Quantum Hall Interferometer in Graphene
Boulder Summer School Lecture 3

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Acknowledgement

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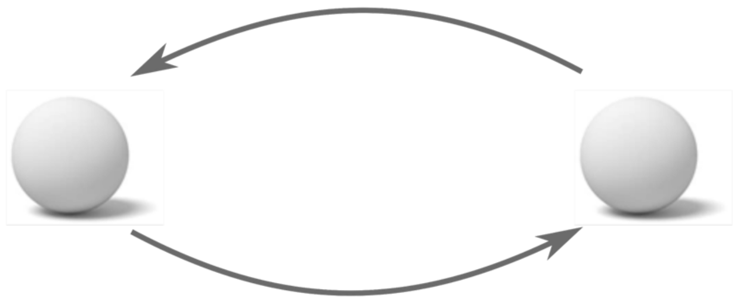
Theory

Bertrand Halperin

Dima Feldman

Anyons in 2-Dimension

Indistinguishable particles



3-Dimension (and higher)

bosons $\psi_{(r_1,r_2)} \rightarrow +\psi_{(r_2,r_1)}$

fermions $\psi_{(r_1,r_2)} \rightarrow -\psi_{(r_2,r_1)}$

2-Dimension is special!

For non-degenerate ground state

$$\psi_{(r_1,r_2)} \rightarrow e^{i\theta} \cdot \psi_{(r_2,r_1)}$$

$\theta = \pm\pi/m$

m = 1,2, 3 ...

Anyon quasiparticles

If the ground state is degenerated,

$$\vec{\psi}_{(r_1,r_2)} \rightarrow U_{12}\vec{\psi}_{(r_2,r_1)}$$

U_{12} : unitary operator

F. Wilczek, Phys. Rev. Lett. **49**, 957 (1982).

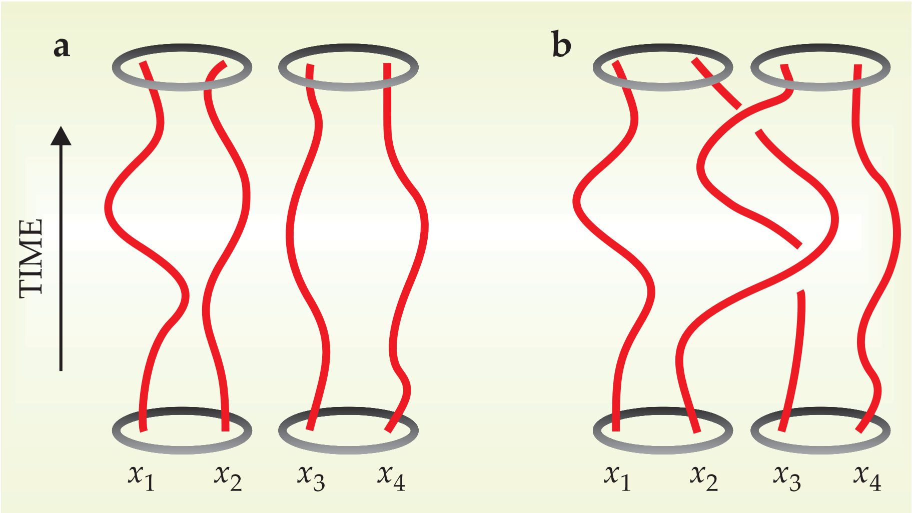
Non-Abelian Anions For Topologically Protected Qubit

Non-abelian anyons

$$\vec{\psi}_{(r_1,r_2)} \rightarrow U_{12}\vec{\psi}_{(r_2,r_1)}$$

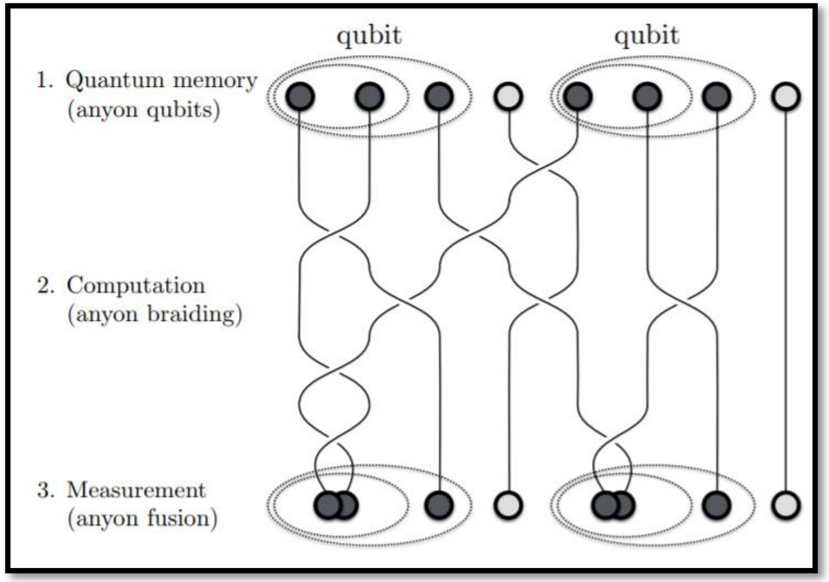
$$U_{12}\,U_{21} \neq U_{21}\,U_{12}$$

By braiding the anyons one can create non-local entangled qubits



Das Sarma, Freedman, Nayak Physics Today (2006)

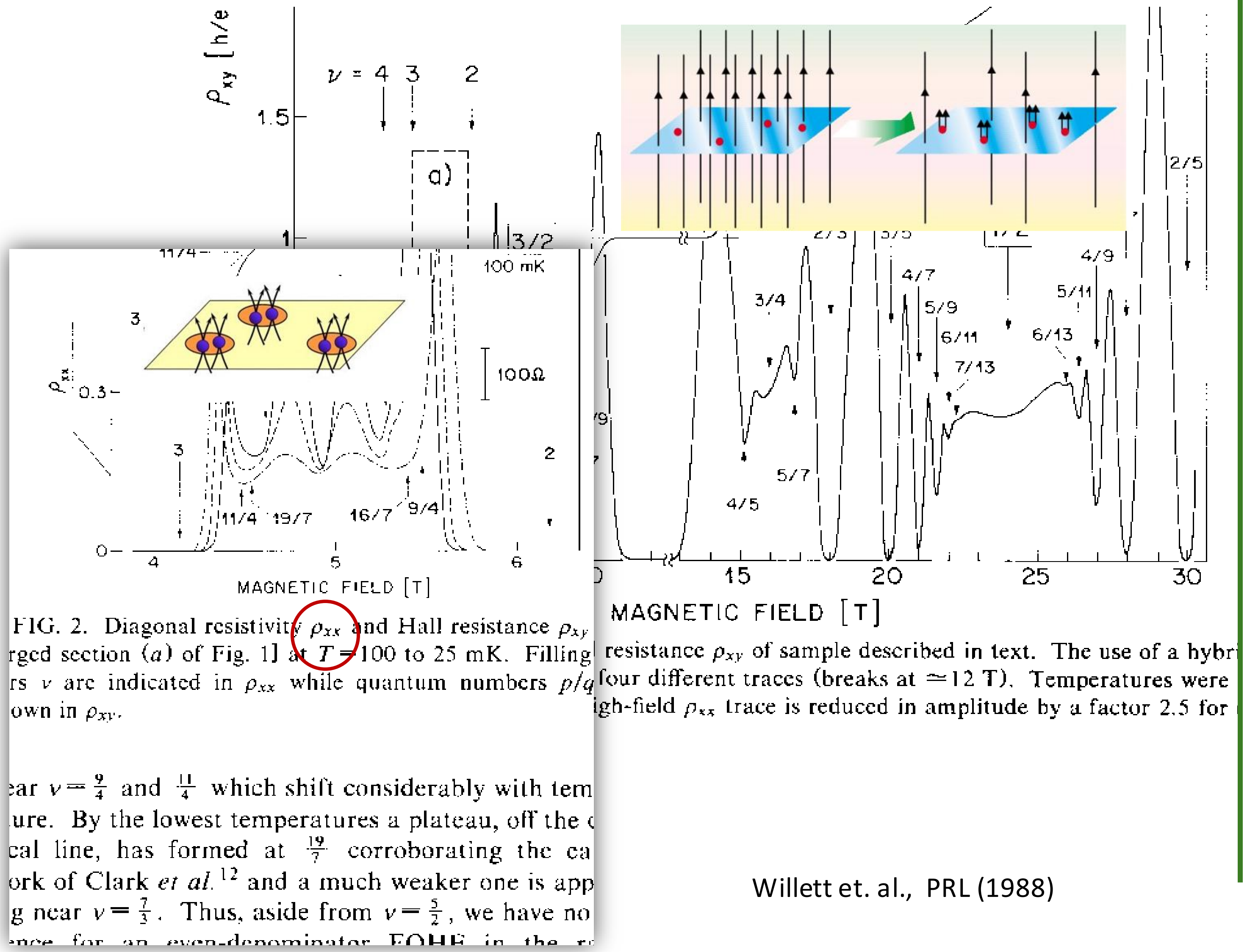
Topologically protected quantum computing with non-abelian anyons



Field and Simula, (2018)

Anyons in Fractional Quantum Hall

Fractional Quantum Hall Effect in electrons in GaAs quantum well



Willett et. al., PRL (1988)

Quasiparticle excitation in FQH states are anyons!

