Statics beyond R_c (interfaces)

1) Characterized by (G.S $T=0$) (also phase low T) (effect of T small) 2 exponents ζ and θ $u \sim x^{\zeta}$

roughness — free-energy fluct exponent

$$\overline{(E_0 - \overline{E}_0)^2} \sim L^{2\theta}$$

sample to sample

L ↓ syst size

↓

T irrelevant $F = -T \ln Z$

$T(\ell) \sim T/\ell^{\theta}$

glass phase fluct $\ell < \ell^*$ $T \sim \ell^{\theta}$

ℓ_θ

$\delta E \sim \ell^{\theta}$

2^{nd} minima $\neq$ by ℓ in same sample

2) these 2 exponents are not independent

localized around GS ($T>0$)

$\updownarrow \ell^{\theta}$

(rare fluct droplet)

th fluct (see 2 level syst arg Sid Nagel)

$$\theta = d - 2 + 2\zeta$$

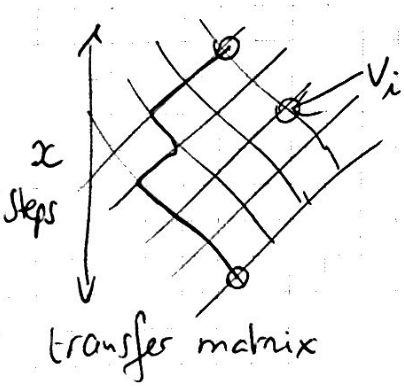
⇖ Statistical Tilt Symmetry (STS)

elastic energy not corrected, c not renormalized $C(L) \equiv C$

$$H_{el} \sim L^{d-2+2\zeta} \sim \delta E$$

$\int d^dx (\nabla u)^2$ ||| L^{θ}

3) $d=1$ directed polymer



V_i

x steps

transfer matrix

$$E_0 = \min_{\gamma} \left(\sum_{i \in \gamma} V_i \right)$$

→ Johansson 2000

$P(V) = e^{-V}$

four models + T. Thiery + PLD 2015 (inverse Beta DP)

$\theta = 1/3$

$$E_0 \simeq \underset{\text{determinist}}{e}\, x + x^{\theta} \underset{\text{random variable}}{\chi}$$

universal ~~dist~~ PDF

Tracy Widom PDF of largest eigenvalue of random matrices

$\theta = 1/3$ $\zeta = 2/3$

proved proved

⟷ KPZ eq in 1d

4) Flory type estimate

elastic energy cost $\sim L^{d-2} u^2$

disorder gain ?

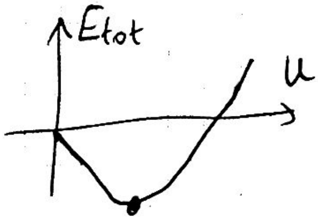
deformation u over scale L

Larkin $\int d^dx\, f(x)\, u \sim - L^{d/2} u$

balance

$u \sim L^{\frac{4-d}{2}}$

too large



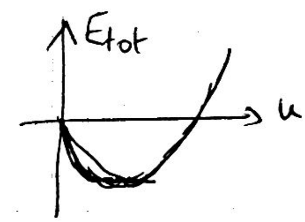
Random field

$$\delta H_{dis} = \int d^dx \left(V(x,u) - V(x,0) \right)$$

$$\overline{\delta H_{dis}^2} \sim L^d \left(R(0) - R(u) \right) \sim L^d |u|$$

balance $E_{tot} = L^{d-2} u^2 - L^{d/2} |u|^{1/2}$

$u \sim L^{\frac{4-d}{3}}$



$$\zeta_F^{RF} = \frac{4-d}{3}$$

general LR $\quad L^{d/2} u^{-\mu/2}$

$$\zeta_F^{LR} = \frac{4-d}{4+\mu} \qquad \mu = -1 \text{ RF}$$

$\rightsquigarrow$ SR $\mu \not\to -1$ $\quad \delta(u) \sim \frac{1}{u}$ $\quad \zeta_F^{SR} = \frac{4-d}{5}$ $\quad$ DP 0.6 exact $2/3$

5) Functional RG (interface / non-periodic / equilibrium static)

(expansion in $\epsilon = 4-d$)

