→ In these 3 lectures I will describe what is called
"disordered elastic systems (DES)"
both static/equilibrium properties and when driven out of eq.
e.g. external force

focus ↔ connections
better discuss both

→ The DES also form what called glass phases
A priori quite ≠ from the glasses that Sid Nagel talked about
Here the glassy properties (E. barriers, slow relax) emerge not
from the interactions of the constituents alone (as in window glass)
but because the system is put in presence of a substrate
with quenched disorder.

→ The interactions within the DES are actually supposed to be simple enough
to be modelled by a ~~simple~~ basic elastic energy (quadratic and
by itself quite boring) but it is the interactions with the
quenched disorder which makes it non-linear, non-trivial
and glassy

Sid discussed similarities and ≠ structural window glass

→ Systems with quenched disorder can also be very complex,
the most famous example being spin-glasses. In fact
there may be at present some convergence with structural window glasses (at least
for theoreticians, common tools led progress → recently jamming
problem hard spheres) so lookely may be considered as
in the same, very difficult class. I will not talk about them.

→ The DES considered as simpler examples of glasses - What
do I mean by simpler? Well for instance their ground states
are complicated but in some cases there are algorithms
to find them in a time polynomial with system size ~~[illegible]~~. This is not the
case for the 3d Edwards Anderson model require exp
(Ising with random ±J couplings)

Also they are a bit more tractable analytically. 2)
In both cases there are mean-field theories, but in addition there ~~are~~ renormalization methods for DES (the functional RG which I will describe a bit)

Since they nevertheless exhibit many glass features → interesting study
give idea more complex

Another big motivation to study them is that there are useful models for a surprisingly large # experimental systems
(~~used~~ fruitfully magnetic systems, CDW, vortex lattices, fluid wetting, fracture crack, earthquakes)

→ So you had an introduction to quenched disorder by Leo and in some sense we will pursue what he started.
He treated the effect of quenched disorder first on phase transitions, and then on ordered states. It is this second part that we will pursue. We will considered small deformations around ordered state, deformations induced by a disorder.

→ let me (as a ~~theoretician~~ theorist) write down the ~~general~~ general model of a DES. It may appear ~~a~~ bit arbitrary at first, but this is the model towards which people have converged.
I will explain why and show examples

~~We will~~
One question we will ask is whether remai of the glassy properties when the system is put in motion

"Standard model" TOE

elastic manifold in a random potential

deformation displacement field $u(x)$ $u = (u_1, .. u_N)$ in $\mathbb{R}^N$ | embedding space

$x = (x_1, .. x_d)$ in $\mathbb{R}^d$ internal space

d ↓ internal dimension

energy functional $H_V[u] = \int d^d x \left(\frac{c}{2} (\nabla u)^2 + V(x, u(x)) \right)$

$\frac{c}{2}(\nabla u)^2 \to H_{el}$ $\qquad$ $V(x,u(x)) \to H_{dis}$

Symbolic quadratic form
S.t $u=0$ is G.S
perfect order
($u(x) = u_0$ invariant)
transl.
c elastic constant

comes from subtrate impurities
↓
~~break~~
Couples to $\vec{u}$
pinning disorder
breaks T.I
$u(x) \to u(x) + u_0$

Leo told you about goldstone modes when Continuous Symmetry broken spont.
TI crystal / rot inv
O(N) spin
no mass pure gradient

gaussian
$\overline{V_i V_j} = C_{ij}$
$\overline{V(x,u)} = 0$

in standard model $V(x,u)$
random function
think of it as gaussian

2pt correlator $\overline{V(x,u)V(x',u')} = \delta^d(x-x') R_0(u-u')$

R_0 ↓ Smooth
Varies
Scale r_f

≠ structural disorder
Crystal size q. random sphere
polydisperse
inv. by translation

$V(x,u)$ lives in $\mathbb{R}^d \times \mathbb{R}^N = \mathbb{R}^D$
$D = d + N$
total space

