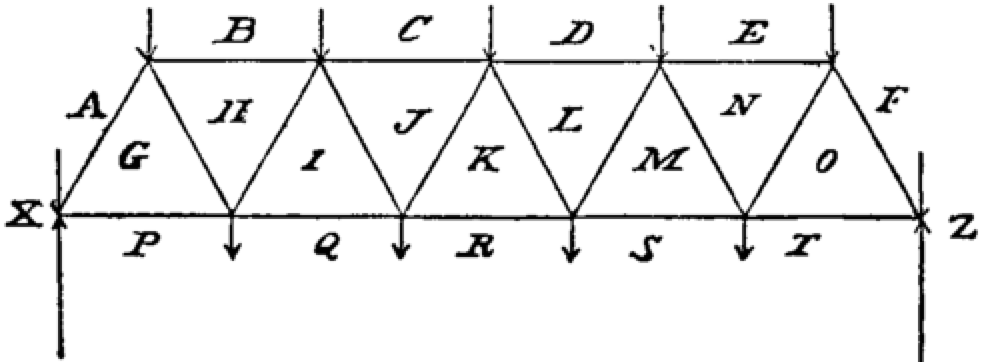
Boulder Figures

Bridges and Buildings

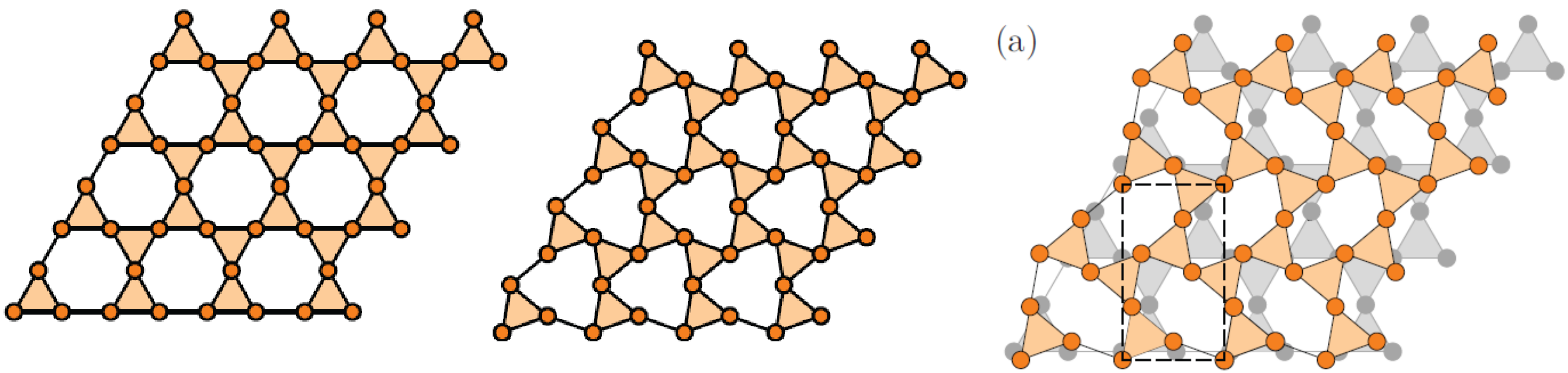
Warren Girder and Warren Truss bridge (1848)



Hancock Tower, Chicago

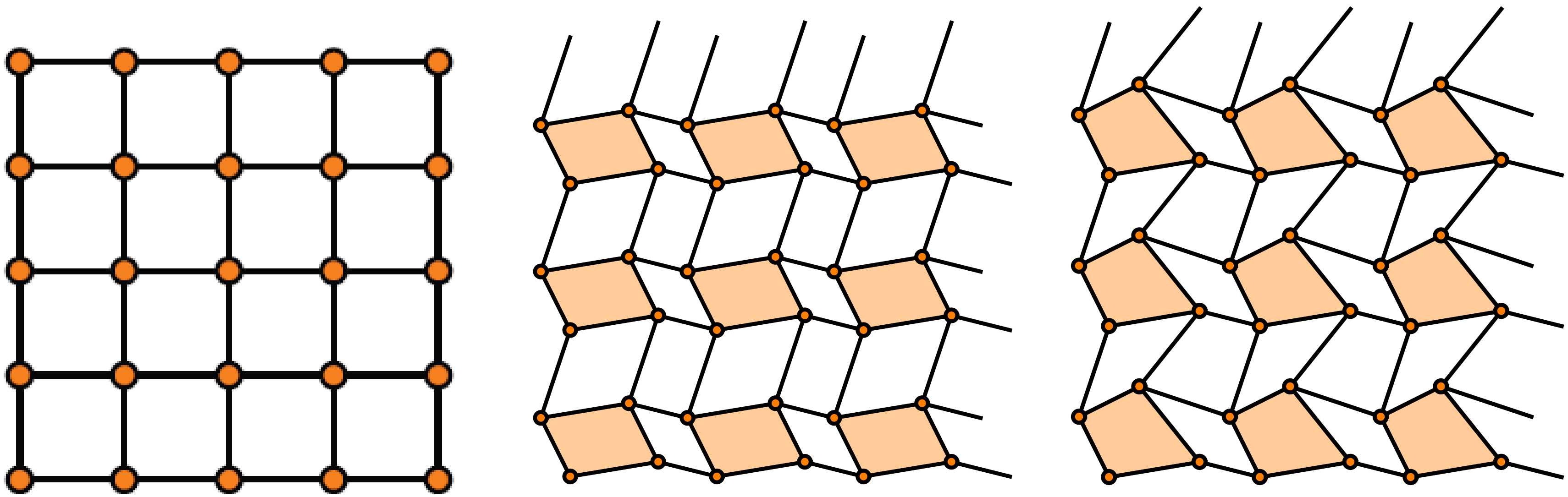
2 D Maxwell Lattices

Maxwell Lattice – one that under periodic boundary conditions have $z=2d$ exactly, i.e., $z=4$ in two-dimensions and $z=6$ in three dimension