$1 \Rightarrow$ $\zeta_{eq}^{RF} = \epsilon/3$ presumably to all orders

idem $\zeta_{eq}^{LR} = \zeta_F^{LR}$ up $\mu < \mu_c < -1$

$T=0$

$R_0(u) \to R_\ell(u)$

$R^*(u)$

$\sim O(\epsilon)$

\#loop = order in ϵ

$\rightsquigarrow$ non-trivial amplitudes in ϵ

$\rightsquigarrow$ $\zeta_{eq}^{RF} = 1$ $\quad d=1$ $\quad D_{\ell c}^{eq} = 2 = 1+1$

$2 \Rightarrow \zeta_{SR} = 0.20829\,\epsilon + 0.00686\,\epsilon^2 + \dots$

	prediction	numerics/exact	improved pred.
$d=3$	0.215 ± 0.004	0.22 ± 0.01	0.214
$d=2$	0.444 ± 0.015	0.41 ± 0.01	0.438
$d=1$	0.687 ± 0.03	$2/3$	$2/3$

1 loop Fisher 86
2 loop Chauve, PLD, Wiese
3 2000-2004

Contact line if it was equilibrium

$\zeta = \epsilon'/3 \qquad \epsilon' = 2-d \qquad d_{uc} = 2$

$= 1/3$

STS

no disorder

how measure elastic coeff?

add field $\uparrow$ $\Rightarrow$ wants to tilt $\downarrow$

$$E(\varphi) := \int d^d x \; c \left(\tfrac{1}{2} (\nabla u)^2 - \varphi \cdot \nabla u \right) \qquad \langle \nabla u \rangle = \varphi$$

$$= \int d^d x \; c \, \tfrac{1}{2} (\nabla u - \varphi)^2 - \tfrac{c}{2} \varphi^2 L^d$$

$$u \to u + \varphi$$

$$E(\varphi) = E(0) - \tfrac{c}{2} \varphi^2 L^d \qquad c \sim \left. \frac{\partial^2 E}{\partial \varphi^2} \right|_0$$

+ disorder $\quad u \to u + \varphi x$

$$V(u, x) \longrightarrow V(u + \varphi x, x) =: W(u, x)$$

$W \equiv V$ in law

$$\overline{W(u,x)\, W(u',x')} = \overline{V(u+\varphi x, x)\, V(u'+\varphi x', x')}$$

$$= \delta^d(x-x')\, R(u - u' + \cancel{\varphi(x-x')})$$

$$\overline{E(\varphi)} = \overline{E(0)} - \tfrac{c}{2} \varphi^2 L^d$$

c unchanged by disorder.

more general

many more relations

(all thermal (connected) correlations are the same as without disorder)

replica $u_a(x) \to u_a(x) + \varphi(x)$

dynamical FT

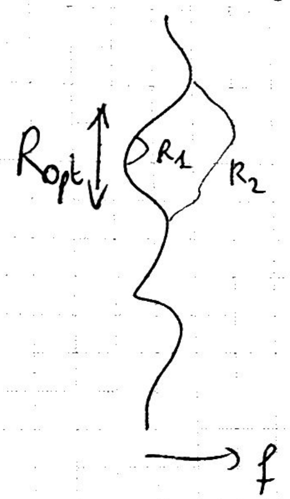
Creep $0 < f \ll f_c \quad T > 0 \qquad v\,? \quad \tau\,?$

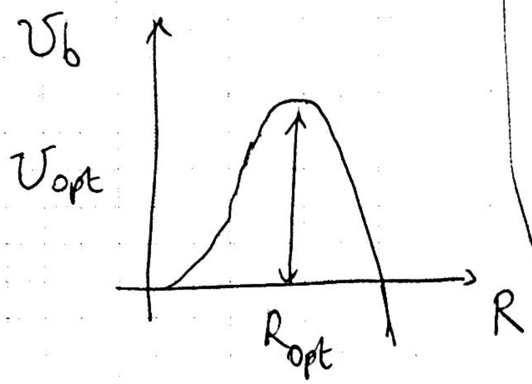
assume
- ~~dynamics~~ limited typical nucleation event
- near equilibrium, activated dyn optimal barrier

$$U_b \sim R^{\psi} - f\, u(R)\, R^d \qquad u(R) \to R^{\zeta}$$

$$R_{opt}^{d+\zeta-\psi} \sim \frac{1}{f}$$

$$R_{opt} \sim f^{-\frac{1}{(d+\zeta-\psi)}} \sim f^{-\frac{1}{2-\zeta}}$$





$$U_b \sim f^{-\mu} \qquad \mu = \frac{\psi}{d+\zeta-\psi}$$

$$\begin{cases} \psi = \theta = d - 2 + 2\zeta_{eq} \\ \zeta = \zeta_{eq} \end{cases}$$