- Symmetries
- relevance
  irrelevance
  large scale (Leo)

note
Statistical TI
$x-x'$, $u-u'$

~~interfaces N=1~~
~~lines with d=1~~
~~in higher dimensions~~

+ equation of motion (eqmo)
involve $H \to$ forces
calculate

pinning force $F(x,u) = -\partial_u V(x,u)$

driven dyn F acquire life of their own... -- bare model --

REMARKS

→ leo explained

→ annealed disorder = another degree of feedom which interacts w system it has his dynamics → equilibrates

→ quenched disorder

"frozen in" degree of feedom

time scale for change much larger

"1 random config"

→ one observe only 1 sample one $V(x,u)$ realization

has well defined statistical property

→ disorder averages..

→ translational averages..

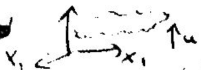
$\langle S_i S_j \rangle = \prod t_i$

1d ISing

Non SA

$\ln \langle \rangle$ is

A bit abstract, let us go to examples + experiment



4)

$\rightarrow u(x)$ scalar $N = 1$ interfaces — line $u(x)$ — surface height $u(x_1, x_2)$ .. $x \in \mathbb{R}^d$

$\rightarrow N > 1$

- lines in higher dim (transverse d°f)
- crystals

periodic
versus non periodic
CDW $N=1$

Start with exp. DW ferromagnet

Simple Model	Phenomena	Exp !

Exp

Best is start experiment

exp: DW in ferromagnet Ferre et al. Orsay PRL 98

film Co 0.5 nm Pt/Co/Pt

↓

Ising ↑z

exchange
e⁻-repulsion
Heisenberg

$H = -J\, \vec{S}_r \cdot \vec{S}_{r'}$

$J > 0$

crystal anisotropy $-K S_z^2$

crystal field
spin orbit

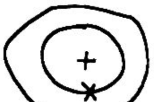
uniaxial
cell Co
hexagonale

all directions inequivalent
symmetries

→ interactions dipolaires $1/r^3$

weaker but LR | boundaries demag

→ apply H_z reverse domains grow from few nucleation centers to circular shape



- Observe magnetooptics imaging
+ microscope
1 μm resolution

Kerr reflected
Faraday transmitted

Kerr effect rot pol light/B

CCD Charge coupled device
photoelectric → analogic

Bloch DW

$\mathcal{E}_{el} = 4(JK)^{1/2} L_z$

E/length - tension - minimize elastic energy

width $\sqrt{\frac{A}{K}} \sim 8\,\mathrm{nm}$

→ observe DW → rough

static → quenched disorder (energy)

impurities = surface steps

terrace ± 0.2 nm

$\xi \sim 10$ nm

$\overline{(u(x+\ell) - u(x))^2}^{\,trans} \sim \ell^{2\zeta}$

$\zeta = 0.69 \pm 0.07$

2 orders mag

$\ell_c^{est} \simeq 25\,\mathrm{nm}$

move + stop → thermal eq (?)

$\ln 10 = 2.3$

Model

$u(x)$

w.r.t average direction
directed line

$H_{el}[u]$

$u(x) = 0$ G.S

+ SR disorder

$H_{dis}[u]$

→ thermal activ. motion
energy barriers

$\tau \sim e^{-\frac{E_b}{T}}$

$v \sim e^{-\frac{U_c}{T}\left(\frac{f_c}{f}\right)^{\mu}}$

barriers diverge
$f \to 0$ ∞ statics
GLASS

→ increase H_z

DW feels force/length ~ H

δu

$\Delta E \sim - H L \delta u$

Oe ≡ G

Should grow ∞ size
pinned by impurities
(average)

Velocity measured

pulse τ e.m response coils

$H_c \simeq 692\,\mathrm{Oe} \sim f_c$

velocities v 0.3 nm/s
40 m/s

11 orders of magnitude in v!

$\mu \simeq 1/4$

Phenomena

→ pinned
→ roughness

$\overline{(u(x) - u(x+\ell))^2} = u_c^2 \left(\frac{\ell}{\ell_c}\right)^{2\zeta}$ disorder?

