

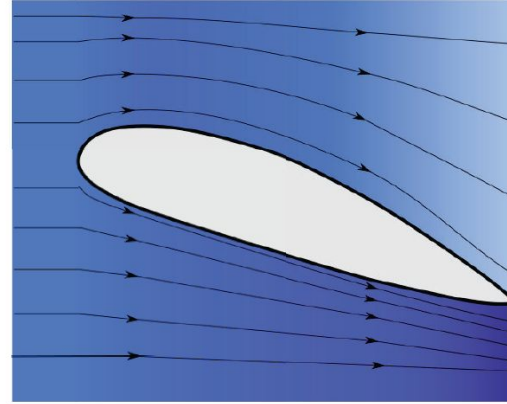
Electron fluids

Leonid Levitov (MIT)

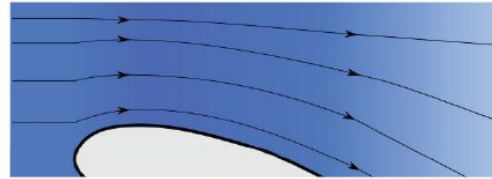
Boulder Summer School 2025



Fluids: chaotic on microscales, orderly on macroscales (conservation laws!)



**Fluids: chaotic on microscales, orderly
on macroscales (conservation laws!)**



Hydrodynamics: theory of everything

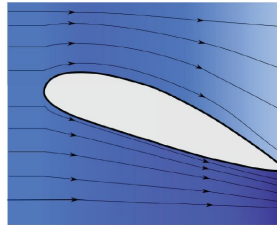


Hydrodynamics: theory of everything

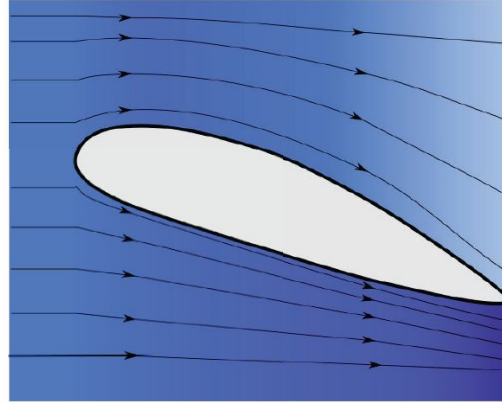


Hydrodynamic description (big picture)

- Space and time symmetries: Euclidean P, T, O(3), spin SU(2)
- Continuous (global) symmetries result in (local) conservation laws
- Local transport equations
- Example: Fermi liquids $\partial_t n + \nabla \cdot j = 0$, $\partial_t p + \nabla \cdot \Pi + \gamma = enE$, where n and p – particle and momentum density – conserved quantities, j and Π – current and stress tensor (aka Fermi pressure) – functions of conserved quantities, γ – momentum dissipation rate (disorder or phonons)
- Separation of time scales – nonconserved quantities quickly erased from system memory (Boltzmann). Ordered behavior from chaotic behavior



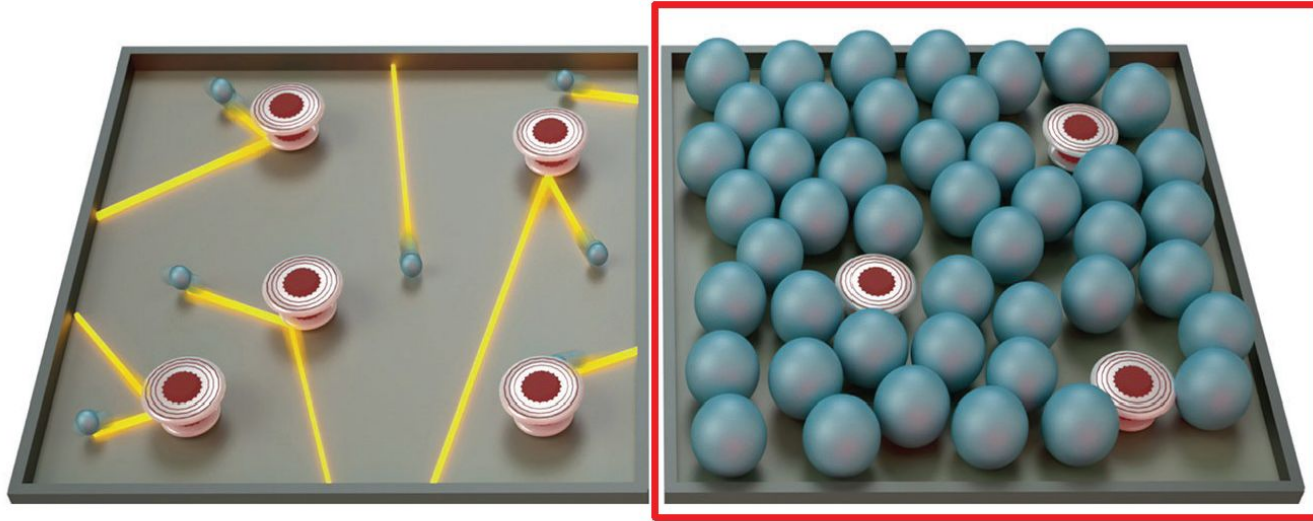
Fluids: chaotic on microscales, orderly on macroscales (conservation laws!)



Ergodicity and separation of scales

- Short-time memory for nonconserved quantities, long-time memory for conserved quantities
- Markovian picture (hydrodynamics justification and validity)
- Interesting non-Markovian effects:
 - in classical gases (Dorfman and Cohen),
 - in quantum systems with disorder: quenching of diffusion, Anderson localization and weak localization (Gorkov, Larkin, Khmel'nitskii),
 - many-body localization,
 - many others
- Manifestations:
 - nonlocal transport equations,
 - kinetic coefficients with long-time memory,
 - infinite or diverging thermalization rates

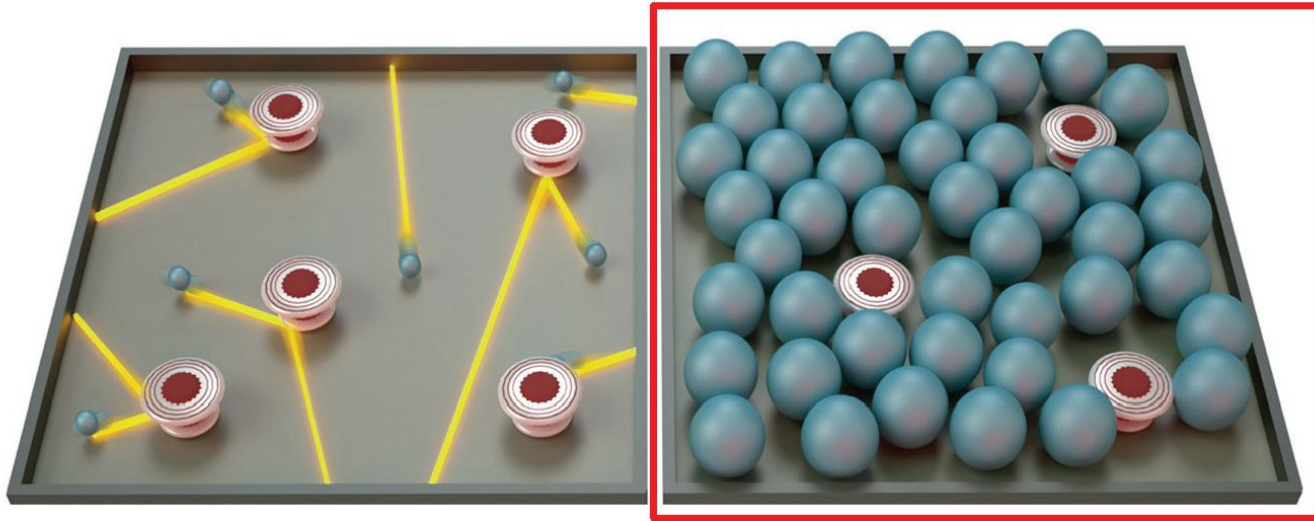
Is hydrodynamics ever relevant **in metals**?



credit: Andy Lucas; from: Jan Zaanen, Science 2016

- High-mobility electron systems (graphene, GaAs 2DES, PdCoO₂, etc):
- Non-Fermi liquids, high-T_c superconductors, strange metals

Is hydrodynamics ever relevant **in metals**?



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Literature

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Yu *et al.* Negative temperature derivative of resistivity.. The Gurzhi Effect? *PRL* **52**, 368 (1984).

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R. N. Gurzhi, A. N. Kalinenko, and A. I. Kopeliovich, *PRL* 74, 3872 (1995)

H. Buhmann, L. W. Molenkamp, *Physica E* 12, 715-718 (2002)

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LL & Falkovich, Electron viscosity... negative nonlocal resistance, *Nat Phys* 12, 672 (2015)

Bandurin *et al.* Negative local resistance caused by viscous... *Science* **351**, 1055 (2016).

Crossno *et al.* Observation of the Dirac fluid... *Science* **351**, 1058 (2016).

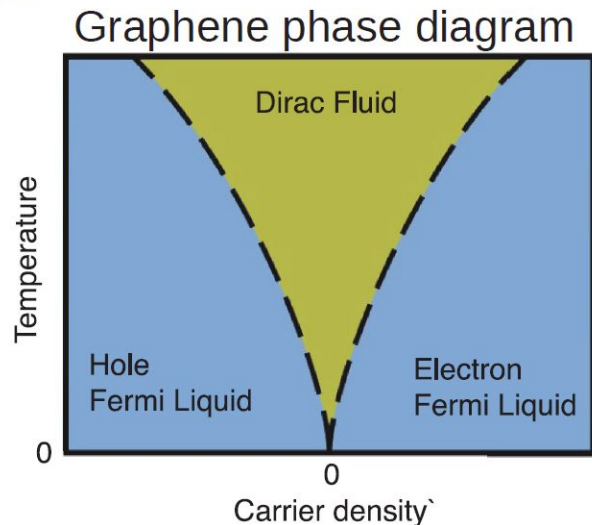
Moll *et al.* Evidence for hydrodynamic electron flow in PdCoO₂. *Science* **351**, 1061 (2016).

Gooth *et al.* Electrical and Thermal Transport at the Planckian Bound of Dissipation in WP₂ (2017)

...

Viscous electron fluids in 2D systems

- Strong interactions (enhanced in 2D, graphene)
- Graphene: weak electron-phonon scattering, no Umklapp ee scattering
- Fast p-conserving ee collisions, shear viscosity



Sheehy and Schmalian, PRL
99, 226803 (2007)

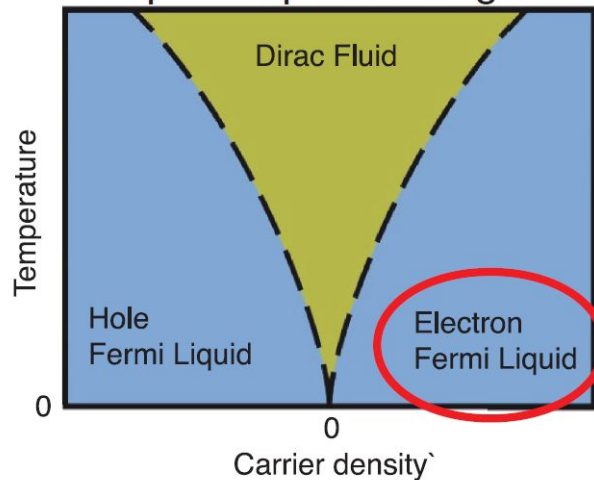
Guo et al. 1607.07269 1612.09239 Bandurin et al. 1703.06672,
Ledwith et al. 1708.01915, 1708.02376

Viscous electron fluids in 2D systems

- Strong interactions (enhanced in 2D, graphene)
- Graphene: weak electron-phonon scattering, no Umklapp scattering
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- **Signatures of viscous effects?**
- **New collective phenomena?**

Graphene phase diagram



Sheehy and Schmalian, PRL
99, 226803 (2007)

Guo et al. 1607.07269 1612.09239 Bandurin et al. 1703.06672,
Ledwith et al. 1708.01915, 1708.02376

Why to be interested in electron hydrodynamics?

