# Turing instability in electron fluids

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# Have you ever thought about how zebras get their stripes?

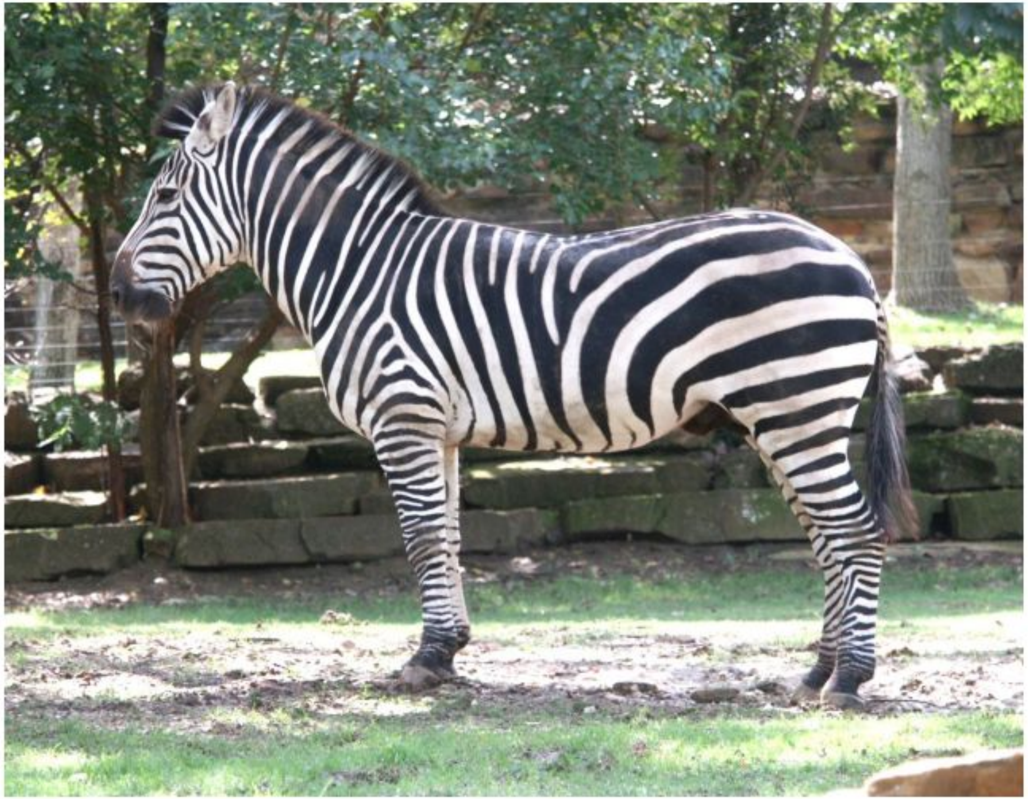


Photo taken from whozoo.org

## **Alan Turing** (1912-1954)

Photo from wikipedia.org

- In 1952, Turing put forth a model for spatial pattern formulation of chemicals reacting and diffusing throughout tissue.
- The model is a system of partial differential equations known as the *Reaction-Diffusion Model*.
- Such spatial patterns in the chemicals are thought to play a role in the determination of the type of melanin is produced by the melanocytes, and thus impact the formation of patterns in the mammalian coats.