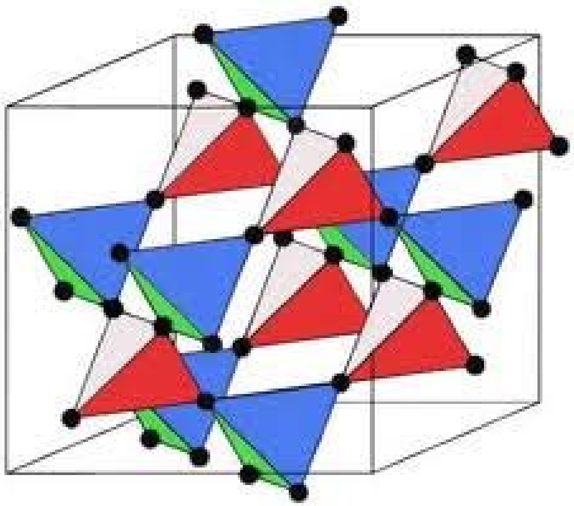
Kagome Lattices

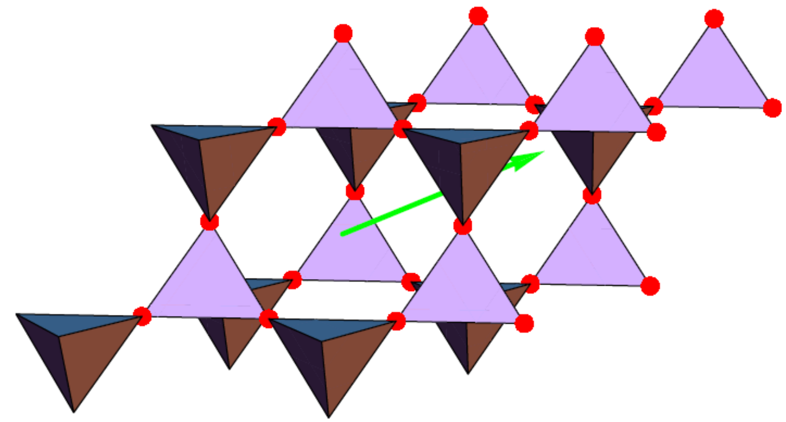
Square-Based 2D Maxwell Lattices

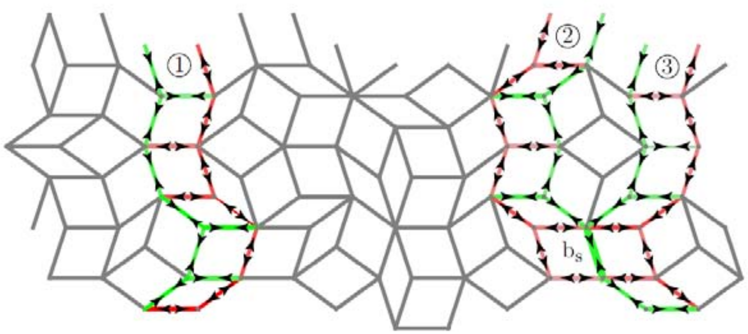


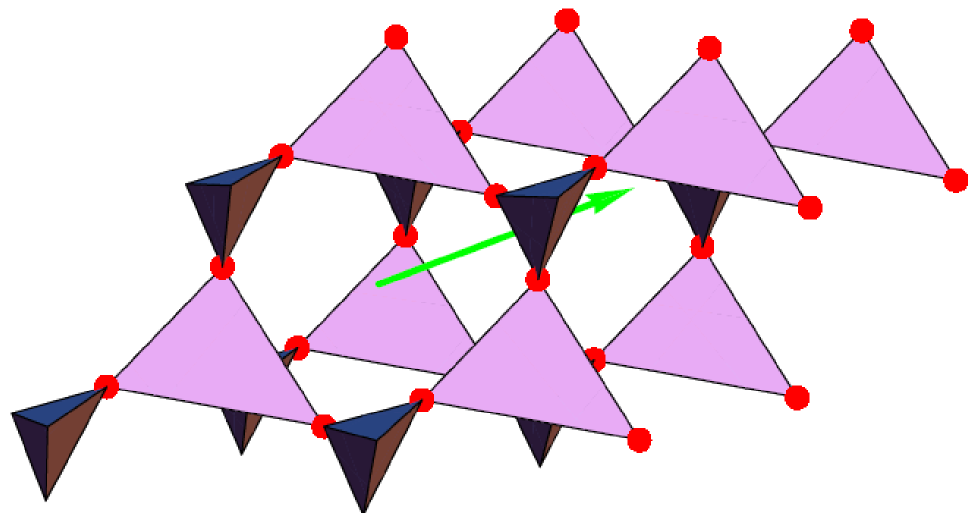
Zeb Rocklin, Bryan Chen, Martin Falk, TCL, and Vincenzo Vitelli – in preperation

3D Maxwell Lattice: Pyrochlore and Distorted Phyrochlore

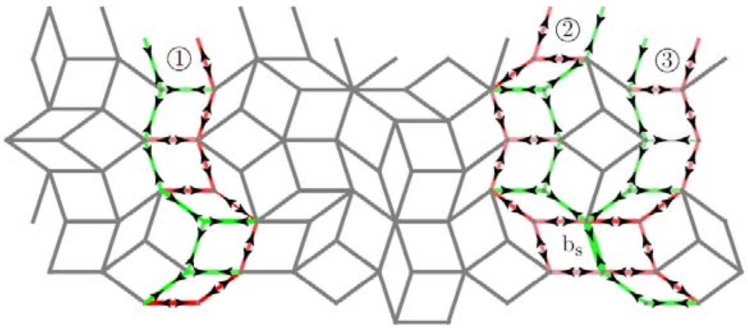


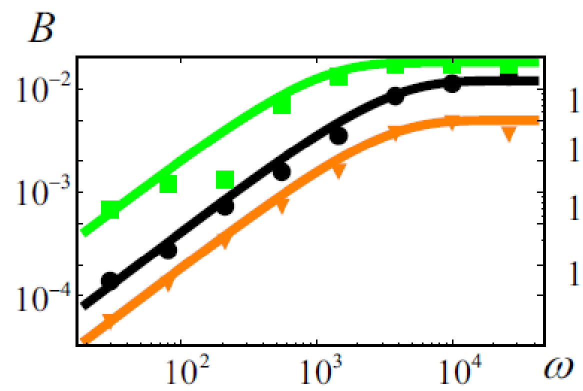


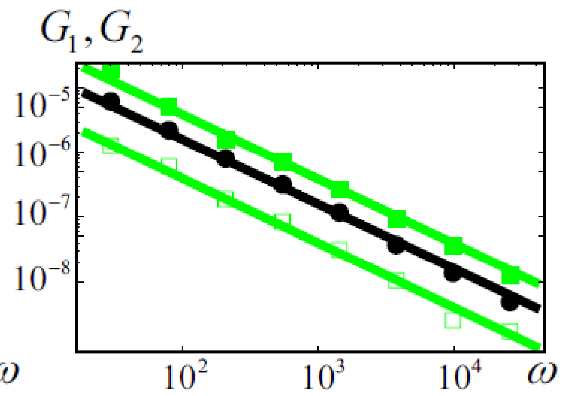


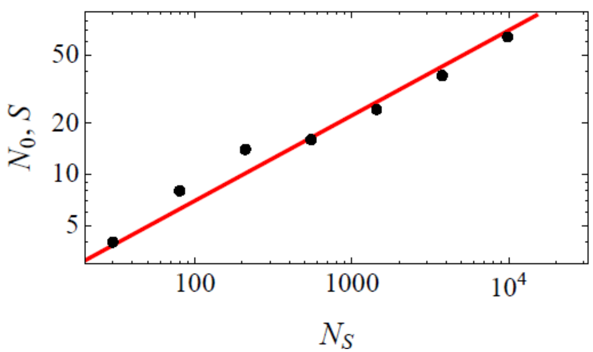


Penrose Tiles









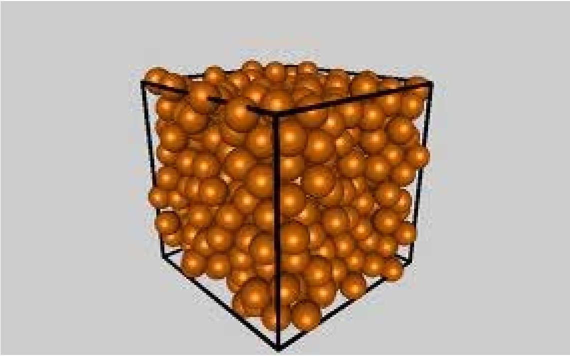
Unrandomized: N0 =S~ $N^{1/2}$ but SSS do not overlap with elasticity: all moduli are zero.

Stenull, TCL, PRL **113**, 158301 (2014).

Goodrich, Dagois-Bohy, Tighe, van Hecke, Liu, and Nagel, Phys. Rev. E **90**, 022138 (2014).

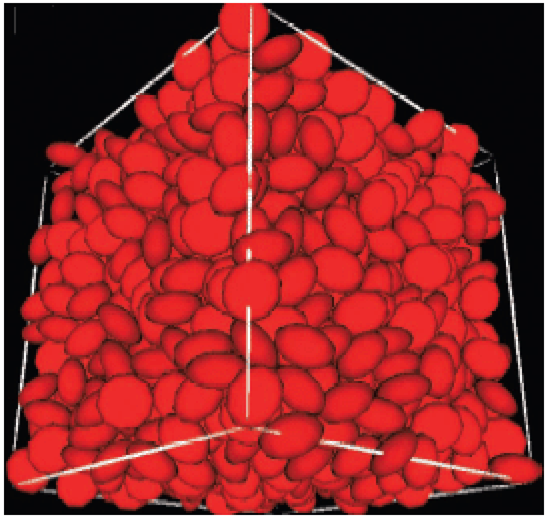
Randomize: N0 = 2 = S (Black Curves). Bulk modulus and one shear modulus nonzero. Shear Modulus vanishes with increasing N (not \omega). Required because large N limit is isostropic. If one shear modulus is zero, the other must be: result, only the bulk modulus survives. Remove one bond, S=N0-1=1: Only one positive elastic Const - B (Orange curve). Add one bond: S=N0+1=3: All elastic distortions stabilized for any finite N.