- Viscous transport: A new regime showing a counterintuitive behavior: **carrier collisions assist conduction**. Compare to motional narrowing in spin resonance (Van Fleck and Anderson) or collision-narrowing in optics (the Dicke effect)
- **Conductance grows with T**: $R(T=0) > R(T \neq 0)$. Other instances: Kondo impurity scattering or localization ($dG/dT > 0$ reflects spin correlations or suppression of quantum coherence)
- This lecture: **a non-Fermi-liquid temperature dependence** in electron hydrodynamics. Surprisingly, the measured T dependence is **linear** rather than T^2 .
Explanation?



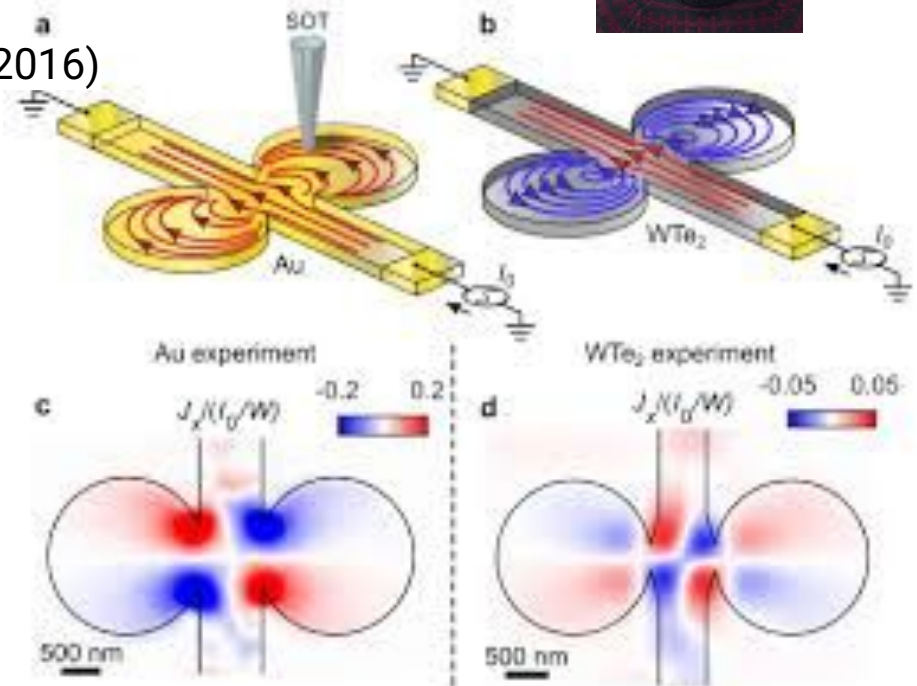
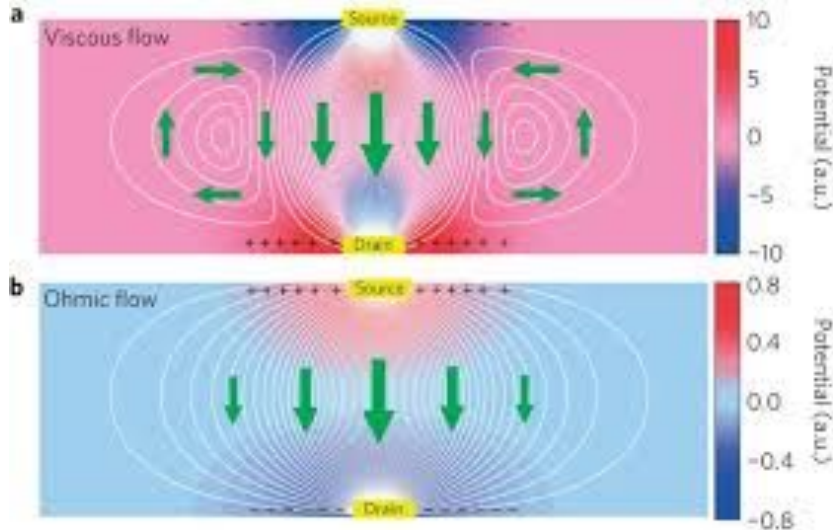
Serhii
Kryhin '22

Why to be interested in electron hydrodynamics?

Vortices in electron fluids studied by scanning probe (Zeldov group, Weizmann Institute).

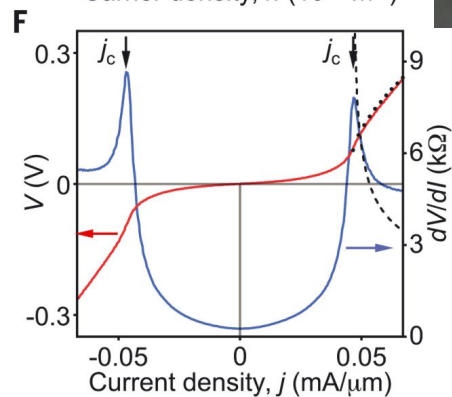
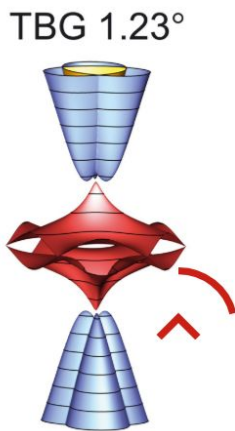
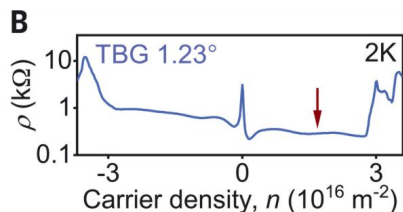
Current flow opposite to E field

LL & Falkovich, Nature Phys 12, 672–676 (2016)

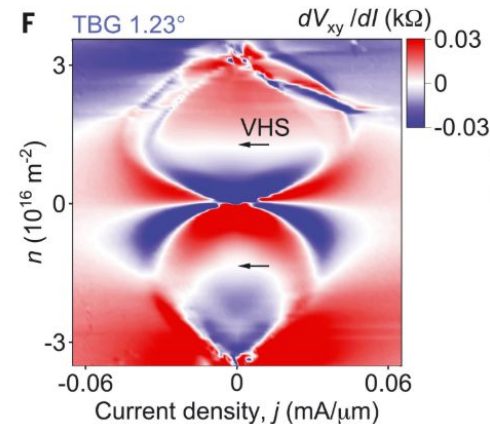
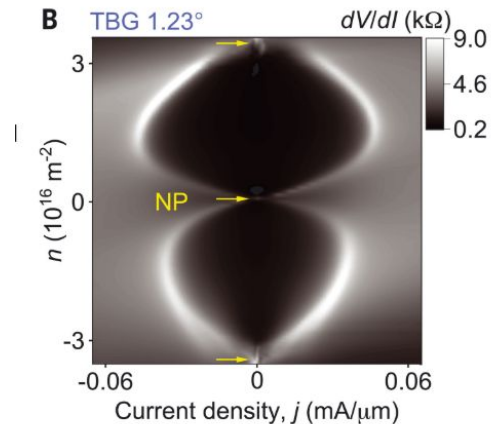


Why to be interested in electron hydrodynamics?

Hydrodynamic instabilities under small, experimentally accessible fields. Current-induced inversion of band occupation (experiments in graphene multilayers and monolayers, moire and non-moire. Current-driven ordering?)



Science 375, 430-433 (2022)
Berdyugin et al, (Geim group)

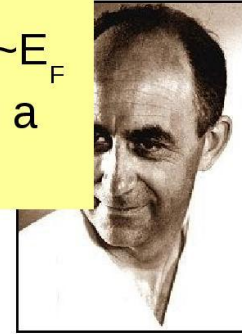
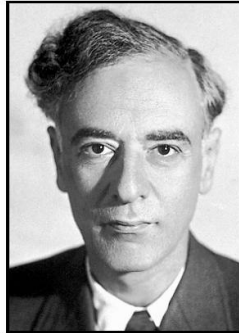


Tomographic electron fluids

- Quasiparticle lifetimes
- Kinetic coefficients
- Nonlocal conductivity
- Tomographic transport
- New phenomena

Quasiparticle lifetimes in Landau Fermi-liquids:

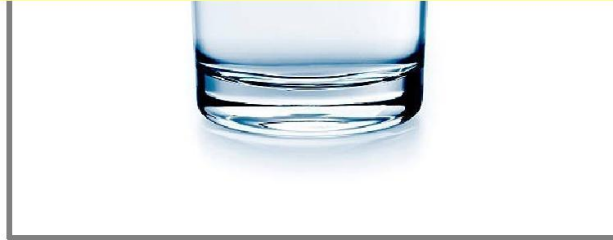
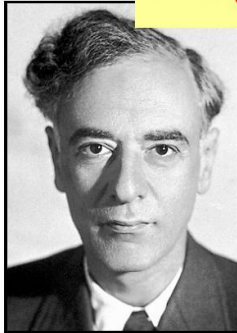
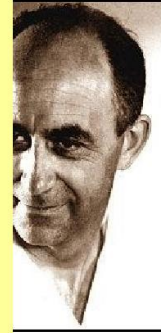
- Fermi sea (filled states with $E < E_F$)
- All the action at the Fermi surface, $E \sim E_F$
- **Quasiparticles**: quasi-free particles in a strongly interacting system



Long-lived excitations, directional memory & e-fluids in 2D

Quasiparticle lifetimes in Landau Fermi-liquids:

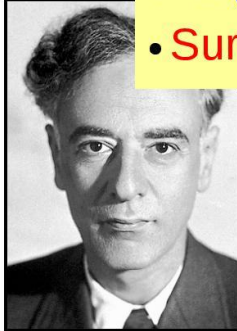
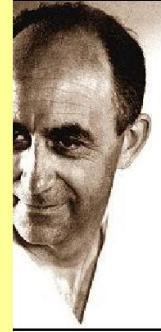
- Fermi sea (filled states with $E < E_F$)
- All the action at the Fermi surface, $E \sim E_F$
- **Quasiparticles**: quasi-free particles in a strongly interacting system
- **Relatively long lifetimes in 3D systems:**
 $\tau \sim 1/(E - E_F)^2$, $\tau \sim 1/(k_B T)^2$
- **Even longer lifetimes in 2D systems:**
 $\tau \sim 1/(k_B T)^4$ for odd-parity excitations



Long-lived excitations, directional memory & e-fluids in 2D

Quasiparticle lifetimes in Landau Fermi-liquids:

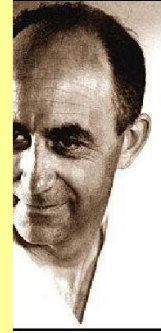
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- **Even longer lifetimes in 2D systems:**
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- **Surprising collective behaviors in e-fluids**



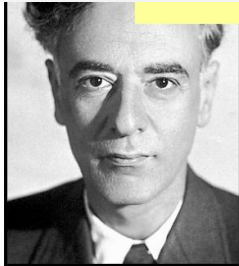
Long-lived excitations, directional memory & e-fluids in 2D

Quasiparticle lifetimes in Landau Fermi-liquids:

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- All the action at the Fermi surface, $E \sim E_F$
- **Quasiparticles**: quasi-free particles in a strongly interacting system
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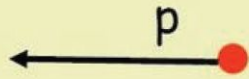


