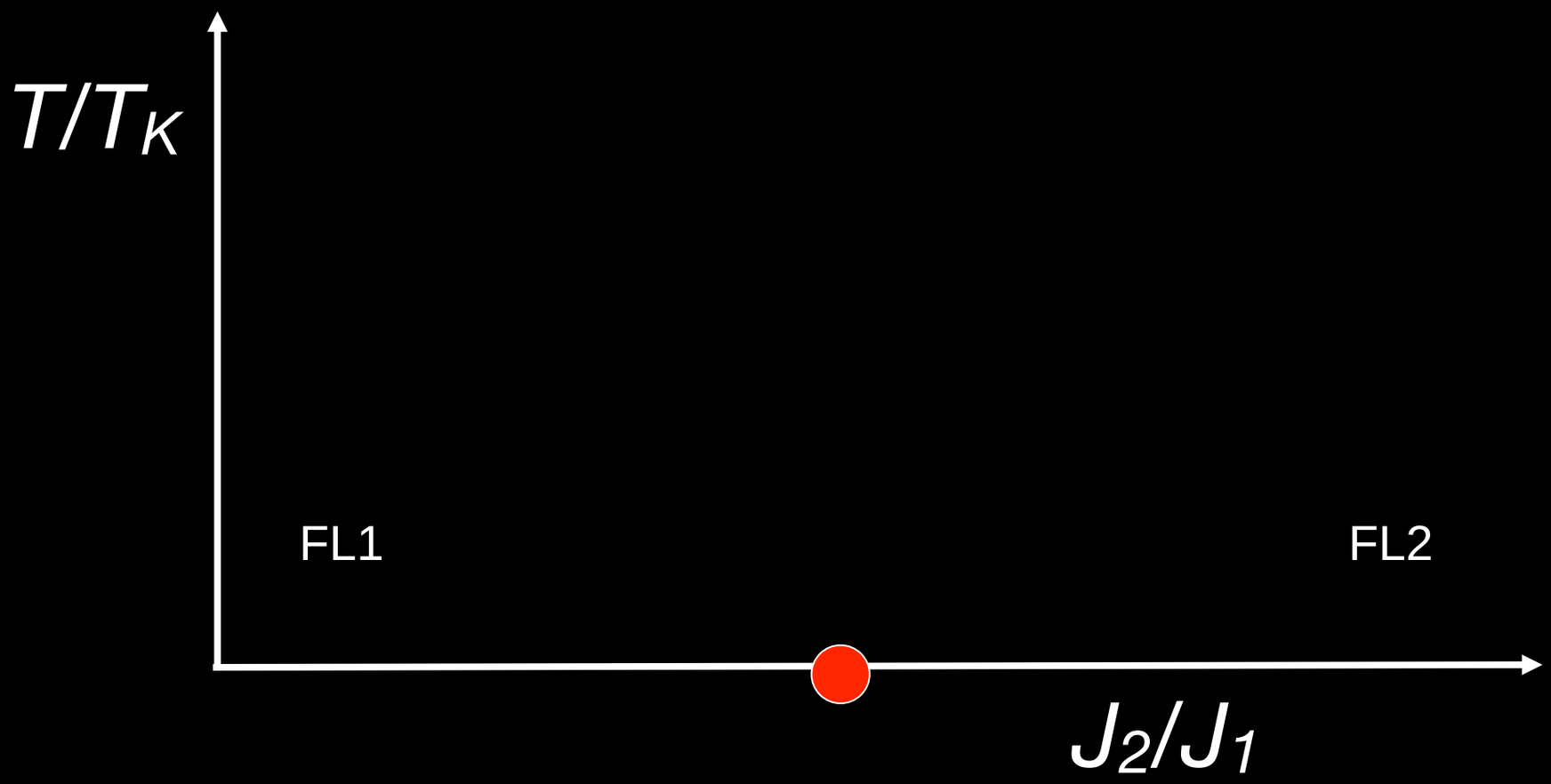
cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Single FS, two channels.

$$\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma\mathbf{k}} \quad c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_j}$$

Impurity: quantum critical point for $J_1 = J_2$

Nozieres and Blandin 1980



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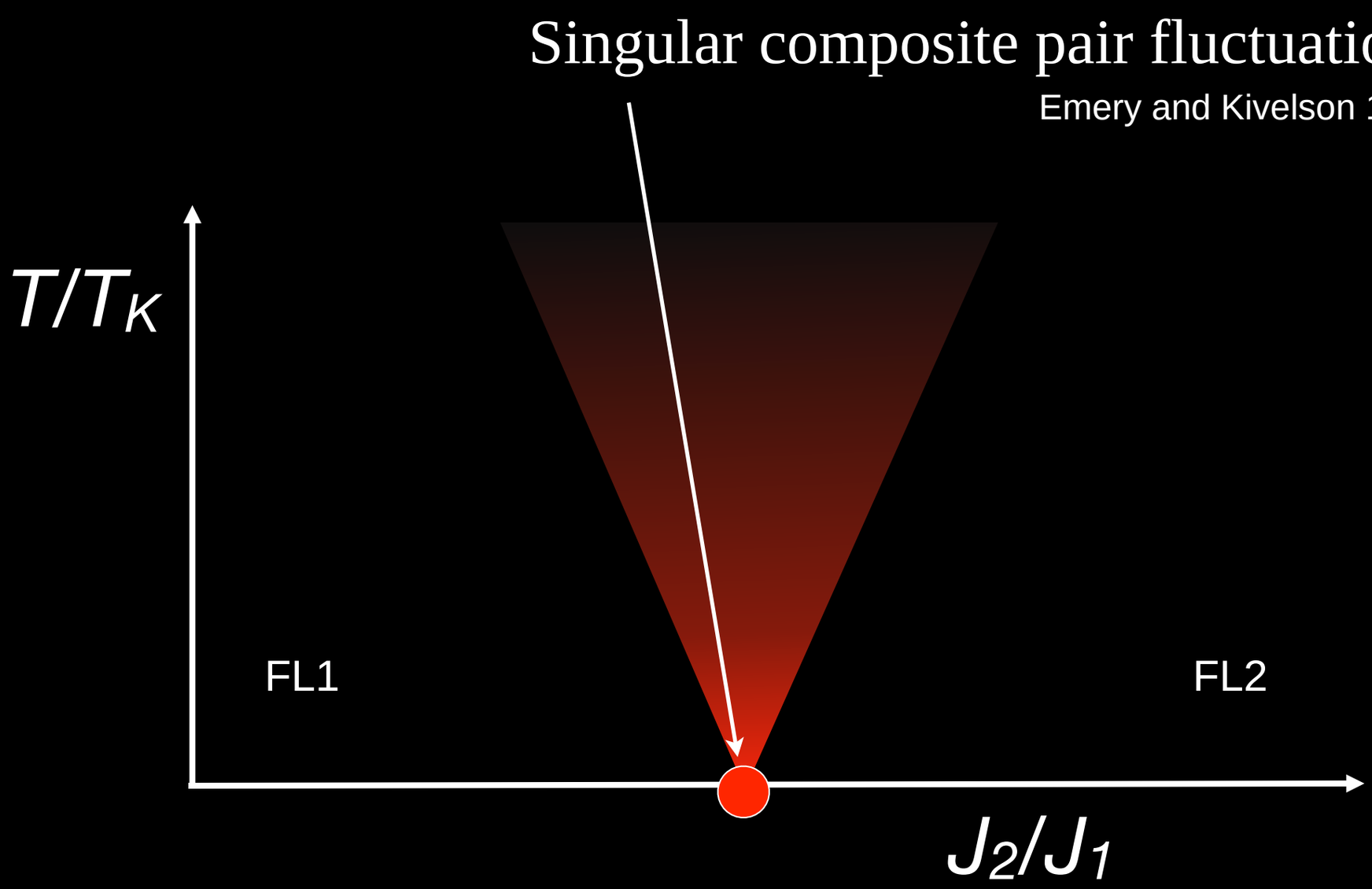

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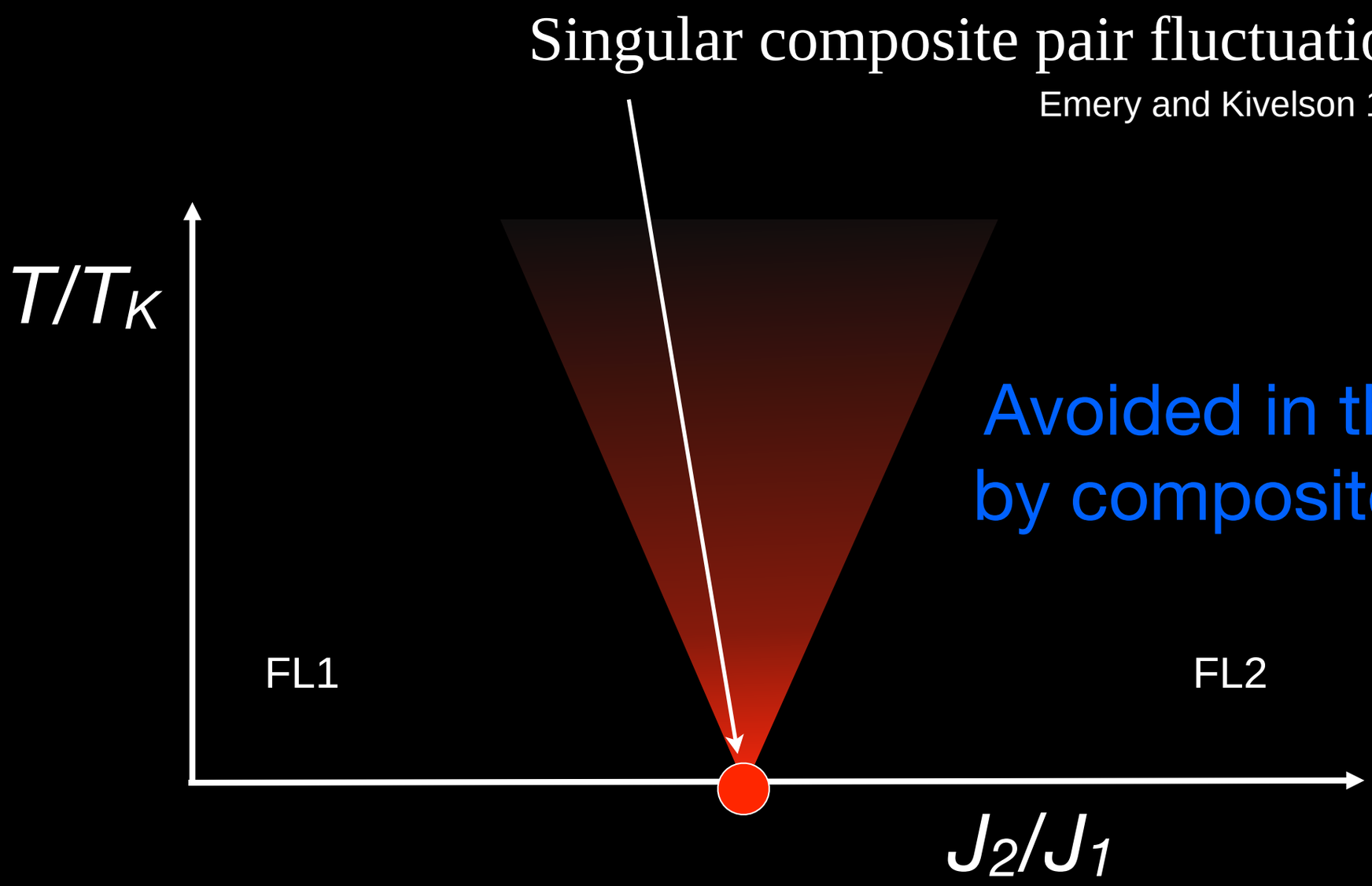

Avoided in the lattice by composite pairing.

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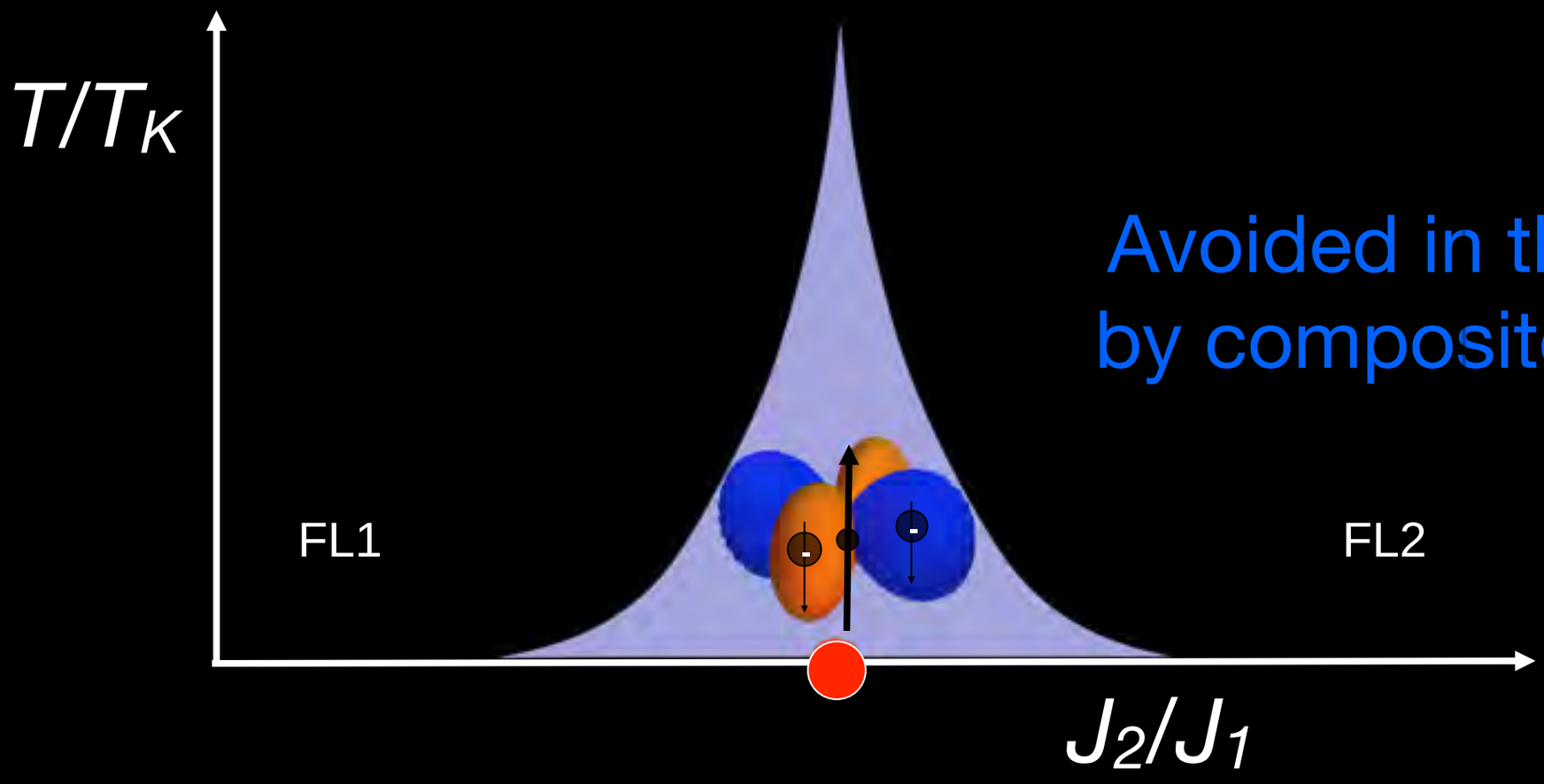
Nozieres and Blandin 1980

Singular composite pair fluctuations

Emery and Kivelson 1992



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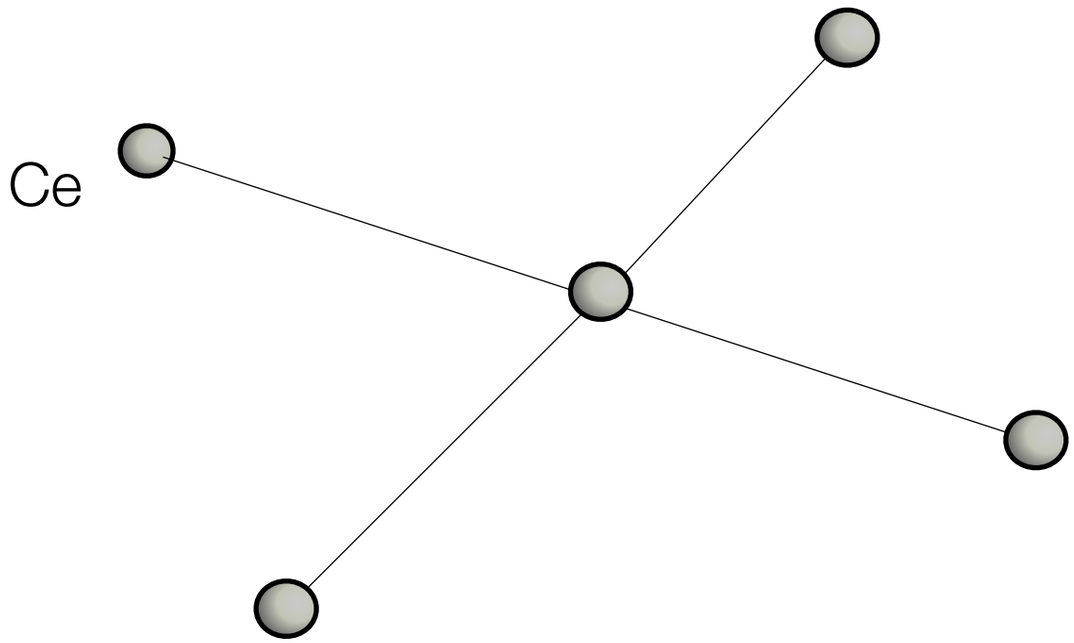
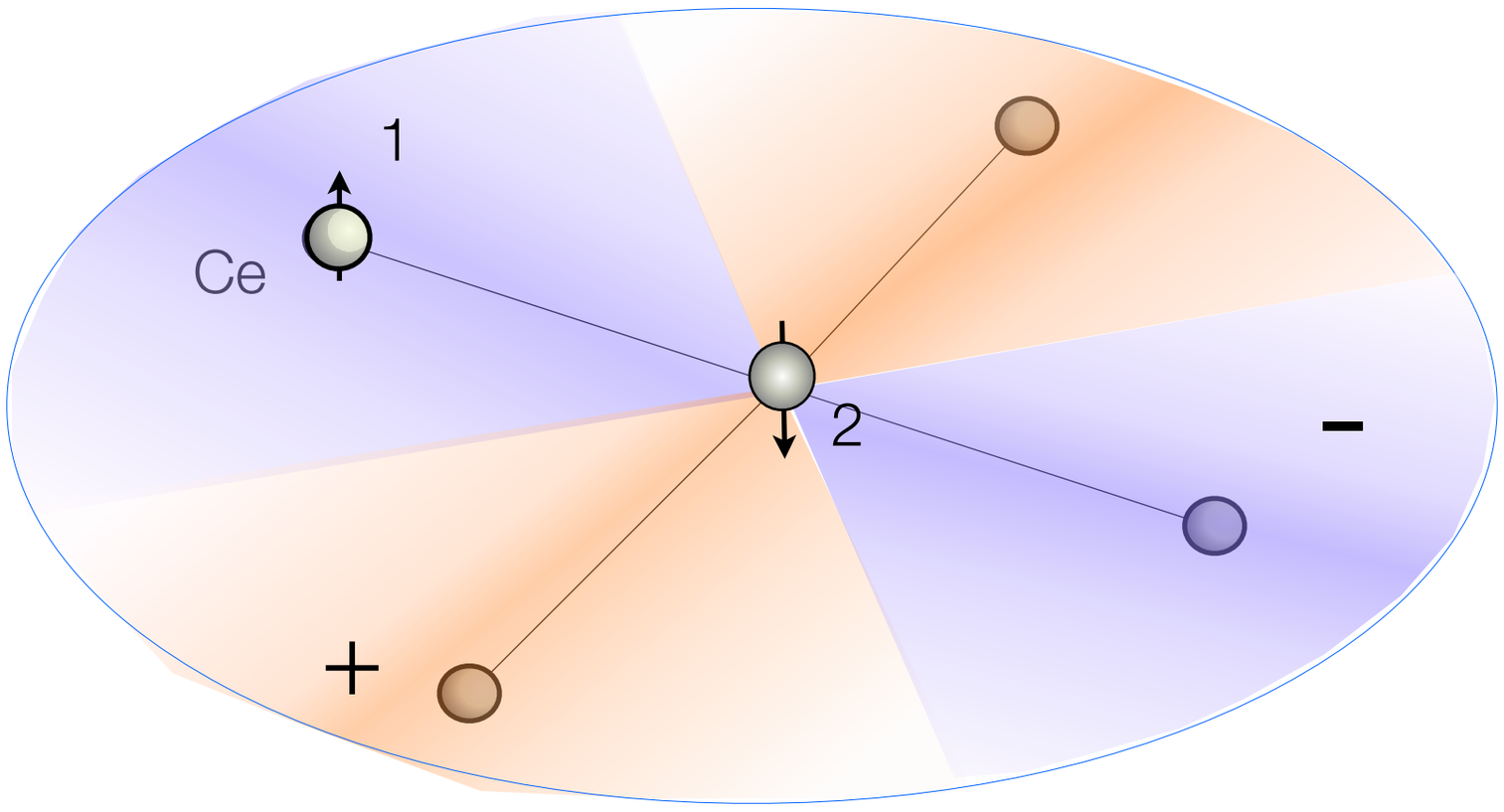
Real-space structure of pair



Real-space structure of pair

Magnetic pair: intercell

$$\Psi_M^\dagger = \Delta_d(1-2) f_\uparrow^\dagger(1) f_\downarrow^\dagger(2)$$



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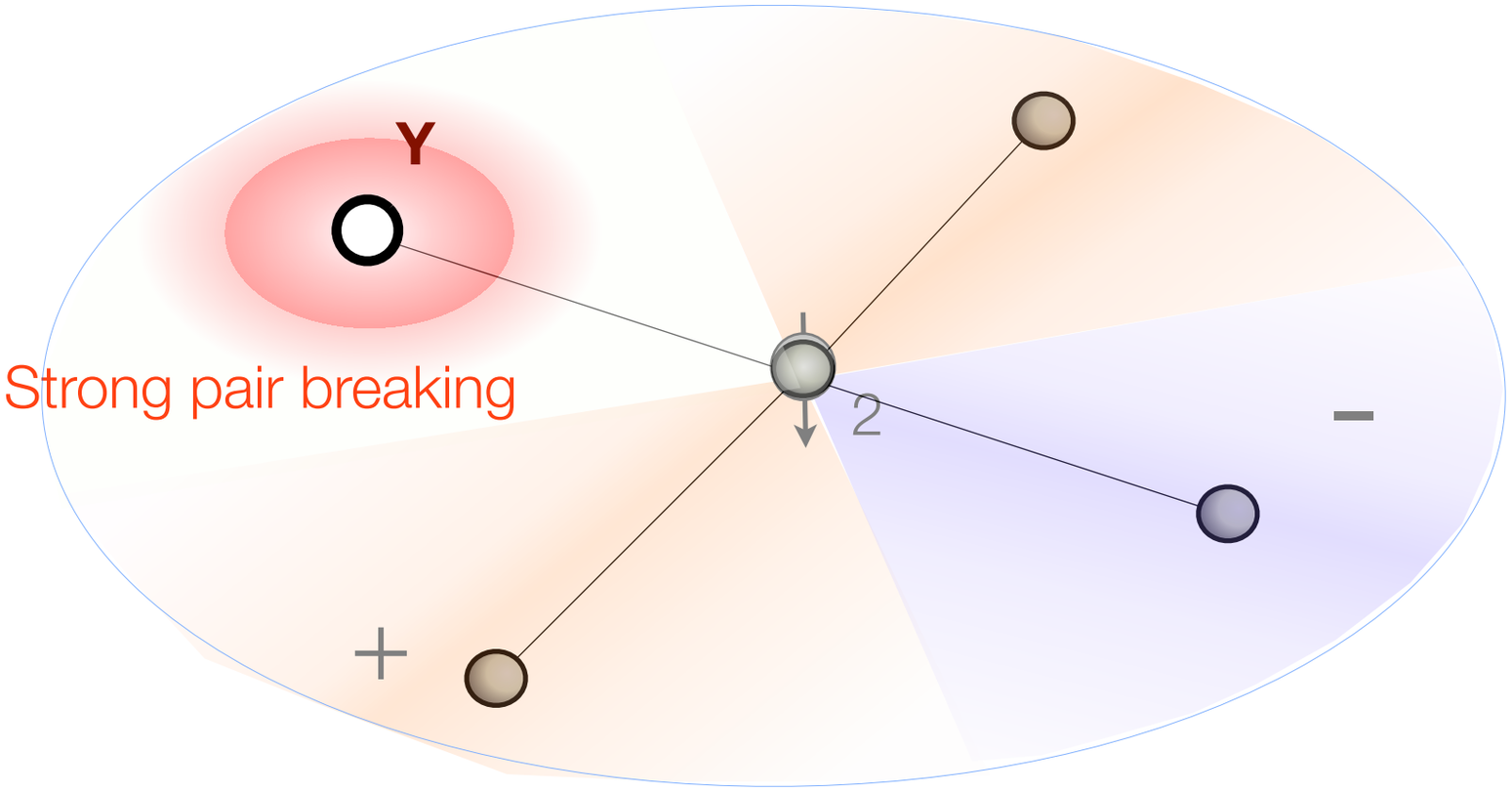

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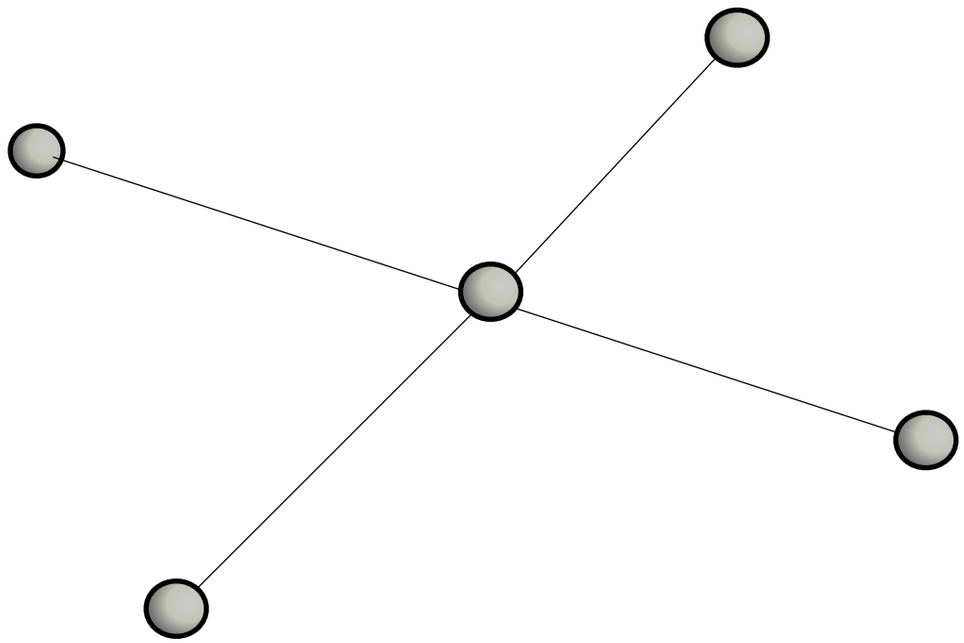

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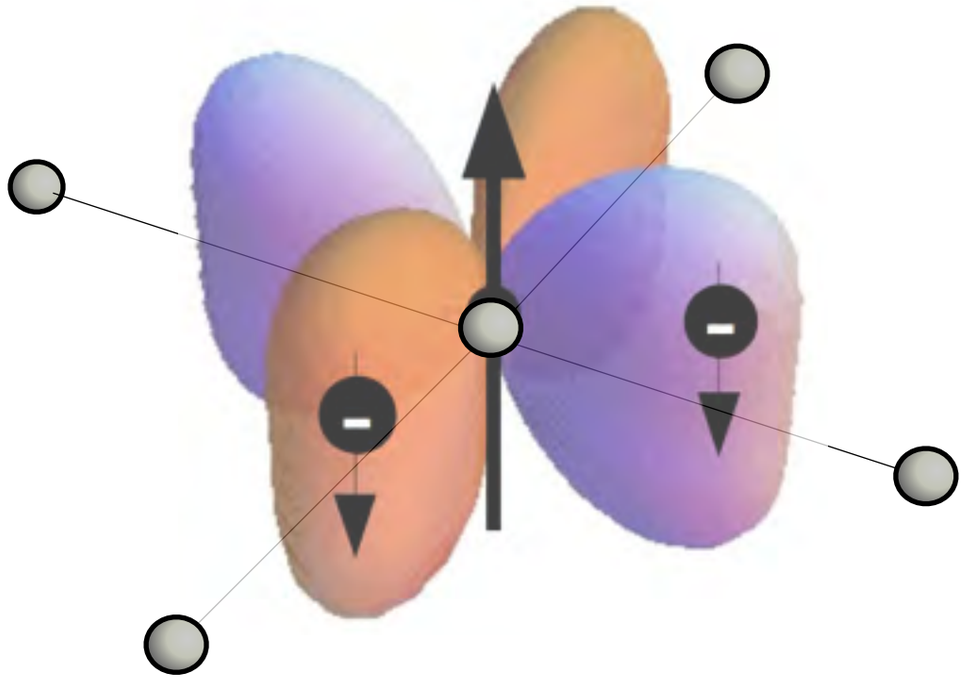

Composite pair

$$\Psi^\dagger_C = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$

Abrahams, Balatsky, Scalapino, Schrieffer 1995

Andrei, Coleman, Kee & Tsvelik PRB (1998)
Flint, Dzero, Coleman , Nat. Phys, (2008)

Real-space structure of pair

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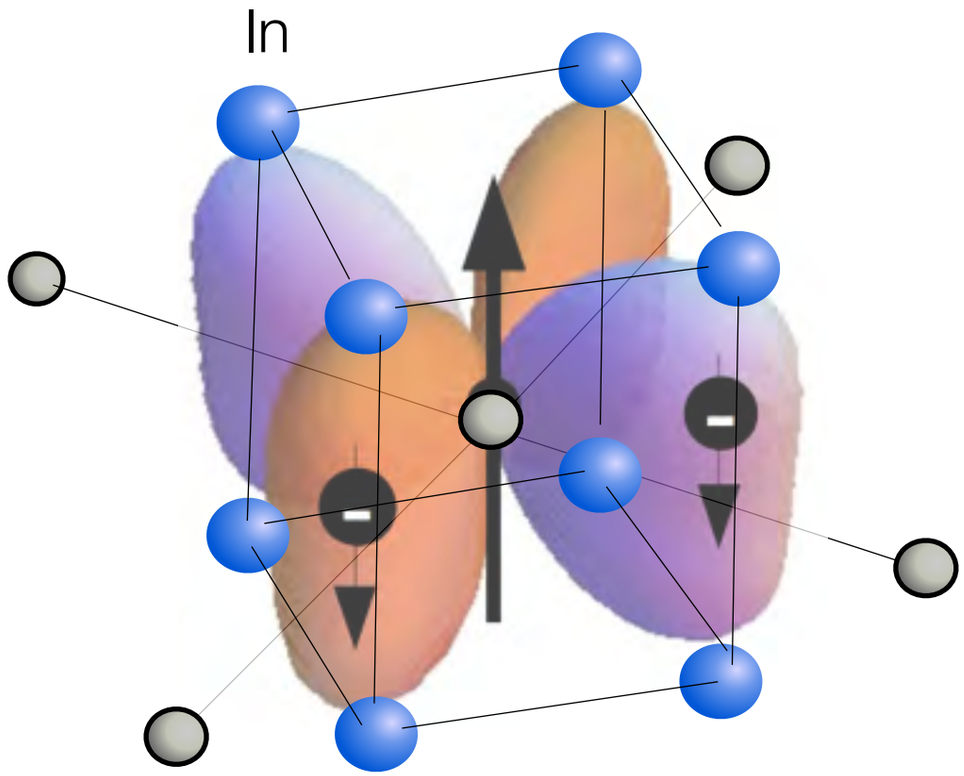

Composite pair**: intra-cell boson**

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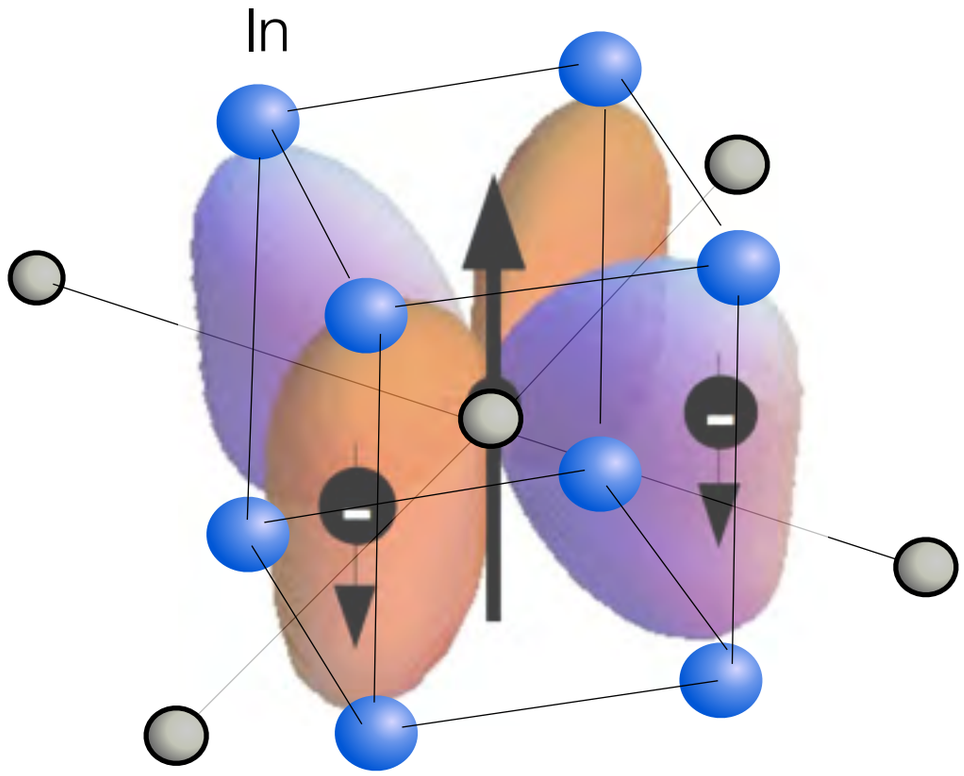

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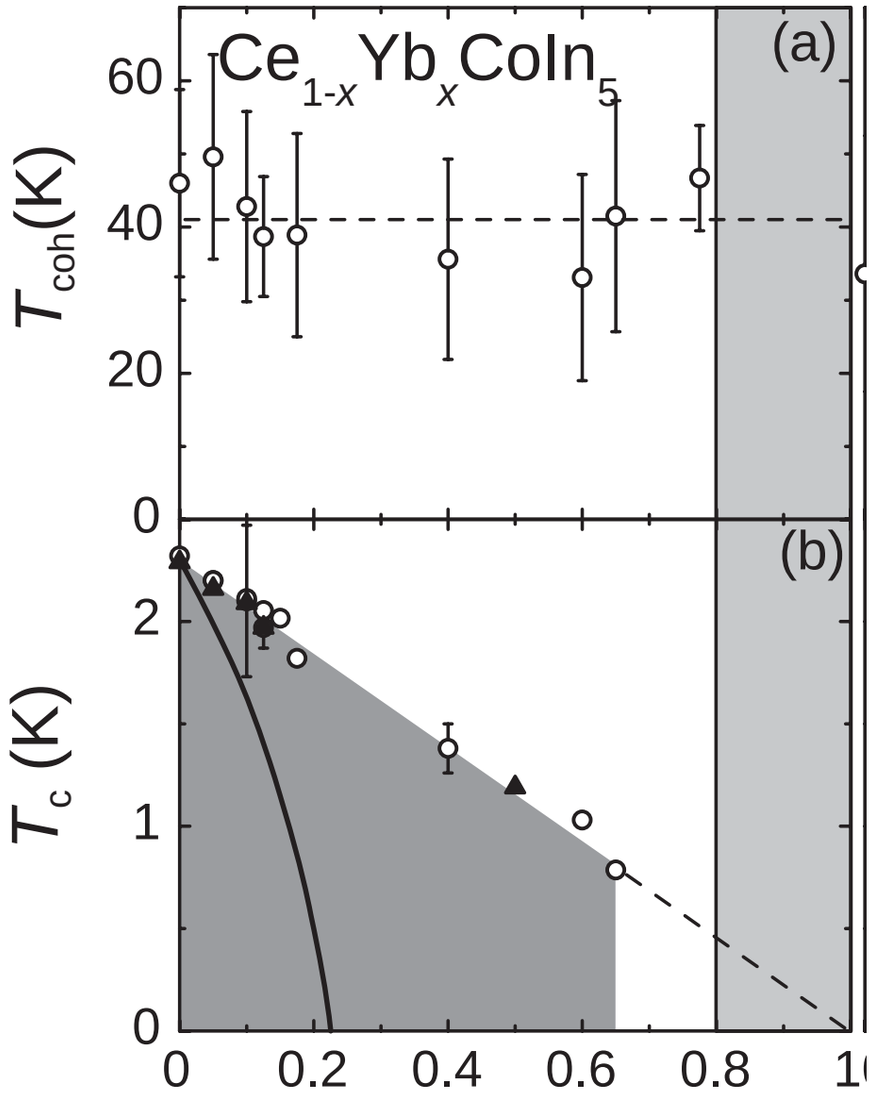
Extreme Resilience to doping on Ce site.



Lei Shu et al, PRL, (2011)

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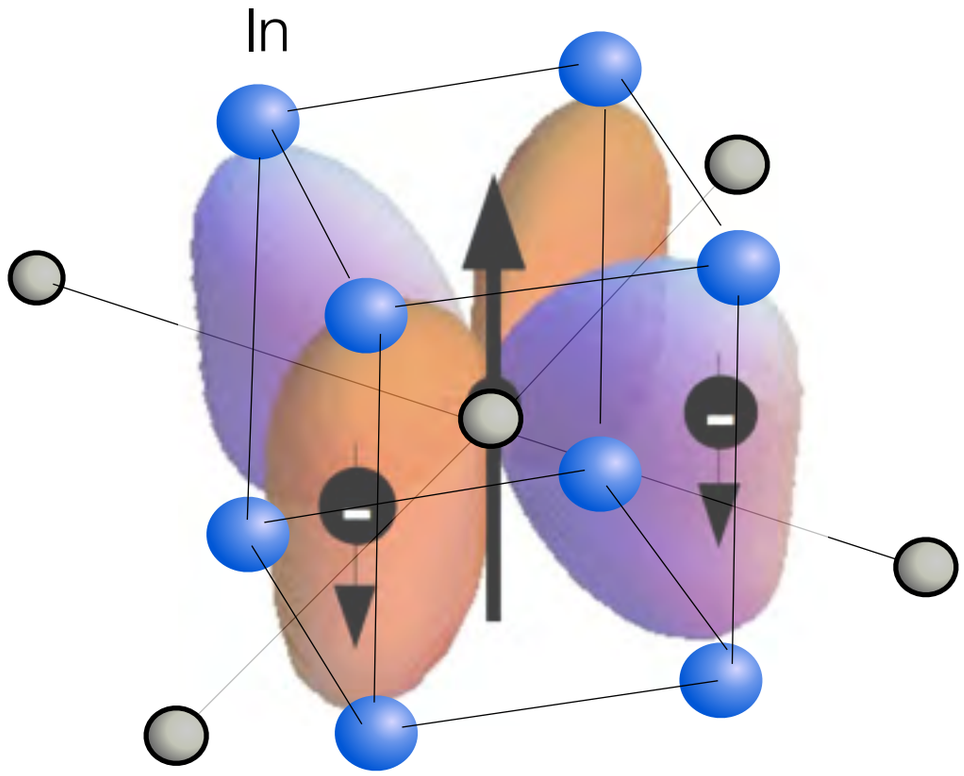

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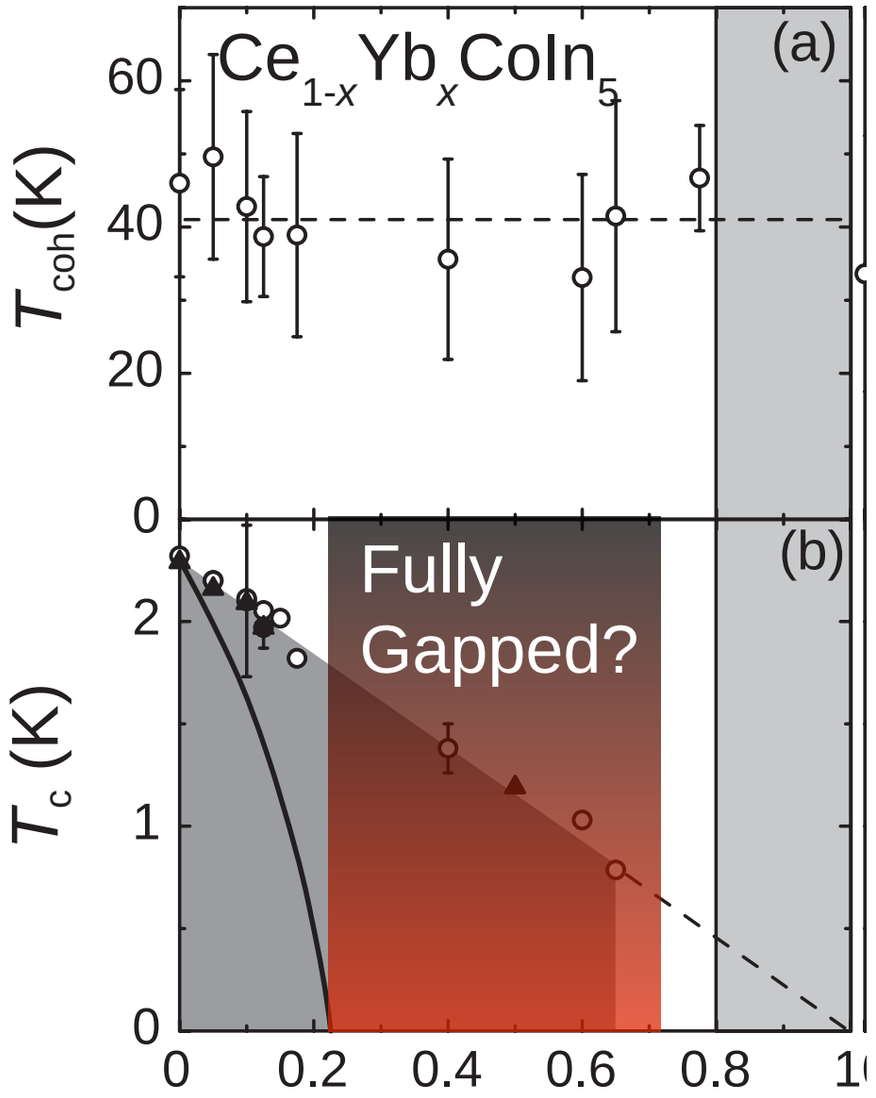
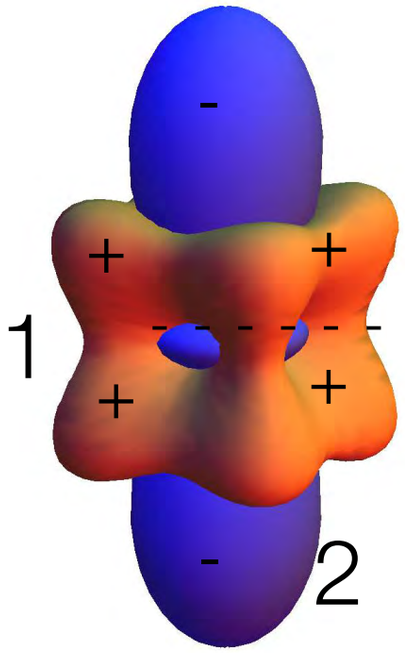


Lei Shu et al, PRL, (2011)

M. Tanatar et al (unpublished)
Erten and PC arXiv1402.7361

4f/5f superconductors



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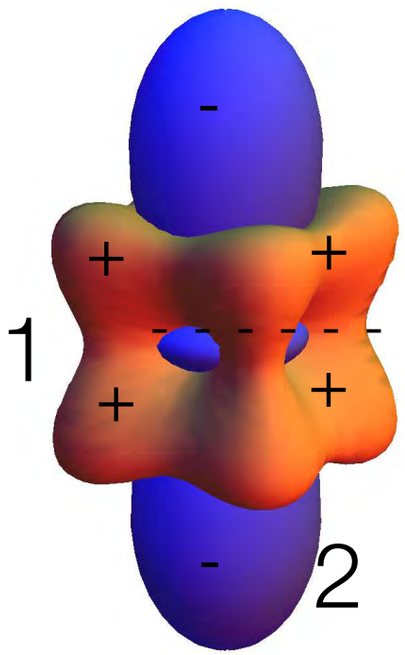
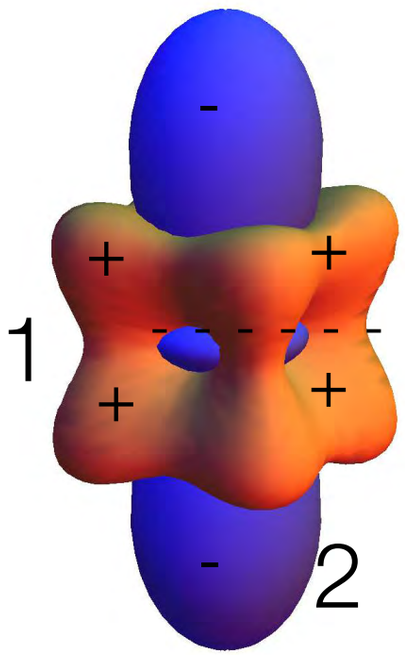
4f/5f superconductors



$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$

$$Q_{zz} \propto \Psi^2_C$$

4f/5f superconductors

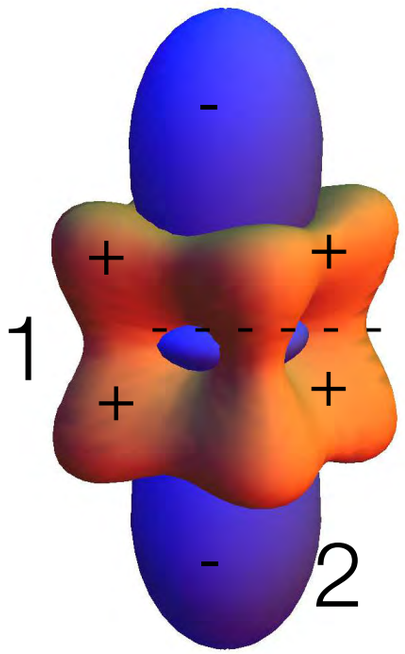


$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$

$$Q_{zz} \propto \Psi^2_C$$

$$\Delta F \propto -Q_{zz} u_{tet}$$

4f/5f superconductors



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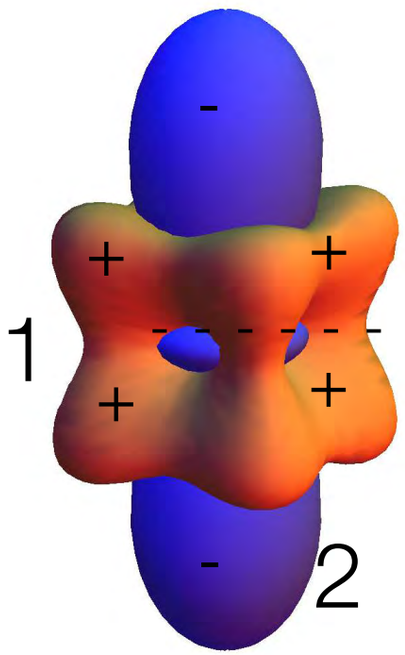
$$\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi^2_C$$

4f/5f superconductors



$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$

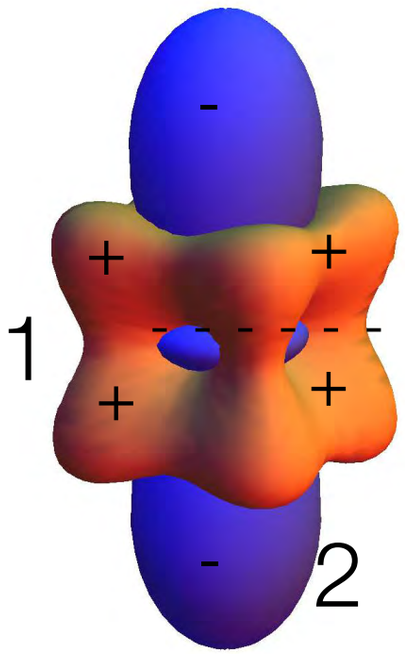
$$Q_{zz} \propto \Psi_C^2$$

$$\Delta F \propto -Q_{zz} u_{tet}$$

$$\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi_C^2$$

$$\Rightarrow T_c = T_{c2} + \lambda u_{tet}$$

4f/5f superconductors



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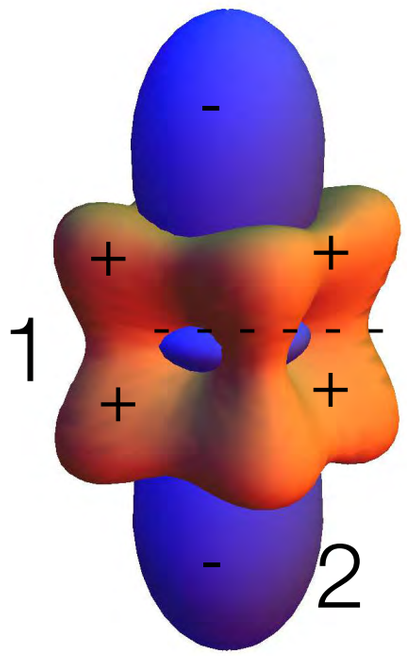
$$\Rightarrow T_c = T_{c2} + \lambda u_{tet}$$

Strain expected to enhance Tc

4f/5f superconductors



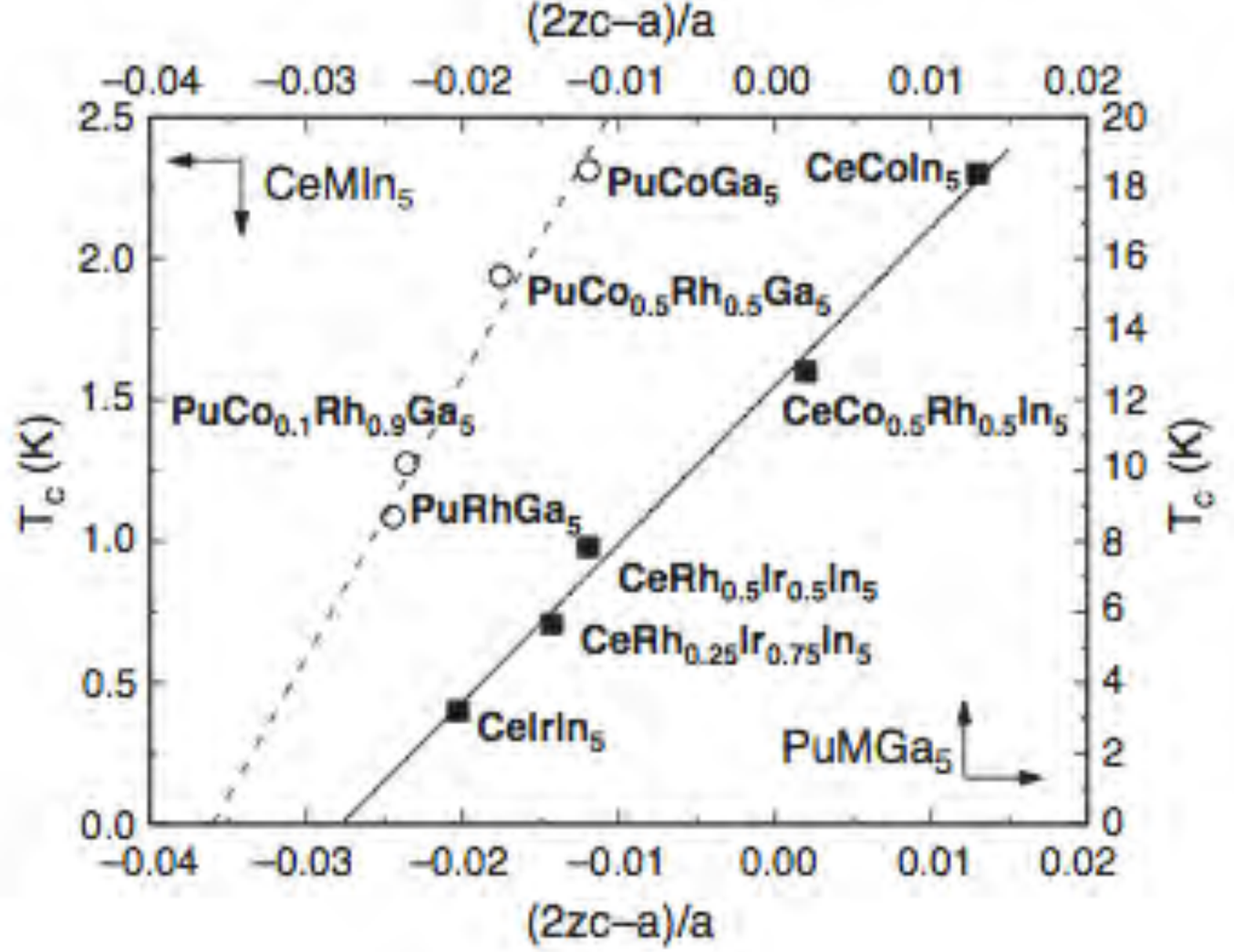
$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$

$$Q_{zz} \propto \Psi^2_C$$

$$\Delta F \propto -Q_{zz} u_{tet}$$

$$\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi^2_C$$

$$\Rightarrow T_c = T_{c2} + \lambda u_{tet}$$



J.L. Sarrao et al. / Physica B 359– 361 (2005) 1144–1146

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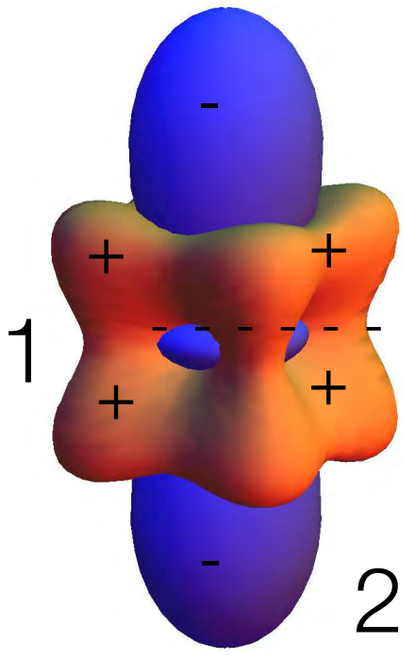

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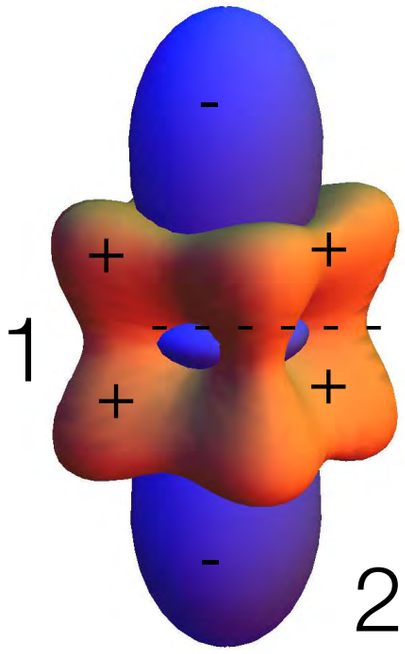

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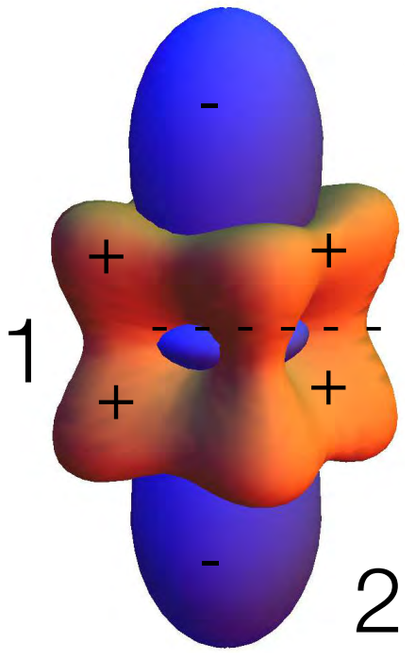

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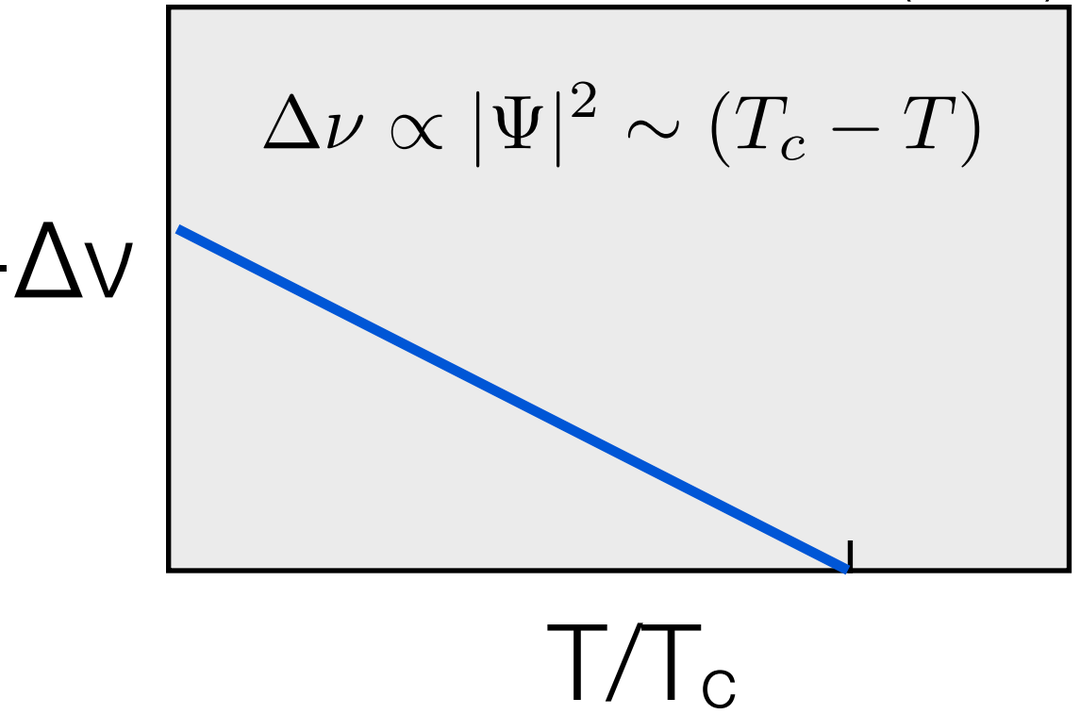
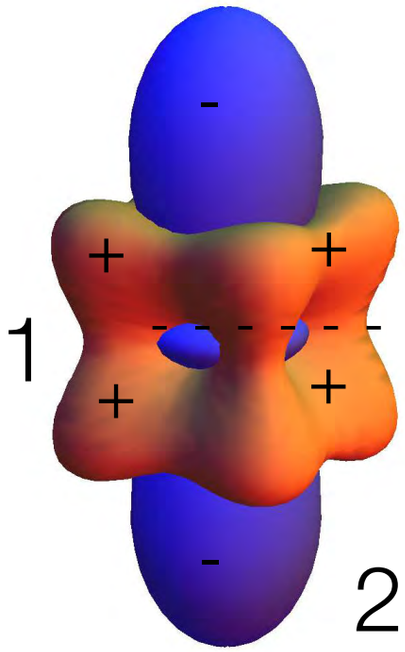
Flint et al, PRB 84, 064054, (2011)





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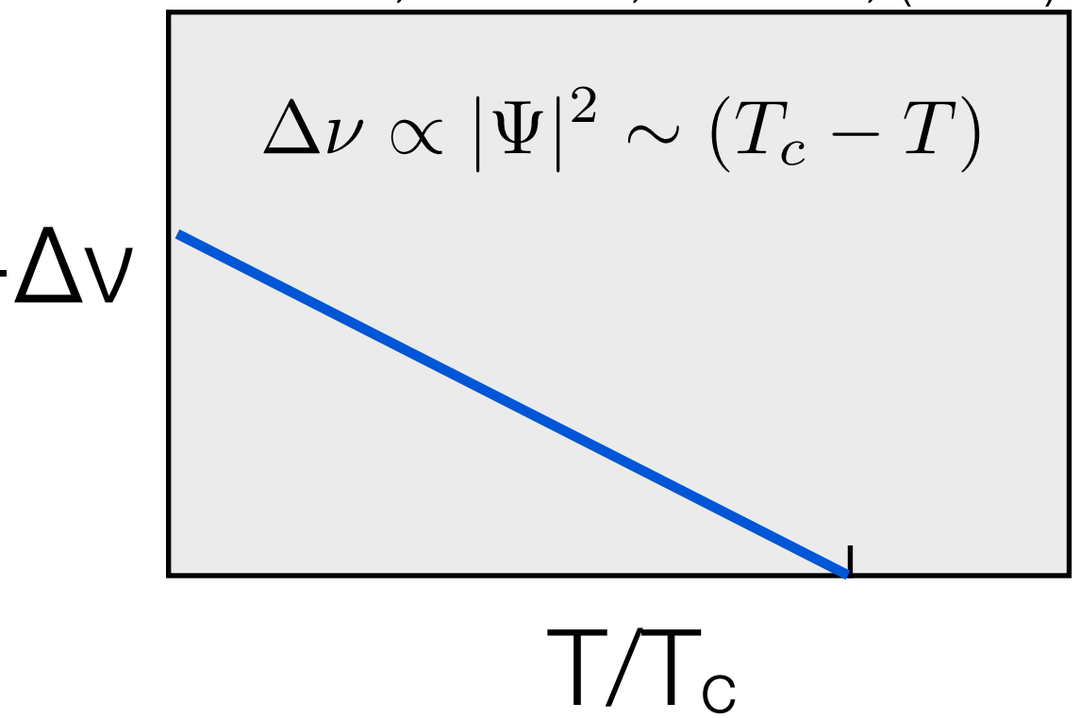
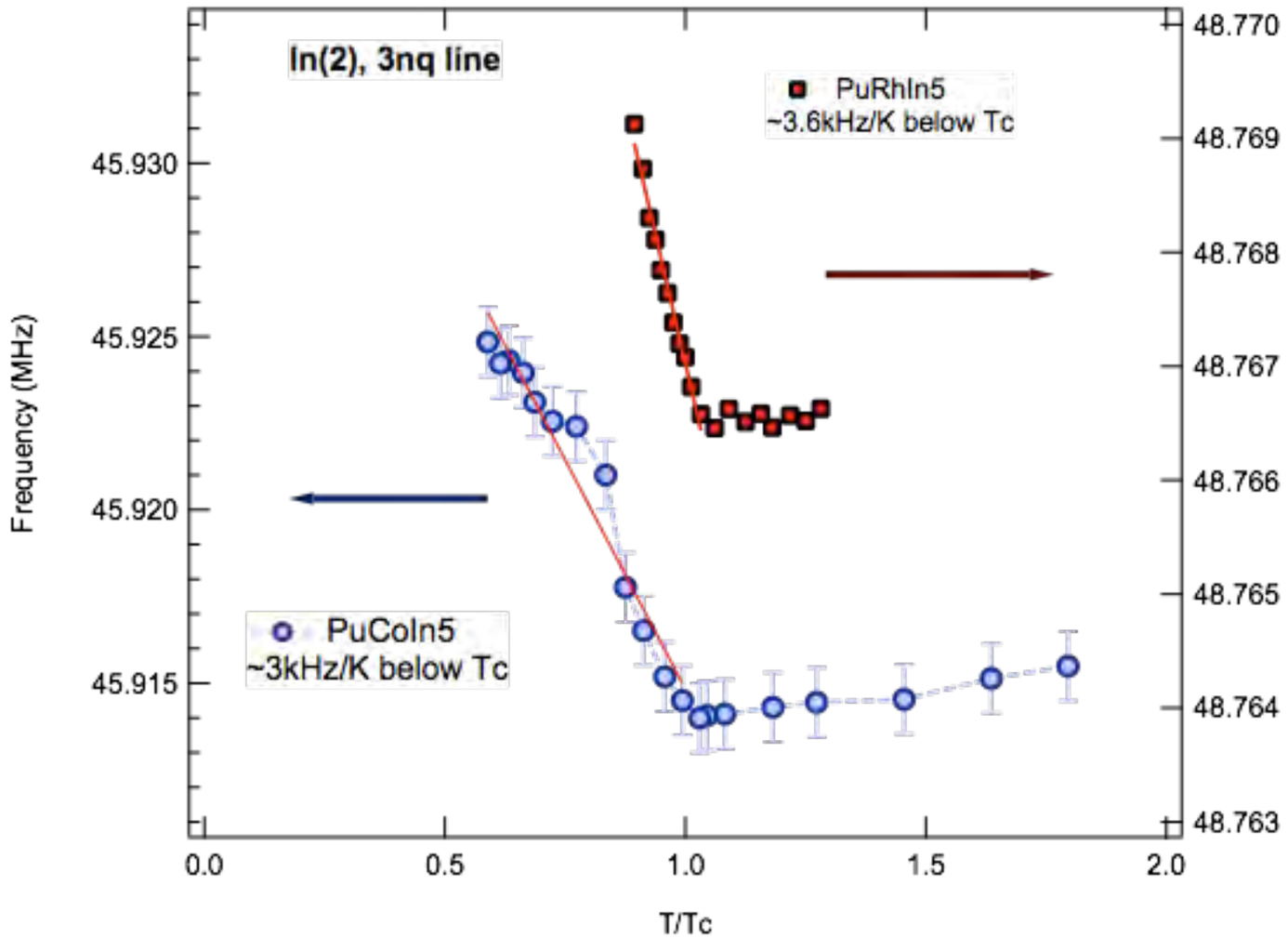
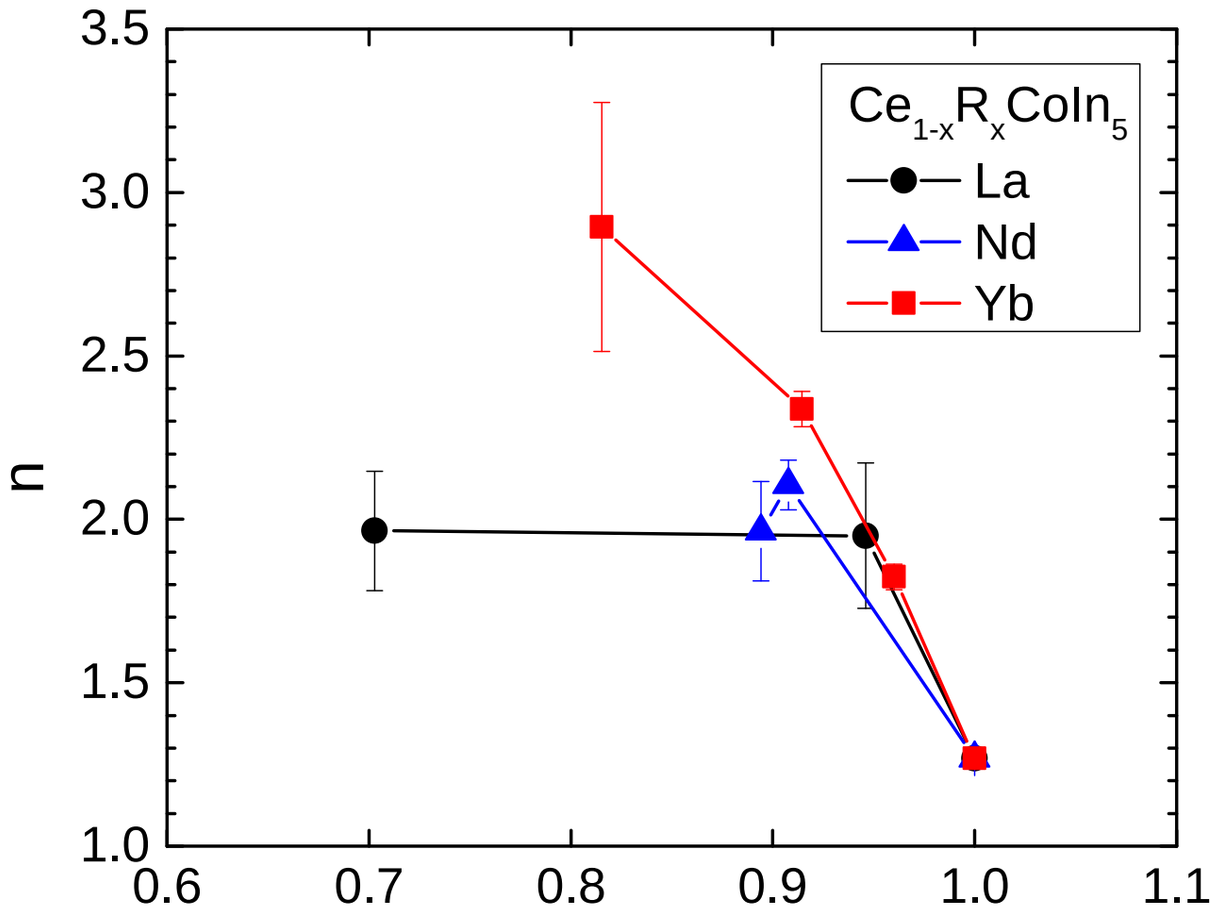
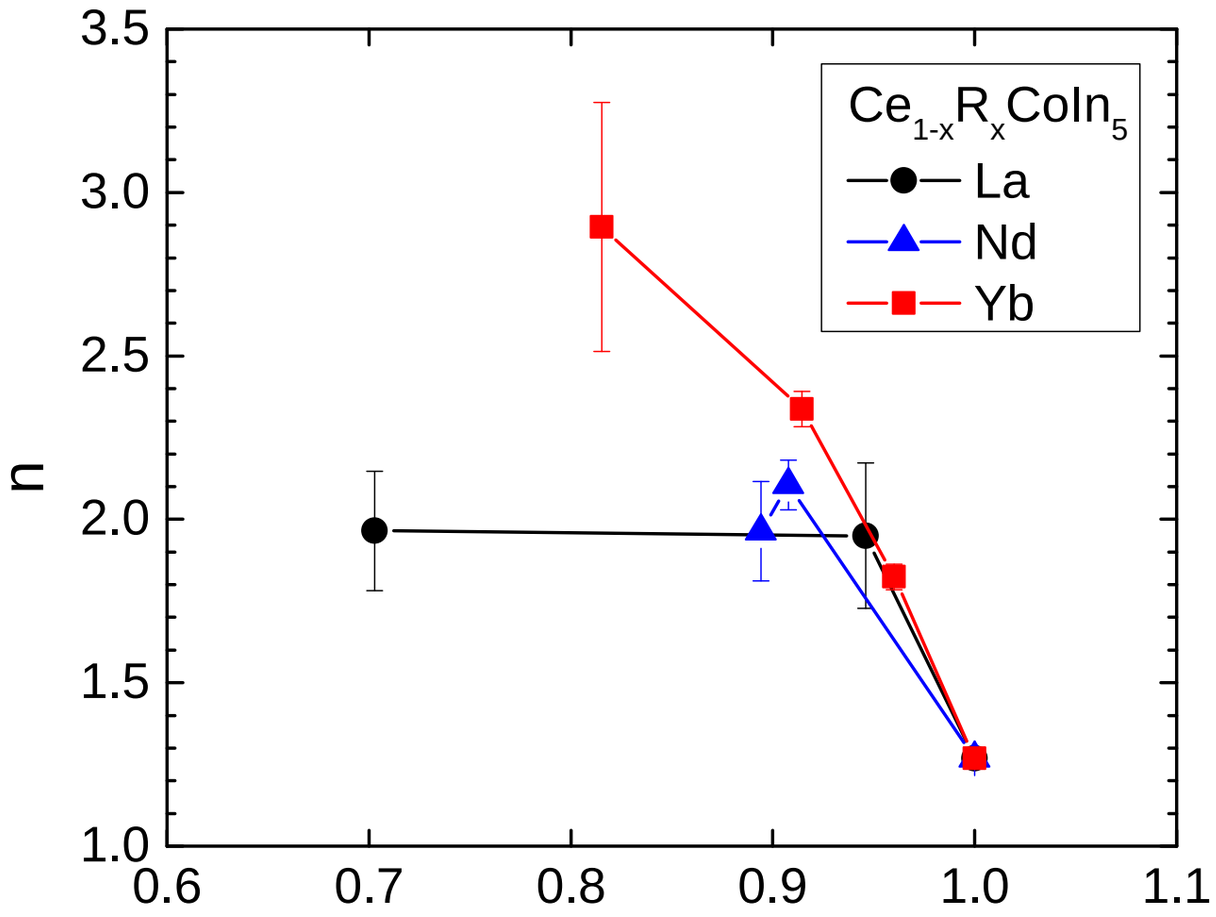


