Superconductivity from repulsion

Andrey Chubukov

University of Wisconsin

School on modern superconductivity, Boulder, CO, July 2014

Three lectures:

1. Superconductivity at weak coupling

Basics: what one needs for SC
Kohn-Luttinger mechanism
Lattice versions of KL story (cuprates, pnictides…)

2. Superconductivity near a QCP

RG: conventional, parquet, functional….
Spin-fermion model, pairing of incoherent fermions
Role of thermal fluctuations: difference between FM and AFM QCP

3. Charge instability as a competitor to SC at a QCP

Charge instability due to spin fluctuations, SU(2) symmetry
Eliashberg-type theory of charge instability
The interplay between charge order and superconductivity

Superconductivity from repulsion

Lecture 1
Superconductivity at weak coupling

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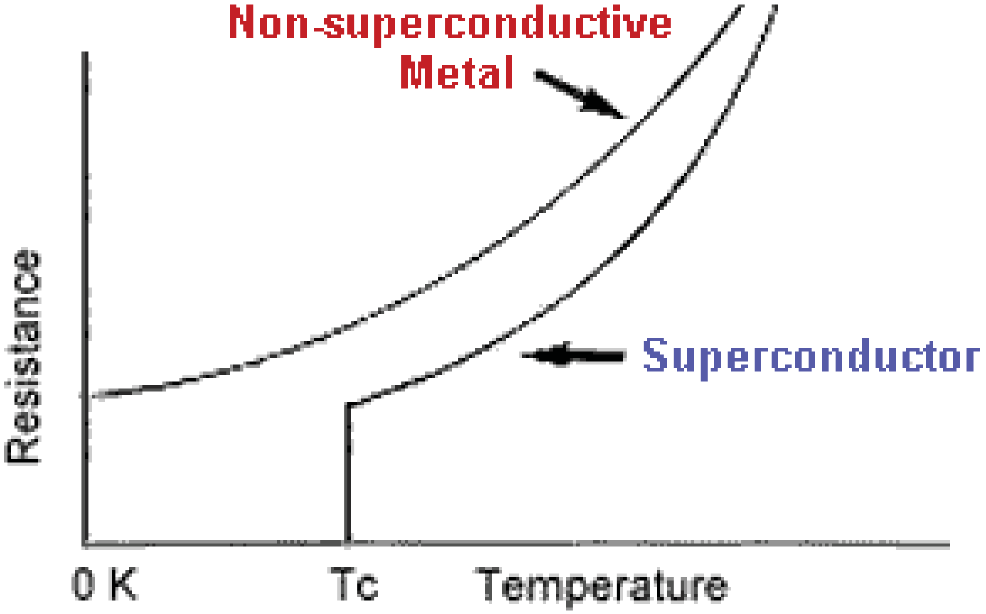
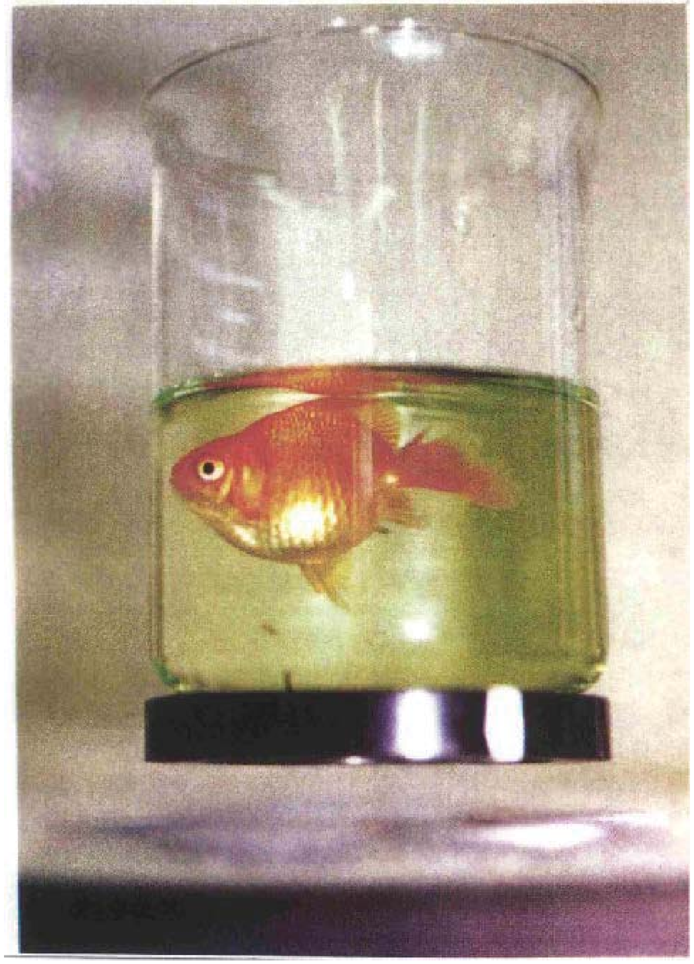
Superconductivity:

Zero-resistance state of interacting electrons

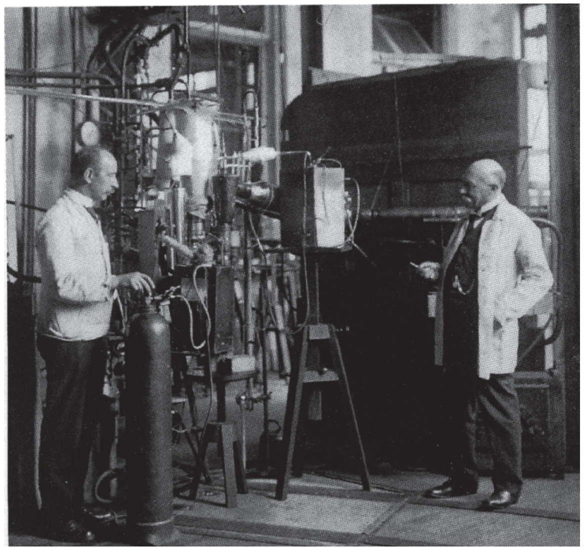


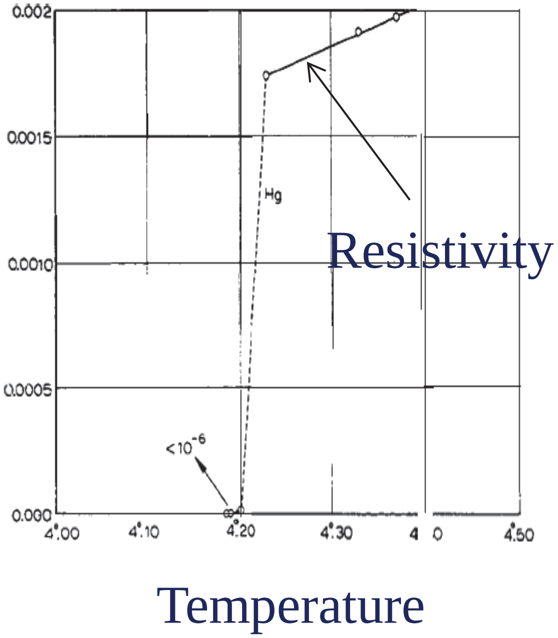
A magnetic field is expelled from a superconductor (Meissner effect)



Superconductivity:

It all started in 1911!





H. KAMERLINGH ONNES, 1853–1926
[From a drawing by his nephew, H. Kamerlingh Onnes]

Nobel Prize 1913

2011 was the 100th anniversary of the discovery of SC

What we need for superconductivity?