Random Close Packing



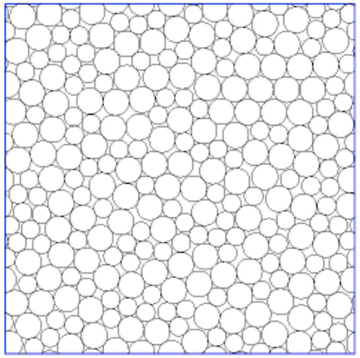
Cherrypit.princeton.edu

Chaikin

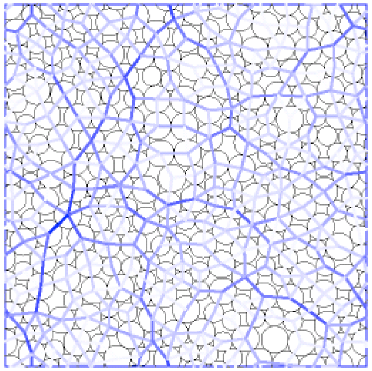


Torquato

non-overlapped
V=0
p=0



T_f=0



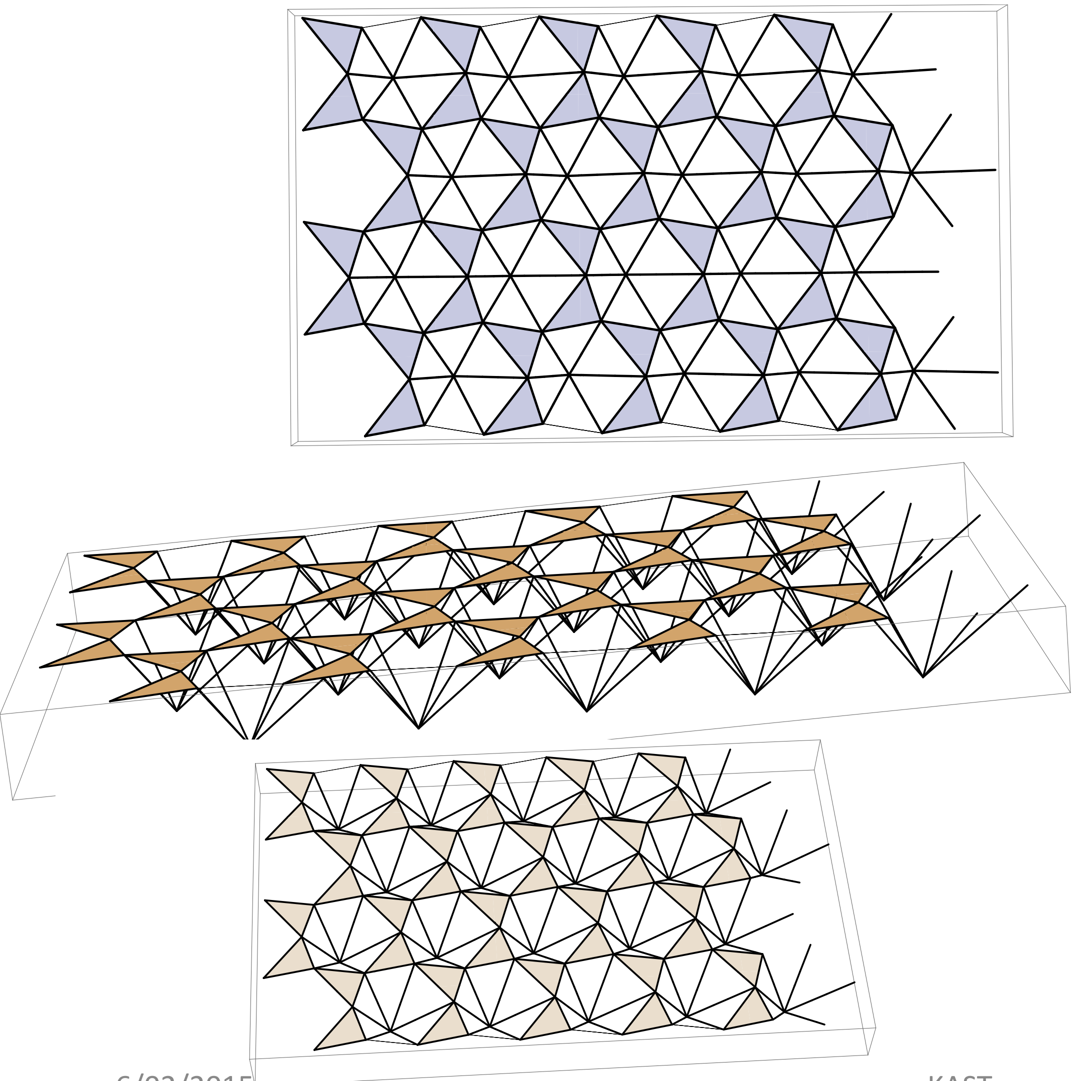
T_f=0

overlapped
V>0
p>0

O'hern, Langer
Liu, Nagel

2d to 3D: "Origami lattices"

d=3, z=6, but a planar structure



Ron Resch: Note curvature

Fig. P 8

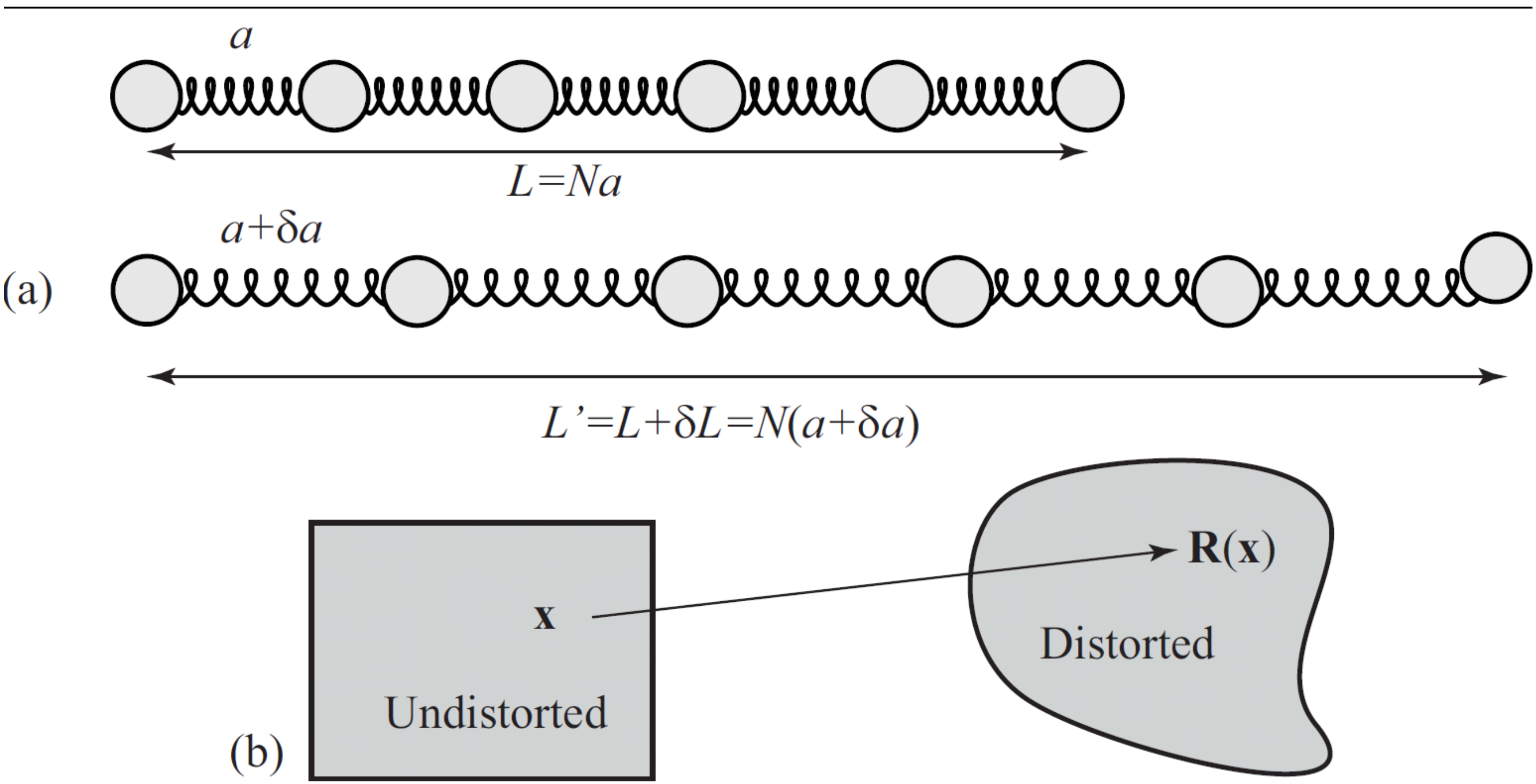
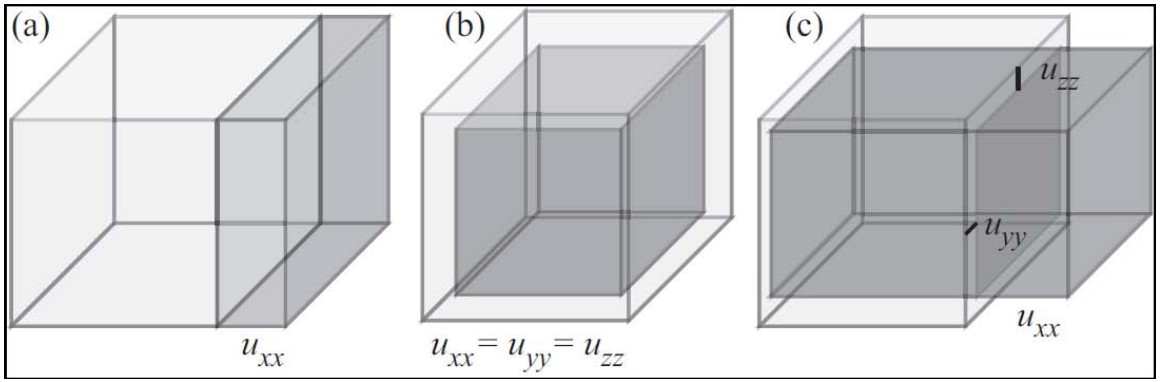
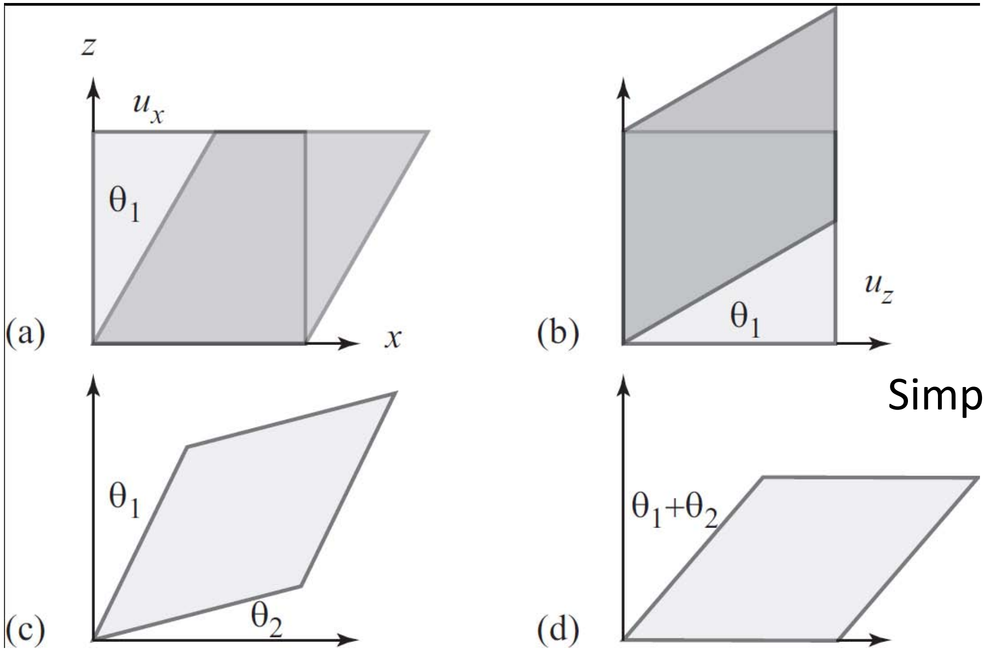