- **Abelian anyons:**

Most of odd denominator fractions

Eg.

$\nu = 1/3$ $\left\{ \begin{array}{l} e^* = e/3 \\ \theta = \pi/3 \end{array} \right.$

$\nu = 2/5$ $\left\{ \begin{array}{l} e^* = e/5 \\ \theta = 2\pi/5 \end{array} \right.$

…

- **Non-Abelian anyons:**

Some of even denominator fractions

Eg. $\nu = 5/2$ $e^* = e/4$

…

Aharonov-Bohm (AB) effect

THE
PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893
SECOND SERIES, VOL. 115, No. 3 AUGUST 1, 1959

Significance of Electromagnetic Potentials in the Quantum Theory

Y. AHARONOV AND D. BOHM
H. H. Wills Physics Laboratory, University of Bristol, Bristol, England
(Received May 28, 1959; revised manuscript received June 16, 1959)

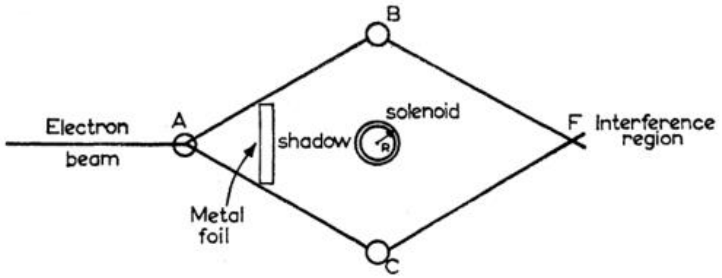


FIG. 2. Schematic experiment to demonstrate interference with time-independent vector potential.

suggests that the associated phase shift of the electron wave function ought to be

$$\Delta S/\hbar = -\frac{e}{c\hbar}\oint \mathbf{A}\cdot d\mathbf{x},$$

where $\oint \mathbf{A}\cdot d\mathbf{x} = \int \mathbf{H}\cdot d\mathbf{s} = \phi$ (the total magnetic flux inside the circuit).

wave packets, the period of the cycle being $\Phi_0 = h/e$. This interference should be reflected in the transport properties of the ring as described by Landauer's formula.[2–4] In this Letter, we describe the first experimental observation of the oscillations periodic with respect to Φ_0 in the magnetoresistance of a normal-metal ring.

Interference effects involving the flux h/e have been previously observed in a two-slit interference experiment involving coherent beams of electrons.[5] Magnetoresistance oscillations in single-crystal whiskers of bismuth periodic in h/e have been reported at low fields for the case where the extremum of the Fermi surface is cut off by the sample diameter.[6] Resistance oscillations of period $h/2e$ (flux quantization) have been seen in superconducting cylinders.[7] Four years ago, magnetoresistance oscillations of period

reason, it is believed that samples much longer than L_ϕ physically incorporate the ensemble averaging.[4,13] Each section (longer than L_ϕ) of a macroscopic sample is quantum-mechanically independent because the electron states are randomized between the sections. The single mesoscopic ring (diameter $< L_\phi$) does not average in this way because the entire sample is quantum-mechanically coherent.[4,13]

There exists a further complication in normal metals; the magnetic flux penetrates the wires composing the device. Stone[14] has shown that the flux in the wire leads to an *aperiodic* fluctuation in the magnetoresistance. This fluctuation was the main complication in interpreting the earlier experiments[15] where the diameter of the ring was not much larger than the widths of the wires. On the basis of the analysis, a prediction was made that, in a ring having an area much larger

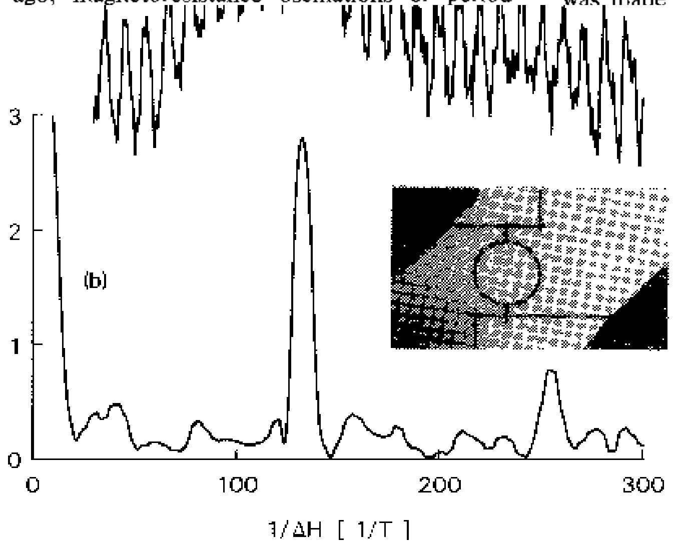


FIG. 1. (a) Magnetoresistance of the ring measured at $T = 0.01$ K. (b) Fourier power spectrum in arbitrary units containing peaks at h/e and $h/2e$. The inset is a photograph of the larger ring. The inside diameter of the nm, and the width of the wires is 41 nm.

$$\phi_0 = \frac{h}{e}$$

Flux quanta

rings (average diameters 825 and 245 nm) and a lone wire (length 300 nm). The samples were cooled in the mixing chamber of a dilution refrigerator, and the resistance was measured with a four-probe bridge