# Turing instability in Reaction-Diffusion model

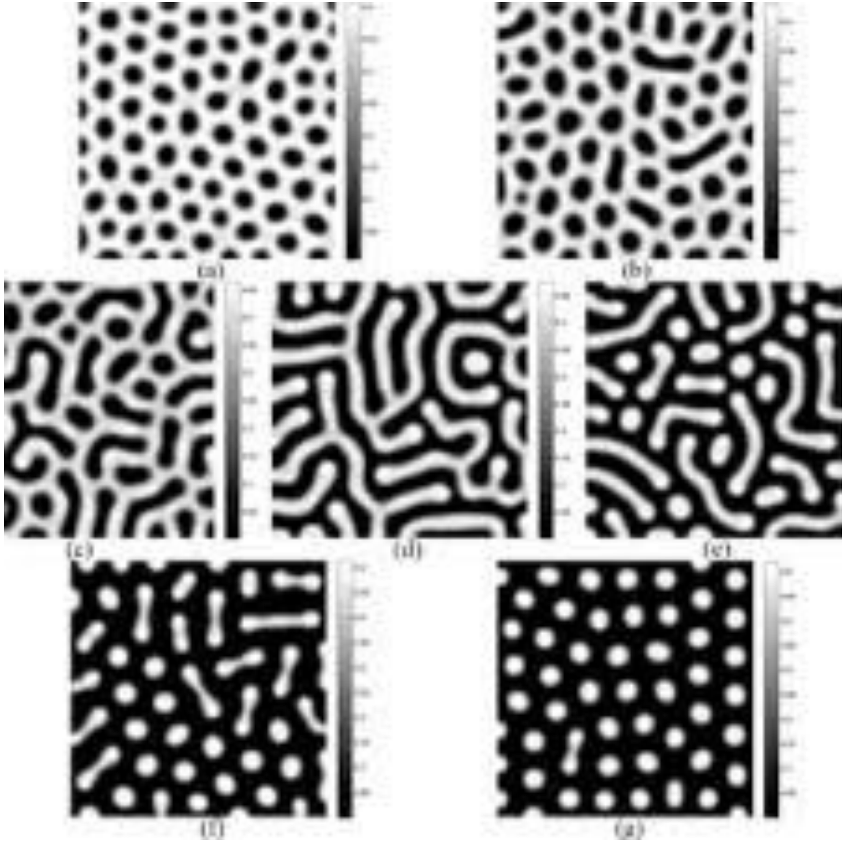
$$
\begin{aligned}
u_t &= \Delta u + \gamma \cdot f(u, v) \\
v_t &= d\Delta v + \gamma \cdot g(u, v).
\end{aligned}
$$

Turing concluded that the Reaction-Diffusion model may exhibit spatial patterns under the following two conditions:

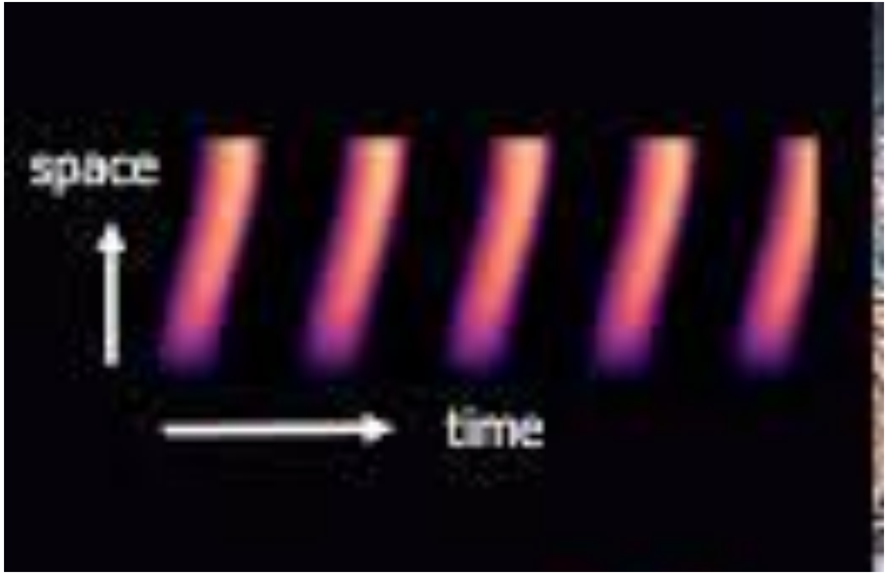
1. the equilibrium solution is linearly stable in the absence of diffusion,
2. the equilibrium solution is linearly unstable in the presence of diffusion.

Such an instability is called a *Turing instability* or *diffusion-driven instability*.