Bauer, G. Koutroulakis Yasuoko,(2014)





$$\lambda_L(T) = \lambda_L(0) + aT^n$$



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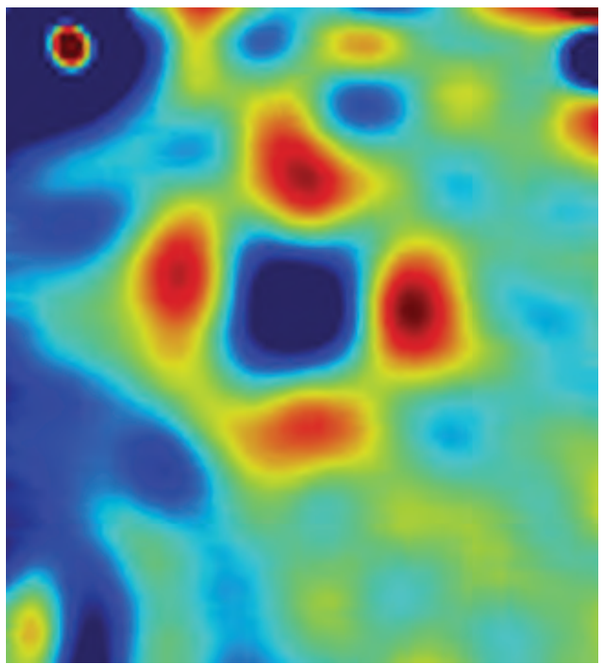
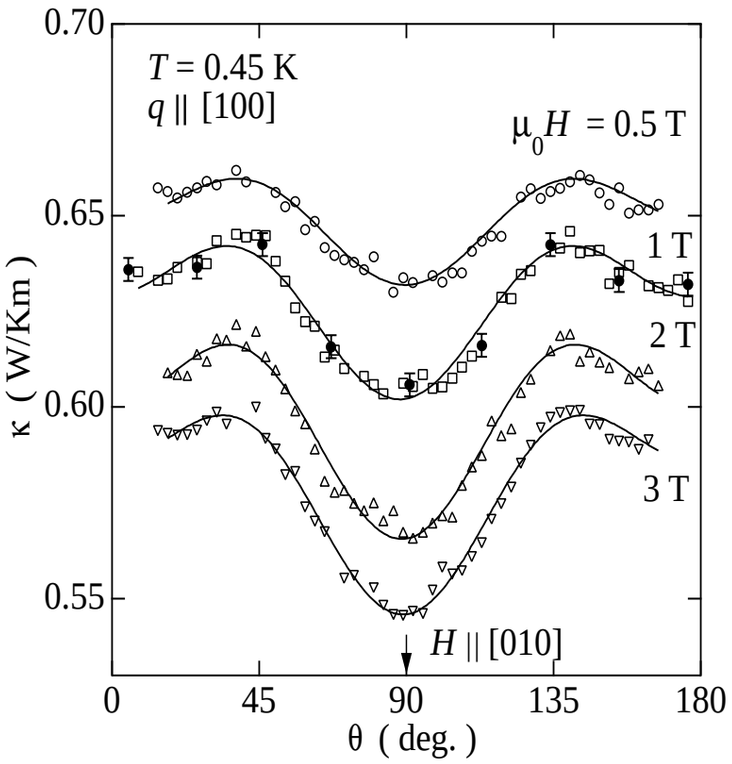
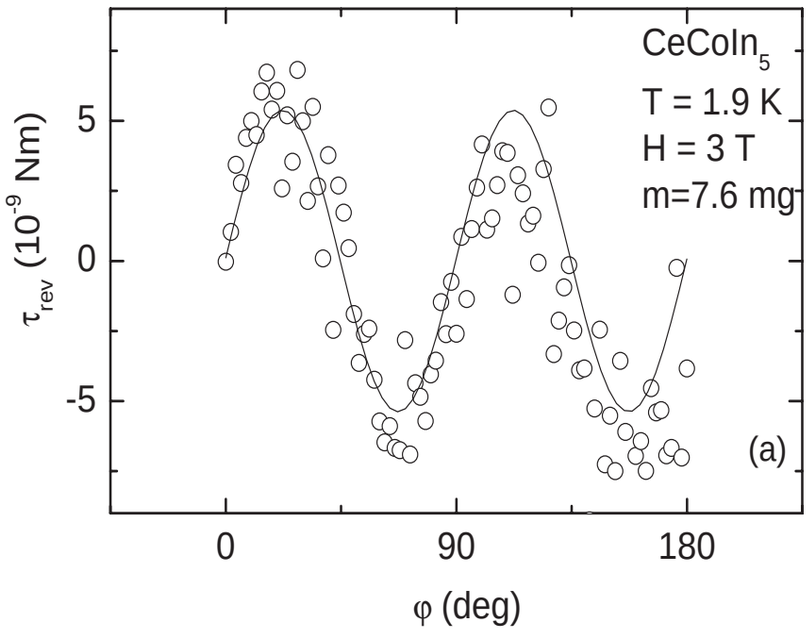


STM: B. Zhou *et al.*
Nature Phys. 9, 474 (2013)



Thermal cond:
K. Izawa *et al.* PRL 87,



Torque magnetometry:
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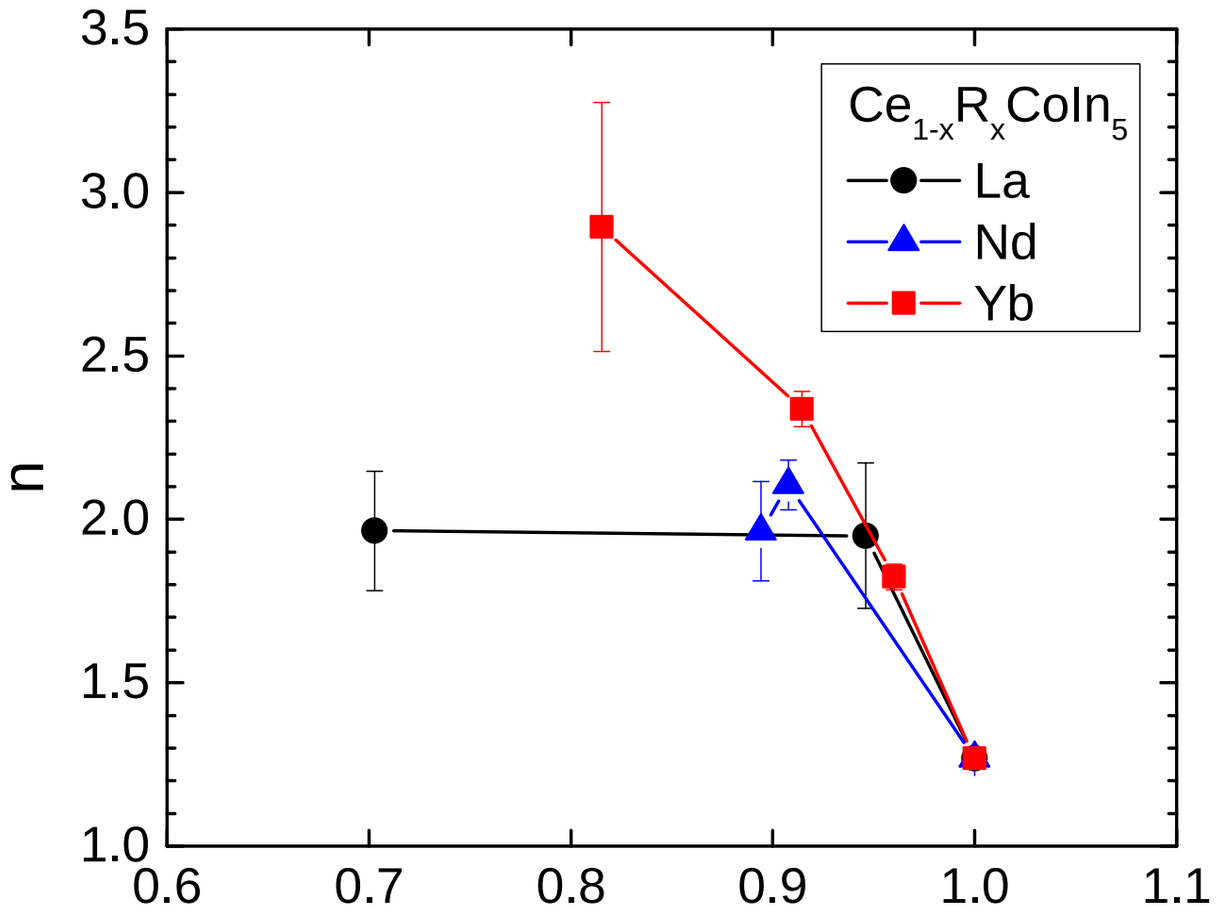

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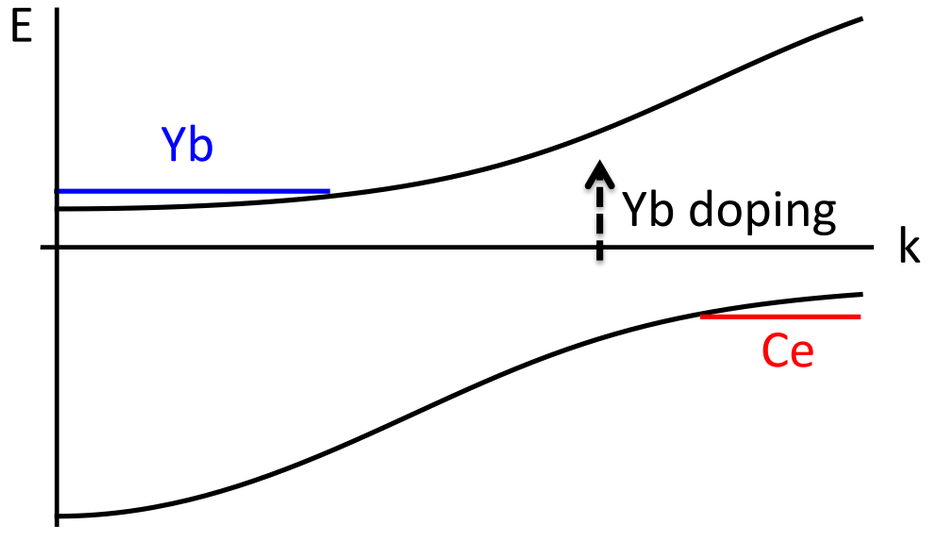

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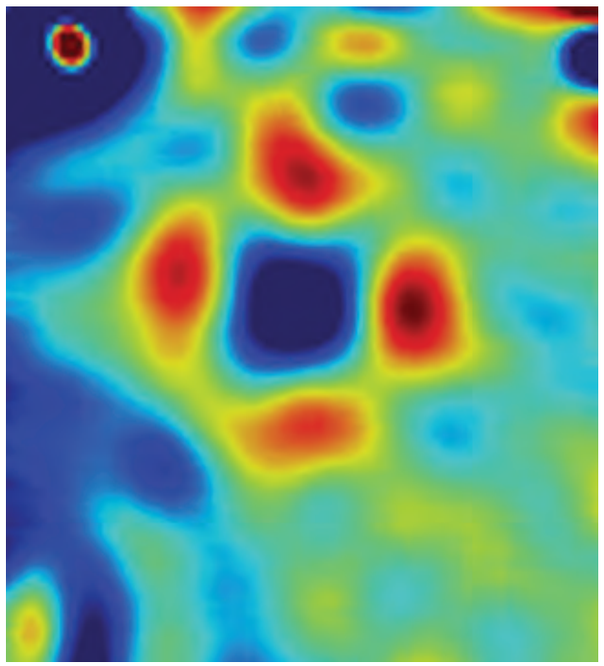

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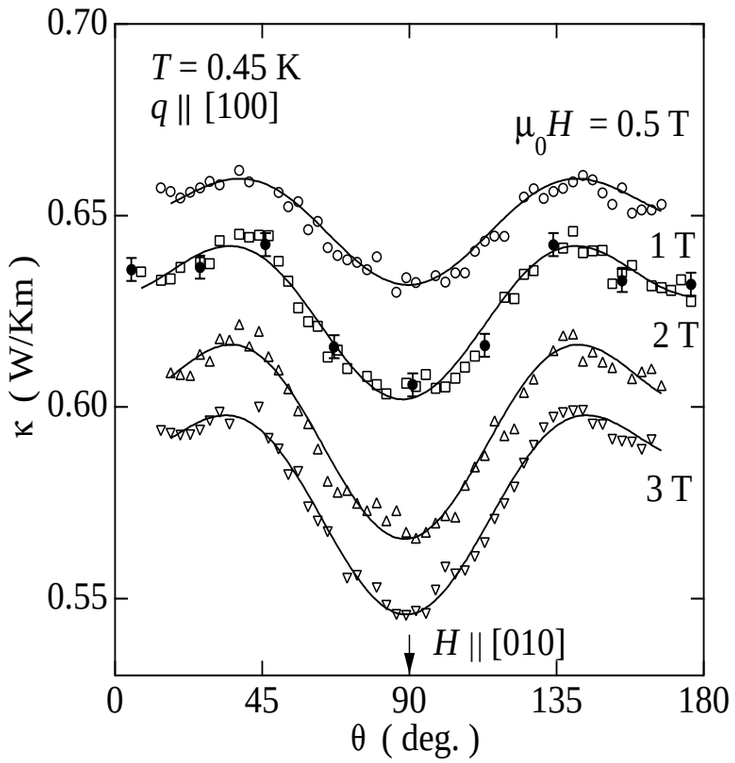

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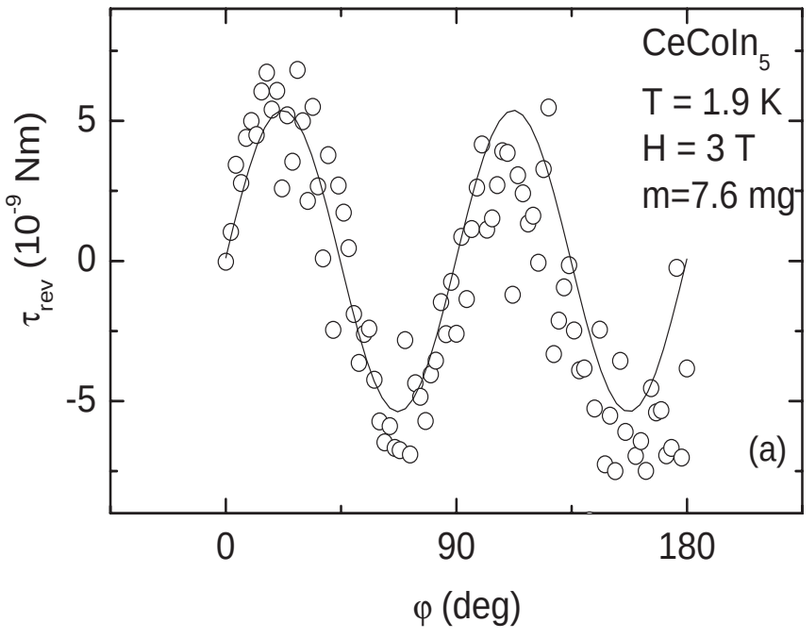

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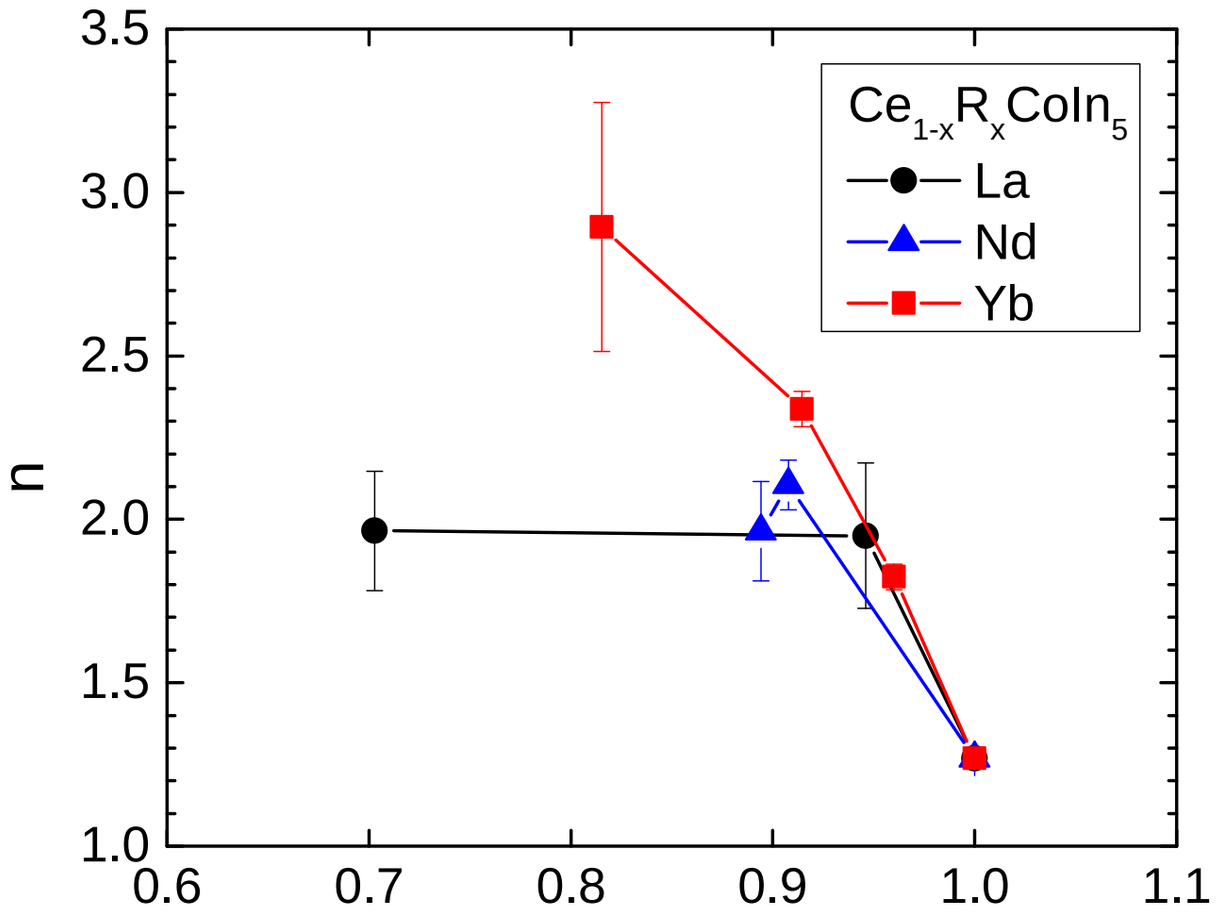

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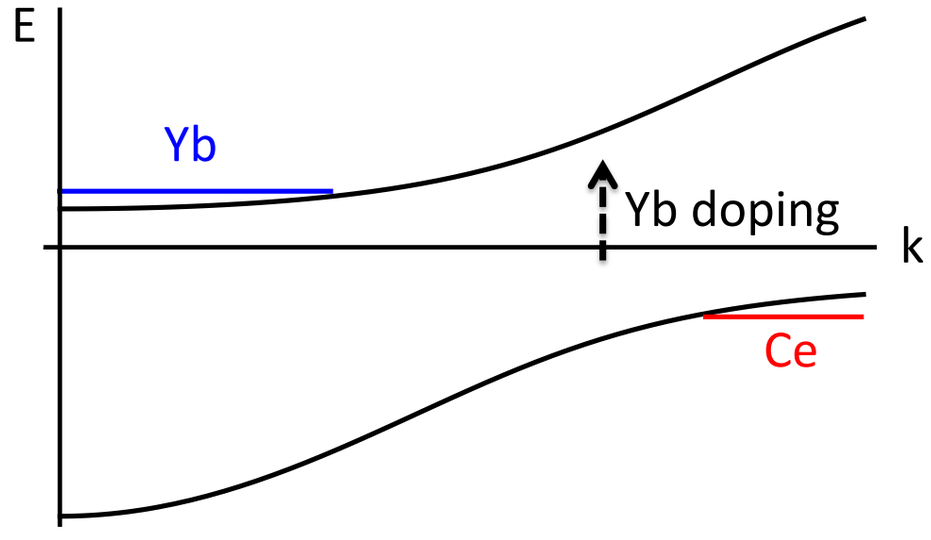
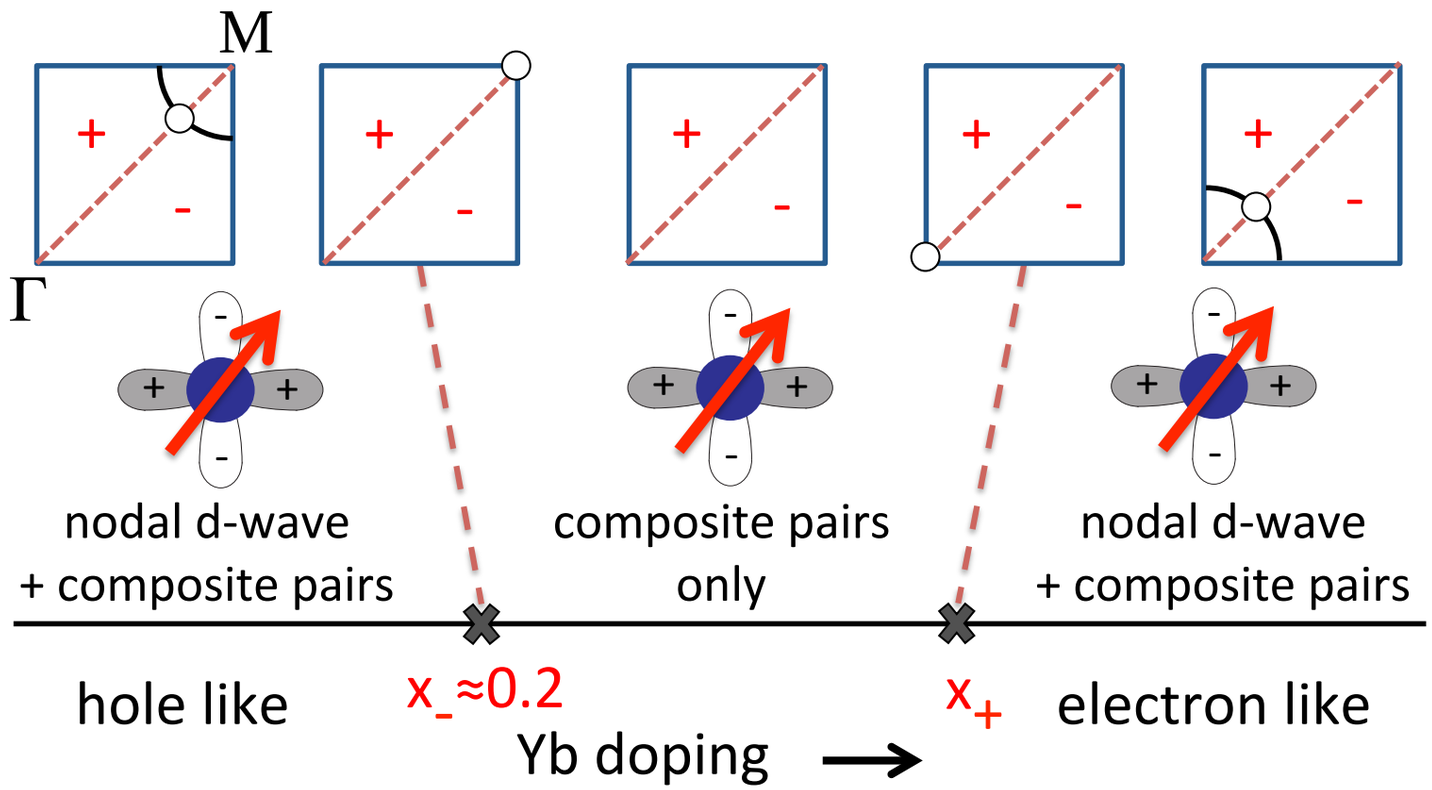




Erten, Flint et al, 2014

$$\rho_s = \rho_0 - \alpha T^{(1-2)} \quad \rho_s = \rho_0 - \beta T^{(3-4)} \quad \rho_s = \rho_0 - \alpha T^{(1-2)}$$

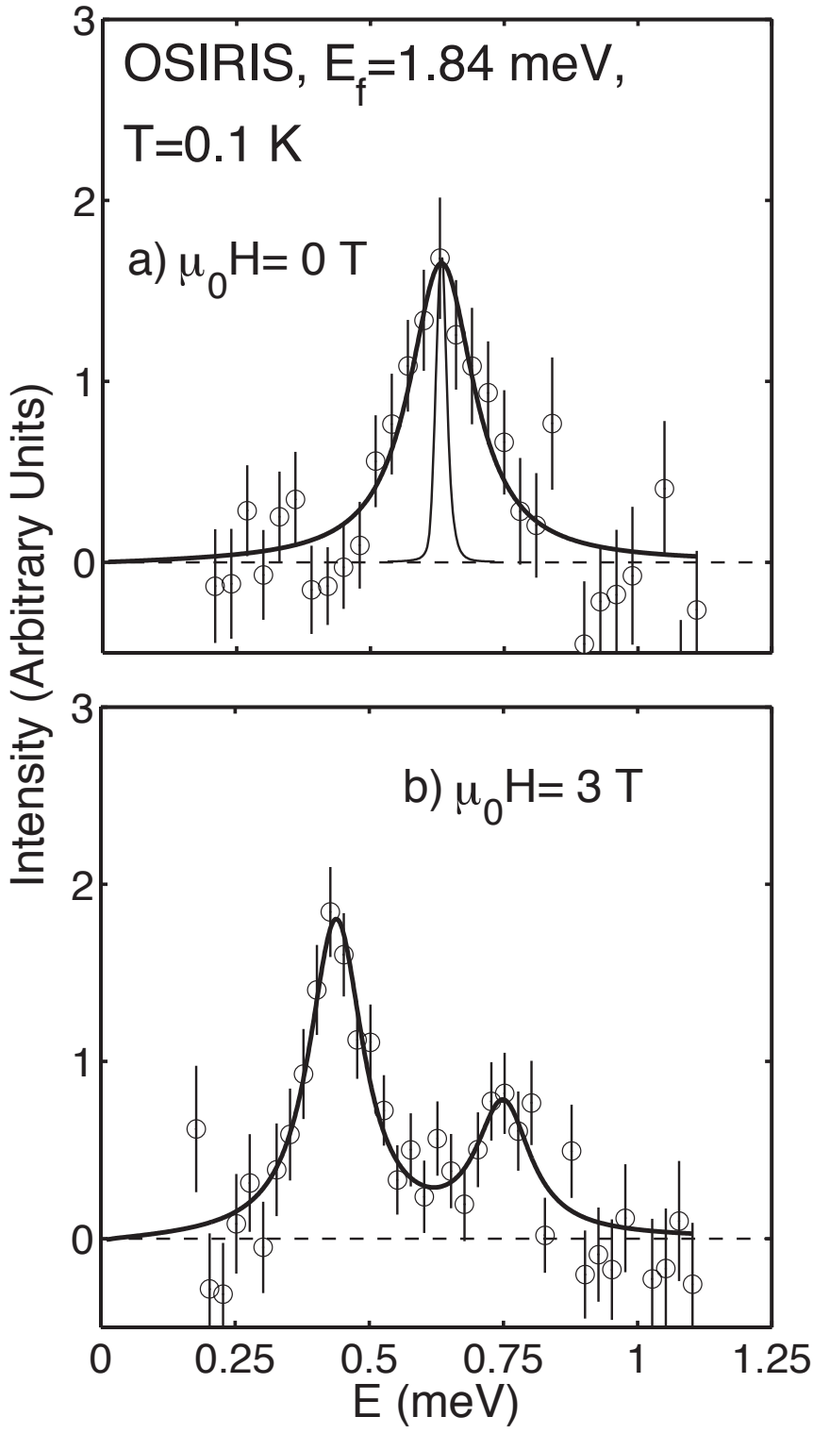
2. arXiv:1203.2189 [pdf, ps, other]

Magnetic field splitting of the spin-resonance in CeCoIn5

C. Stock, C. Broholm, Y. Zhao, F. Demmel, H.J. Kang, K. C. Rule, C. Petrovic

Comments: 5 pages, 4 figures

Subjects: **Superconductivity (cond-mat.supr-con)**



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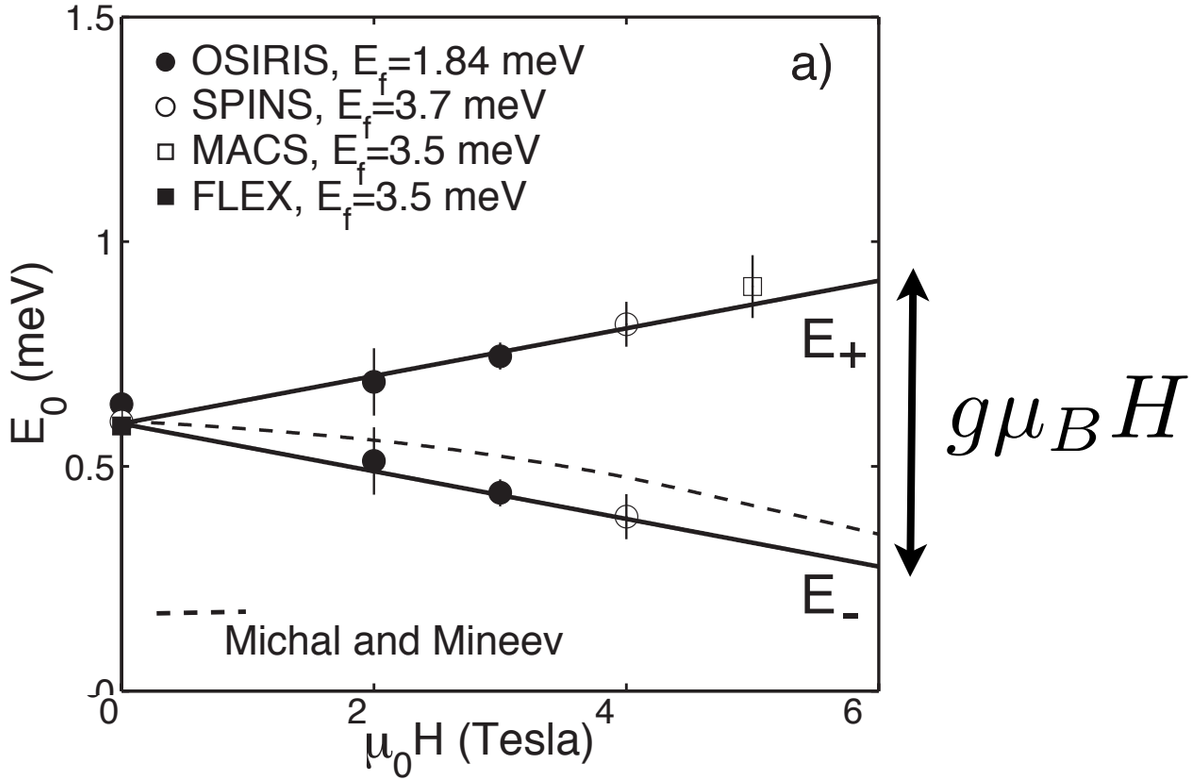
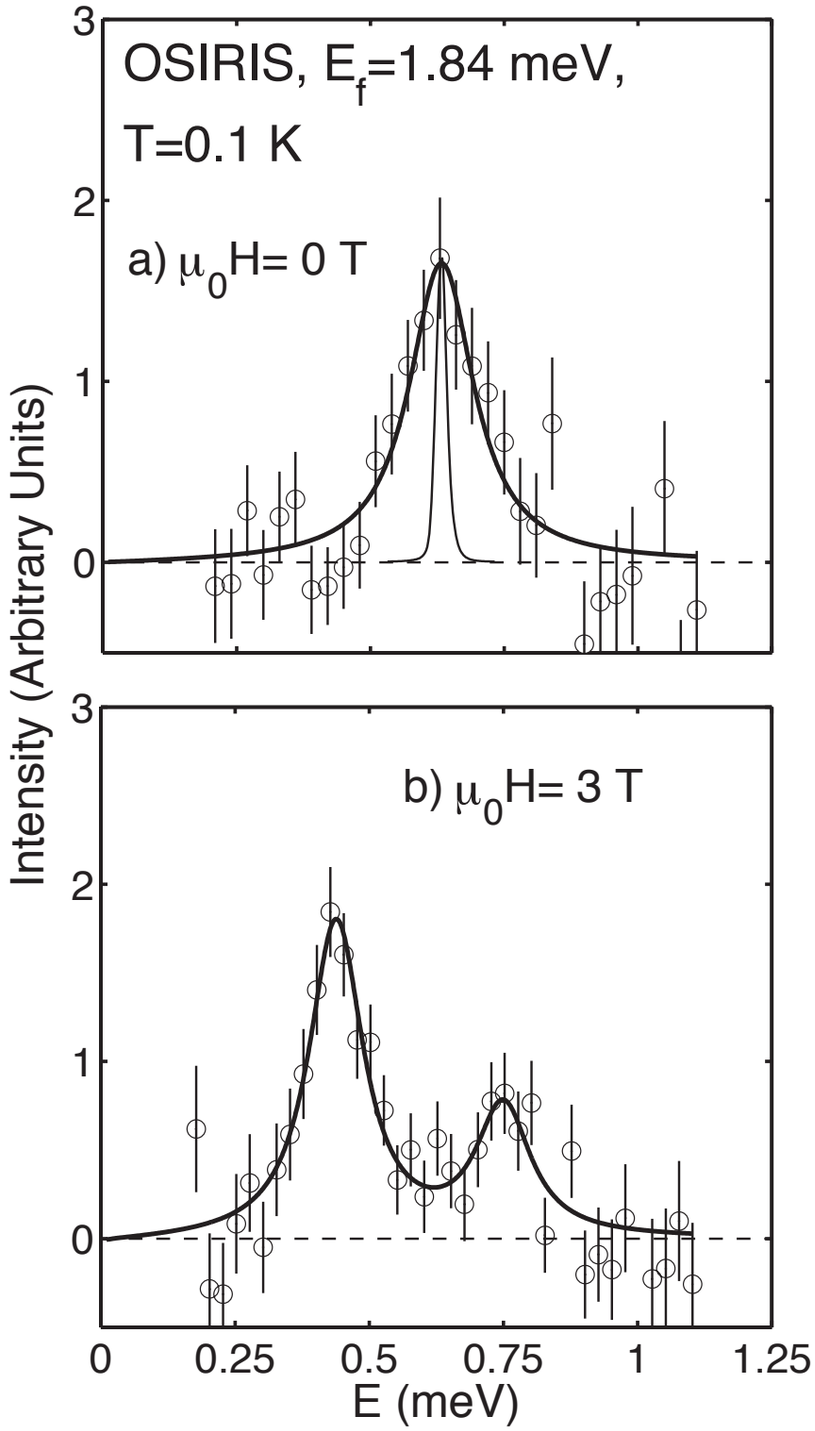


g=0.96 crystal fields give g=0.81



Conclusions

- Convergence of magnetism and superconductivity: require new concepts over and beyond spin fluctuation theory. TREMENDOUS POTENTIAL FOR DISCOVERY.

- 115 heavy fermion superconductors suggest a new kind of pairing: composite pairing, robust against disorder on magnetic site.

- Could the same phenomenon occur in d-electron materials, at much higher temperatures?

Hastatic Order in URu_2Si_2

Hasta: Spear (Latin). A new kind of spinor order.

Piers Coleman
Center for Materials Theory
Rutgers, USA

Rebecca Flint
(MIT)

Premi Chandra
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P.Chandra, R. Flint and P.C
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- Hidden order in URu_2Si_2

- Giant Ising Anisotropy and implications

- Hastatic Order.

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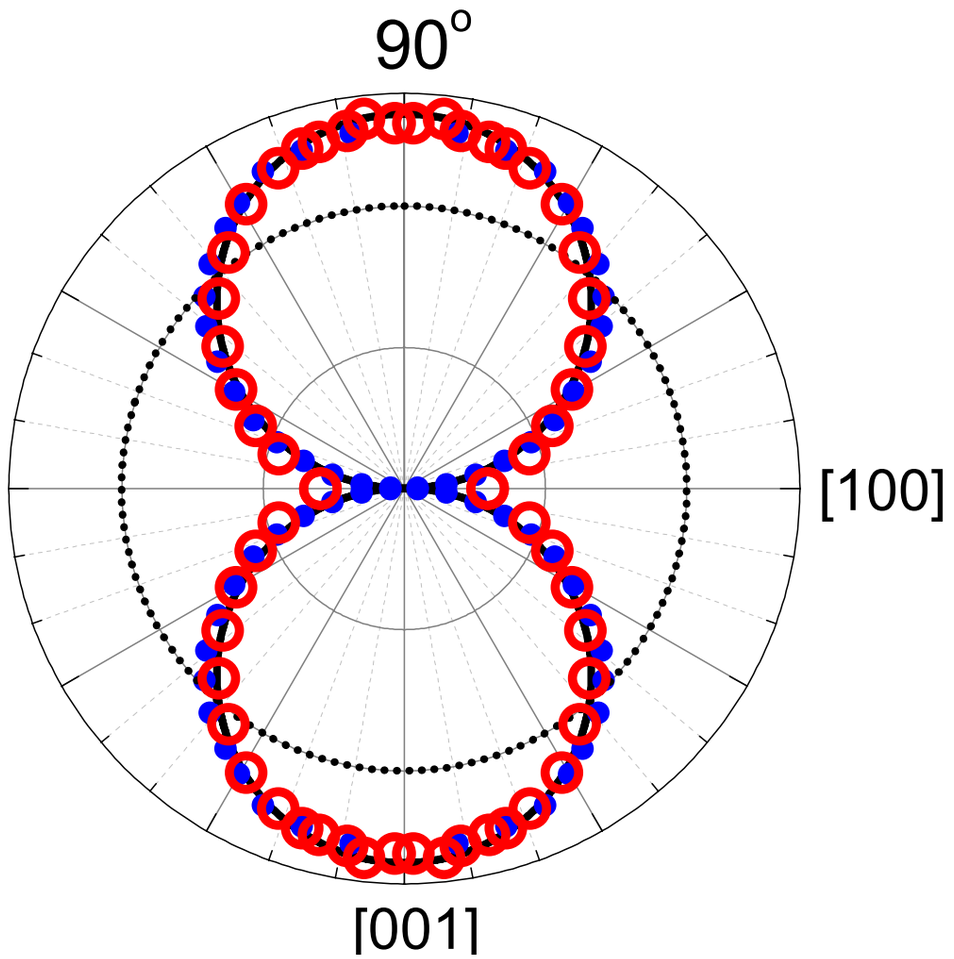

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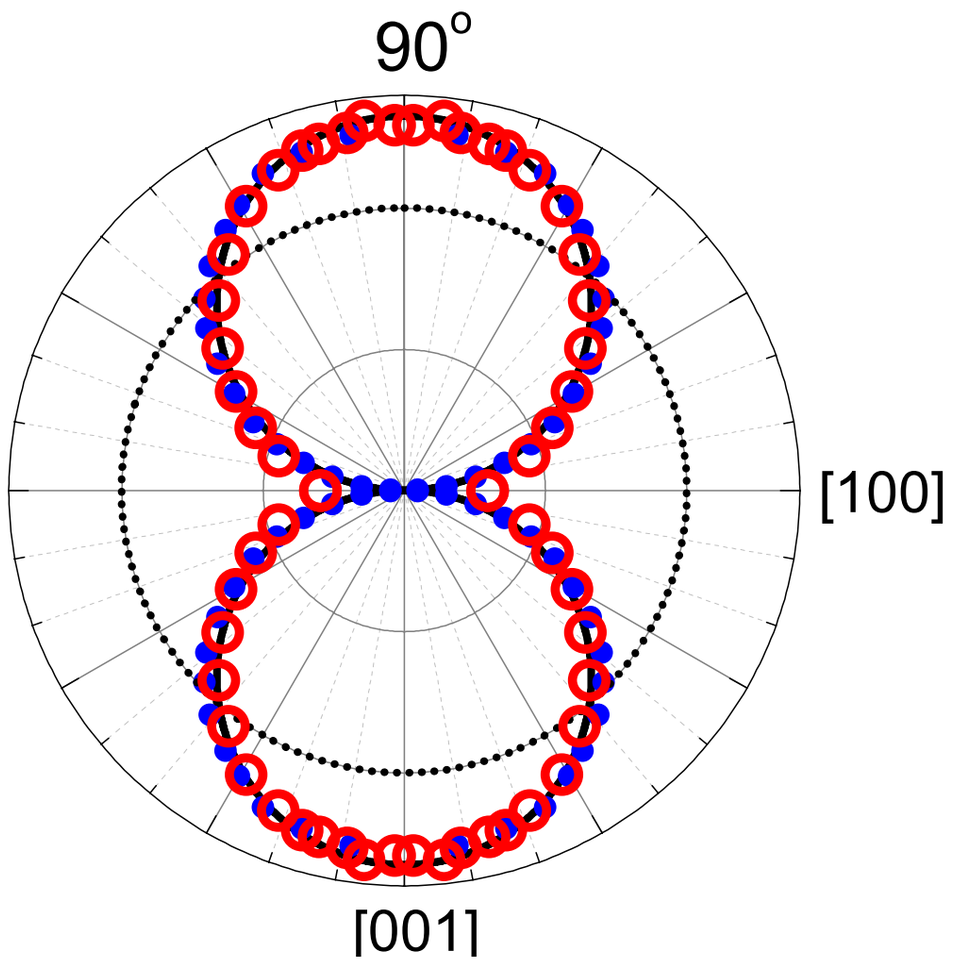

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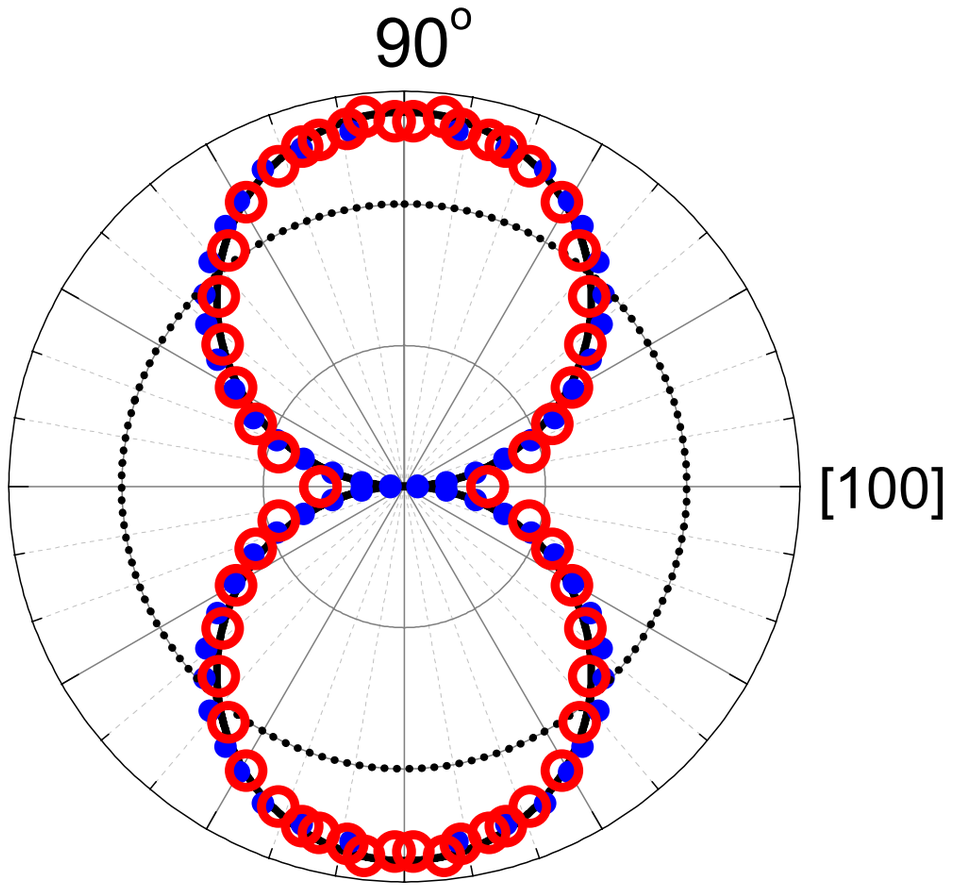
Take-home:
Observation of (Perfect) Ising Quasiparticles



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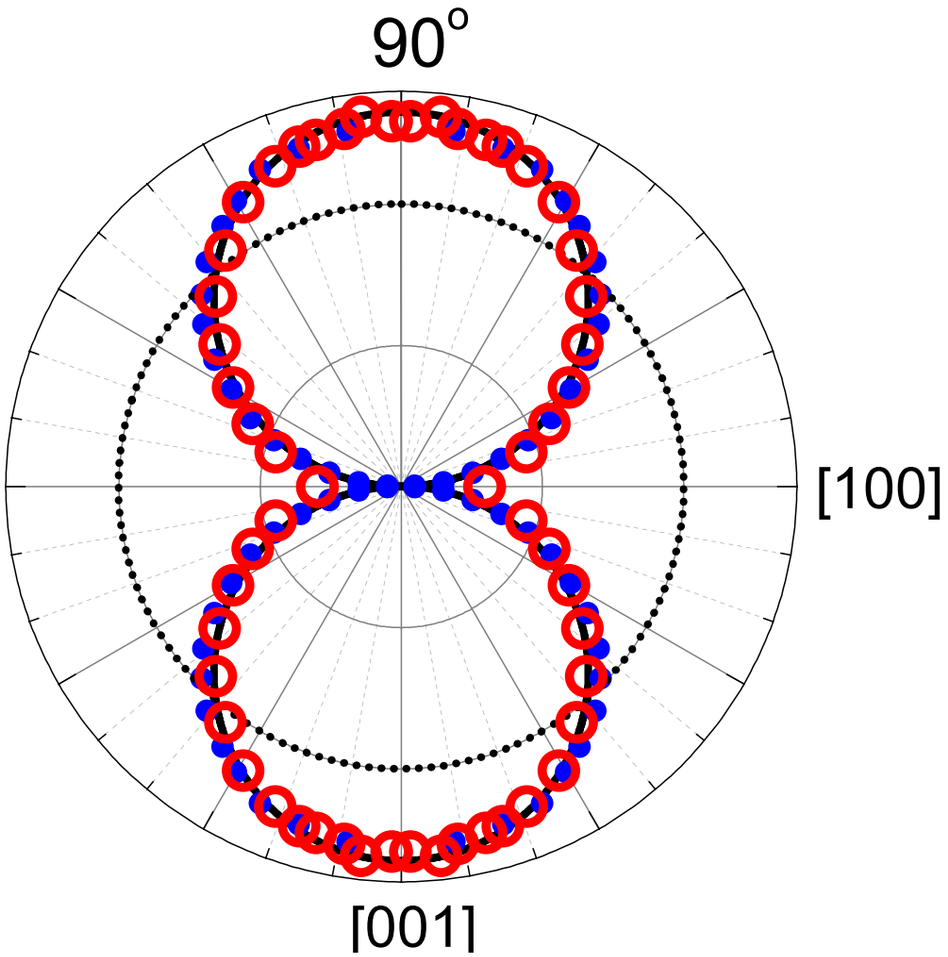
Take-home:

Observation of (Perfect) Ising Quasiparticles

↓

Many body hybridization

Spin 1/2 e $\rightleftharpoons$ Integer spin ($J_z=\pm1$) Doublets



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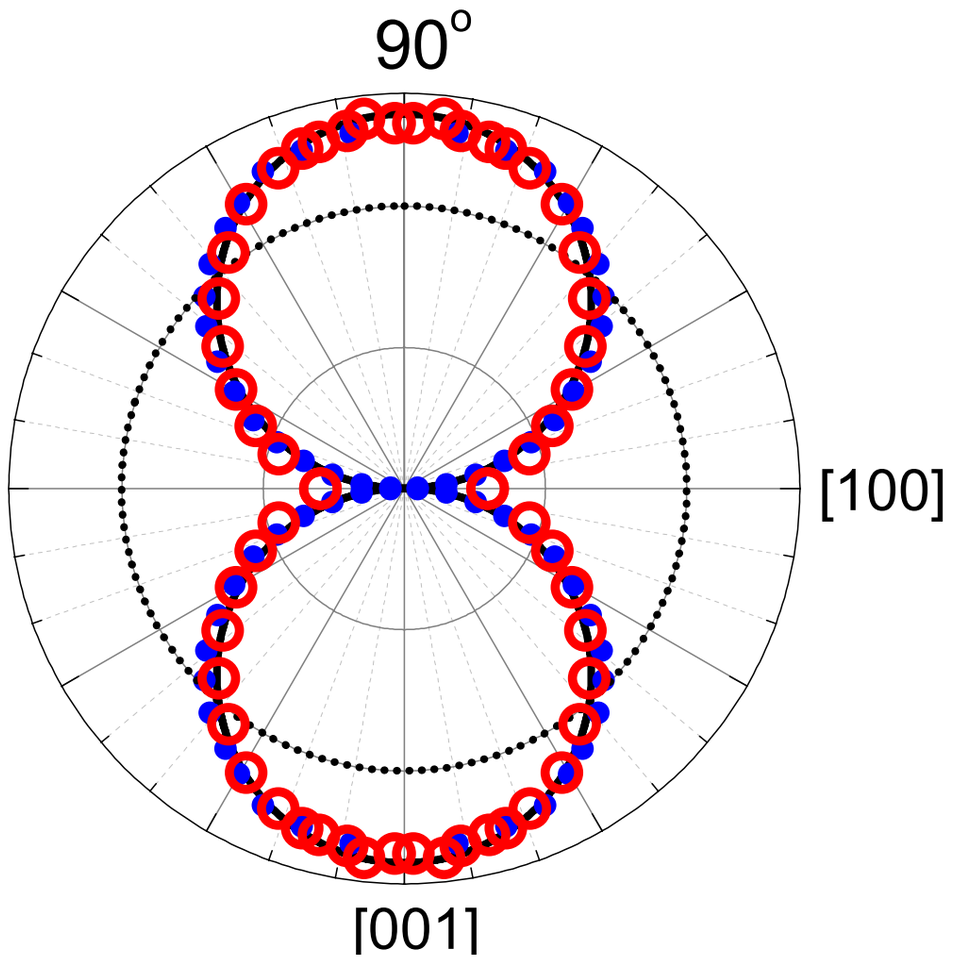
Proposal: Order parameter is a spinor



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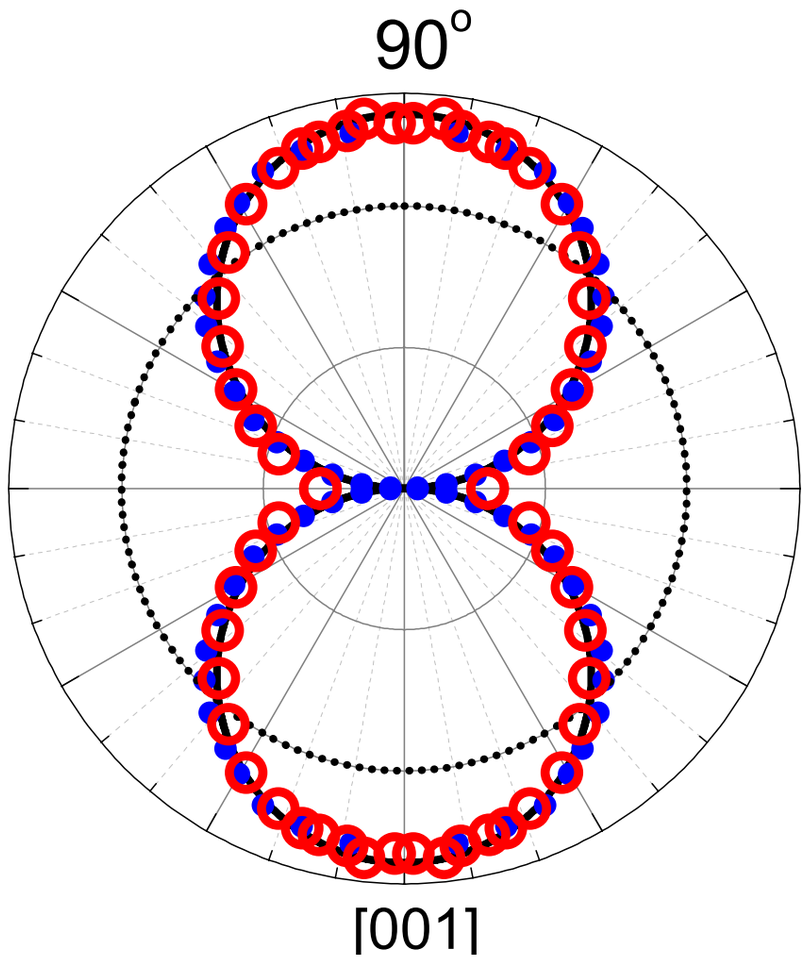
↓

Proposal: Order parameter is a spinor
(mixing J & J+1/2, breaking double time reversal)

$$\Theta^2 = (-1)^{2J}$$



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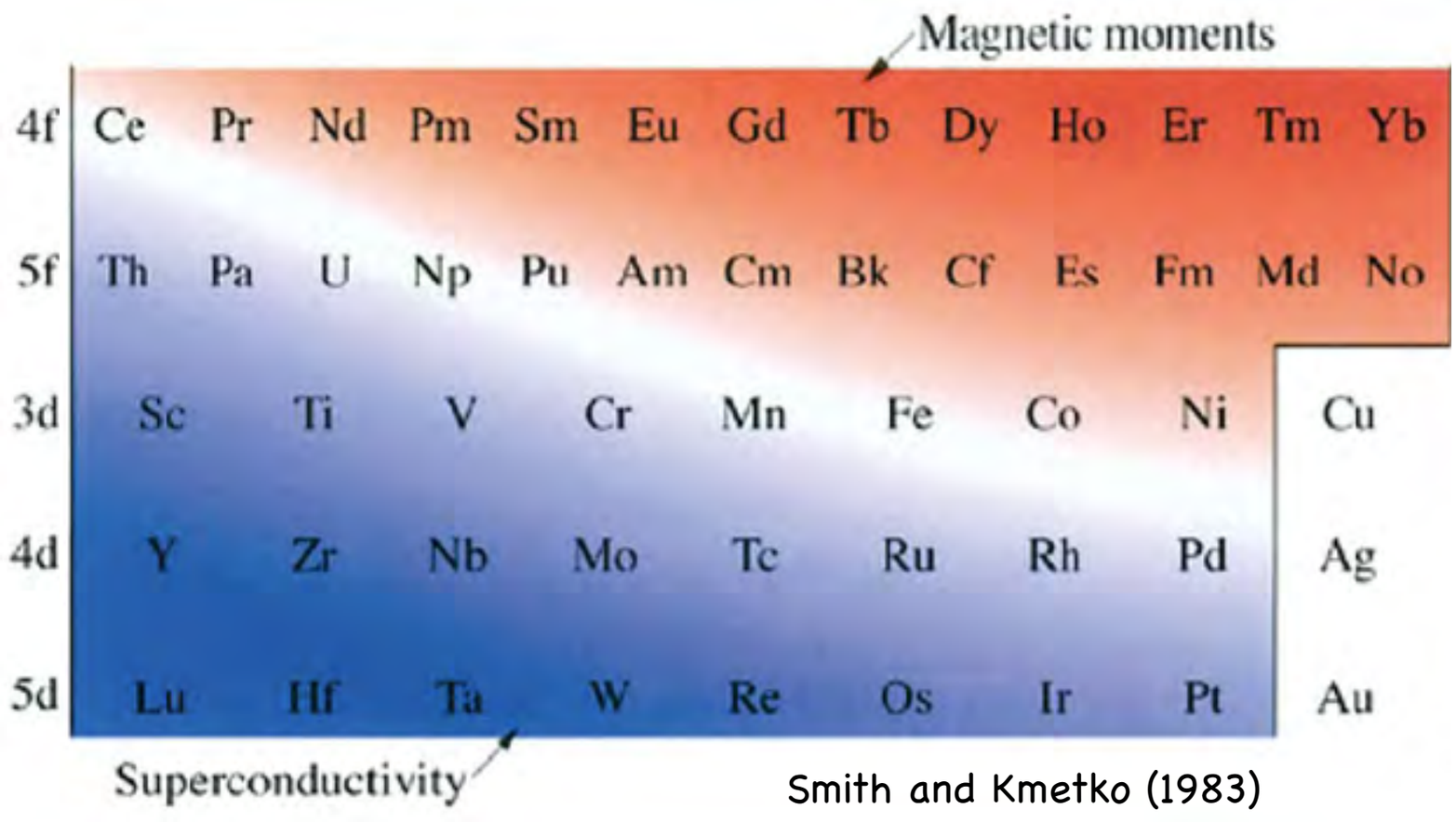
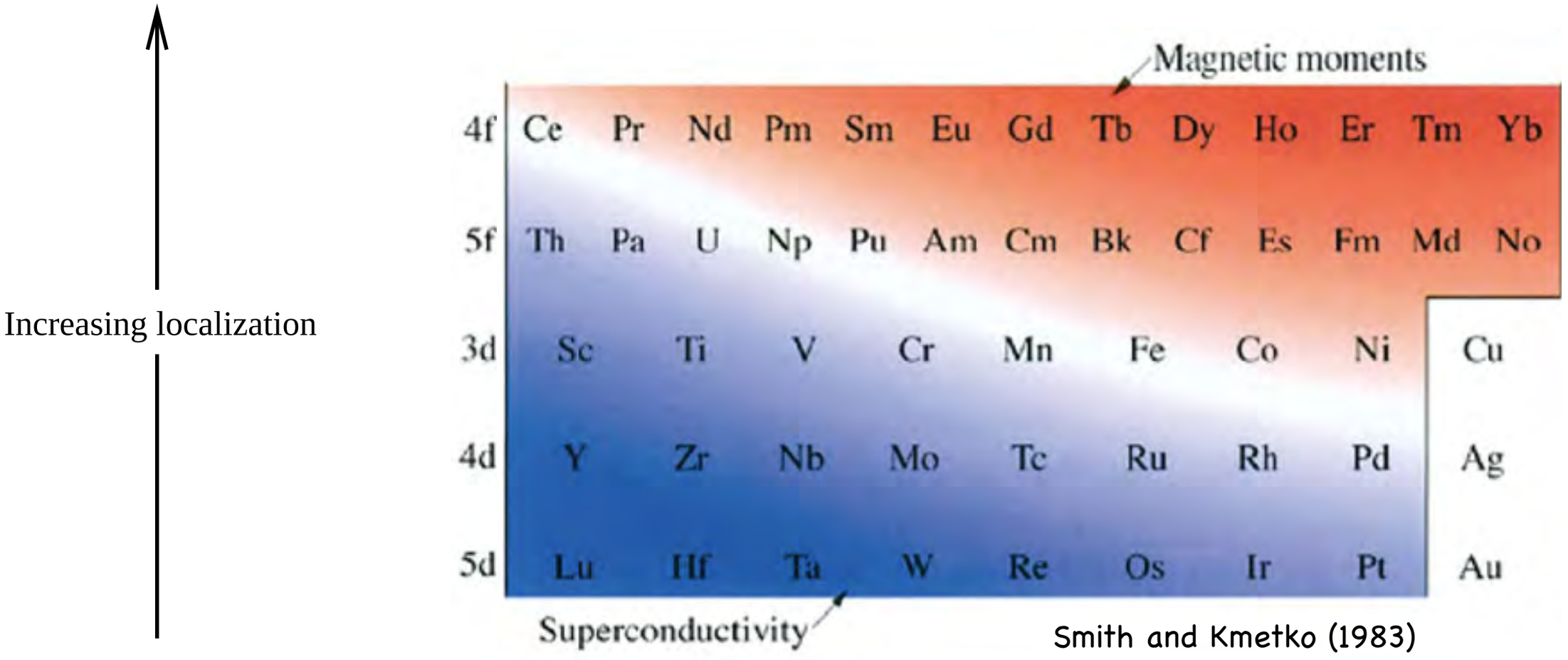
URu_2Si_2: The Hidden Order Mystery

Kmetko-Smith Phase Diagram



Smith and Kmetko (1983)