One cannot live in society and be free from society (V I Lenin)

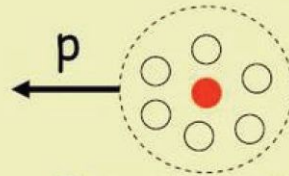


Long-lived excitations, directional memory & e-fluids in 2D

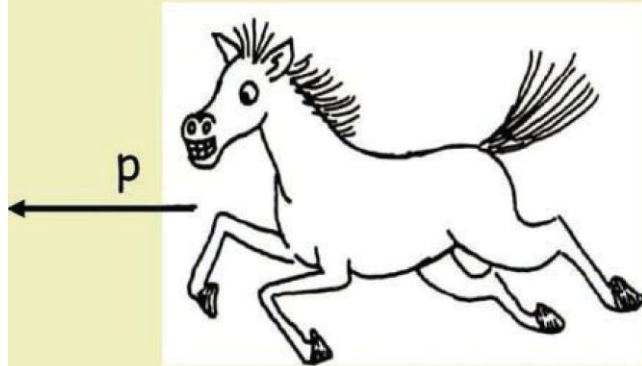
Quasiparticle concept



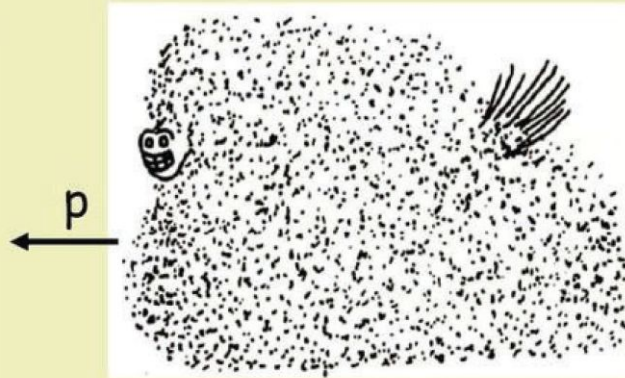
Real particle



Quasi particle



Real horse

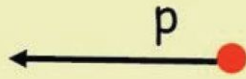


Quasi horse

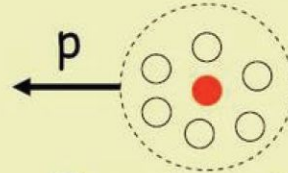
R. D. Mattuck, a guide to Feynman Diagrams in the MB problem, Dover, 1976

A quasiparticle has an **effective mass**, **selfenergy** (energy and lifetime).

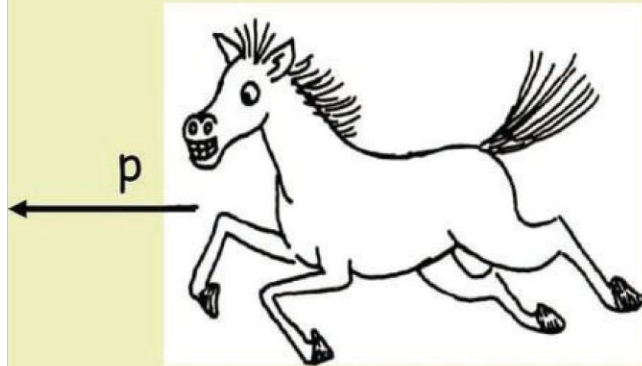
Quasiparticle concept



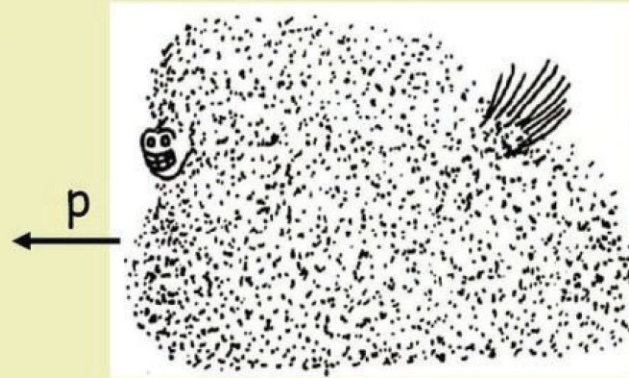
Real particle



Quasi particle



Real horse

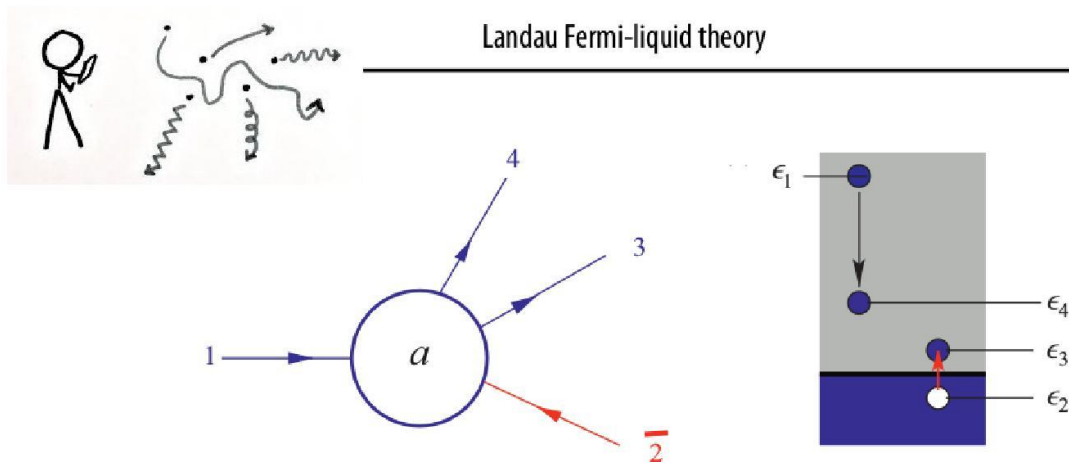


Quasi horse

R. D. Mattuck, a guide to Feynman Diagrams in the MB problem, Dover, 1976

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The phase space argument

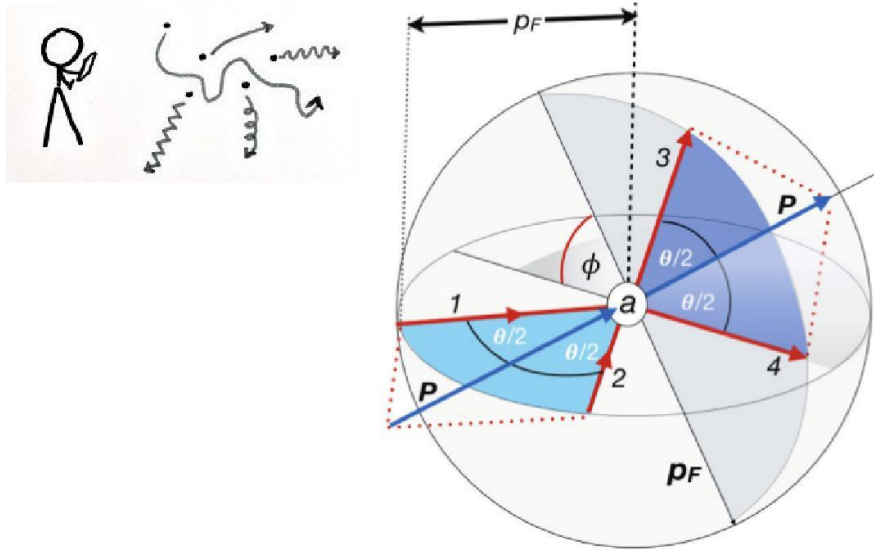


$$\gamma \sim \int \int \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4) f(\epsilon_2)(1-f(\epsilon_3))(1-f(\epsilon_4)) \sim \max[\epsilon_1^2, T^2]$$

Kinematics of ee scattering:

In 3D angular relaxation not a bottleneck (and thus does not matter)

Landau argument works



Kinematics of ee scattering:

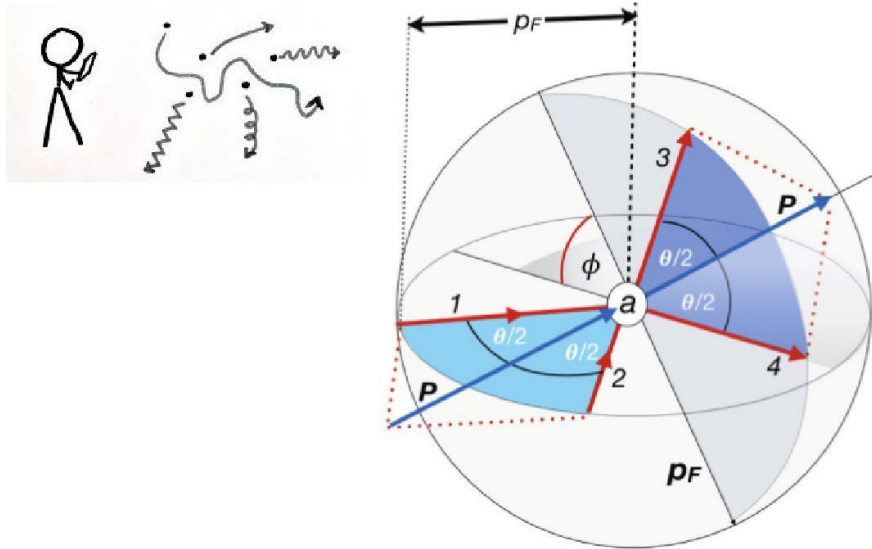
In 3D angular relaxation not a bottleneck (and thus does not matter)

Landau argument works

However, in 2D it does matter!

angular relaxation IS a bottleneck

revision of Fermi-liquid theory required



Tomographic electron fluids

- Quasiparticle lifetimes
- Kinetic coefficients
- Nonlocal conductivity
- Tomographic transport
- New phenomena

Team



Patrick Ledwith '19



Lev Kendrick '19



Haoyu Guo '18



Andrey Shytov
(Exeter, UK)