# Turing pattens variety

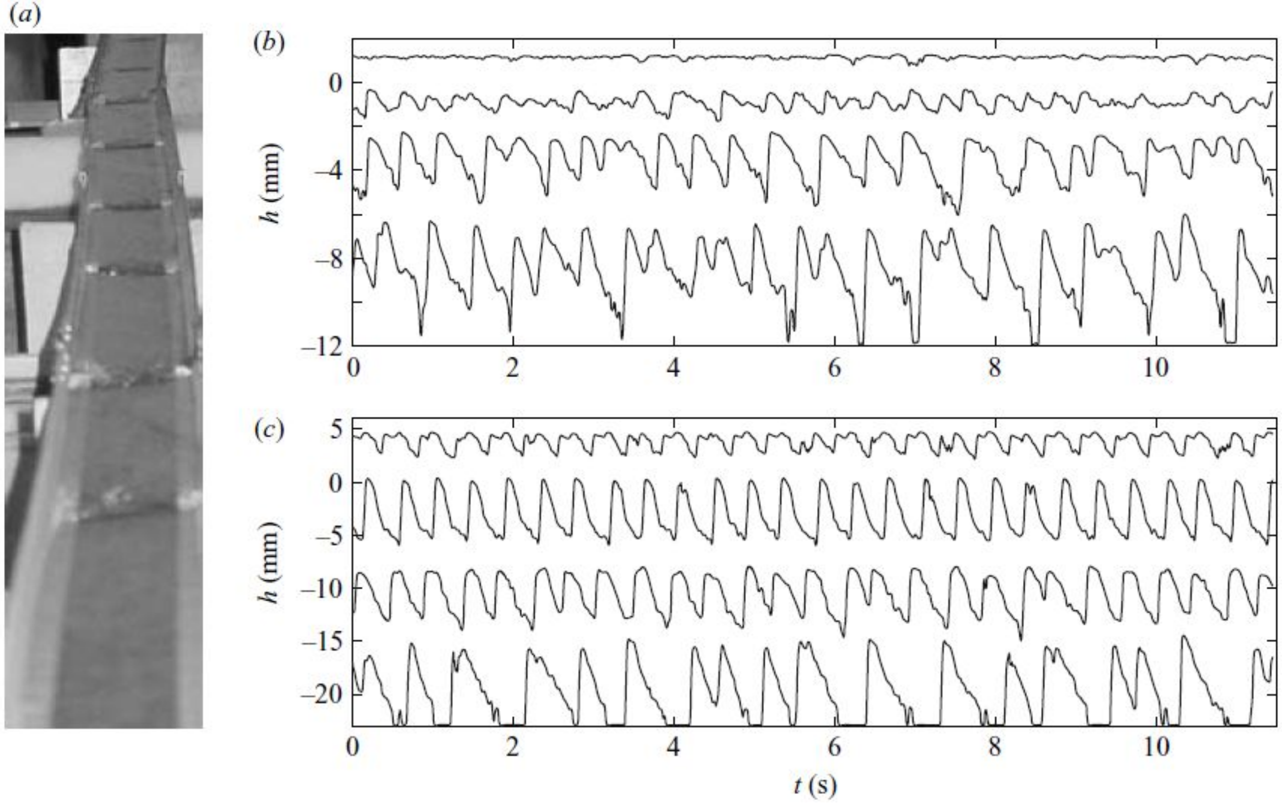



# Can Turing instability be triggered by current?




That would correspond to zebras in space-time.

# Roll waves (driven by Kapitsa instability)




From: Balmforth and Mandre, Dynamics of roll waves, J. Fluid Mech. (2004)

# Roll waves vs. self-sustained waves

In fluids: capillary waves and gravity waves
In solids: plasma waves, phonons

- Self-sustained waves exist even if the fluid is not moving as a whole,
- Propagate with the velocity v=d$\omega$/dk
- Exist when $\omega$(k)$\gg\gamma$ (weak damping)
- In contrast, roll waves (or, Kapitsa waves) occur when the fluid is moving, but are absent in a still fluid
- Excited even when $\omega$(k)$\ll\gamma$ (strong damping)
- Propagate with (approximately) the flow velocity, downstream

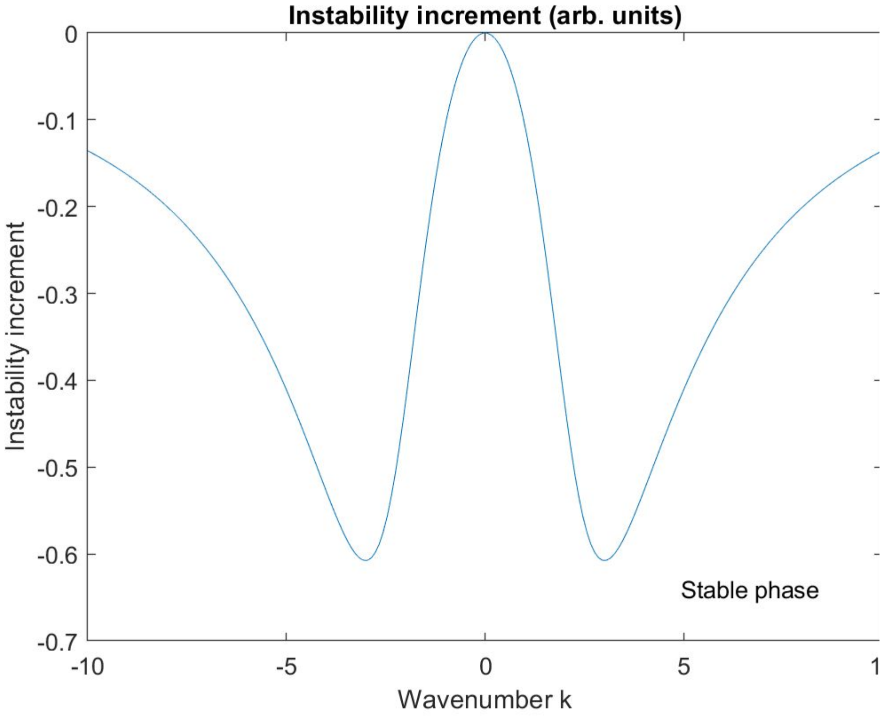
## Electronic Kapitsa instability in a dissipative flow