Drude theory for metals predicts that resistivity should remain finite at T=0

Ohm's law

$$j = \frac{ne^2\boxed{\tau}}{m_e}E$$

$$j = \sigma E = \frac{E}{\rho}$$

$$\rho = \frac{m_e}{ne^2\boxed{\tau}}$$

If the system has a macroscopic condensate $\boxed{\Xi = |\Xi| e^{i\varphi}}$, then there is an additional current $\boxed{j \propto \nabla\varphi}$, which is not accompanied by energy dissipation and exists in thermodynamic equilibrium

Once we have a condensate, we have superconductivity

For bosons, the appearance of a condensate is natural, because bosons tend to cluster into one quantum state at zero momentum (Bose-Einstein condensation)

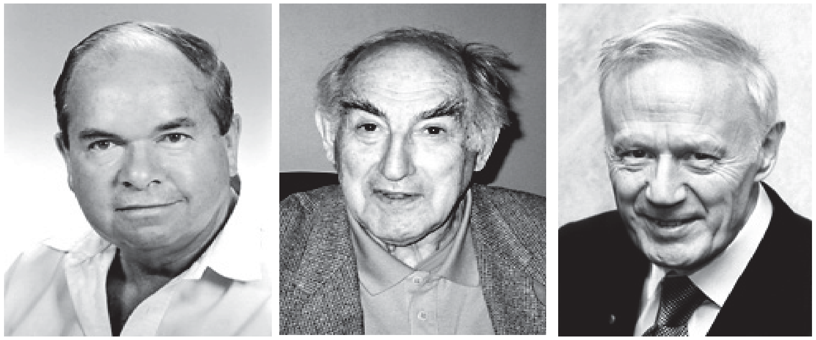
But electrons are fermions, and two fermions simply cannot exist in one quantum state.

However, if two fermions form a bound state at zero momentum, a bound pair becomes a boson, and bosons do condense.

We need to pair fermions into a bound state.

J. Bardeen, L. Cooper, R. Schrieffer

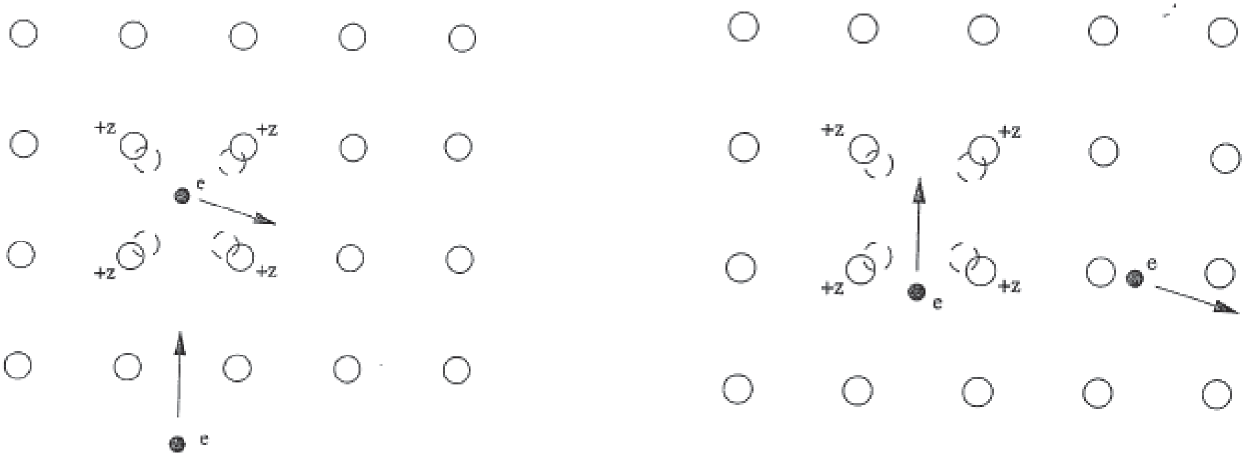
Nobel Prize 1972



A. Abrikosov, V. Ginzburg, A. Leggett

Nobel Prize 2003

Two electrons attract each other by exchanging phonons –
quanta of lattice vibrations



Phonon-mediated attraction competes with Coulomb repulsion between electrons and under certain conditions overshadows it

Phonon superconductivity was one of the greatest successes of physics of the 20th century

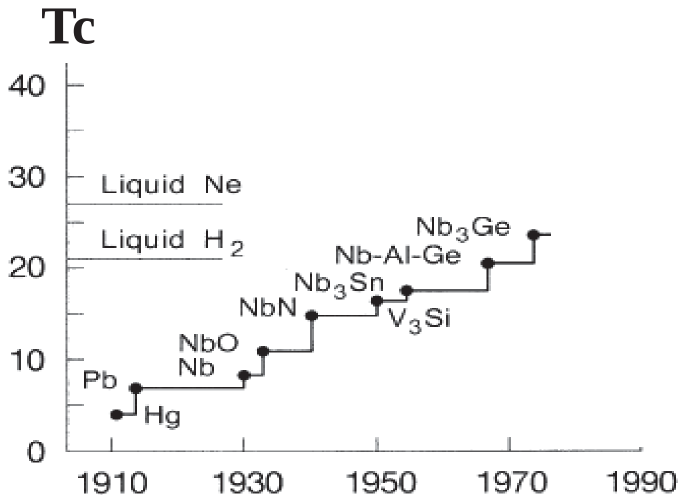
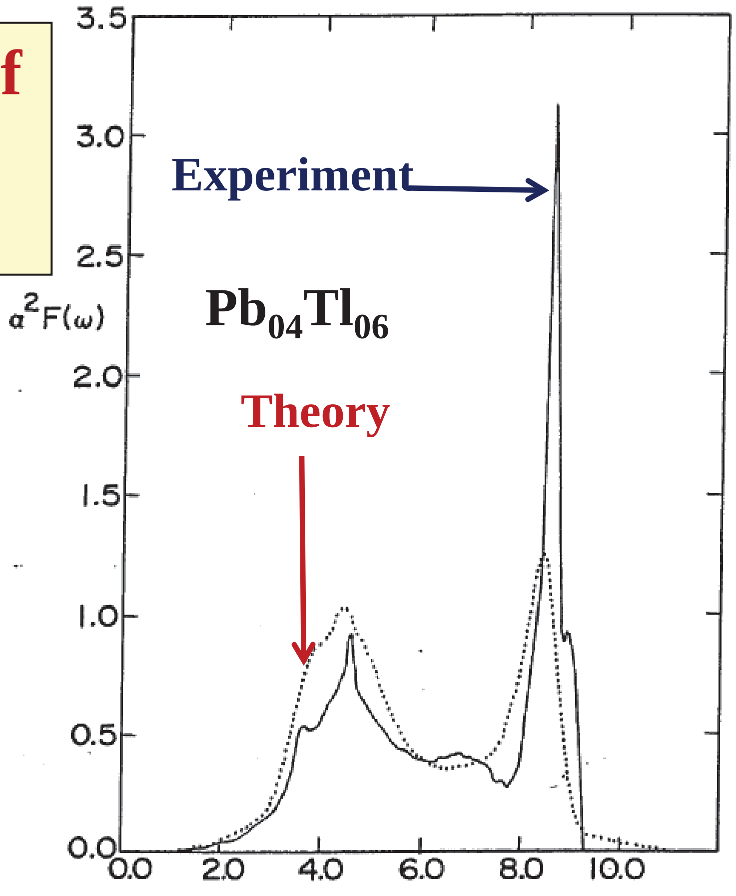


Fig. 1. Evolution of the superconductive transition temperature subsequent to the discovery of the phenomenon.