Fig. P9



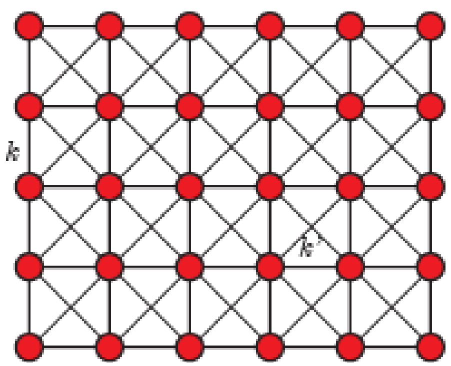
(b) Isotropic compression

(c) Pure Shear



Simple Shear

Square Lattice II (P.10)



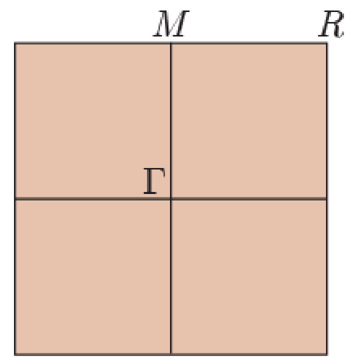
$$C_{11} = (k + k');$$

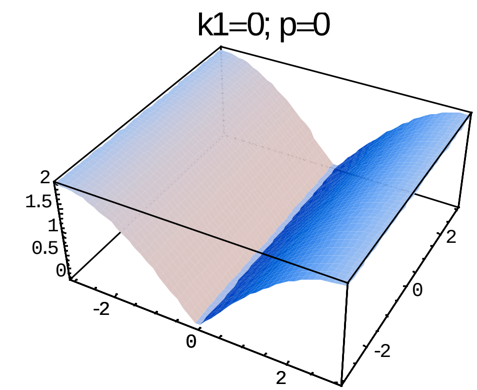
$$C_{44} = k'; \quad C_{12} = k'$$

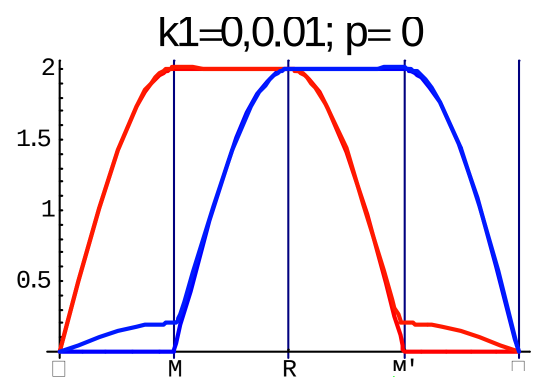
$$\omega_M^2\left(\frac{G_0}{2}, q_x\right) = 4k' + k q_x^2 a^2$$

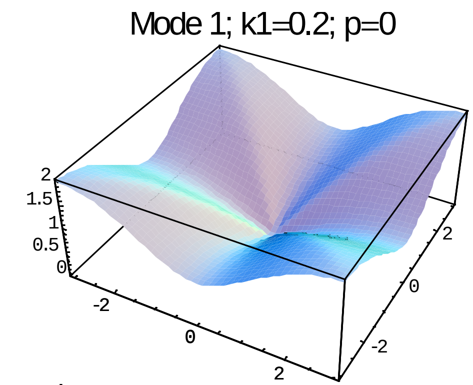
$$\xi = \ell^* \sim \sqrt{\frac{k}{k'}}$$

$$\omega^* = 2\sqrt{k'} = \sqrt{k}(a / \xi)$$

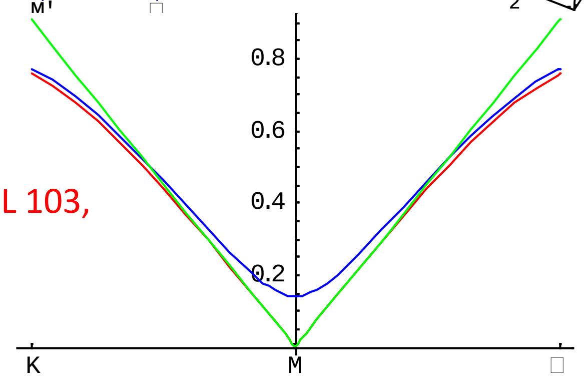






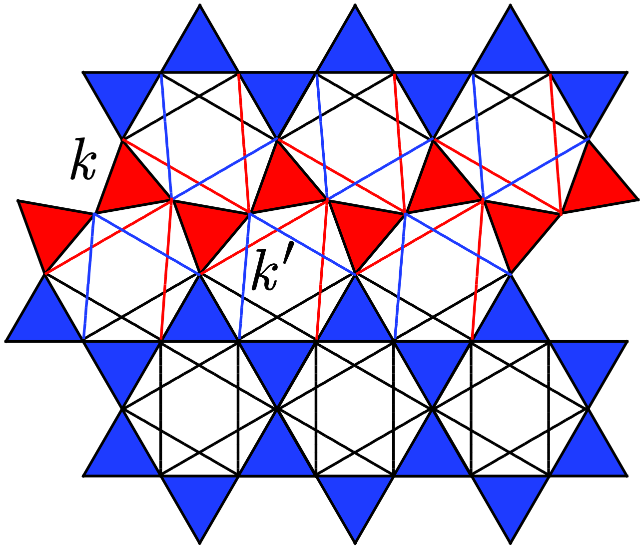


Souslov, Liu, TCL, PRL 103, 205503 (2009)



Kagome Lattice I (P.11)

z=4 with periodic boundary conditions



NN: spring constant k
NNN: spring constant k'

$k'=0$: $z=2d=4$: isostatic
No. of zero modes ~ perimeter:
As in square lattice, expect 1D modes

Uniform Elasticity: kagome supports both compression and shear, even though there are $N^{1/2}$ zero modes! And deformations are affine!