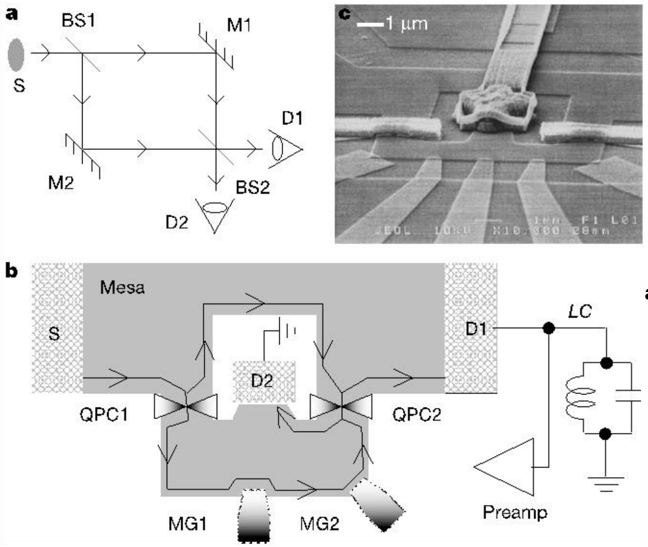
Quantum Hall Interferometer

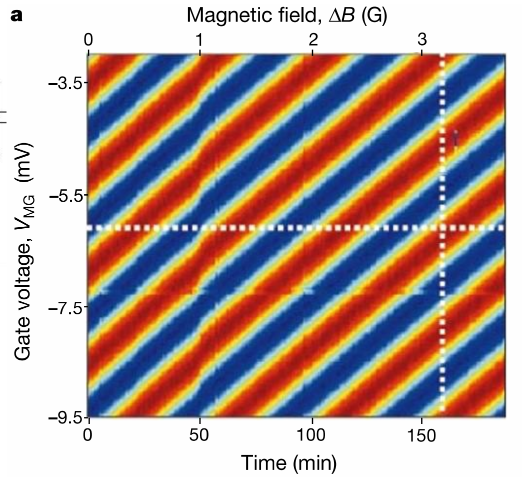
NATURE | VOL 422 | 27 MARCH 2003 |

An electronic Mach–Zehnder interferometer

Yang Ji, Yunchul Chung, D. Sprinzak, M. Heiblum, D. Mahalu & Hadas Shtrikman

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel





PRL **108,** 256804 (2012) PHYSICAL REVIEW LETTERS week ending 22 JUNE 2012

Fabry-Perot Interferometry with Fractional Charges

D. T. McClure,[1] W. Chang,[1] C. M. Marcus,[1] L. N. Pfeiffer,[2] and K. W. West[2]

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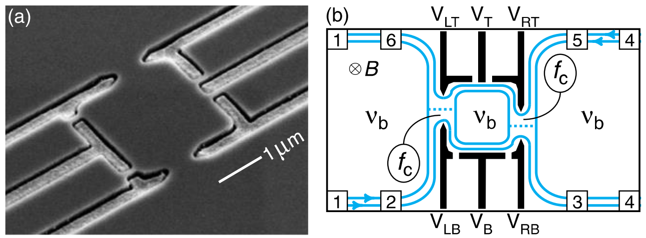
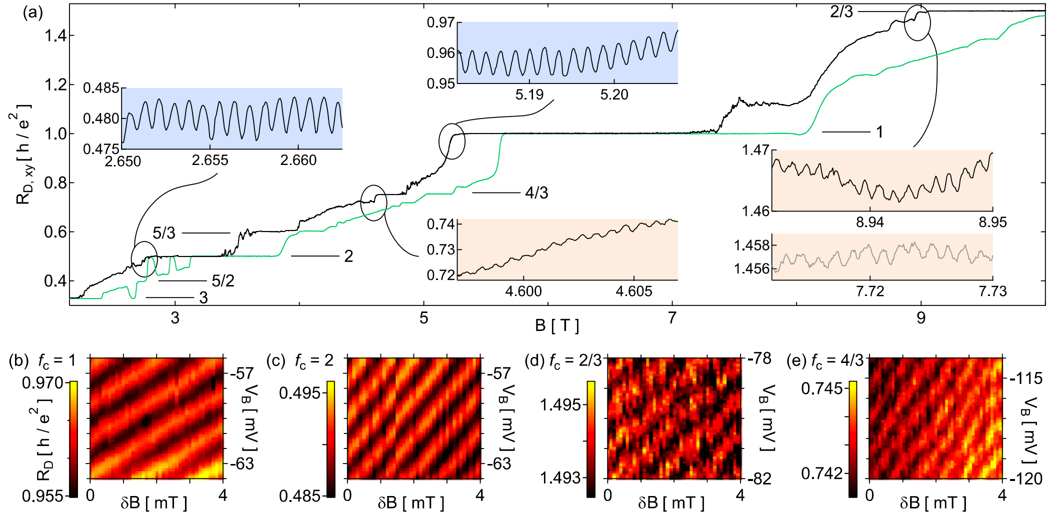
[2]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

(Received 2 December 2011; published 19 June 2012)





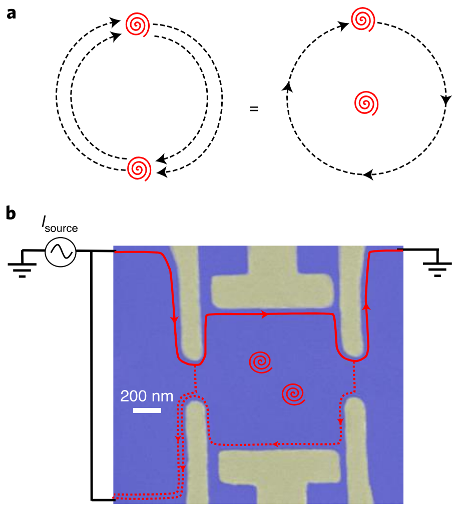
Braiding Abelian Anyons (2020)

Anyon Fabry-Perot interferometer

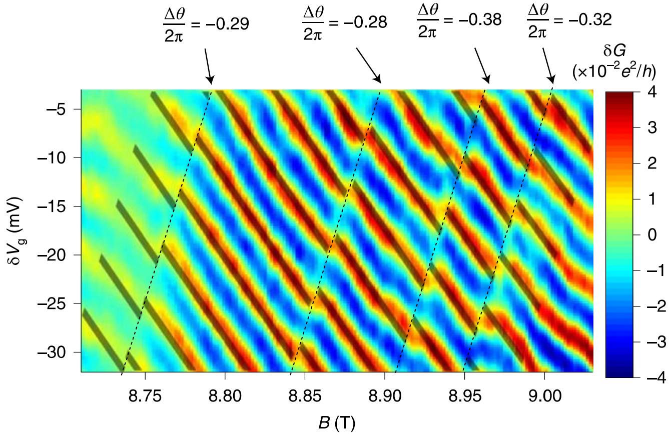
nature physics ARTICLES
https://doi.org/10.1038/s41567-020-1019-1
Check for updates

Direct observation of anyonic braiding statistics

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] and M. J. Manfra[1,2,3,4,5]



$$\theta = 2\pi \frac{e^*}{e} \frac{A_I B}{\Phi_0} + N_{qp}\theta_{anyon}$$



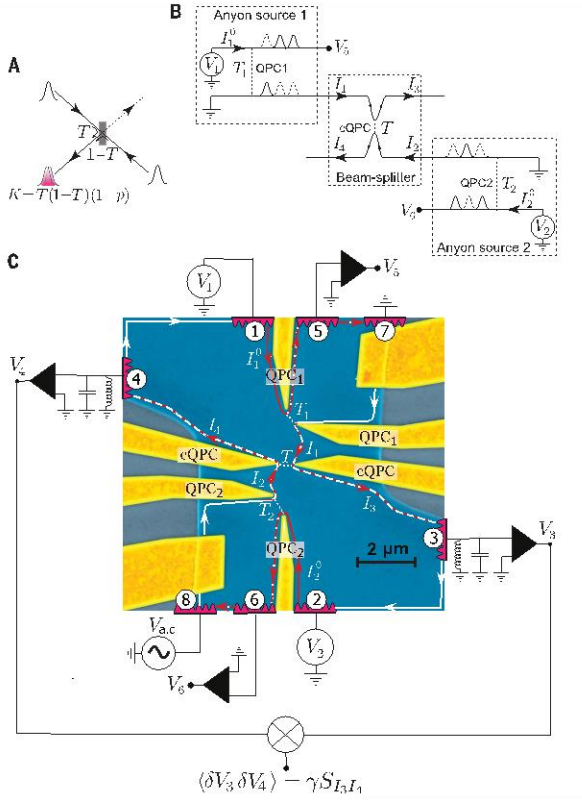
Anyon Hong-Ou-Mandel Experiment

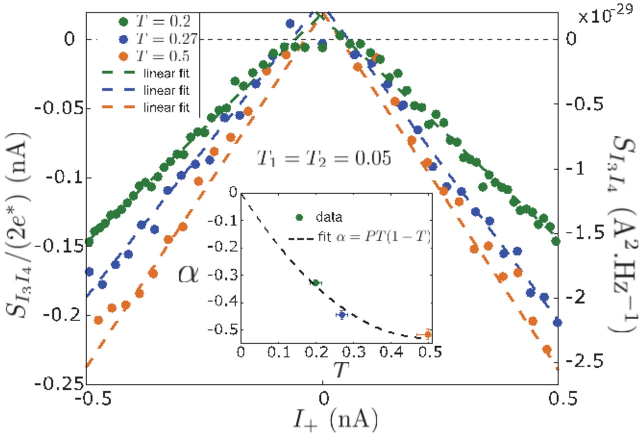
RESEARCH
Bartolomei et al., Science 368, 173–177 (2020) 10 April 2020