$$\boxed{\mu = \frac{\theta}{2-\zeta_{eq}}}$$ CREEP exponent

$d = 1 \quad \mu = \frac{1/3}{2 - 2/3} = \frac{1/3}{4/3} \qquad \mu = 1/4$

DW creep OK

$$U_b(R) = U_c \left(\frac{R}{R_c}\right)^{\theta} - f\, r_f \left(\frac{R}{R_c}\right)^{\zeta} R^d$$

$$U_{opt} = U_c \left(\frac{f_c}{f}\right)^{\mu}$$

$\longrightarrow$ what happens after barrier crossing ?

Kolton, Krauth, Rosso

$\rightsquigarrow$ depinning

$$\frac{1}{v} = \frac{1}{T}\int_0^{+\infty} dz\, e^{-fz/T} \left\langle e^{(V(u+z)-V(u))/T} \right\rangle_u$$

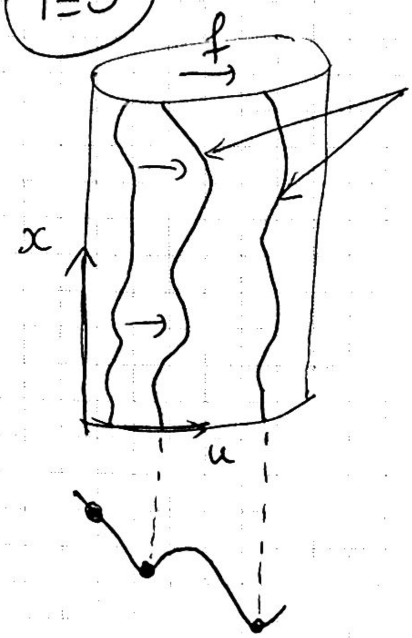
$$\langle A(u) \rangle_u = \lim_{L\to\infty} L^{-1} \int_0^L du\, A(u)$$

V. Vinokur, PLD Physica C (1995)

cond-mat/9501131

Depinning (T=0)

$f < f_c$



metastable state = local minima
zero force

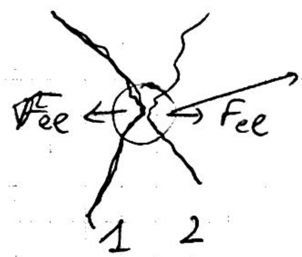
$$c\nabla^2 u(x) + F(x,u(x)) + f = 0 \quad \forall x$$

blocking configurations

any initial condition
to the left ends up there

Can almost be thought as particles

Middleton no_crossing theor.
if 1 starts left of 2
same dyn
1 can never cross 2



same F
f

$\nabla^2 u_1 < \nabla^2 u_2$

$F_{ee}^1 < F_{ee}^2$

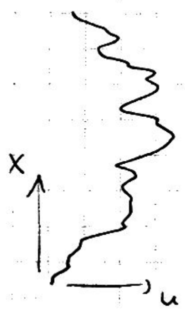
Rosso
Krauth
2002
found way
to enumerate
B.C
last B.C

1 turn of cylinder

$f \uparrow$ bloch. conf disappear

$f = f_c^-$ last B.C disappears

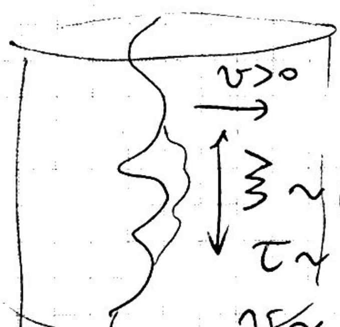
$f = f_c^{F,cyl}$



$u \sim x^{\zeta_{dep}}$

roughness of last B. conf

$f > f_c$



diverging
length scale
time scale
$f \to f_c^+$

$\xi \sim (f-f_c)^{-\nu}$

$\tau \sim \xi^z$

$v \sim (f-f_c)^\beta$

Middleton
guarantees unique steady state

ξ correl. length of $\dot{u}(x,t)$

τ timescale

(β sometimes called θ)

$$v \sim u/\tau \sim \xi^{\zeta - z} \sim (f-f_c)^\beta$$

$$\boxed{\beta = \nu(z - \zeta)}$$

3 exp.