Mathematical apparatus to study superconductivity

L.P. Gorkov **G.M. Eliashberg** **Yoichiro Nambu**

New era began in 1986: cuprates

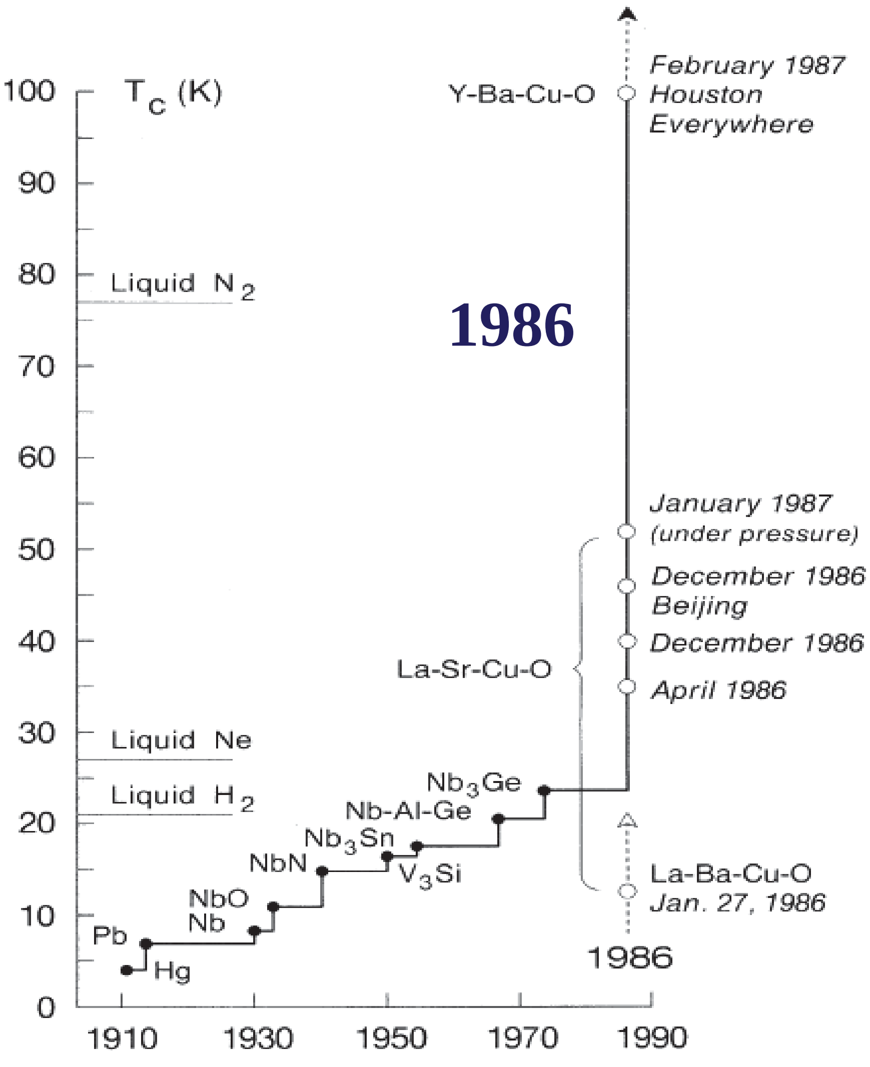
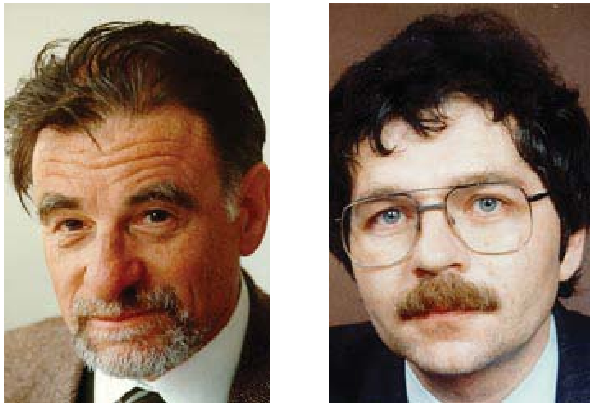


Fig. 1. Evolution of the superconductive transition temperature subsequent to the discovery of the phenomenon.



Alex Muller and Georg Bednortz

Nobel prize, 1987

New breakthrough in 2008: Fe-pnictides

$LaFeAsO_{1-x}F_x$, $Tc = 26K$

$SmFeAsO_{1-x}F_x$, $Tc = 43K$

Hideo Hosono

Letter

Nature 453, 761-762 (5 June 2008) | doi:10.1038/nature07045

Superconductivity at 43 K in SmFe

X. H. Chen1 (#a1), T. Wu1 (#a1), G. Wu1 (#a1), R. H

1. Hefei National Laboratory for Physical Sciences at Microsca

Correspondence to: X. H. Chen1 (#a1) Correspondence and reque

Since the discovery of high-transition-tem been devoted to exploring the origins of th from Bardeen–Cooper–Schrieffer theory1 copper oxide superconductors. The highes (ref. 2 (/nature/jo La–Nd, Sm and G $LaO_{1-x}F_xFeAs$ (r superconductivit measurements r high-temperatur

Letter

Nature 453, 903-905 (12 June 2008) | doi:10.1038/nature07058; Received 2

ved 23 April 2008; Accepted 5 May 2008; Published online 4 June 2008

$FeAsO_{0.85}F_{0.15}$

H. Chen2 (#a2) & C. L. Chien1 (#a1)

imore, Maryland 21218, USA

artment of Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

Letter

Nature 459, 64-67 (7 May 2009) | doi:10.1038/nature07981; Received 4 November 2008; Accepted 13 March 2009