MESOSCOPIC PHYSICS

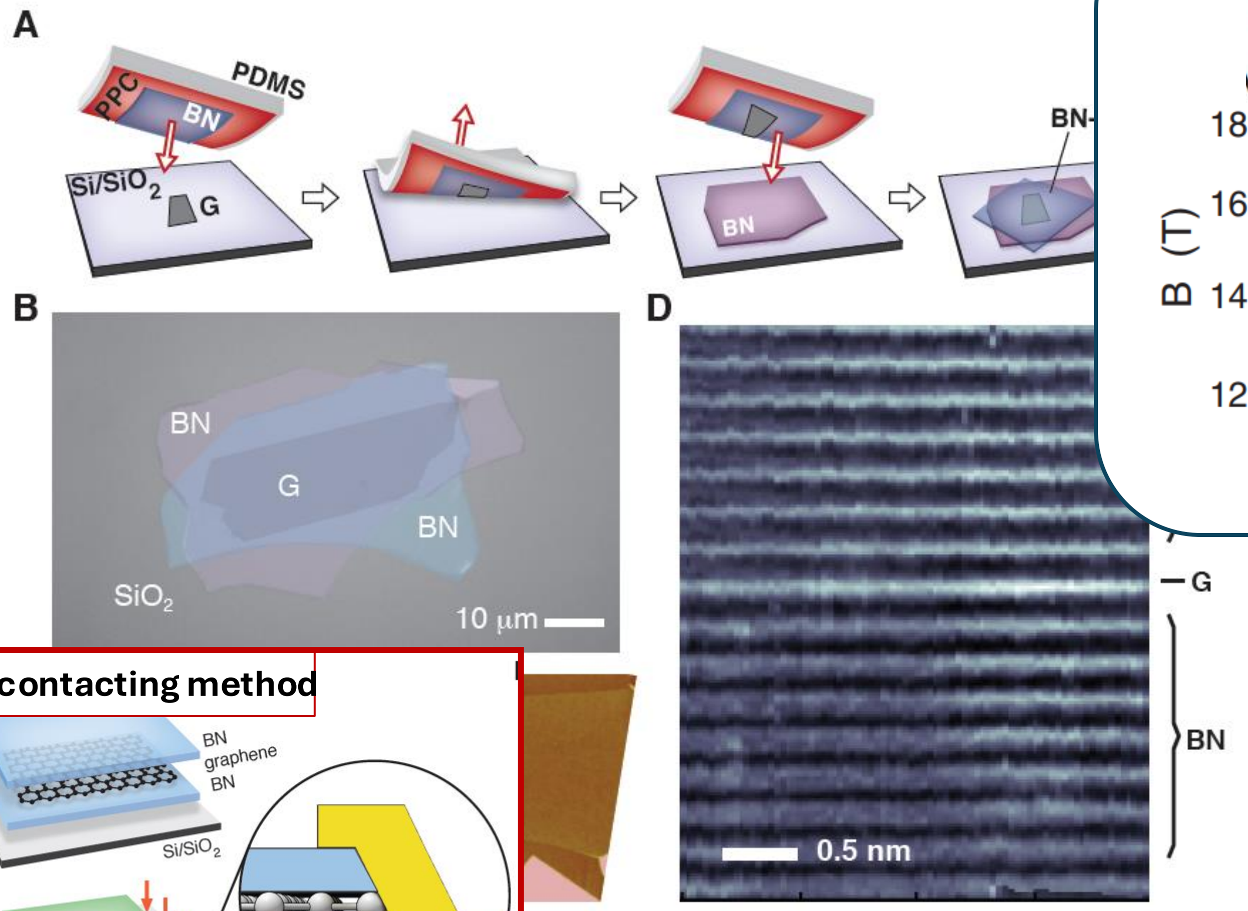
Fractional statistics in anyon collisions

H. Bartolomei[1]*, M. Kumar[1]*†, R. Bisognin[1], A. Marguerite[1]‡, J.-M. Berroir[1], E. Bocquillon[1], B. Plaçais[1], A. Cavanna[2], Q. Dong[2], U. Gennser[2], Y. Jin[2], G. Fève[1]§

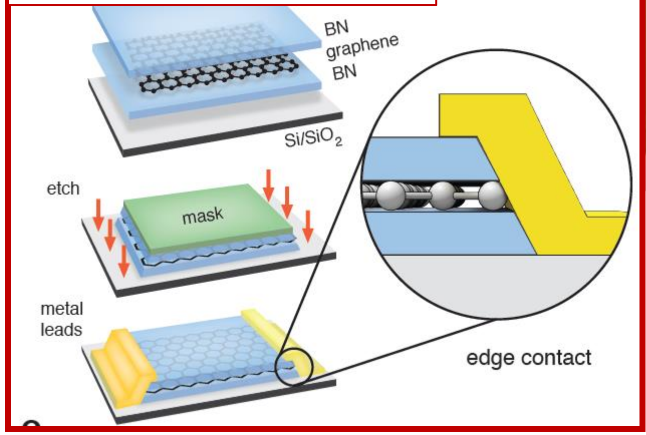




High Quality Graphene Channel in hBN vdW Structures



Edge contacting method



Even denominator FQHE in bilayer graphene

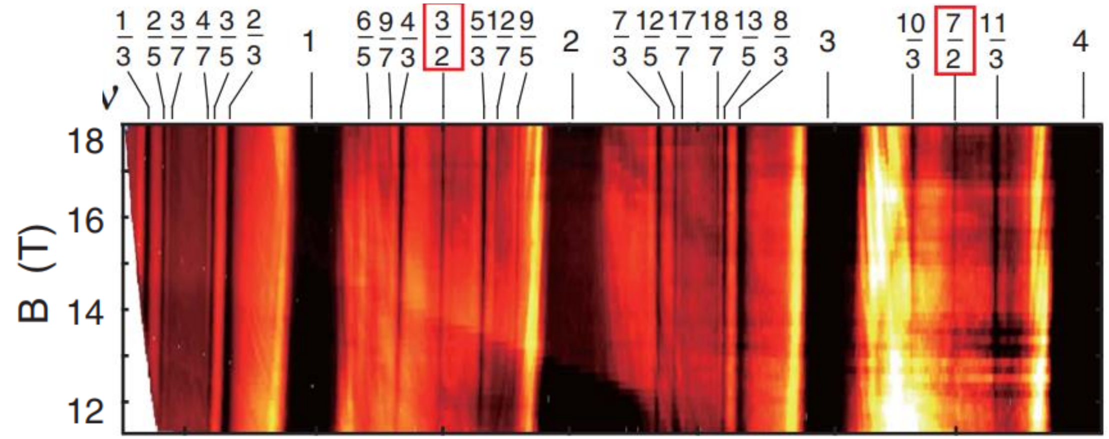
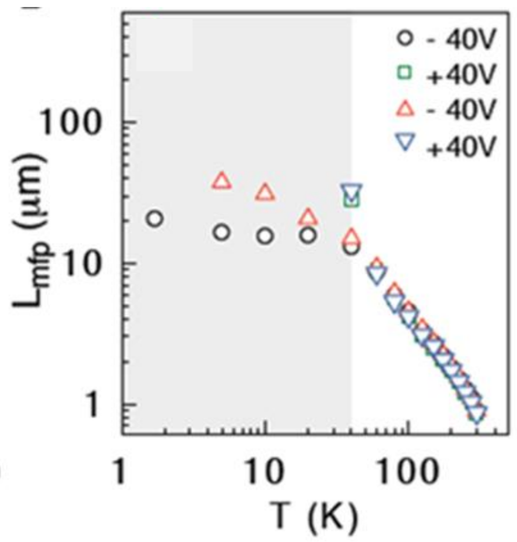


figure from Li et al *Science* (2017), see also Zibrov et al *Nature* (2017)