STS $\quad c\nabla^2 u \sim f - f_c$

$$\boxed{\nu = \frac{1}{2-\zeta}} \Rightarrow \text{2 indep exp } \zeta, z$$

Expected renormalizations

Correct $\equiv$ due to disorder $\sim \Delta$

$$\eta \partial_t u(x,t) = c \nabla^2 u(x,t) + F(x, u(x,t)) + f$$

η: Corrections to friction $\eta \sim L^{z-2}$

c: uncorrected

F: Correct. to pinning force correlator; flow of $\Delta(u)$ yield ζ

f: Corrections produce $-f_c$

Mean-field

$\zeta = 0$

$\nu = 1/2$

$z = 2$

$\beta = 1$

$d \geq 4$

$d = 4$ logs

Fisher 85

self consist model of a particle

result depends on smoothness of potential seen by particle $\Leftrightarrow$ arbitrariness

not first principle

FRG $\equiv$ first principle

Contains MFT

Functional RG

$\epsilon = d_{uc} - d \quad d_{uc} = 2\alpha$

unless said otherwise $\alpha = 2 \quad d_{uc} = 4$

story (statics)

- Larkin $R_0(u) = w u^2 \to \zeta_L = \frac{4-d}{2}$

invariance of H_{el}/T
$T \to T L^{-\theta}$

power counting $R^{(2p)}(0)\, u^{2p} \sim L^{4-d+(2p-4)\zeta}$

function relevant $d<4$

Same in dynamics $\Delta_0(u) = -R_0''(u)$

$\overline{F(x,u) F(x',u')} = \Delta_0(u-u')\,\delta^d(x-x')$

Fisher +
Natterman +
flow of function

- what does that mean doing RG

define renormalized of effective action
parameter η, Δ correlator

1) Wilson RG $\equiv$ integrate over $e^{-\ell}\Lambda_\ell < k < \Lambda$

$\frac{H}{T} \equiv S \to S_{eff}$ or dynamics $\longrightarrow$ effective action (1 loop) $\to$ rescaling fixed point (UV div at d_{uc})

2) $d < d_{uc}$ + at critical point no UV div (continuum limit exist)
but IR divergences L

$\frac{\partial}{\partial \ln L} \underset{\Gamma}{\underset{|||}{S_{eff}}} = \beta(\Gamma)$

η_L, Δ_L
$\Downarrow$
FP

- even more physical $\to$ here $\Delta(u)$ measurable — these are the physical ones!
- higher loop
- exact RG $\hookrightarrow$ anomalies
ambiguities..
non-analytic $\overline{\text{cusp}}$
effect. action

put ~~elastic~~ interface in quadratic well

$$H_V[u] + \int \frac{m^2}{2} (u(x) - w)^2 \, d^d x$$

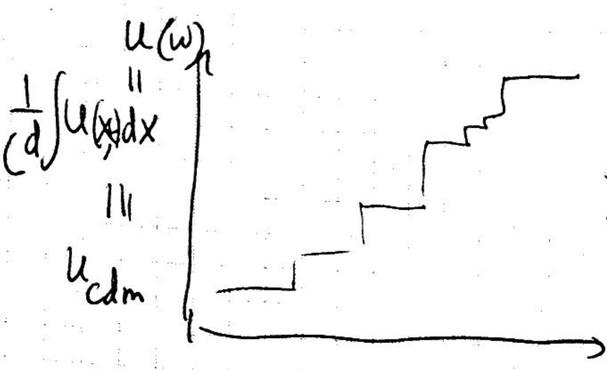
statics

$W(t) = vt$

$$\eta \partial_t u = \nabla_x^2 u + F(x,u) + \underbrace{m^2(w - u)}_{\downarrow\, f}$$