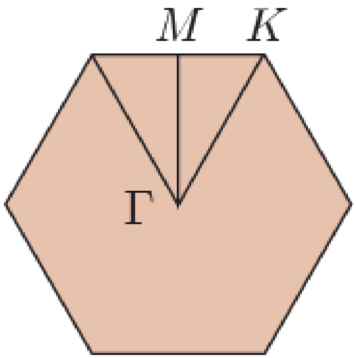
$$N_c = N_x N_y ; N_s = 3N_c$$
$$N_B = 6(N_x - 1)(N_y - 1) + 4(N_x - 1) + 4(N_y - 1) + 3$$
$$N_0 = 2N_s - N_b = 2(N_x + N_y) - 1$$

$$f_{\text{hex}} = \tfrac{1}{2}\lambda u_{ii}^2 + \mu u_{ij}^2$$

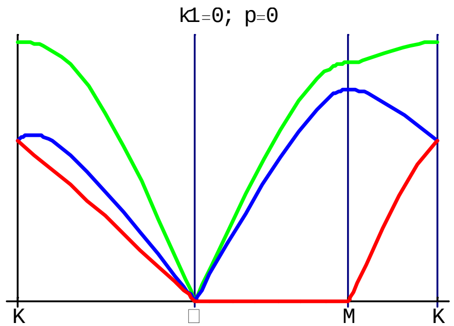
$$\lambda = \mu = \frac{3}{16}k$$

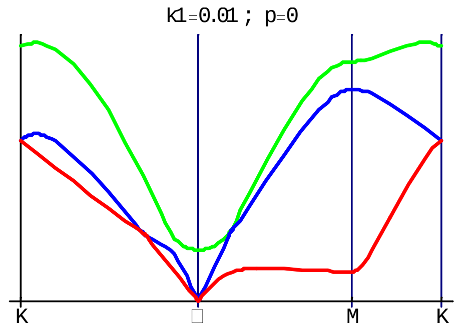
$$B = \lambda + \mu; \quad G = \mu$$

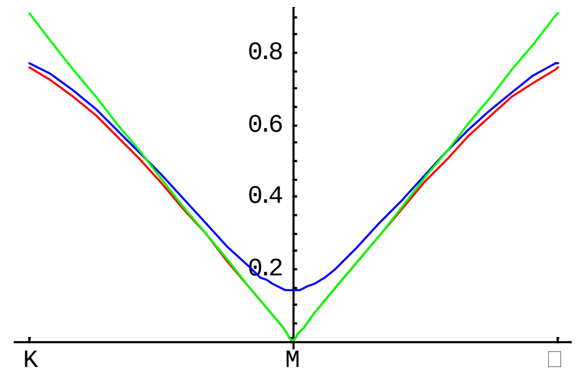
Souslov, Liu, TCL, PRL 103, 205503 (2009)

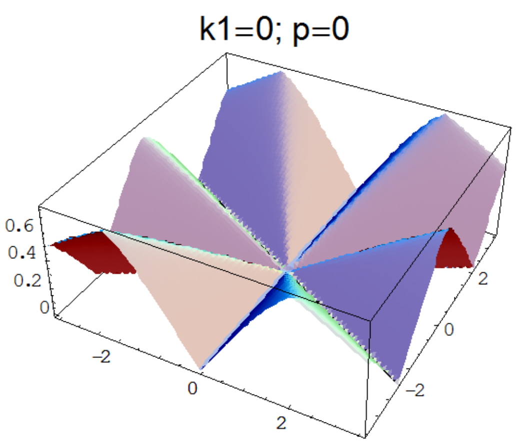


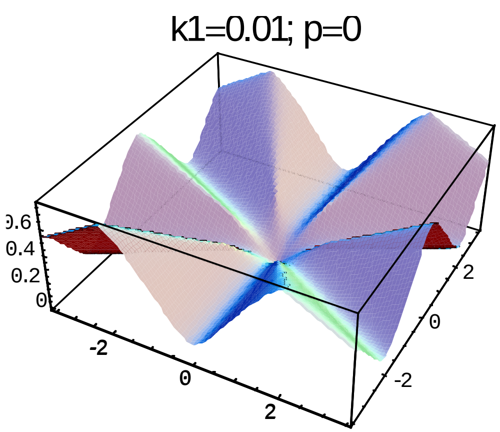
Kagome Lattice II (P.12)











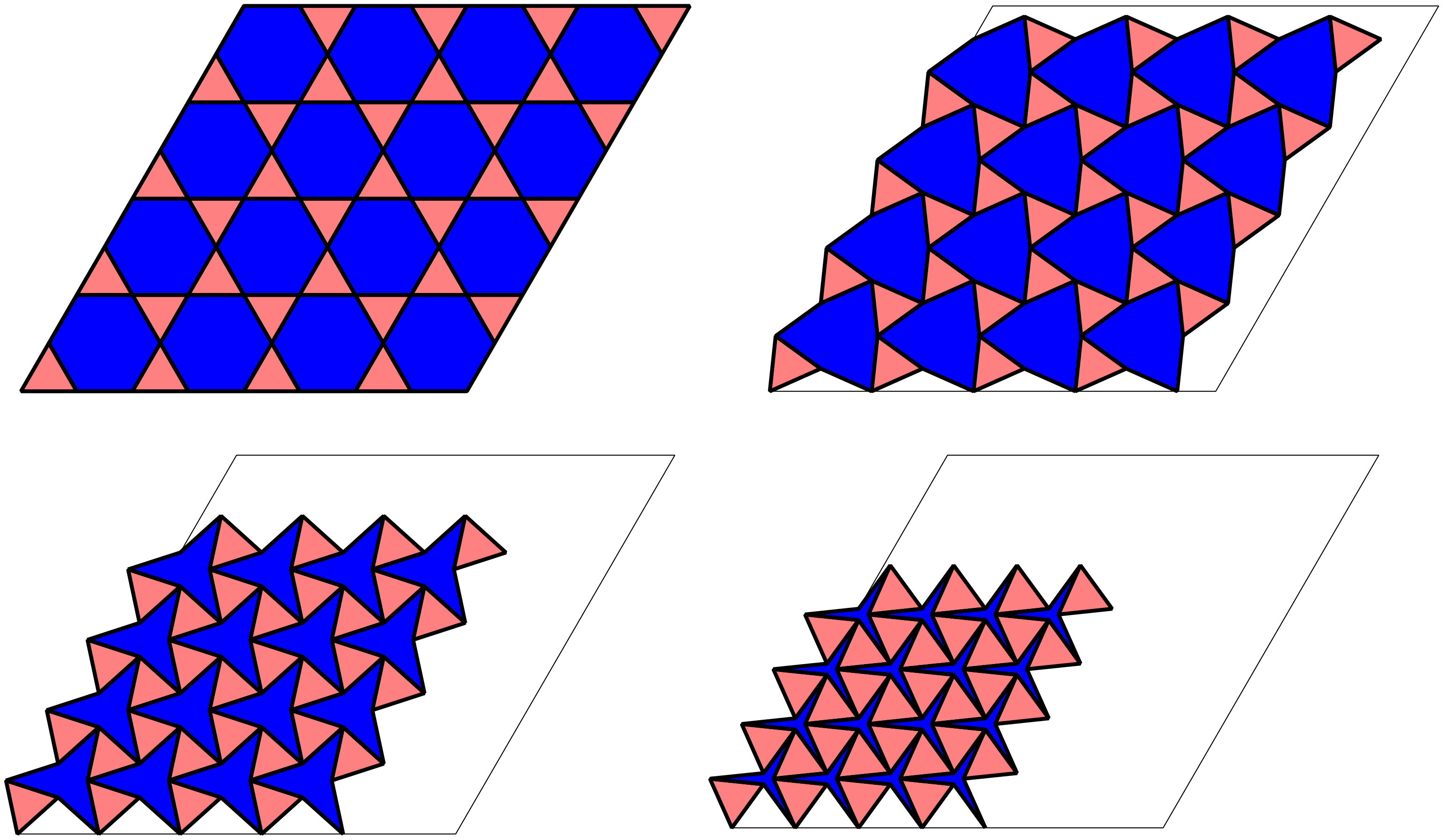
$$\omega_M^2(\frac{G_0}{2}, q_x) = 4k' + \frac{3}{16} k q_x^2$$

$$\omega_\Gamma^2(q) = 6k' + \frac{1}{16} k q^2$$

$$\xi = \ell^* \sim \sqrt{\frac{k}{k'}}; \quad \omega^* = 2\sqrt{k'}$$

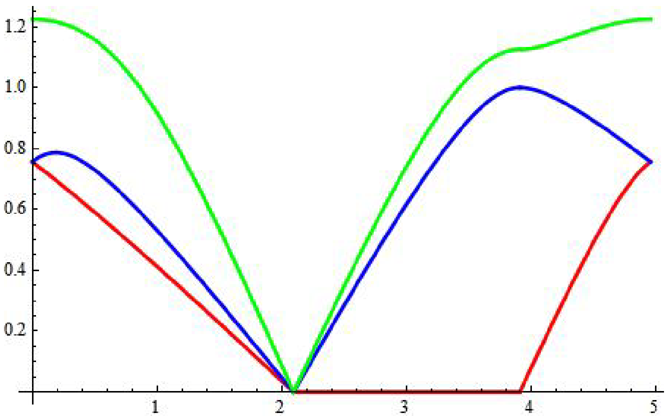
Souslov, Liu, TCL, PRL **103**, 205503 (2009)