Kmetko-Smith Phase Diagram

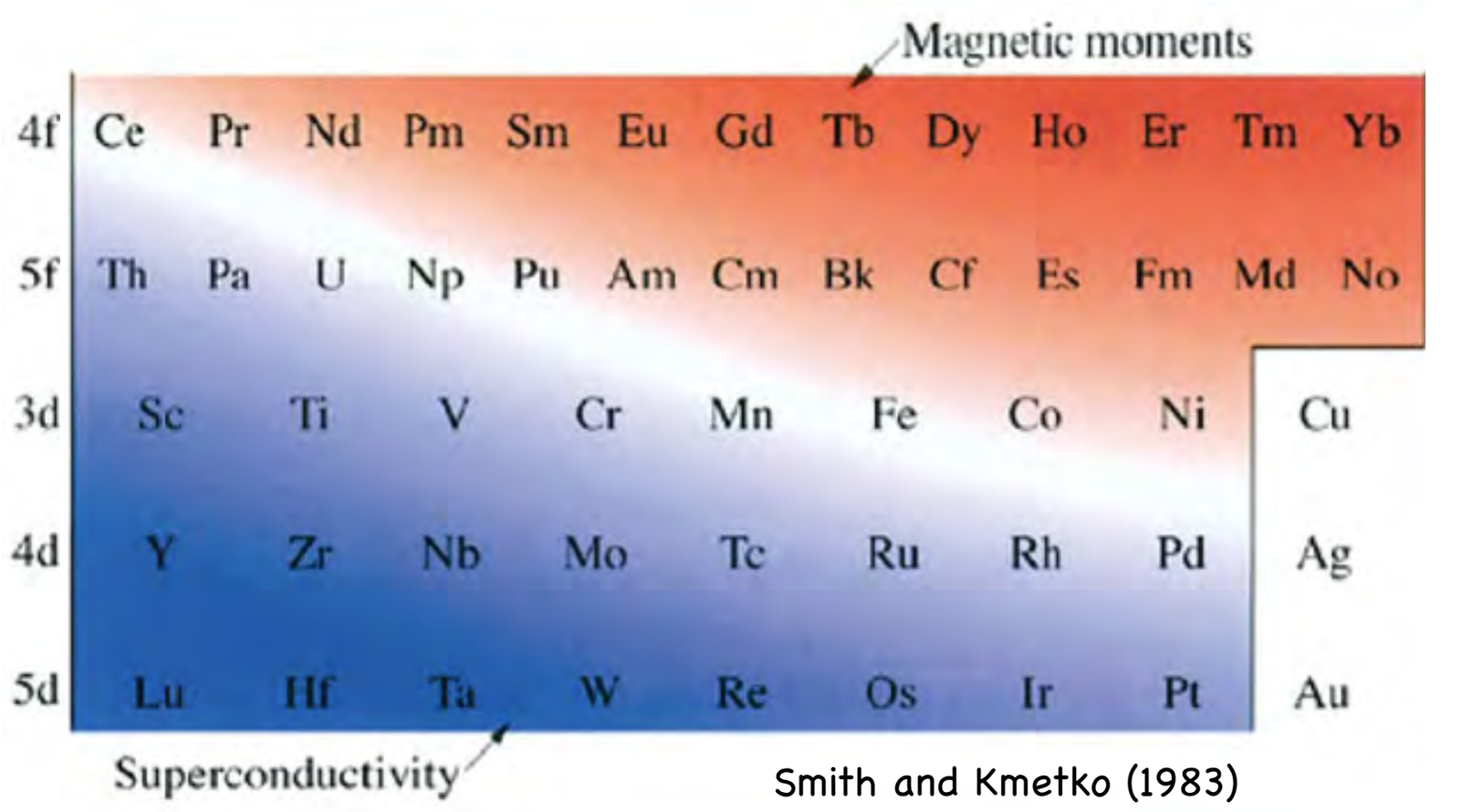


Smith and Kmetko (1983)

Kmetko-Smith Phase Diagram

Increasing localization

Increasing localization



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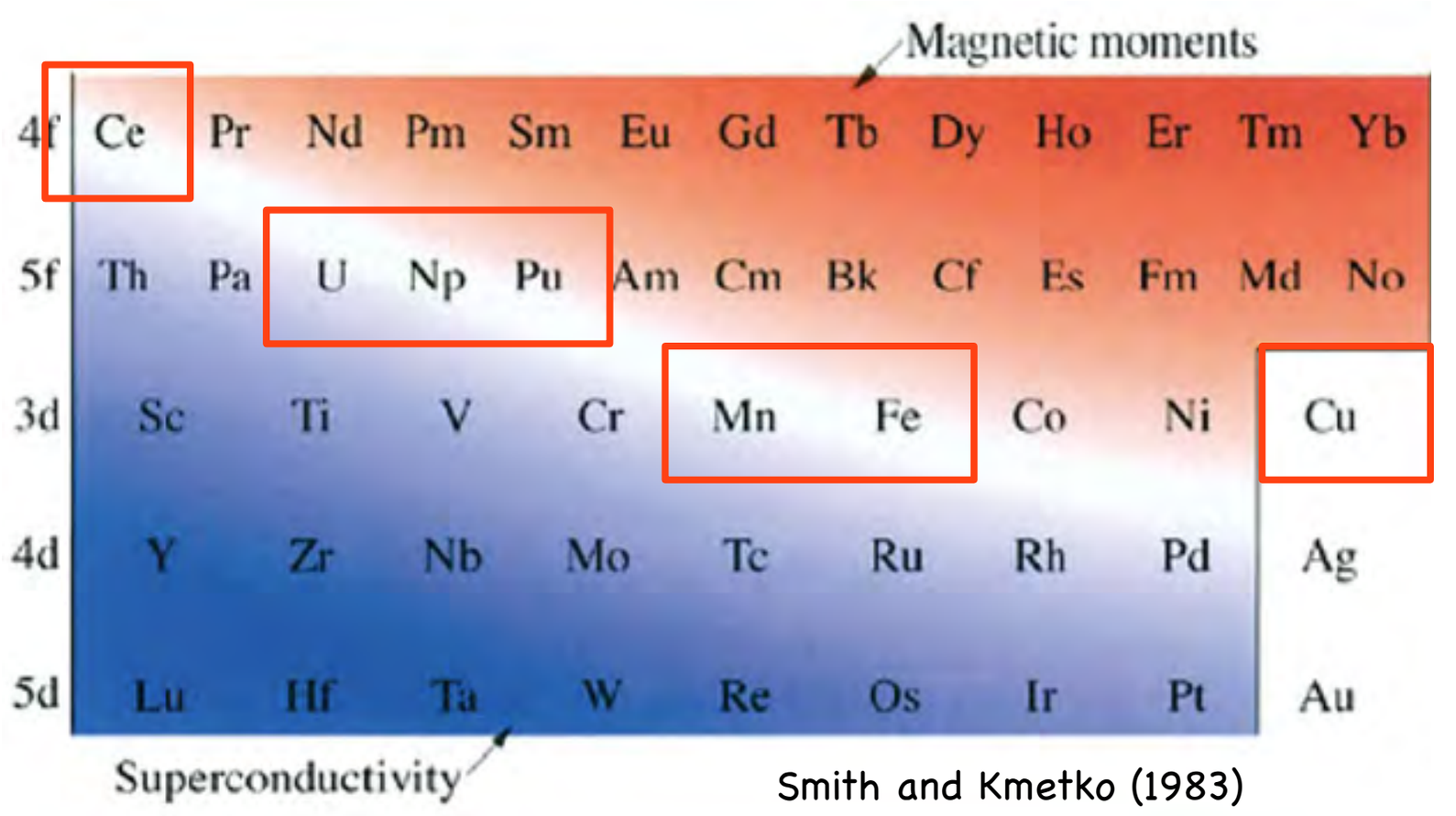

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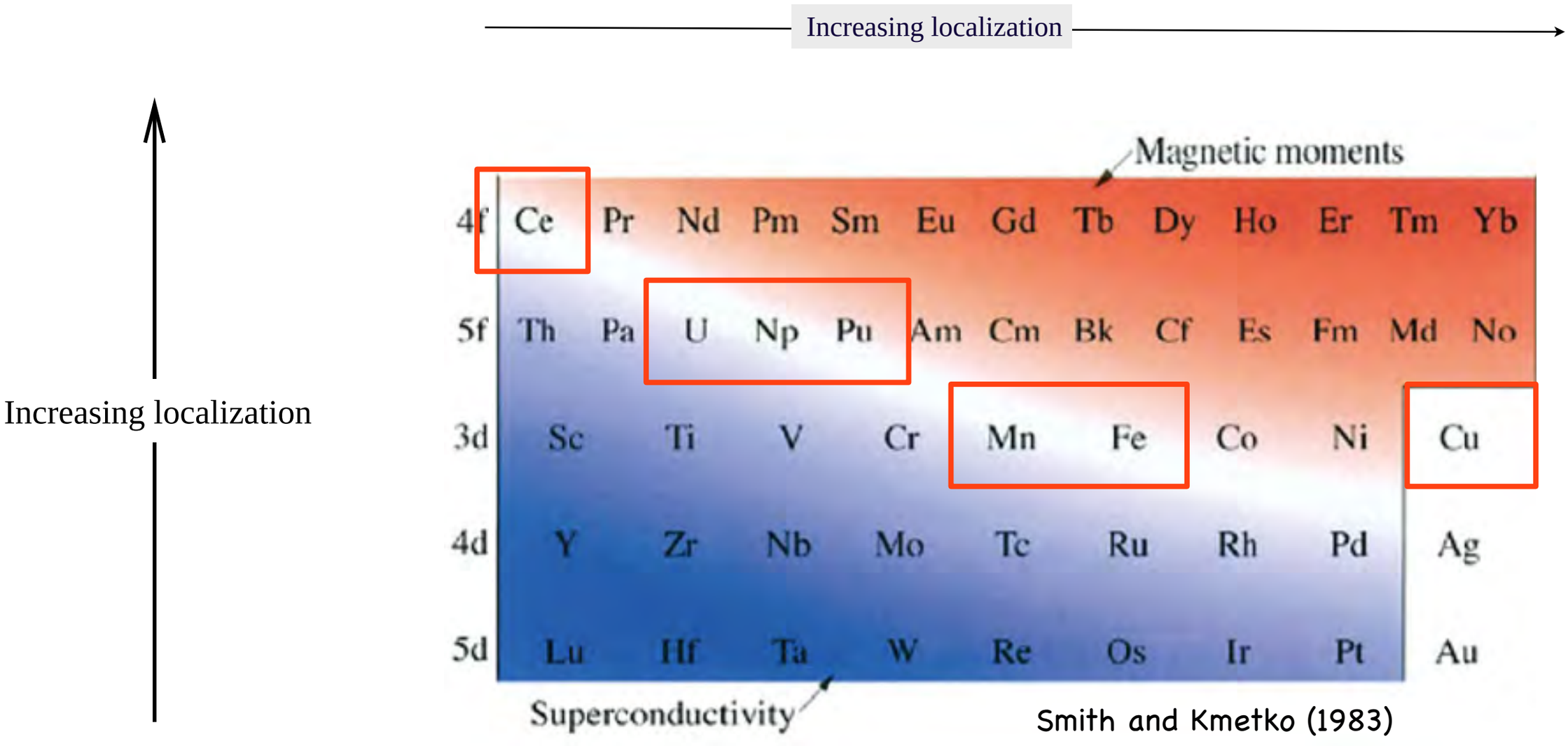
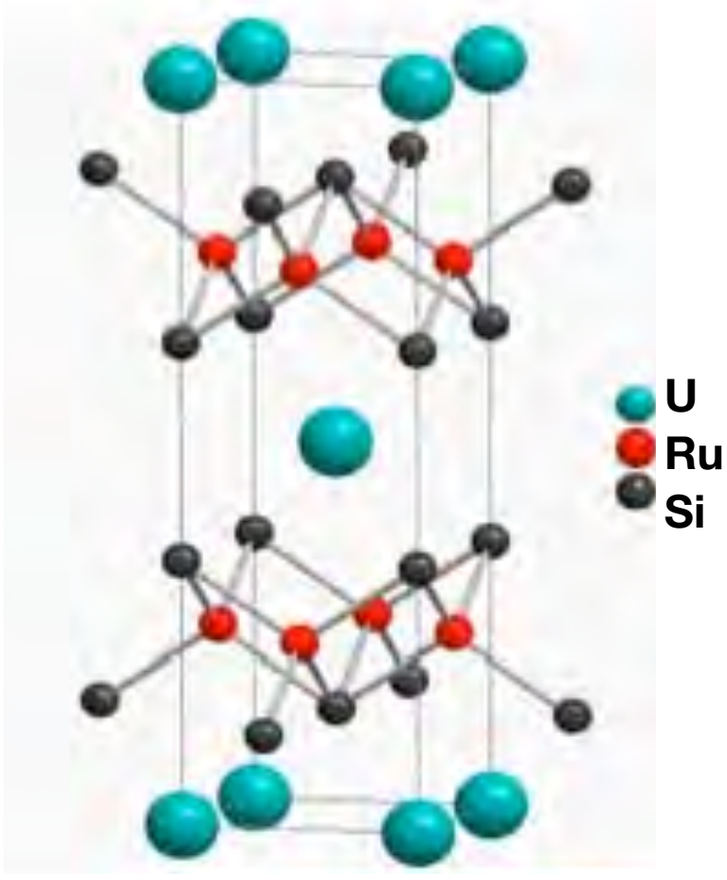


A lot of action at the brink of localization.

Hidden Order in URu_2Si_2



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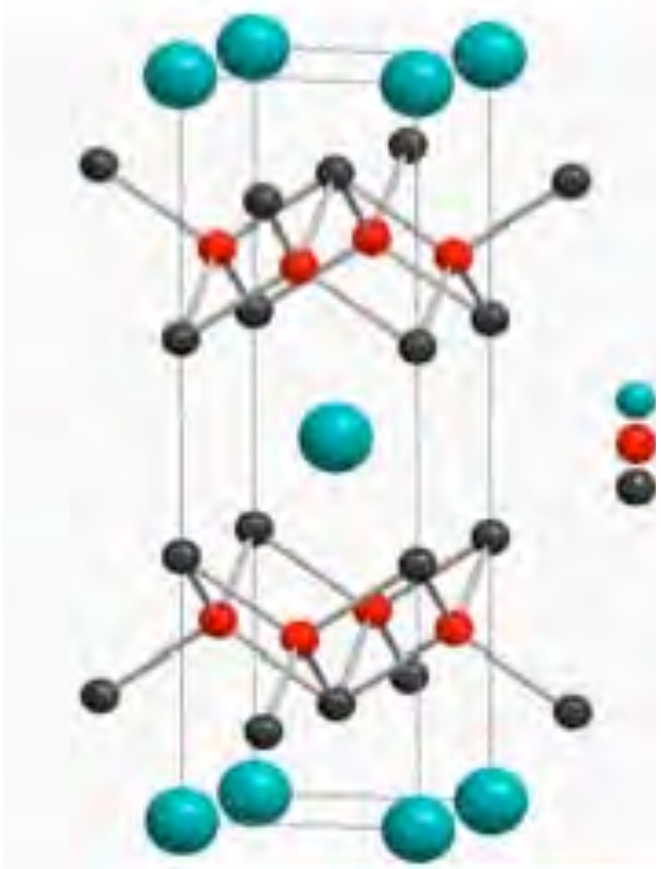

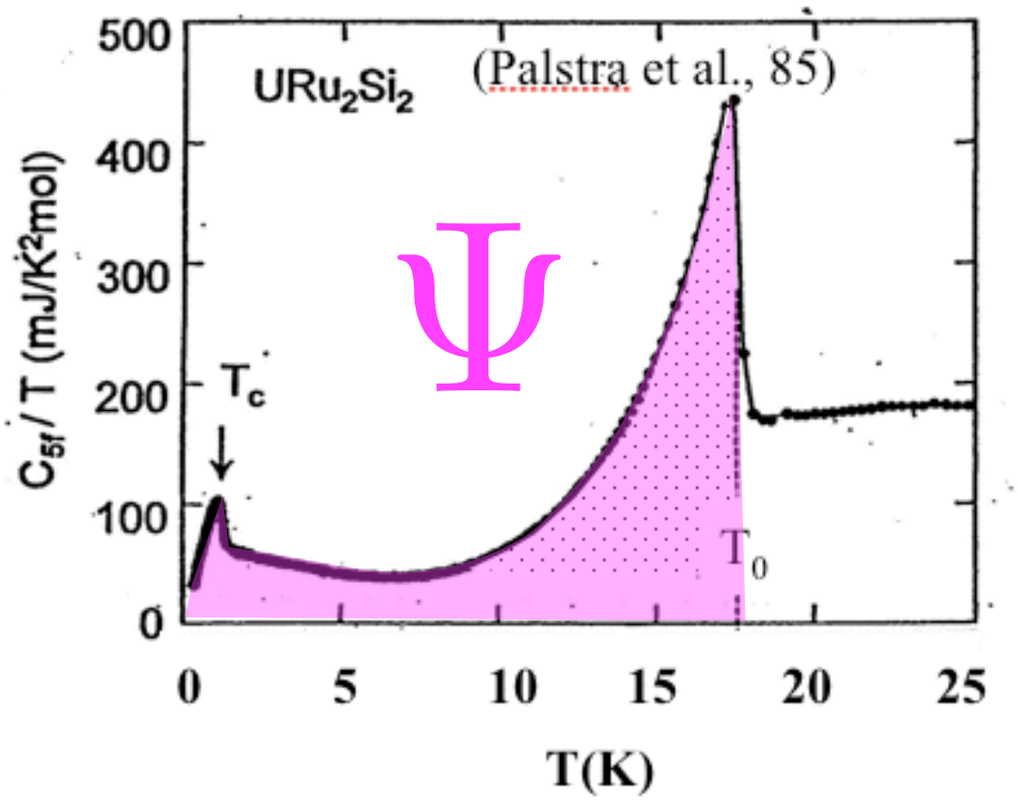

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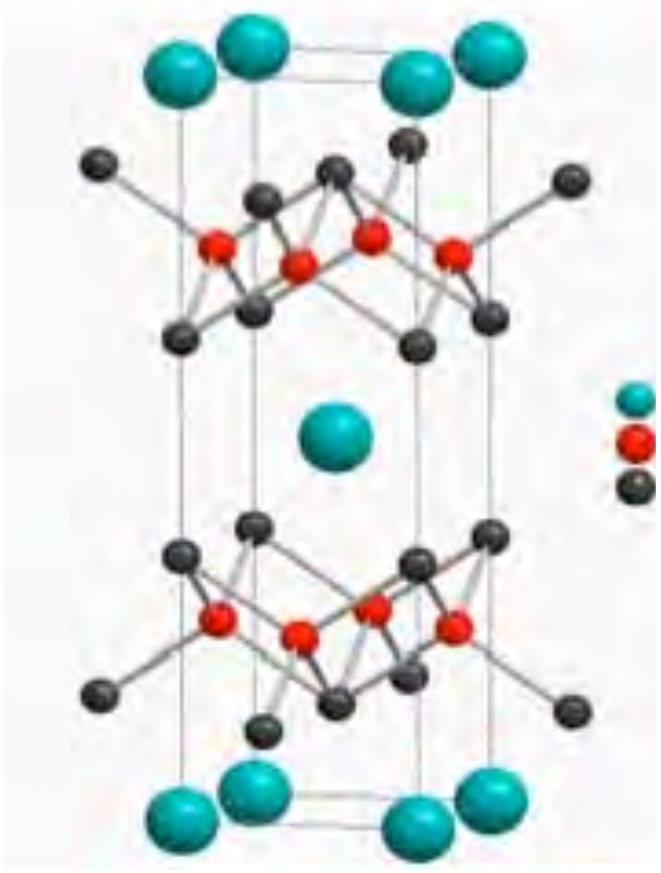
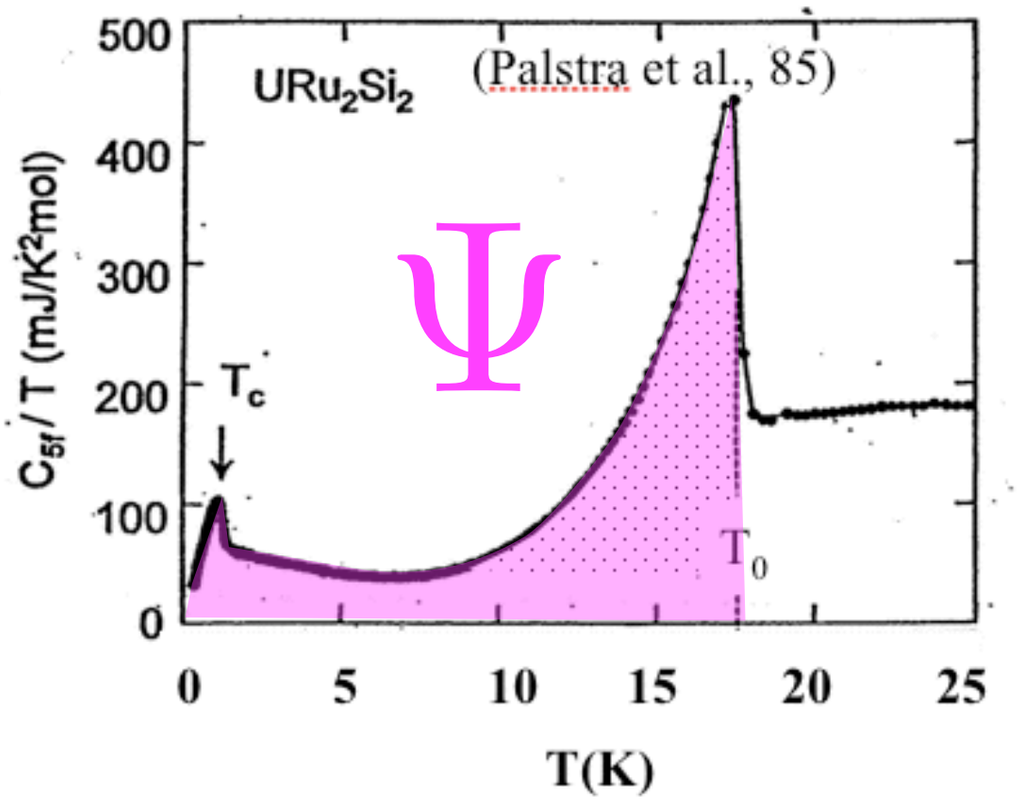


$$\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$$



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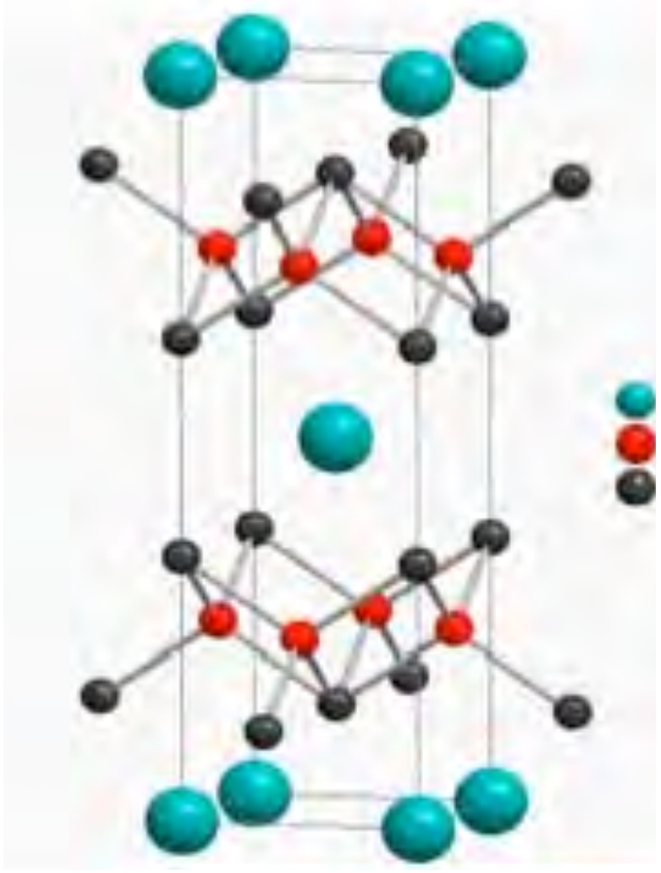
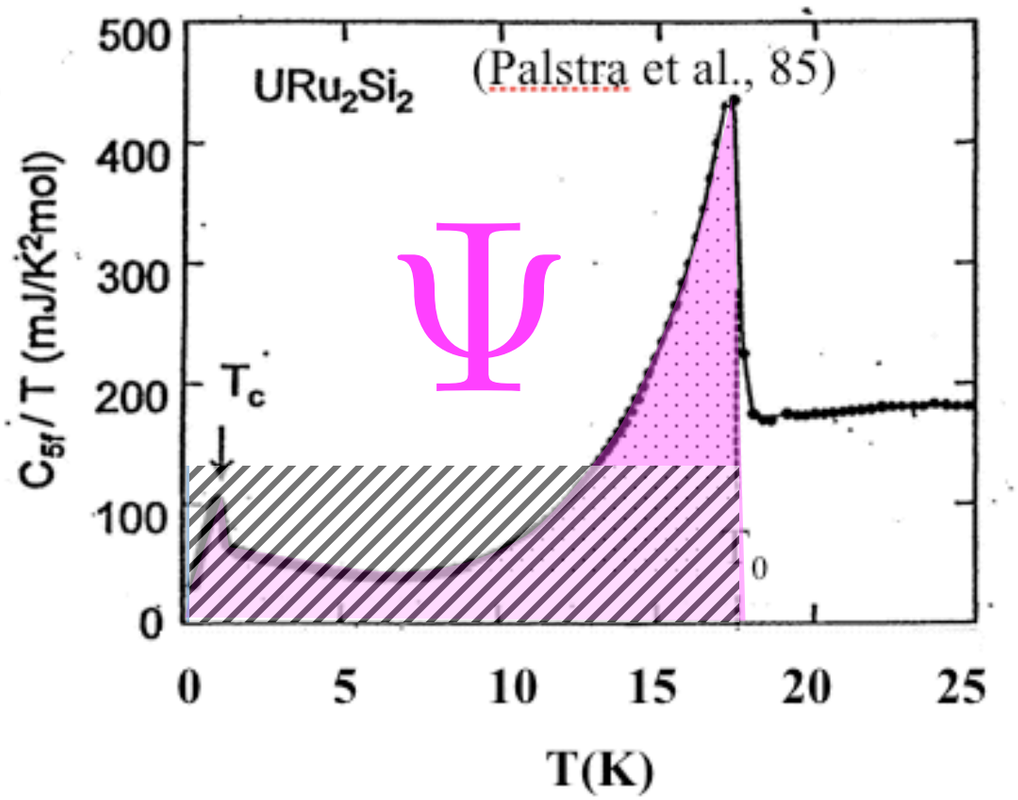


$$\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$$



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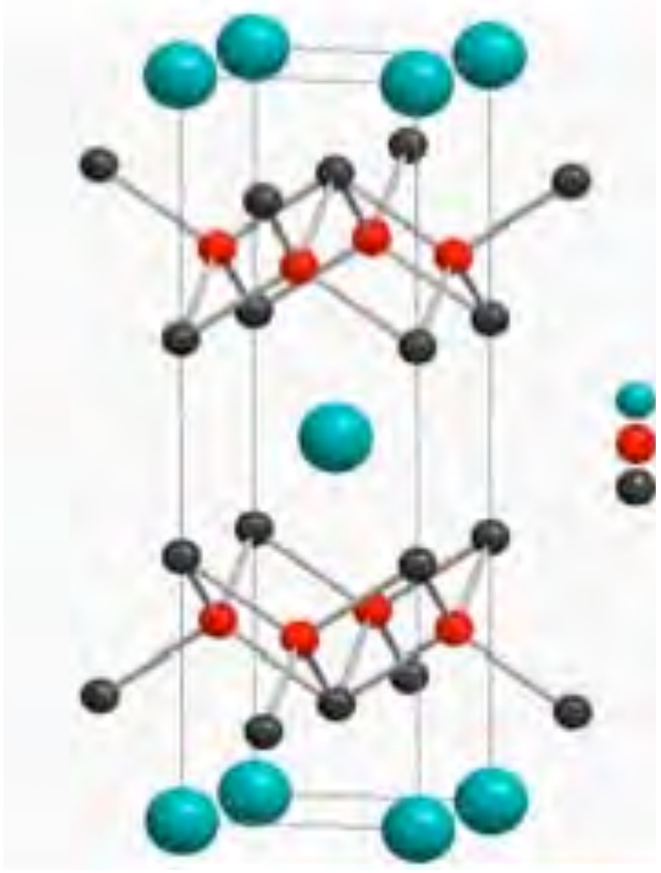

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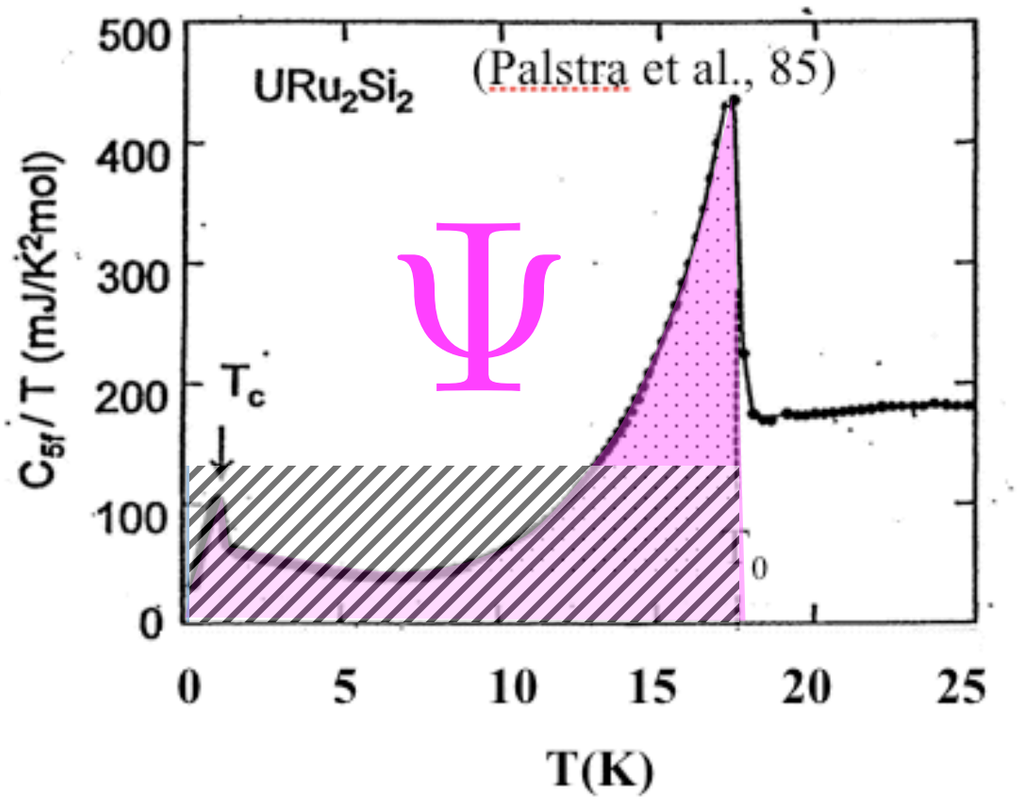
=0.14 x 17.5 K

=2.45 J/mol/K

=0.42 R ln 2



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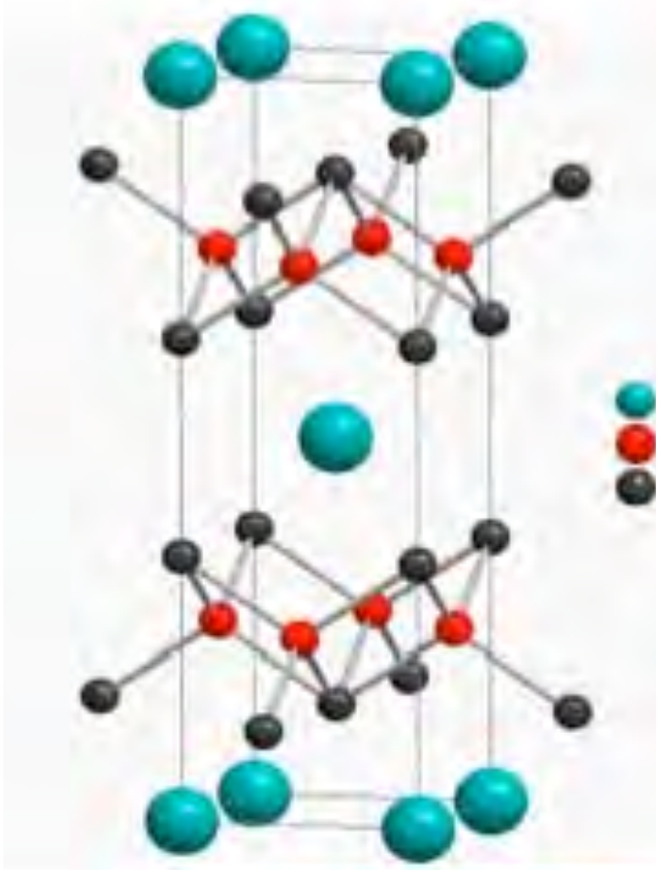

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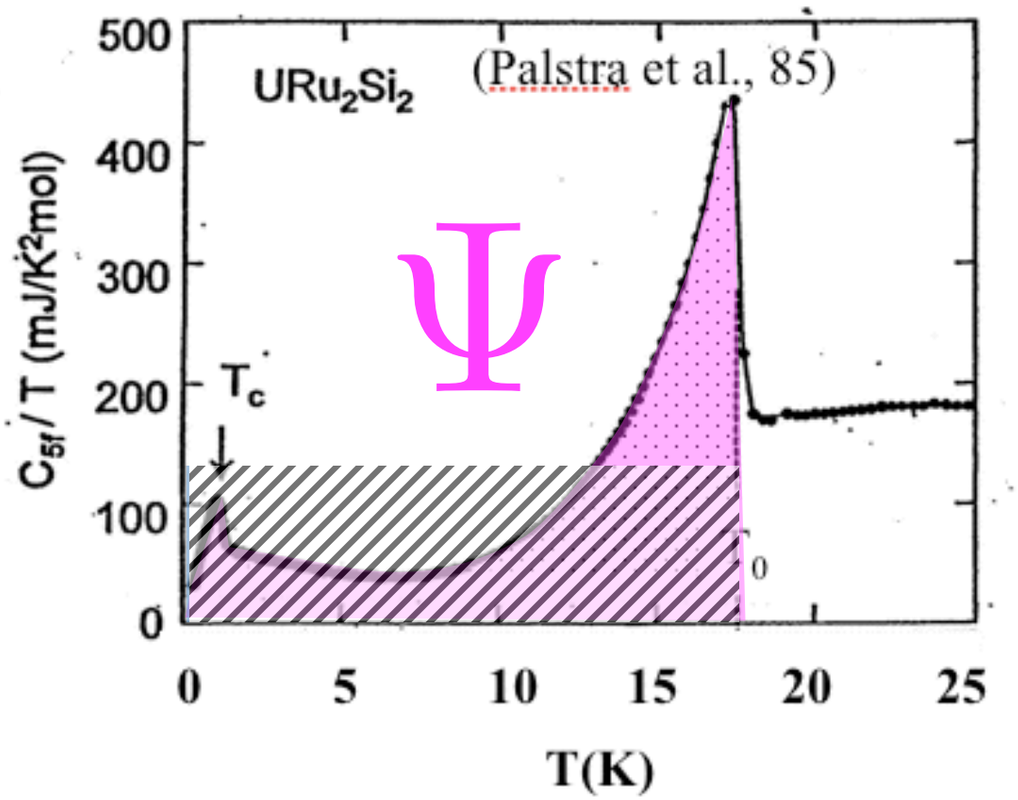

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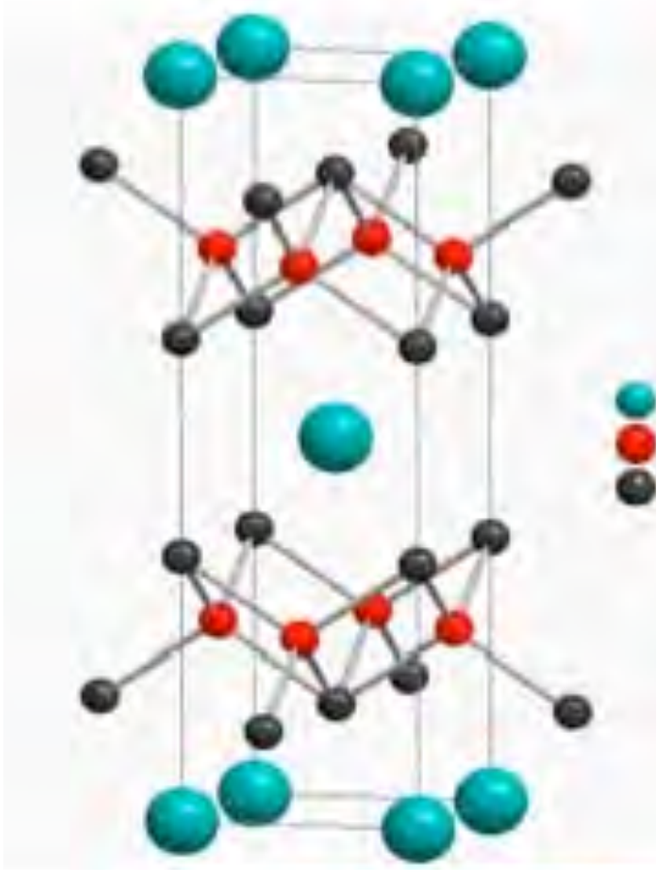

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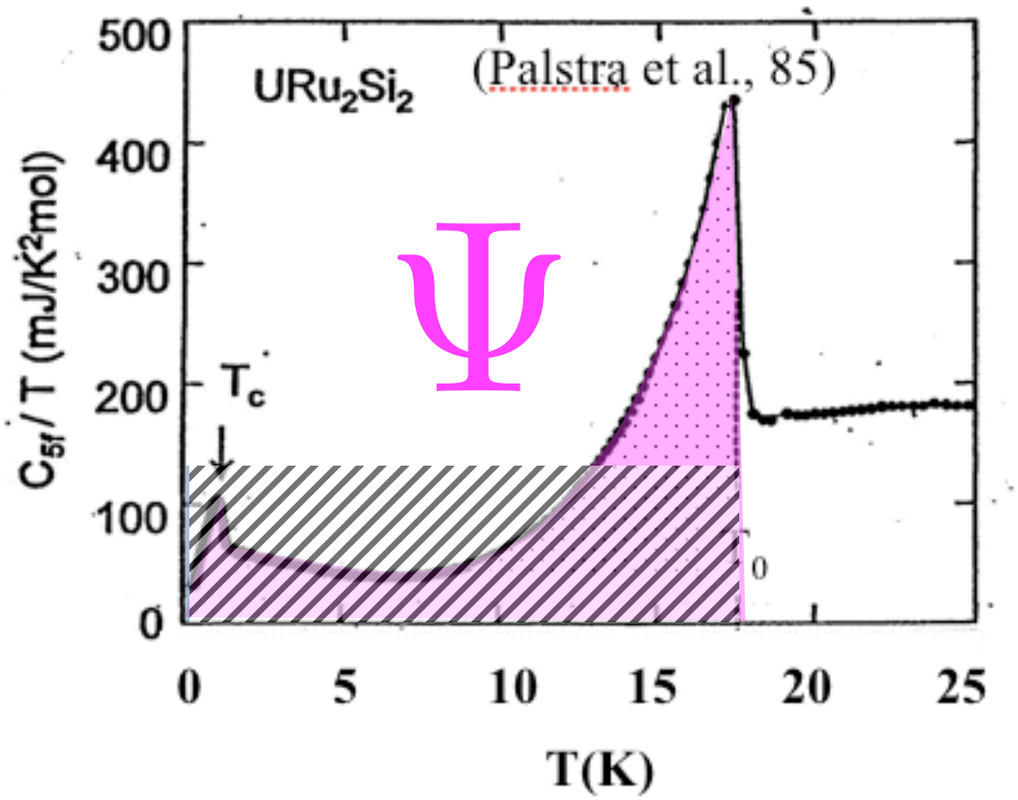
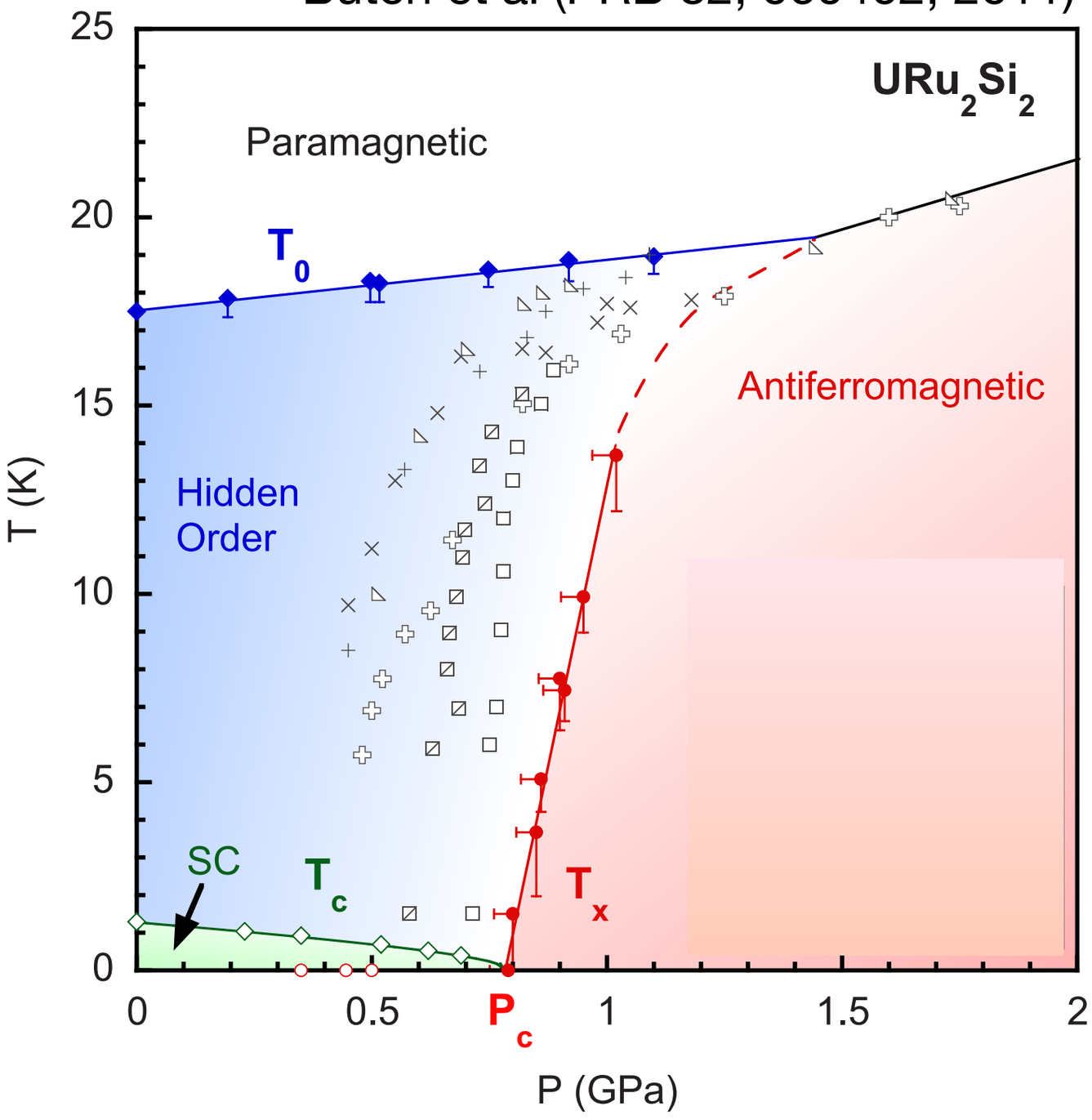


What is the nature of the hidden order?

High pressures, high fields

Rutgers
Center for Materials Theory

Butch et al (PRB 82, 060402, 2011)



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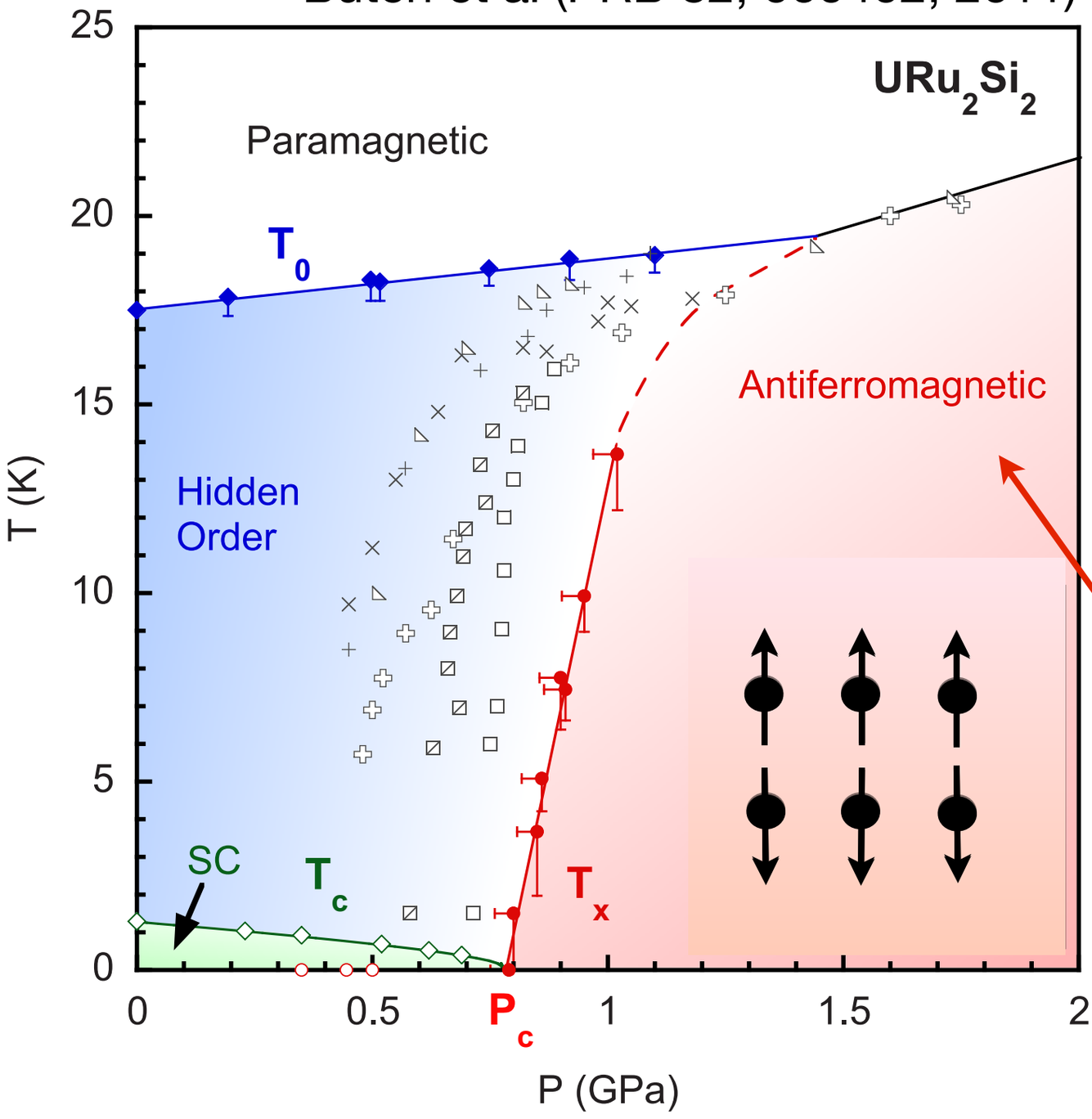

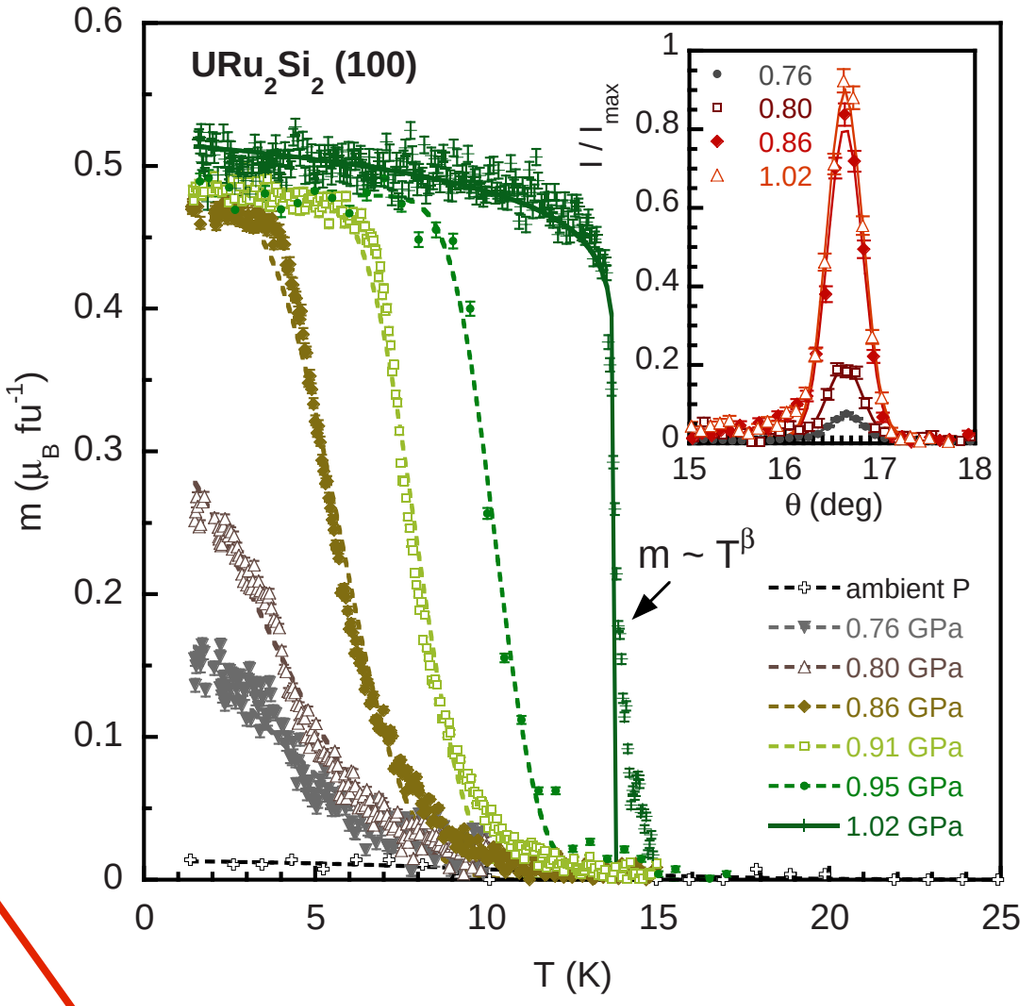

Ising order, present in LMAF,

High pressures, high fields

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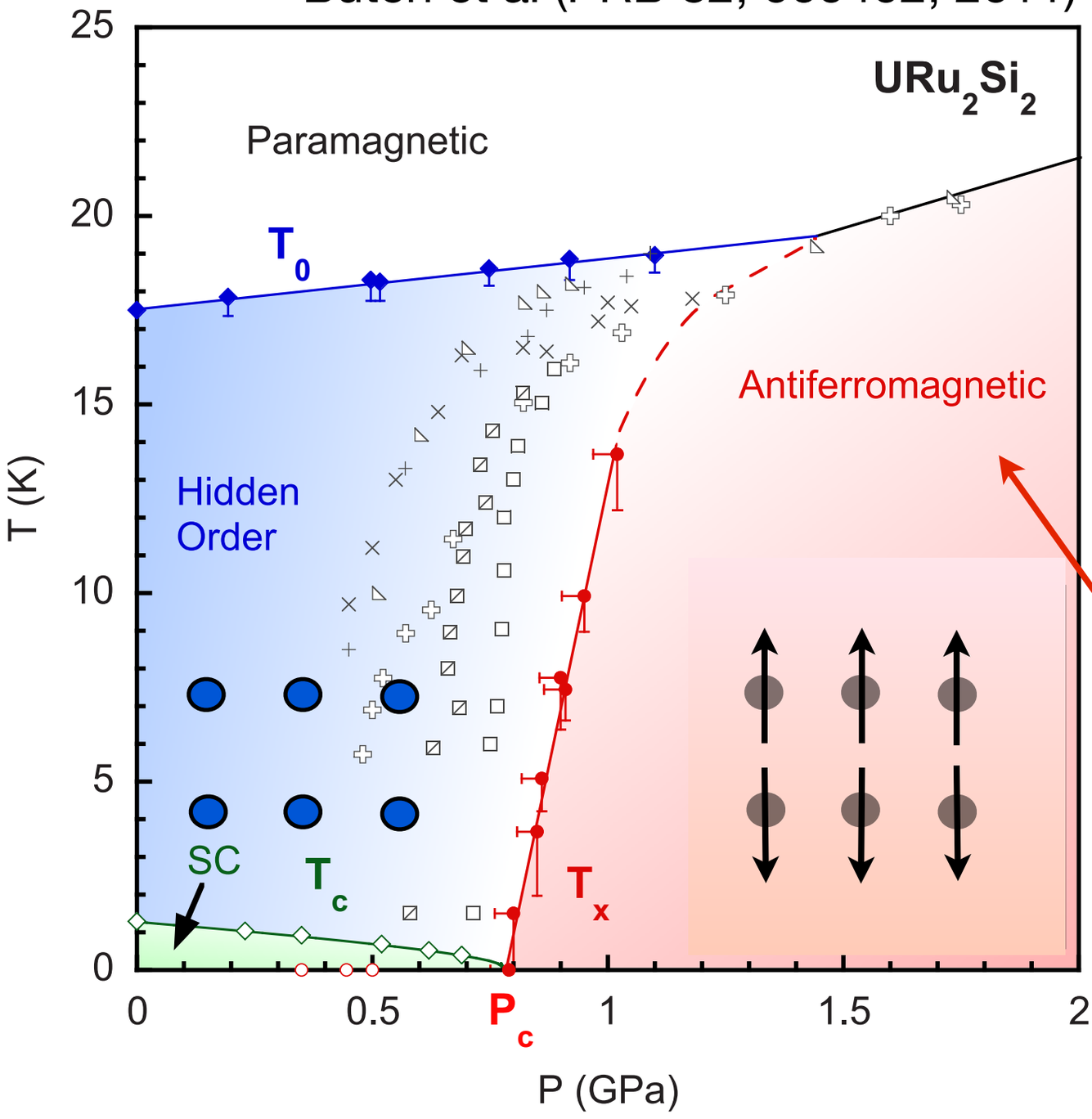

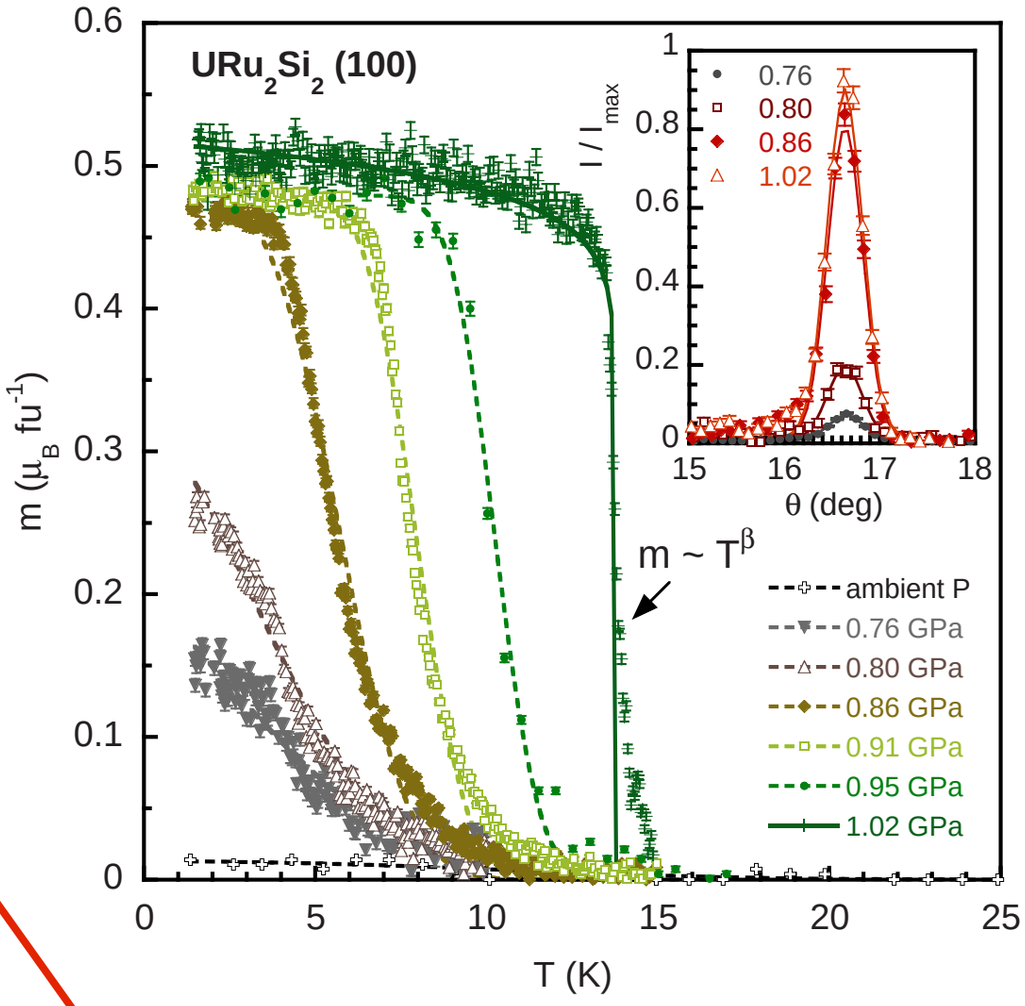

Ising order, present in LMAF, vanishes in the hidden order state. (NMR,MuSR).

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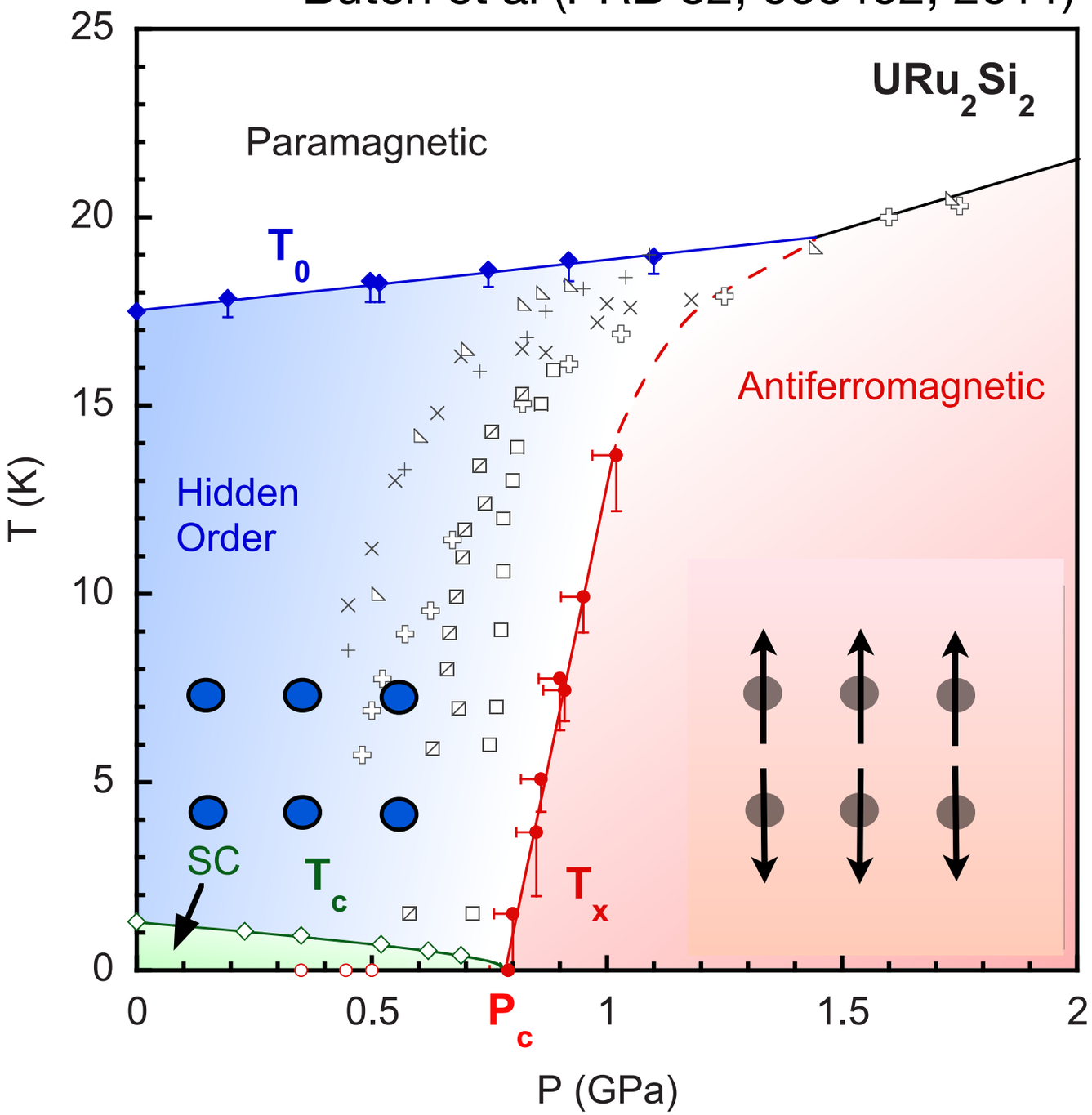
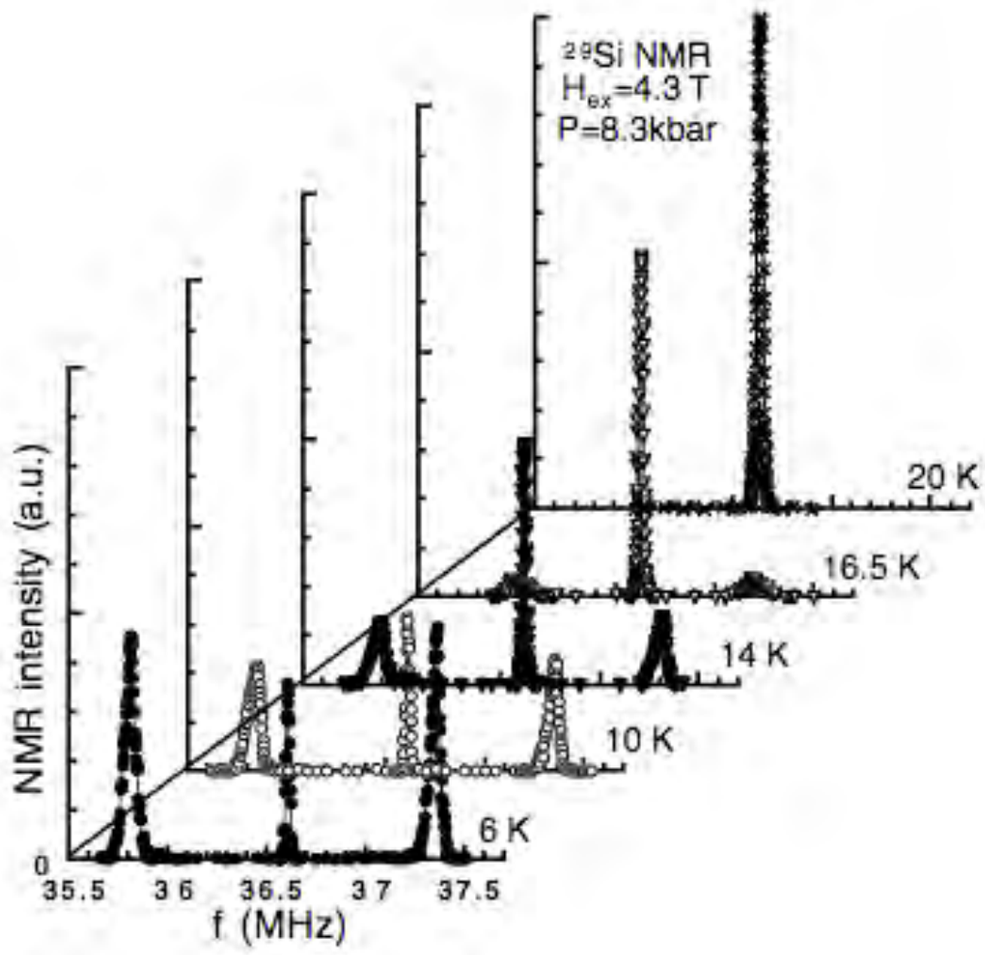


McElfresh et al. (87), Fisher et al (90), Luke et al (94), Amitsuka et al. (99)



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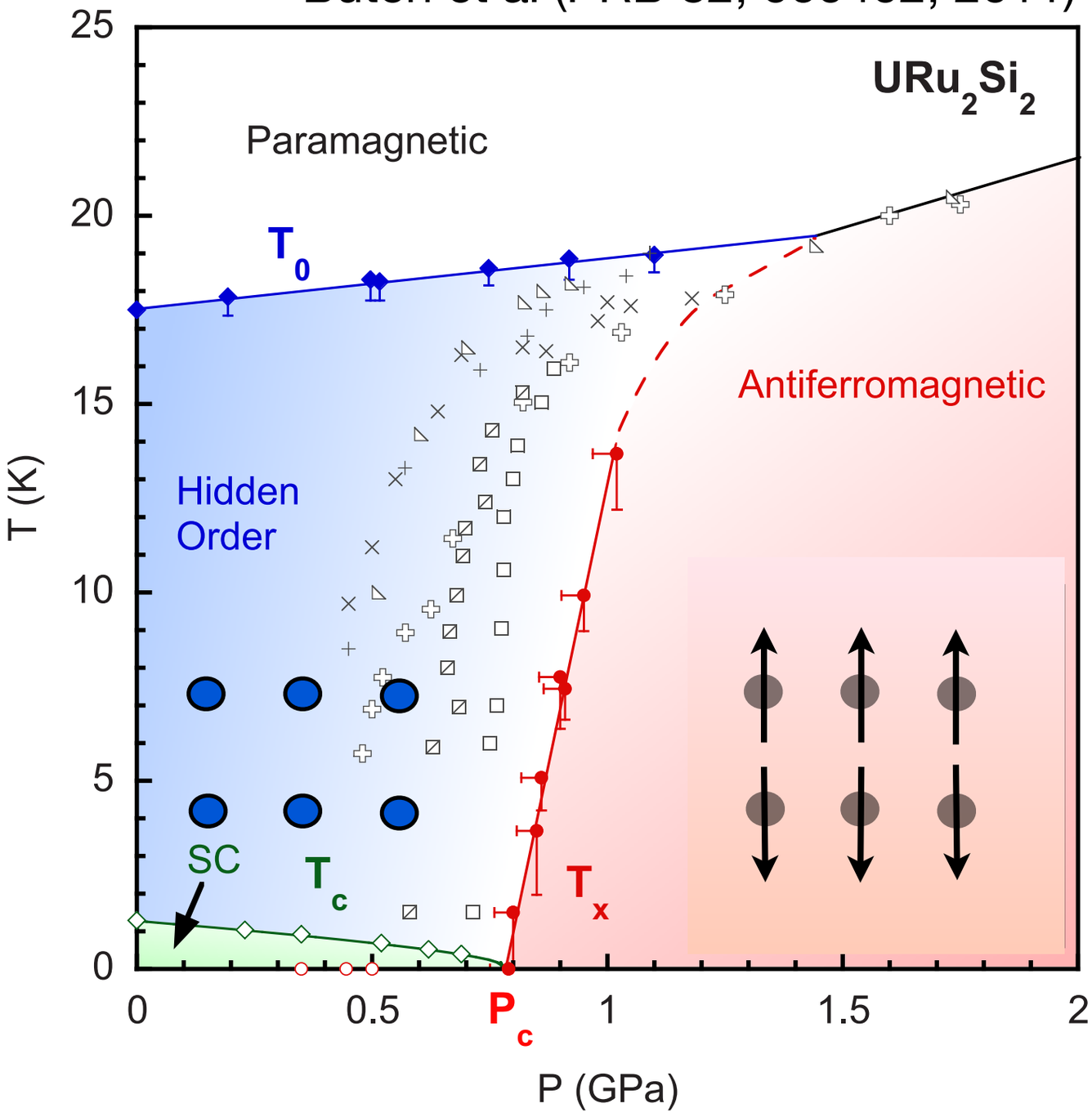

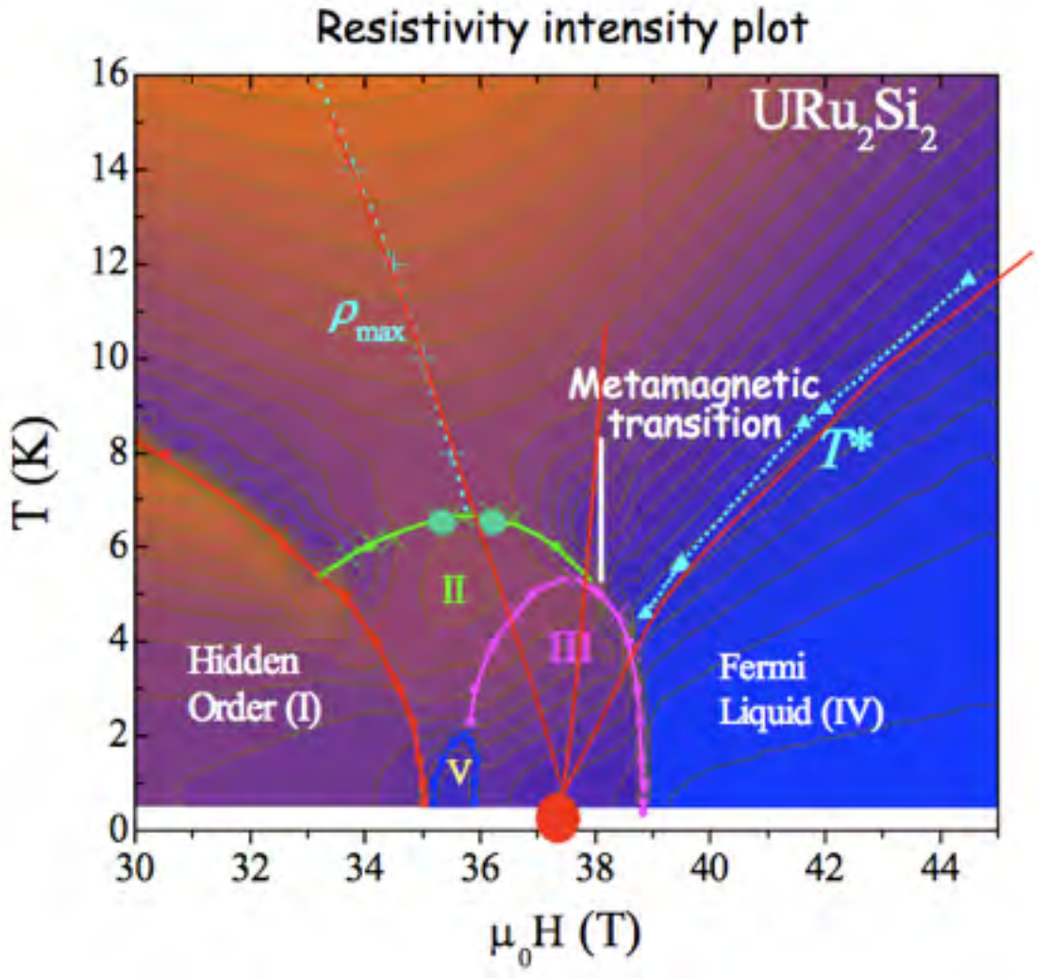

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What is the nature of the hidden order?