A large iron isotope effect in $SmFeAsO_{1-x}F_x$ and $Ba_{1-x}K_xFe_2As_2$

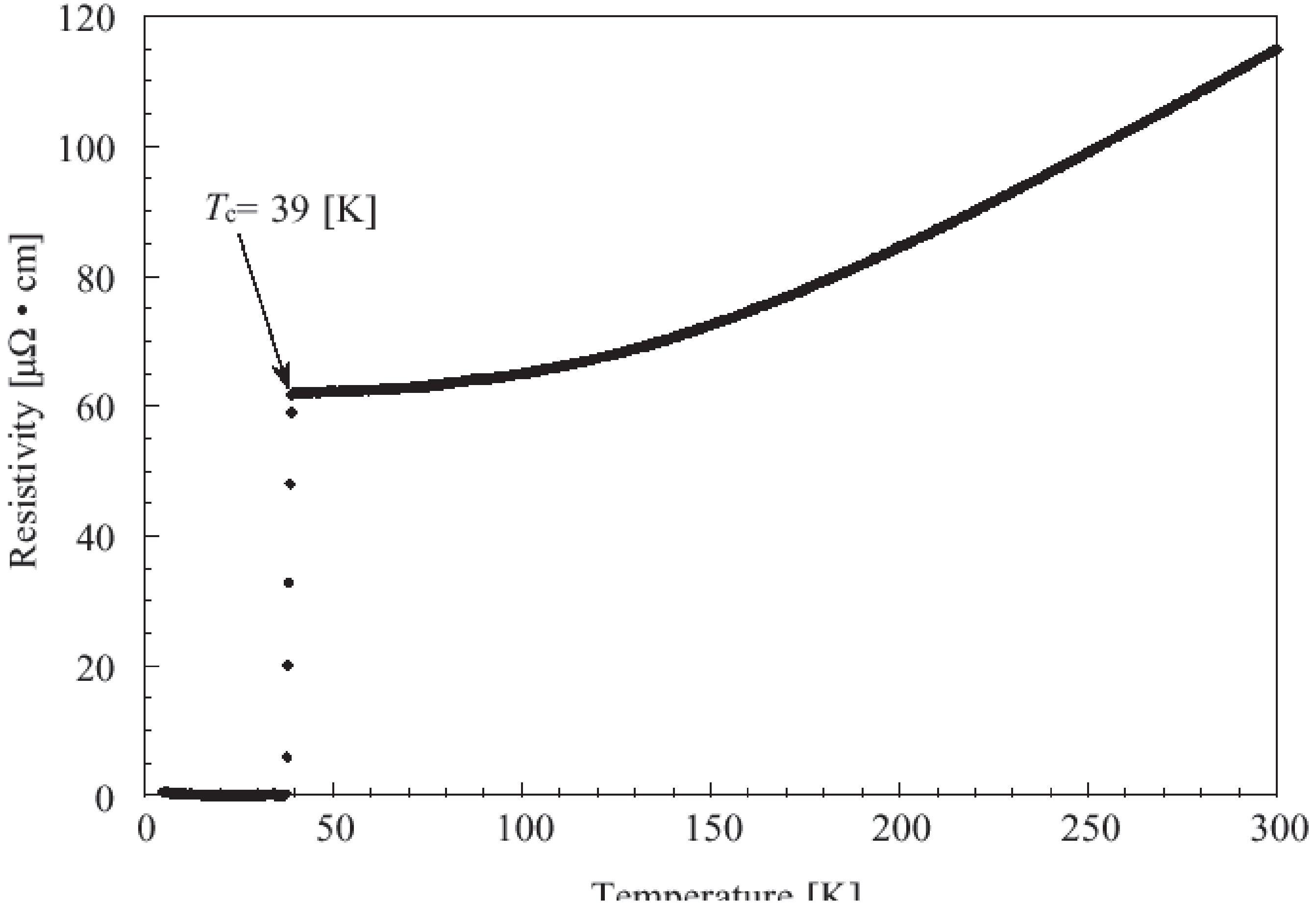
R. H. Liu1, T. Wu1, G. Wu1, H. Chen1, X. F. Wang1, Y. L. Xie1, J. J. Ying1, Y. J. Yan1, Q. J. Li1, B. C. Shi1, W. S. Chu2,3, Z. Y. Wu2,3 & X. H. Chen1

Is only high Tc relevant? No

MgB_2: A phonon Superconductor at 40 K

Fig.4 Nagamatsu et.al

T_c=39 K Akimitsu et al (2001)



Then what is relevant?

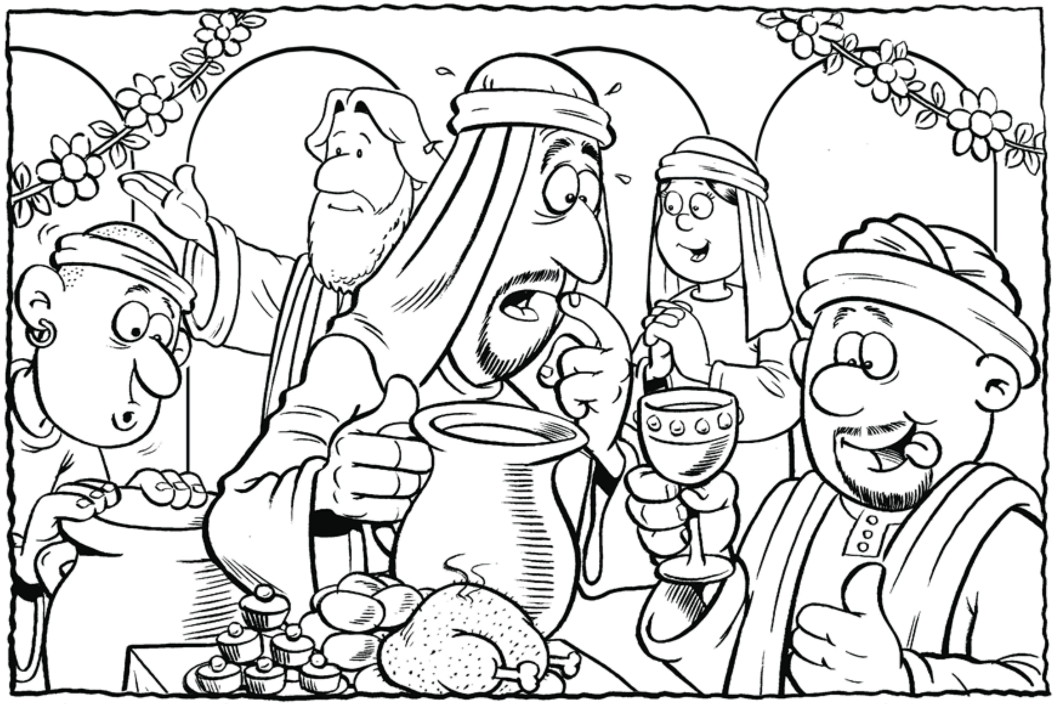
In Cuprates, Fe-pnictides, as well as in
Ruthenates (Sr_2RuO_4) ,
Heavy fermion materials ($CeIn_5$, UPl_3, $CePd_2Si_2$),
Organic superconductors ($(BEDT\text{-}TTF)_2\text{-}Cu[N(CN)_2]Br$)
…

electron-phonon interaction most likely is NOT responsible for the pairing, either by symmetry reasons, or because it is just too weak (Tc would be 1K in Fe-pnictides)

If so, then the pairing must somehow come from electron-electron Coulomb interaction, which is repulsive

Superconductivity from repulsive interaction

How one possibly get superconductivity out of repulsion?



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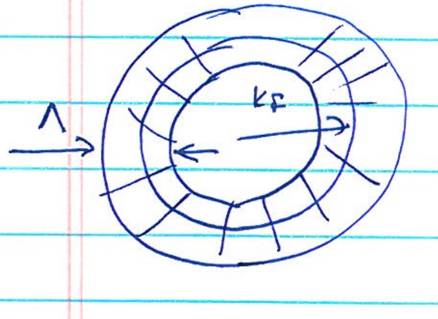
The ABC of theoretical treatment of superconductivity
(let’s do this on the blackboard)

Addition I

ABC of theoretical treatment of superconductivity (SC)

Lecture by A. Millis: SC comes from fermions in a shell of size Λ around the Fermi surface.

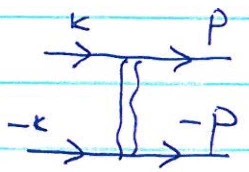


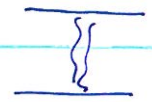
Consider fully renormalized interaction $U(\vec{k}-\vec{p})$, where $\vec{k}$ and $\vec{p}$ are at the Fermi surface, i.e

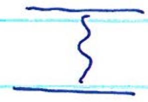
$$|\vec{k}| = |\vec{p}| = p_F$$

Fully renormalized $\equiv$ we integrated out all fermions outside a shell Λ.

For the pairing, we need



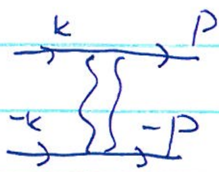

≡ fully renormalized interaction.


bare interaction

<u>Basics</u>

for fermions, $\Psi(1,2) = -\Psi(2,1)$, hence if the total spin of two fermions is $S=1$, orbital momentum L is odd, if $S=0$ (a singlet state), L is even.