→ small thermal fluctuations

$\mathcal{E}_{dis} \gg T$

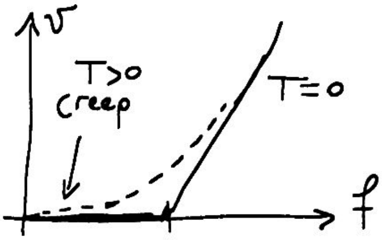
thermal equil. ?

→ pinned

laisser tableau

→ $v(f)$

→ depinning transition



Sharp transition
T=0
order parameter
v

Creep, T, easier to see in films thin
→ weaker pinning
→ less stiff
less magnetostat.

Build a simple model $u(x)$ w.r.t average direction directed line

1) STATIC

energy cost for deformations $H_{el}[u]$

$$u_q = \int_x e^{iqx} u(x)$$
$$\int_q \equiv \int \frac{d^dq}{(2\pi)^d} \qquad \int_x \equiv d^dx$$

1) elastic energy $u(x) = 0$ G.S $u = u_0$

Small def $H_{el}[u] = \int_x \frac{1}{2} c (\partial_x u)^2 = \int_q \frac{1}{2} c q^2 u_{-q} u_q$ line energy line tension surface tension

c elastic constant

$\partial_x u$ small → where is this energy coming from?

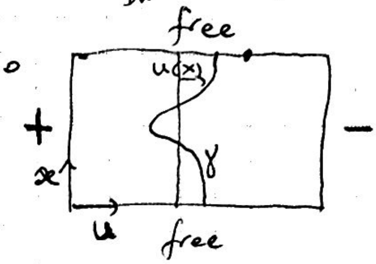
$u = u_0$ can be large

inv. transl.

$E_{DW} \sim$ length $= c \int dx \sqrt{1 + (\partial_x u)^2} \simeq c\,\ell + H_{el} + \dots$

$+ \int dx \frac{c}{2} (\partial_x u)^2 + \dots$

$h = 0$



$S_i = \pm 1$ example Ising model

$H = -J \sum_{i<j} S_i S_j$ lattice a_0

$J > 0$ ferro

energy cost $2J$ per unsatisfied +/− bond

→ length $c = 2J/a_0$

here Bloch domain wall

valid $q_x \ll 1/a_0$

Short scale overhangs, bubbles

→ $c(T)$ in ferrophase

1') external field $\Delta E = -h\,\ell\,\delta u$

$H[u] = -\int dx\, f.u$ if stops then PINNED

2) disorder energy

modifies energy cost locally DW

a) random bond, SR disorder

$$H = -\sum_{\langle ij \rangle} (J + \delta J_{ij}) S_i S_j$$

$b \equiv ij \qquad \overline{\delta J^2} = \sigma^2$

$b = (x, u)$ $\qquad E_{DW} = \sum_{b \in \gamma} J + \delta J_b$

$b \in \gamma \quad = (x, u(x))$

$$E_{dis} = \sum_x \delta J_{(x, u(x))} = \int \frac{d^dx}{a_0^d} \delta J(x, u(x))$$

disorder averages

$$H_{dis}[u] = \int dx\, V(x, u(x))$$

$$\overline{V(x,u)\, V(x',u')} = \delta_{a_0}(x - x')\, \delta_{r_f}(u - u')$$

→ SR

r_f → max (r_f, thickness) ↔ thermal, $r_f(T)$

Leo Lecture b) random field, LR disorder

general model $\int_q \frac{1}{2} \Phi(q) u_{-q} u_q + \int_x V(x, u(x))\, d^dx$

$\Phi(q) \to c q^2$ or q^μ $\quad \mu < 2$

SR or LR fct

$$\overline{V(u,x)\, V(u,x')} = \delta^d(x - x')\, R_0(u - u')$$

7

gaussian proba
and roughness

$$\mathcal{P}[u] \sim e^{-\frac{1}{2}\int_q \phi(q)\, u_{-q} u_q} \qquad u_{-q} = u_q^*$$