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25 Years of Theoretical Proposals

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Landau Theory

Shah et al. ('00) "Hidden Order"

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Landau Theory

Itinerant

Shah et al. ('00) "Hidden Order"

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Okuno and Miyake '98 (composite)

Tripathi, Chandra, PC and Mydosh, '02 (orbital afm)

Dori and Maki, '03 (Unconventional SDW)

Mineev and Zhitomirsky, '04 (SDW)

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Rau and Kee '12 (Rank 5 pseudo-spin vector)

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Local	Barzykin & Gorkov, '93 (three-spin correlation)
	Santini & Amoretti, '94, Santini ('98) (Quadrupole order)
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	Kasuya, '97 (U dimerization)
	Kiss and Fazekas '04, (Rank 3 octupolar order)
	Haule and Kotliar '09 (Rank 4 hexa-decapolar)
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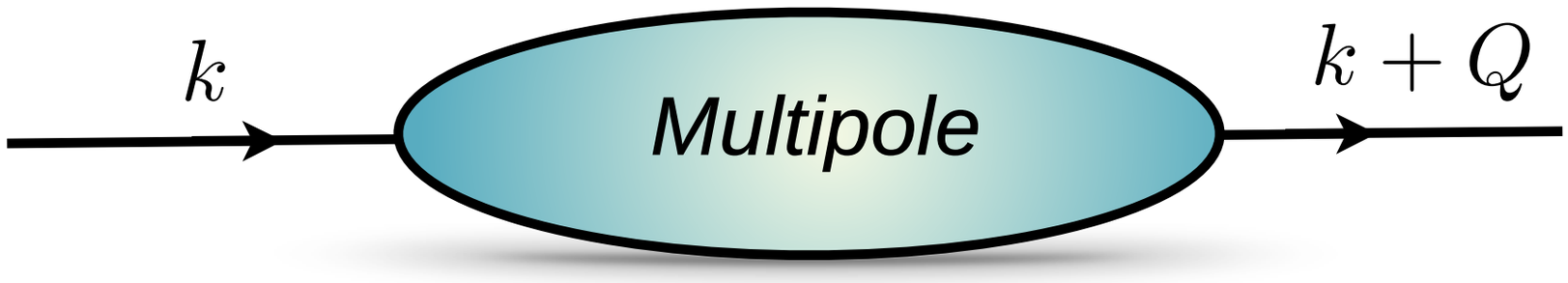
	Pepin et al '10 (Spin liquid/Kondo Lattice)
Kondo Lattice	Dubi and Balatsky, '10 (Hybridization density wave)

a)



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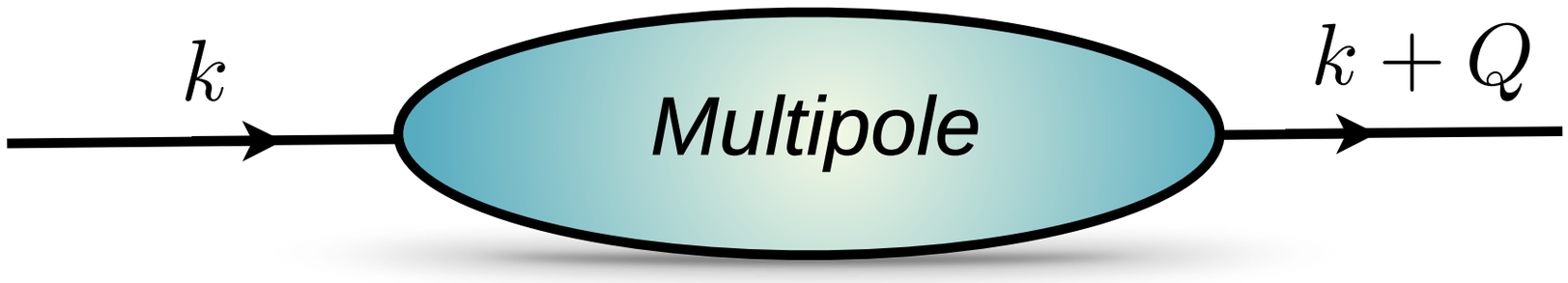

$$\langle c^{\dagger}_{\mathbf{k}+Q\alpha} c_{\mathbf{k}} \rangle = \Psi_{\alpha\beta}(\mathbf{k}) = \begin{cases} \psi_0(\mathbf{k}) & \text{CDW} \\ \vec{d}(\mathbf{k}) \cdot \vec{\sigma}_{\alpha\beta} & \text{SDW} \end{cases}$$

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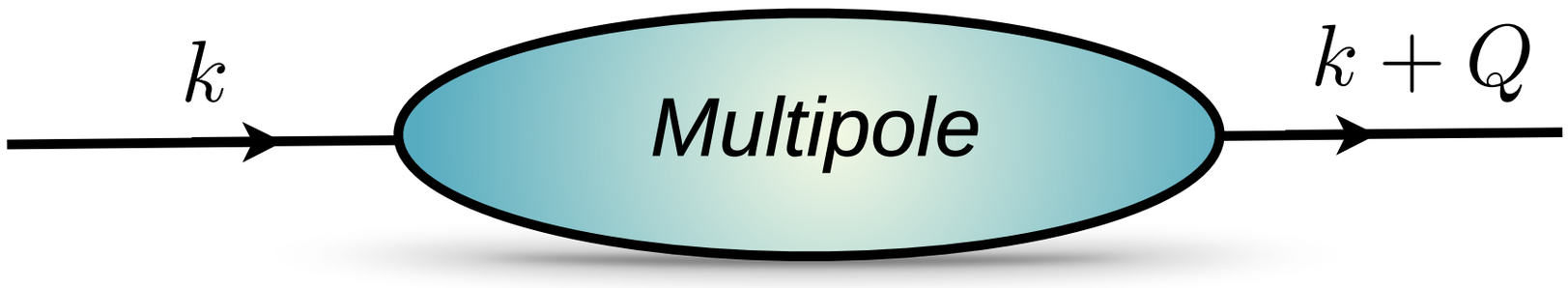

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$$\Psi(R\mathbf{k}) = D(R)\Psi(\mathbf{k})$$

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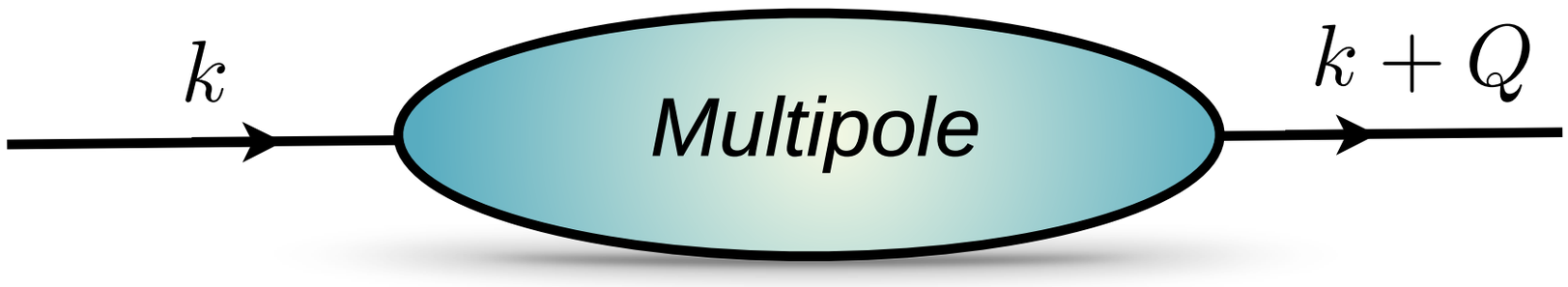

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OP	L	S	Name

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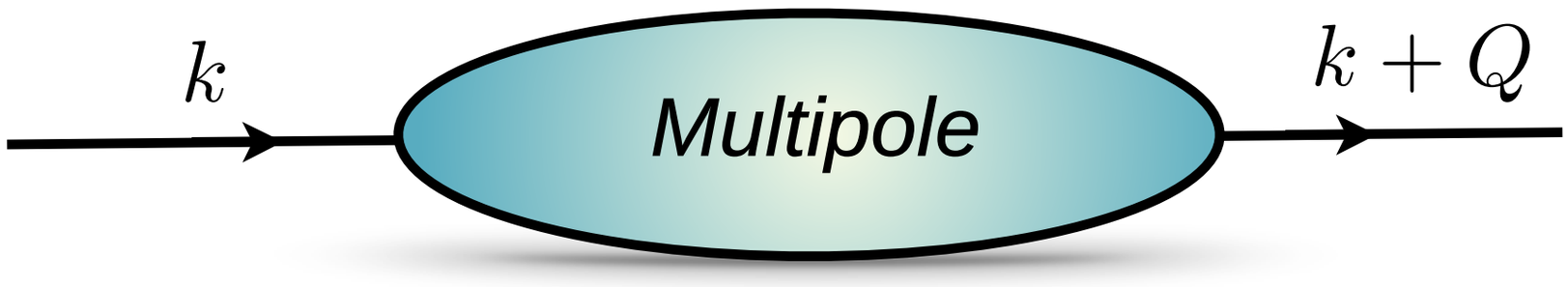

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OP	L	S	Name
1	0		Scalar (CDW)

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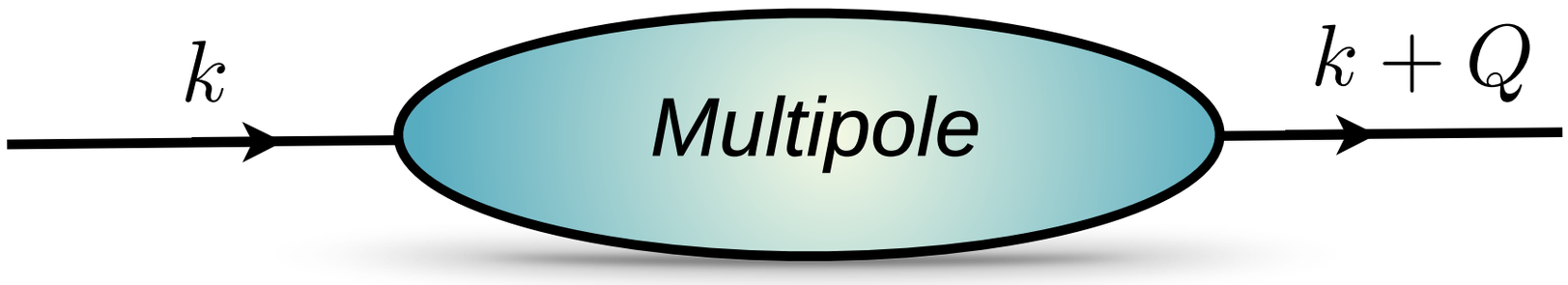

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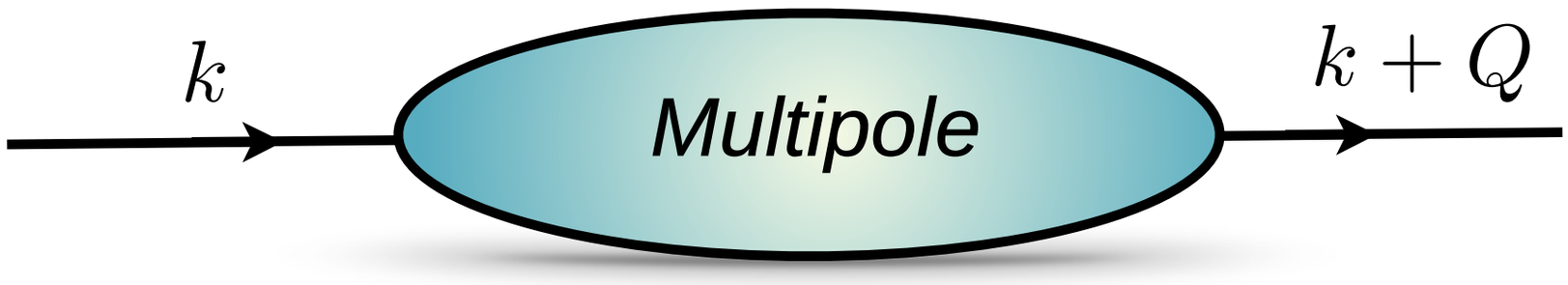
OP	L	S	Name
1	0		Scalar (CDW)
m		1	AFM

a)



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OP	L	S	Name
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k_x	1		Vector (c.f nematic)

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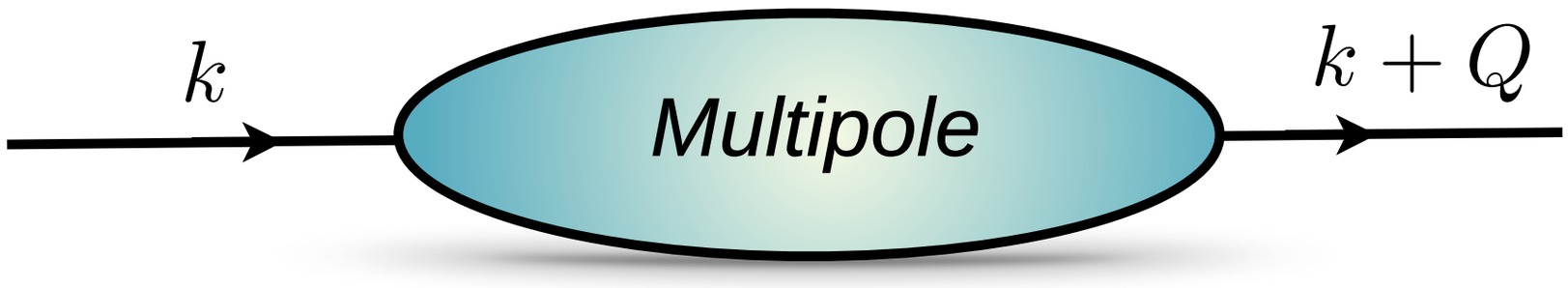

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$k_x k_y k_z$	3		Octopole

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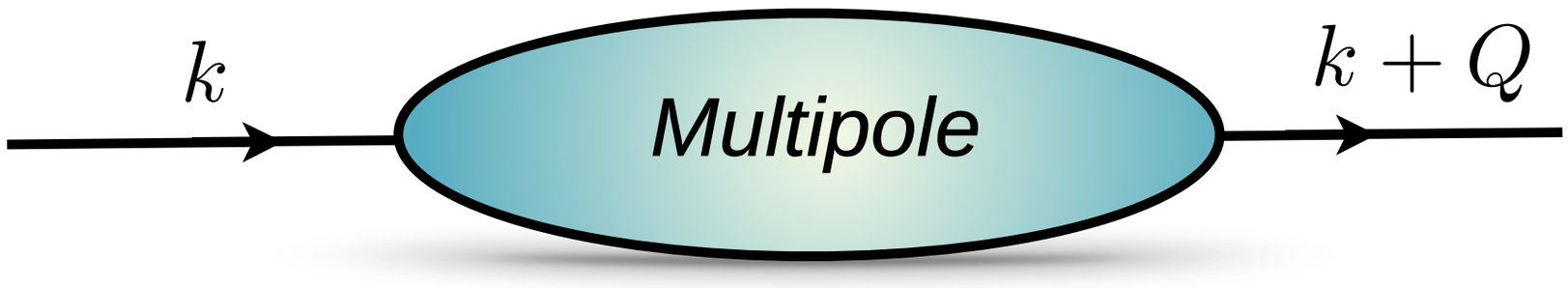

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k_x	1		Vector (c.f nematic)
$k_x k_y k_z$	3		Octopole
$k_x k_y (k_x^2 - k_y^2)$	4		Hexdecapole.

Can Electron Order Parameters carry 1/2 integer Spin?

$$f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$$

Landau Theory

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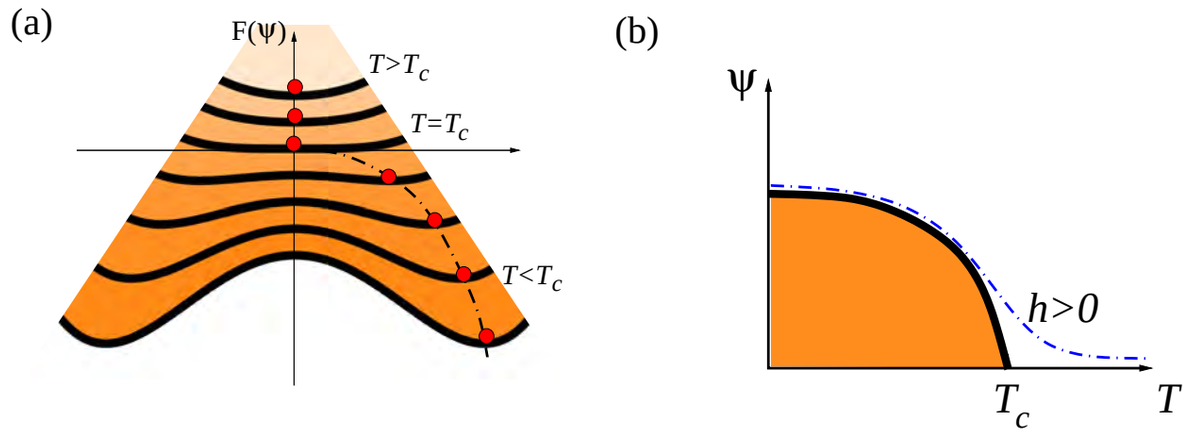

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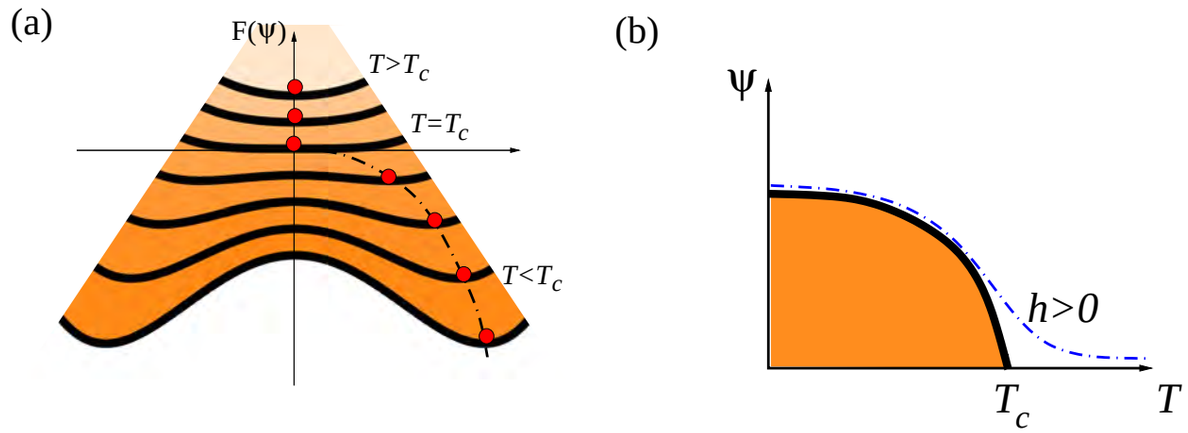

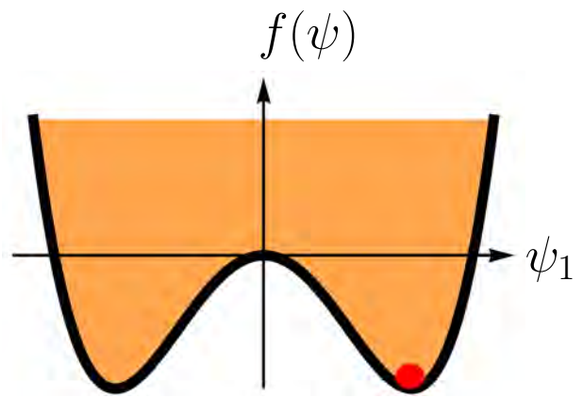

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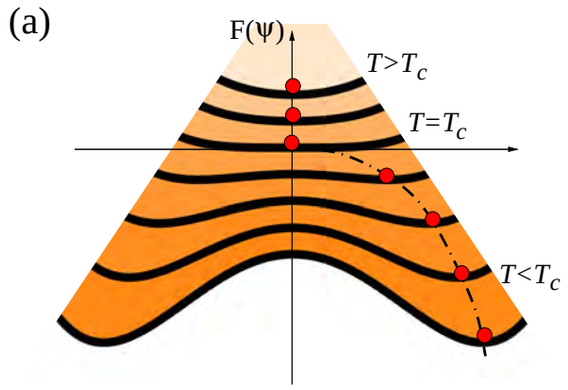

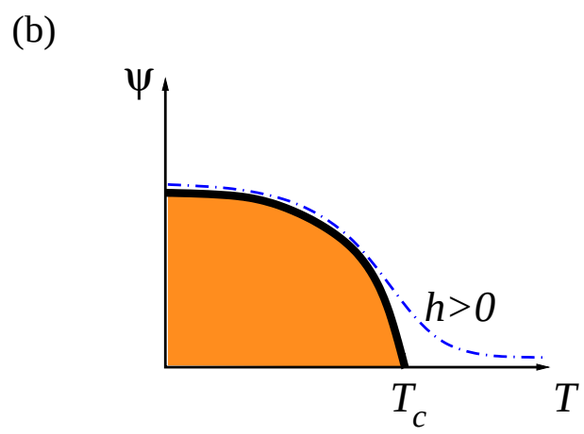

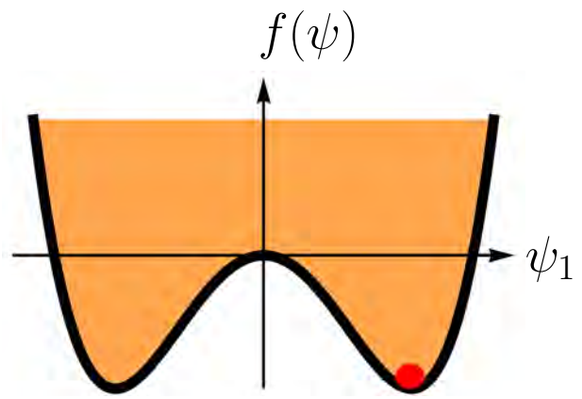

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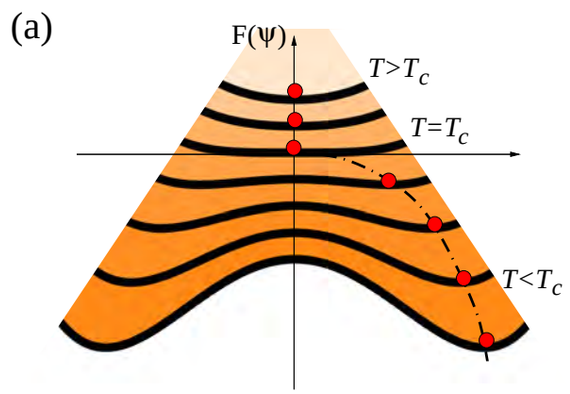
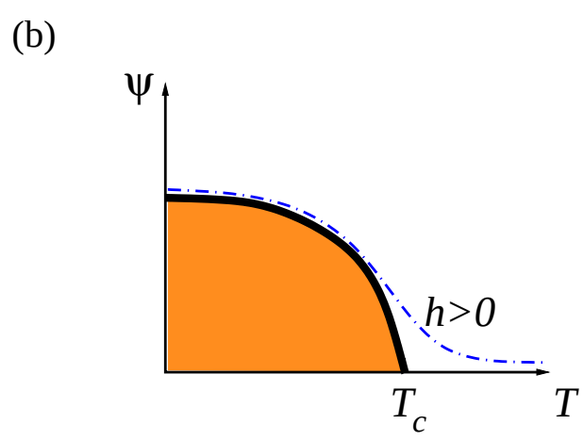
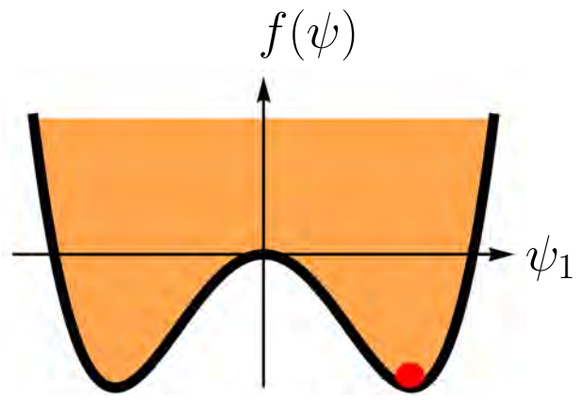
$$[H, R] = 0$$

Symmetry







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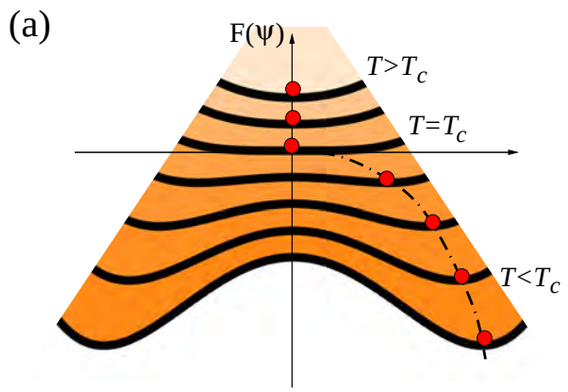
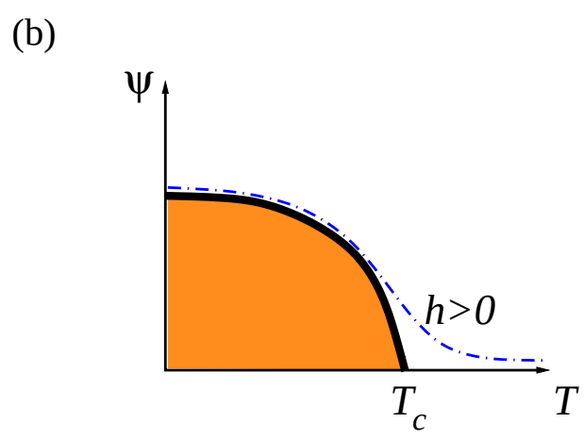


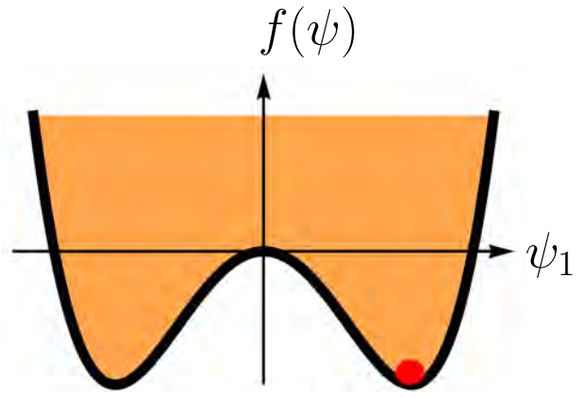


$$[H, R] = 0$$

Symmetry

$$F[\psi_R] = F[\psi]$$

$$\psi \xrightarrow{R} \psi_R = D(R).\psi$$

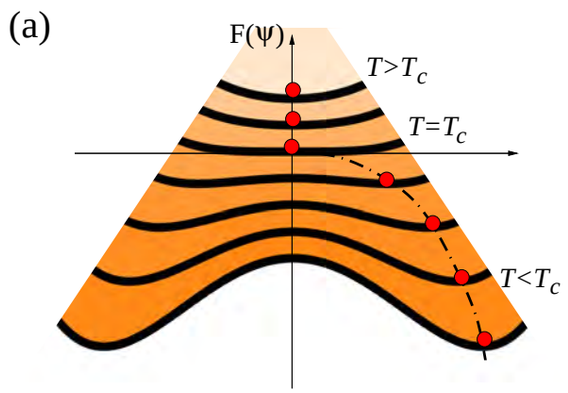


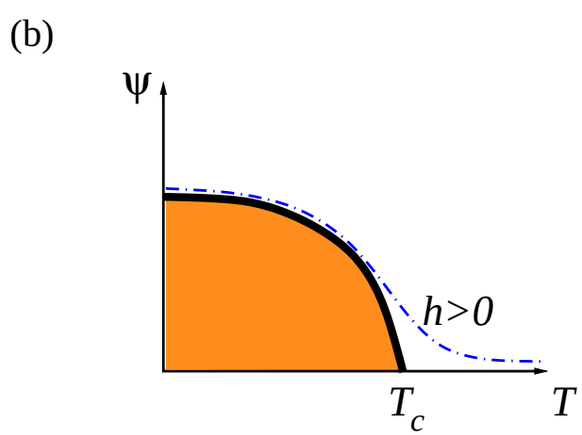
Broken Symmetry

Can Electron Order Parameters carry 1/2 integer Spin?

$$f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$$

Landau Theory





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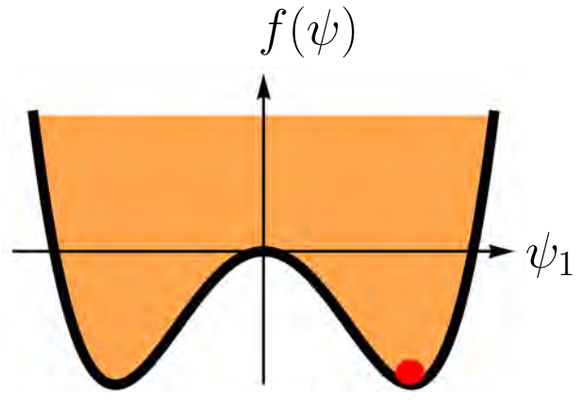
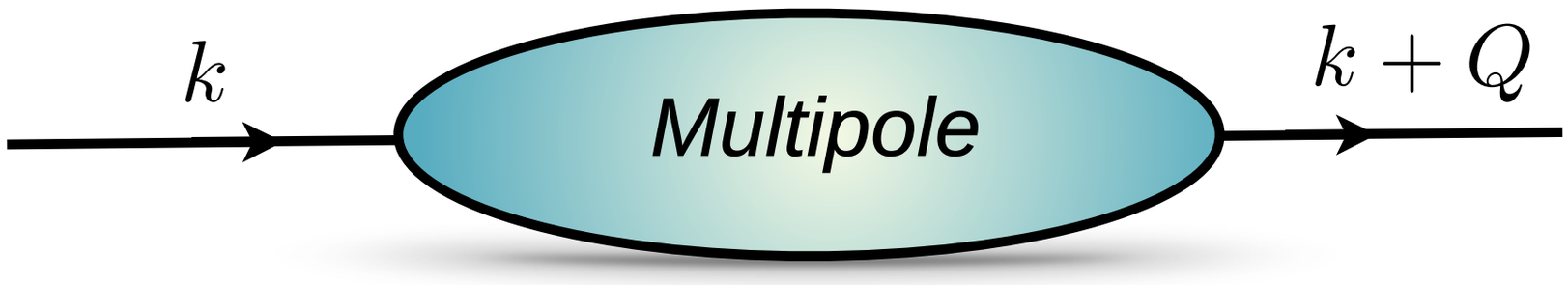


Broken Symmetry

Can an electron OP- ever transform like a spinor, or 1/2 integer particle?

a)



$$\langle c^{\dagger}_{\mathbf{k}+Q\alpha} c_{\mathbf{k}} \rangle = \Psi_{\alpha\beta}(\mathbf{k}) = \begin{cases} \psi_0(\mathbf{k}) & \text{CDW} \\ \vec{d}(\mathbf{k}) \cdot \vec{\sigma}_{\alpha\beta} & \text{SDW} \end{cases}$$

$$\Psi(R\mathbf{k}) = D(R)\Psi(\mathbf{k})$$

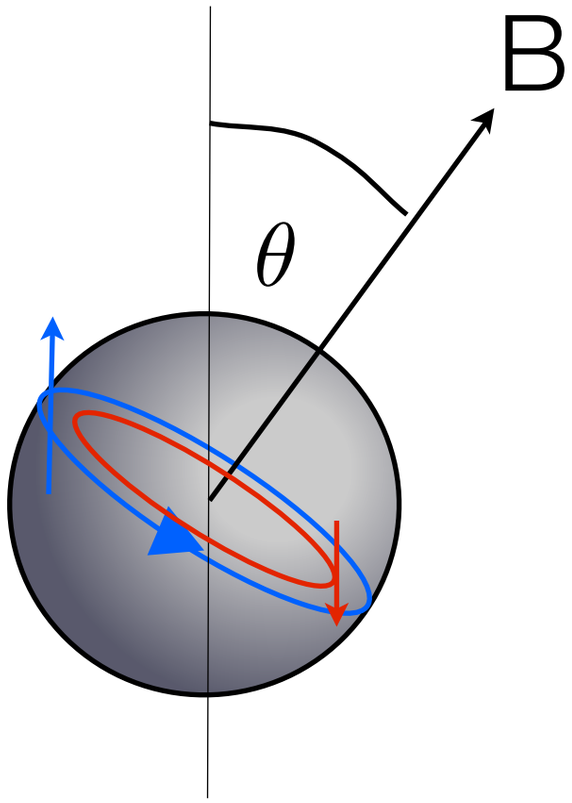
b)

$$\psi = \sqrt{\text{Multipole}} = \text{Spinorial OP}$$

The Giant Ising Anisotropy.

Quantum Oscillations and Spin Zeros

Quantum Oscillations and Spin Zeros



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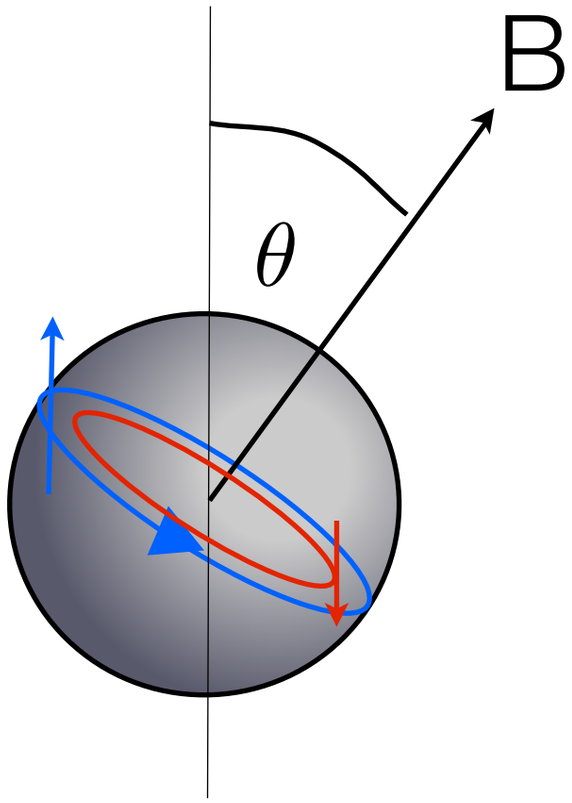
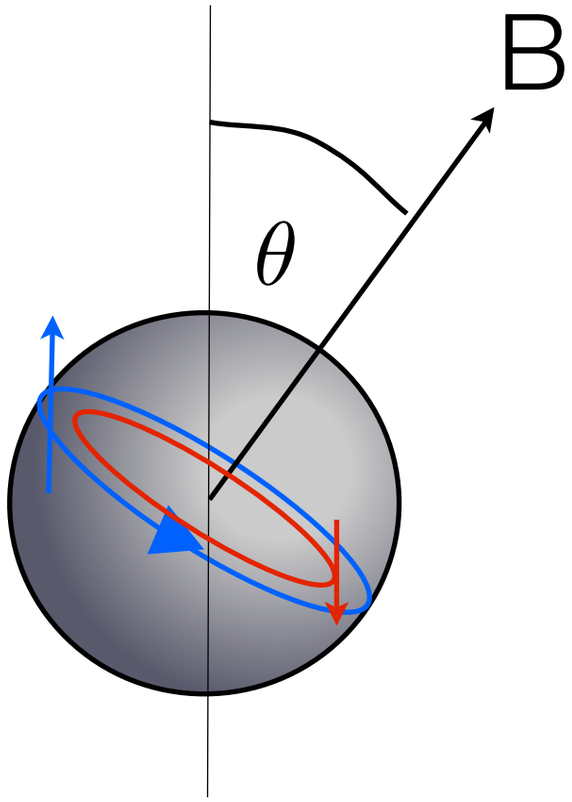


$$M \propto \cos\left[2\pi \frac{\text{Zeeman}}{\text{cyclotron}}\right]$$

Quantum Oscillations and Spin Zeros



$$\frac{g}{2}\frac{eB}{2m_e}$$

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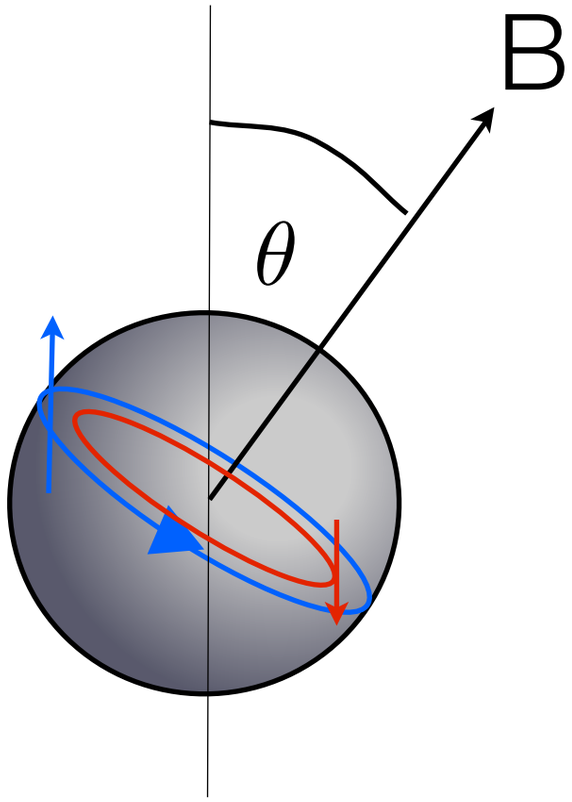

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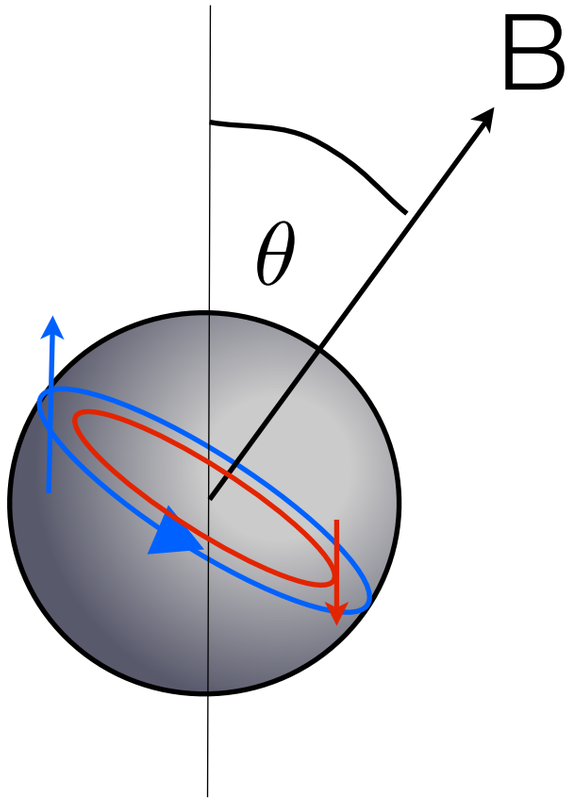

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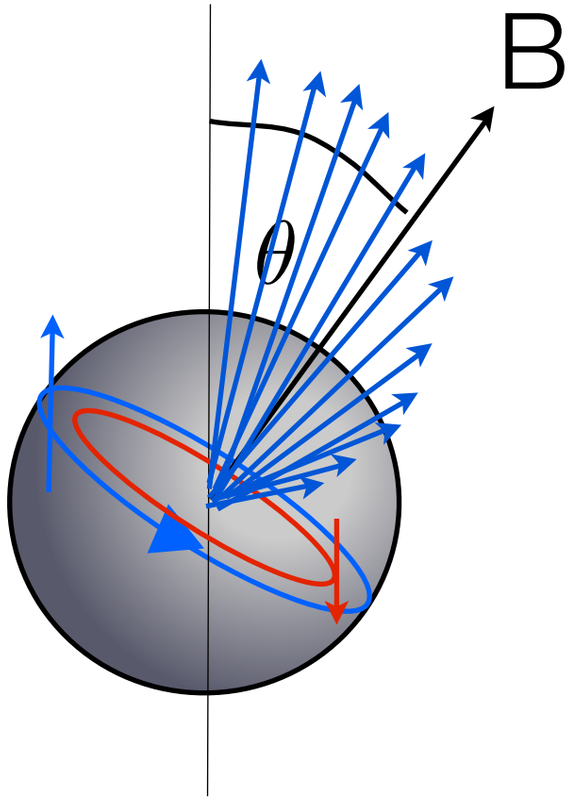

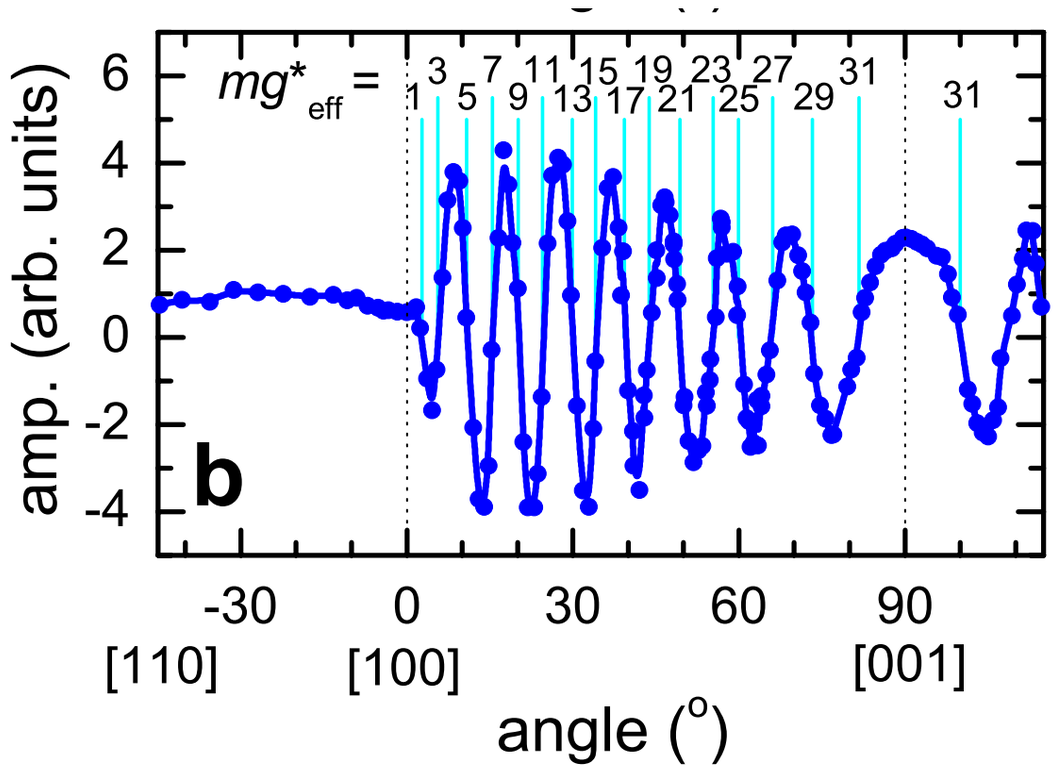

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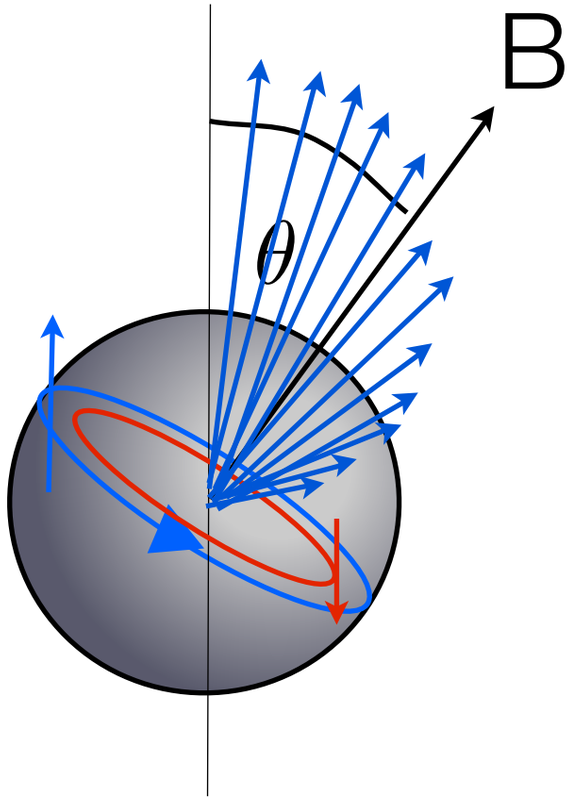

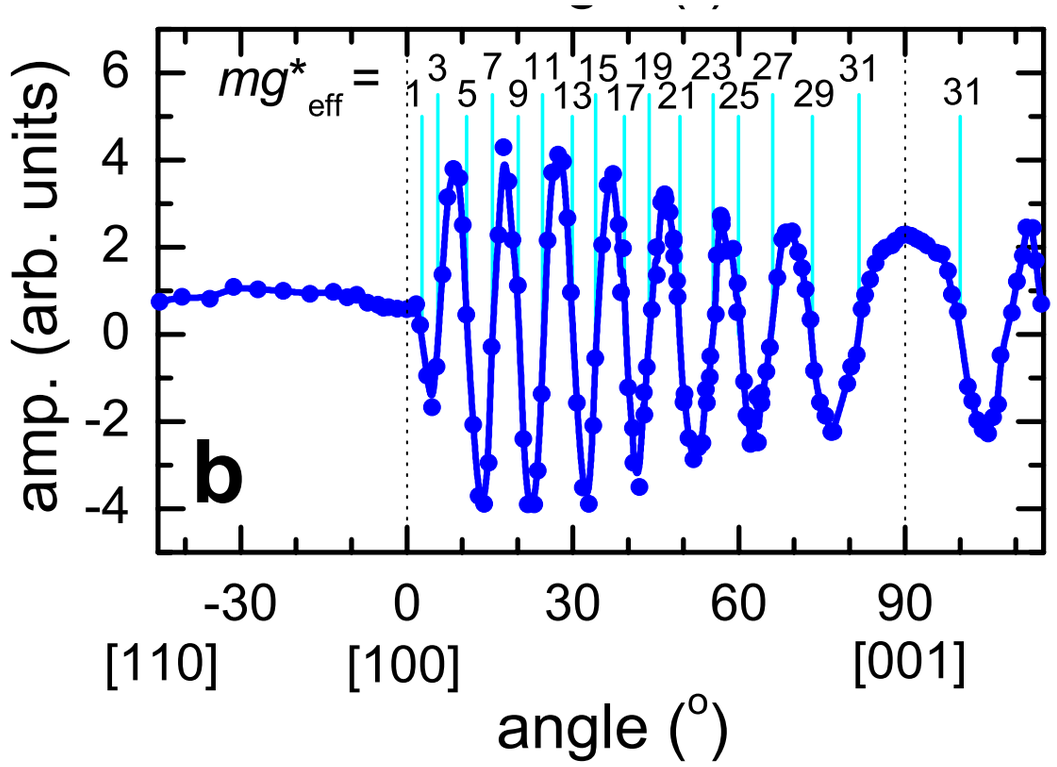

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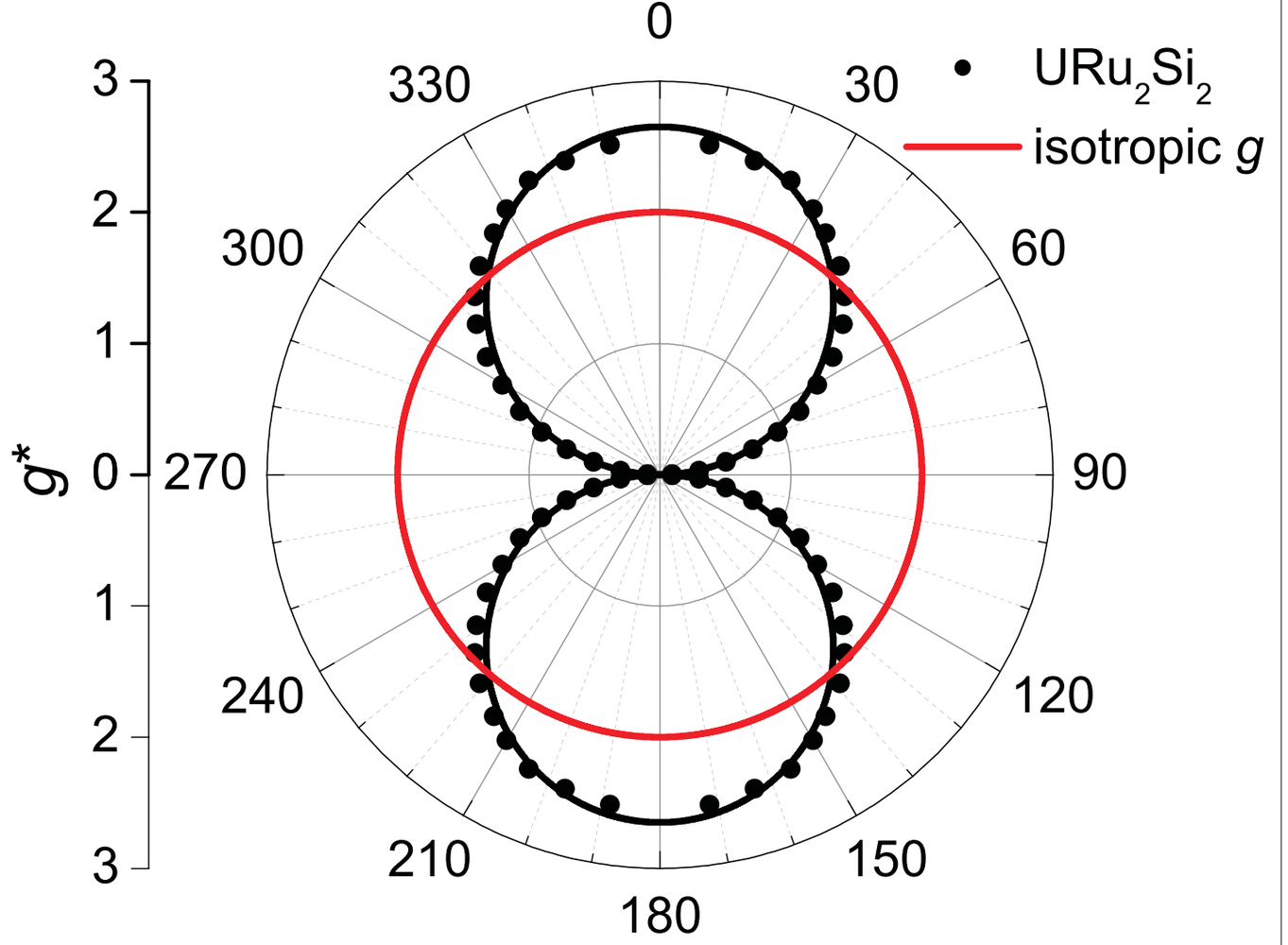

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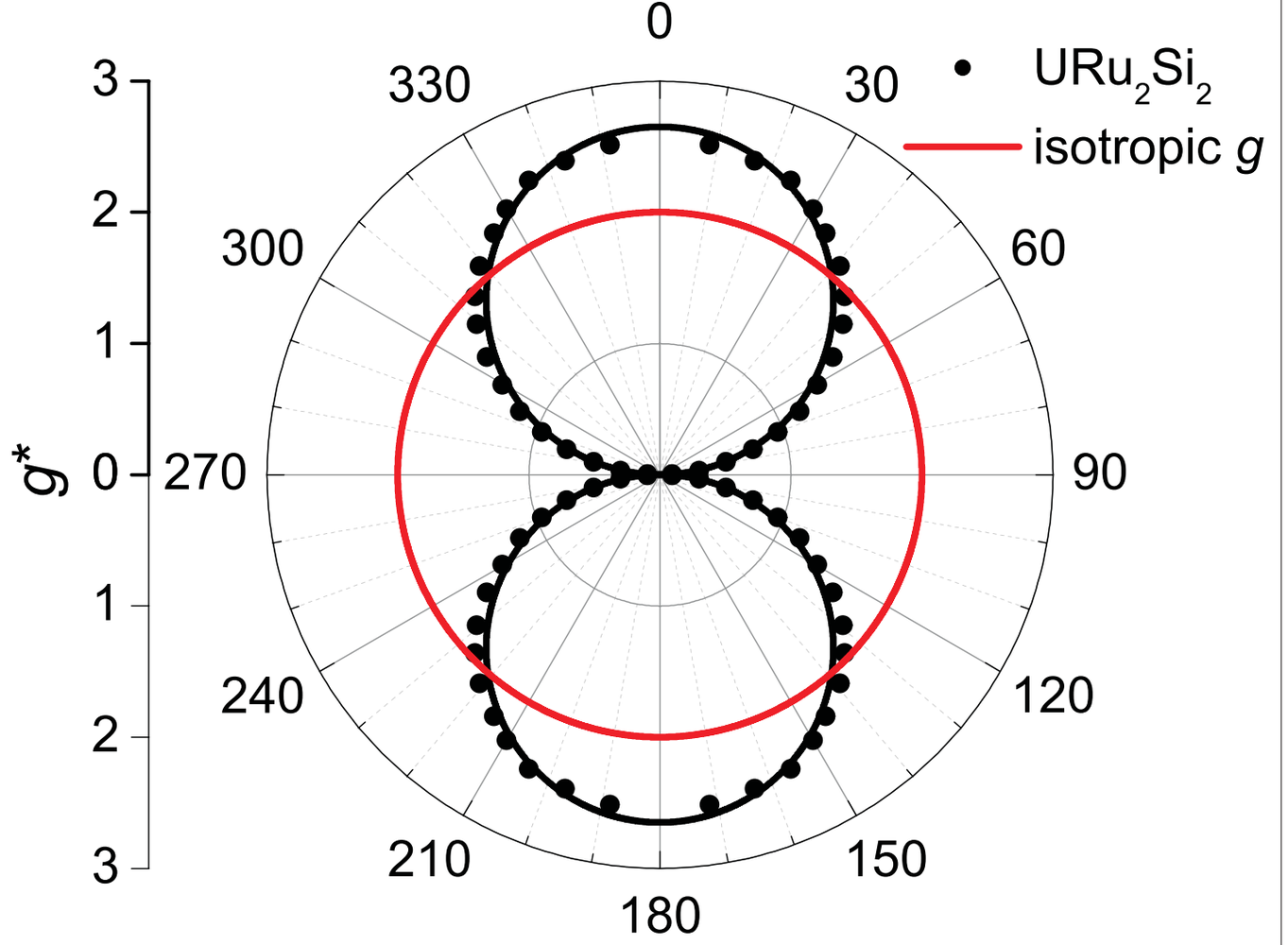

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Quasiparticle with giant Ising anisotropy > 30.
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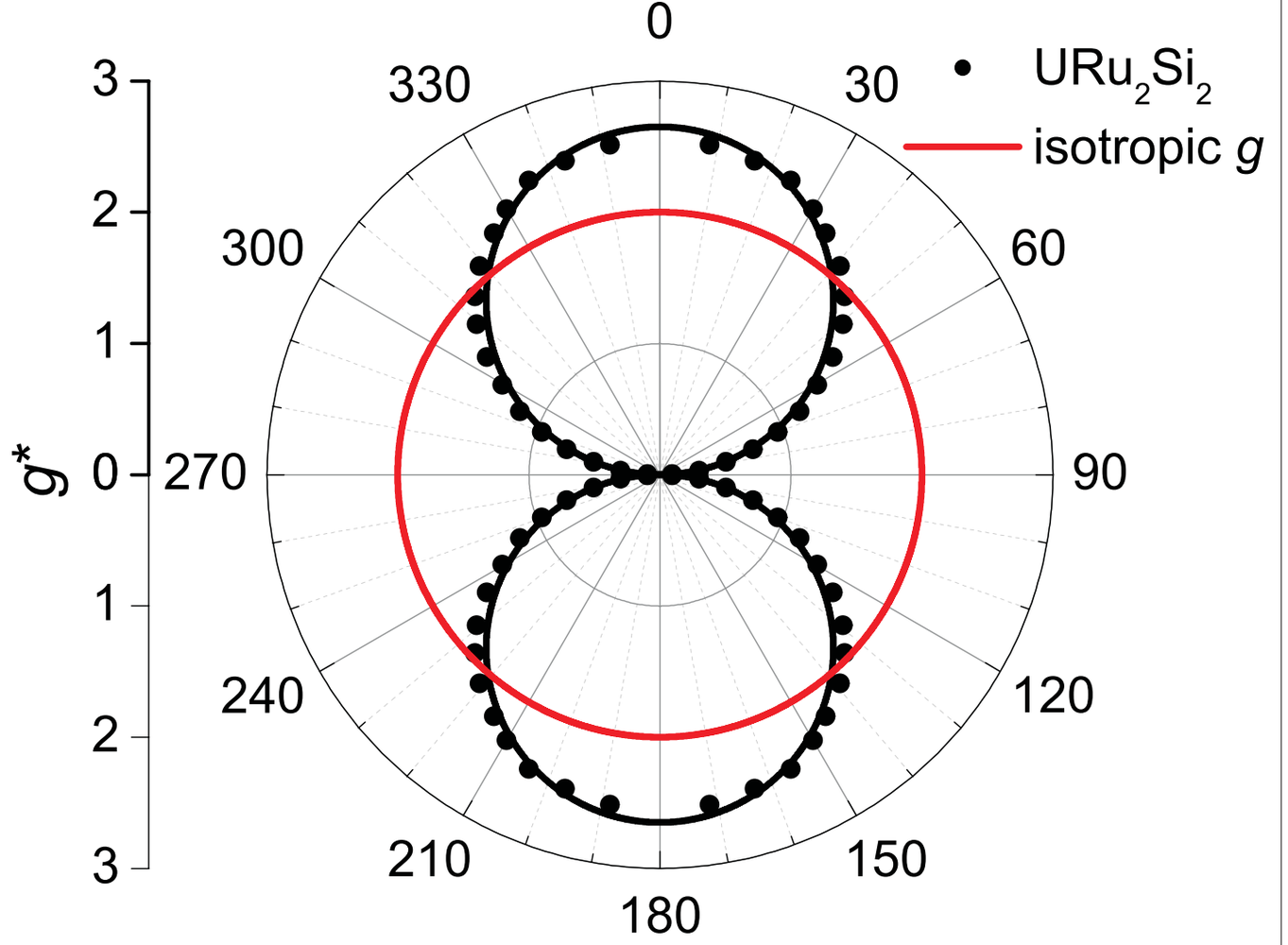

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Superconductivity: Giant Ising Anisotropy

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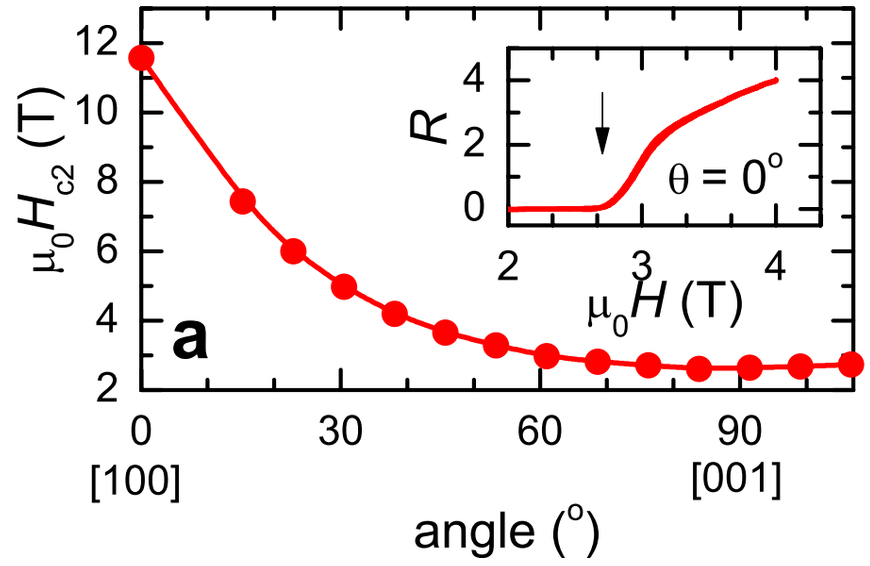

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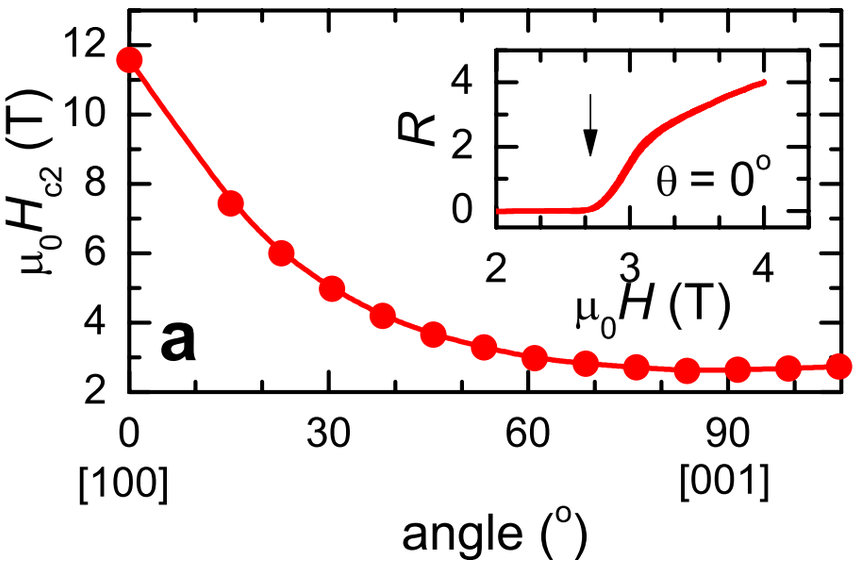

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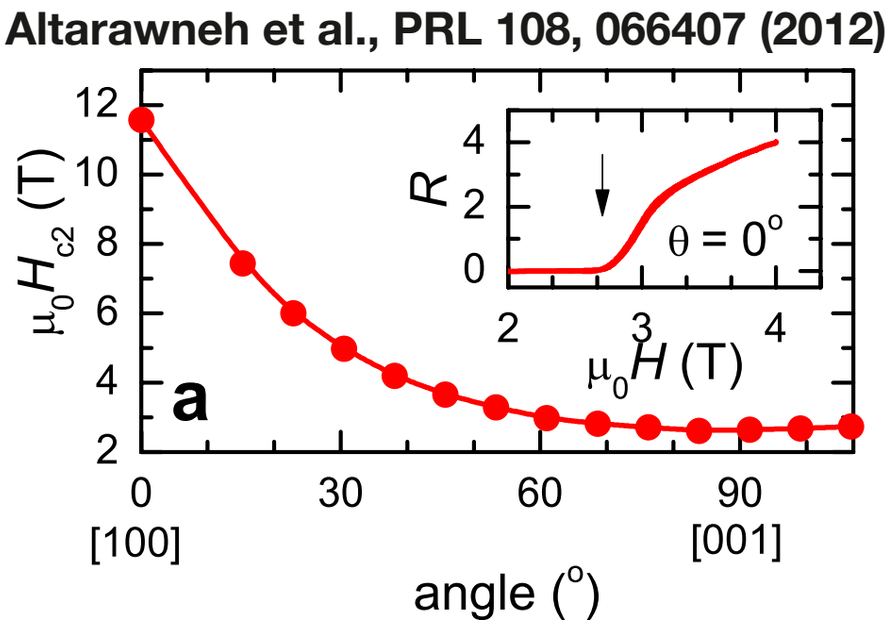