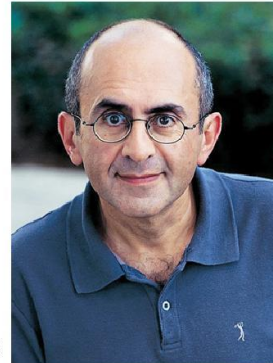


Serhii Kryhin '22

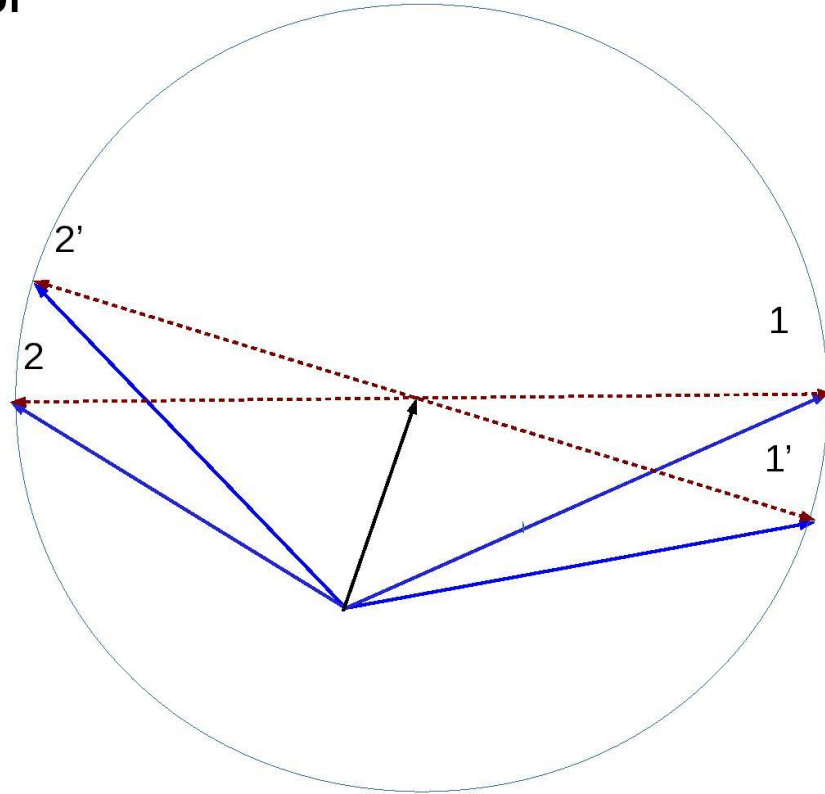


Qiantan Hong '21

Gregory Falkovich
Weizmann Institute

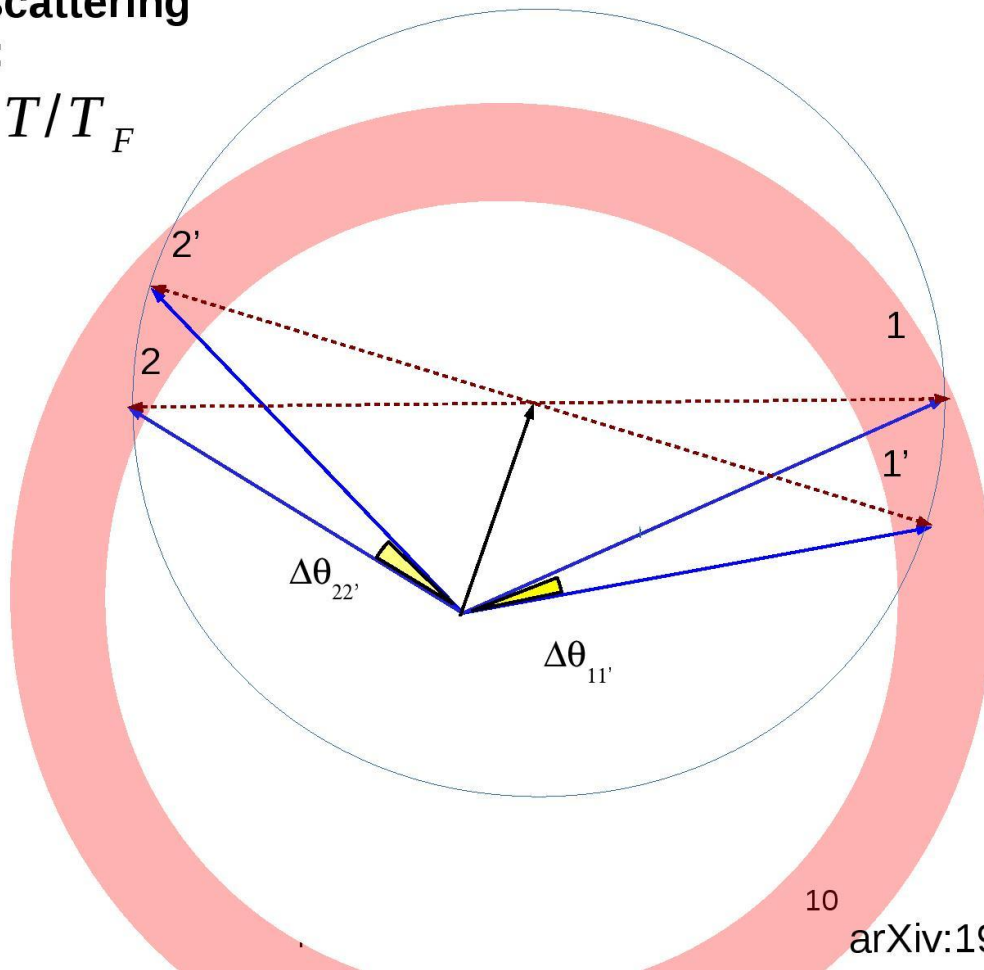


Kinematics of two-body collisions:



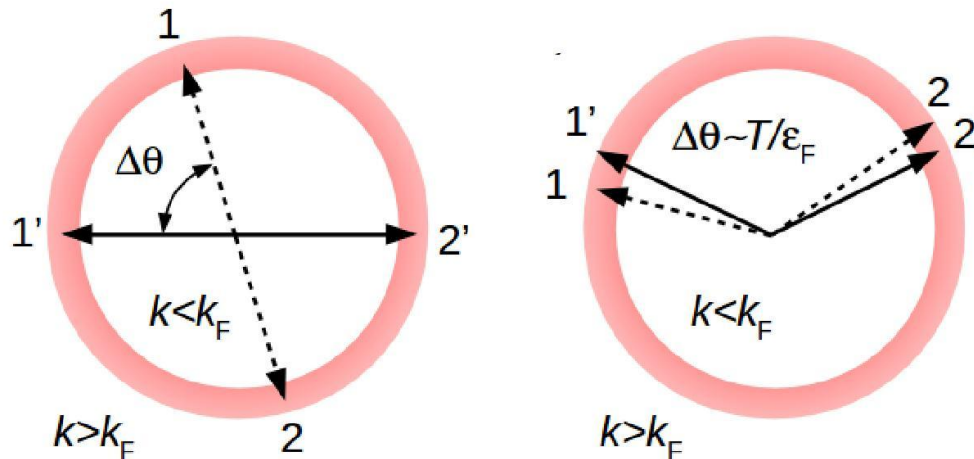
Small scattering
angles:

$$\Delta\theta \sim T/T_F$$



New behavior in 2D

- Momentum conservation and fermion exclusion single out two types of collisions: a) head-on, and b) small-angle
- Angular relaxation dominated by (near) head-on collisions.
- The **even-parity** and **odd-parity** parts of momentum distribution, $\delta f(p)=\delta f(-p)$ & $\delta f(p)=-\delta f(-p)$ relax at different rates



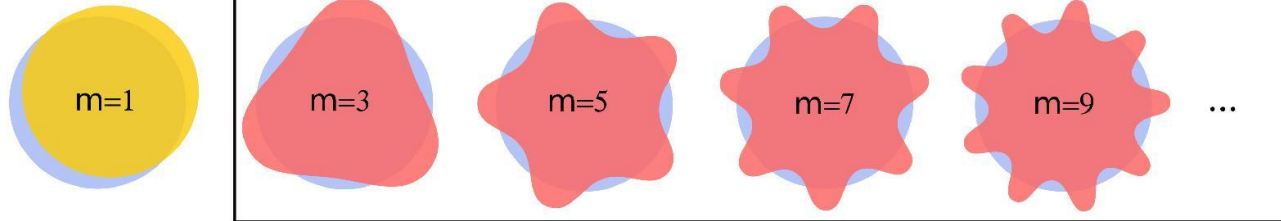
Long-lived excitations, directional memory & e-fluids in 2D

Even and odd harmonics

- The **even-parity** and **odd-parity** parts of momentum distribution, $\delta f(p)=\delta f(-p)$ & $\delta f(p)=-\delta f(-p)$ relax at different rates.
- Relaxation rates for the $\delta f(p)$ harmonics of the **odd** and **even** parity can differ by orders of magnitude: $\gamma'/\gamma \sim (T/T_F)^2$, $\gamma \sim T^2/T_F$

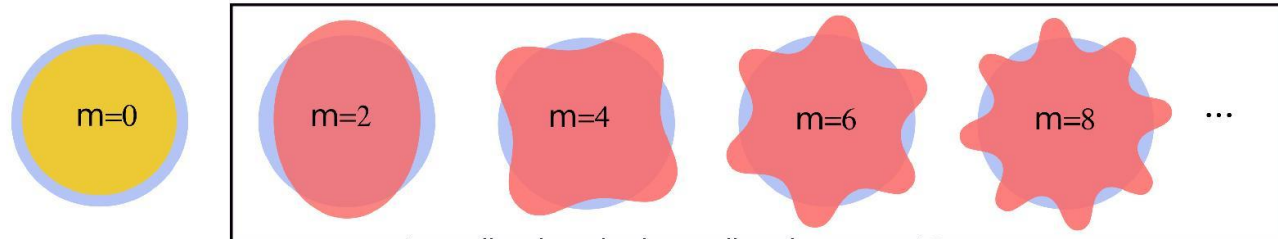
a) Odd-parity modes

Long-lived



b) Even-parity modes

Short-lived



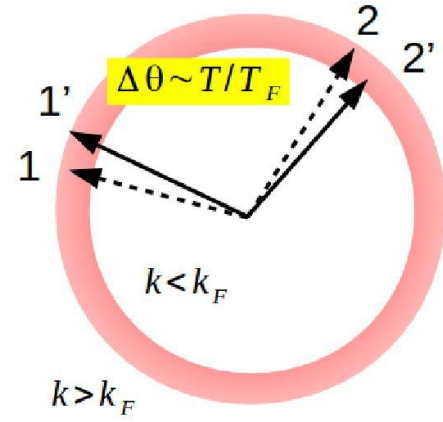
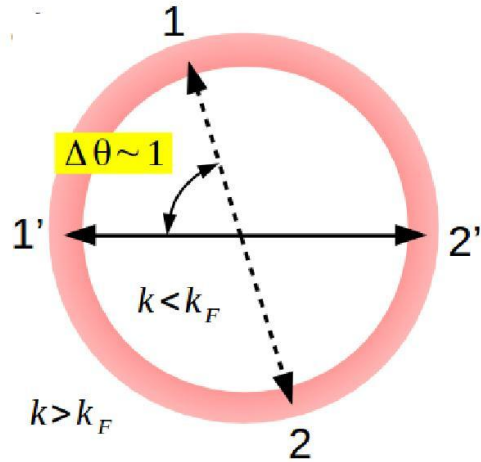
The rates γ_m

Estimating the rates

$$\gamma_{\text{even}} \sim R_* \frac{T^2}{T_F^2}$$

$$\gamma_{\text{odd}} \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2$$

Angular diffusion

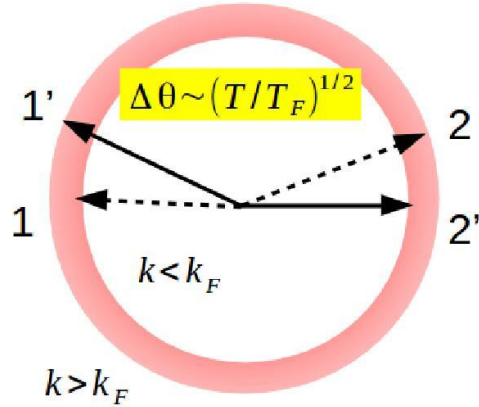
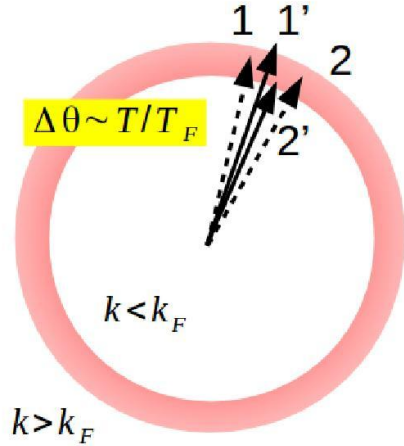


The odd- m rates:

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2$$

Naively:

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 = R_* \frac{T^3}{T_F^3} m^2$$