To incorporate this fact, it is convenient to consider anti-symmetrized interaction



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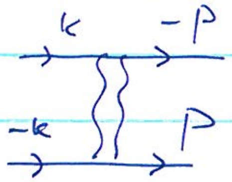
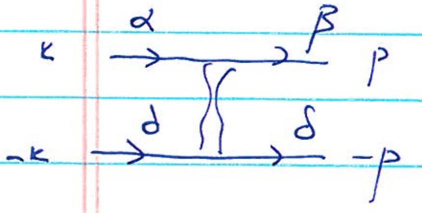
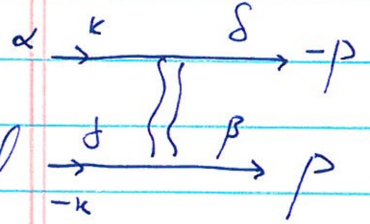


$U(k-p)$ $\qquad$ $U(k+p)$

When we include spins:



: $\delta_{\alpha\beta}\,\delta_{\gamma\delta}$ (density-density interaction)



anti-symmetrized part

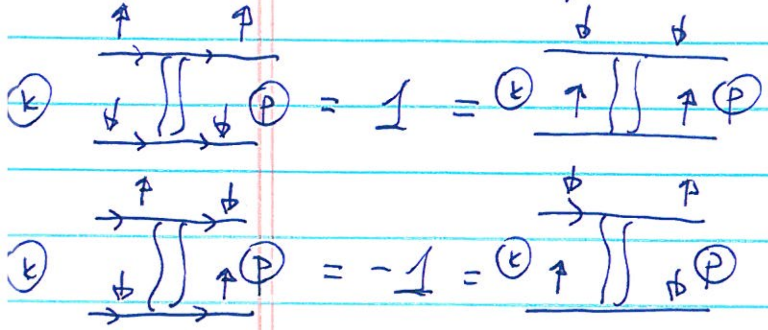
$$\Gamma_{\alpha\gamma,\beta\delta}(k,p) = U(k-p)\,\delta_{\alpha\beta}\,\delta_{\gamma\delta} - U(k+p)\,\delta_{\alpha\delta}\,\delta_{\beta\gamma}$$

We can re-write this as

$$\Gamma_{\alpha\gamma,\beta\delta}(k,p) = \left[U(k-p) - U(k+p)\right]\left(\frac{\delta_{\alpha\beta}\,\delta_{\gamma\delta} + \delta_{\alpha\delta}\,\delta_{\beta\gamma}}{2}\right)$$

$$+ \left[U(k-p) + U(k-p)\right]\left[\frac{\delta_{\alpha\beta}\,\delta_{\gamma\delta} - \delta_{\alpha\delta}\,\delta_{\beta\gamma}}{2}\right]$$

Now recall: if we have $\frac{1}{2}\left(\delta_{\alpha\beta}\,\delta_{\gamma\delta} - \delta_{\alpha\delta}\,\delta_{\beta\gamma}\right)$:



$$\equiv \underset{k}{\frac{1}{\sqrt{2}}\left(\uparrow\downarrow - \downarrow\uparrow\right)} \times \underset{p}{\frac{1}{\sqrt{2}}\left(\uparrow\downarrow - \downarrow\uparrow\right)}$$

In other words:

$$\frac{\delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}}{2} \equiv \text{singlet}: \quad U_s = U(k-p)+U(k+p)$$

$$\frac{\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}}{2} \equiv \text{triplet}: \quad U_t = U(k-p)-U(k-p)$$

In isotropic case 

$$U(k-p) = \sum_{l} P_l(\cos\theta)\, U_l$$

U_s : even l, U_t - odd l

as it should be, indeed.

Note also that antisymmetrized interaction can be re-expressed as

$$\left[\ \delta_{\alpha\delta}\delta_{\beta\gamma} = \frac{1}{2}\left[\delta_{\alpha\beta}\delta_{\gamma\delta} + \vec{\sigma}_{\alpha\beta}\vec{\sigma}_{\gamma\delta}\right]\right]$$

$$\Gamma_{\alpha\gamma,\beta\delta}(k,p) = \left[U(k-p) - \tfrac{1}{2}U(k+p)\right]\delta_{\alpha\beta}\delta_{\gamma\delta} - \frac{U(k-p)}{2}\vec{\sigma}_{\alpha\beta}\vec{\sigma}_{\gamma\delta}$$

i.e.
$$\begin{cases} U_{charge} = U(k-p) - \frac{1}{2}U(k+p) \\ U_{spin} = -\frac{1}{2}U(k+p) \end{cases}$$

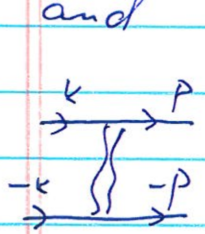
Comparing the expressions for U_s, U_t and for U_{charge}, U_{spin}, we obtain

$$\left.\begin{aligned} U_s &= U_{charge} - 3U_{spin} \\ U_t &= U_{charge} + U_{spin} \end{aligned}\right\}$$ compare with lectures by A. Millis

* * *

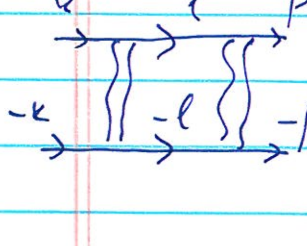
Now let's recall what we need for superconductivity.

Let's momentarily set $U = \text{const}$ and neglect spin indices



$\equiv U$

the density of states per spin



$= -U^2\,\Pi(0)\log\frac{\Lambda}{T}$

$$\Pi(0) = \begin{cases} \dfrac{mk_F}{2\pi^2}, & 3D \\ \dfrac{m}{2\pi}, & 2D \end{cases}$$

Summing up ladder series of diagrams, we obtain

$$u \Rightarrow u - u^2\Pi(0)\log\frac{\Lambda}{T} + u^3\Pi^2(0)\log^2\frac{\Lambda}{T} + \dots$$

$$= \frac{u}{1 + u\Pi(0)\log\frac{\Lambda}{T}}$$

When $u > 0$, $u \Rightarrow \frac{1}{\Pi(0)\log\frac{\Lambda}{T}} \to 0(!)$

When $u < 0$, $u \to \frac{u}{1 - |u|\Pi(0)\log\frac{\Lambda}{T}}$

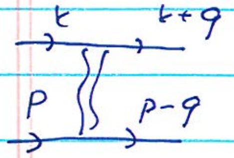
Dressed interaction diverges at

$$T \sim \Lambda e^{-\frac{1}{\lambda}}, \quad \lambda = |u|\Pi(0)$$

By itself, divergence doesn't tell us that there is an instability.

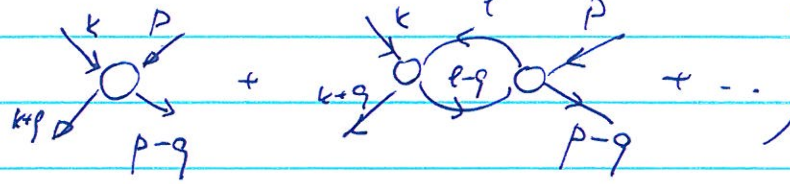
Example: zero sound, coming from forward scattering.

If I take



and dress it up in particle - hole channel

(schematically:



)

we get (in 2D) at finite ω and q

$$U \rightarrow \frac{U}{1 + U\Pi(\text{[illegible]})\left(1 - \frac{\omega}{\sqrt{\omega^2 - (v_F q)^2}}\right)}$$

This leads to zero sound:

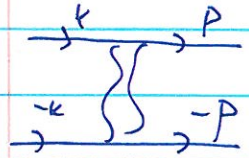
$$U \rightarrow U_{eff}(q, \omega) \sim \frac{1}{\omega^2 - v^2 q^2 + i\delta}$$

Pole in the lower frequency ½ plane!

when the pole is in the lower frequency ½ plane, excitations *do not diverge* (zero sound does not destroy Fermi liquid)

Homework problem.