$$\int_q \equiv \int \frac{d^d q}{(2\pi)^d}$$

$$u_q = \int d^d x\, u(x) e^{iqx} = \int_x u_x e^{iqx}$$

$$\int_x \equiv \int d^d x$$

$$\langle u_q u_{q'} \rangle_\rho = \underbrace{2\pi \delta^d(q+q')}_{\equiv\, \delta_{q',-q}} \frac{1}{\phi(q)}$$

gen

$$e^{-\frac{1}{2}\int_q \sum_{\alpha\beta} u^\alpha_{-q} \phi(q)_{\alpha\beta} u^\beta_q}$$

$\alpha\beta$ components N
replica...

$$\langle u_q u_{q'} \rangle_\rho = \delta_{q',-q} \left(\phi(q)^{-1}\right)_{\alpha\beta}$$

$$\phi(q) = \frac{1}{A} q^\alpha \qquad \langle u_q u_{q'} \rangle_\rho = \frac{A}{q^\alpha} \delta_{q',-q}$$

ex: thermal fluct $\left| \begin{array}{l} A = \frac{T}{c} \\ \alpha = 2 \end{array}\right.$ $\qquad \langle u_q u_{q'} \rangle_\rho = \frac{T}{c q^2} \delta_{q',-q}$

TI Set 1

$$\langle u_x u_0 \rangle_\rho = A \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^\alpha} e^{iqx} \qquad \frac{1}{L} < q < \Lambda = \frac{1}{a_0}$$

(survives order ~~broken~~ TS broken)

$\langle u_0^2 \rangle \sim A \Lambda^{d-\alpha} \qquad d > \alpha \qquad$ bounded

$\langle u_0^2 \rangle \sim A \ln(\Lambda L) \qquad d = \alpha \qquad$ critical dim

$\langle u_0^2 \rangle \sim A L^{\alpha-d} \qquad d < \alpha$

$$\langle (u_x - u_0)^2 \rangle_\rho = 2A \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^\alpha}(1 - \cos qx)$$

$\underset{x\to\infty}{\sim} 2A\Lambda^{d-\alpha} = 2\langle u_0^2 \rangle \; (- x^{\alpha-d}) \qquad d > \alpha \qquad$ bounded

$\underset{x\to\infty}{\sim} A \ln\left|\frac{x}{a_0}\right| \qquad d = \alpha$

$\underset{x\to\infty}{\sim} A\, x^{\alpha-d} \qquad d < \alpha \qquad \boxed{\zeta = \frac{\alpha-d}{2}}$

$L = \infty$
can be taken
except $\alpha \geq d+2$

$x^2 L^{\alpha-d-2} = x^2 L^{2(\zeta-1)}$

metric constraint becomes very relevant

$$\boxed{\alpha = d + 2\zeta}$$

Q: see pinned ?
how pinned and also at equilibrium ?

energy minimum locally – does not fluct. thermally
see it frozen
$T \ll E_{dis}$

Q: what would be ζ if thermal eq no disorder
random walk

$\langle u_q u_{q'} \rangle \sim T/cq^2 \, \delta_{q',-q} \to x$ $\qquad \zeta_{th} = 1/2 \qquad u \sim x^{1/2}$