Contac Resistance: <100 Ω μm

Mobility > 10^6 cm^2/Vsec

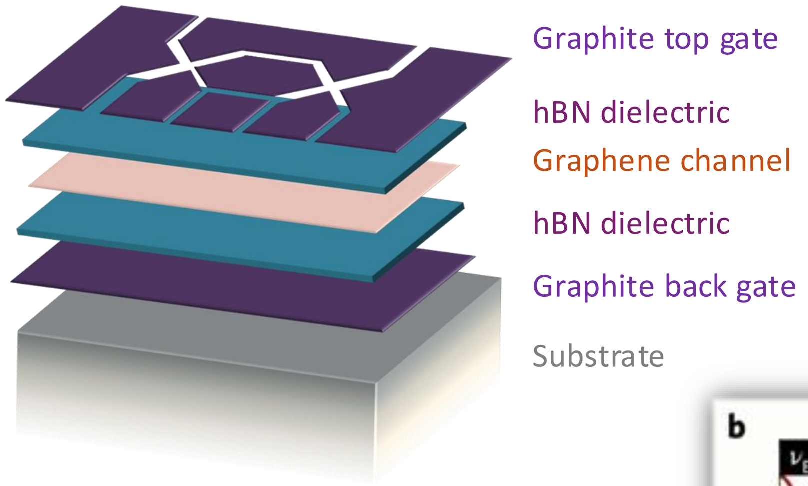
Mean free path > 10 μm

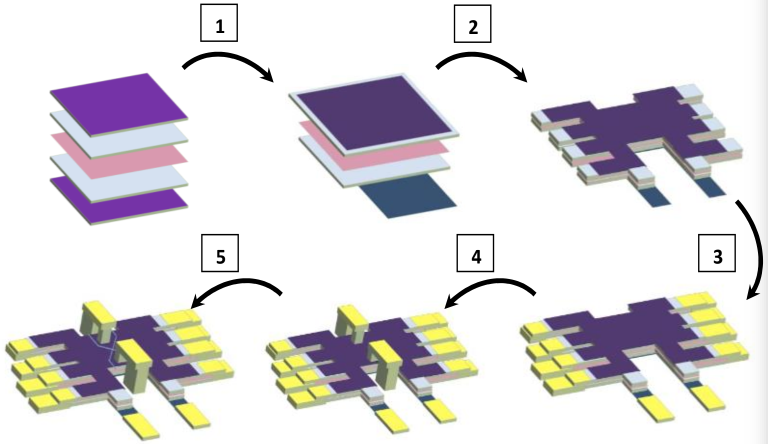


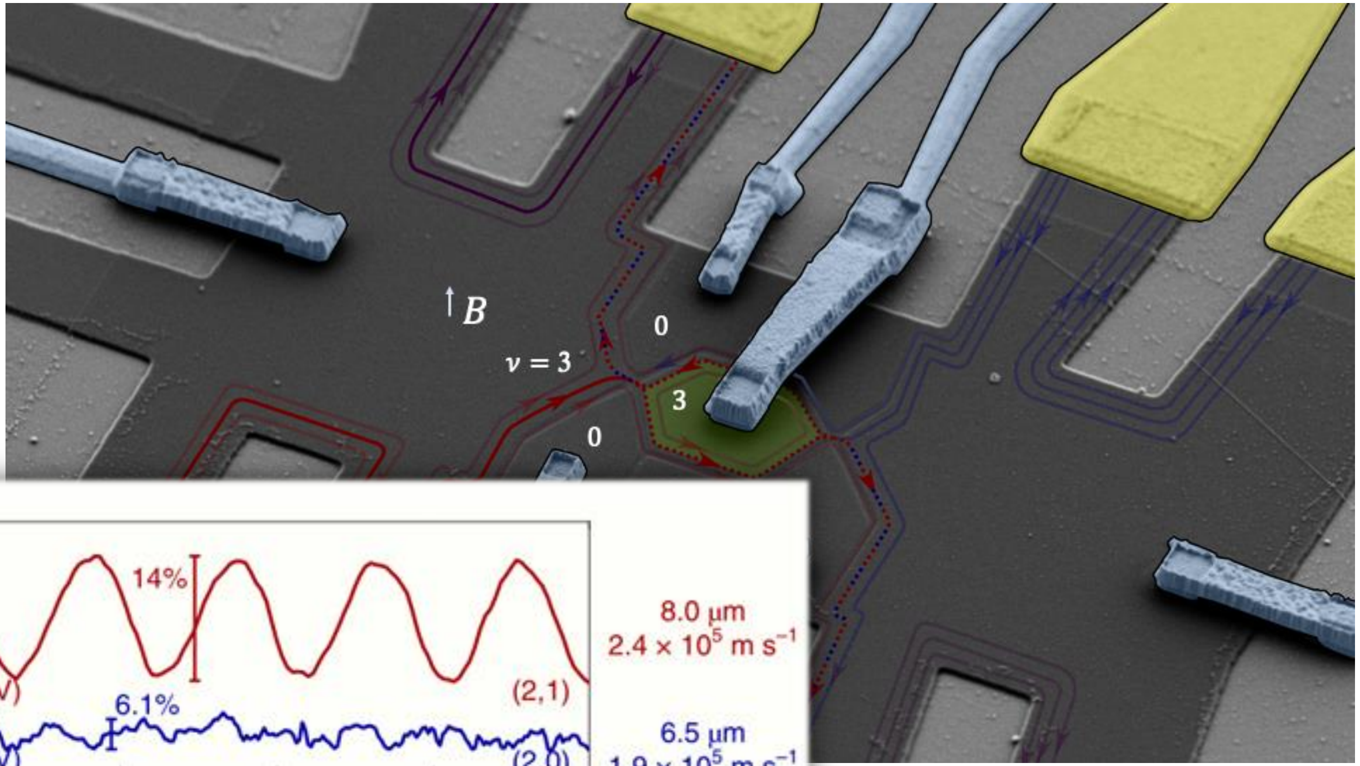
L. Wang *et al*, Science (2013)

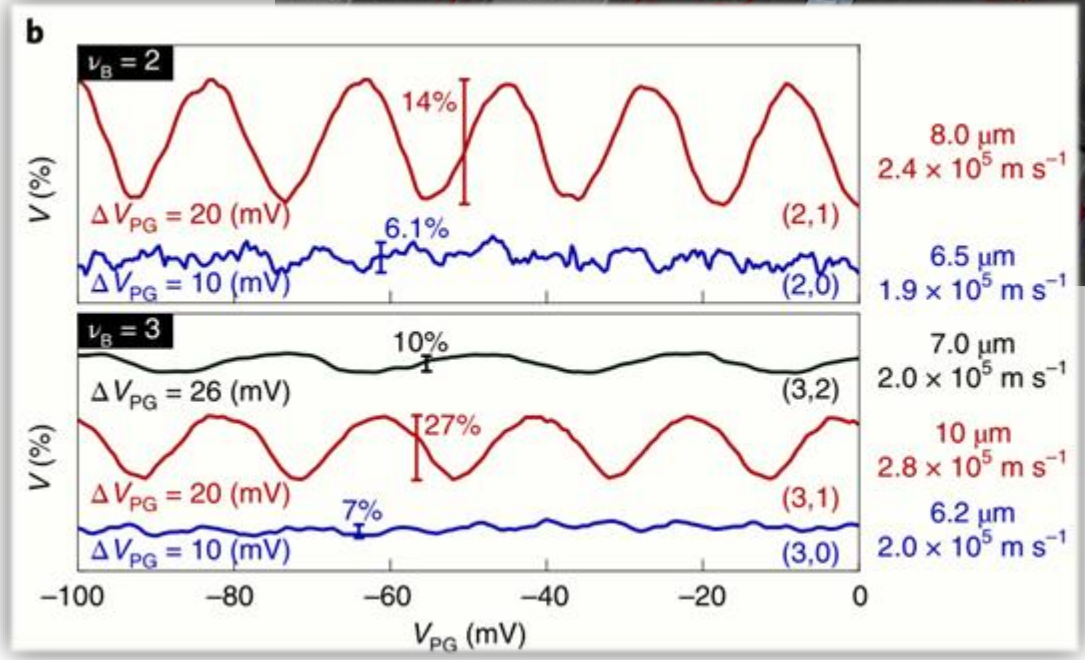
Graphene Based Quantum Hall Interferometer

Multiple vdW stacks for device fabrication





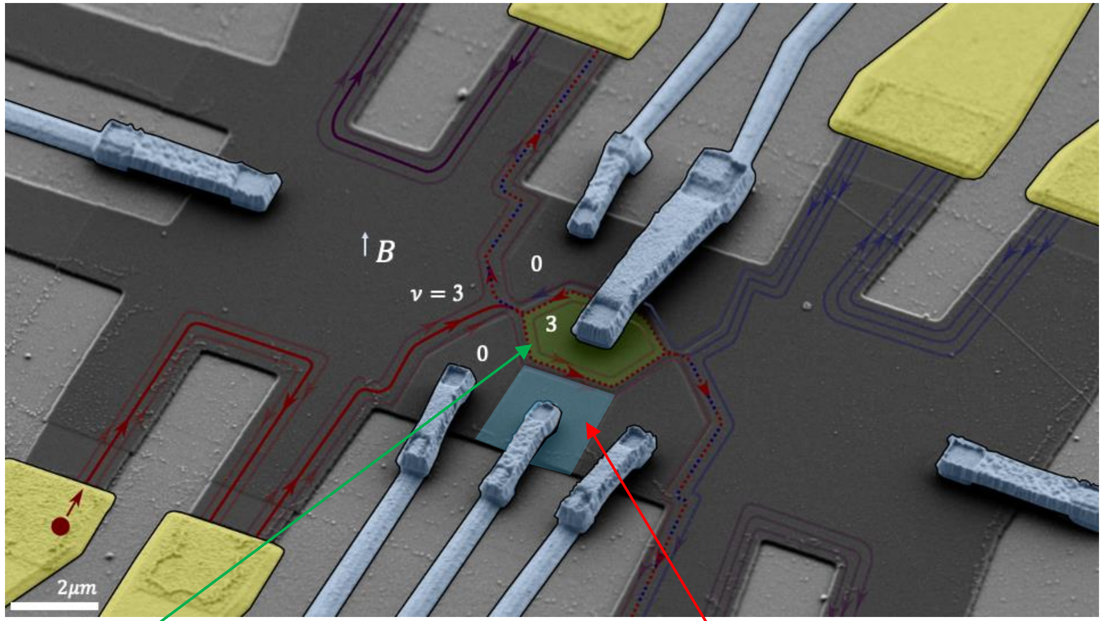




Electron Fabry-Perot Interference in IQH regime

Ronen*, Werkmeister* et al., Nature Nano (2021), Similar results by C. Déprez et al., Nature Nano (2021) ; bilayer graphene result by Jun Zhu's group Fu et al., Nano Letters (2023)