Fermi liquid hydrodynamics $\partial_t n + \nabla j = 0$, $\partial_t p + \nabla \Pi + \gamma p = enE$, where n and p – particle and momentum density – are conserved quantities, j and $\Pi$ – current and stress tensor (here, Fermi pressure) – are functions of conserved quantities, $\gamma$ is momentum dissipation rate (disorder or phonons)

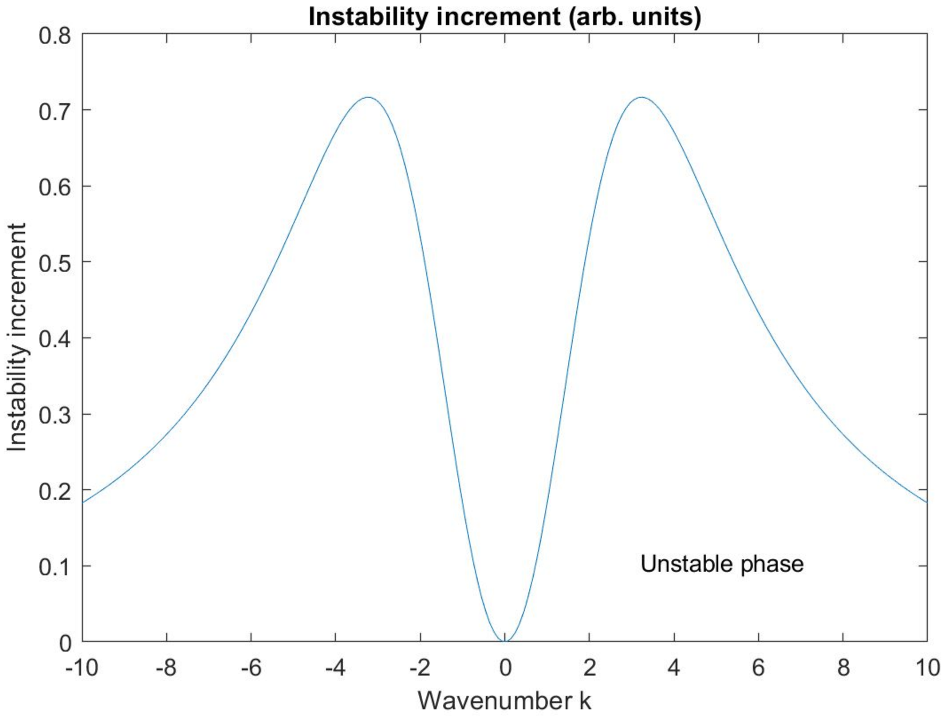
Derive the instability dispersion relation