Consider



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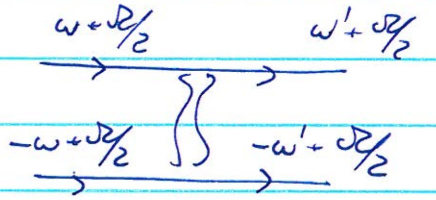


and compute dressed $V_{eff}(\Omega)$

Show that the pole in Ω is in the *upper* frequency ½ plane

Such a pole means that fluctuations *grow in time* and destroy a Fermi liquid.

Back to generic interaction

$U(k-p)$ with spin indices.

<u>Homework problem II</u>

Show that

a) singlet & triplet channels decouple

b) within each channel, there is complete decoupling between different l, if the system is rotationally invariant.

As a result: $$U_{eff}^{(l)} = \frac{U_l}{1 + U_l \Pi(0) \log \frac{\Lambda}{T}}$$

~~It is~~ and

$U_{eff}^{(l)}(\Omega)$ has a pole in the upper half-plane if $U_l < 0$

Conclusion:

it is sufficient to have just one V_ℓ to be negative for the system to be superconducting.

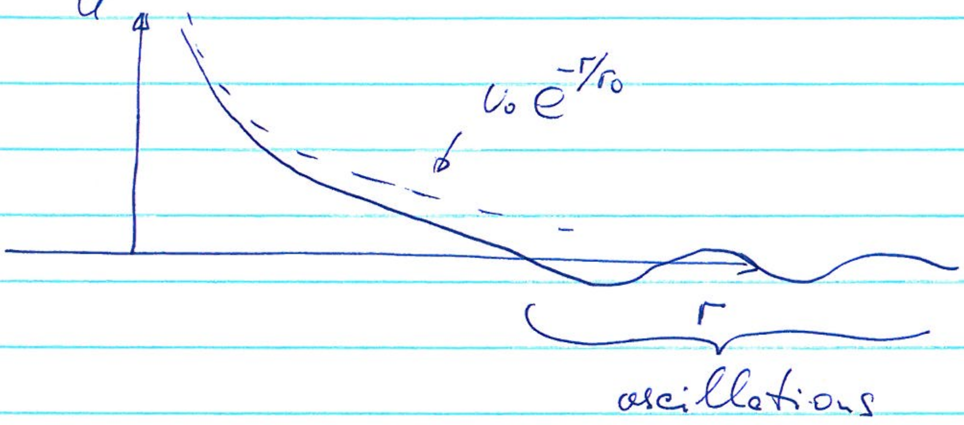
But it is easier to say than to obtain.

If we take $U(r) = U_0\, e^{-r/r_0}$, do Fourier transform to $U(q) \sim \frac{1}{q^2 + r_0^{-2}}$ and compute V_ℓ of $U(q = k - p)$,

we get all $V_\ell > 0$

(complete repulsion!)

Screened Coulomb potential:

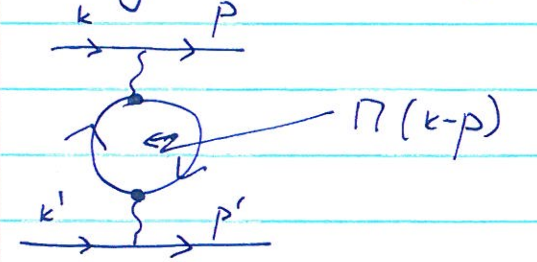


(occasional over-screening)

These are called Friedel oscillations:

Where they come from:

Screening



$\Pi = \Pi(\Theta)$: near $\Theta = \pi$ $\Pi \sim (\pi-\Theta)^2 \log(\pi-\Theta)$

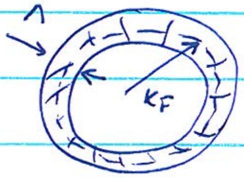
(non-analytic)

If we find l for which over-screening wins over screening, we get an attraction.

Larger l $\equiv$ larger distances.

Kohn-Luttinger story (1965)

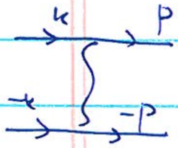
we have



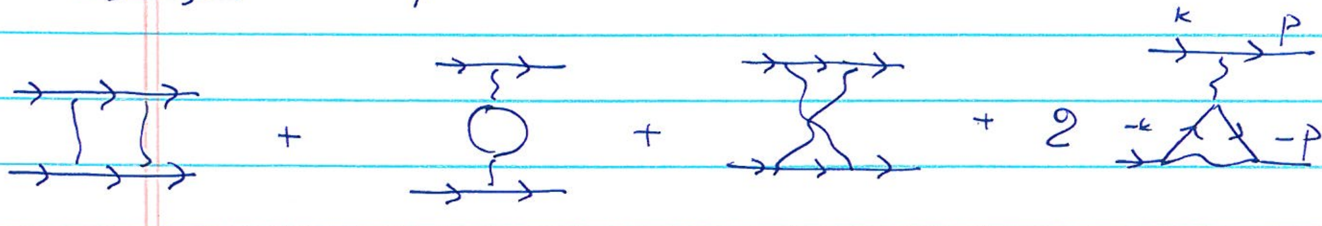
Let's integrate outside of Λ

Let's start doing this perturbatively. Our goal is to obtain $U(k-p)$ at low energies.

we have:

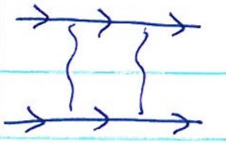


$\equiv U_0(k-p)$ } first order in U_0



and, after we evaluate $U(k-p)$ to second order in U_0, we ~~an~~

1) anti-symmetrize
2) compute components U_ℓ
3) check if any of them are negative

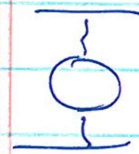


$$\equiv U^2 \log \frac{E_F}{\Lambda}$$

This is what will eliminate artificial scale Λ from the expression for T_c (lecture by Steve Kivelson)

$$T_c \to \overset{E_F}{\cancel{\Lambda}} e^{-1/\lambda}$$

Other 3 diagrams



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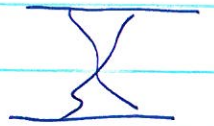

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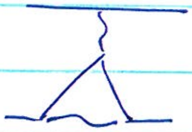
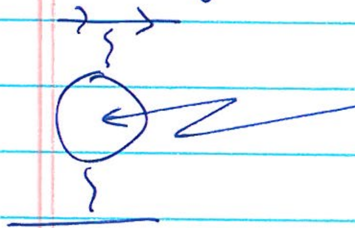


represent screening

We cannot compute these diagrams at arbitrary $U(k-p)$ (we later compute them for $U = \text{const}$), but we can play a different game

Namely, we know



$$\Pi(k-p) = \Pi(\theta) \equiv \Pi(\pi) - \alpha(\theta-\pi)^2 \log(\pi-\theta)$$

This gives some $U_{eff} \propto U^2 \Pi(\theta)$.

If we now ~~take~~ compute U_ℓ (partial amplitudes of the interaction with momentum ℓ), we find $U(\ell) \propto 1/\ell^4$

If we take $U(k-p) = U(q) \sim \dfrac{1}{q^2 + r_0^{-2}}$,

we find $U_\ell \propto e^{-\ell}$

Hence, at large enough ℓ,

non-analytic part is the most efficient.