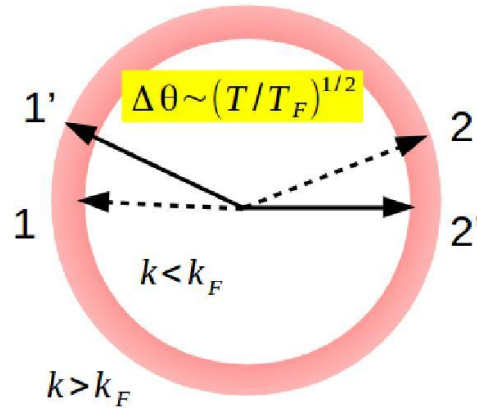
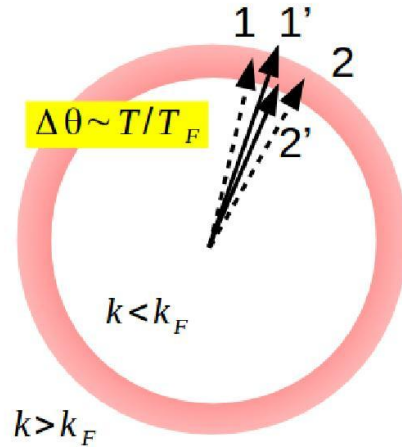
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Naively:

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Is this true? Not quite



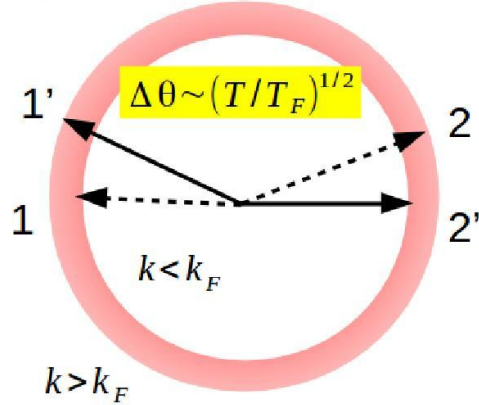
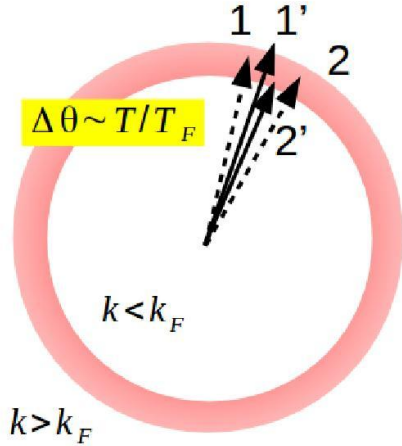
The odd- m rates:

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2$$

Actually: Angular superdiffusion

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} (\Delta\theta^2 m^2)^2 = R_* \frac{T^4}{T_F^4} m^4$$

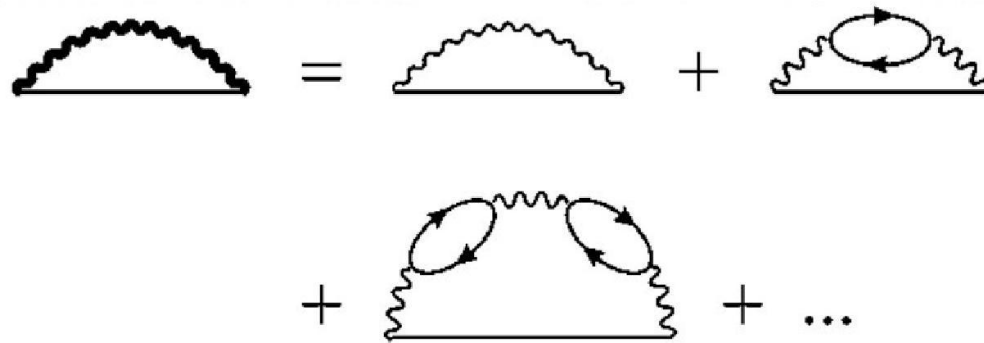
Origin: correlated angular shifts in ee scattering



cf. lifetimes from selfenergy in 2D

$$\gamma = -2 \Sigma''(\epsilon, p) \sim T^2 \ln(1/T)$$

Chaplik 1971
Hodges, Smith, Wilkins 1971
Bloom 1975
Giuliani, Quinn 1982
Menashe, Laikhtman 1996
Zheng, DasSarma 1996
Chubukov, Maslov 2003



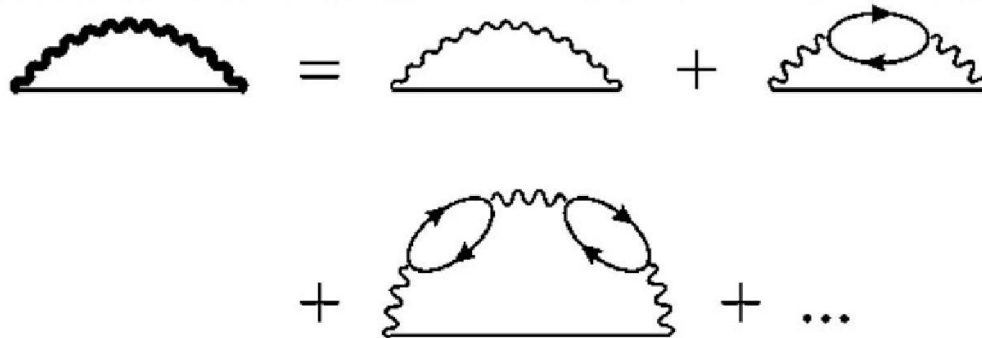
Long-lived excitations, directional memory & e-fluids in 2D

cf. lifetimes from selfenergy in 2D

$$\gamma = -2 \Sigma''(\epsilon, p) \sim T^2 \ln(1/T)$$

Dominated by the fast pathways (rapid decays)
and by (near) head-on collisions,
Insensitive to slowly decaying modes

Chaplik 1971
Hodges, Smith, Wilkins 1971
Bloom 1975
Giuliani, Quinn 1982
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Long-lived excitations, directional memory & e-fluids in 2D

Lifetime of two-dimensional electrons measured by tunneling spectroscopy

S. Q. Murphy,* J. P. Eisenstein, L. N. Pfeiffer, and K. W. West

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(Received 24 October 1994; revised manuscript received 7 June 1995)

For electrons tunneling between parallel two-dimensional electron systems, conservation of in-plane momentum produces sharply resonant current-voltage characteristics and provides a uniquely sensitive probe of the underlying electronic spectral functions. We report here the application of this technique to accurate measurements of the temperature dependence of the electron-electron scattering rate in clean two-dimensional systems. Our results are in qualitative agreement with existing calculations.

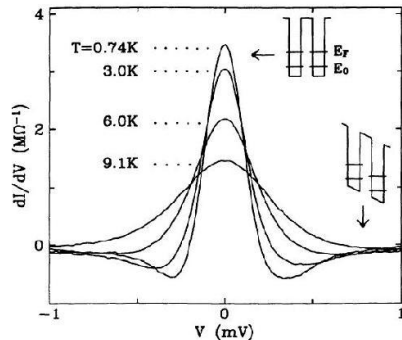


FIG. 1. Typical 2D-2D tunneling resonances observed at various temperatures in a sample with equal densities ($N_s = 1.6 \times 10^{11} \text{ cm}^{-2}$) in the two 2DES's. Insets show simplified band diagrams on and off resonance.

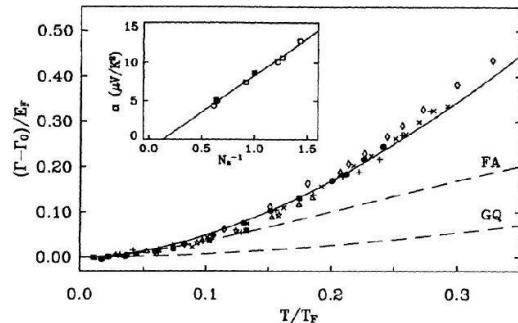


FIG. 3. Tunnel resonance width vs temperature for all samples (having eight different densities). On dividing Γ by T_F and the resonance width (minus the zero-temperature limit Γ_0) by E_F all the data collapse onto a single curve. The dashed lines are the calculations of GQ (Ref. 18) and FA (Ref. 20). The solid line is $6.3 \times$ GQ. Inset: Coefficient of T^2 term in Γ vs inverse density N_s^{-1} (in units of 10^{-11} cm^2).

Lifetimes of individual
modes with even and odd
 m , a direct calculation

Kinetic equation, how expansion in $T/T_F \ll 1$ fails

Linearize near equilibrium $f(\mathbf{p}) = f_0(\mathbf{p}) - \frac{\partial f_0}{\partial \epsilon} \eta(\mathbf{p})$, $f_0(1 - f_0) \frac{d\eta_1}{dt} = I_{ee} \eta$

$$I_{ee} \eta = \sum_{21'2'} \frac{2\pi}{\hbar} |V|^2 F_{121'2'} \delta_{\epsilon_1 + \epsilon_2 - \epsilon_{1'} - \epsilon_{2'}} \delta_{\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_{1'} - \mathbf{p}_{2'}}^{(2)} (\eta_{1'} + \eta_{2'} - \eta_1 - \eta_2) \quad F_{121'2'} = f_1^0 f_2^0 (1 - f_{1'}^0)(1 - f_{2'}^0)$$

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Focus on individual angular harmonics

$$\eta(\mathbf{p}, t) = e^{-\gamma_m t} e^{im\theta} \chi_m(x) \quad -\gamma_m f_0(1 - f_0) \chi_m(x) = I_{ee} \chi_m(x)$$

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Integrating over angles yields

$$f_0(1 - f_0) \frac{d\chi(x_1)}{dt} = gT^2 \int dx_2 dx_{1'} dx_{2'} F \delta(x_1 + x_2 - x_{1'} - x_{2'}) [\chi(x_1) - \chi(x_2)]$$

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Introduce Fourier transform in the energy variable

$$\chi(x) = 2 \cosh \frac{x}{2} \zeta(x) \quad \zeta(x) = \int dk e^{ikx} \psi(k)$$

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Obtain a 1D Schrodinger equation with a secanth potential (Poschl-Teller problem)

$$\partial_t \psi(k) = gT^2 \left[\left(\frac{\pi^2}{2} - \frac{\pi^2}{\cosh^2 \pi k} \right) \psi(k) - \frac{1}{2} \psi''(k) \right]$$

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Zero modes, one per each odd m

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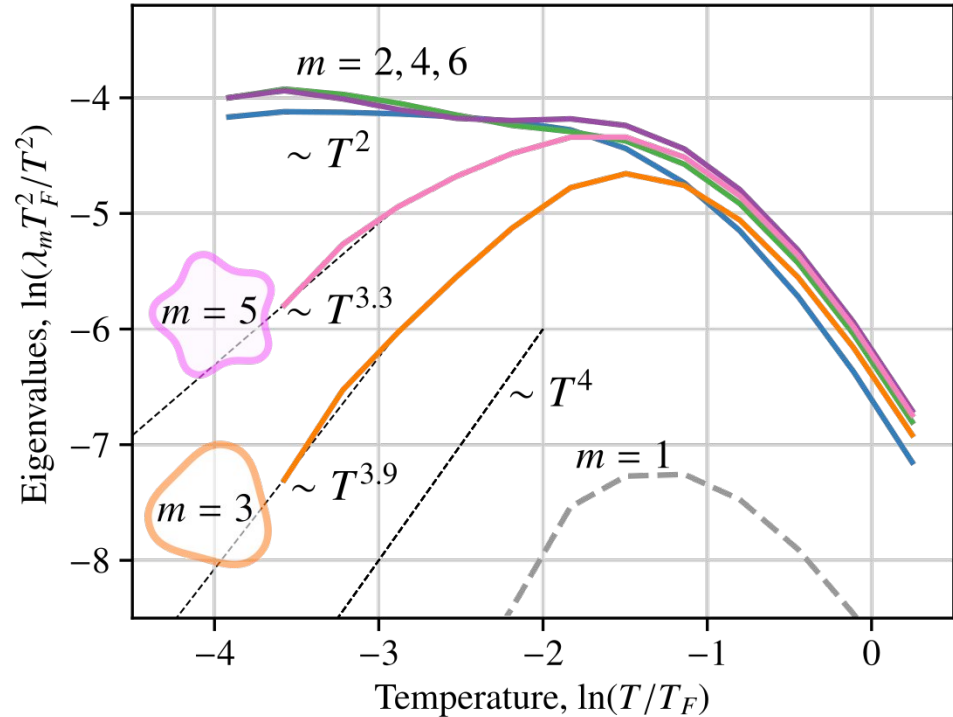
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Infinite lifetimes at order T^2