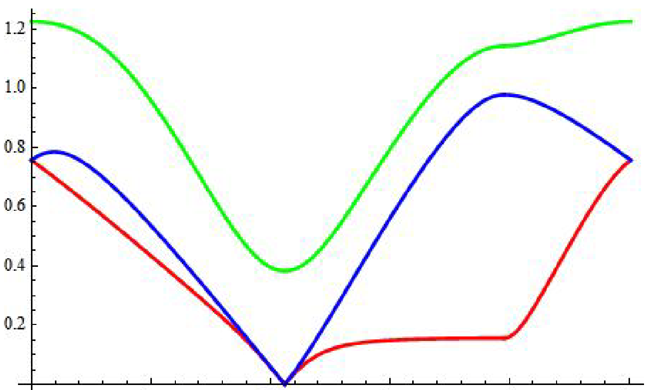
Twisted Kagome I (P13)

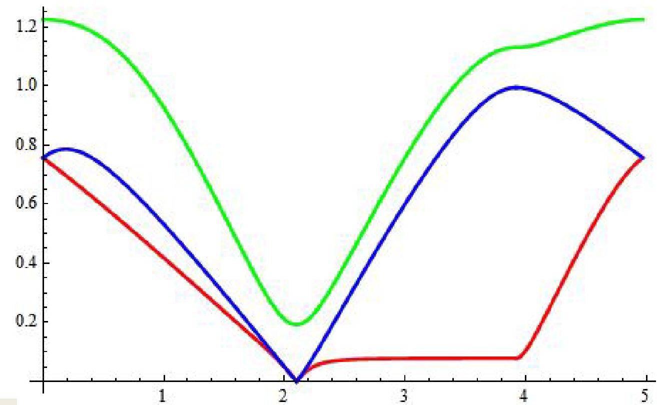


Grima, Alderson, Evans phys. stat. sol. **242**, 561–575 (2005)

Twisted Kagome Phonon Spectrum (P14)