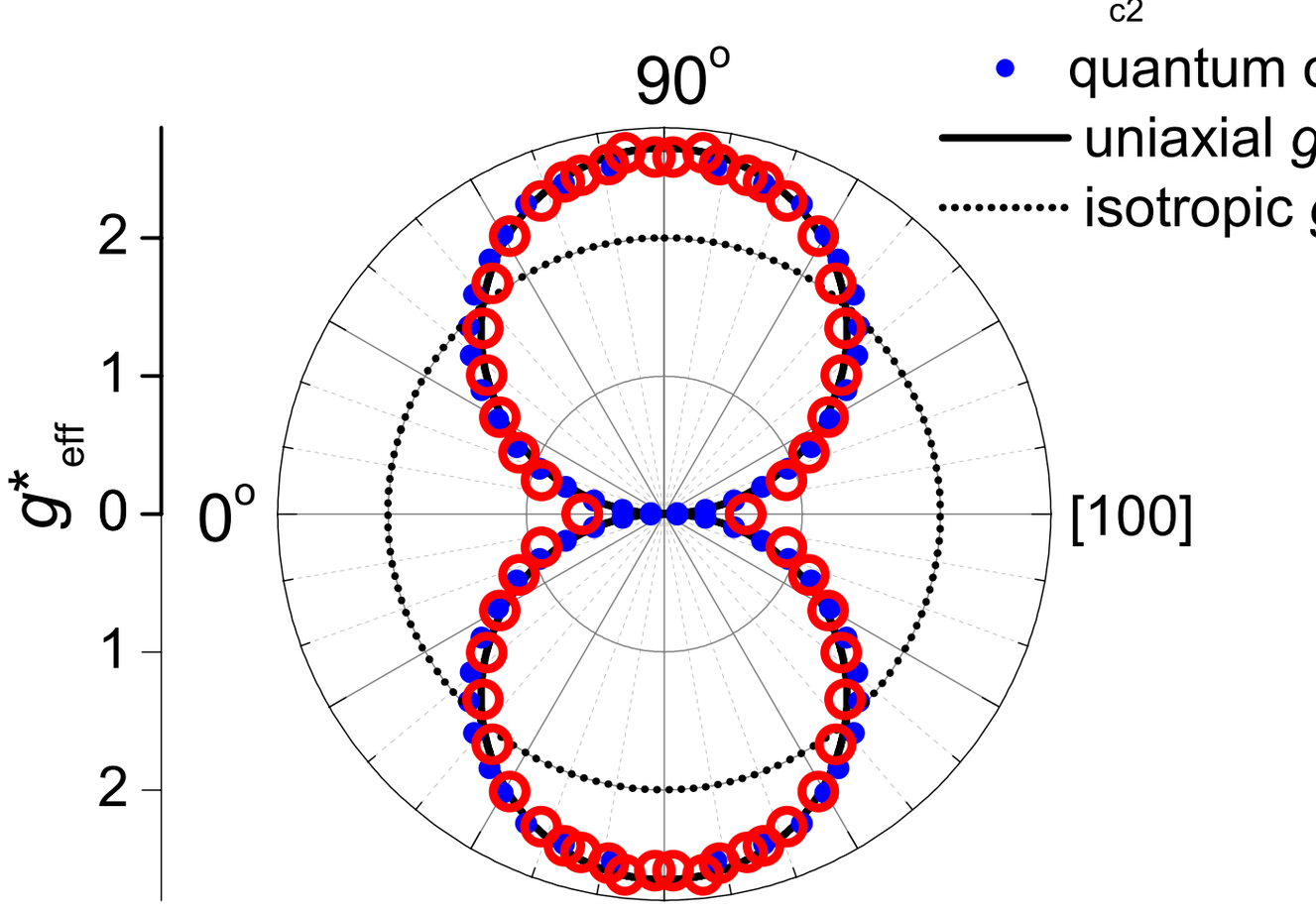

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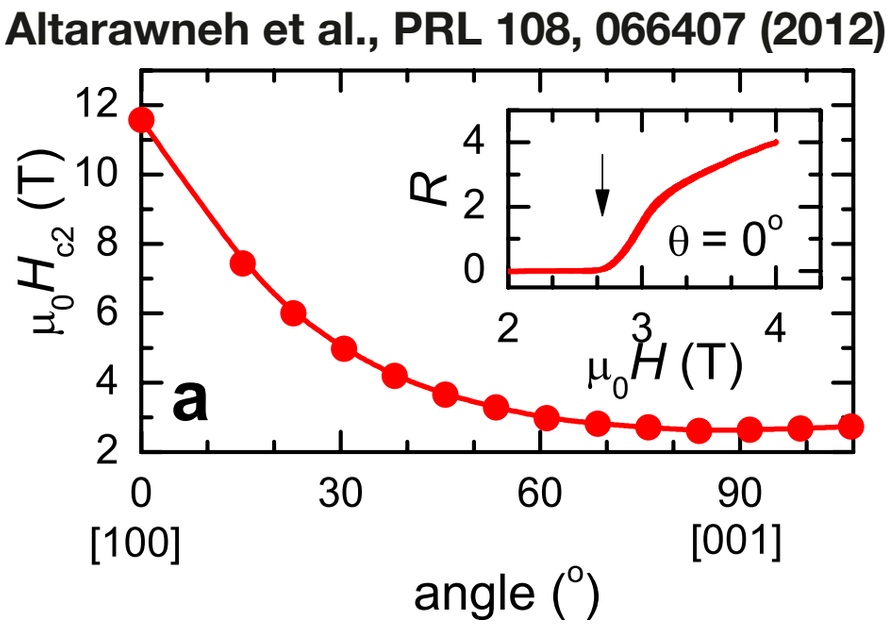

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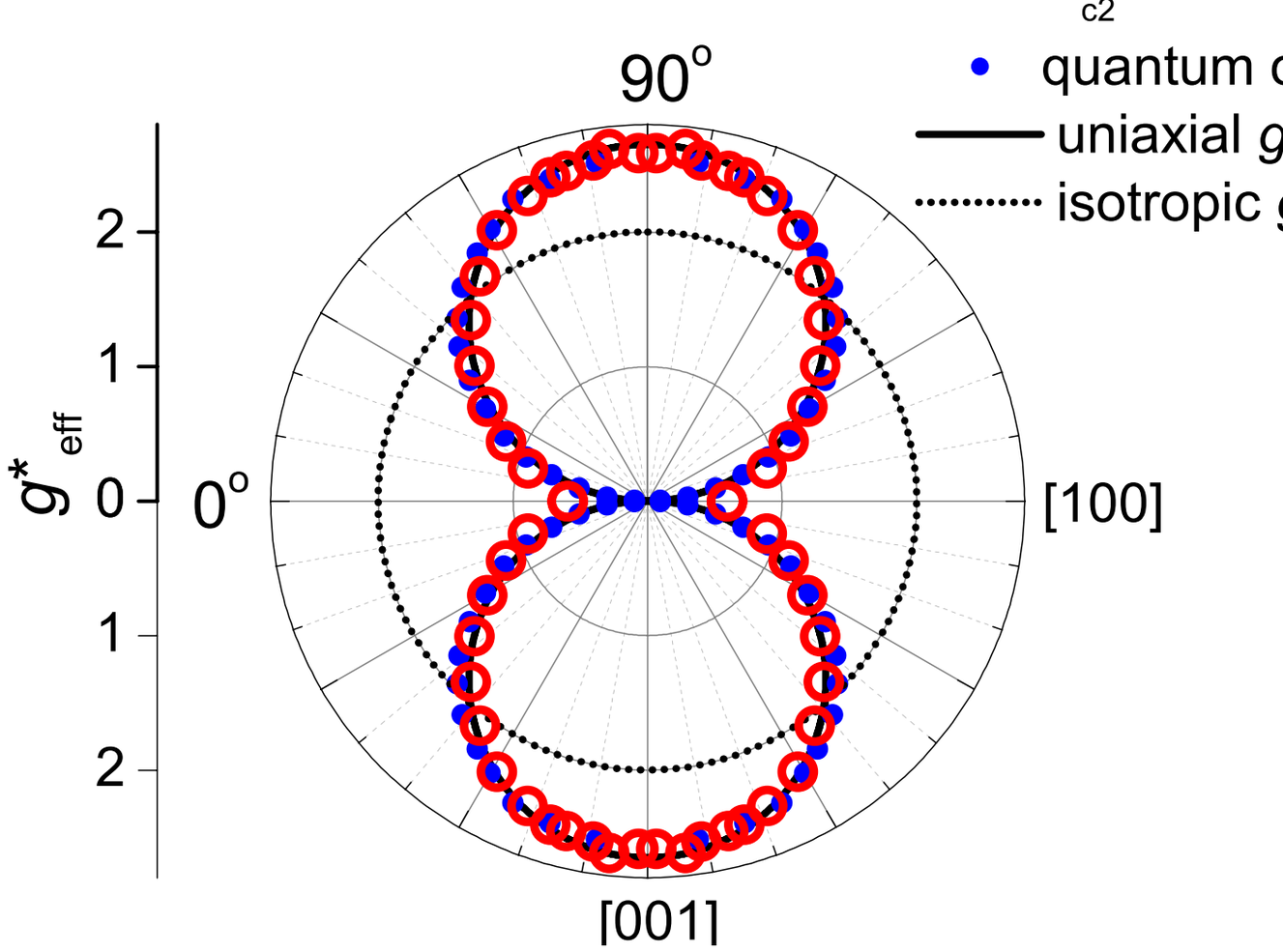

<u>Ising QP's pair condense.</u>

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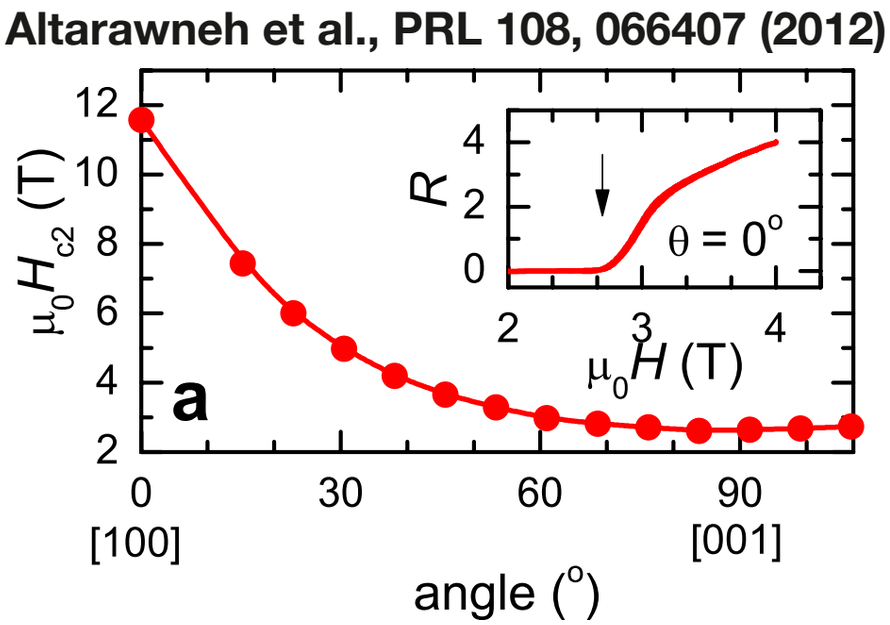

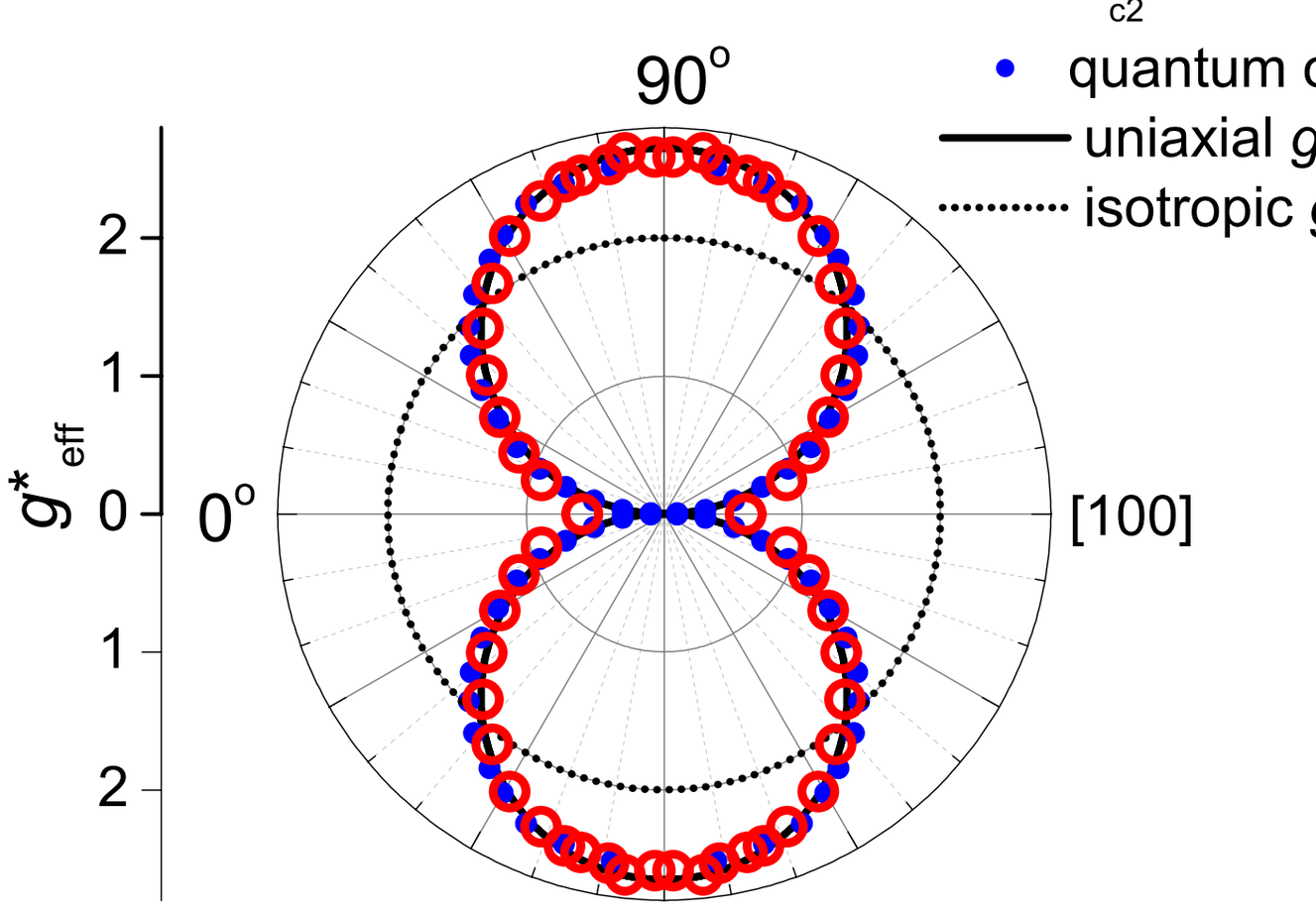

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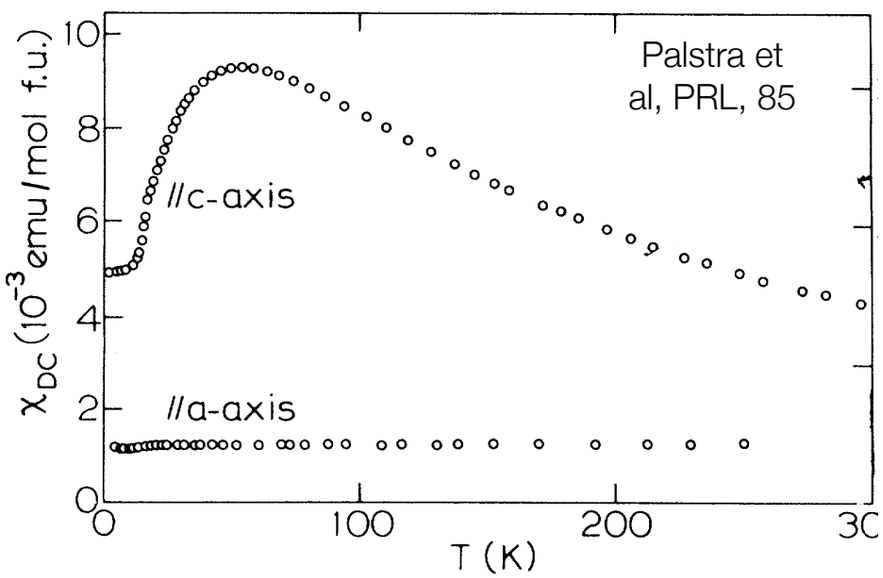

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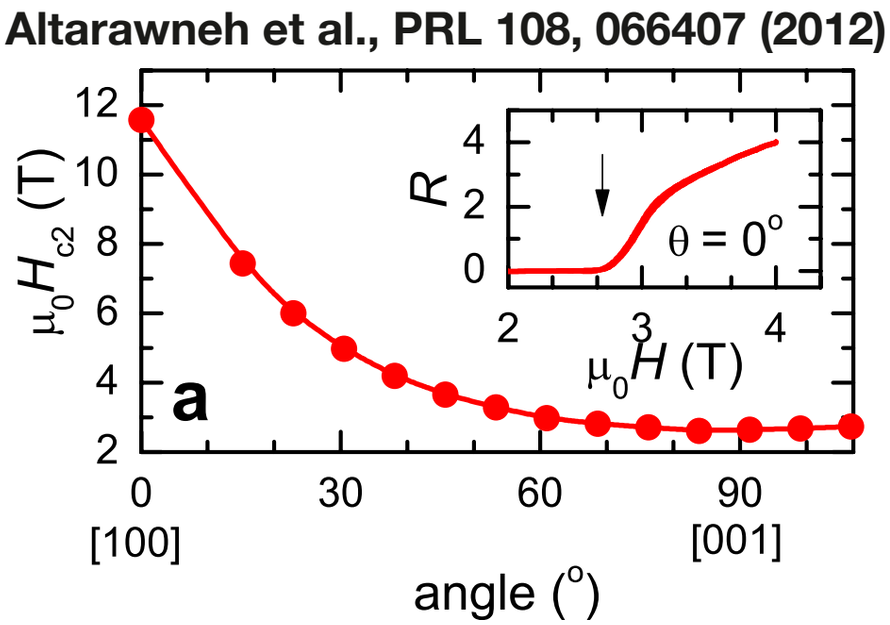

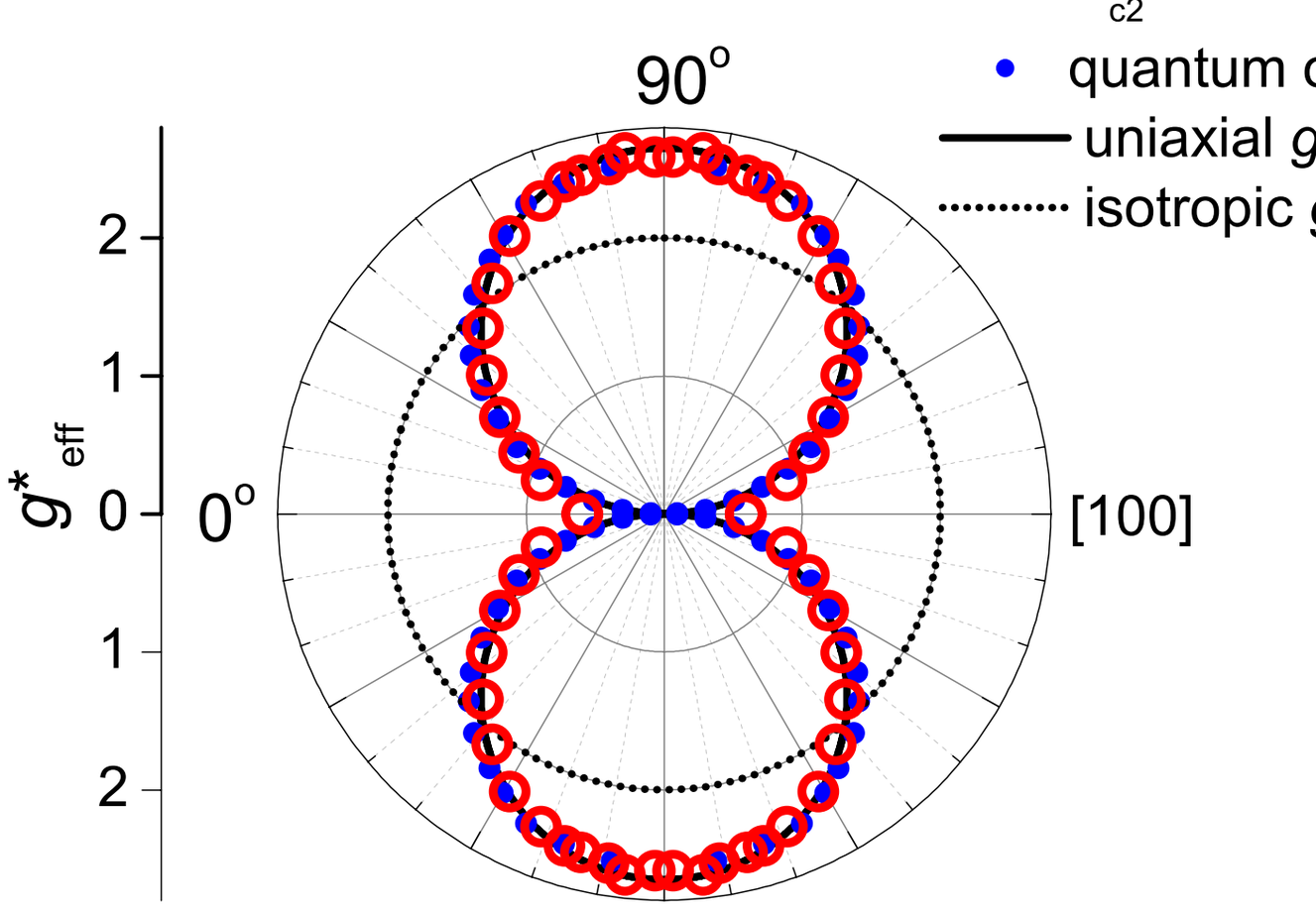

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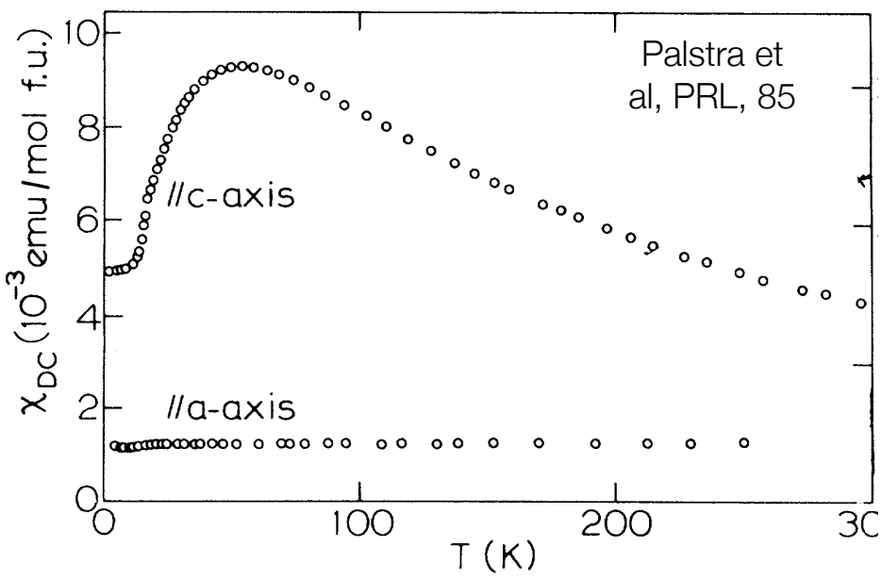

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Electrons hybridize with Ising 5f state to form Landau quasiparticles.

$$\langle \mathbf{k}\sigma | J_{\pm} | \mathbf{k}\sigma' \rangle = 0$$

“Ising Order: detailed analysis.”

Symmetry Implications of giant Ising anisotropy

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Giant Ising anisotropy indicates quasiparticle hybridization
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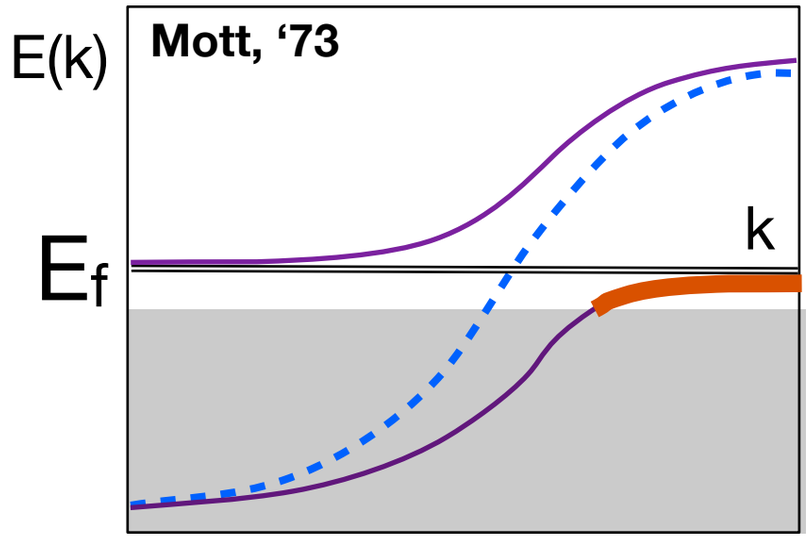

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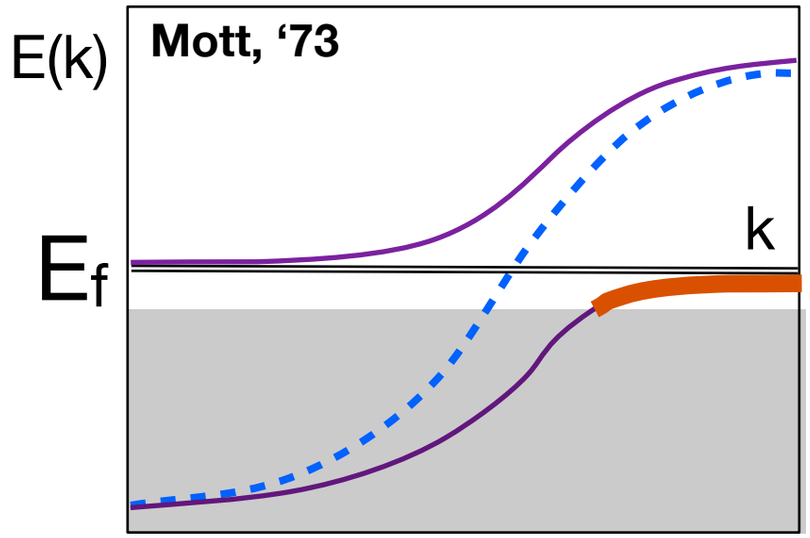

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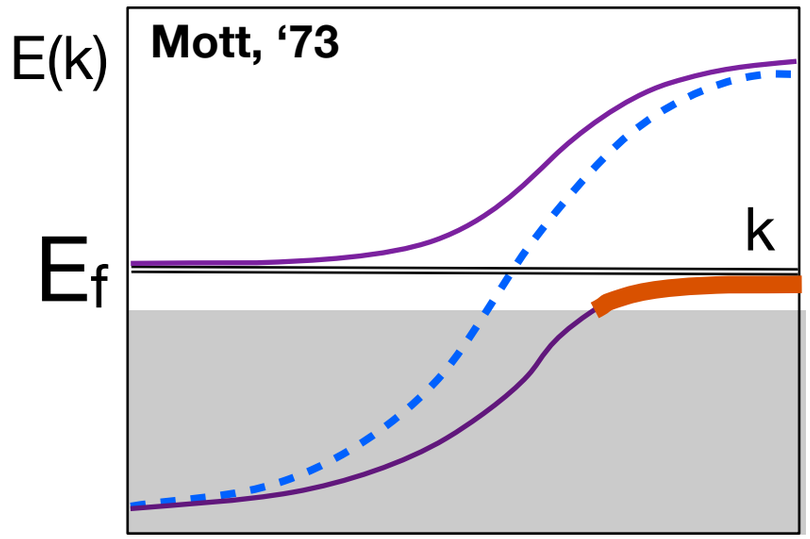

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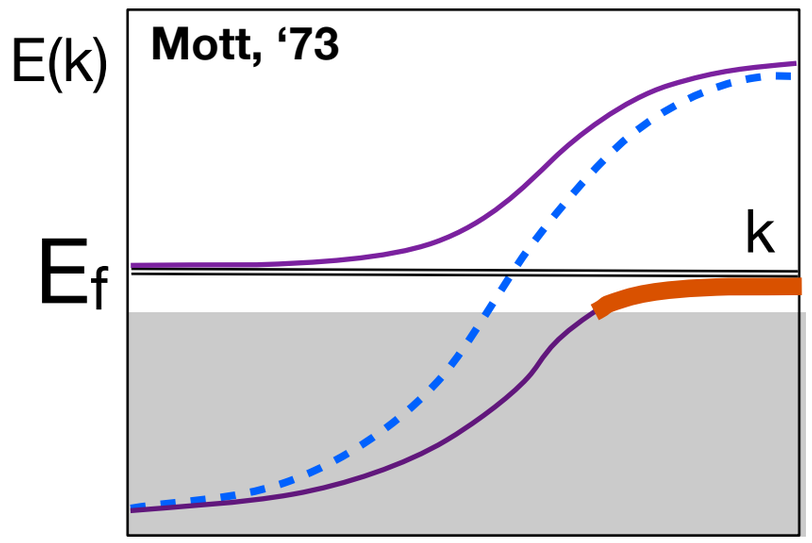

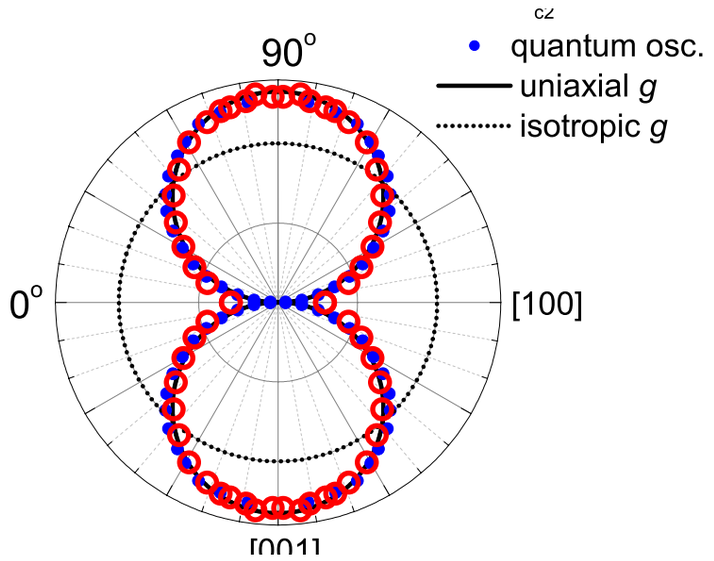

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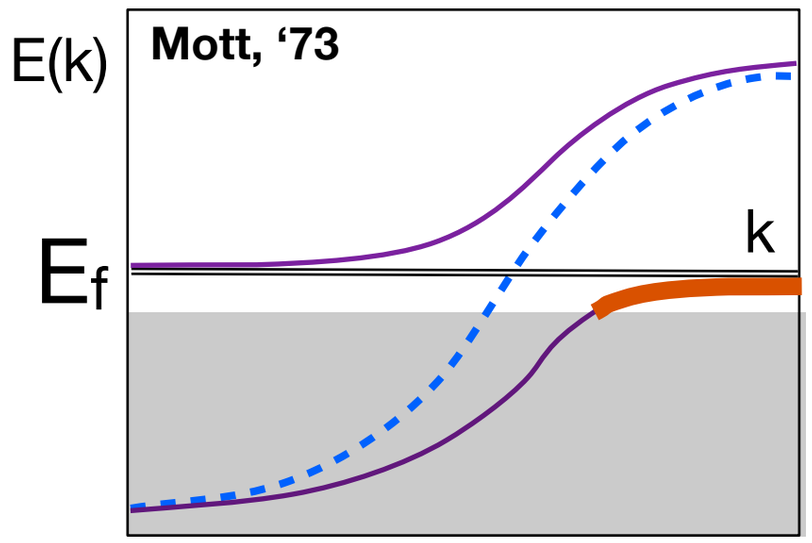

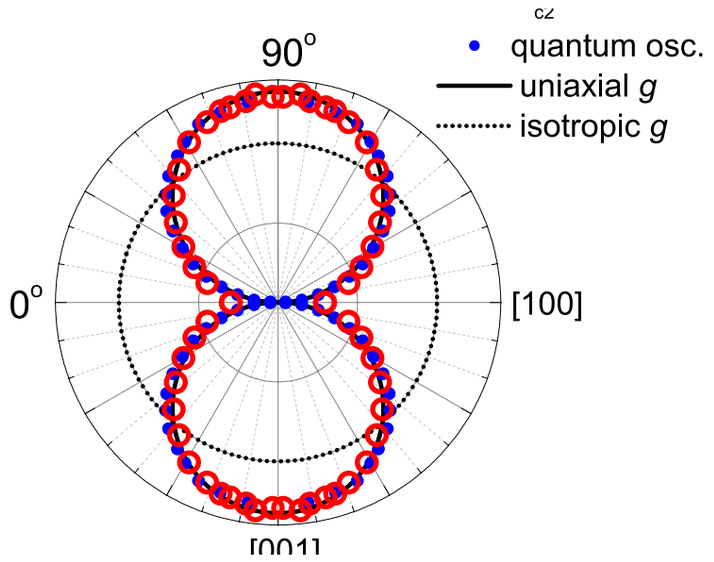

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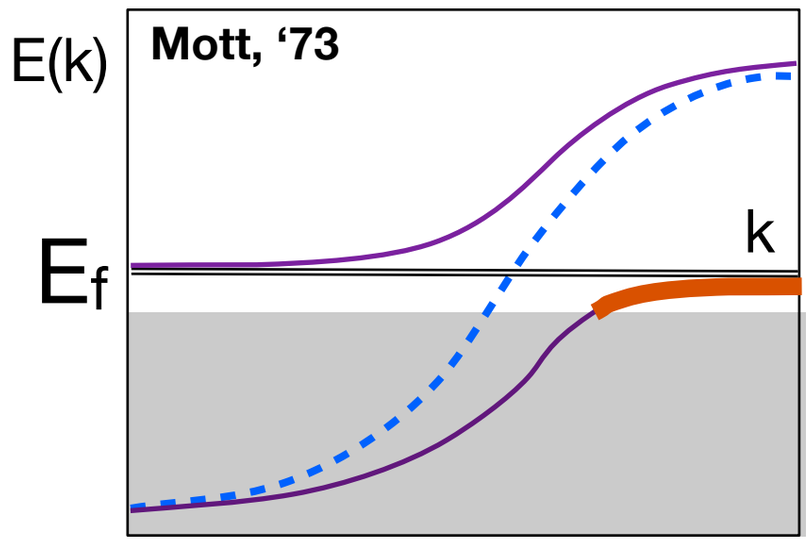

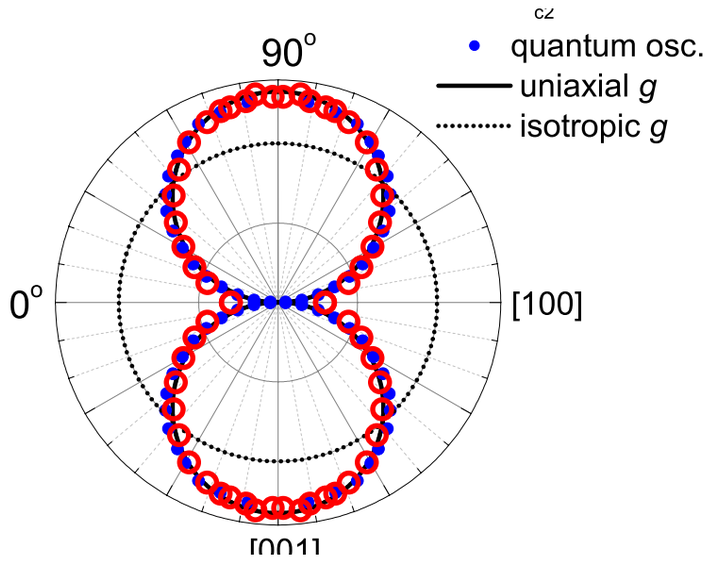

4 fold symmetry
mixes states differing by ~~4~~ *4n*

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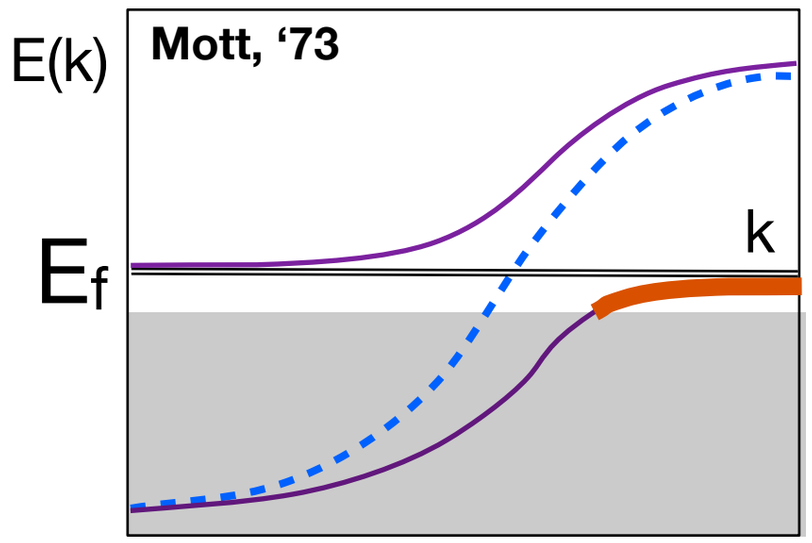
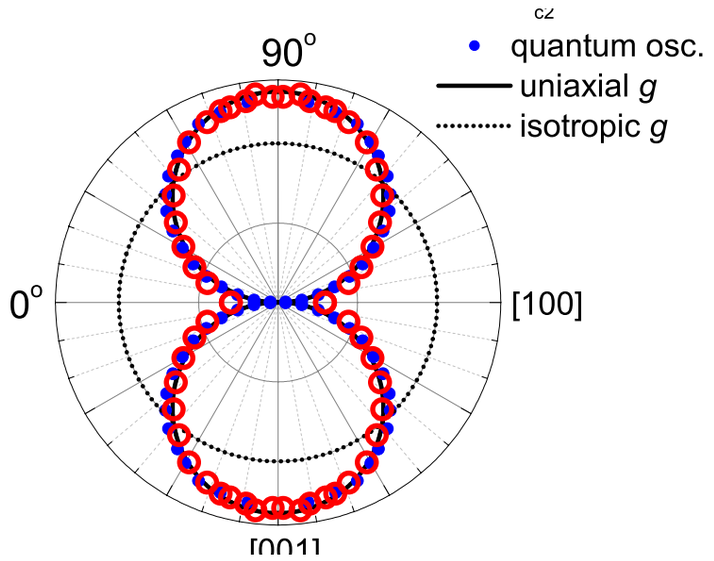
$$\mathcal{H} = (|\mathbf{k}\sigma\rangle V_{\sigma\alpha}(\mathbf{k})\langle\alpha| + \text{H.c})$$





4 fold symmetry
mixes states differing by ~~4n~~

$$|+\rangle = \sum_n a_n |M - 4n\rangle$$

$$|-\rangle = \sum_n a_n |-M + 4n\rangle$$

$$\sum_{n,n'} a_n a_{n'} \langle M - 4n|J_{\pm}|-M + 4n'\rangle = 0$$

Symmetry Implications of giant Ising anisotropy

$$\langle +|J_{\pm}|-\rangle = 0$$

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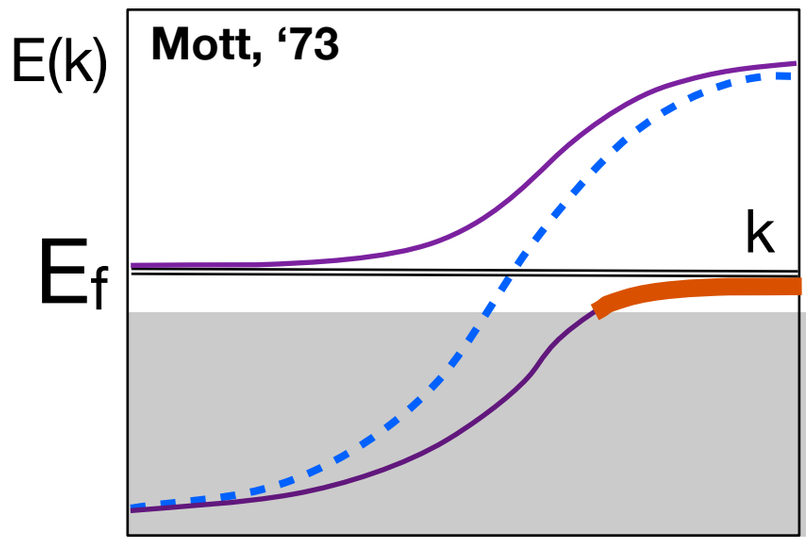

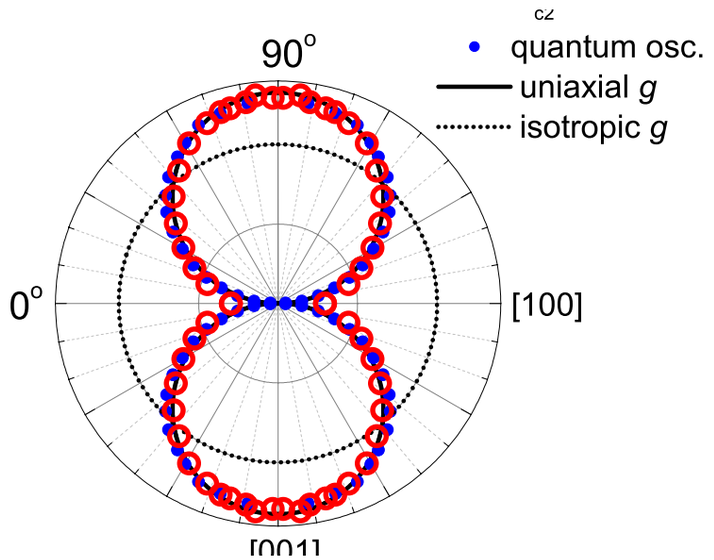

Must vanish

4 fold symmetry mixes states differing by ~~4~~ *4n*

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Symmetry Implications of giant Ising anisotropy

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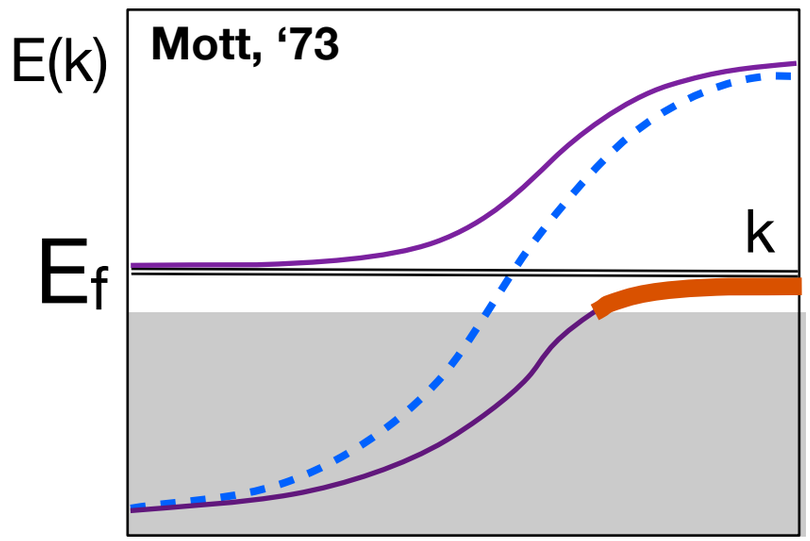

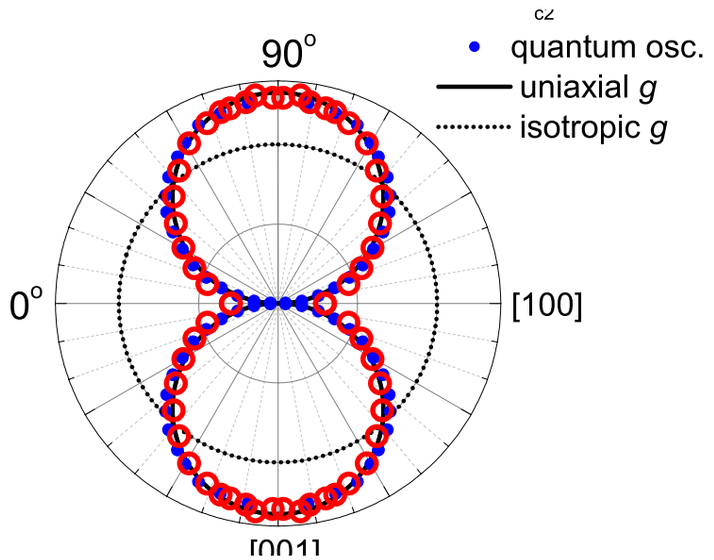

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$$M - 4n \neq -M + 4n' \pm 1$$

Symmetry Implications of giant Ising anisotropy

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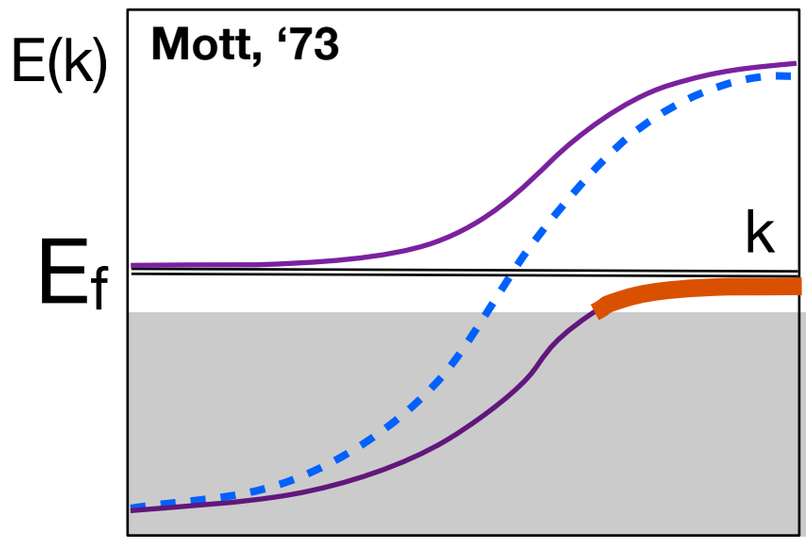

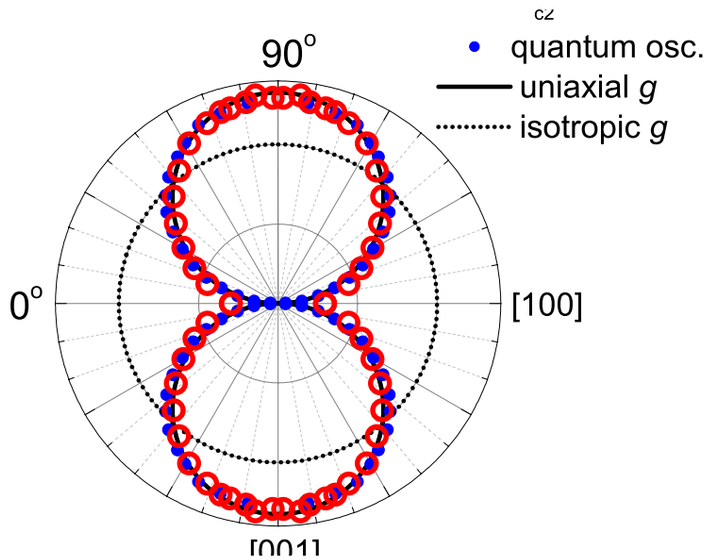

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$$M \neq 2(n - n') \pm \tfrac{1}{2}$$

Symmetry Implications of giant Ising anisotropy

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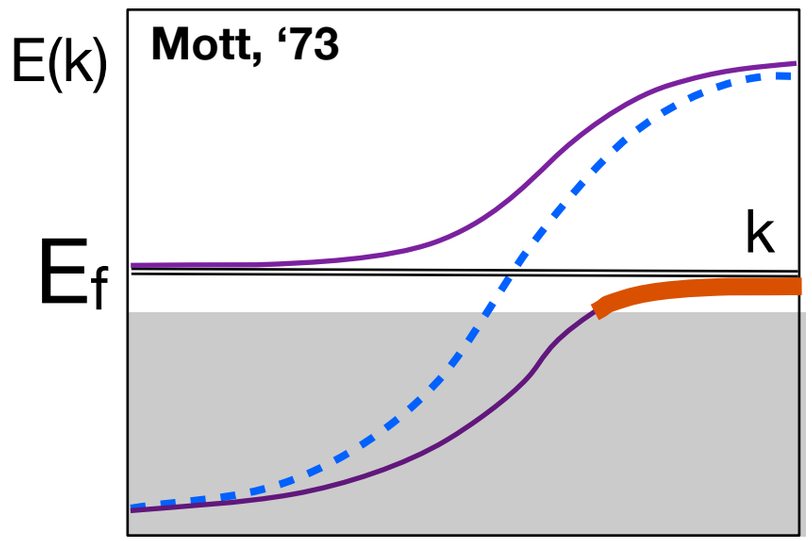

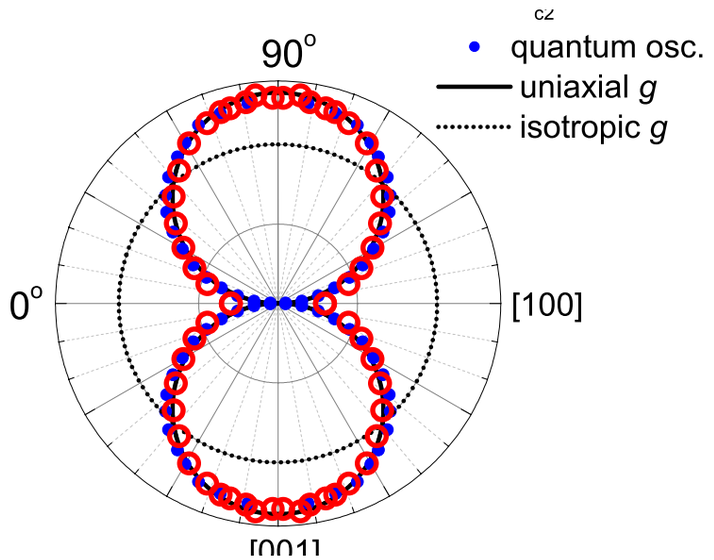

Must vanish

4 fold symmetry mixes states differing by ~~4~~ 2

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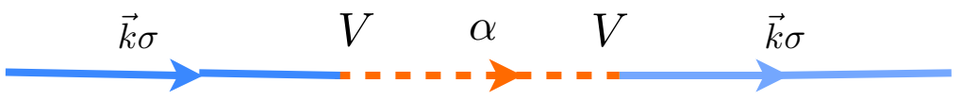
$$M \neq 2(n - n') \pm \tfrac{1}{2}$$

$\therefore$ Integer spin M

Symmetry Implications of giant Ising anisotropy



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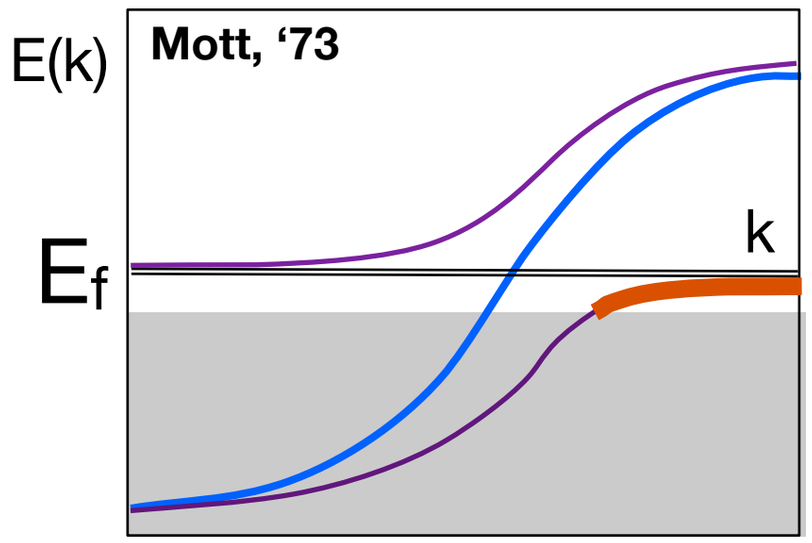
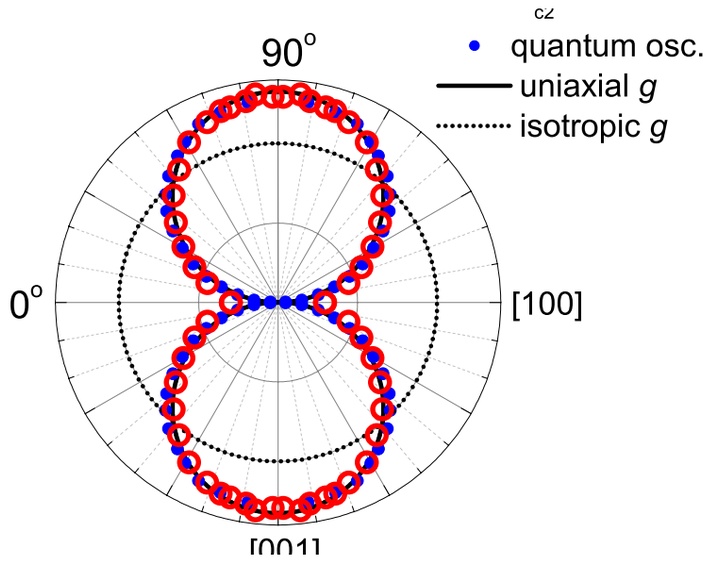
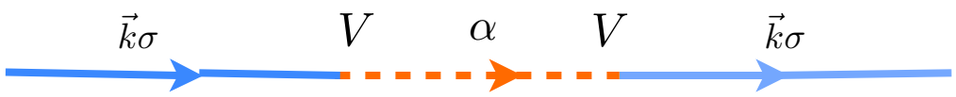




∴ Integer spin M

Symmetry Implications of giant Ising anisotropy



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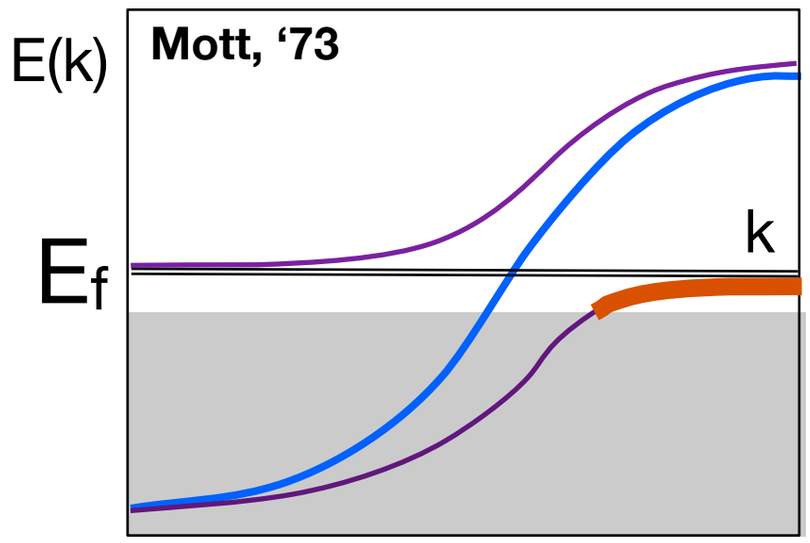
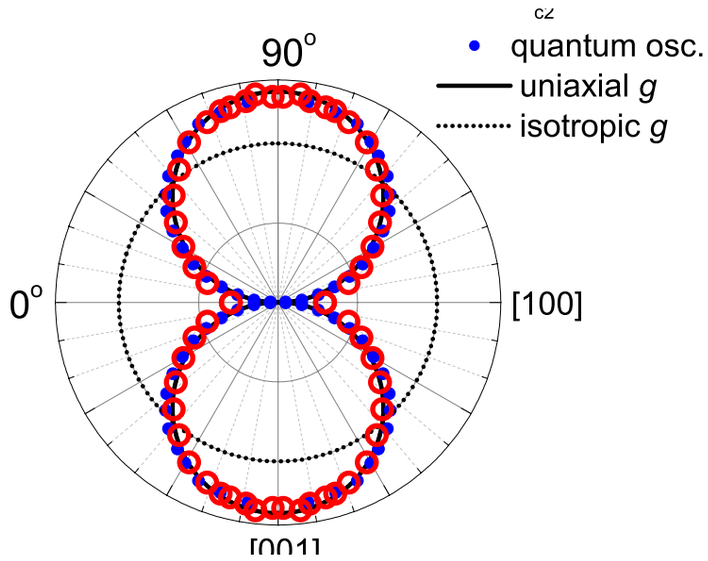


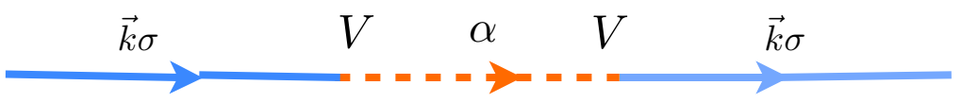


$$|\Gamma,\pm\rangle = a|\pm 3\rangle + b|\mp 1\rangle$$

"Γ_5" non-Kramers doublet $5f^2$

$\therefore$ Integer spin M

Symmetry Implications of giant Ising anisotropy



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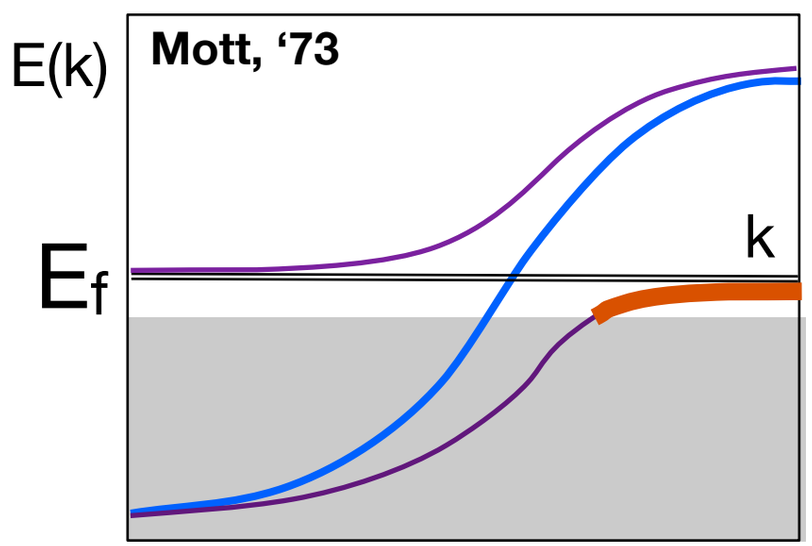
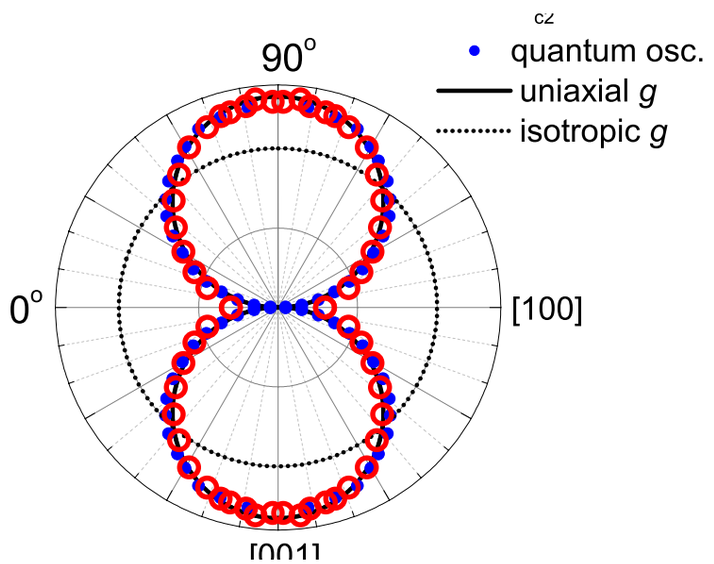
Integer





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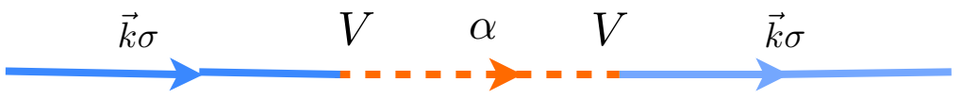
Symmetry Implications of giant Ising anisotropy



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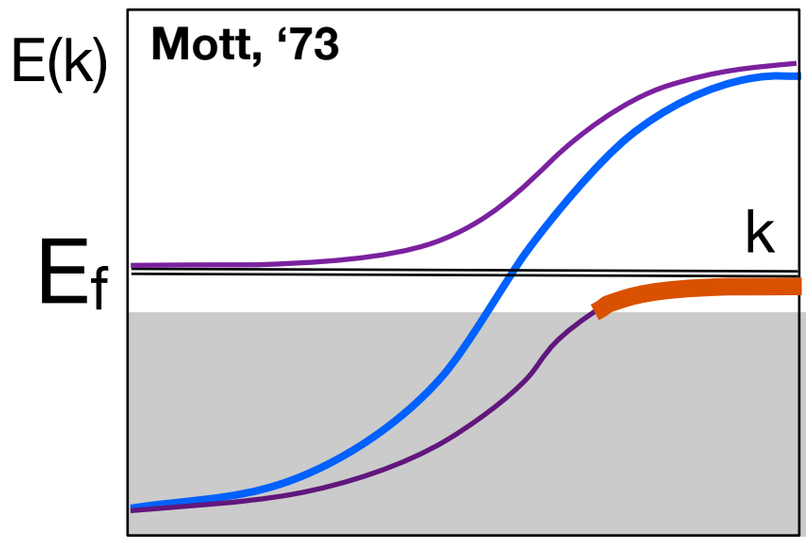
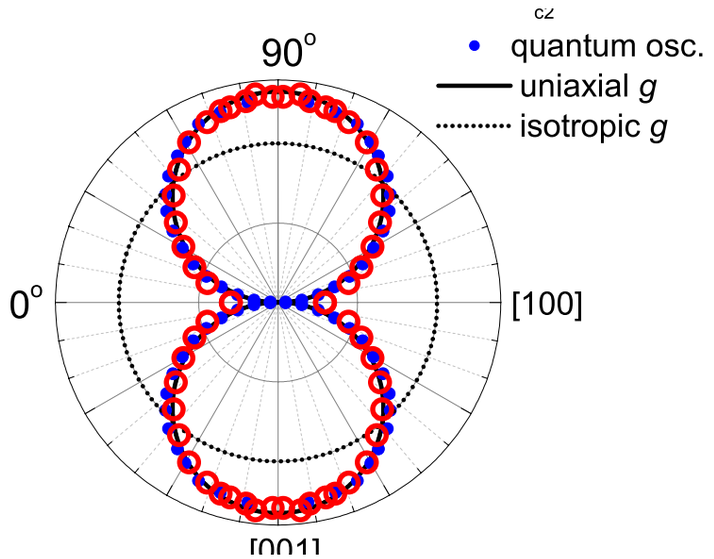
Integer

1/2 integer





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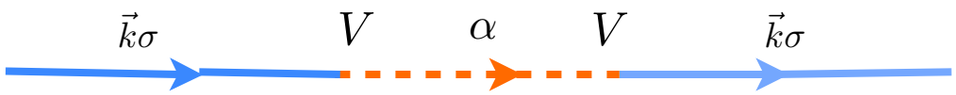
Symmetry Implications of giant Ising anisotropy



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1/2 integer

Integer

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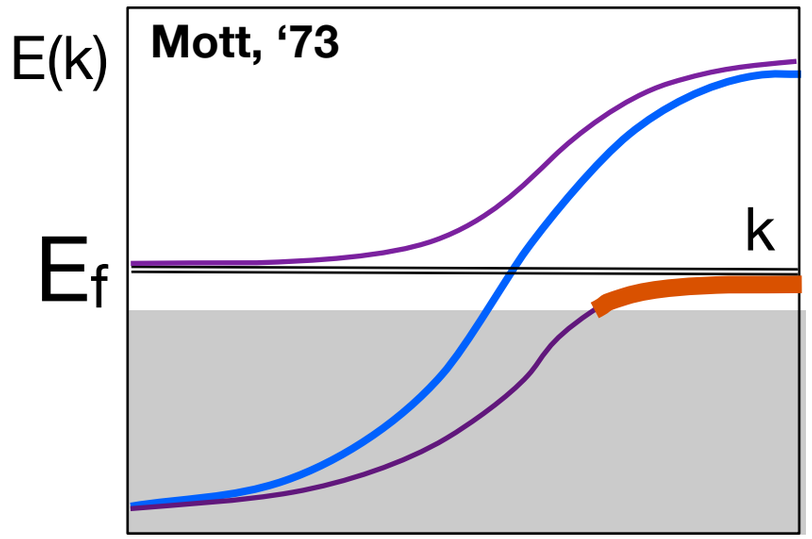

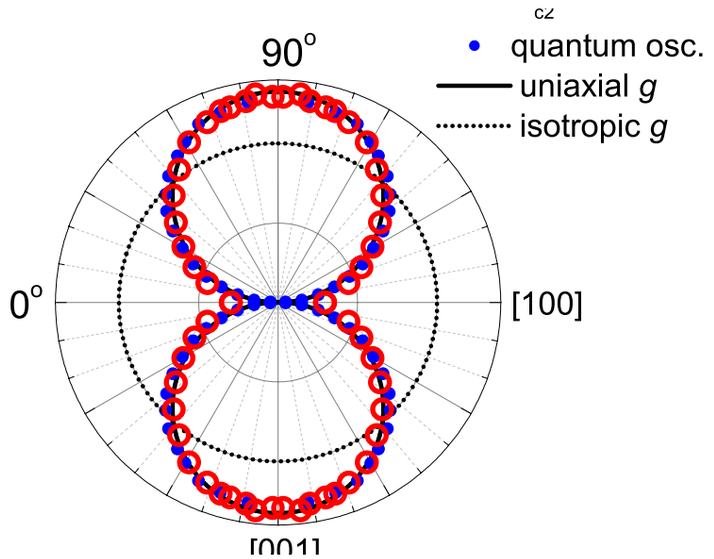

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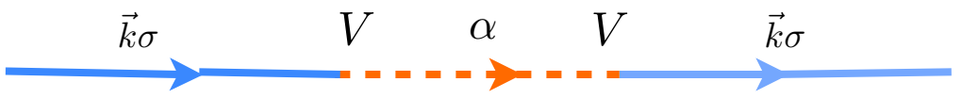

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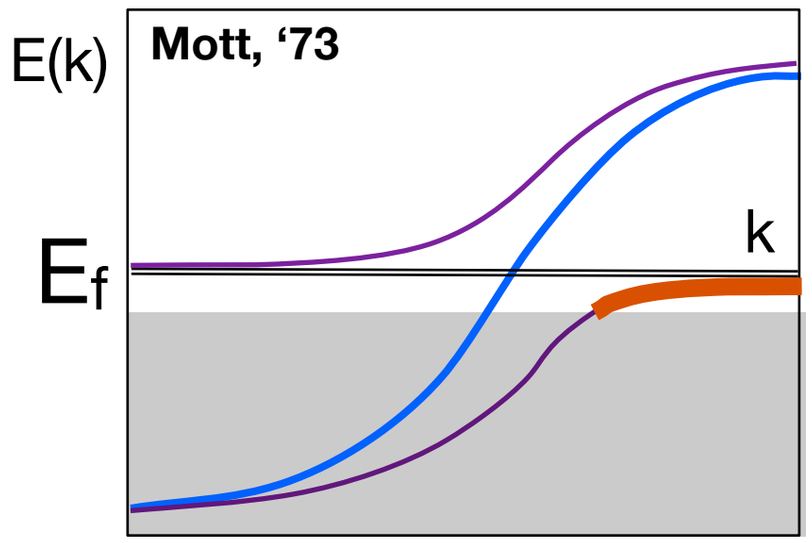

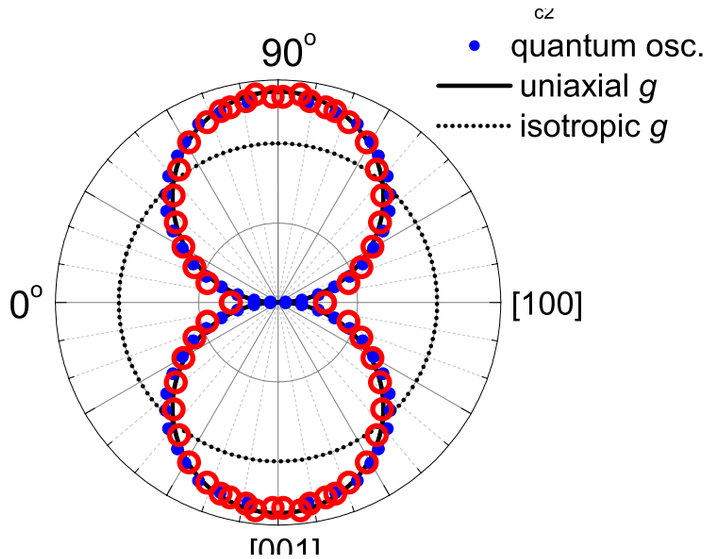

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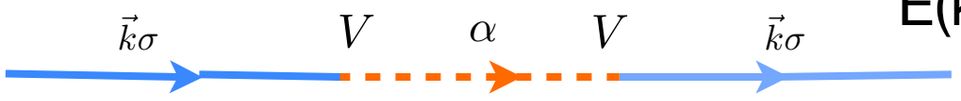
Hybridization is a spinor.