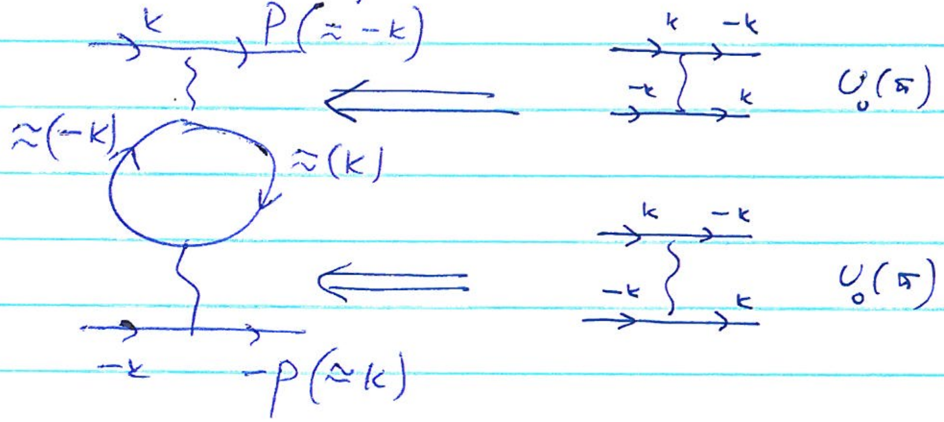
Then the task is simpler:

we need to understand what

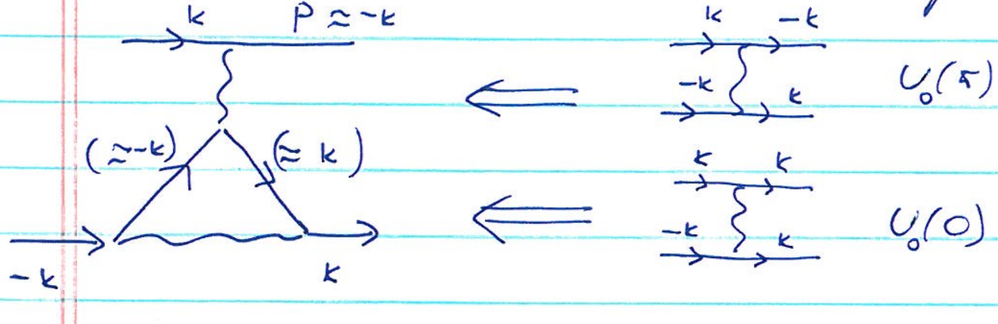
momenta in the bubble contribute to non-analyticity.

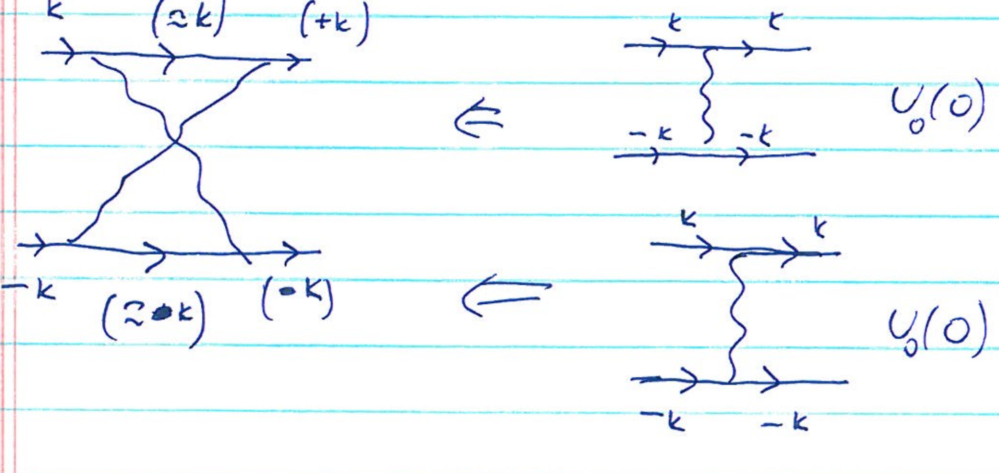
Homework # 3. Show that

$(\pi-\theta)^2 \underline{\log(\pi-\theta)}$ in the polarization bubble come from



Look at other two diagrams:





Do bookkeeping job: construct antisymmetrized interaction to order U_0^2

singlet $$U_{\ell=2m} = 2U_{0,\ell=2m} + \frac{\alpha}{\ell^4}\left[2U_0^2(\pi) - 2U_0(0)U_0(\pi) - U_0^2(0)\right]$$

$$\alpha > 0$$

$$U_{\ell=2m+1} = 2U_{0,\ell=2m+1} \bullet \frac{\alpha}{\ell^4}\left[2U_0^2(\pi) - 2U_0^2(0)U_0(\pi) + U_0^2(0)\right]$$

Recall: negative U_ℓ means attraction!

At large ℓ, $U_{0,\ell} \sim e^{-\ell}$ is irrelevant.

Now: $$2U_0^2(\pi) - 2U_0(0)U_0(\pi) + U_0^2(0) =$$

$$= U_0^2(\pi) + \left(U_0(\pi) - U_0(0)\right)^2 > 0.$$

Hence, $U_{\ell=2m+1}$ is definitely negative for $\ell \gg 1$.

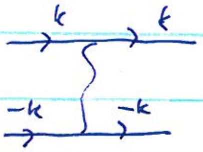
Next: the calculations have been done in perturbation theory.

Statement (Kohn & Luttinger, 1965): higher order terms only transform $U(0)$ & $U(\bar{\pi})$ into fully renormalized interactions

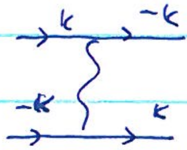
Because $U(0)$ & $U(\bar{\pi})$ are *some* numbers, $u_{\ell=2n+1}$ definitely remains negative

For advanced consideration:

Fully renormalized



$\equiv U(0)$ &



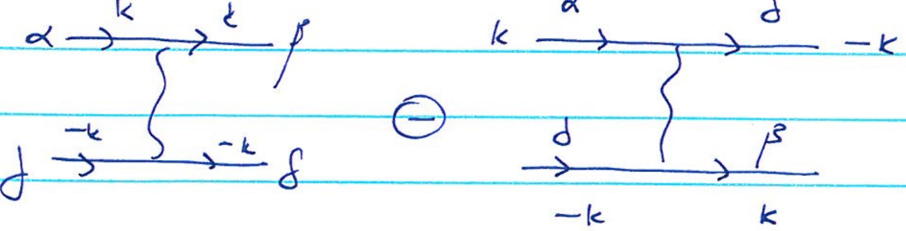
$\equiv U(\bar{\pi})$

are expressed via spin and charge components of the *backscattering* amplitude.