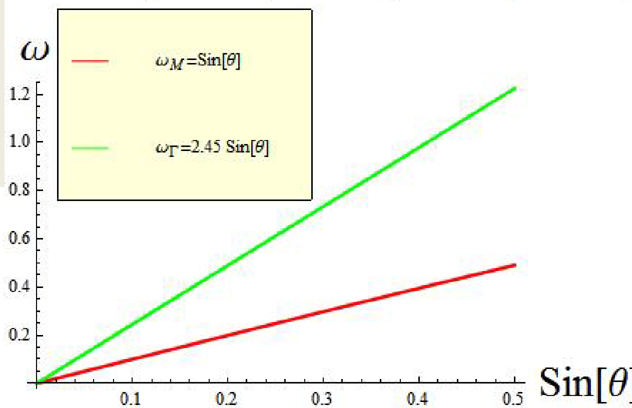






$$\omega_\theta \sim |\sin\theta|$$

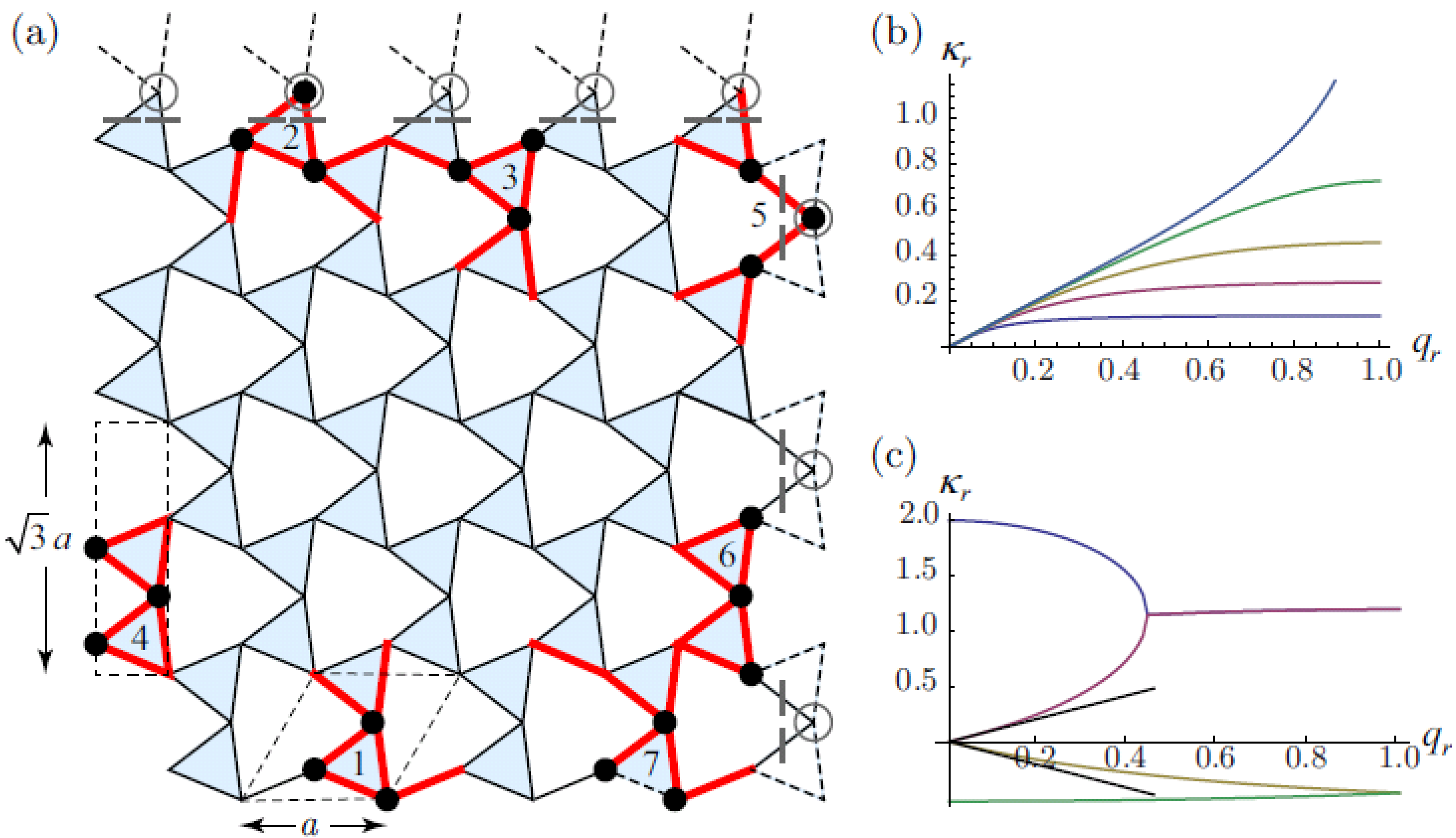
$$l_\theta \sim |\sin\theta|^{-1}$$



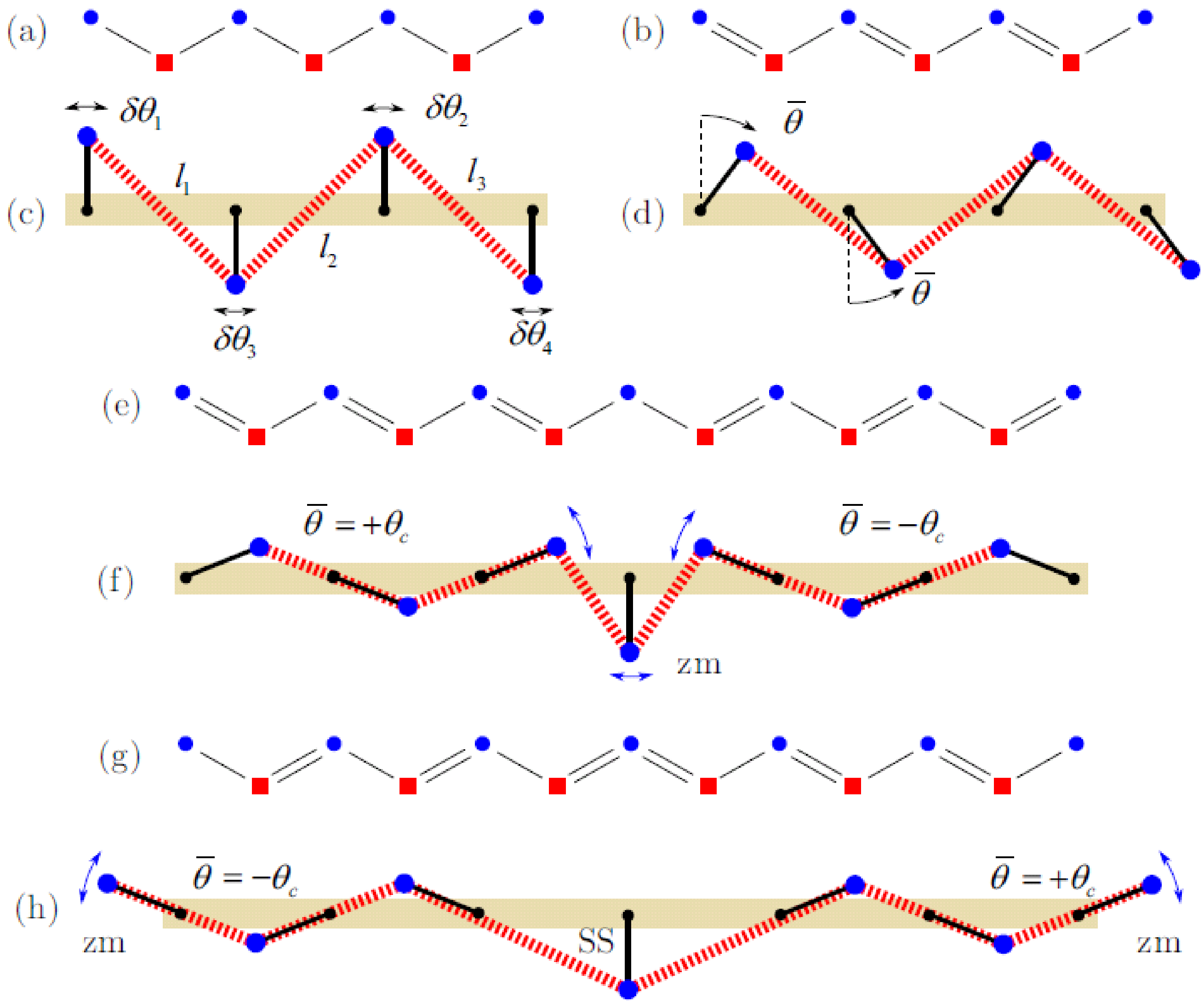
There are no zero modes in the periodic spectrum even though Maxwell rule for free BC says there should be ~$N^{1/2}$.

$\sin^2[\theta]$ acts like k' of NNN spring!

P15

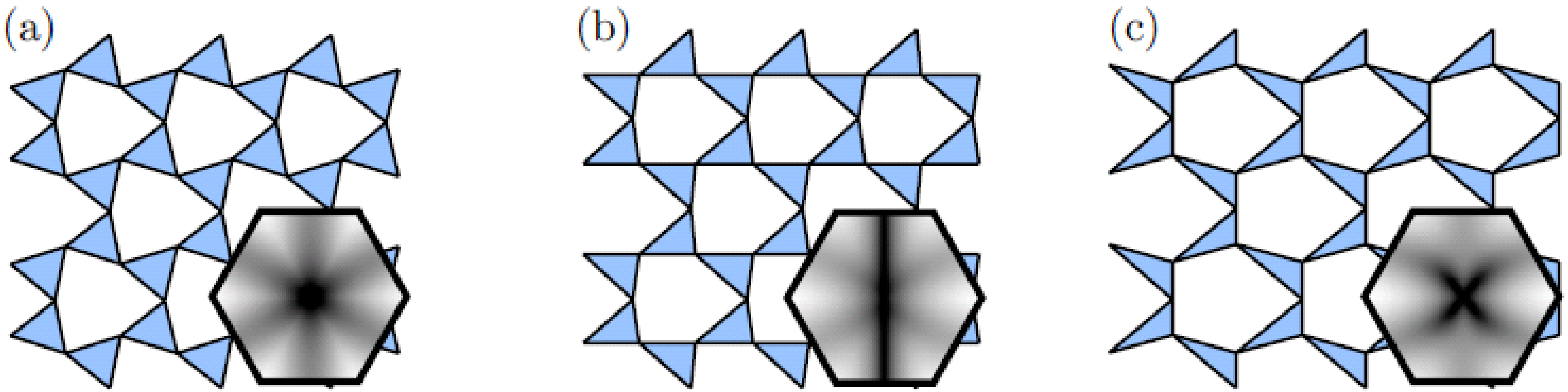


P16

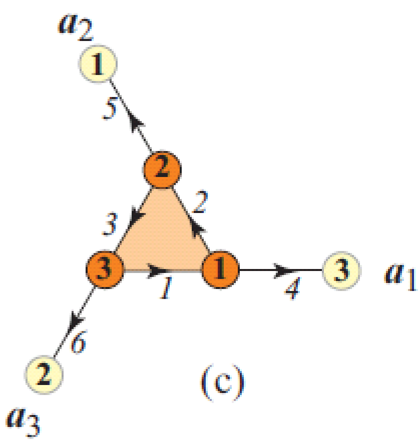


(a)
(b)
$\delta\theta_1$
$\delta\theta_2$
l_1
l_3
(c)
l_2
$\delta\theta_3$
$\delta\theta_4$
$\bar{\theta}$
(d)
$\bar{\theta}$
(e)
$\bar{\theta} = +\theta_c$
$\bar{\theta} = -\theta_c$
(f)
zm
(g)
$\bar{\theta} = -\theta_c$
$\bar{\theta} = +\theta_c$
(h)
zm
SS
zm