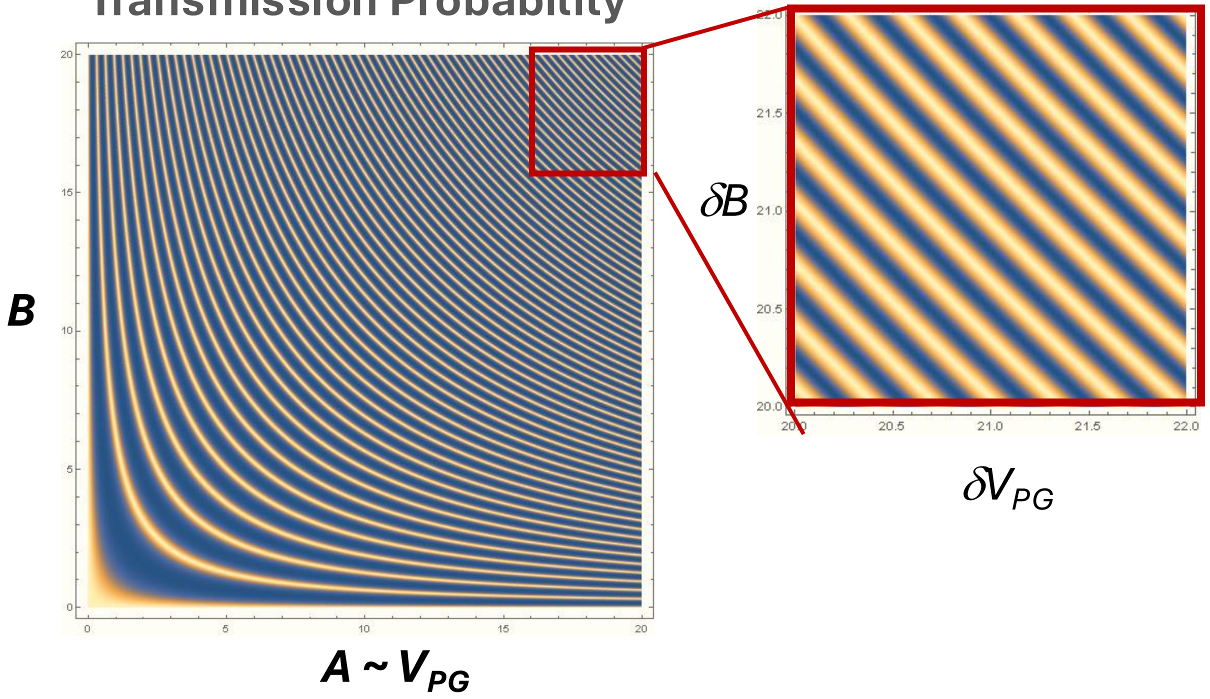
AB Effect in QH Fabry-Perot Interferometer



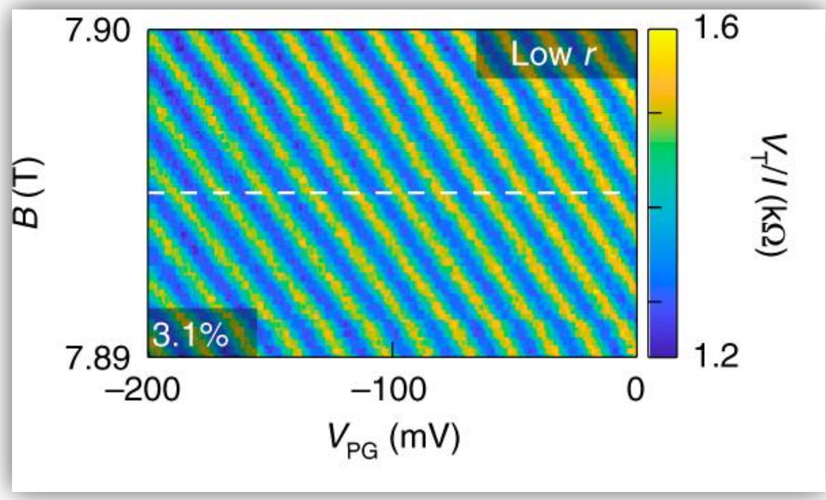
Interferometer area *A*

Plunger gate V_{PG} to control area

Transmission Probability



Ronen*, Werkmeister* et al., Nature Nano (2021),



AB Phase

$$\frac{\phi}{2\pi} = \frac{AB}{\phi_0}$$

Transmission Probability

$$T(\phi) = |t|^2 = \frac{|t_1|^2|t_2|^2}{1 + |r_1|^2|r_2|^2 - 2|r_1 r_2|^2 \cos(\phi)}$$

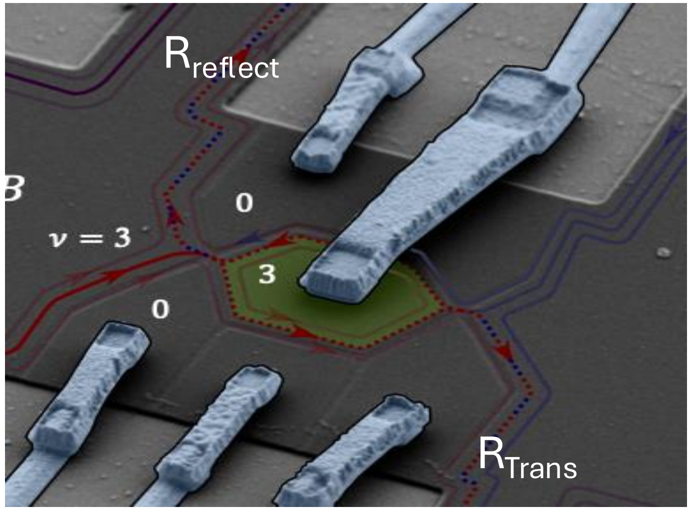
$$\phi_0 = \frac{h}{e}$$

Flux quanta

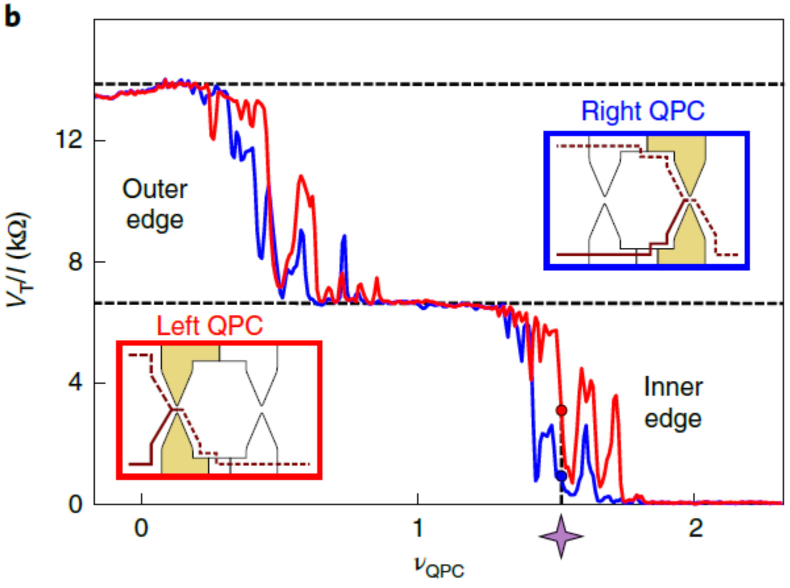
Graphene Quantum Hall Point Contact as Charge Sensor