Symmetry Implications of giant Ising anisotropy



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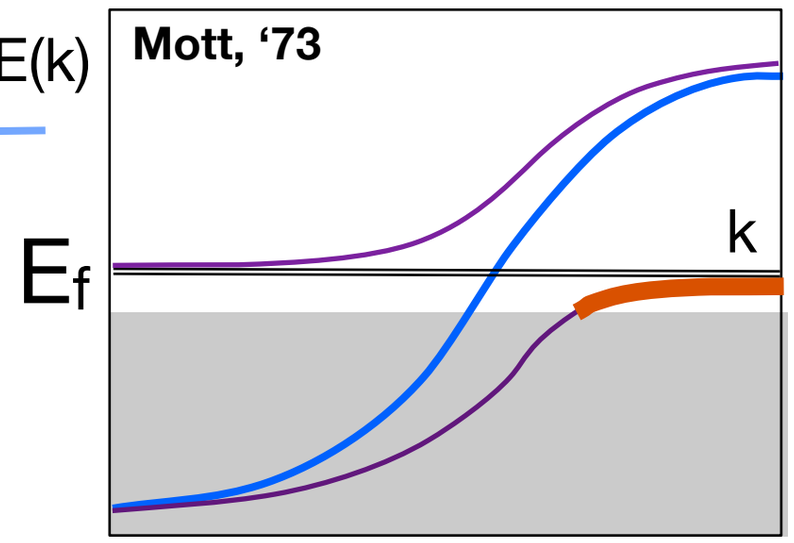

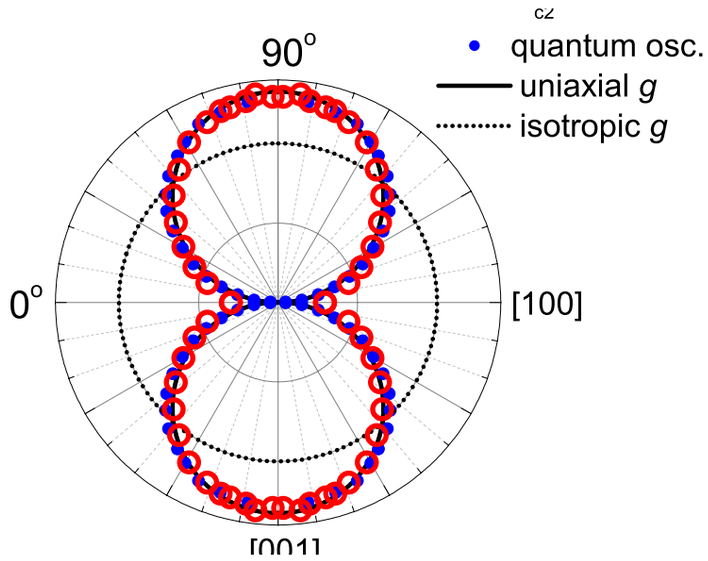

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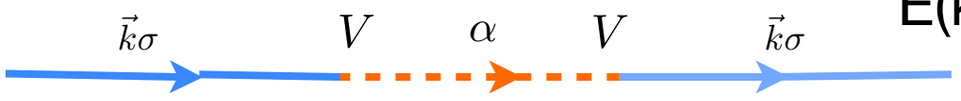
Symmetry Implications of giant Ising anisotropy



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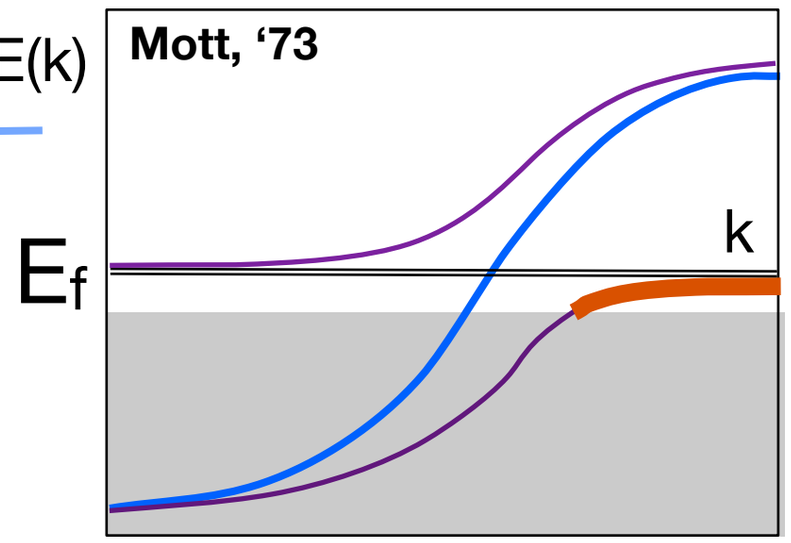

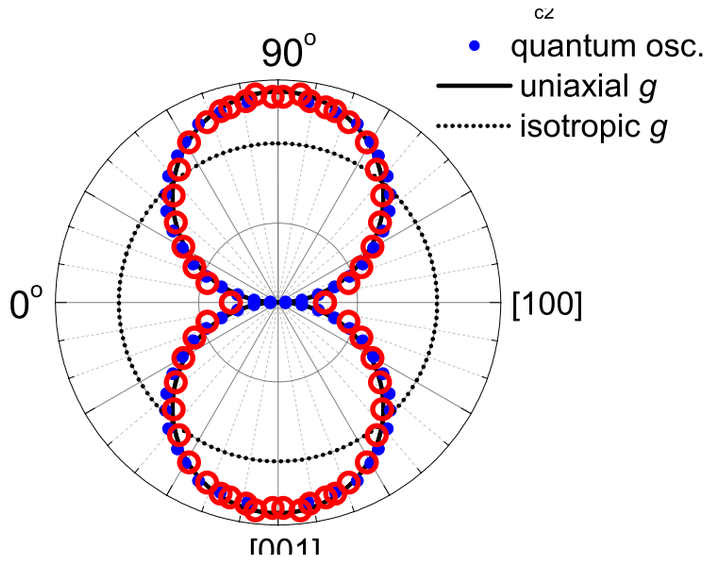

Kramers index K: quantum no of ***double*** time reversal θxθ = θ^2.

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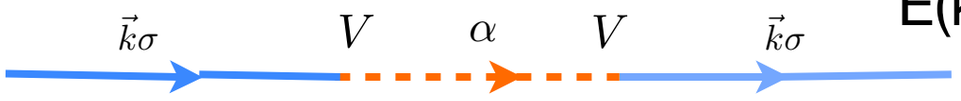
Symmetry Implications of giant Ising anisotropy



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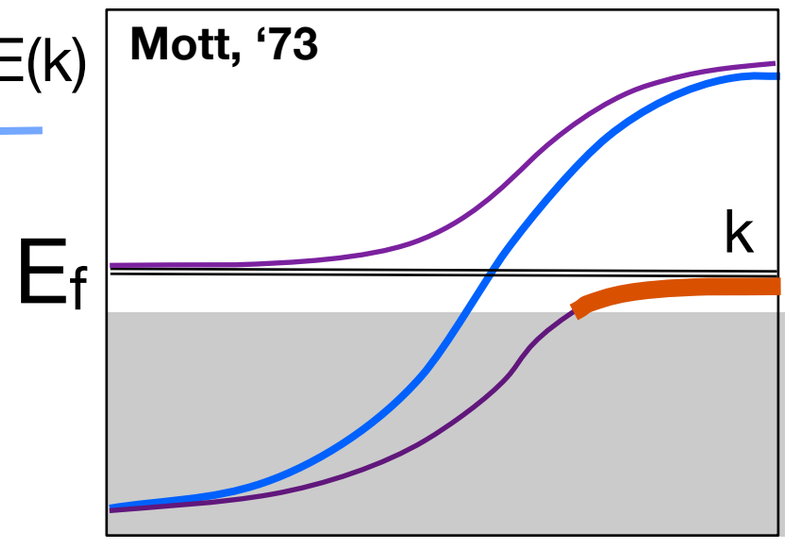

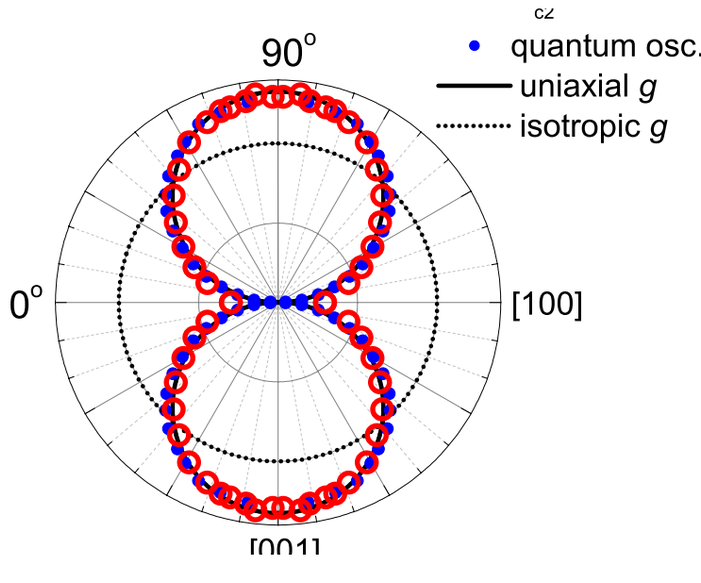

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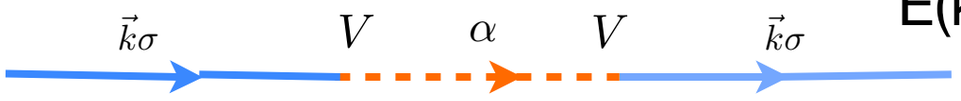

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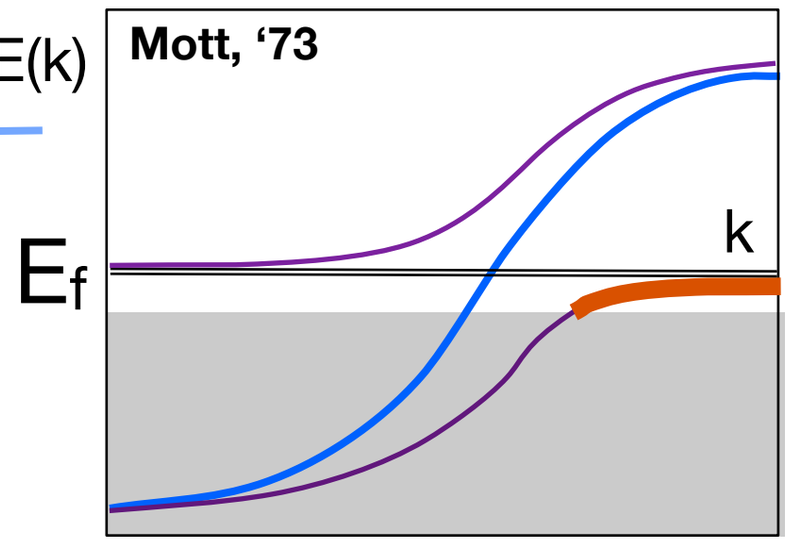

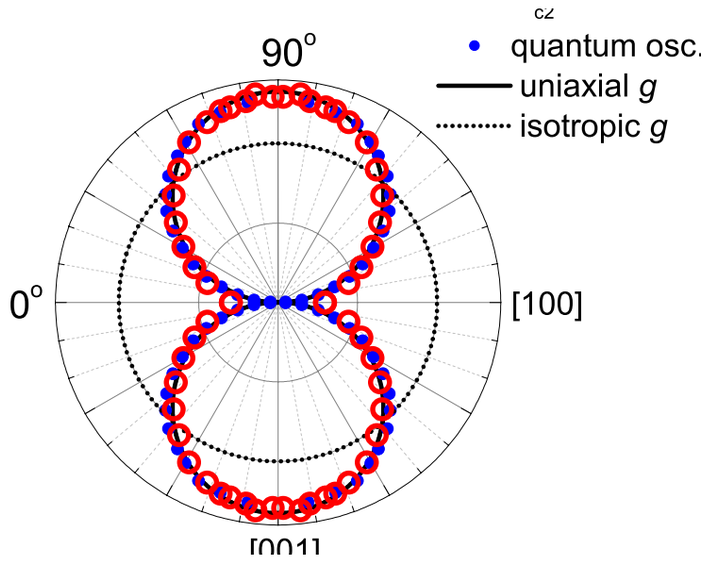

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Half-integer spins change sign, integer spins do not.

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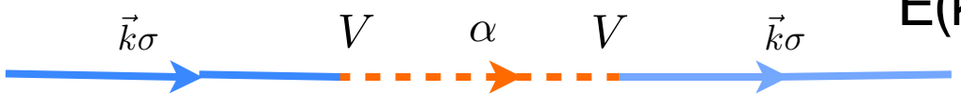
Symmetry Implications of giant Ising anisotropy



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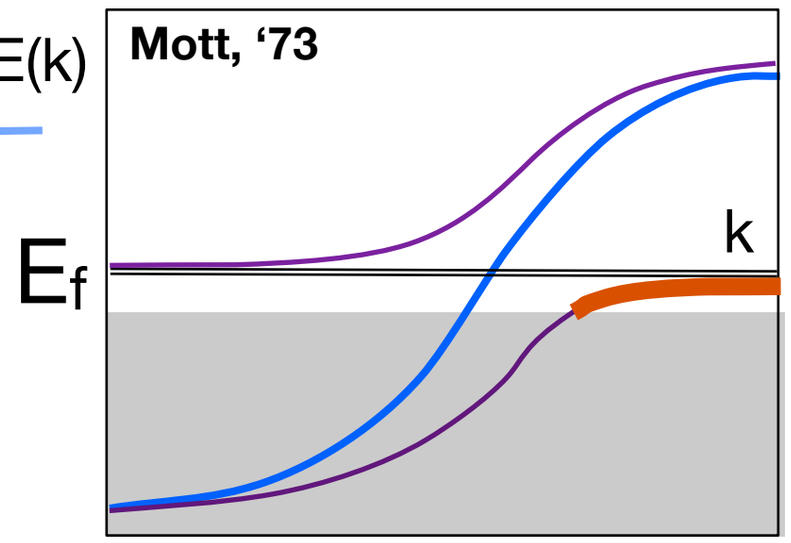
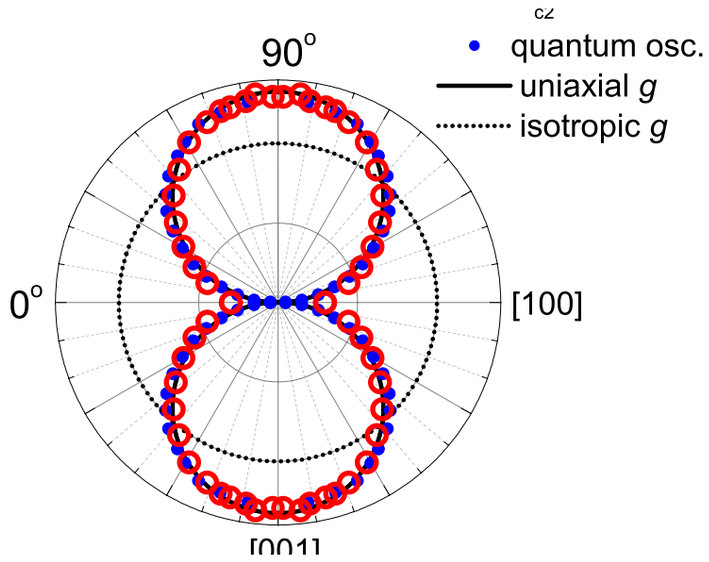
-1





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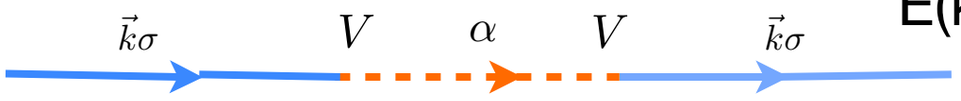

$$|\alpha\rangle \equiv |\pm\rangle$$

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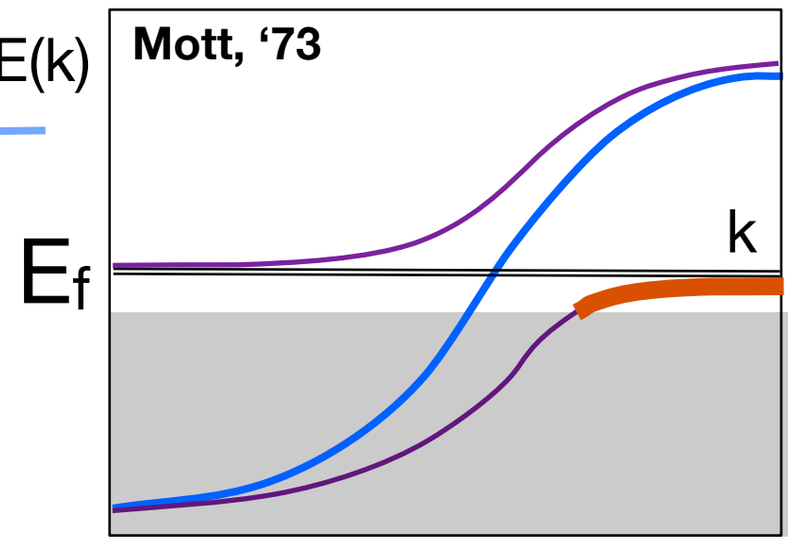
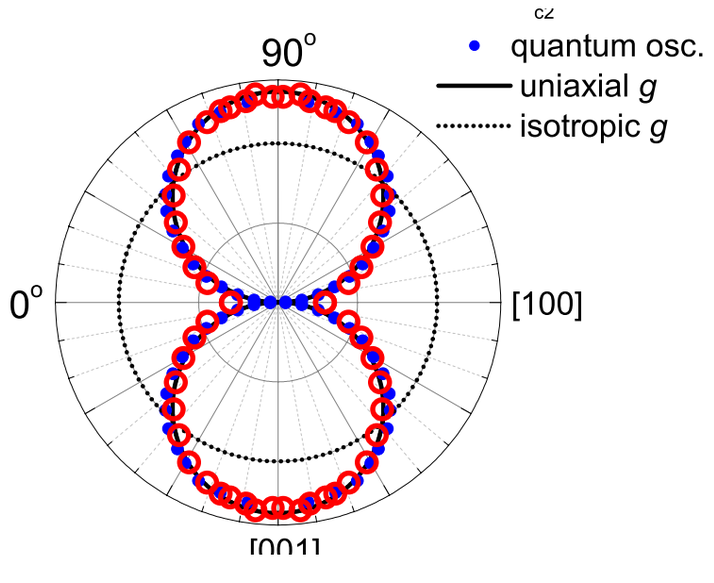
-1 +1





Kramers index K: quantum no of ***double*** time reversal θxθ = θ^2.

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Half-integer spins change sign, integer spins do not.

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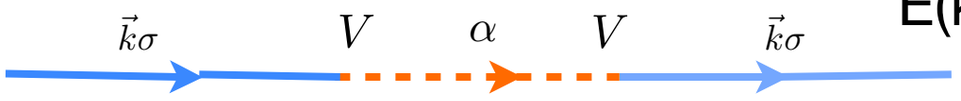

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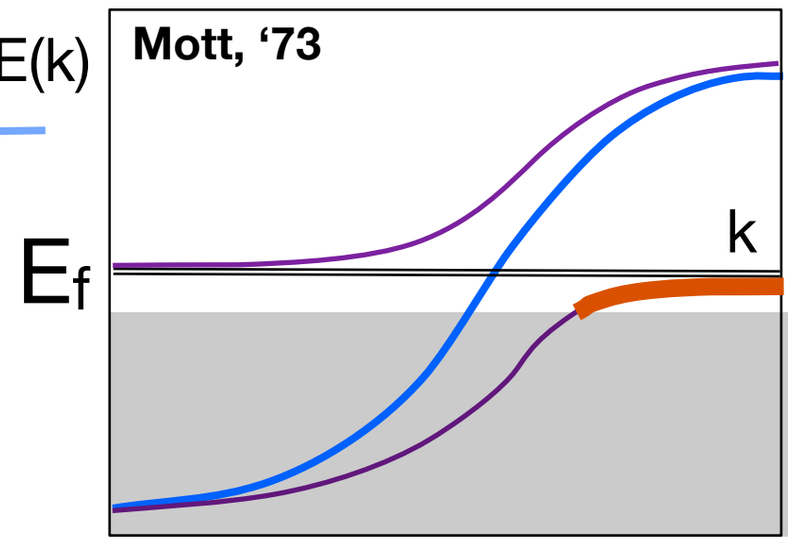

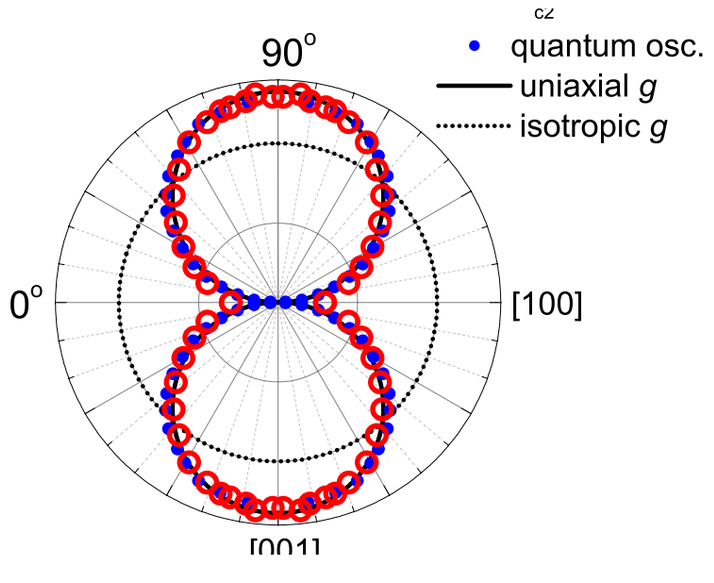

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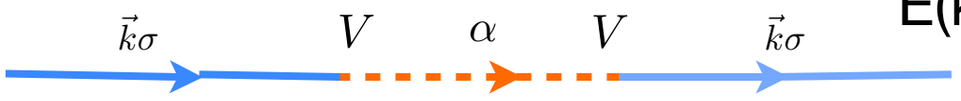

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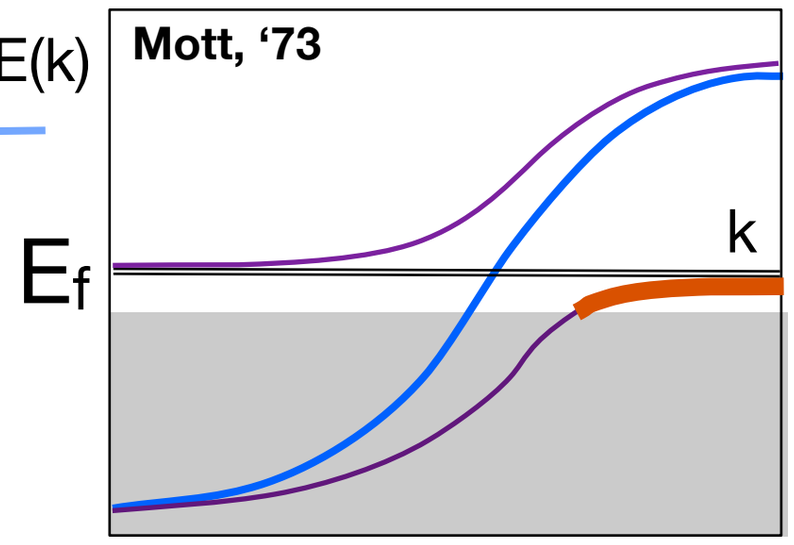

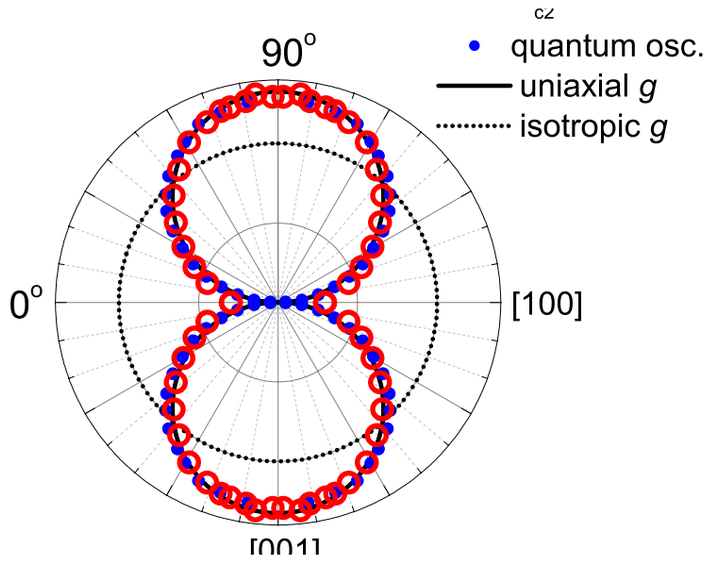

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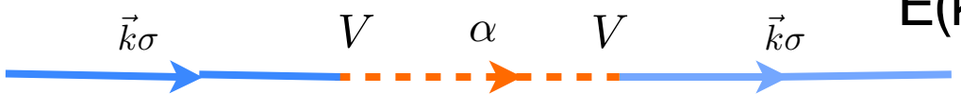

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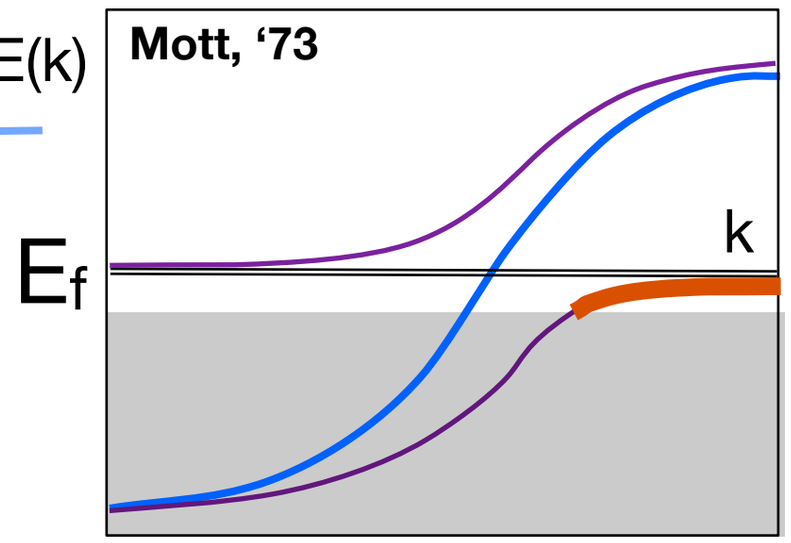

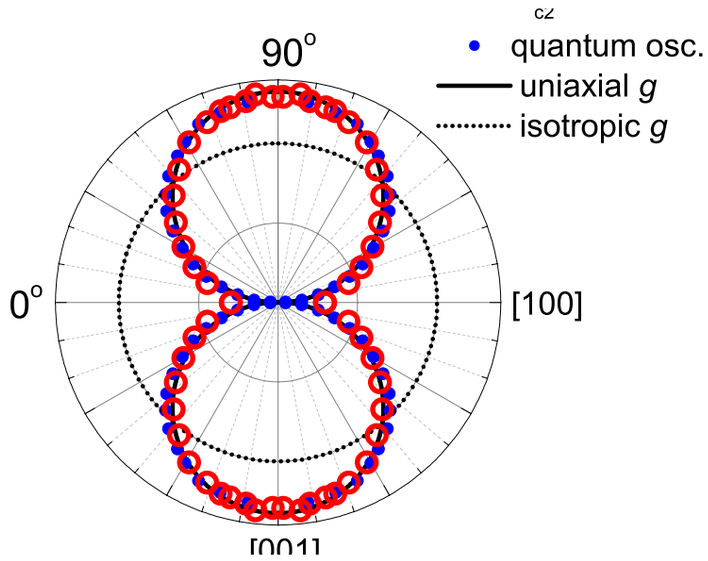

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Hybridization transforms as a 1/2 integer spin.

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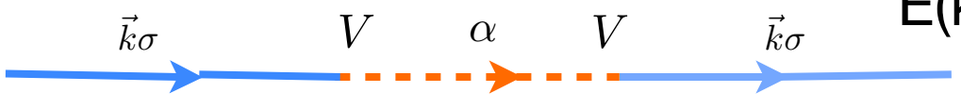
Symmetry Implications of giant Ising anisotropy



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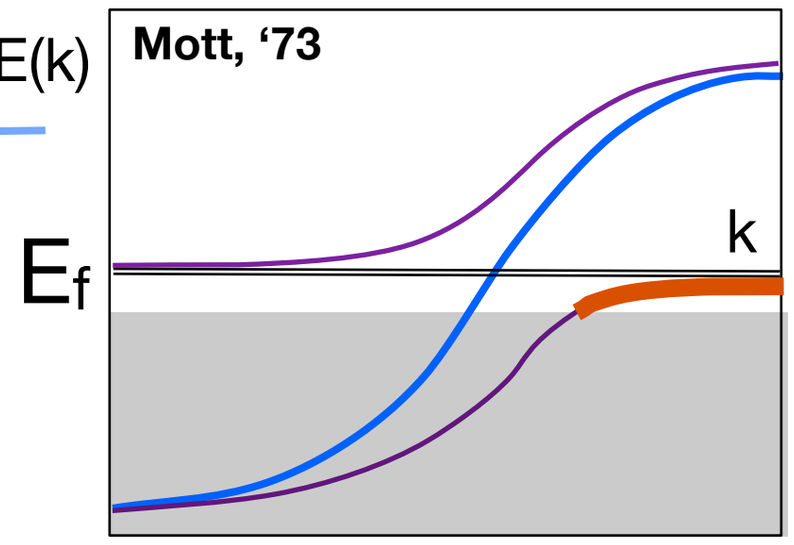

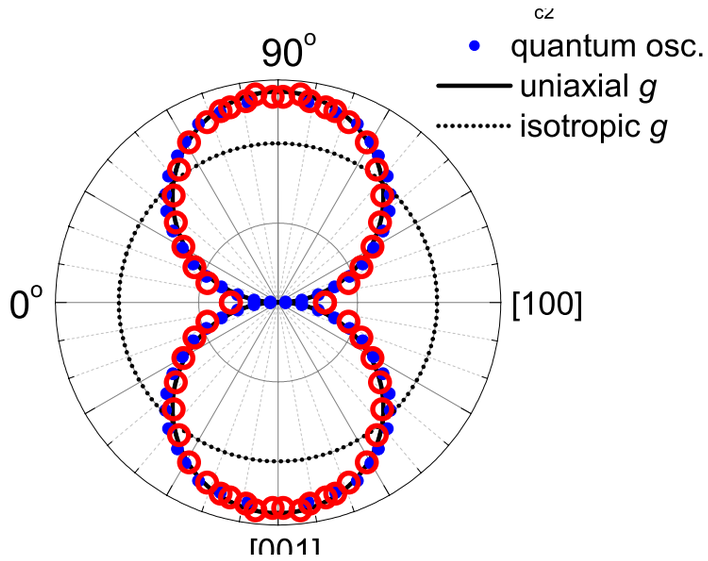

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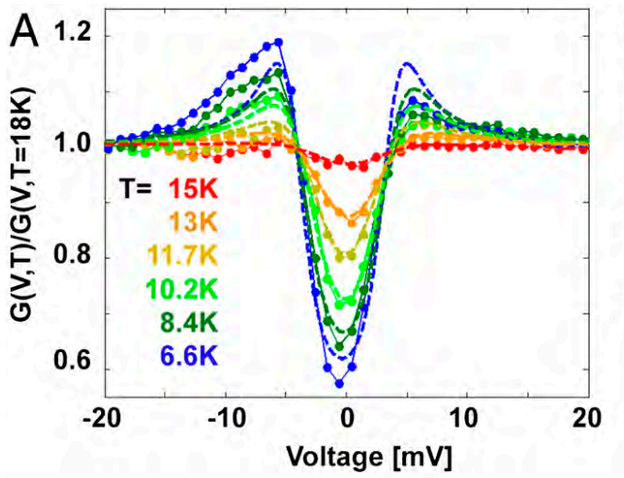
Hybridization transforms as a 1/2 integer spin.
Unlike magnetism, it breaks **double time reversal.**
A new kind of order parameter.

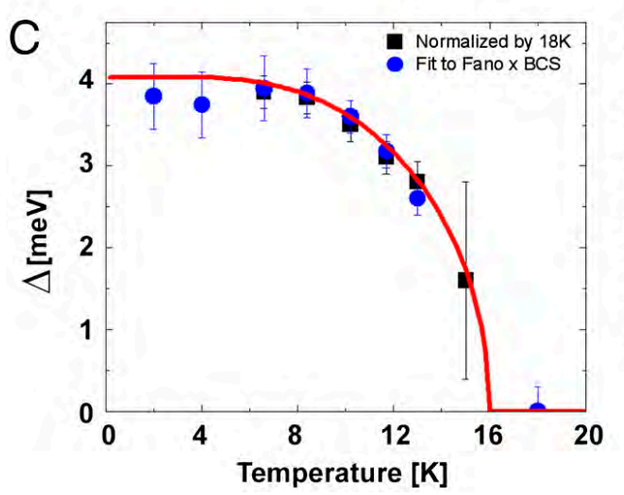
Hybridization is a spinor.

Support for hybridization as the order parameter.

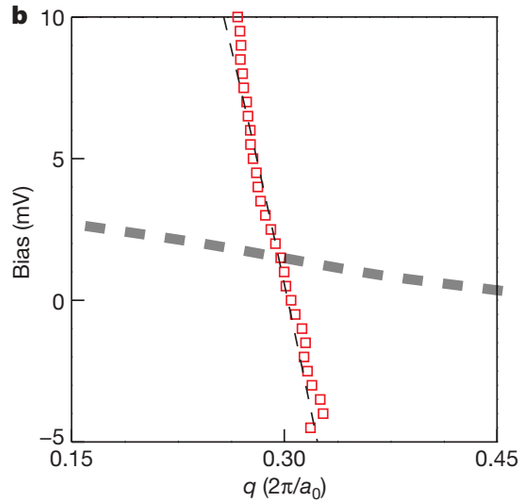
Spectroscopy: H-gap in STM/Optics

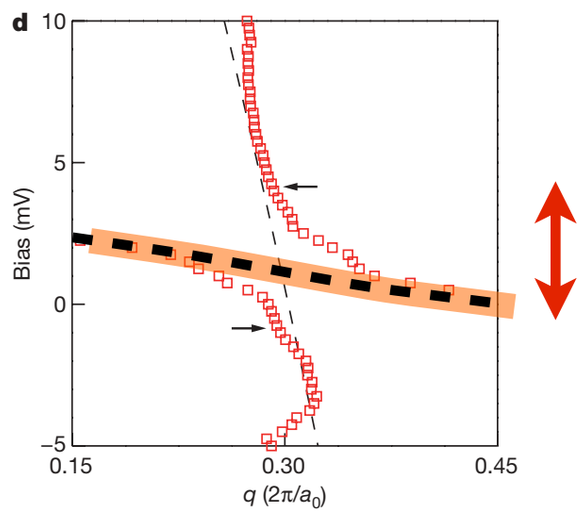
Aynajian* et al, PNAS (2010)





Schmidt*, Hamidian et al, Nature (2010)





Spectroscopy
H-gap in STM/Optics

Aynajian* et al,
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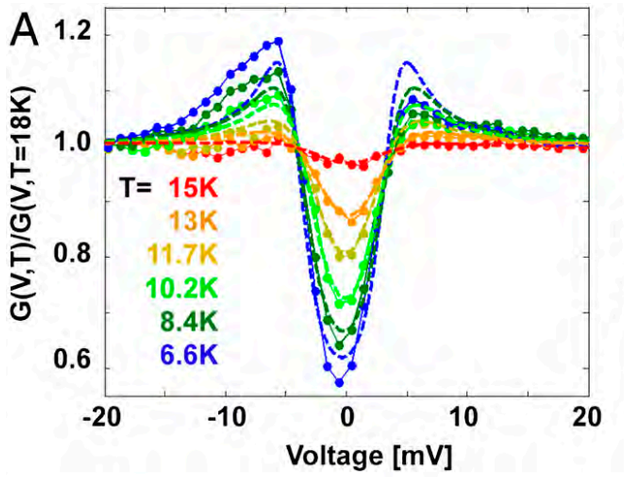

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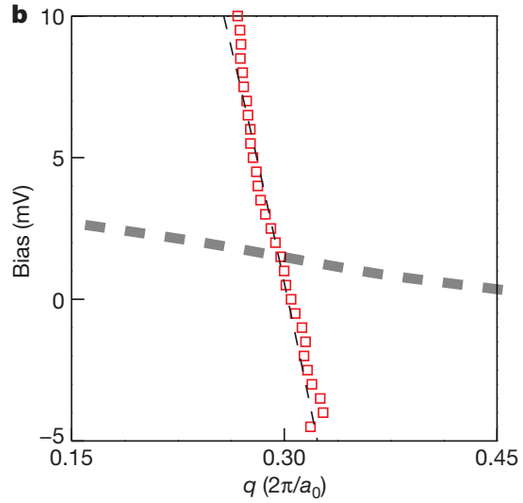

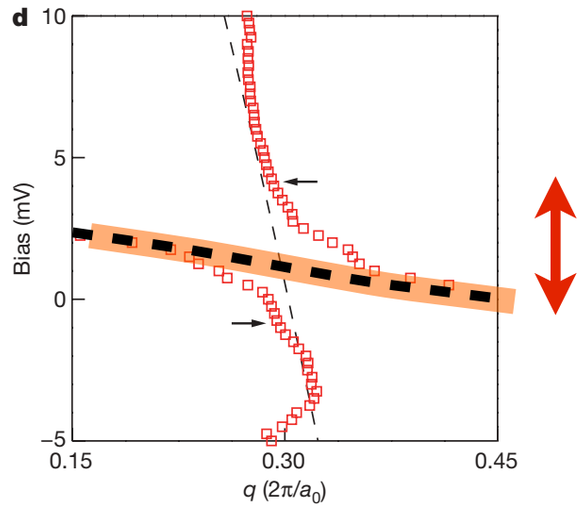

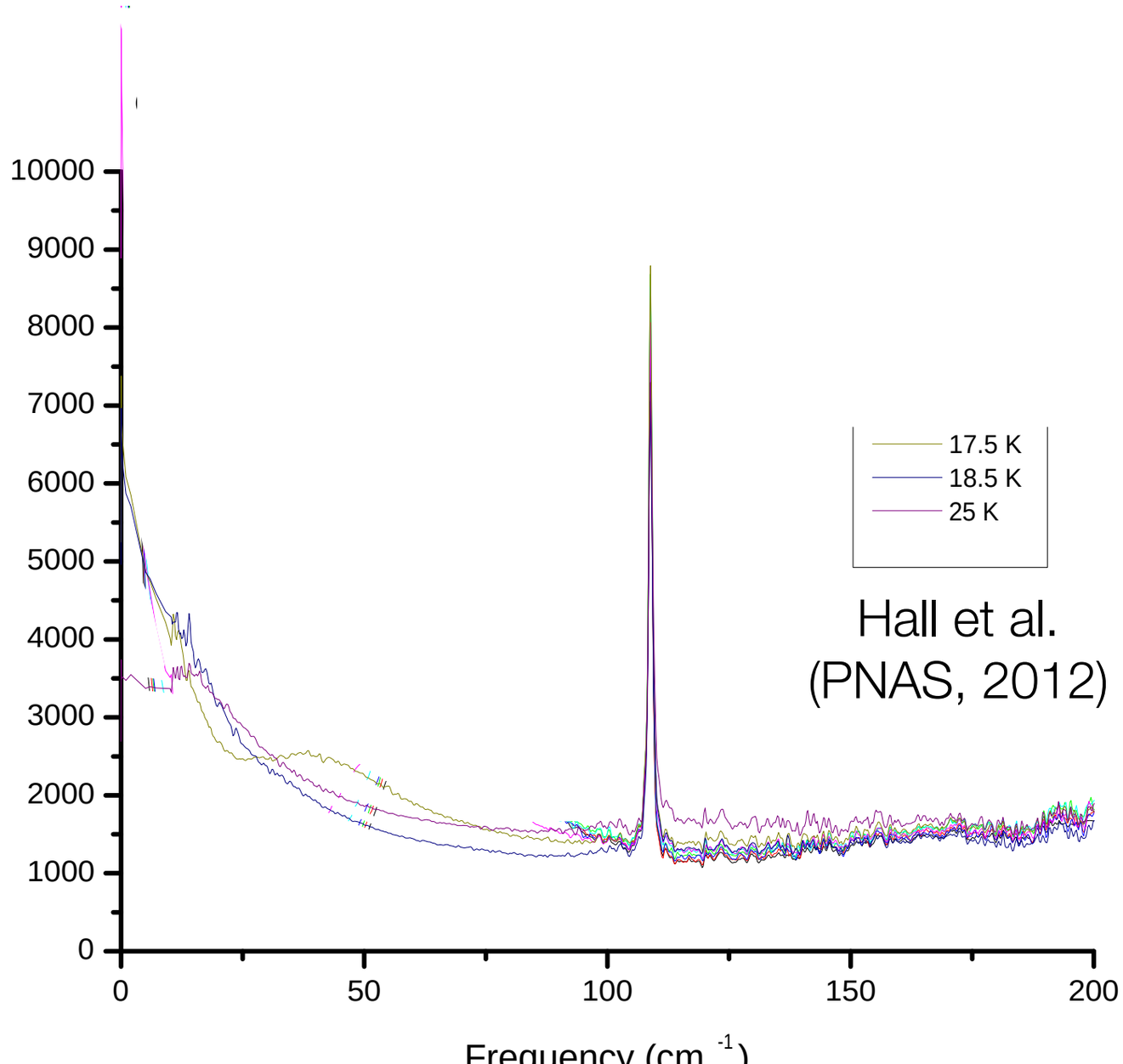

Hall et al.
(PNAS, 2012)

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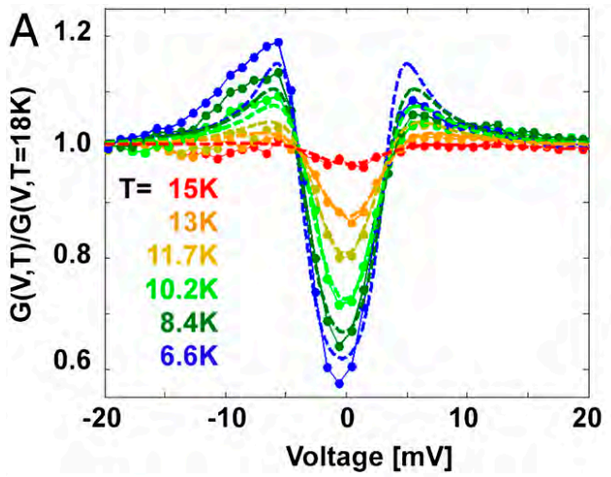

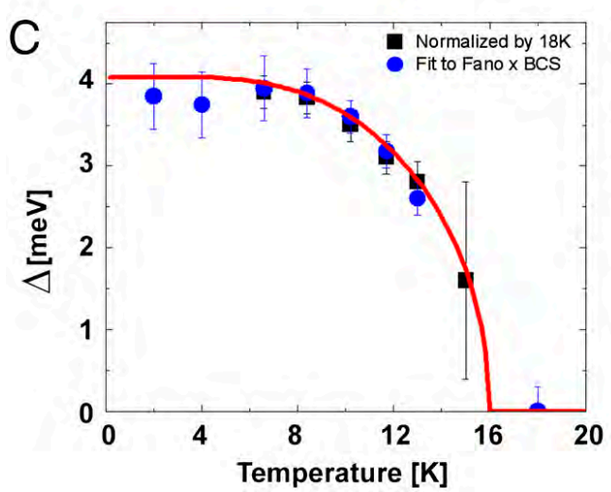

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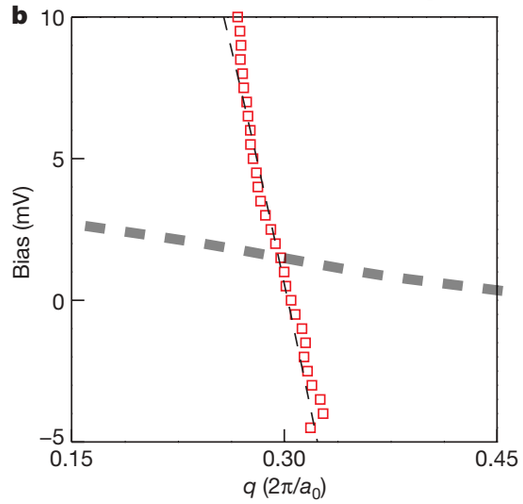

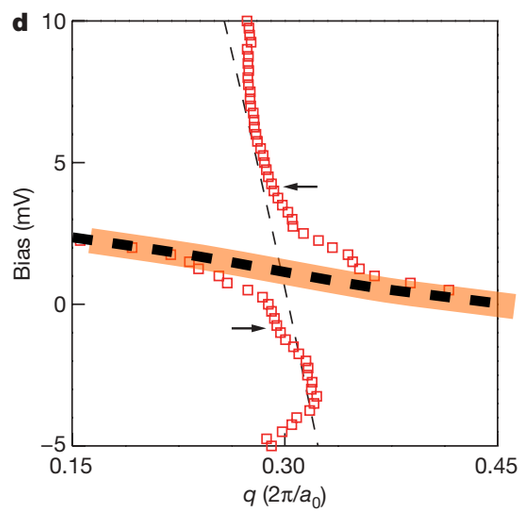

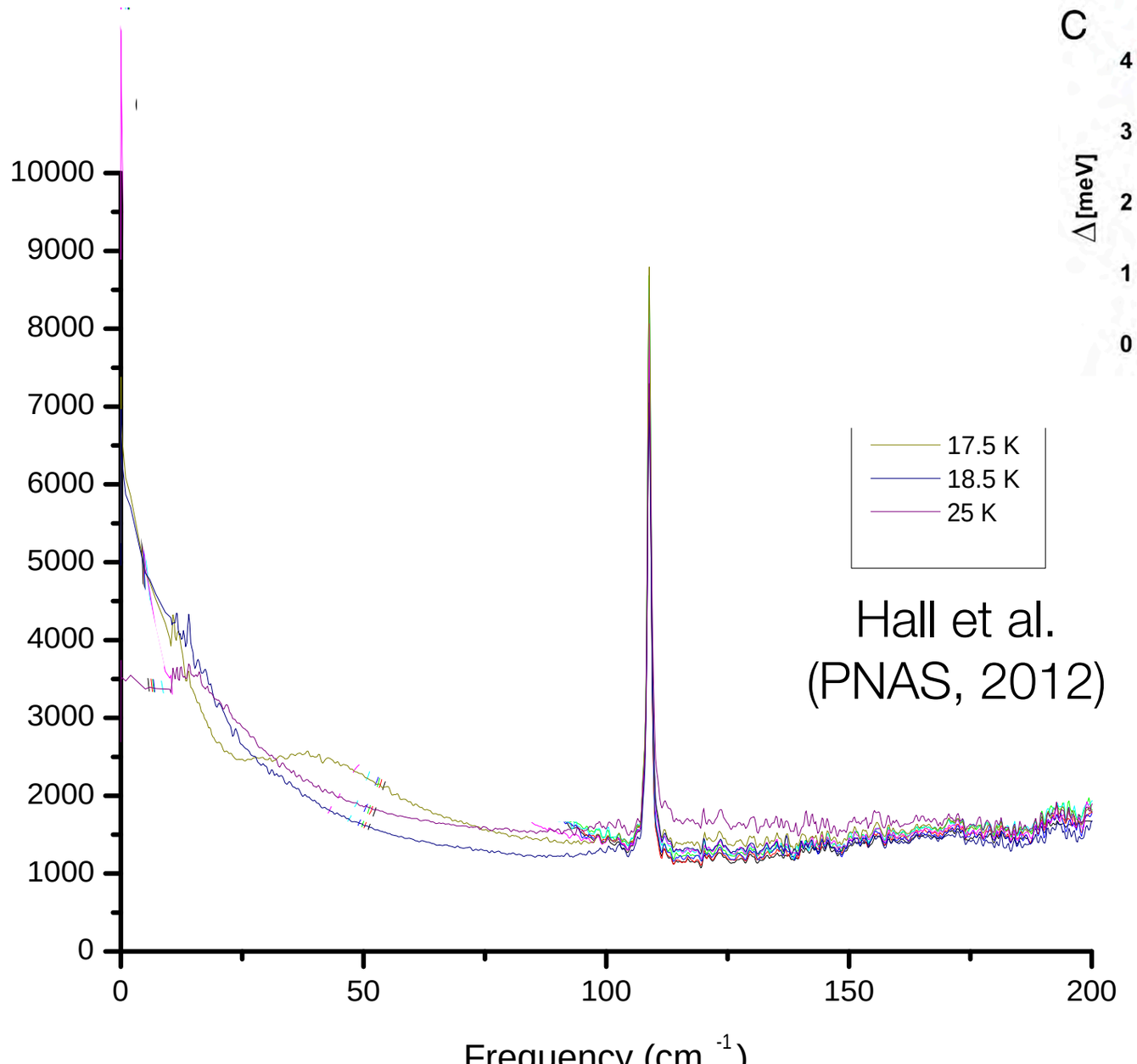

Hall et al.
(PNAS, 2012)

Spectroscopy
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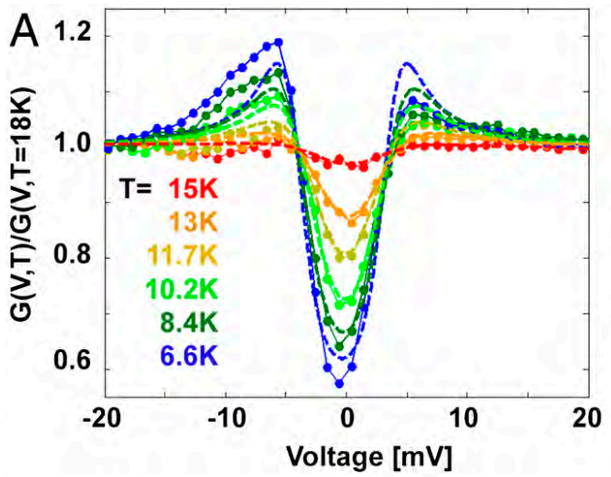

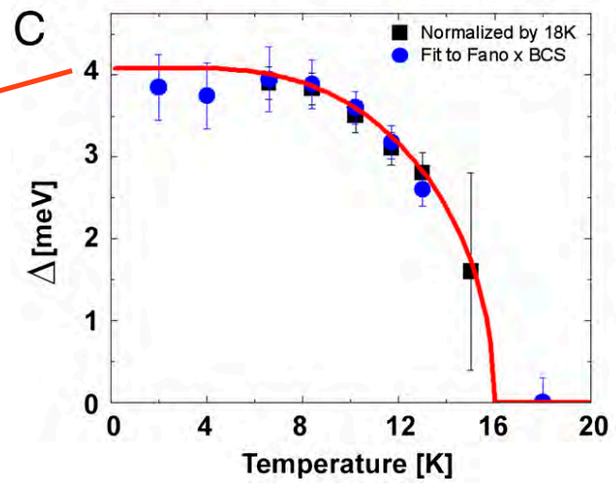

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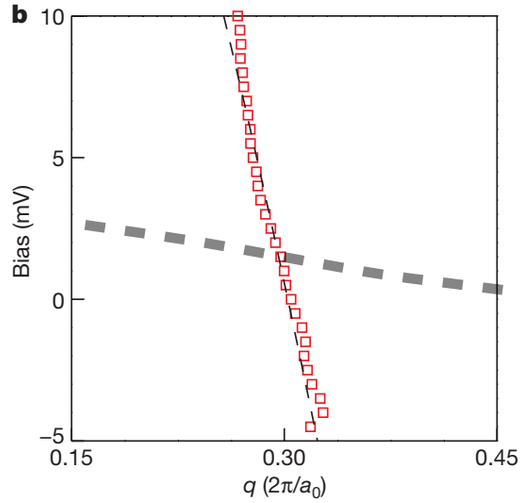

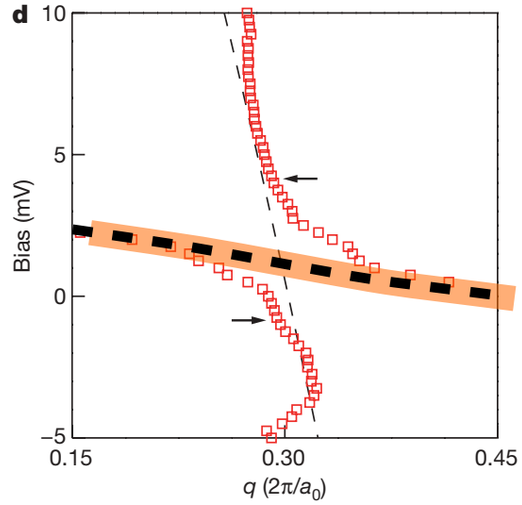

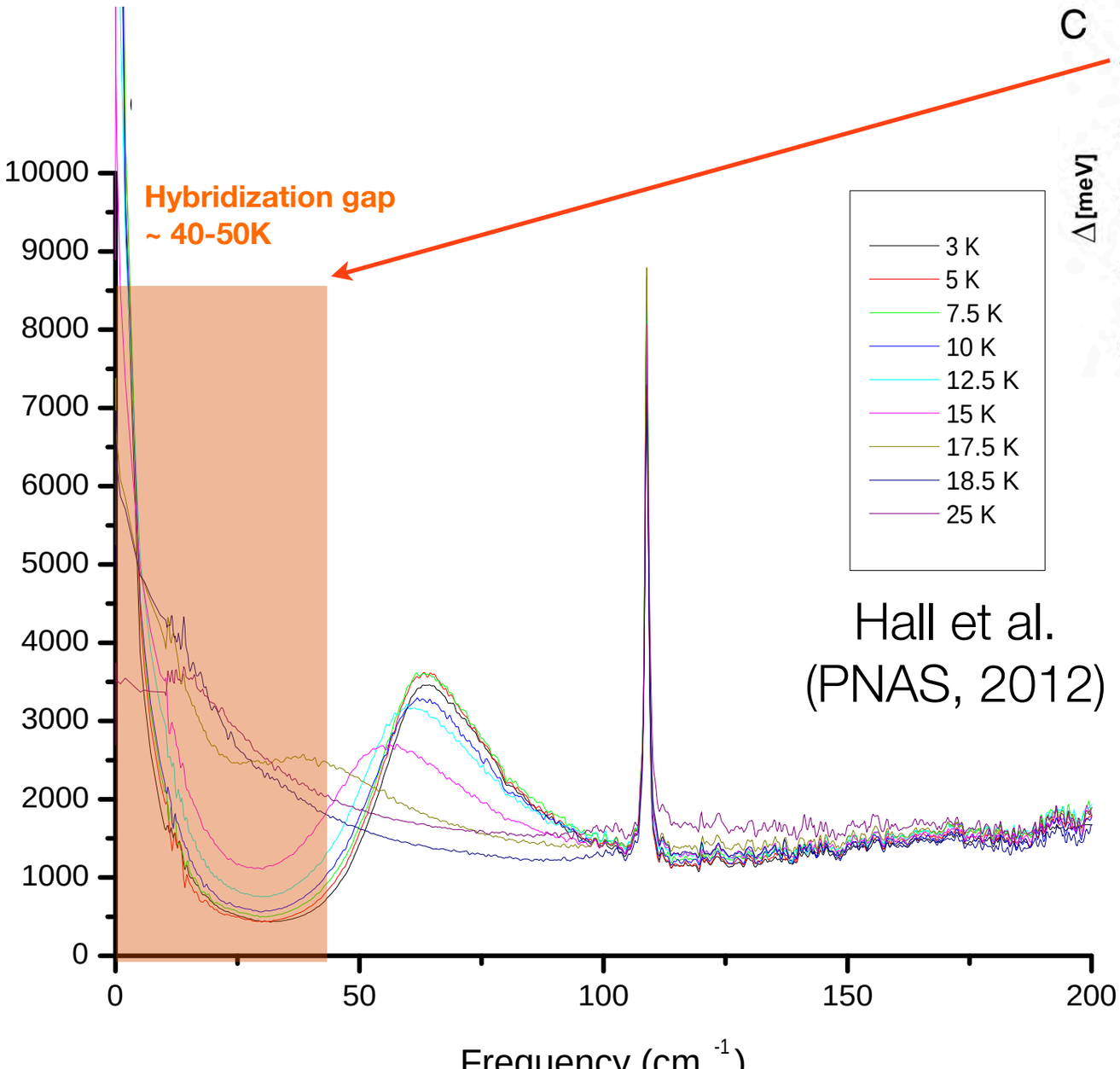

Hall et al.
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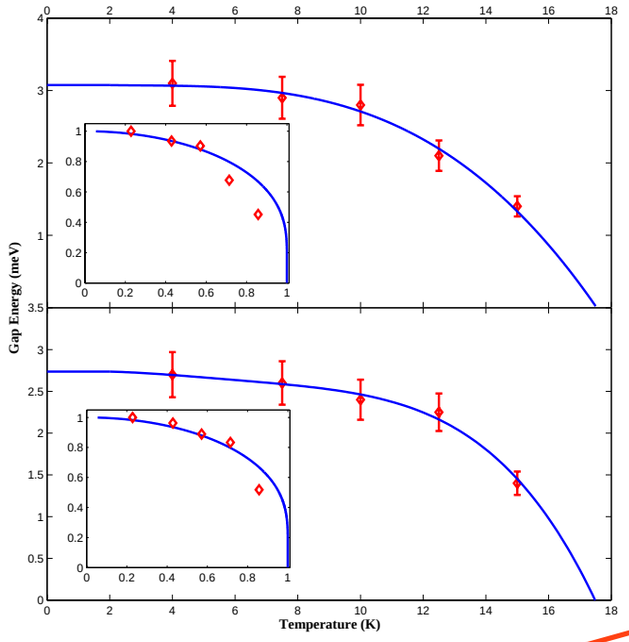
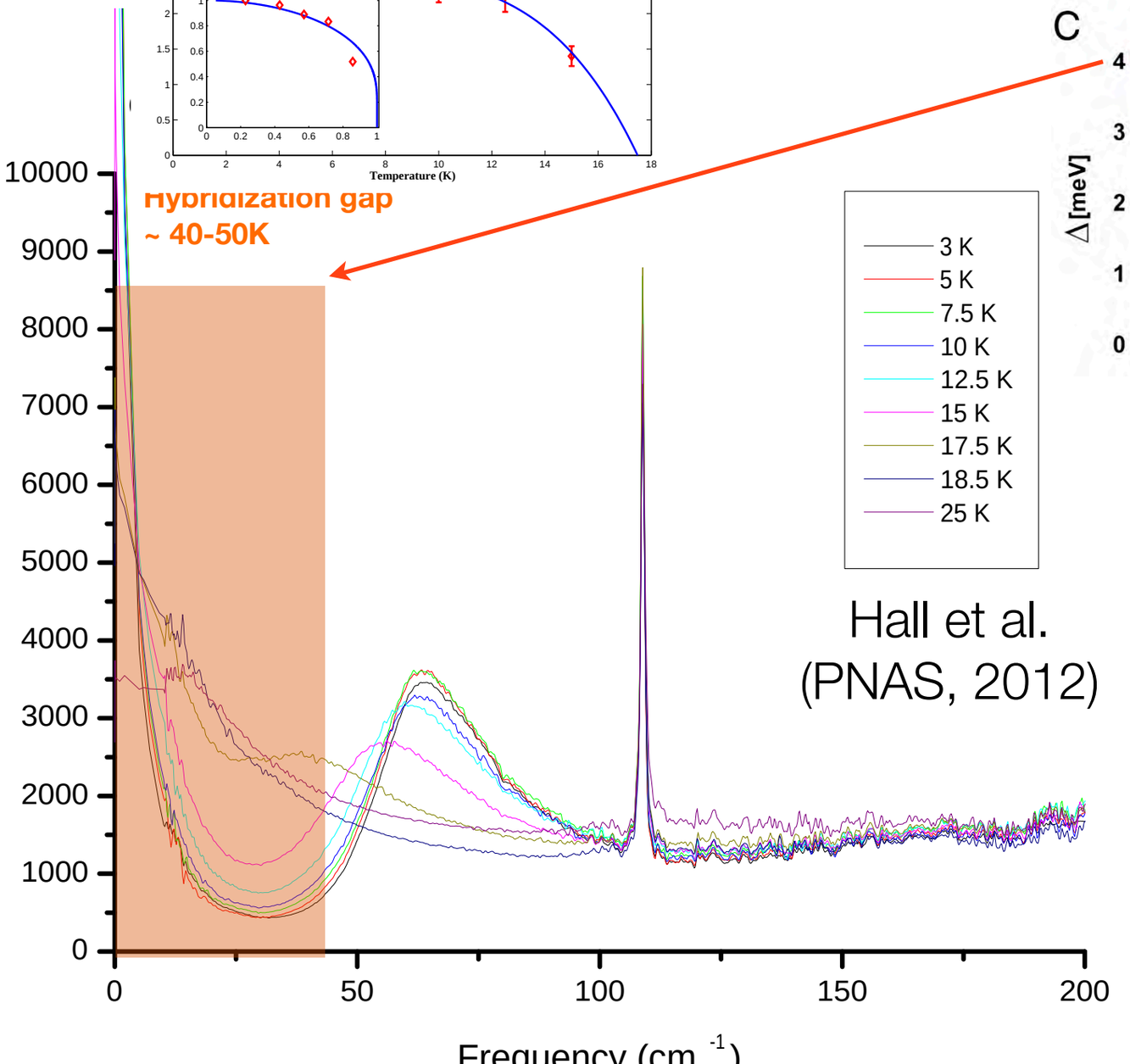
Spectroscopy
H-gap in STM/Optics





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Aynajian* et al,
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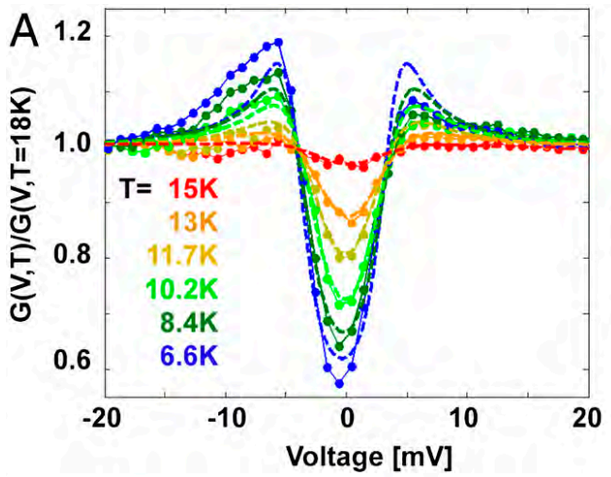