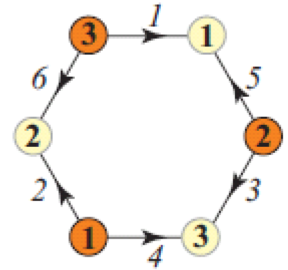
P17



P18

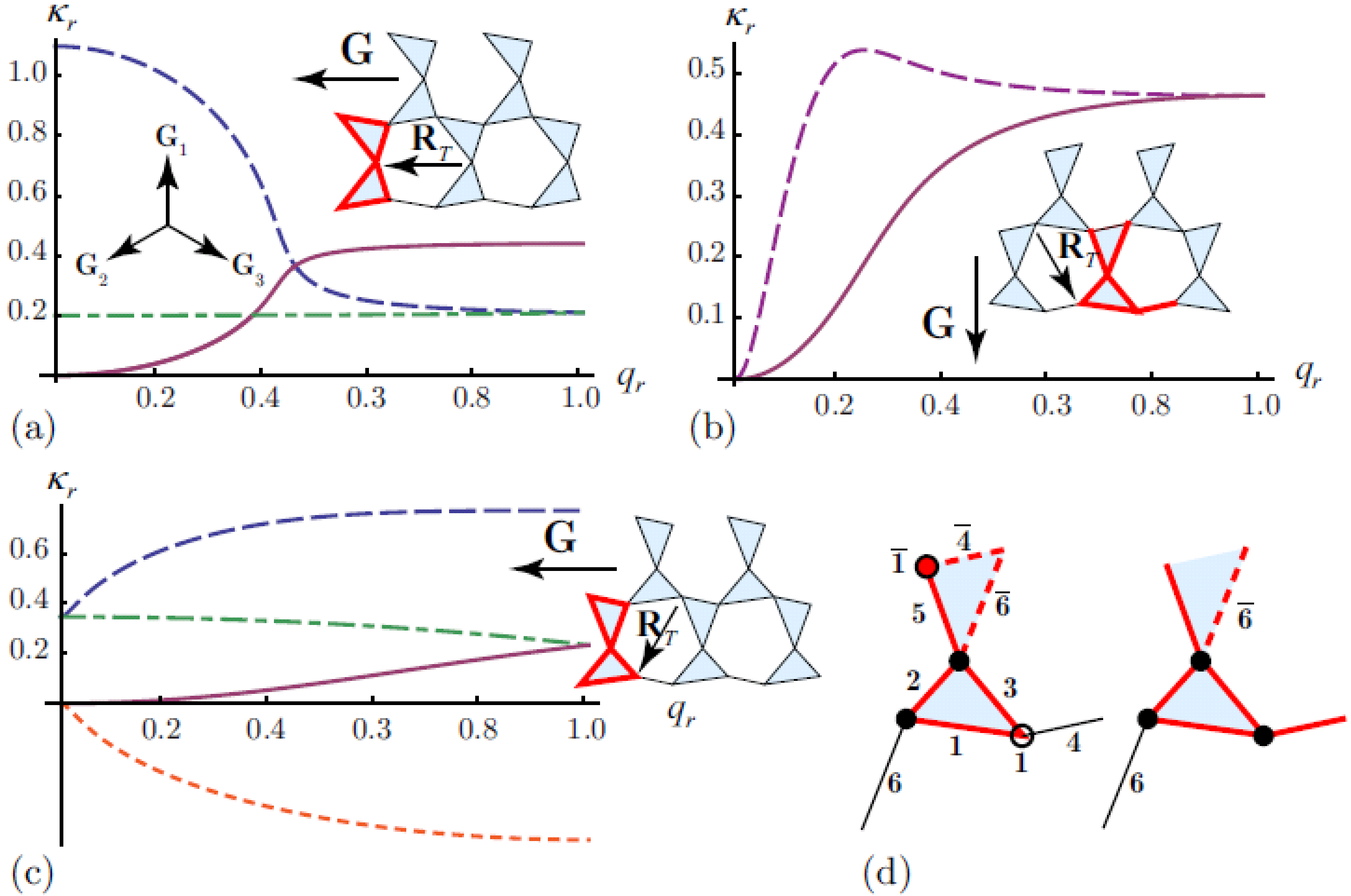


(c)



(d)

P19

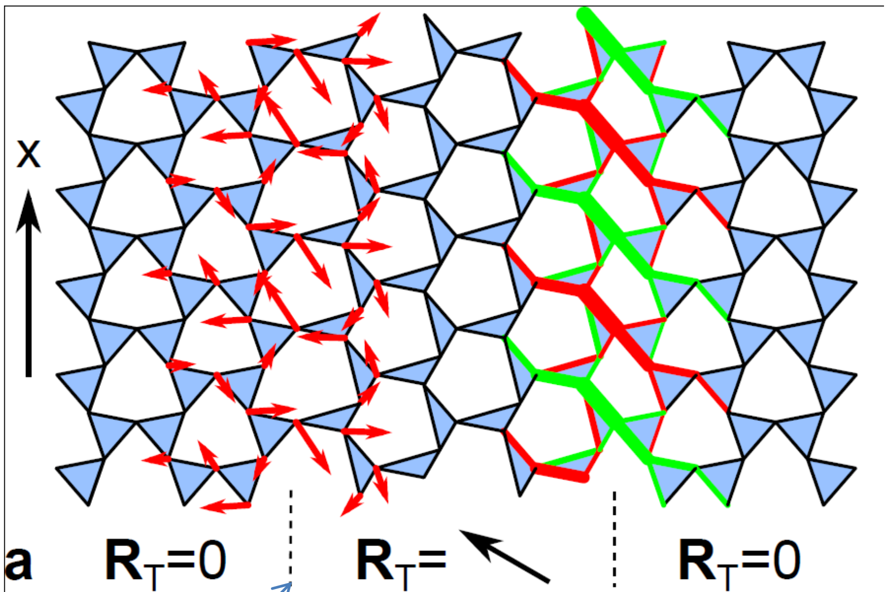


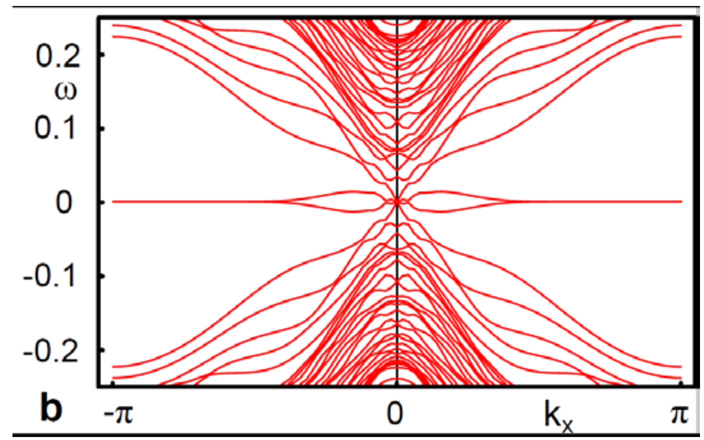
κ_r
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.3
0.8
1.0
q_r
G
$\mathbf{R}_T$
$\mathbf{G}_1$
$\mathbf{G}_2$
$\mathbf{G}_3$
(a)
κ_r
0.5
0.4
0.3
0.2
0.1
0.2
0.4
0.3
0.8
1.0
q_r
$\mathbf{R}_T$
G
(b)
κ_r
0.6
0.4
0.2
G
$\mathbf{R}_T$
0.2
0.4
0.3
0.8
1.0
q_r
(c)
$\bar{1}$
$\bar{4}$
$\bar{6}$
5
2
3
1
1
4
6
$\bar{6}$
6
(d)

Domain Wall (P20)

$$\mathbf{G} = -\mathbf{b}_1 \qquad \mathbf{G} = -\mathbf{b}_1$$

Kane and TCL, Nature Physics 10, 39 (2014)





Modes of full H with zero modes from *Q* (States of self stress) and form Q^T (zero modes)

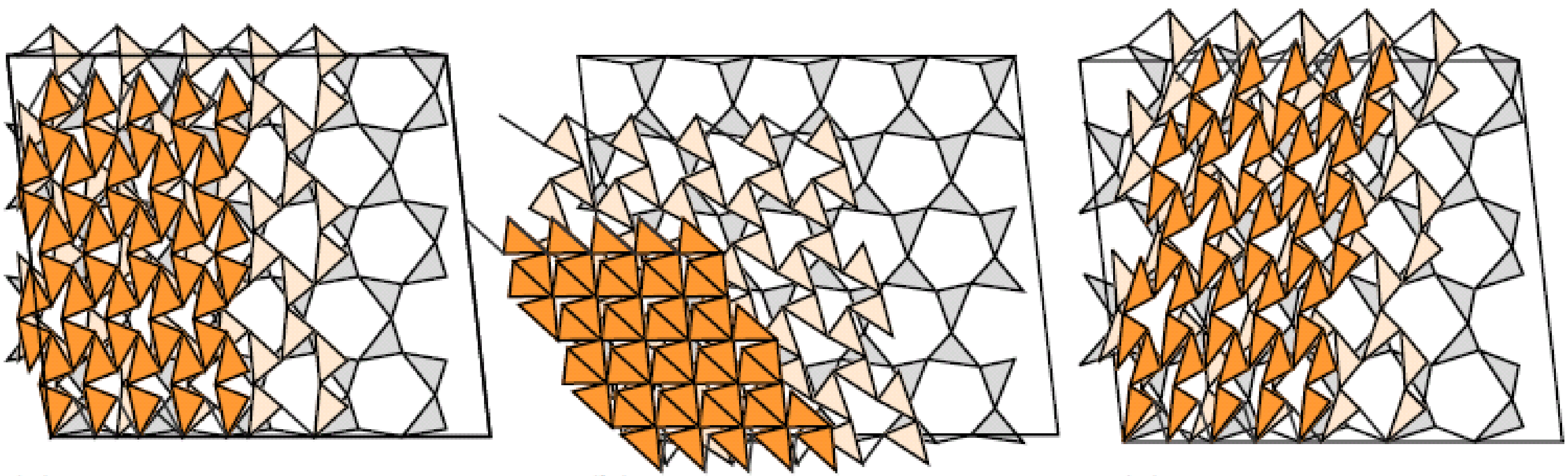
$$\nu_T = \frac{\mathbf{G}\cdot(\mathbf{R}_T^l - \mathbf{R}_T^r)}{2\pi} = 1$$

zero modes

$$\nu_T = \frac{\mathbf{G}\cdot(\mathbf{R}_T^l - \mathbf{R}_T^r)}{2\pi} = -1$$

States of Self Stres

P21



(a) (b) (c)