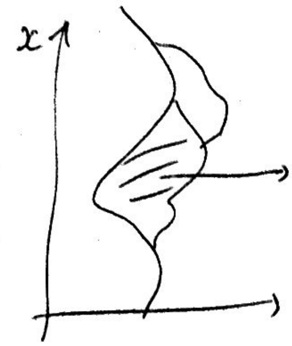
$u \equiv u(x,t)$

$f_c(m) = m^2 \overline{(w - u_{cdm})}$



not smooth avalanches

$w = vt$



avalanche size

$\to S$ area

$v \sim 0^+$ very slow quasistatic

renorm disorder corr

$$\overline{(u(w)-u(w'))^2} \sim S^2 |w| \quad \text{proba}$$

$$\overline{(u(w)-w)(u(w')-w')}^{\,c} = L^{-d}\,\frac{\Delta_m(w-w')}{m^4}$$

$w \to vt$

def of

$$\Delta_m(u) = m^{\epsilon-2\zeta}\,\tilde{\Delta}(u m^{\zeta})$$

$\tilde{\Delta} \to$ FP dim-less

$\Delta_b \equiv \Delta_{m=+\infty}$

Δ_0

$m \downarrow$ critical

$\ell \equiv \ln \frac{1}{m} \equiv \ln L$

$-m\partial_m$

$$\partial_\ell \tilde{\Delta}(u) = (\epsilon-2\zeta)\tilde{\Delta}(u) + \zeta\tilde{\Delta}'(u) \qquad \text{rescaling } O(\Delta)$$

$O(\Delta^2)$ one loop (ϵ)

$$-\frac{\partial^2}{\partial u^2}\left(\frac{\tilde{\Delta}(u)^2}{2} + \tilde{\Delta}(u)\tilde{\Delta}(0)\right)$$

$\tilde{\Delta}' \equiv \frac{\partial}{\partial u}\tilde{\Delta}$

$O(\Delta^3)$ 2 loop (ϵ^2)

$$+\frac{1}{2}\frac{\partial^2}{\partial u^2}\left(\tilde{\Delta}'^2\left(\tilde{\Delta}-\tilde{\Delta}(0)\right)\right)$$

$$+\frac{\lambda}{2}\tilde{\Delta}'(0^+)^2\,\tilde{\Delta}''(u)$$

$$R_{q,t} = \frac{1}{\eta} e^{-\frac{1}{\eta}(q^2+m^2)t}$$

$$\boxed{c=1}$$

$$\delta f = \int_{q,t>0} R_{q,t}\, \Delta'(vt)$$

$\downarrow v \to 0^+$ quasi-static depinning

$$-\delta f = \delta f_c = \int^{\Lambda} \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 \cancel{+m^2}} \Delta'(0^+) \sim \frac{W}{r_f} R_c^{2-d}$$

$\Delta'(0^+) \downarrow$ $\Delta(0) \to W/r_f$ $\Lambda \equiv 1/R_c$ $\quad ||| \quad f_c^{\text{Larkin}}$

$$\delta\eta = -\int_q R_{qt}\, t\, \Delta''(vt)$$

$\Downarrow v = 0^+$

$$\delta(\ln\eta) = \frac{\delta\eta}{\eta} = \int_q \frac{d^d q}{(2\pi)^d} \frac{\Delta''(0^+)}{(q^2+m^2)^2} \sim A_d \underbrace{m^{-\epsilon} \Delta''(0^+)}_{\tilde\Delta''(0^+)}$$

$$\partial_\ell \equiv -m\partial_m \qquad -m\partial_m \ln\eta = 2 - z - \tilde\Delta''(0^+)$$

$$z = 2 - \tilde\Delta'''(0^+)$$

1) ζ determined from reparing existence of FP with $\tilde\Delta^*(u)$ SR / LR bc at ∞

2) $z = 2 - \tilde\Delta^{*''}(0^+)$

3) Correlations $\overline{u_{-q} u_q} \simeq \frac{\Delta_m(0)}{(q^2+m^2)^2} + O(\Delta^2)$

$$\frac{\tilde\Delta^*(0)}{q^{d+2\zeta}} \longleftarrow \frac{m^{\epsilon-2\zeta}}{m^4}\tilde\Delta^*(0) \sim \left.\frac{\Delta_m(0)}{m^4}\right|_{m\to q}$$

to one loop OK

1 loop

a. if $\tilde{\Delta}(u)$ analytic $\Rightarrow$ $\partial_\ell \tilde{\Delta}(0) = (\epsilon - 2\zeta)\tilde{\Delta}(0)$

to all orders LARKIN

(dim-red)

b. $\partial_\ell \tilde{\Delta}''(0) = \epsilon \tilde{\Delta}''(0) - 3\tilde{\Delta}''(0)^2$