# Electronic Kapitsa instability–increment vs. k

Stable phase

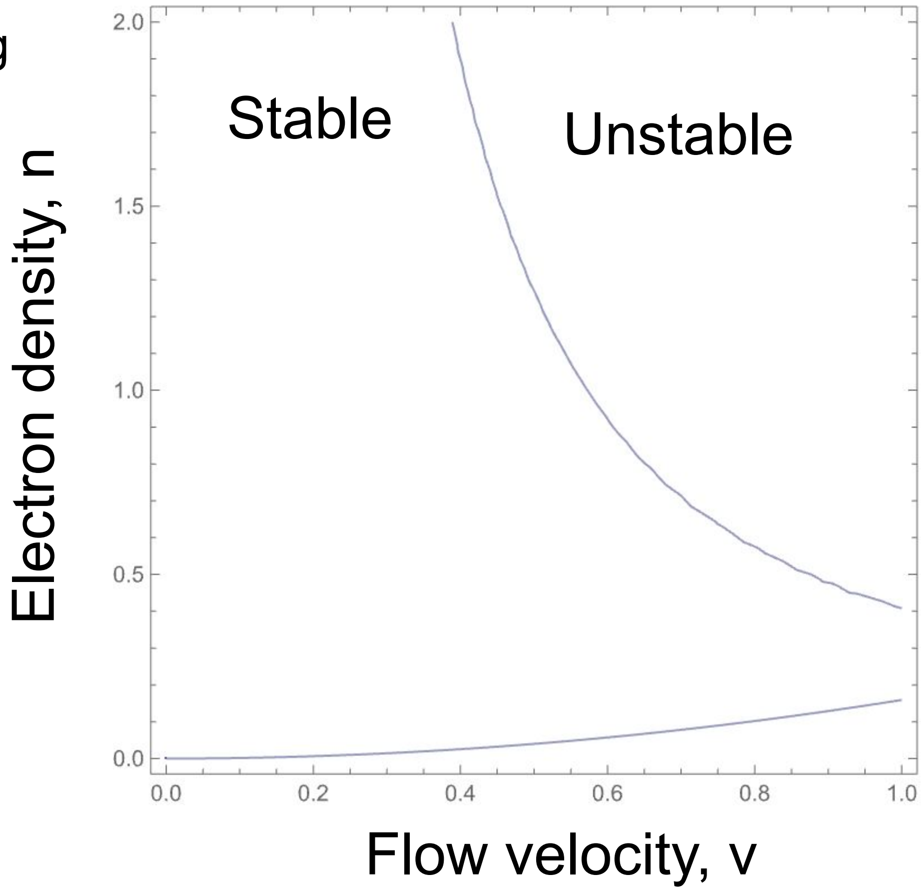



Unstable phase



# Electronic Kapitsa instability–Phase diagram

Credit: Prayoga Liong

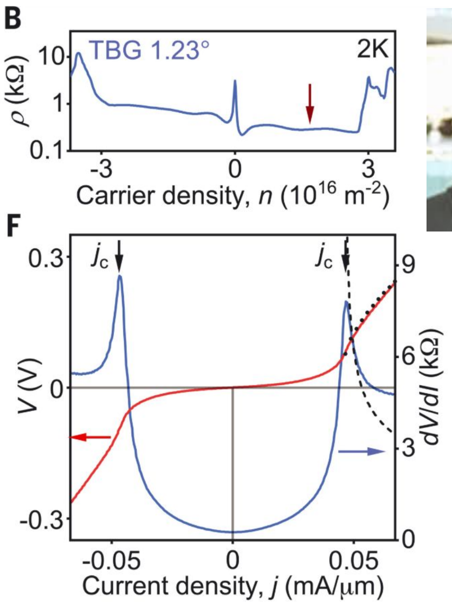


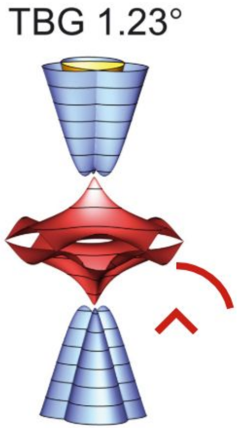

$v \ll v_F$ (Slow flow)

$v = v_F$ (Fast flow)

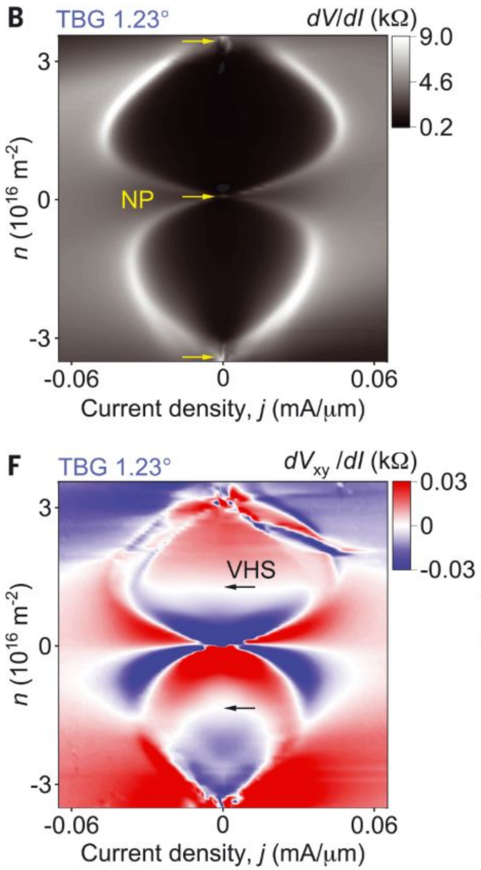
# Possible relation with current-induced criticality?

**Hydrodynamic instabilities** under small, experimentally accessible fields. Current-induced inversion of band occupation (experiments in graphene multilayers and monolayers, moire and non-moire. Current-driven ordering, instability?




Science 375, 430-433 (2022) Berdyugin, Krishna Kumar et al, (Geim group)



Questions?