1st Gen

Local gate defined quantum dots and point contacts under magnetic fields



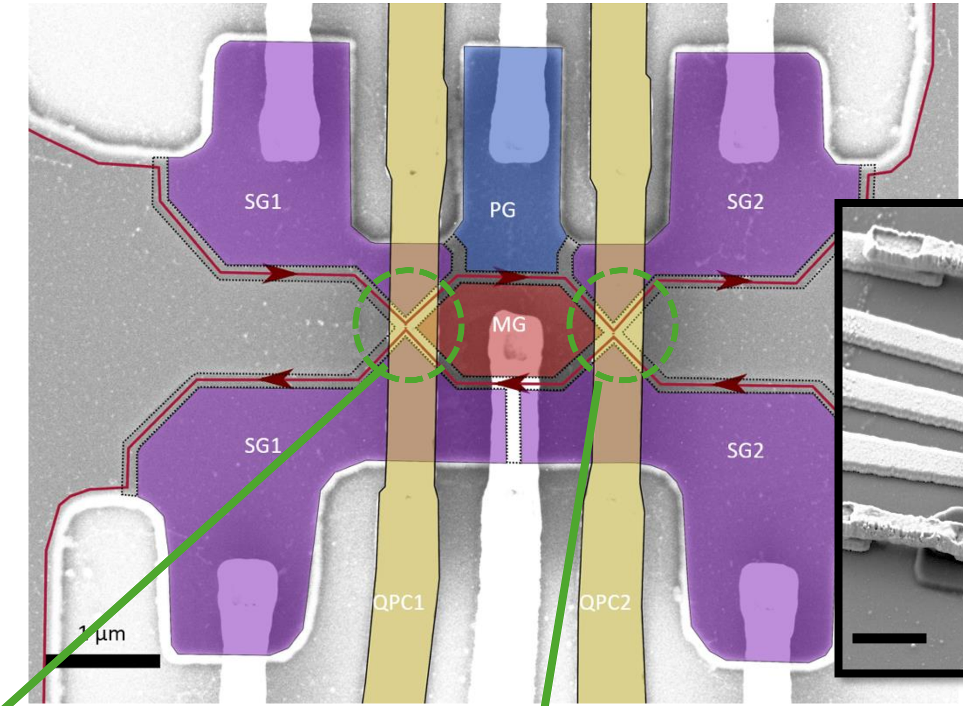
6 T



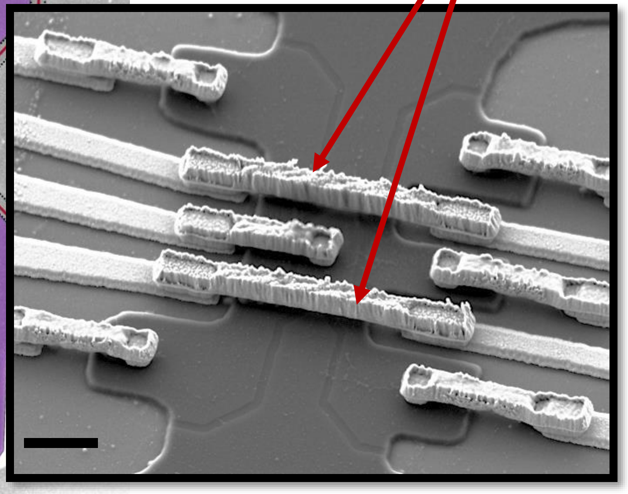
Ronen*, Werkmeister* et al., Nature Nano (2021)

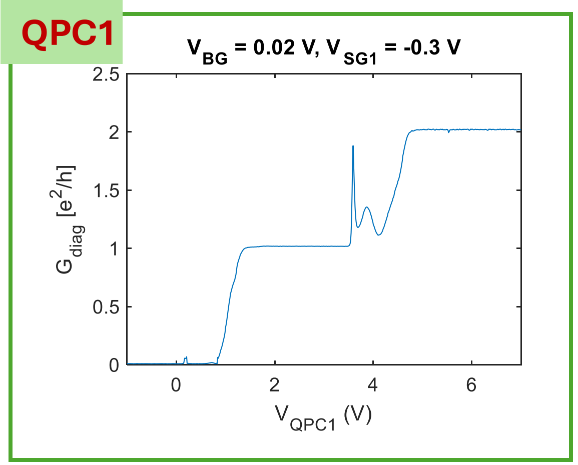
Airbridge overhang gates for individual QPC controls

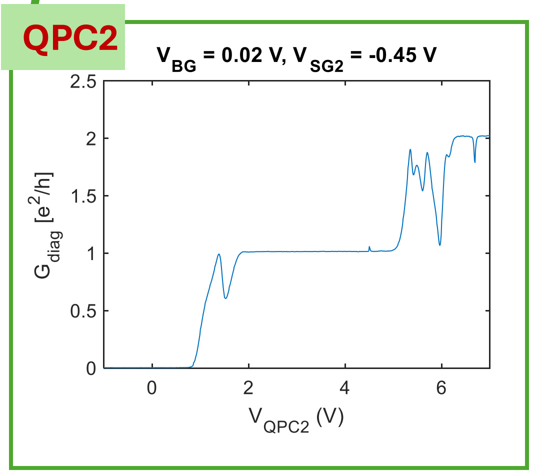
2nd Generation



Airbridge suspended gates







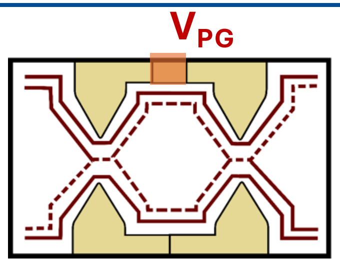
Excellent charge stability and independent control has been demonstrated!