Numerically diagonalize the linearized collision operator

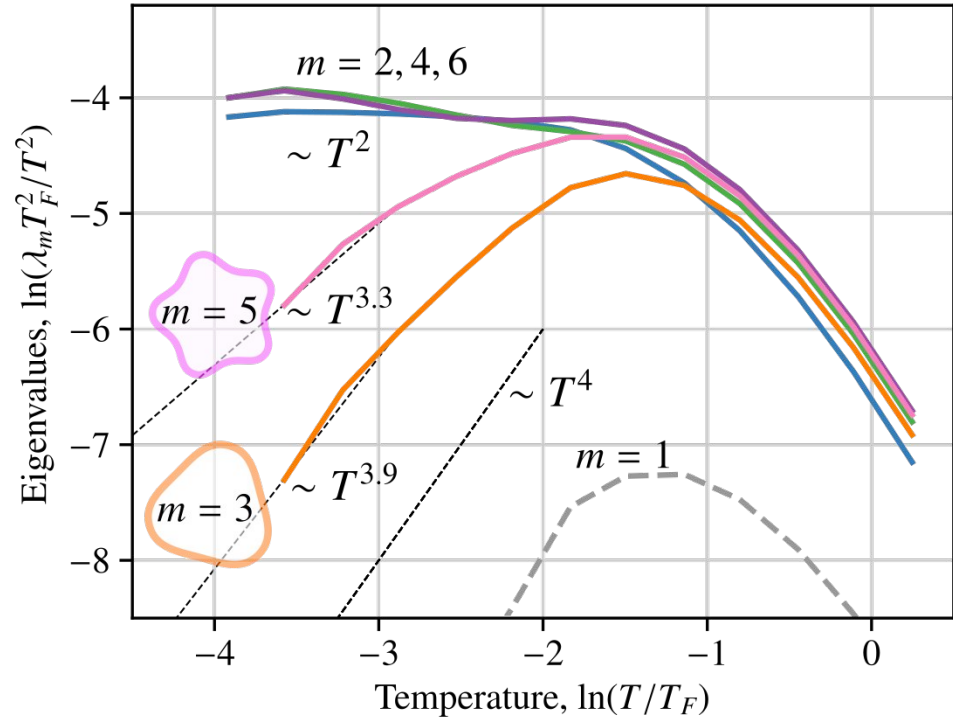
A method that does not rely on a small parameter $T \ll T_F$



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Long-lived excitations: super-Fermi-liquid lifetimes for odd- m harmonics

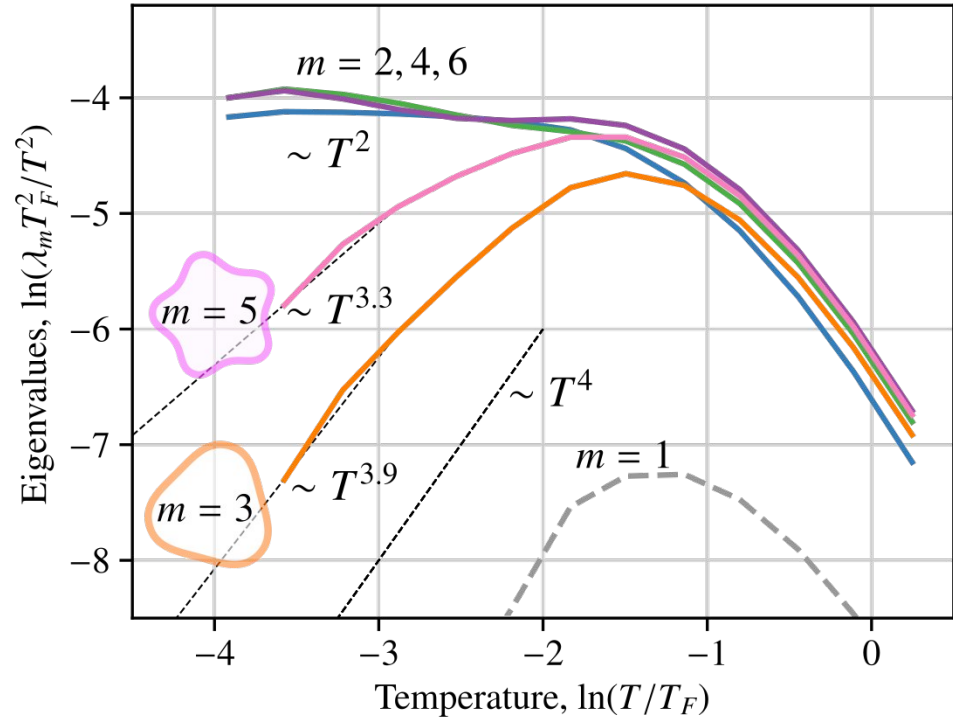


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Scaling $\gamma_m \sim T^\alpha$, $\alpha \sim 4$



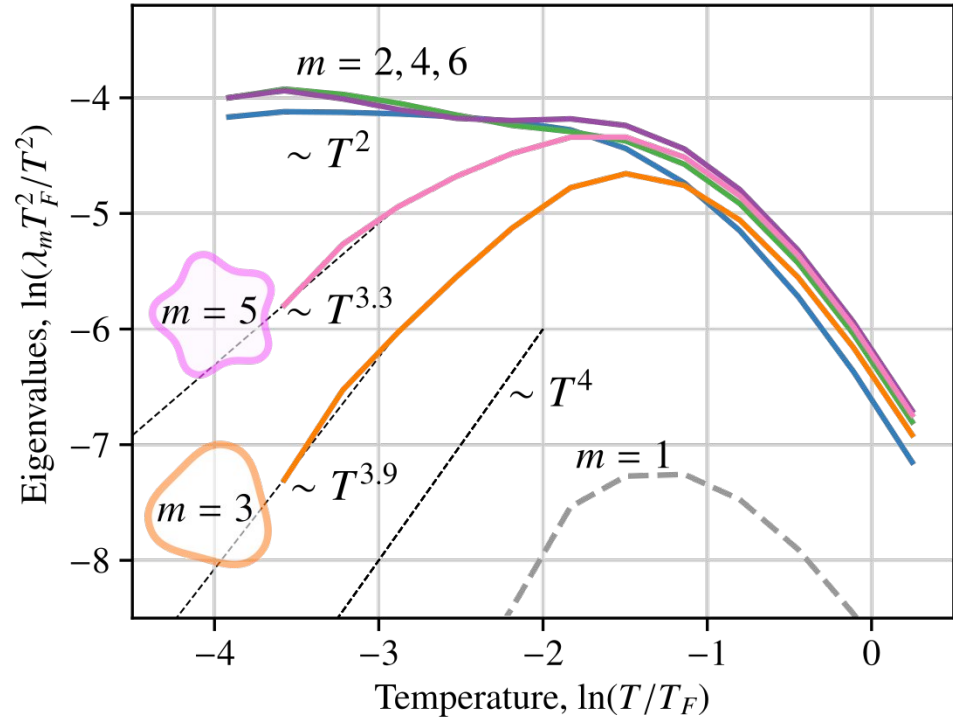
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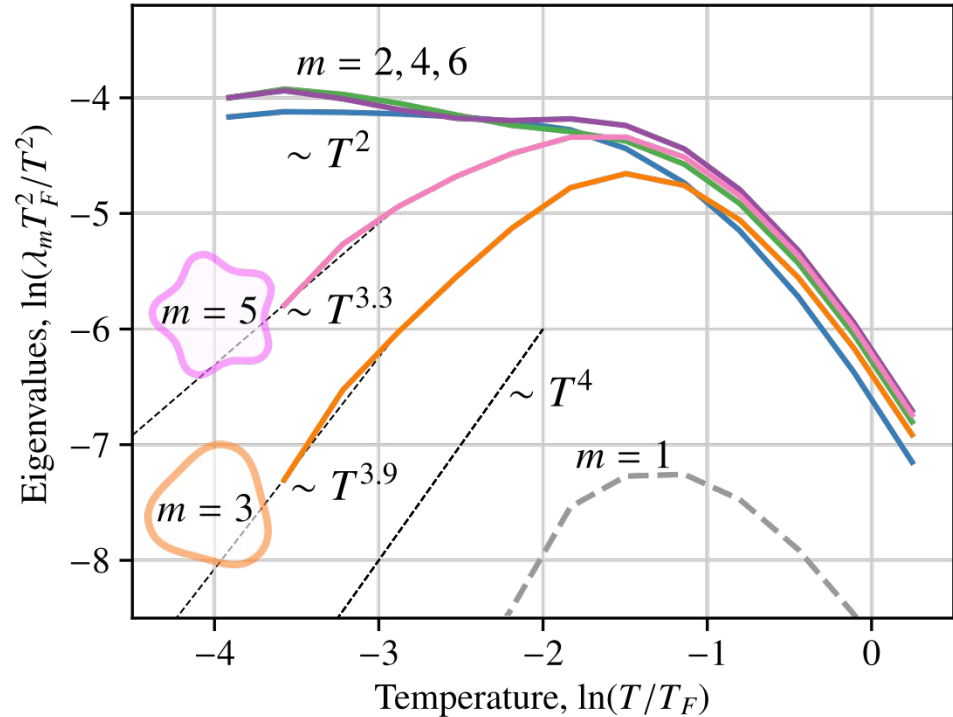
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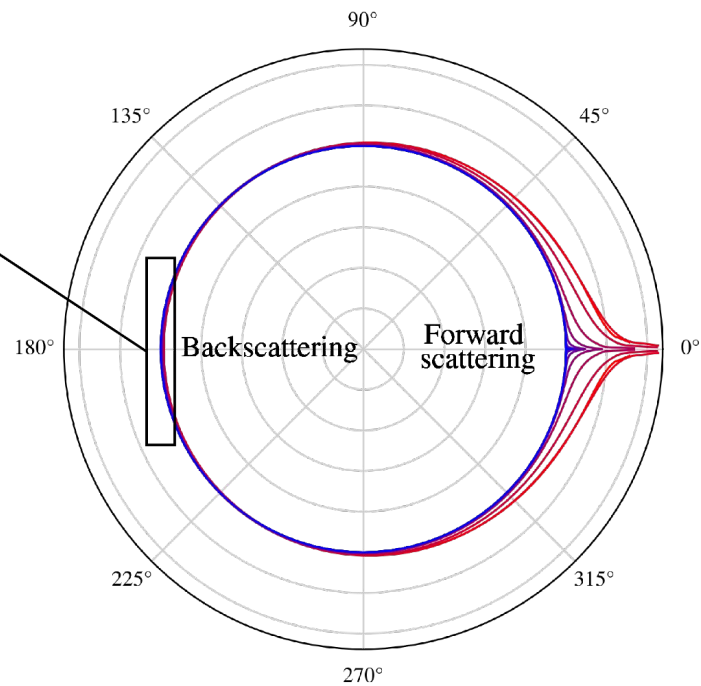
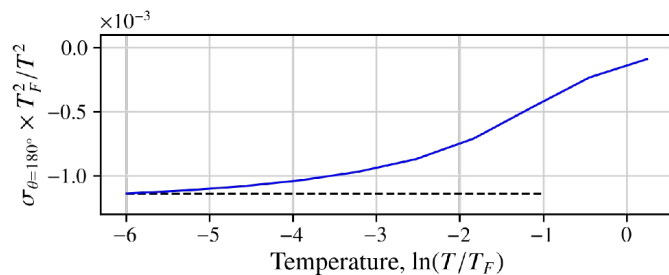
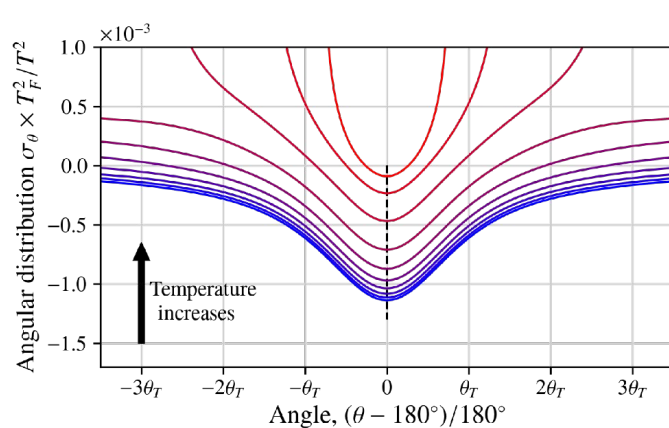
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A hierarchy of time scales: $\gamma_{m \text{ odd}} \ll \gamma_{m \text{ even}}$