(see pp. 21-22)

At $p = -k$, this function is called __backscattering__

Antisymmetrize :



We have $U(0)\,\delta_{\alpha\beta}\delta_{\gamma\delta} - U(\pi)\,\delta_{\alpha\gamma}\delta_{\beta\delta}$

$$= \left[U(0) - \frac{U(\pi)}{2}\right]\delta_{\alpha\beta}\delta_{\gamma\delta} - \frac{U(\pi)}{2}\vec{\sigma}_{\alpha\beta}\vec{\sigma}_{\gamma\delta}$$

or
$$\begin{cases} U(\pi) = -2\,U^{spin}_{back} \\ U(0) = U^{charge}_{back} - U^{spin}_{back} \end{cases}$$

$$2U_0^2(\pi) - 2U_0(0)U_0(\pi) + U_0^2(0) =$$

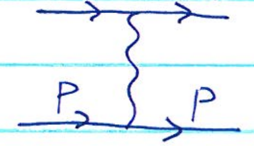
$$= 8\,U^{spin}_{back} + 4U^{spin}_{back}\left(U^{charge}_{back} - U^{back}_{spin}\right)$$

$$+ \left(U^{charge}_{back} - U^{back}_{spin}\right)^2 =$$

$$= 5\left(U^{spin}_{back}\right)^2 + \left(U^{charge}_{back}\right)^2 + 2U^{spin}_{back}\,U^{charge}_{back}$$

$$= \left(U^{charge}_{back} + U^{back}_{spin}\right)^2 + 4\left(U^{back}_{spin}\right)^2$$

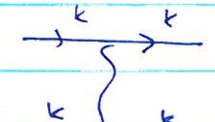
In general, when we talk about interactions at small ~~moment~~ momentum transfer, we introduce



and call this $\bar{U}(k-p)$

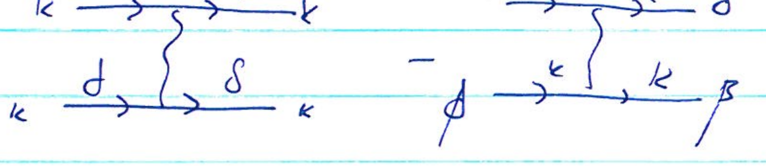
This is the function which determines properties of a Fermi liquid (lecture by A. Millis)

$\bar{U}(0) \equiv$



true forward scattering (all particles move along the same line)

Anti-symmetrize:



$=$

$$= \bar{U}(0)\,[\delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}]$$

Back to Kohn-Luttinger

It is essential that renormalizations into $U_\ell \sim 1/\ell^4$ occur outside of a window $|\varepsilon| < \Lambda$

In other words, & internal fermions in the bubble must be outside the Λ range around Fermi surface.

This restriction leads to

$$T_c \sim f(\ell)\, e^{-\ell^4(\#)}$$

KL: applied the result to ^3He and set $\ell = 2$ (d-wave)

get $T_c \sim (\ )\, e^{-17} \sim (\cdot\cdot)\, 10^{-40} \equiv 0$!

Next: Fay & Layzer, 1968 :

they returned to pert. theory and applied KL result to $U_0(q) \equiv U_0$ (Hubbard model)

To first order in U_0, we have repulsion in S-wave channel and nothing in all other channels

To order U^2:

$$\left.\begin{aligned} U_{\ell=2m} &= \frac{\alpha}{\ell^4}\left[-U_0^2\right] \\ U_{\ell=2m+1} &= -\frac{\alpha}{\ell^4}\left[U_0^2\right] \end{aligned}\right\} \text{both are negative, the magnitude grows as } \ell \text{ gets smaller}$$

Because this is pert. theory, we can compute U_ℓ explicitly:

$$U_1 = -\frac{U^2 N(0)}{5\pi^2}\left[2\log 2 - 1\right] < 0$$

$$U_2 \sim U_1/16 \ll U_1$$

$$\boxed{\text{Result: p-wave pairing}}$$

Interesting: for ^3He:

$$T_c^{kl} \sim \mathcal{E}_F \left(10^{-3}\right) \sim (mK)$$

[experiment: $T_c \sim 3mK$]

—— . —— . —— . —— .

Additional remark: the pairing (p-wave) in ^3He is often attributed to ferromagnetic spin fluctuations:

Let's recall from p. 4.

$$\left.\begin{aligned} U_{charge} &= U(k-p) - \tfrac{1}{2} U(k+p) ; \\ U_{spin} &= -\tfrac{1}{2} U(k+p) \end{aligned}\right\} \begin{aligned} U_t &= U_{charge} + U_{spin} \\ U_s &= U_{charge} - 3U_{spin} \end{aligned}$$

To first order in U:

$$\left.\begin{aligned} U_{charge} &= u_0/2 \\ U_{spin} &= -u_0/2 \end{aligned}\right. \Rightarrow U_t = 0$$

To second order in U

$$U_{charge} \approx \frac{u_0}{2} \ominus \beta U_0^2 N(0) \Rightarrow \text{decreases}$$

$$U_{spin} \approx -\frac{u_0}{2} - \beta U_0^2 N(0) \Rightarrow \text{increases}$$

In a simple ladder approximation.

$$U_{charge} = \frac{U_0}{2} \frac{1}{1 + 2\beta U_0 N(0)}$$

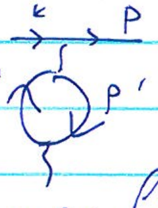
$$U_{spin} = -\frac{U_0}{2} \frac{1}{1 - 2\beta U_0 N(0)}$$

$$U_t = U_{charge} + U_{spin} = \ominus \frac{U_0}{2} \left\{ \frac{1}{1 - 2\beta U_0 N(0)} \ominus \frac{1}{1+2\beta U_0 N(0)} \right\}$$

$$\simeq U_{spin}.$$

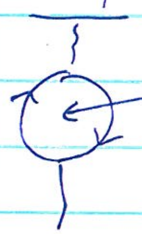
Conclusion: higher order terms (often re-phrased as spin-fluctuations) only enhance the attraction in p-wave channel (in isotropic systems), the attraction itself emerges already at second order, due to KL effect.

NB: for $l=1$, typical momenta in

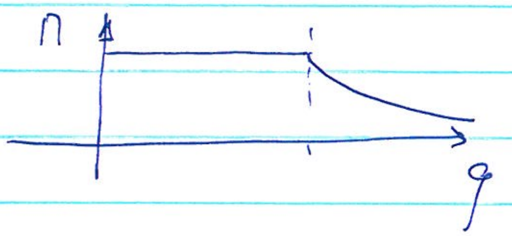


the bubble (k' & p') are of order p_F away from the FS.

A word about <u>2D</u>:



$\Pi(q)$, $q = k - p$

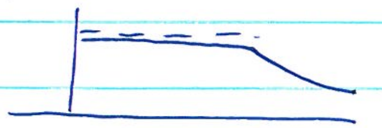


For $q < 2p_F$ (i.e when k and p are on the FS)

Then, $U_{\ell \neq 0} \equiv 0$

<u>What to do?</u>

a) go to next order (u^3)



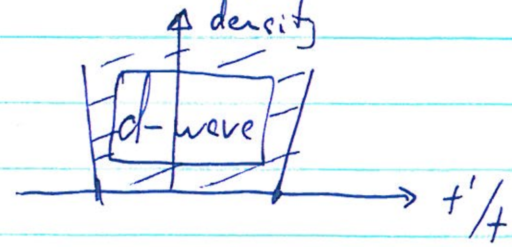
$\Pi(q) \equiv f(q)$

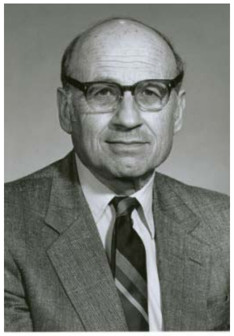
~~The~~ $U_\ell = - \alpha u^3/\ell^2$ (attraction)

p-wave in isotropic systems

b) include higher order terms in the dispersion and stay with order u^2

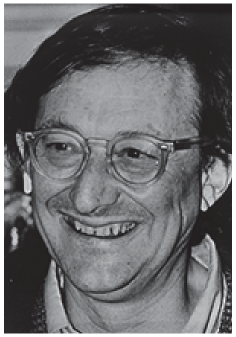
Then the result is; d-wave for most of parameters (t, t')





Walter
Kohn

Joaquin
Luttinger



For the rest of my lectures I will explore KL idea that the effective pairing interaction is different from a bare repulsive U due to screening by other fermions and may have attractive components in some channels

- cuprates
- doped graphene
- Fe-pnictides

Each case will represent different lattice version of KL physics

To continue