$\tilde{\Delta}''(0) \to -\infty$

<0 smooth

finite scale R_c

known to Larkin

c. $\tilde{\Delta}(u)$ develops cusp beyond R_c

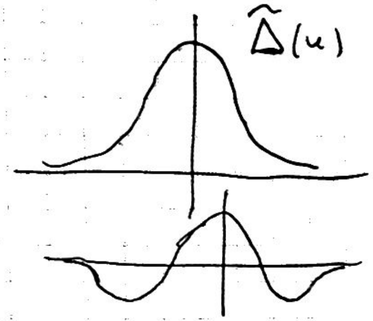
Fisher $\tilde{\Delta}(u)$ becons NA

but remain perturbative FP $O(\epsilon)$

$$\partial_\ell \tilde{\Delta}(0) = (\epsilon - 2\zeta)\tilde{\Delta}(0) - \tilde{\Delta}'(0^+)^2$$

RF

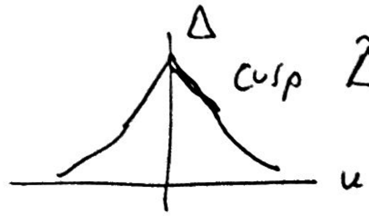
RB



$\longrightarrow$

$R > R_c$

$m < m_c \equiv 1/R_c$



d. FRG eq same to one loop for statics and depinning!

$\zeta_{RF} = \epsilon/3$

$\zeta_{SR} = 0.20..\epsilon$

"argue" $\zeta_{dep} = \epsilon/3$

SR flows to RF

$\downarrow$

Narayan-Fisher argued to all orders

$\alpha = 2 \quad \zeta_{d=1} = 1$

$\alpha = 1 \quad \zeta_{d=1} = 1/3$

seen later

no solid untill Rosso-Krauth algo

need 2 loop!

<u>2 loop</u>

$\lambda_{stat} = -1 \Rightarrow$ potentiality $\exists \tilde{\Delta}^* \int \tilde{\Delta}^* = 0$ $\underline{\underline{SR}}$

$R(u) = SR$

$\lambda_{dep} = 1 \Rightarrow \int \tilde{\Delta} \neq 0$ irreversibility

depinning
$\Rightarrow$ unique universality class

$$\zeta_{dep} = \frac{\epsilon}{3}(1 + 0.1433\epsilon) > \zeta_{NF}$$

$$z = 2 - \frac{2}{9}\epsilon - 0.04321\epsilon^2 + \dots$$

---

		prediction	numerics
$d=1$	$\zeta_{dep}^{\alpha=1}$	$0.\cancel{4}4 \pm 0.05$	0.39 ± 0.002
	$z_{dep}^{\alpha=1}$	0.7 ± 0.1	0.74 ± 0.03

ζ_{dep} $\underline{\alpha = 2}$			
	$d=3$	0.38 ± 0.02	0.34 ± 0.01
	2	0.82 ± 0.1	0.75 ± 0.02
	$\boxed{1}$	$\boxed{1.2 \pm 0.2}$	$\boxed{1.25 \pm 0.01}$

RFIM
interfaces destroy

β	1	$d=4$	
	0.84	$d=3$	
	0.53	$d=2$	
	0.2 ± 0.2	$d=1$	$0.2 - 0.4$

ν	$1/2$
	0.62
	0.85
	1.25

1) Cusp $\Leftrightarrow$ avalanches

$$\frac{\Delta'(0^+)}{m^4} = \frac{\langle S^2 \rangle}{2\langle S \rangle}$$

exact relation
if all motion
in avalanches
(tested in CL
experiment)

$\Rightarrow$ calculation

$$P(S) \sim S^{-\tau}$$

MF $\tau = 3/2$

$$\boxed{\tau = 2 - \frac{2}{d+\zeta} \\ = \tau_{MF}}$$

$O(\epsilon)$

ε^2

2) $\partial_\ell T_\ell = -\theta T_\ell$ $\qquad T_\ell = T m^\theta$

$$\partial_\ell \tilde{\Delta}(u) = \beta(\tilde{\Delta}) + T_\ell \tilde{\Delta}''(u)$$

$\downarrow$

rounds the cusp
at scale $u \sim T_\ell = T m^\theta$

$$\partial_\ell \ln \eta = -\tilde{\Delta}''(0)$$

$$\sim \frac{1}{T_\ell} \sim \frac{1}{T} e^{\theta \ell}$$

$$\eta(\ell) \sim \exp\left[\frac{1}{T} E_b(L)\right)$$

$\downarrow$

L^θ

$\psi = \theta$