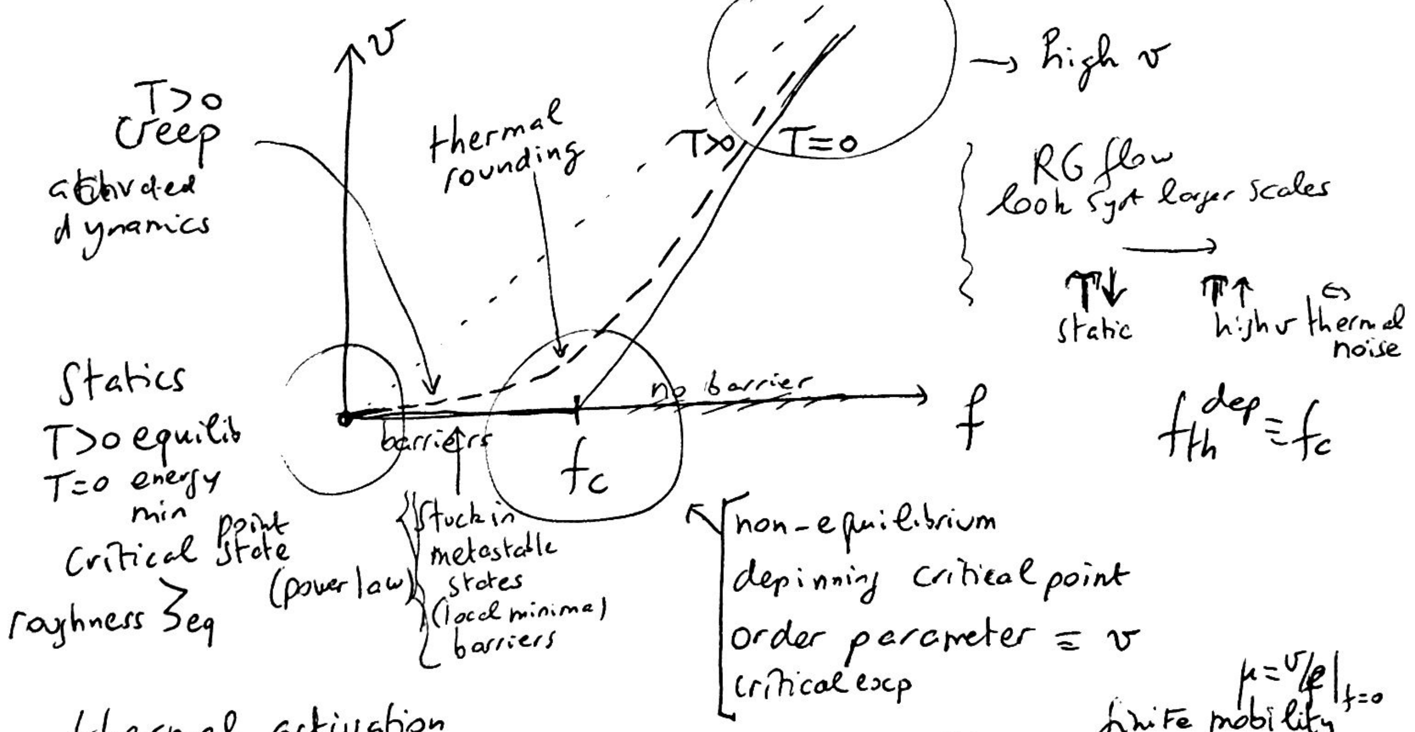
what value ?

Q: why larger $\qquad \zeta > \zeta_{th} = 1/2$

line want to extend to visit favorable regions
balance between disorder energy gain from disorder spot finding good
and elastic energy cost.

$D = 1+1 \qquad \zeta_{eq}^{exact} = 2/3 \qquad$ see later why

$\zeta \simeq 0.69 \pm 0.07 \qquad 0.66$



thermal activation

$\tau \sim e^{+E_b/T}$ $\qquad$ 1) $v \sim v_0 \, e^{-E_b/T}$ $\quad$ finite mobility $\mu = v/f|_{f=0}$, finite barrier

$\qquad\qquad\qquad\qquad v_0 \sim f$

$E_b \underset{f \to 0}{\approx} U_c (f_c/f)^{\mu}$ $\qquad$ creep 2) $v \sim v_0 \, e^{-\frac{U_c}{T}\left(f_c/f\right)^{+\mu}}$ $\quad$ glassy diverging barriers go back to model

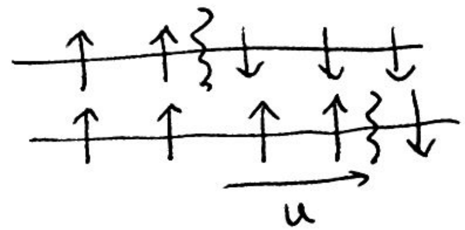
$R(n)$

$\Rightarrow$ explain RFIM

$\Rightarrow$ limitations

2) random field disorder   RFIM   Ising + $h_i S_i$ $\overline{h_i h_j} = g\,\delta_{ij}$ (4