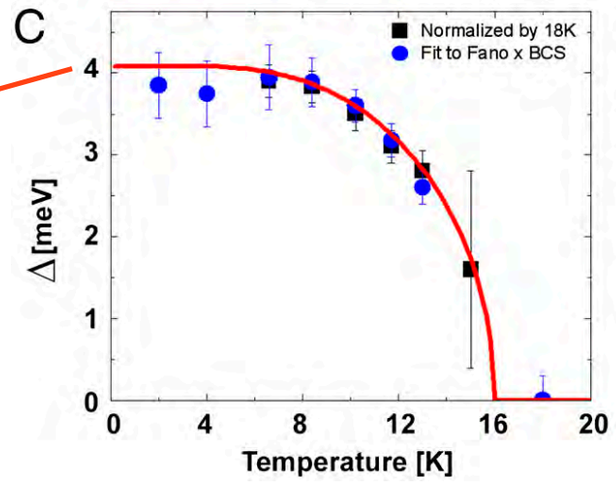

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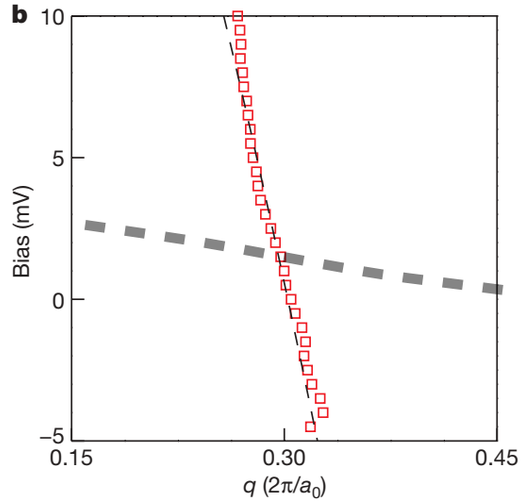

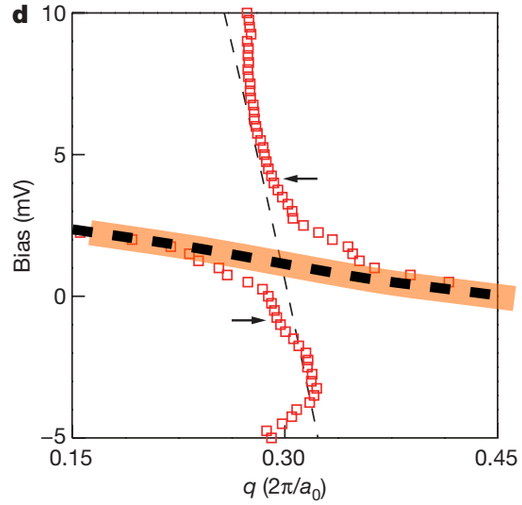

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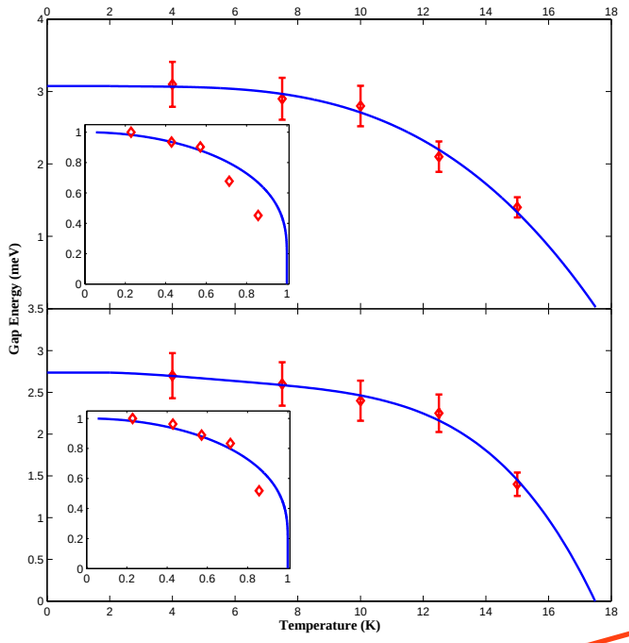

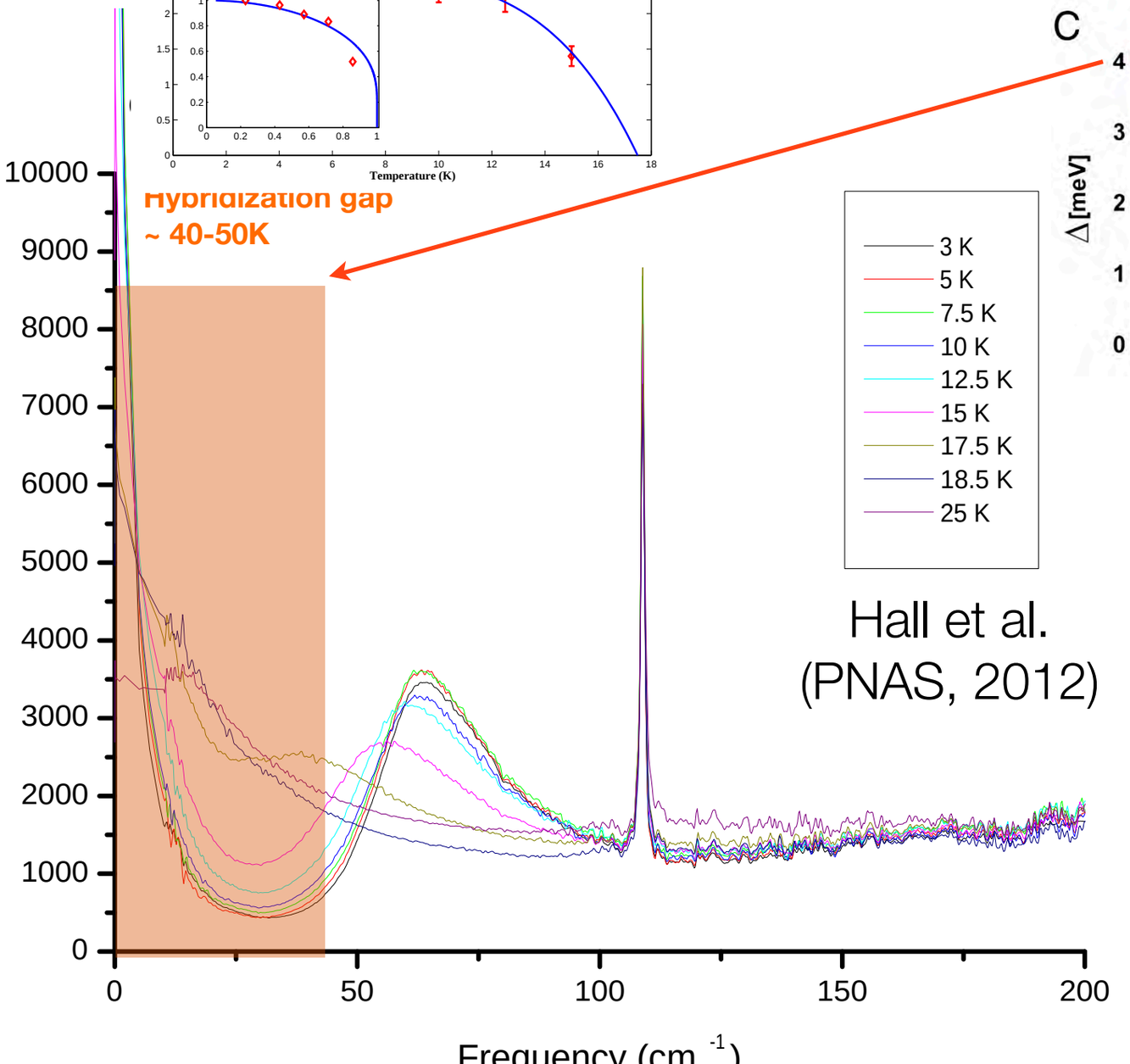

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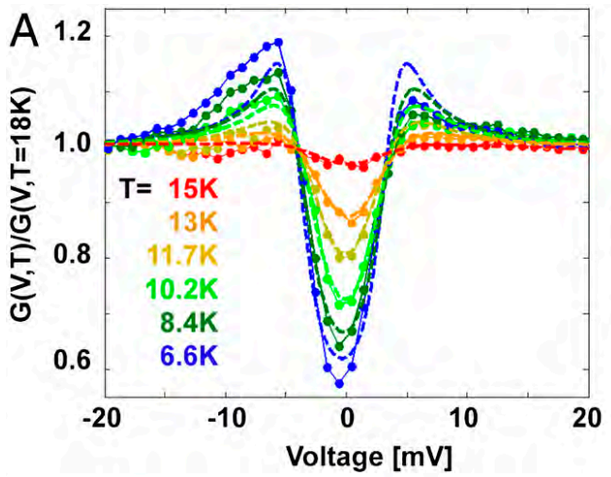

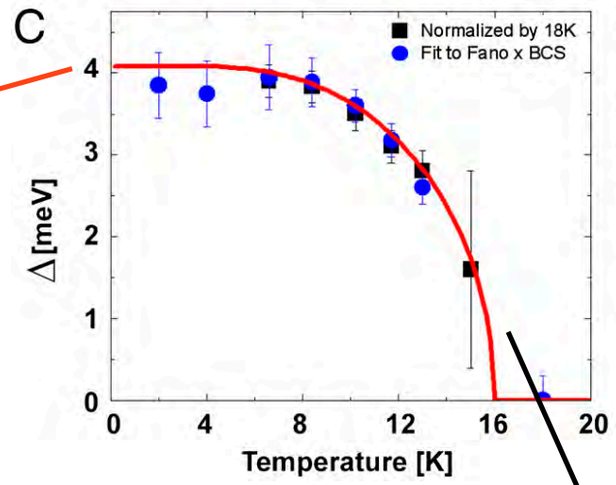

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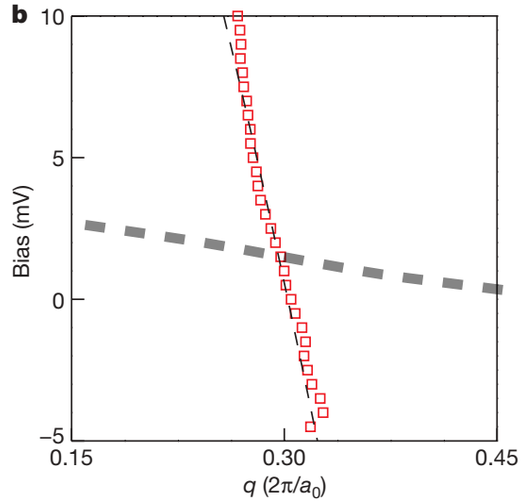

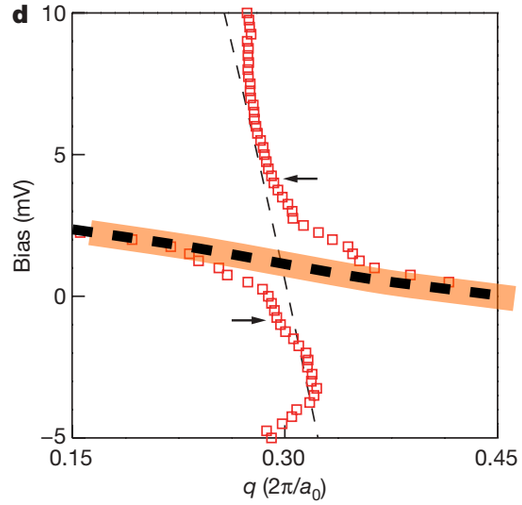

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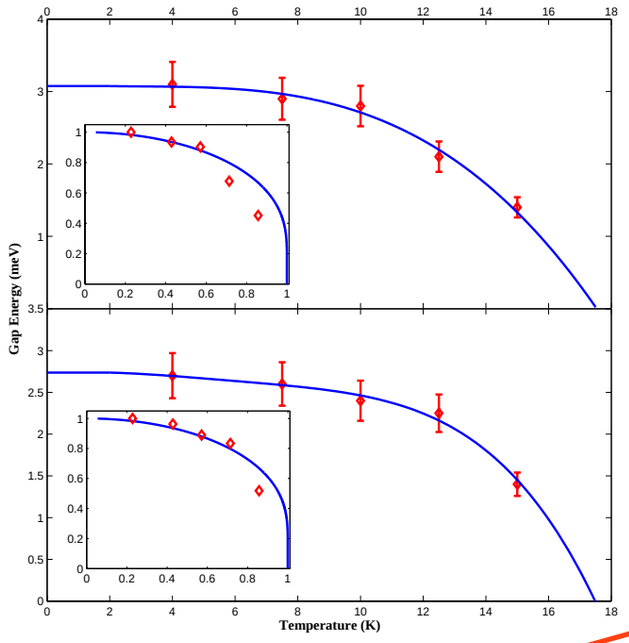
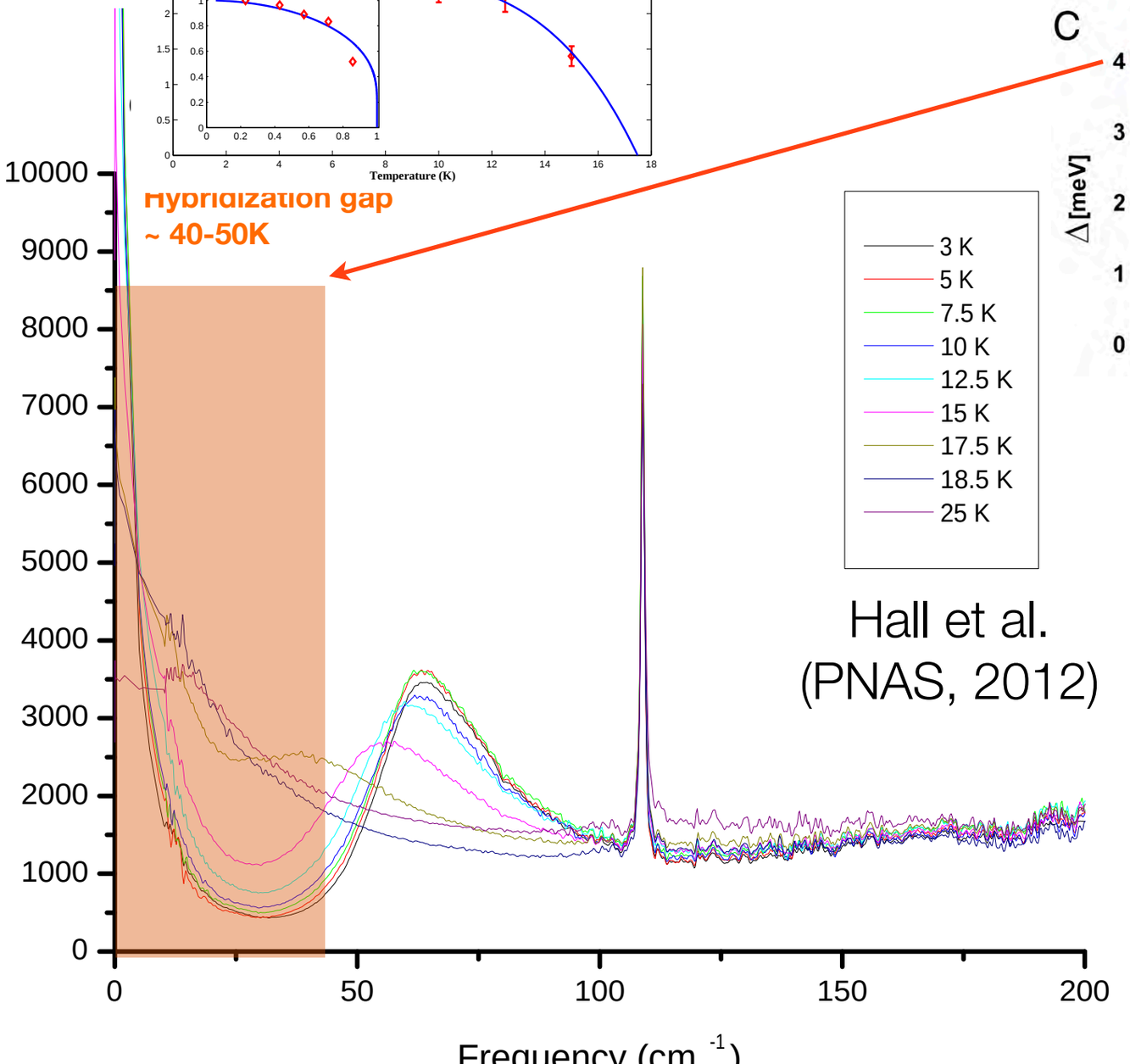
Spectroscopy H-gap in STM/Optics





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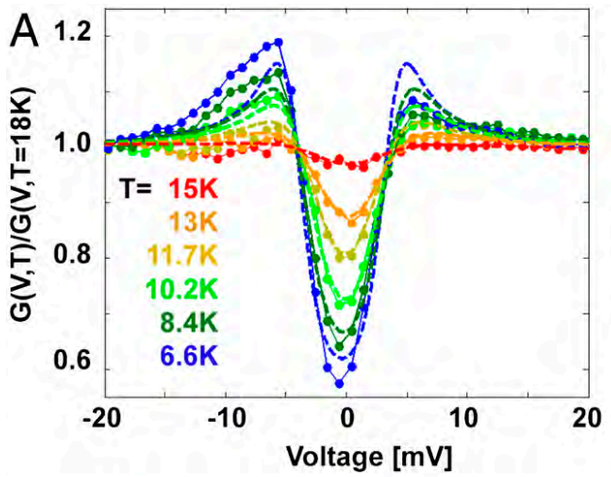

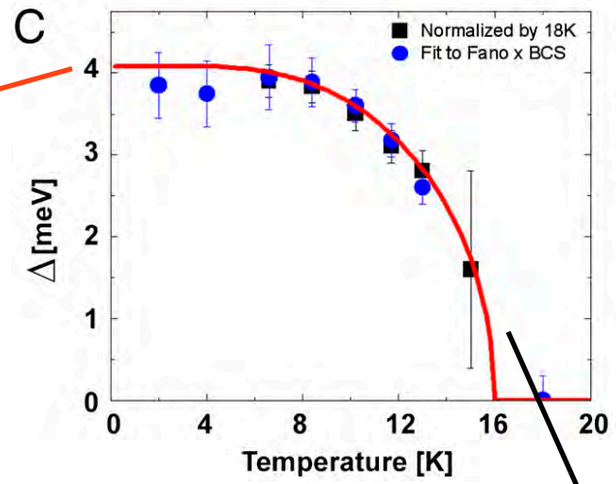

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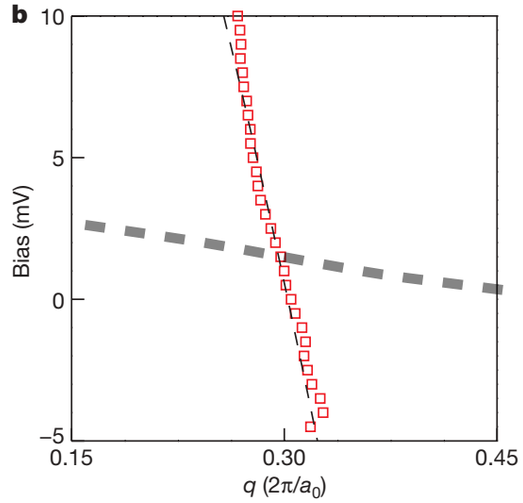
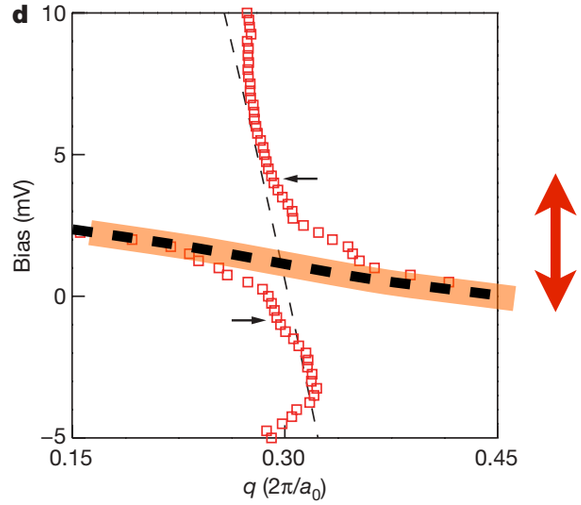




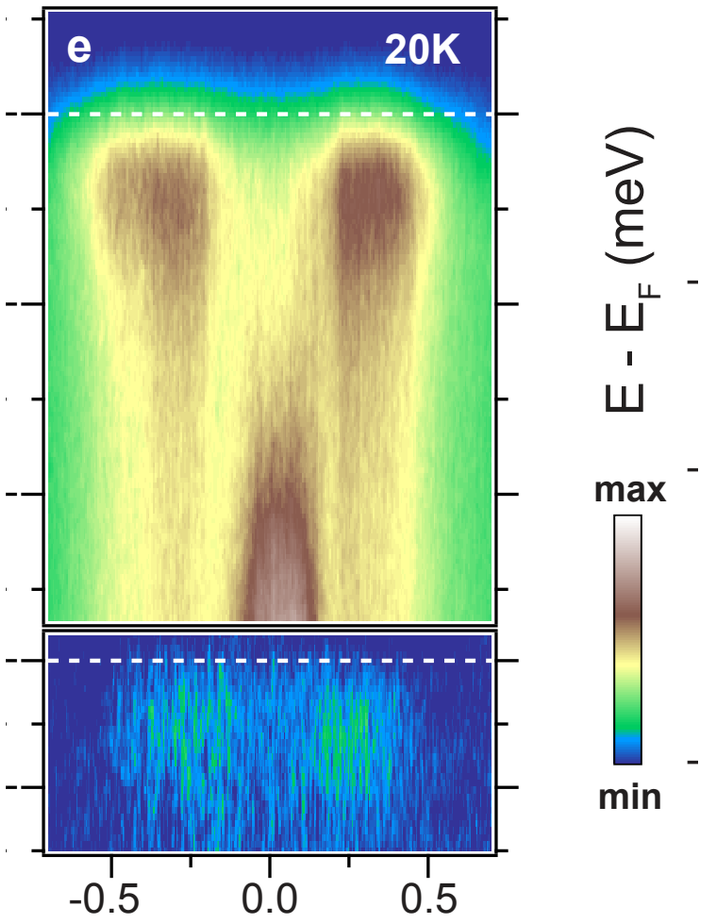
$$\Psi_H \sim \left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}$$

Hybridization *develops at T_{HO}*

Morr et al (10), Dubi and Balatsky(10)

High Resolution ARPES

Chaterjee et al, arXiv 1211.5312



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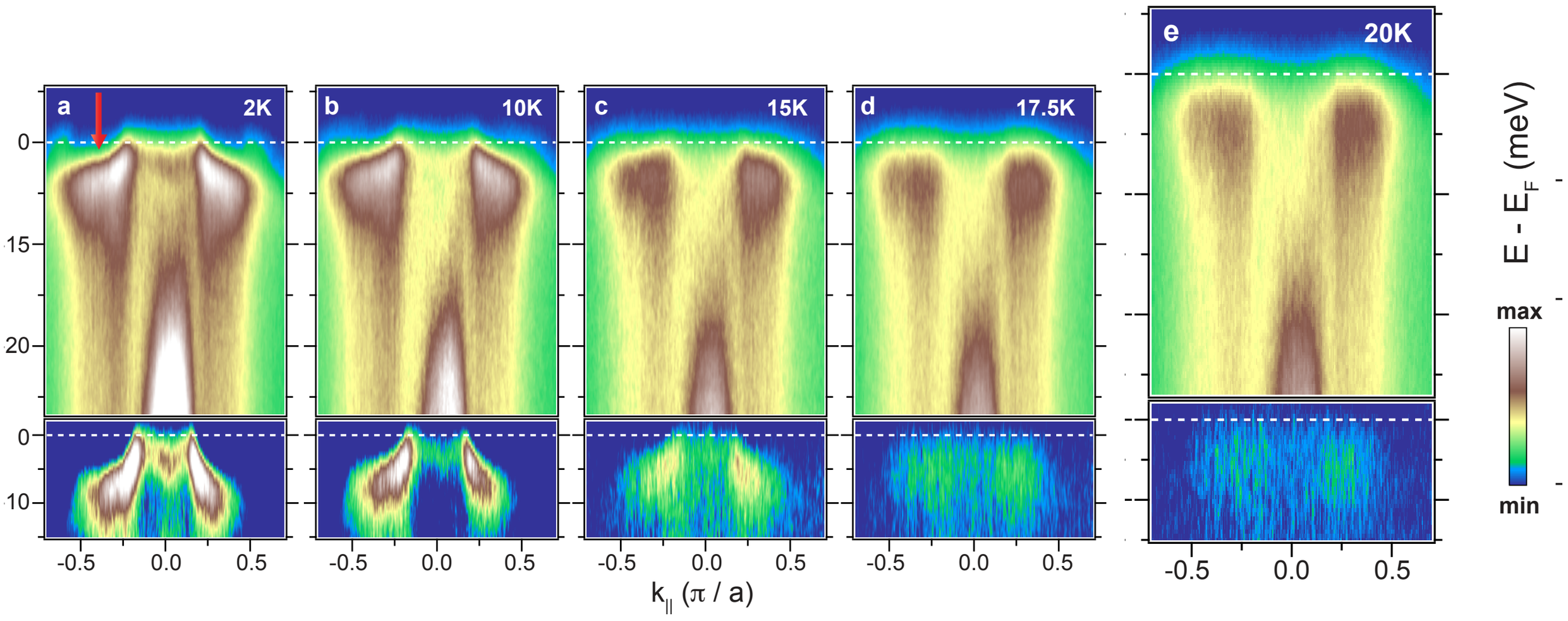

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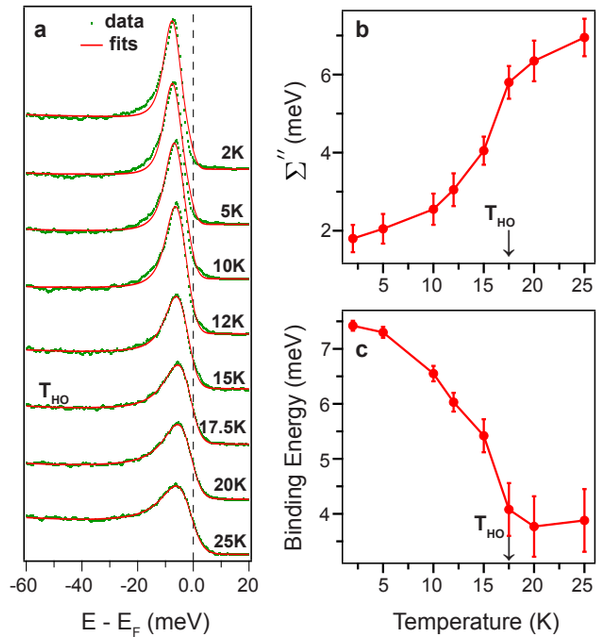

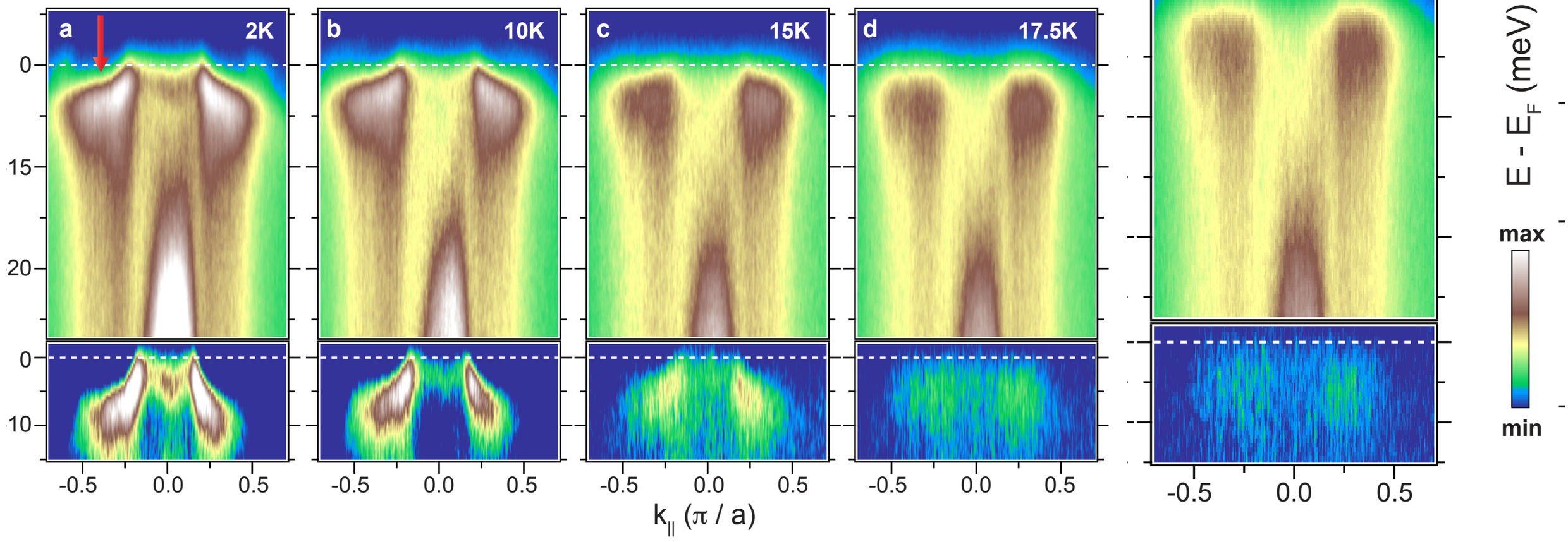

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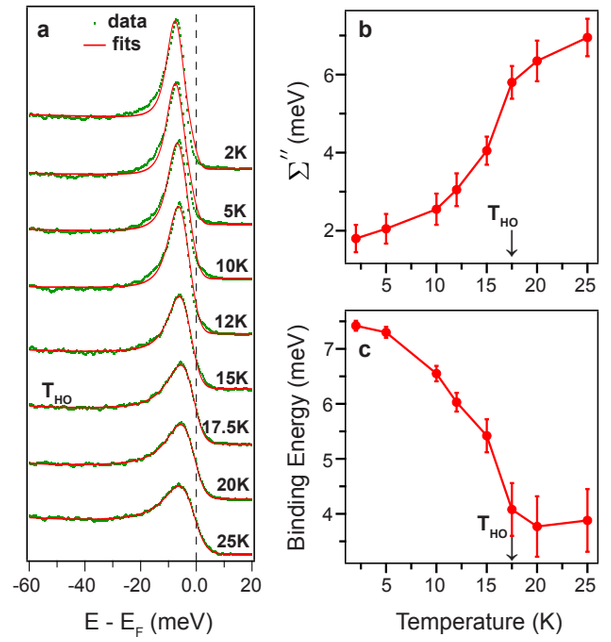

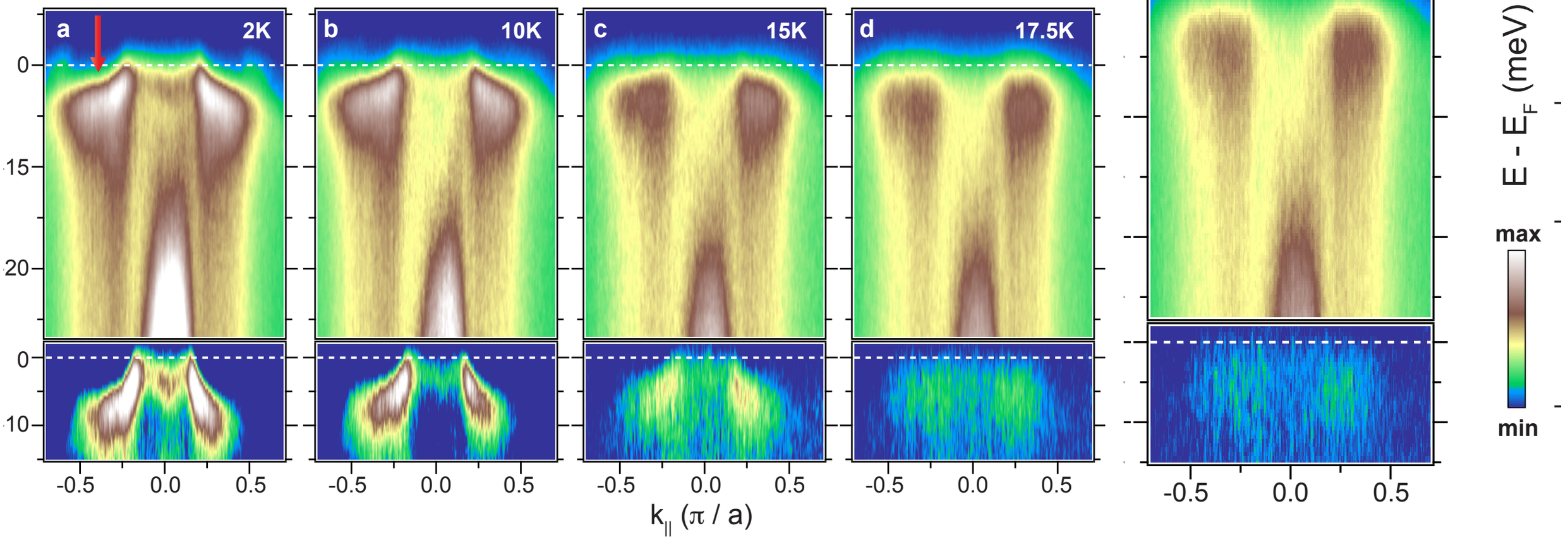

Heavy fermion bands come into focus at T_{HO}

“Hastatic Order”

Landau Theory:

Conventional Landau theory of electron fluids involves the formation of two body bound-states. When the two body bound-state carries a quantum number, the corresponding two body wavefunction transforms non-trivially under the symmetries of the vacuum, and is promoted to an order parameter. Landau order parameters involve even numbers of electrons and carry integer spin.

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e.g. $\overbrace{\psi^\dagger_\alpha(x)\psi_\beta(x)} = \vec{M}(x)\cdot\vec{\sigma}_{\alpha\beta}$ Ferromagnetism=2 body BS

Transforms as vector (S=1)

Hastatic order generalizes Landau’s concept to half integer order parameters (bosons) which violate the spin-statistic theorem.

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$$\overbrace{\psi^\dagger_{\sigma_1}(1)\psi^\dagger_{\sigma_2}(2)\psi_{\bar{\sigma}_3}(3)} = \Gamma^\alpha_{\sigma_1\sigma_2\bar{\sigma}_3}(1,2,3;x)\chi^\dagger_\alpha(x)$$

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Half integer OP

S=1 fermion

“Hastatic” order.

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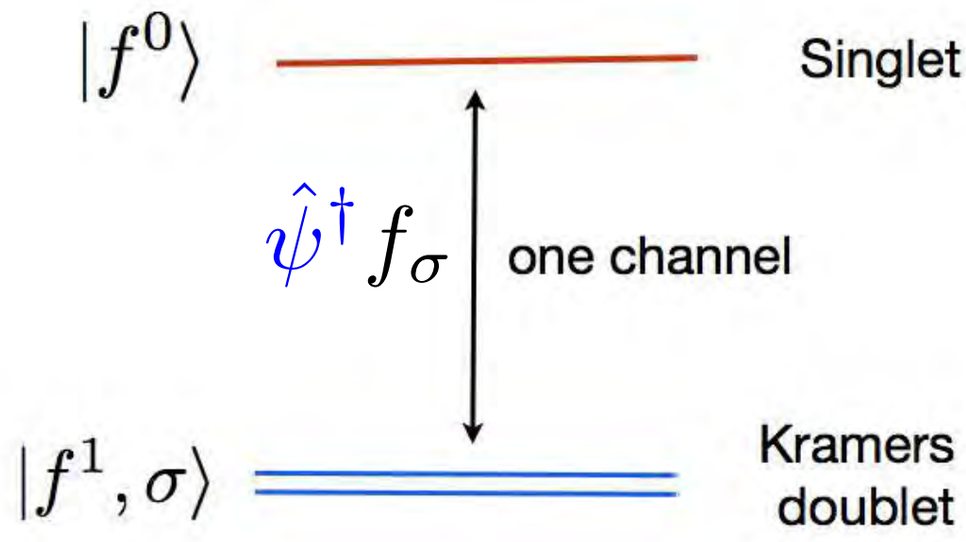

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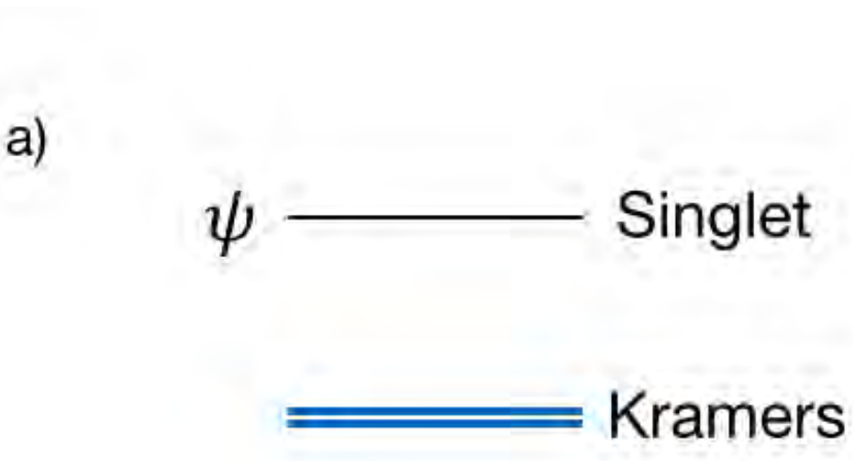

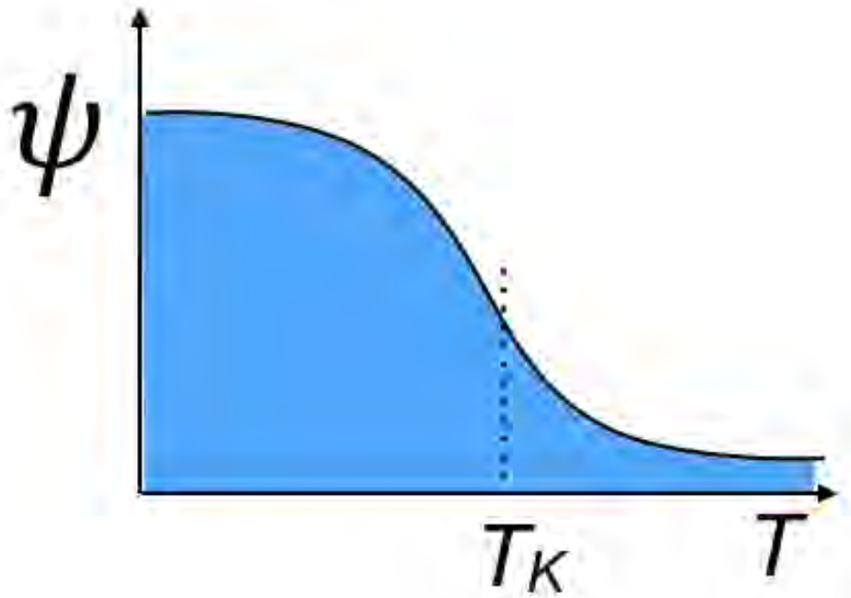

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H. Amitsuka and T. Sakakibara, *J. Phys. Soc. Japan* **63**, 736-47 (1994).

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non-Kramers (K=+1)

Γ_5

$|5f^2, \alpha\rangle = \hat{\chi}^\dagger_\alpha |0\rangle$

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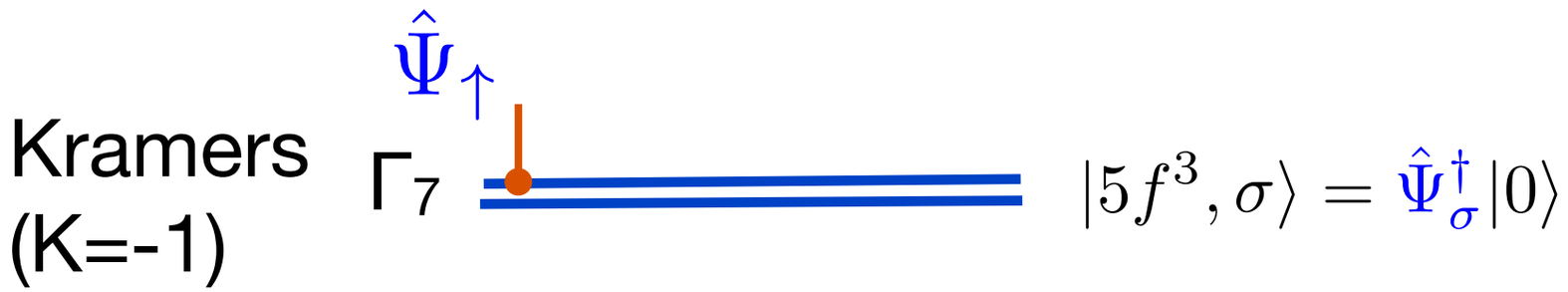

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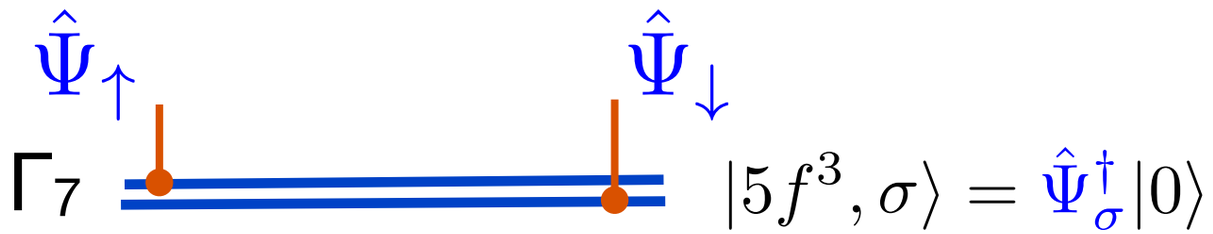

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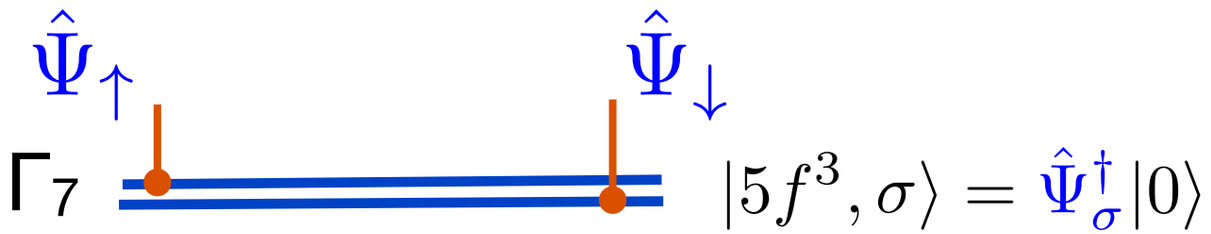

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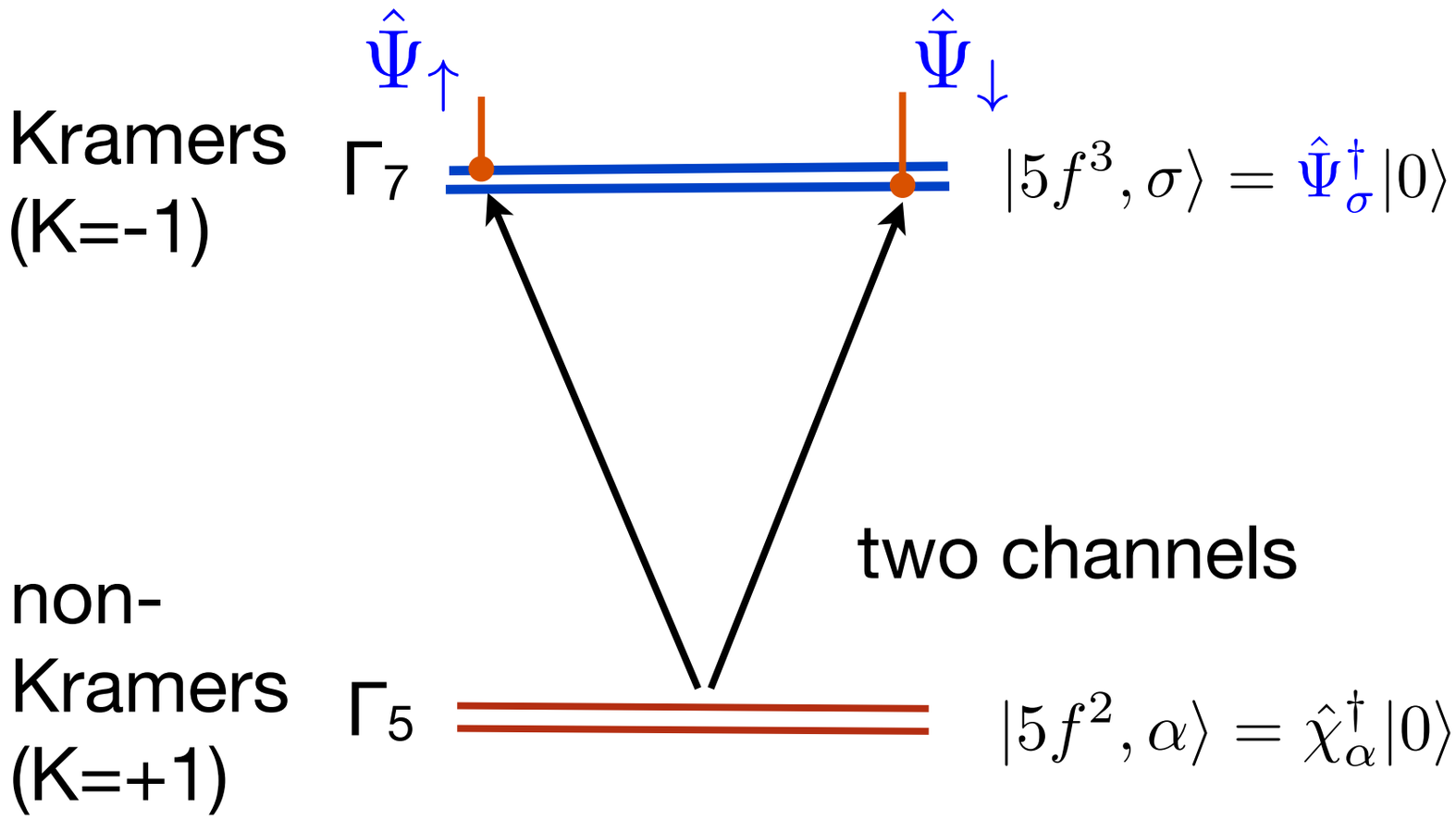

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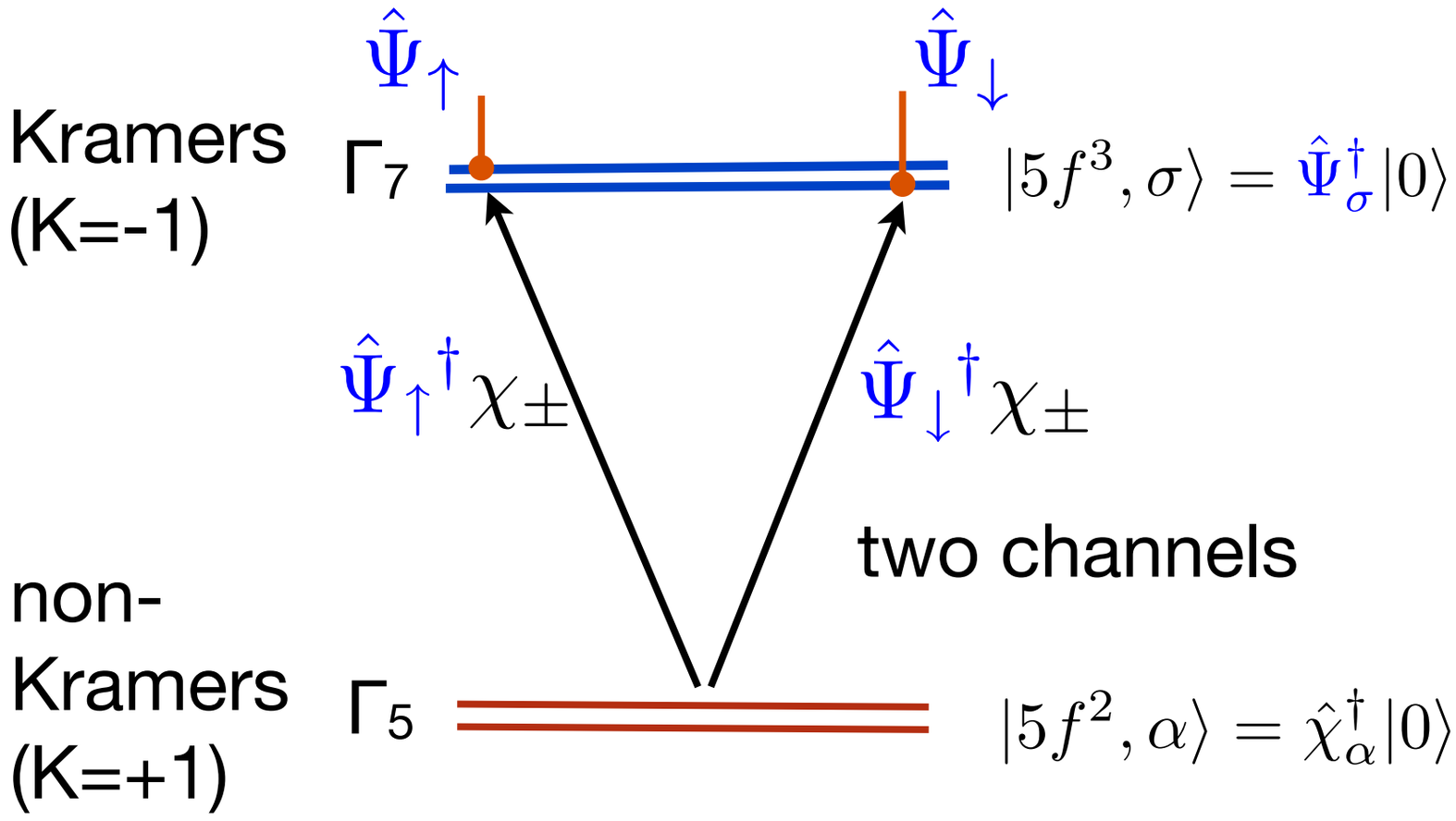

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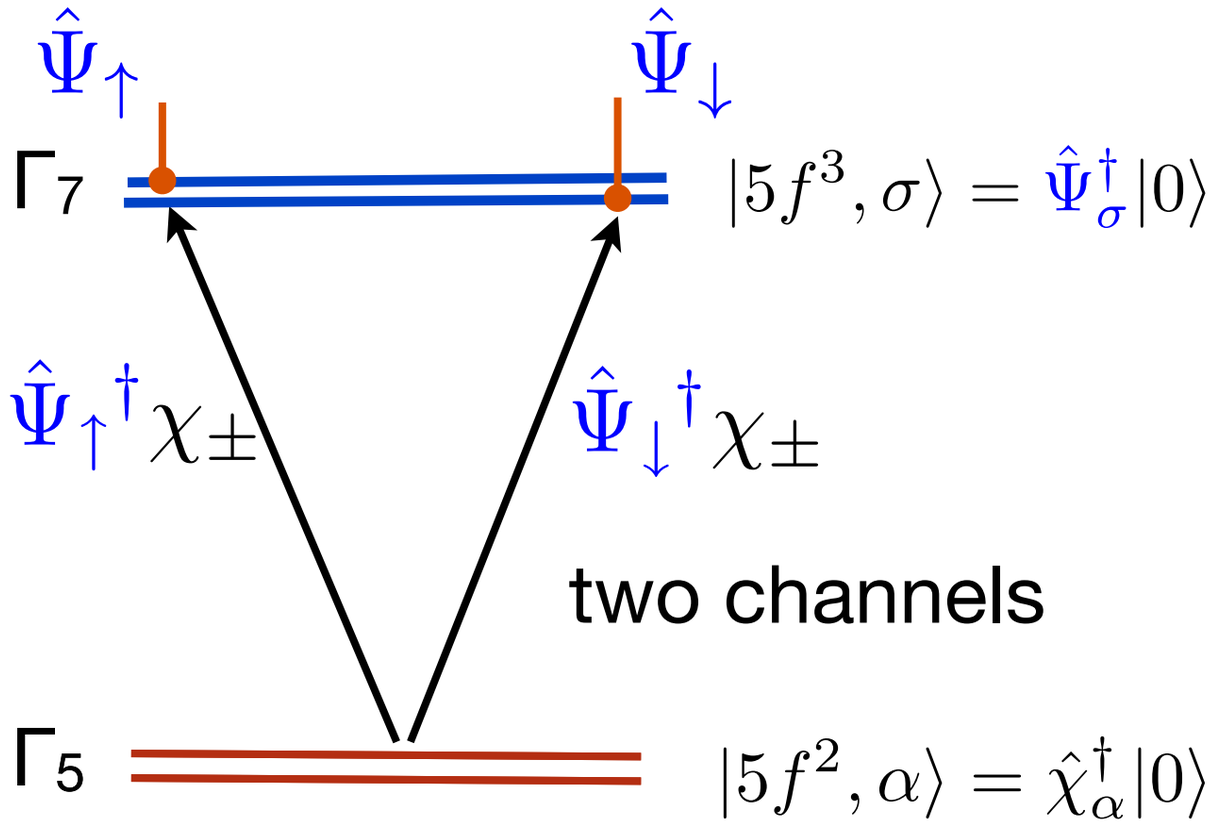

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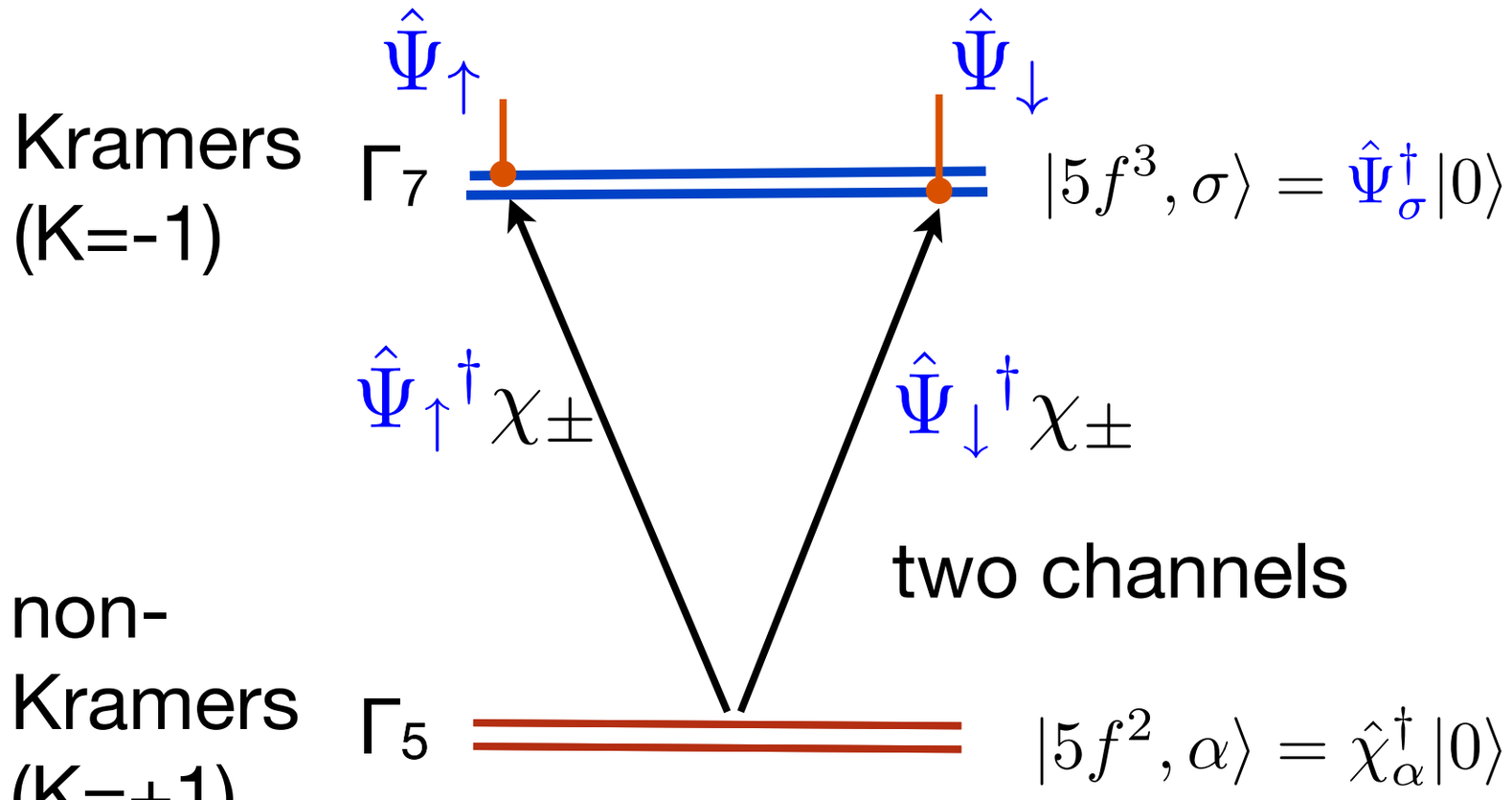

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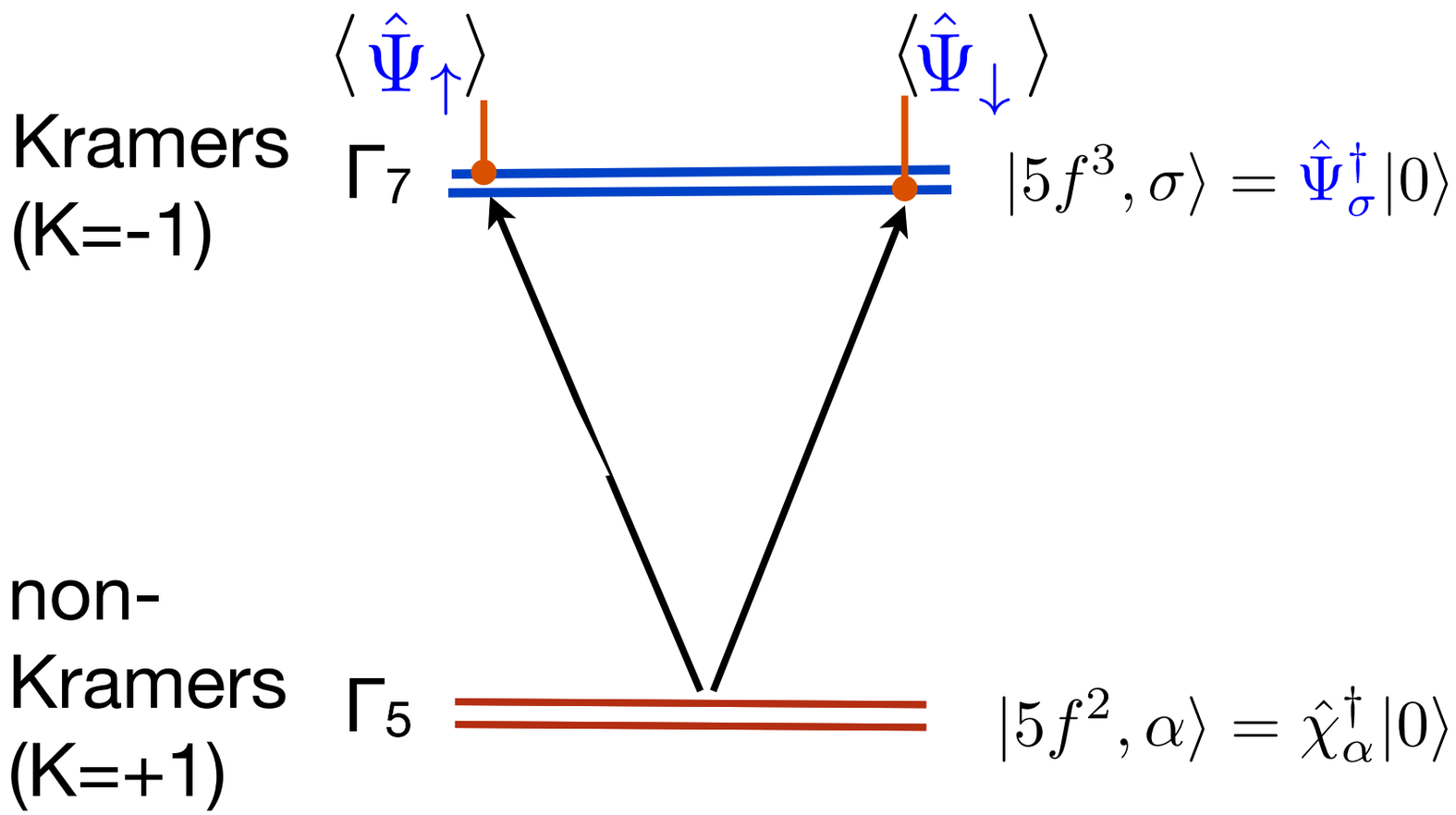

("Magnetic Higgs Boson")

$$\Psi = \begin{pmatrix} \langle \Psi_\uparrow \rangle \\ \langle \Psi_\downarrow \rangle \end{pmatrix}$$

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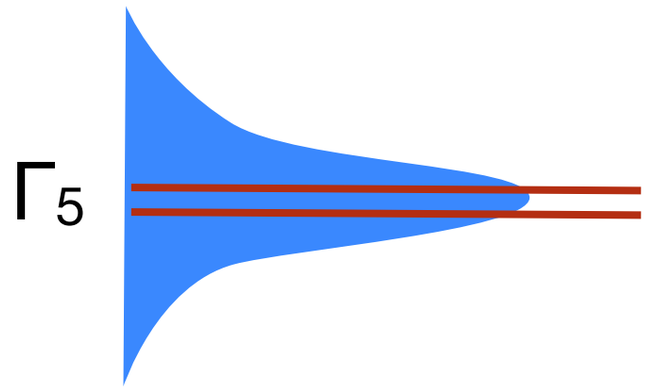

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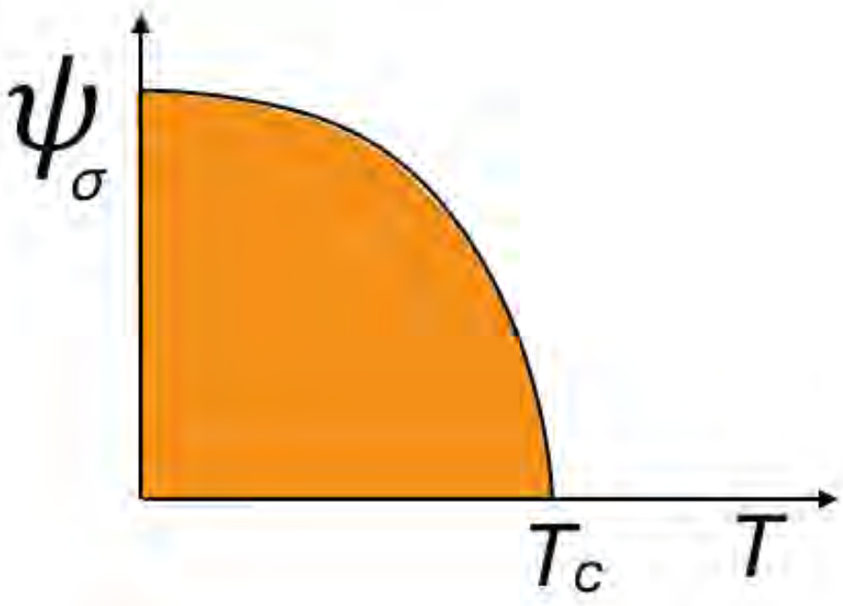
"Hastatic" order.

Quasiparticles acquire the Ising anisotropy of the non-Kramers doublet.



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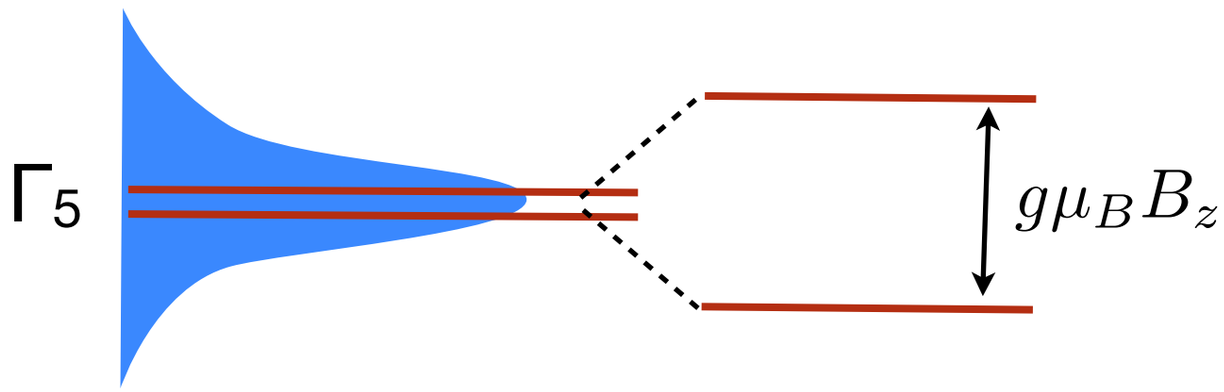

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The origin of the anisotropy in the non-linear susceptibility.