Angular distribution for two-body scattering

$\sim T^2$ for generic angles

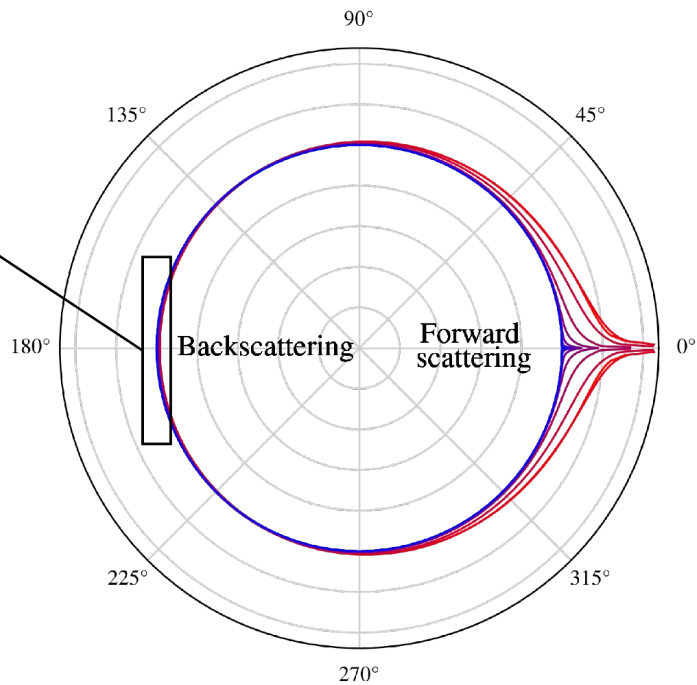
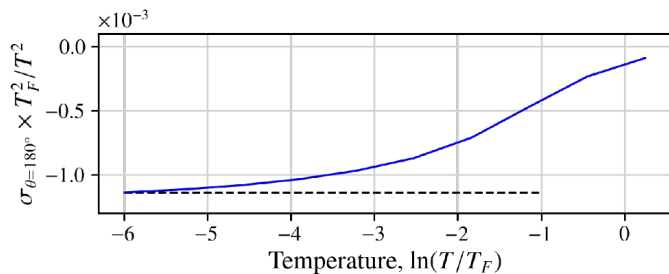
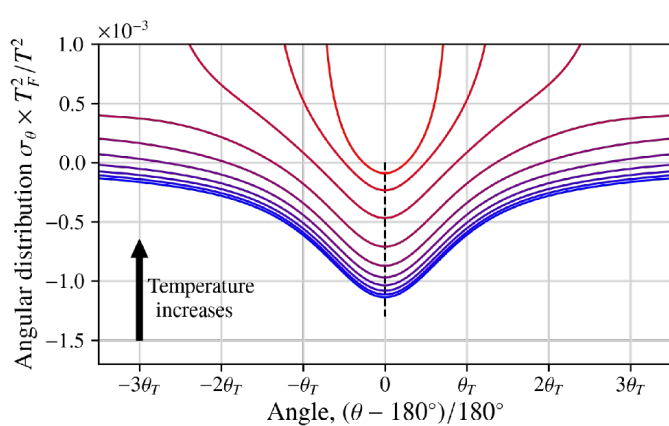


$$\sigma(\theta) = \sum_m e^{im(\theta - \theta_i)} (\gamma_m - \gamma_0)$$

Angular distribution for two-body scattering

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Sharp peaks σ
 $(\theta) \sim T^2/|\theta|$, σ
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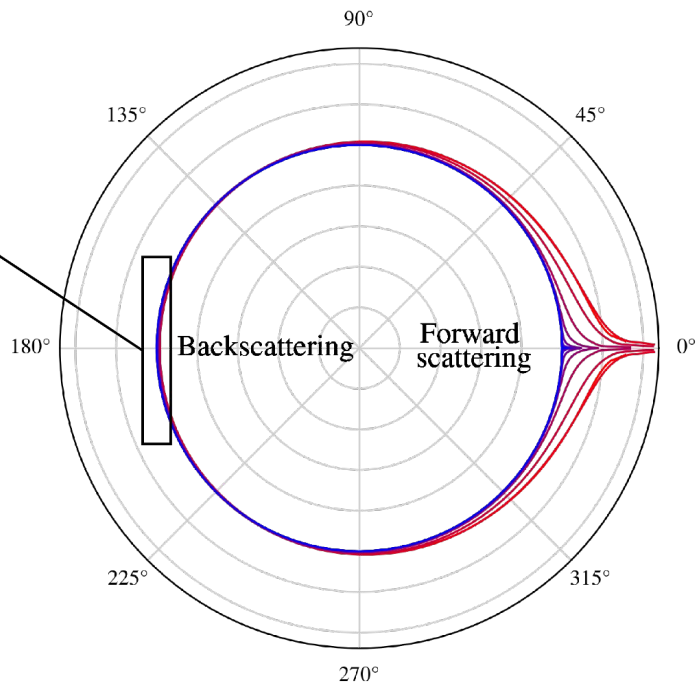
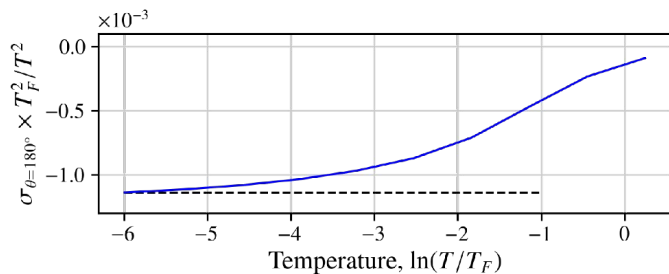
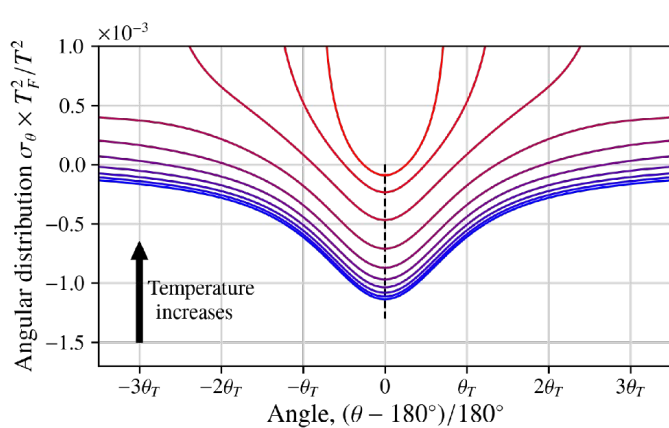
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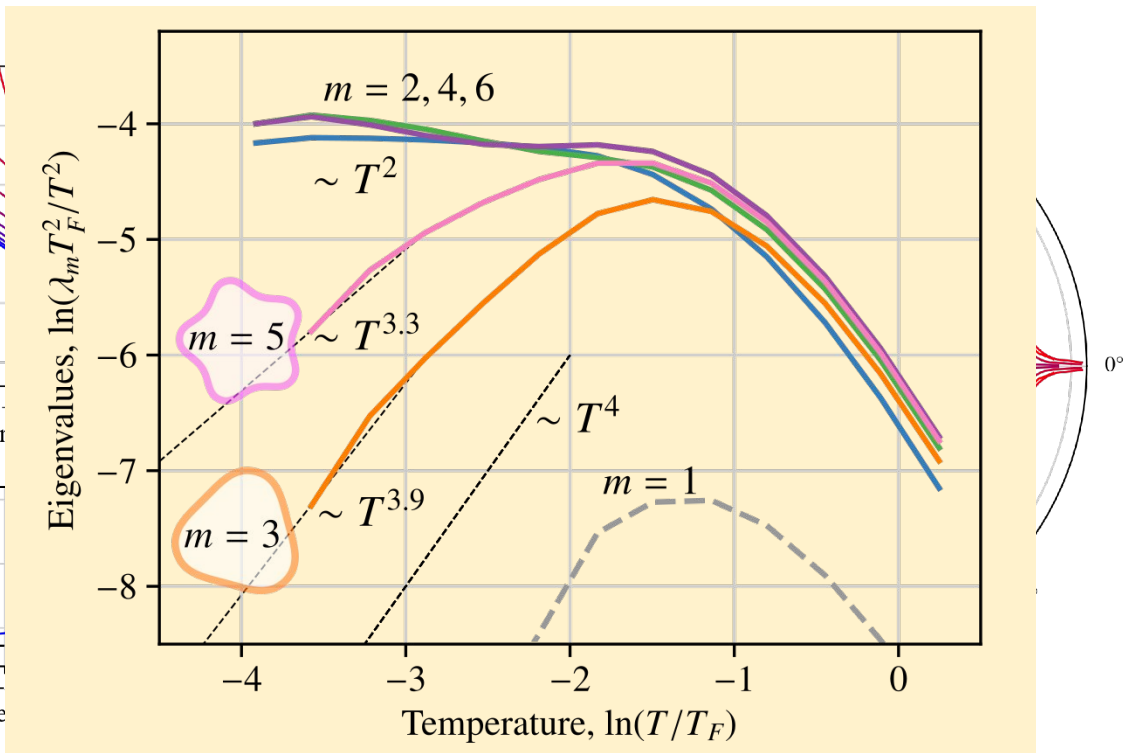
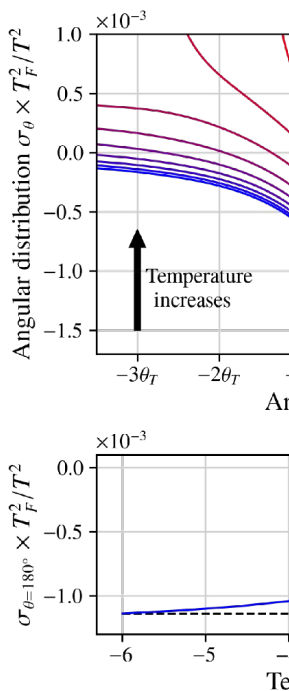
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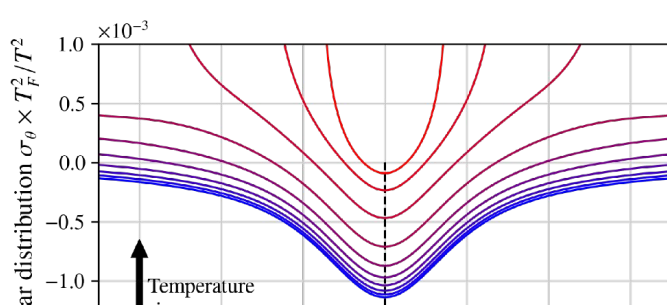
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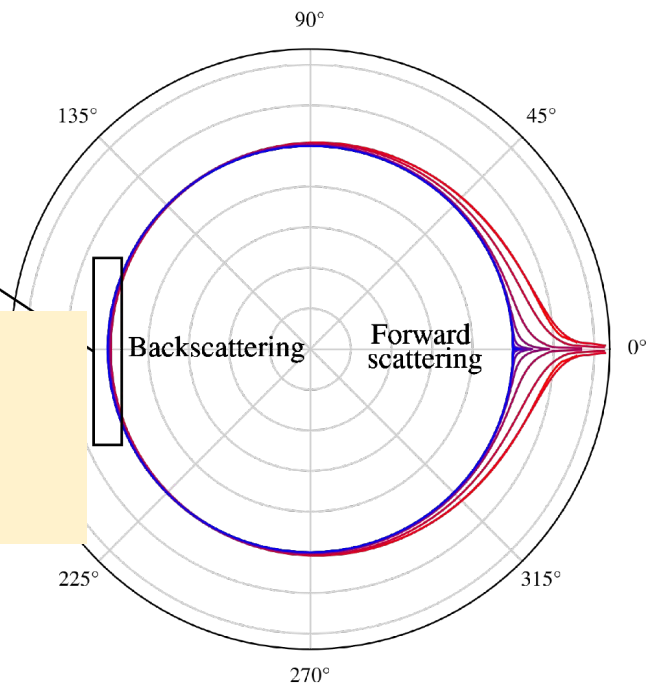
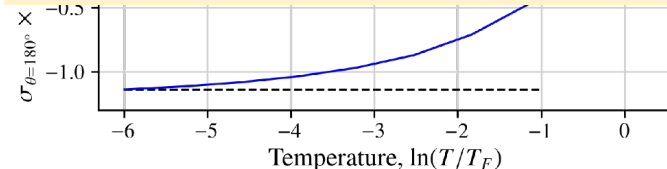
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Log enhancement of total cross-section:
 $\int d\theta \sigma(\theta) \sim T^2 \log(T_F/T)$