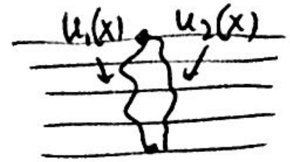
D = 0+1



$$V(u) - V(0) = 2\sum_{i=1}^{u} h_i \quad \text{random walk}$$

$$\overline{(V(u) - V(0))^2} \sim 4g|u|$$

D = 1+1



$$\Delta V = \sum_{x} \sum_{u=u_1(x)}^{u=u_2(x)} 2 h_{x,u}$$

$$= \sum_{x} V(x, u_2(x)) - V(x, u_1(x))$$

$$V(x,u) = 2 \sum_{i=1}^{u} h_{x,i}$$

discret

$$\overline{V(x,u)\,V(x',u')} = \delta_{xx'} R(u-u')$$

$$\overline{(V(x,u) - V(x,u'))^2} = 2\,(R(0) - R(u)) = 4g|u|$$

$$R(u) \underset{u\to\infty}{\sim} -\sigma|u|$$

more general LR

RFIM   D = 3   ferro phase exits (Imbrie,..)

D = 2   destroyed beyond $L_{\text{Imry-Ma}}$

description holds in ferro φ below

→ GO TO MODEL

other systems
interface 2 phases w
random local ≠ in free energy
(ask contact line)

1) ordered state

+ small deformations

displacement field

$\vec{u}(\vec{x})$ — $\in \mathbb{R}^N$ embedding space; $\vec{x} \in \mathbb{R}^d$ internal space

total $D = d + N$

$$H_{el}(u) = \frac{1}{2}\int d^dx \, c(\nabla_x u)^2 = \frac{1}{2}\int_q c q^2 u_{-q} u_q$$

$u_q = \int_x e^{iqx} u(x)$

$\int_x := \int d^dx$

$\int_q := \int \frac{d^dq}{(2\pi)^d}$

$\vec{u} = 0$ G.S perfect order

$\vec{u} = cst$

transl. inv $\vec{u}(x) \to \vec{u}(x) + \vec{u}_0$

($u = \theta x$ costs $\theta^2 L^d$ directed / oriente)

2) substrate impurities couple to $\vec{u}$ pinning disorder

$\neq$ structural disorder cpl ∇u internal no pinning

breaks T.I

$$H_{dis}[u] = \int d^dx \; V(x, u(x))$$

$V(x,u)$ random potential lives in $\mathbb{R}^d \times \mathbb{R}^N$ total $\mathbb{R}^D$, $D = d+N$

in standard model

LATER $V(x,u)$

2 pt correlator $\overline{V(x,u)V(x',u')} = \delta^d(x-x') R_0(u-u')$ $R_b(u)$ bare

$R_0(u)$ smooth varies scale r_f

limitations/universality

Note: standard model

continuum

cautious keep track cutoff a, L | look for continuum limit $L/a \to \infty$

define universality classes

$\Rightarrow$ model $\nabla u \ll 1$, V gaussian (naive)

- higher cumulants V irrelevant | much broader validity u.class

except some extreme situations ($V^{(6)}$ exists $d=1$ $N=1$ talk later)

- higher non linearities in ∇u irrelevant (not in dynamics)

main classes

- $R_0(u)$ SR (random bond)
- $R_0(u)$ LR
  - generic $\sim |u|^{-\alpha}$
  - (random field) $\sim \sigma |u|$
  - log-correlated $\sim \ln|u|$
- $R_0(u)$ periodic (random periodic) same periodicity as lattice

more severe limitations = defects in ordered structure

interface overhangs

crystal vacancies, dislocations, .. $\to$ no single valued $u(x)$

BUT if defects only (paired disloc) below some scales $\to$ $\exists$ a coarse grained $u(x)$ described by this model