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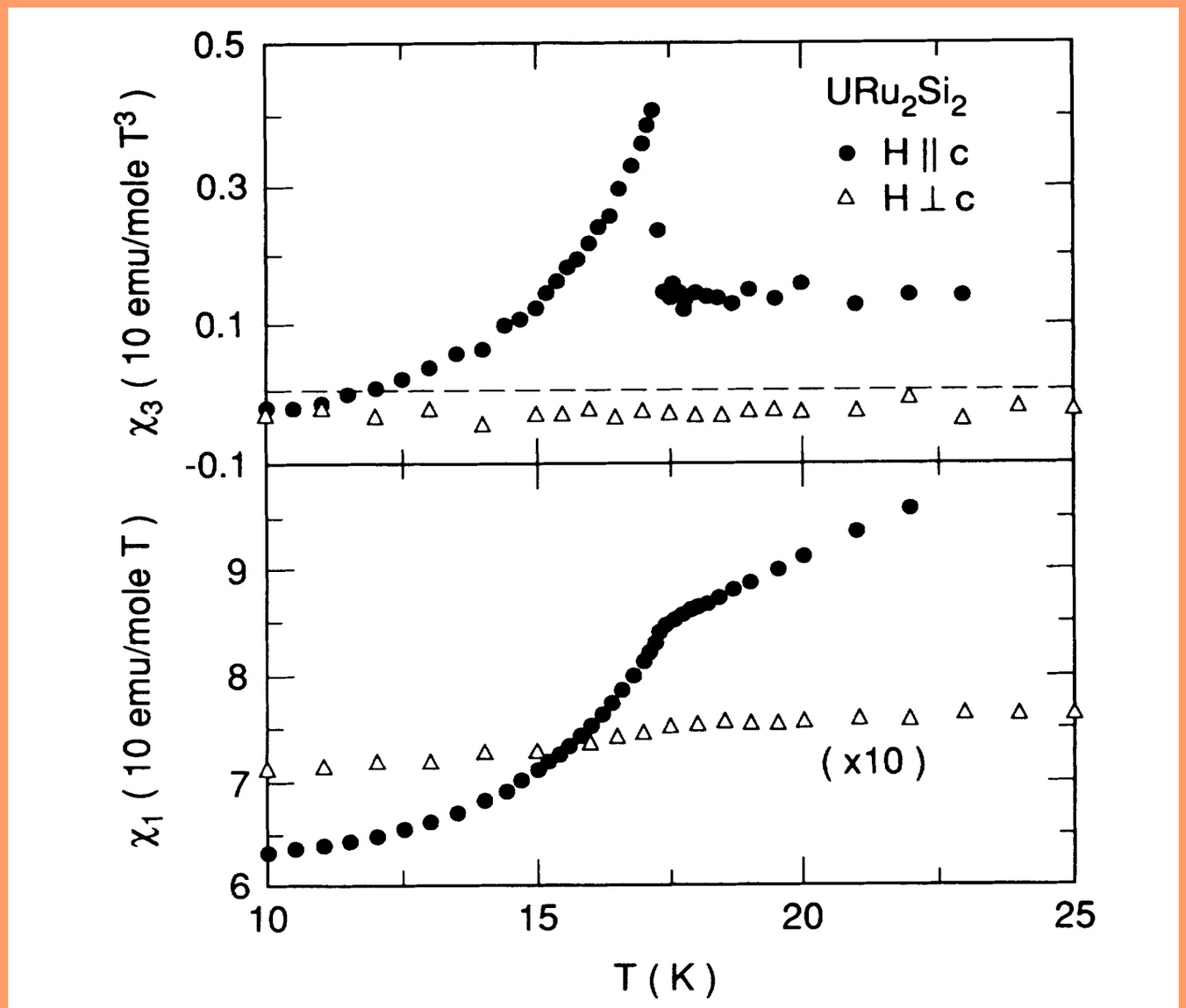
A. Ramirez, P. Coleman, P. Chandra, A. Menovsky, E. Bruck, Z. Fisk and E. Bucher, Physical Review Letters, 68, 2680, (1992).



Landau Theory of Hastatic Order. I.

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AFM: $P>P_c$

$$\Psi_A \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \Psi_B \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Large f-moment

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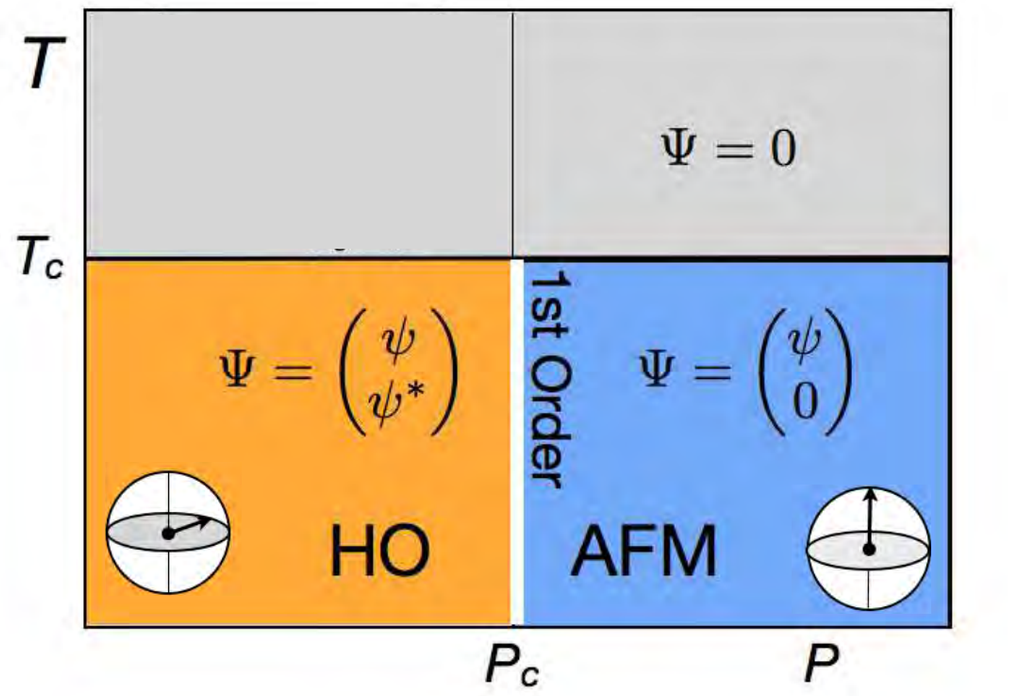

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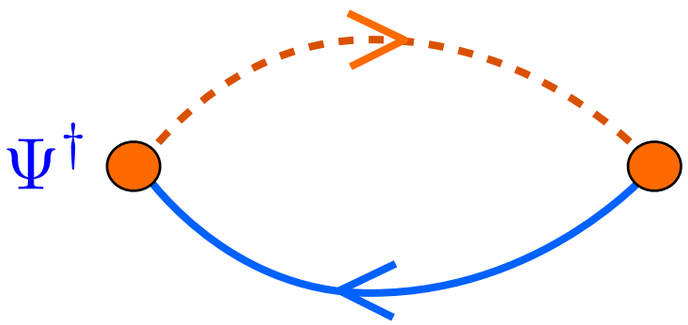

$$= \alpha(T_c - T)|\Psi|^2$$

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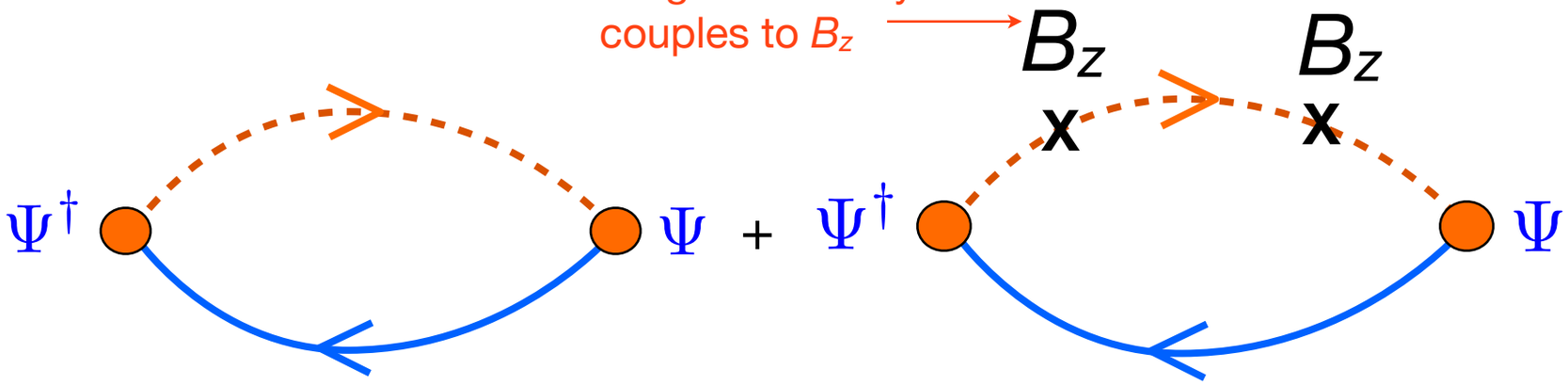
Ising f-state only couples to B_z



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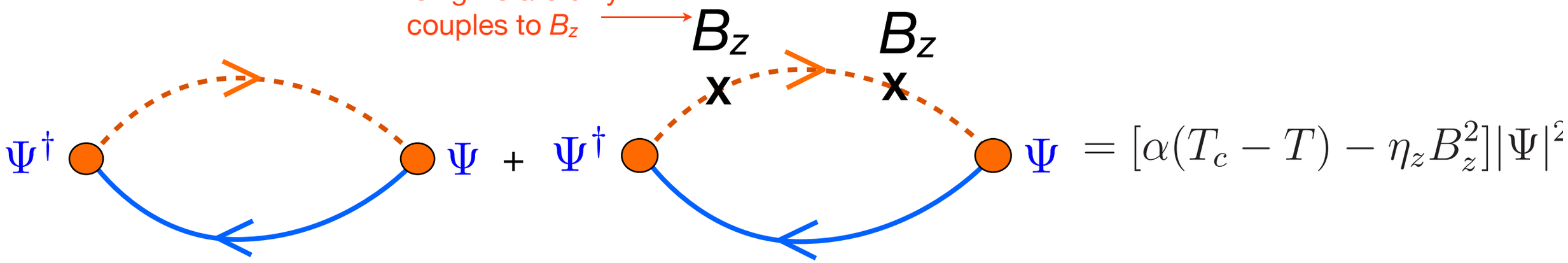

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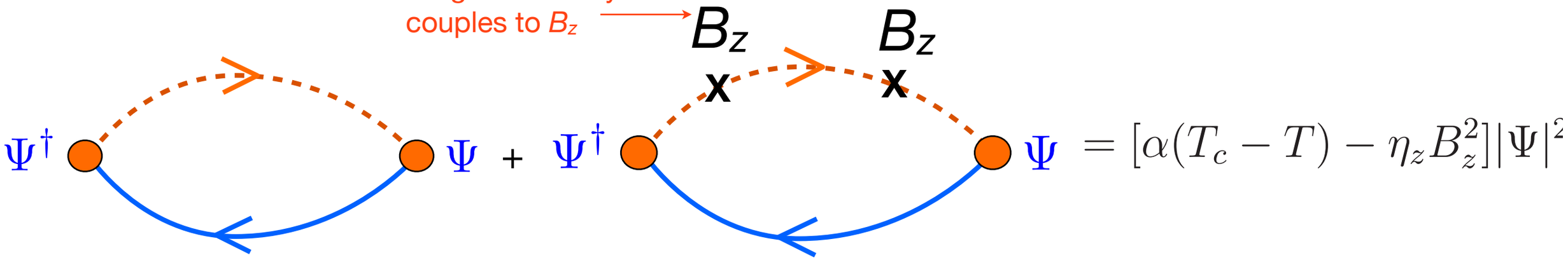

$$f[T,P,B_z] = [\alpha(T_c - T) - \eta_z B_z^2]|\Psi|^2 + \beta|\Psi|^4 - \gamma(\Psi^\dagger \sigma_z \Psi)^2$$

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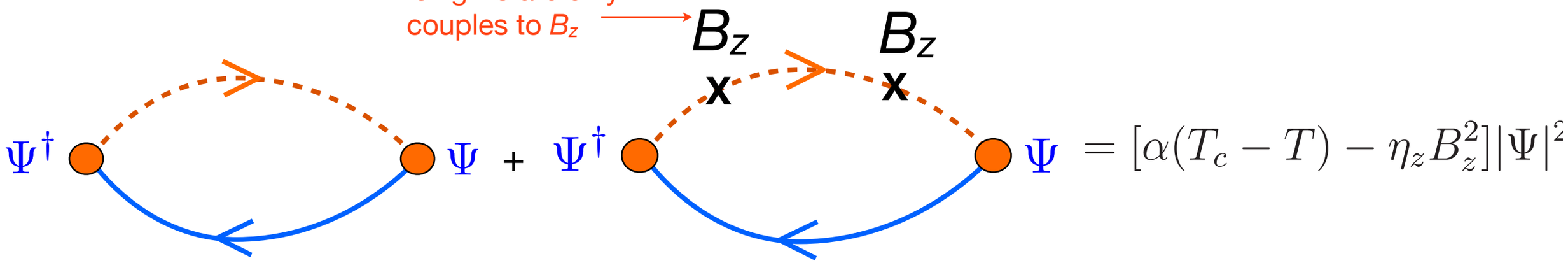

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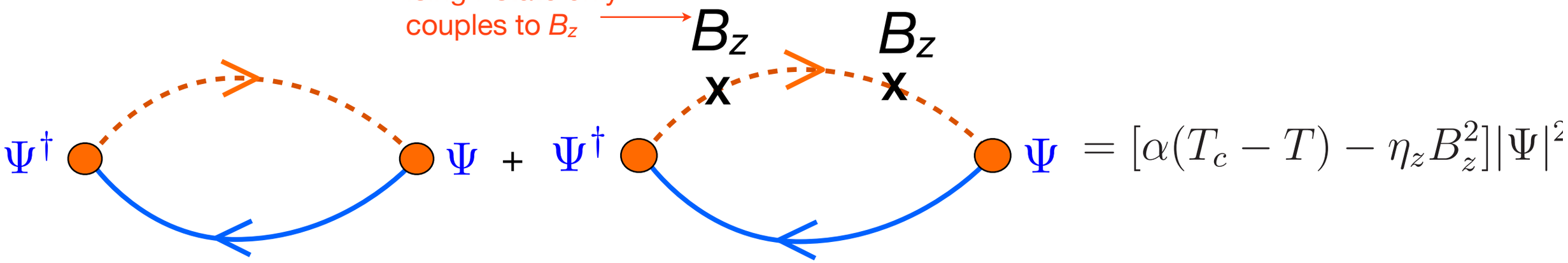

$$f = -\frac{1}{4\beta}\left[\alpha(T_c - T) - \eta_z (B\cos\theta)^2\right]^2$$

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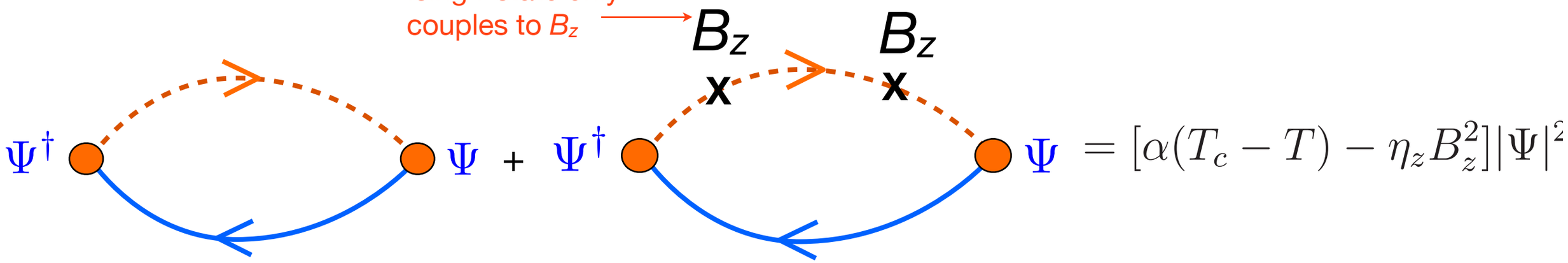

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Landau Theory of Hastatic Order. II. Origin of the non-linear susceptibility

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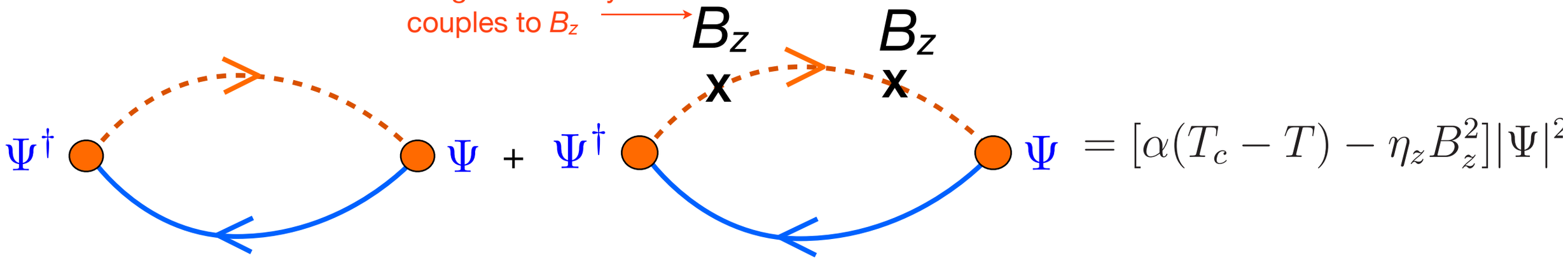

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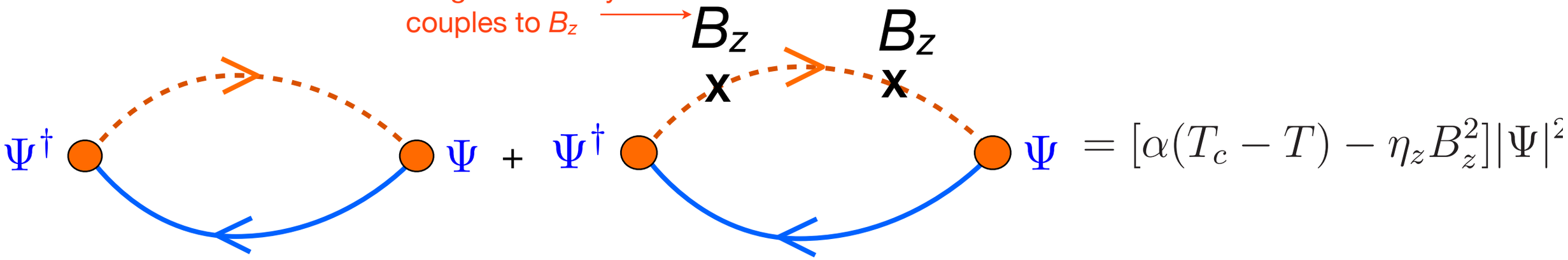

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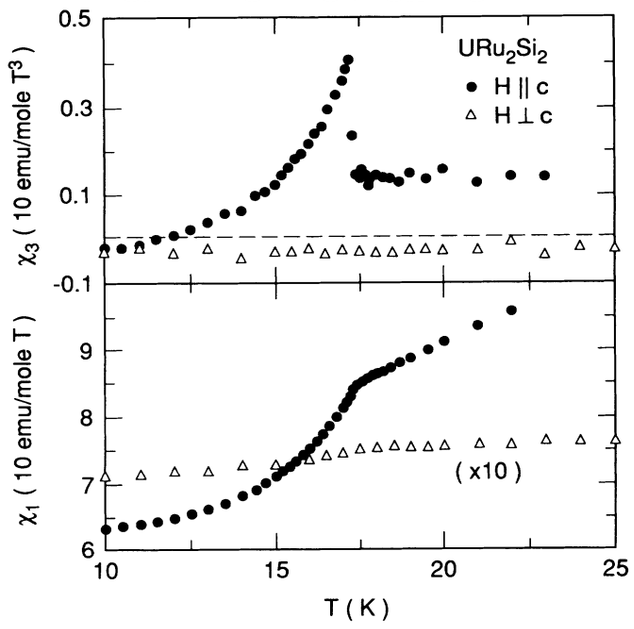
Ramirez et al '92



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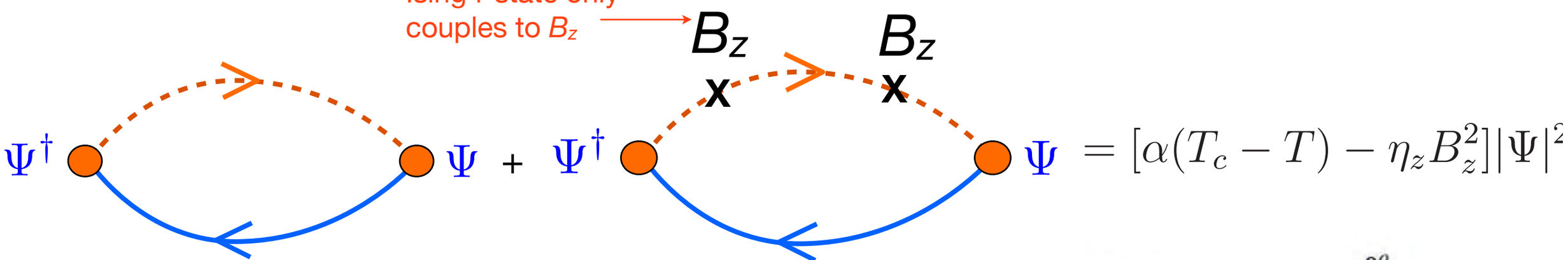

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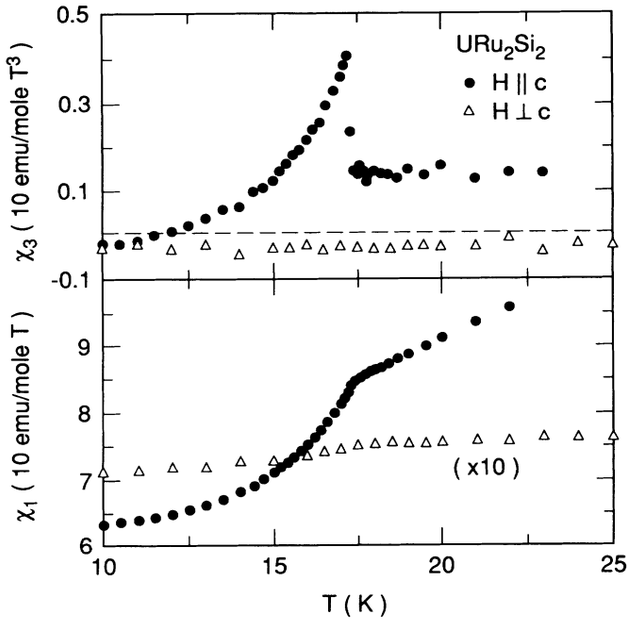
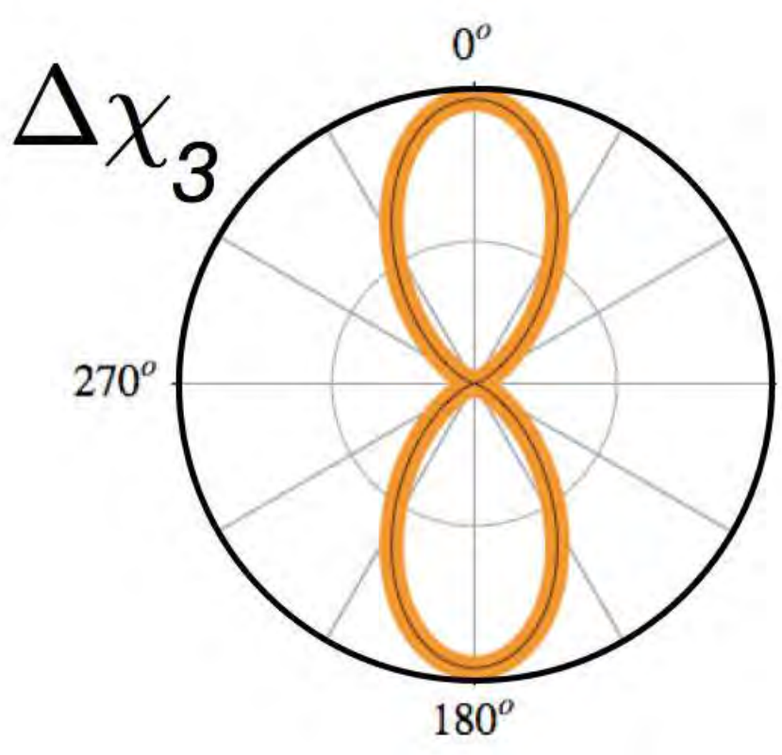


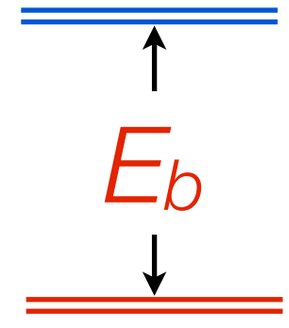


Anisotropy in excess of 1000 predicted.

Microscopic theory of Hastatic Order

$$|\Gamma_7^+, \sigma\rangle \equiv \Psi_\sigma^\dagger |0\rangle$$

$$|\pm\rangle \equiv \chi_\pm^\dagger |0\rangle$$



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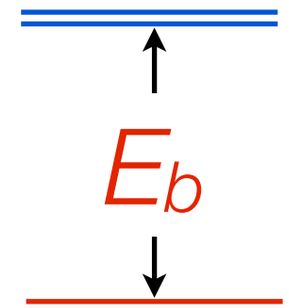

$$5f^2 \rightleftharpoons 5f^1 + e^- \qquad \psi_{\Gamma\sigma}^\dagger(j) = \sum_{\mathbf{k}} \left[\Phi_\Gamma^\dagger(\mathbf{k})\right]_{\sigma\tau} c_{\mathbf{k}\tau}^\dagger \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{R}_j}$$

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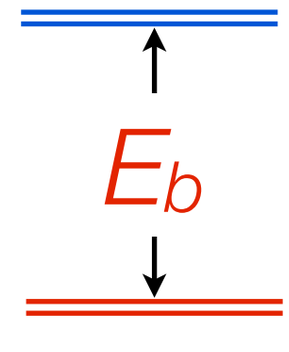

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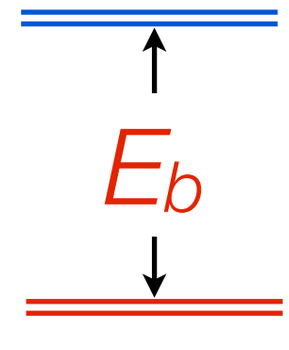

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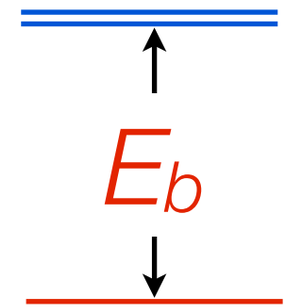

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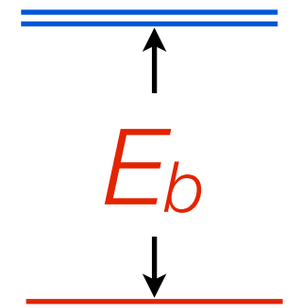

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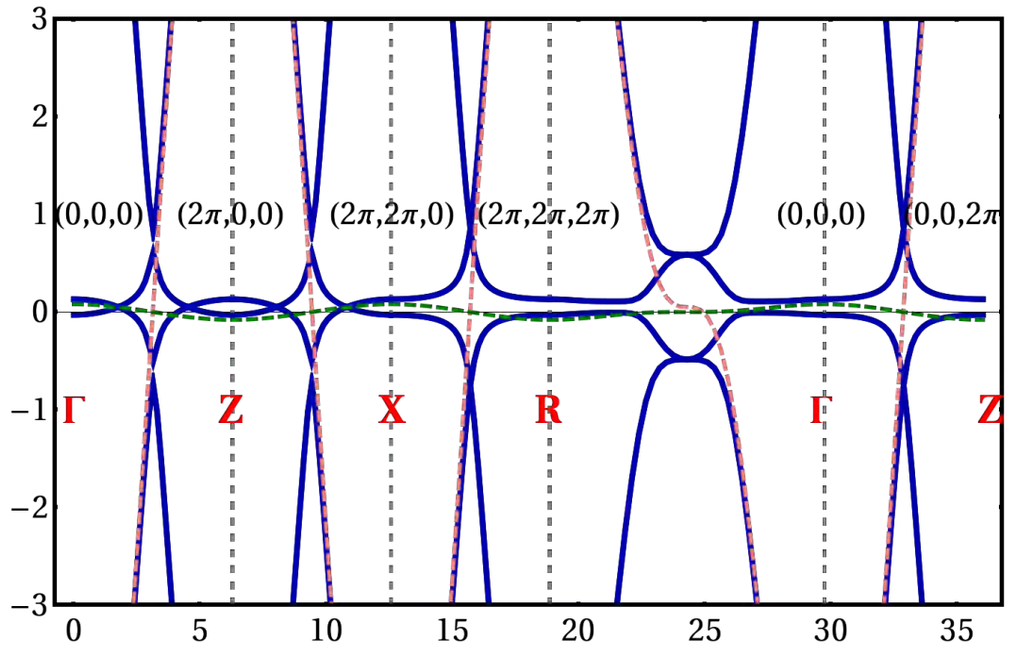
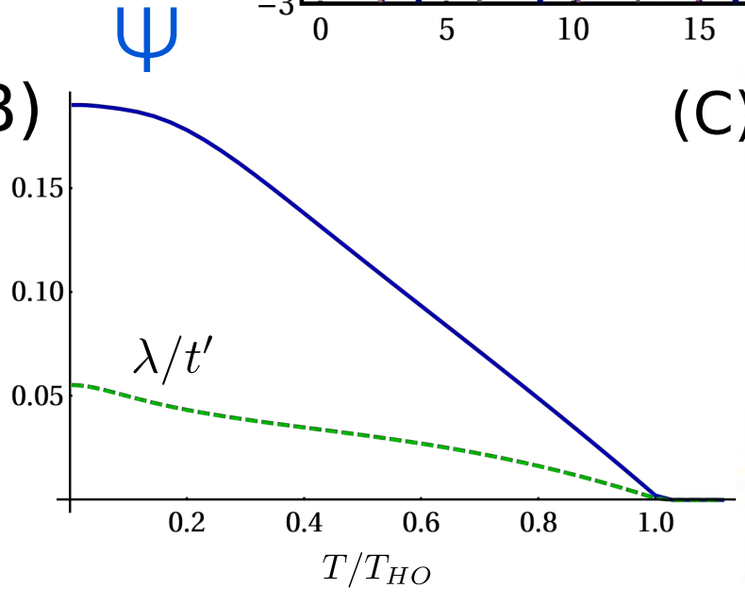
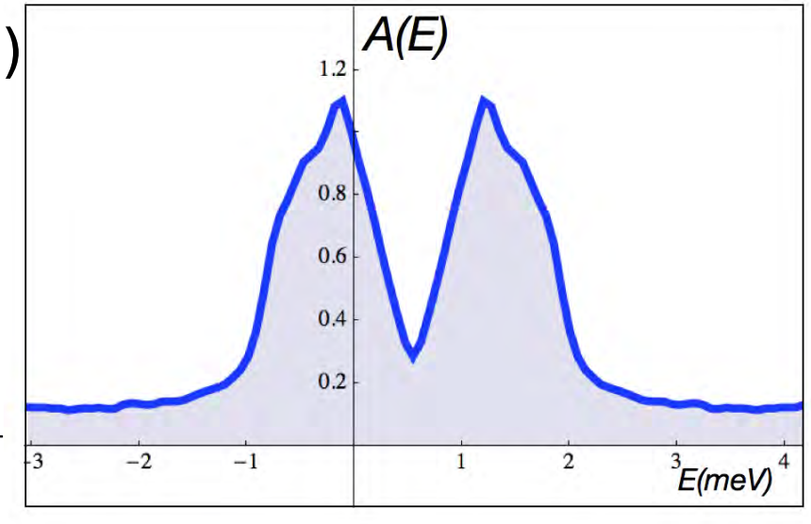
(A)



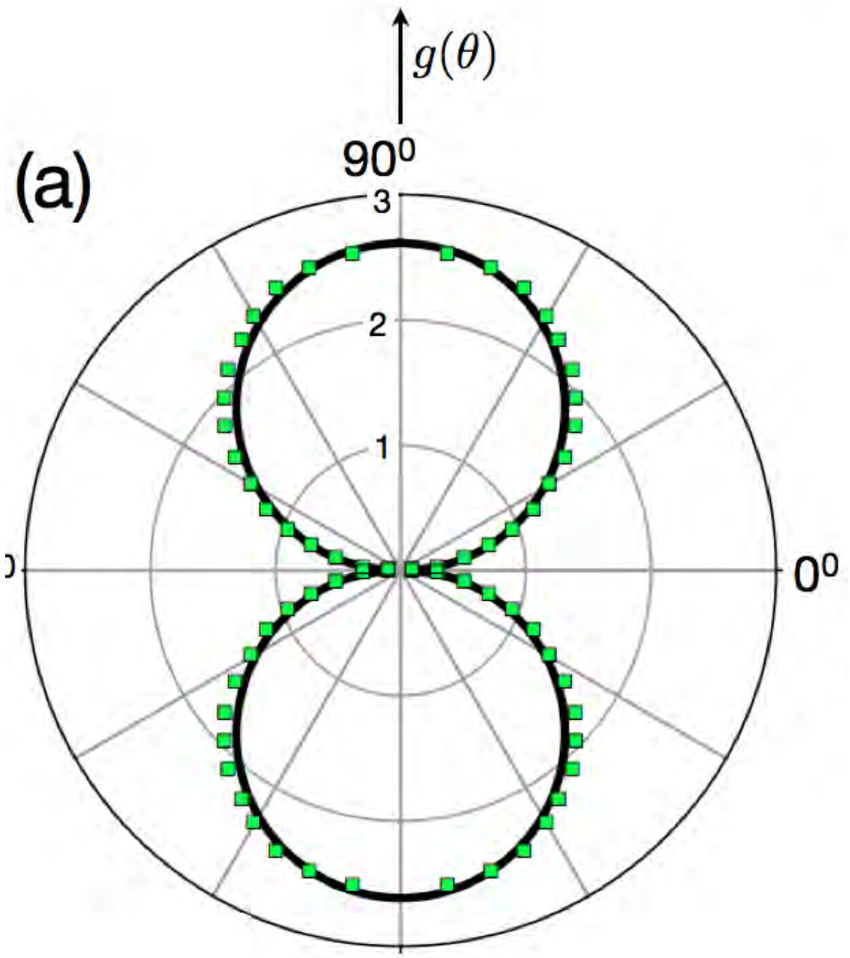
(B)



(C)

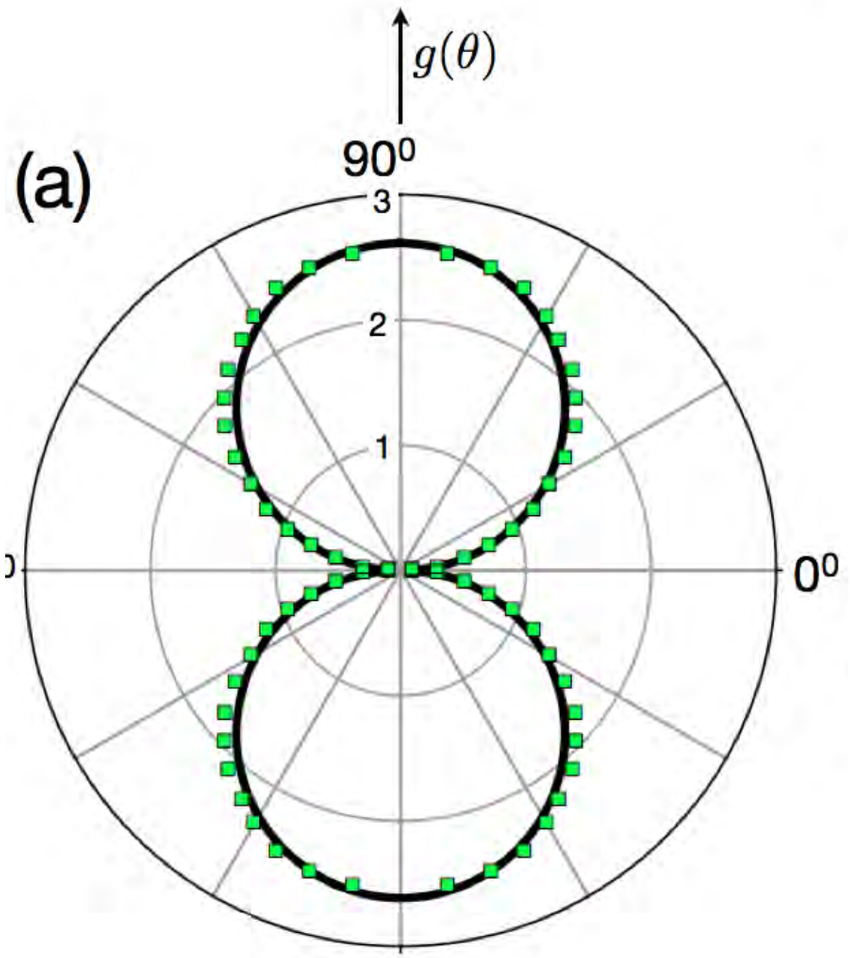


Consistency with experiment.



Computed g-factor Anisotropy

(a)



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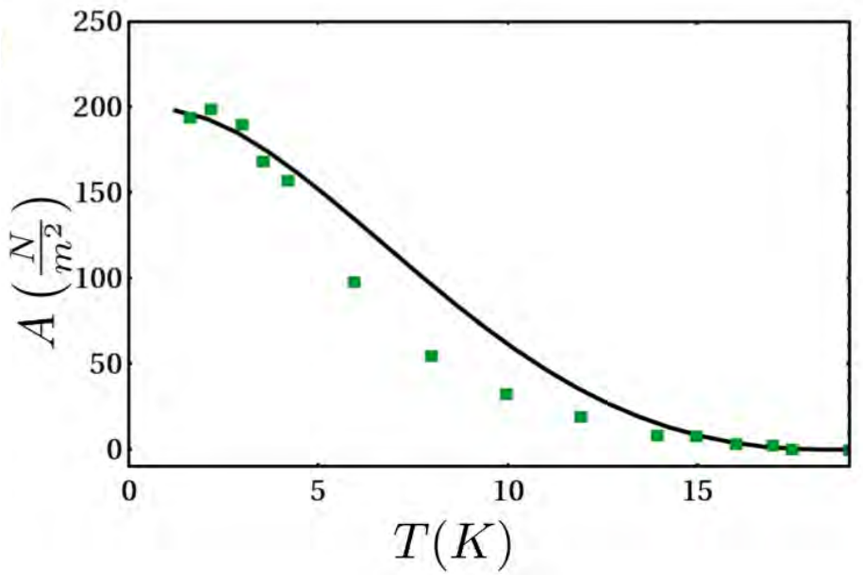

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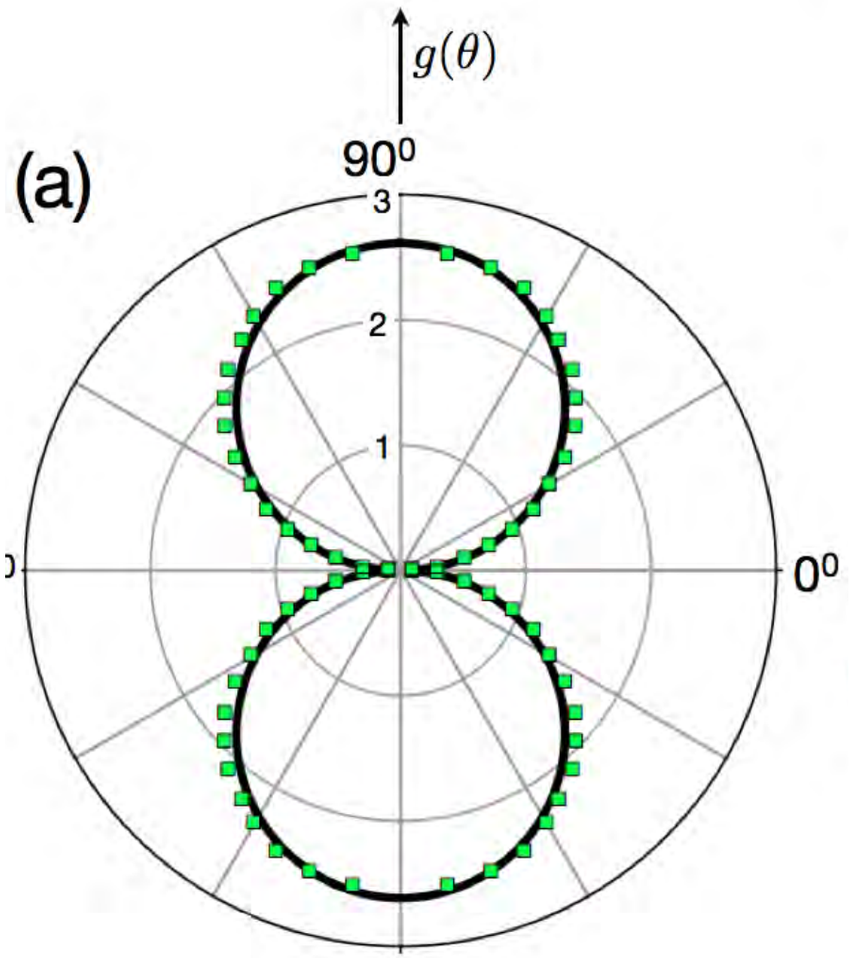
Nematicity: anisotropic X_{xy} consistent with observed susceptibility anomaly.



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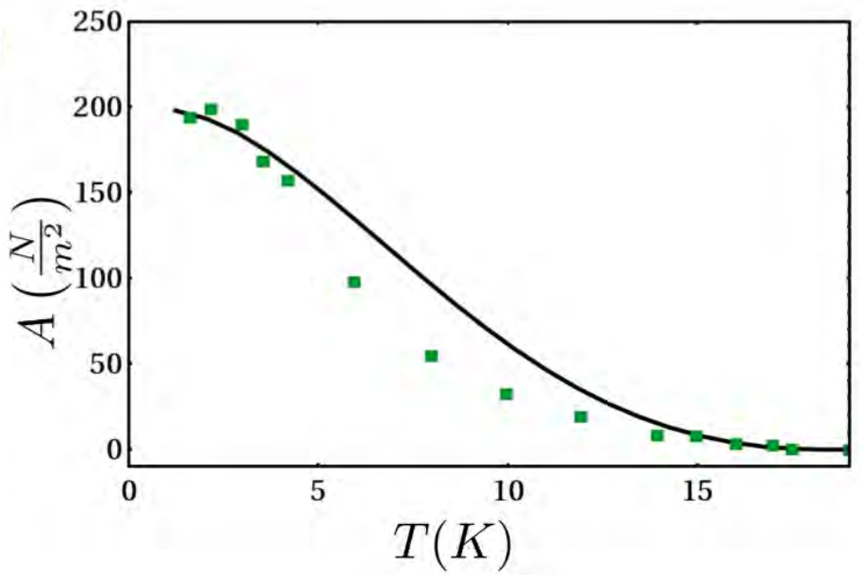

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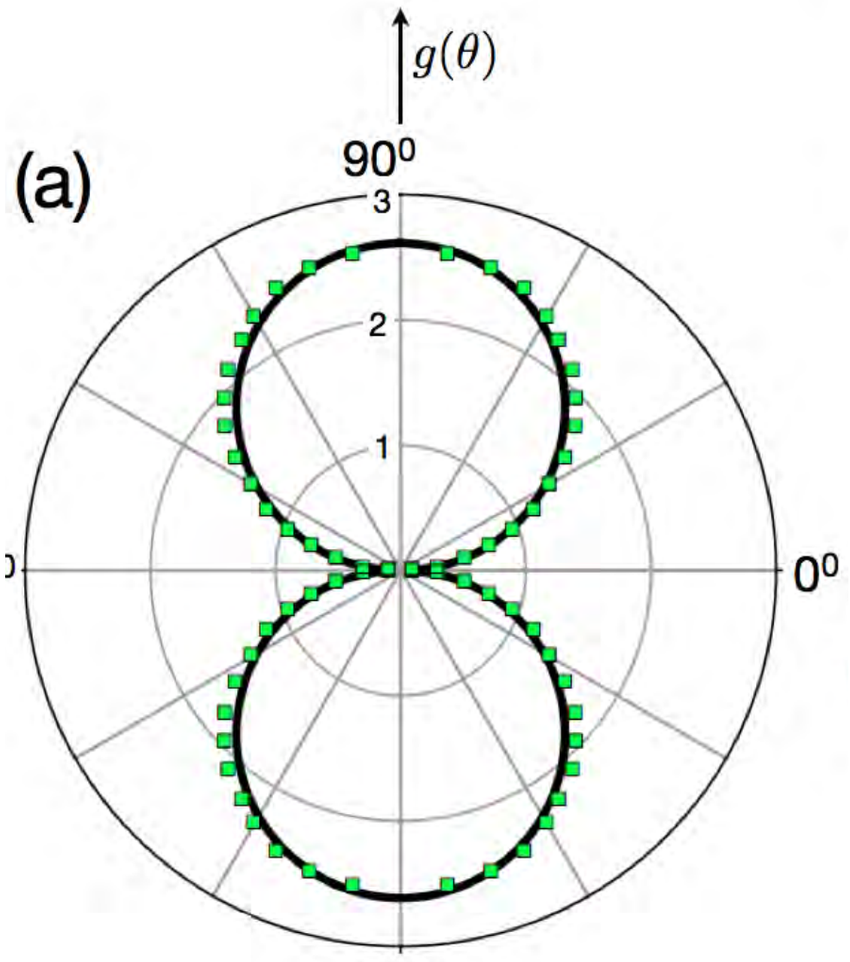

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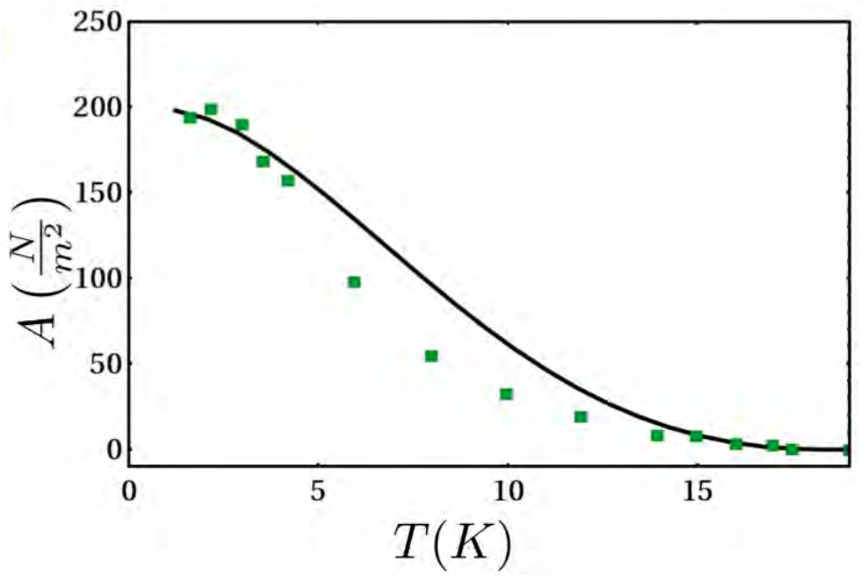

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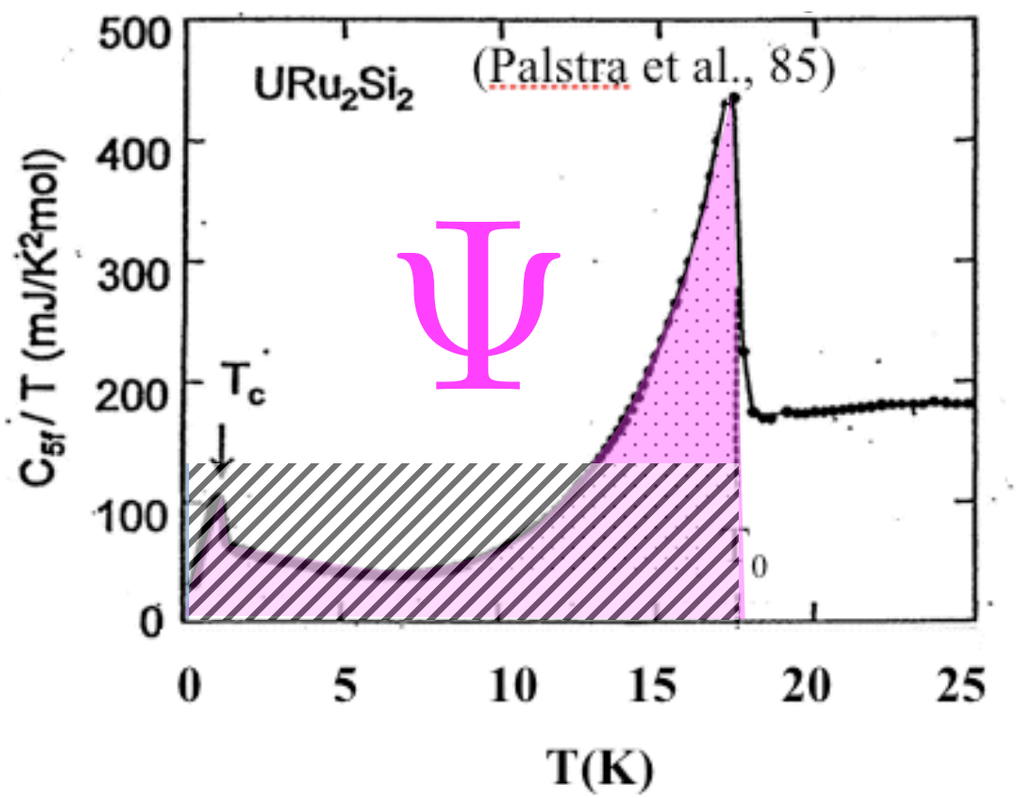
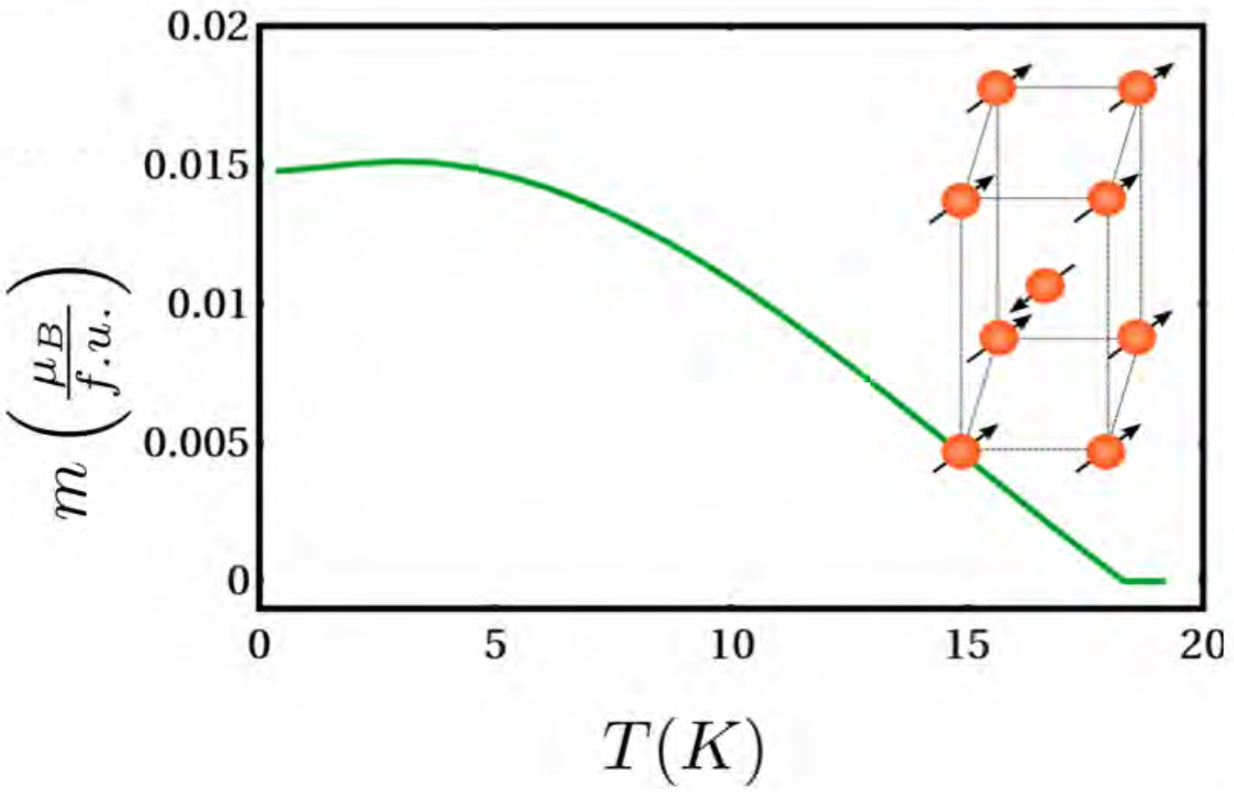


Predictions.

(a) Transverse moment in conduction sea

m~ O(TK/D)~ 0.01μ_B



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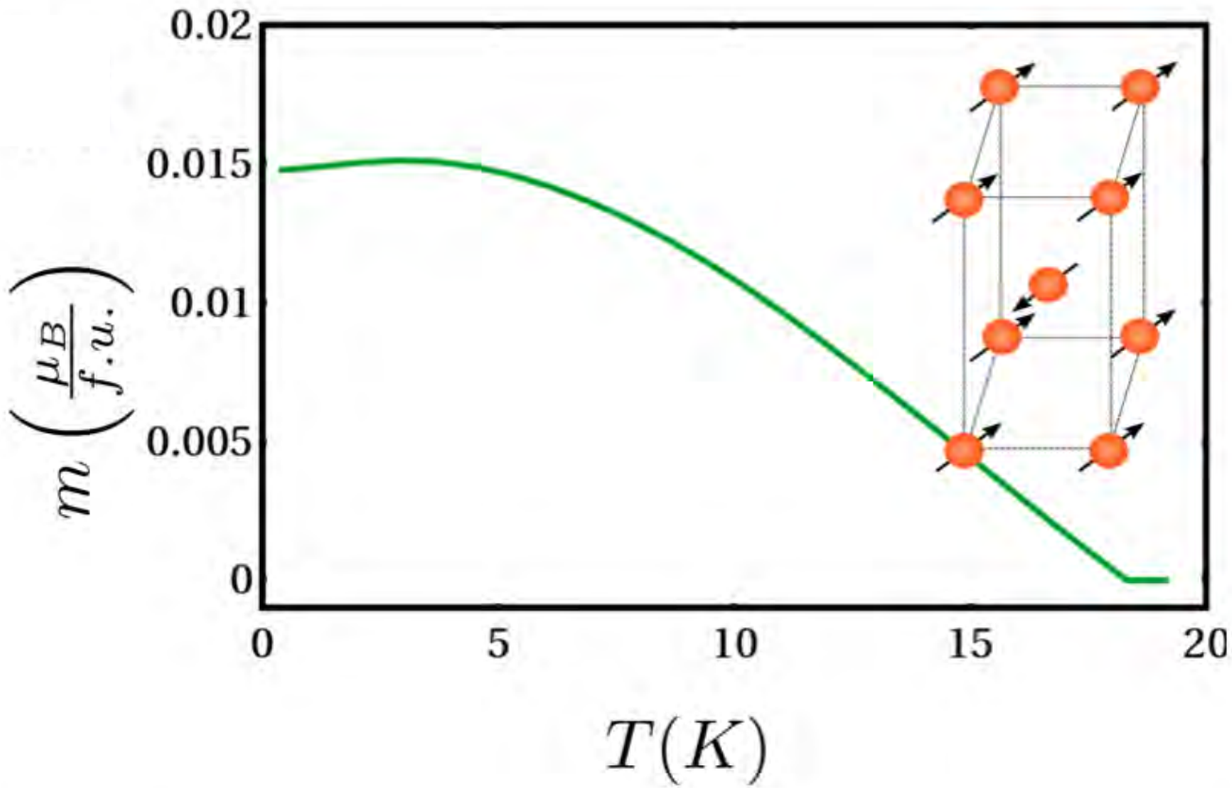
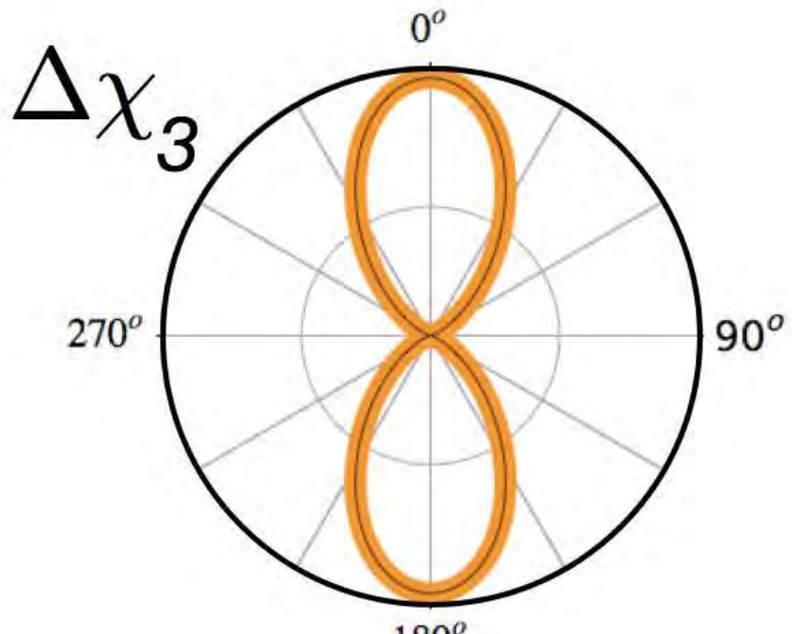


(b) Giant non-linear susceptibility anomaly.



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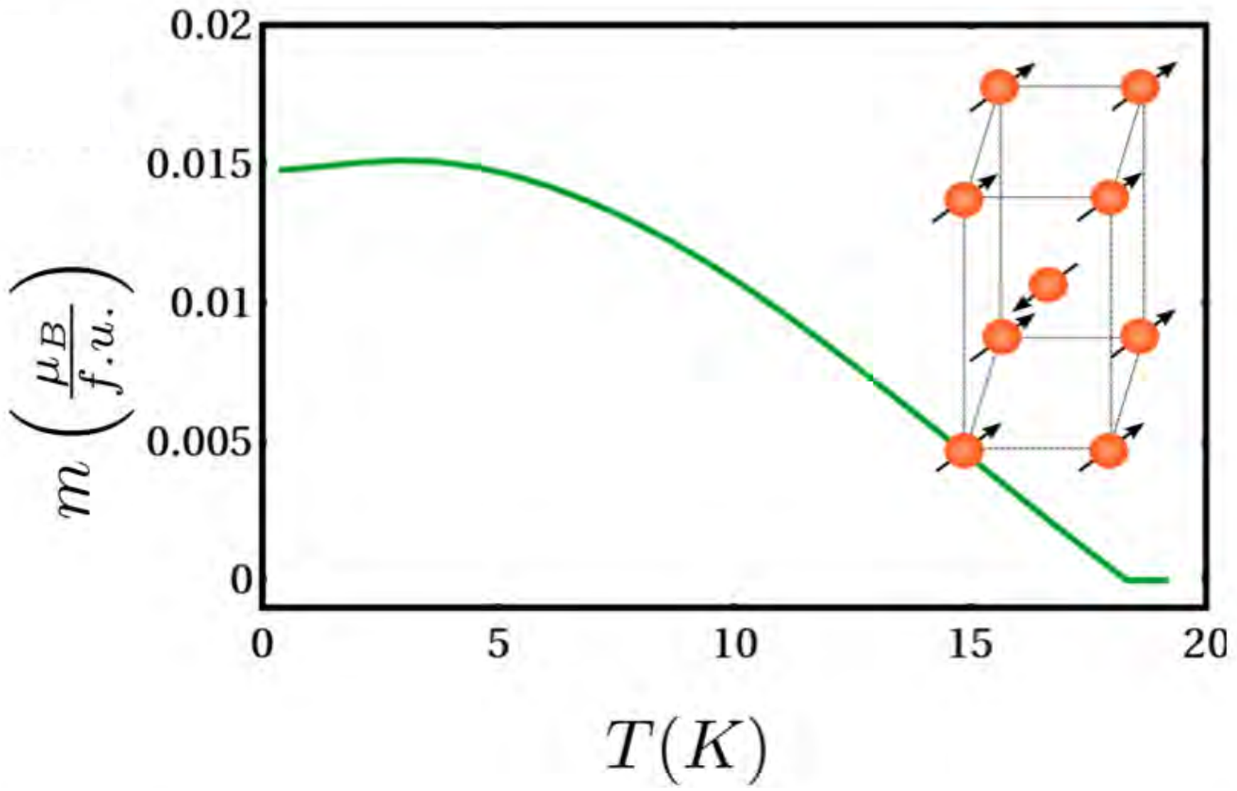

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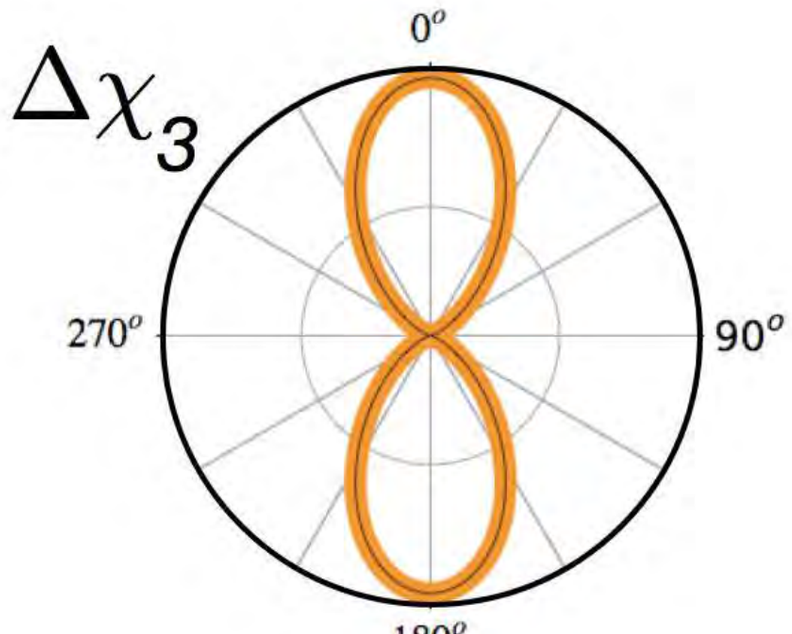
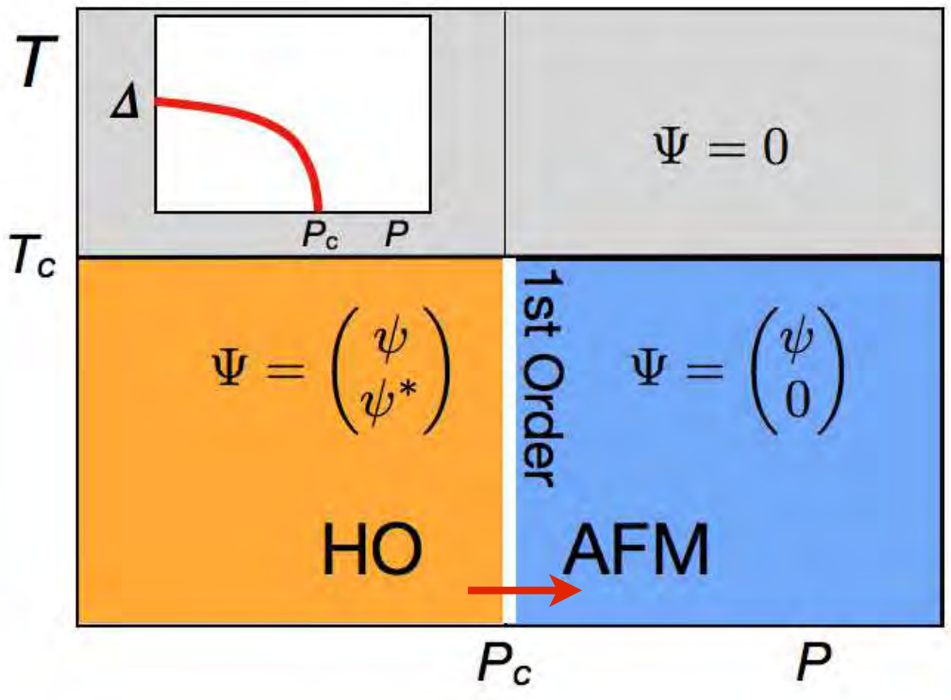
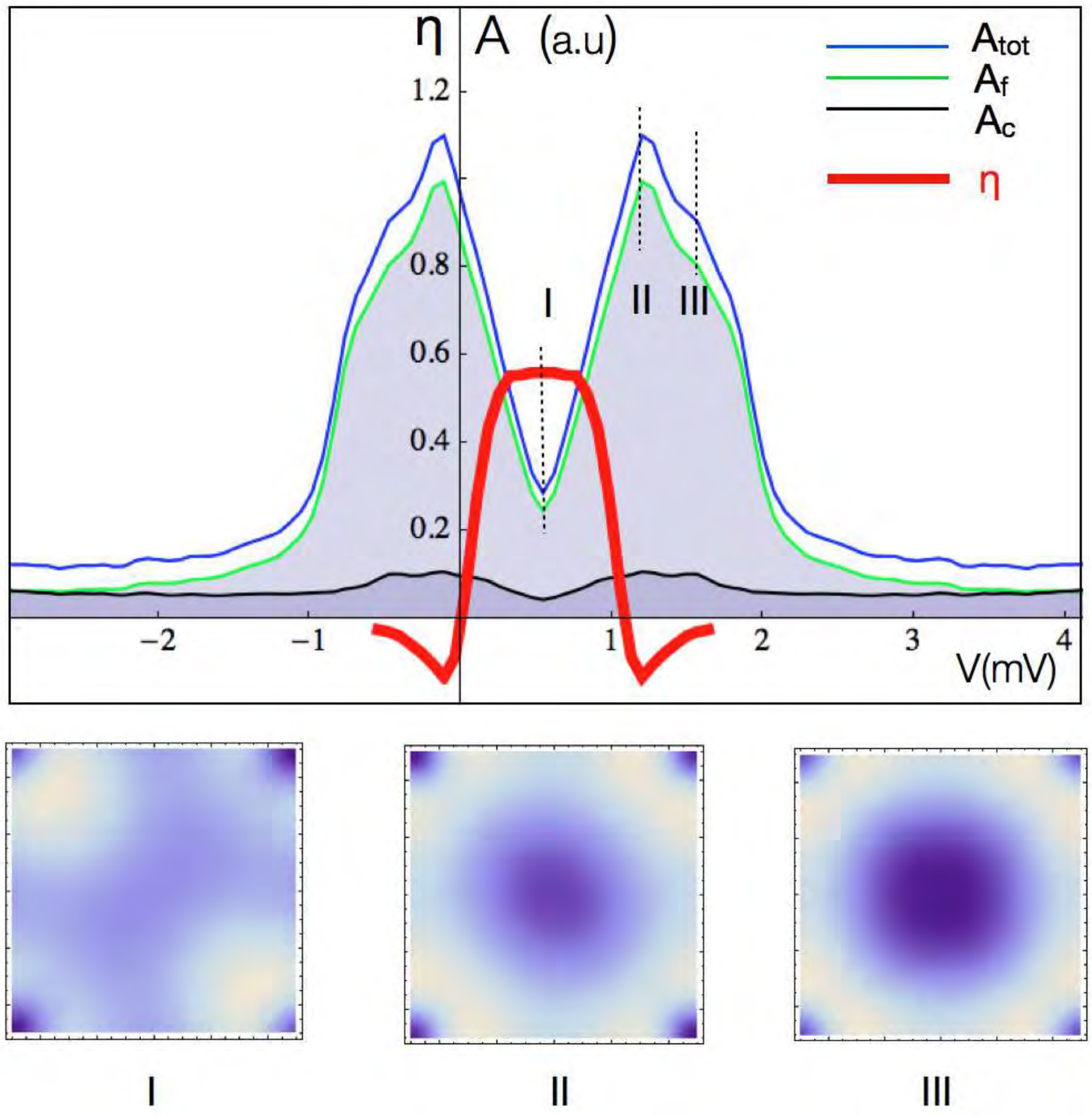




(c) Collapse of gap to Ising fluctuations at 1st order transition line.

(d) Resonant Nematicity in STM



I

II

III

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