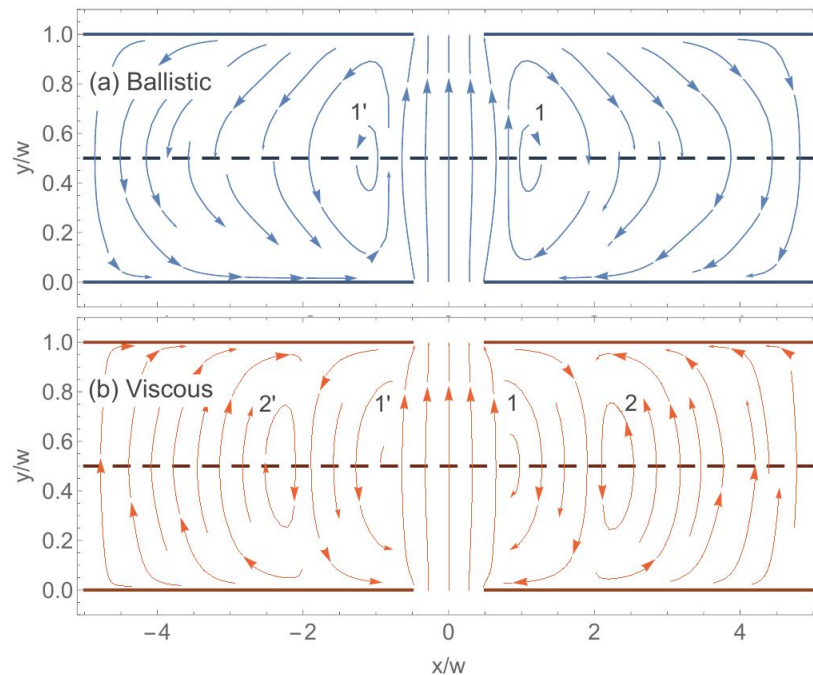


Observables?

$$z = \frac{k^2 v^2}{4}$$

Nonlocal conductivity $j(r) = \int d^2 r' \sigma(r-r') E(r')$

$$\sigma(k) = \frac{D}{\gamma_0 + \Gamma(k)} \quad \Gamma(k) = \frac{z}{\gamma_1 + \frac{z}{\gamma_2 + \frac{z}{\gamma_3 + \dots}}}$$



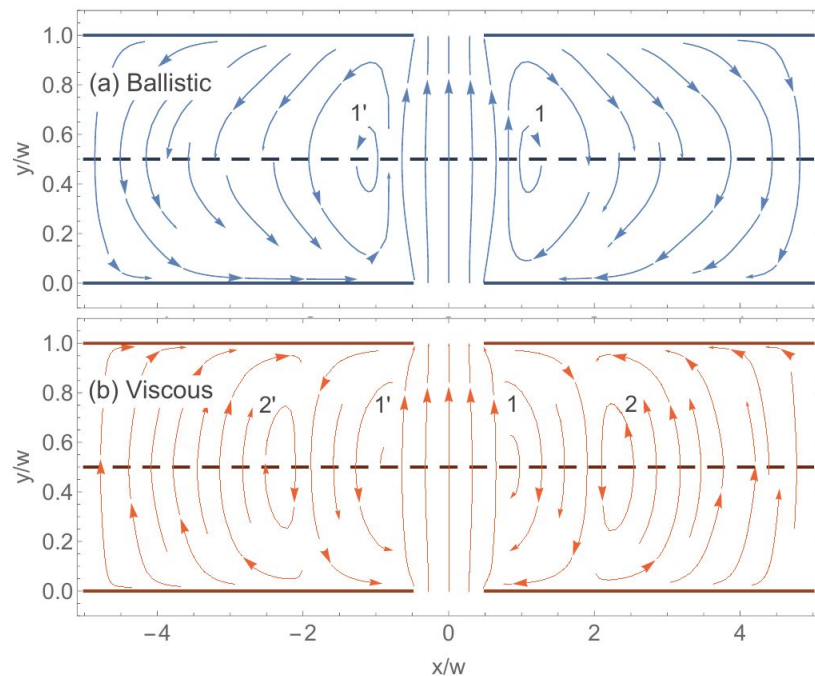
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A continued fraction representation of k-dependent response: $j_k = \sigma(k) E_k$



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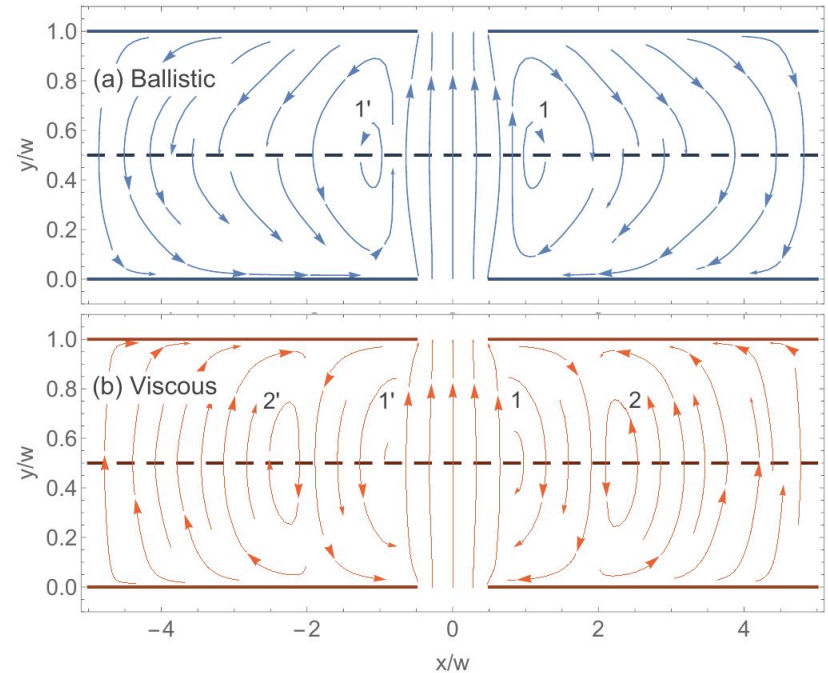
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$\sigma(k)$ determines spatial distribution of vorticity and the sensitivity of vortices to momentum relaxing scattering by disorder & phonons



Vortices in electron fluids, hydro and non-hydro

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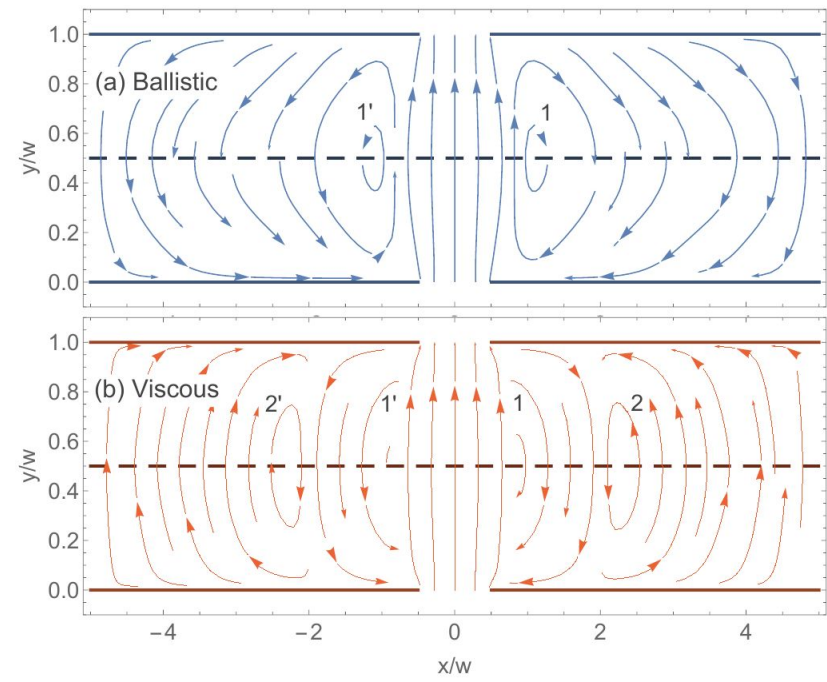
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A continued fraction representation of the k -dependent response: $j_k = \sigma(k) E_k$

$\sigma(k)$ determines spatial distribution of vorticity and the sensitivity of vortices to momentum relaxing scattering by disorder & phonons

The relaxation rates γ_m are a “genetic code” that governs the dispersion $\sigma(k)$

Robustness of vortices



Summary/discussion

Abnormally long-lived excitations in a 2D Fermi gas with super-Fermi-liquid lifetimes

Origin: collinear scattering

Implications: sharp angular distributions of scattered particles, hole backscattering, $\log(T_F/T)$ enhanced Fermi-liquid decay rates for other excitations

Robustness: generic 2-body interactions and particle dispersion, OK for weakly non-circular Fermi surfaces

Manifestations: nonlocal transport, current vortices, angular memory of response functions