Werkmeister et al., arXiv:2312.03150

Interger Quantum Edge Interference

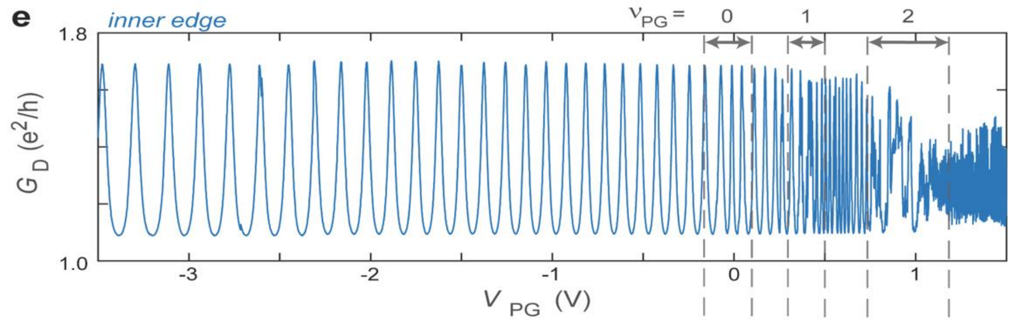
n =2 QH state

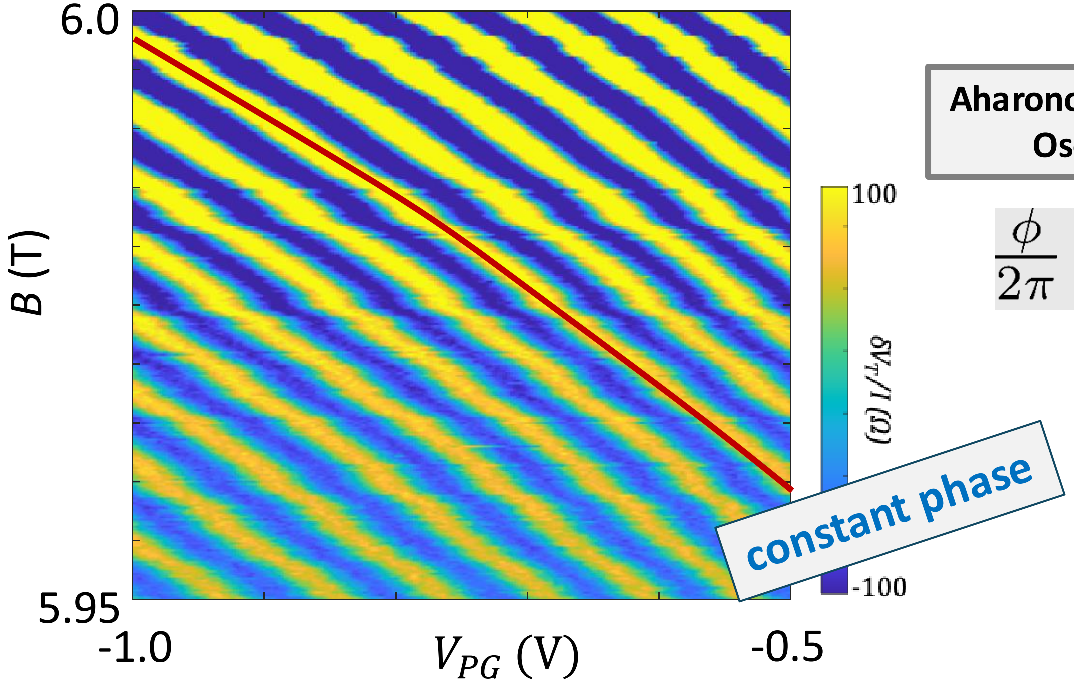
Interference of
inner QH edge



Partition inner edge

6 T @ 20 mK



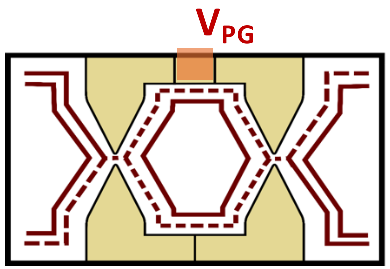


Aharonov-Bohm (AB) Oscillation

$$\frac{\phi}{2\pi} = \frac{AB}{\phi_0}$$

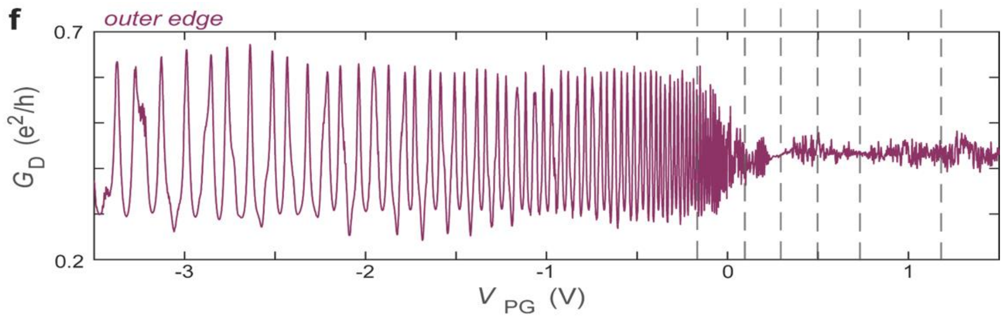
n =2 QH state

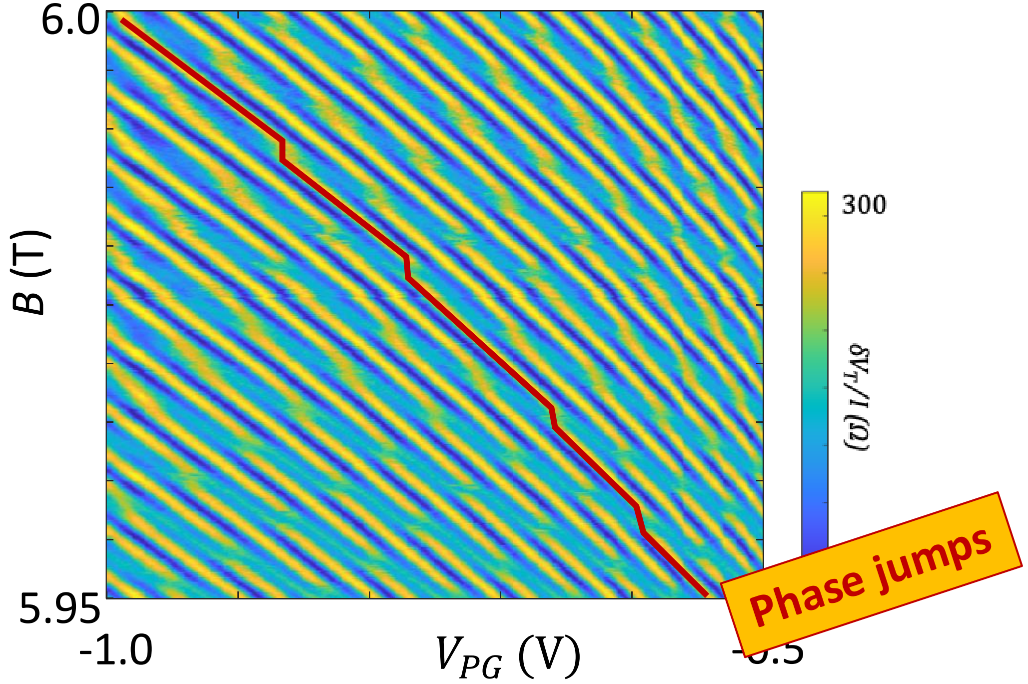
Interference of
Outer QH edge



Partition outer edge

6 T @ 20 mK

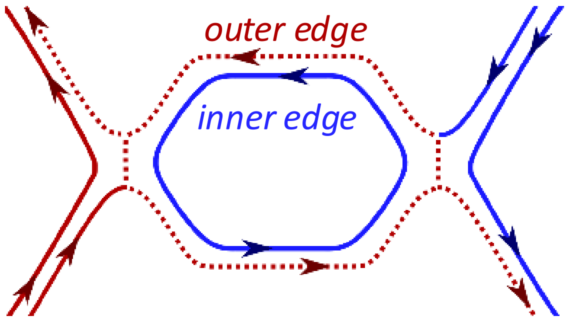


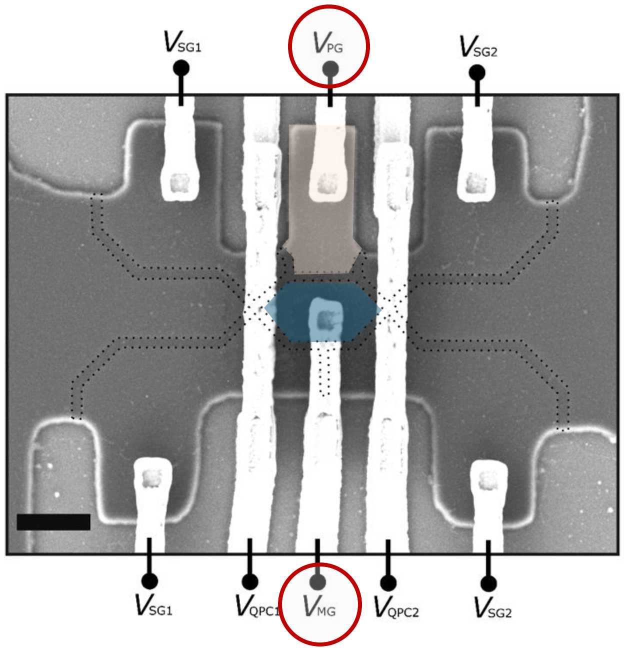


Phase Jumps in Fabry Perot Interference

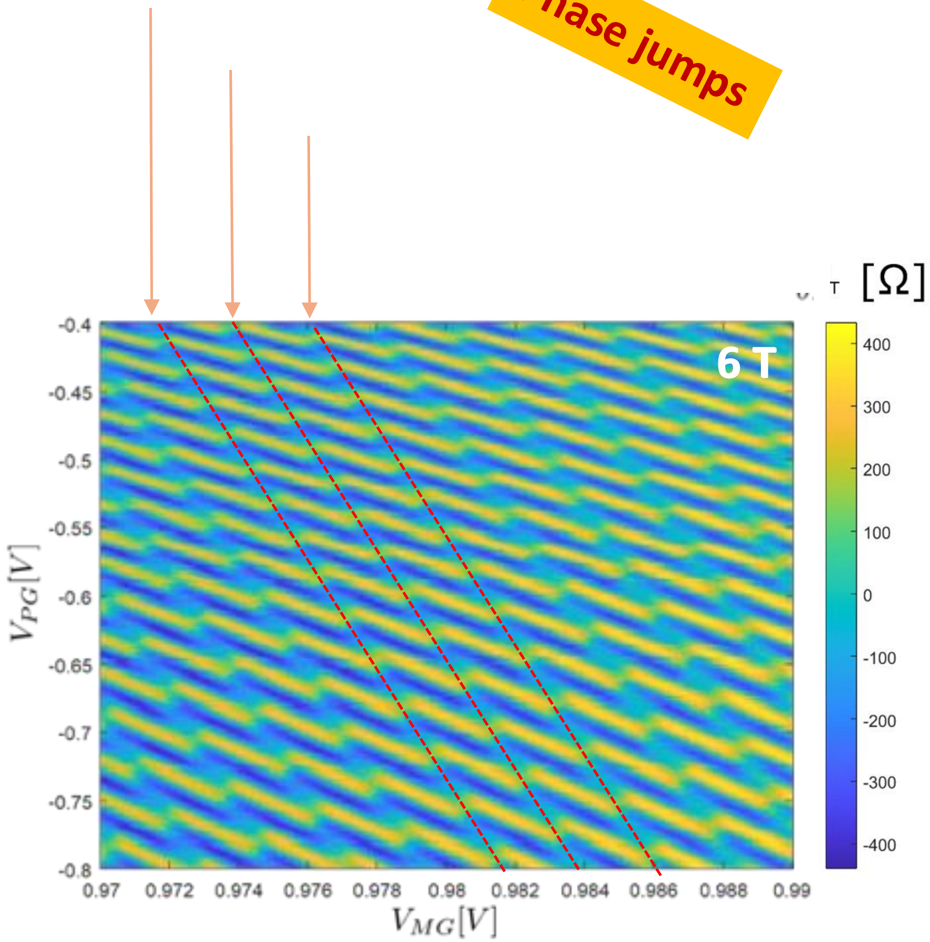
Feldman & Halperin, PRB (2022)

Interference of **outer edge** with **inner edge island**





Phase jumps



Plunger gate and top gate sweep at Constant Magnetic Field

Phase of QH FP Interference

$$\delta\theta = 2\pi\delta N$$

Total charge change in interferometer

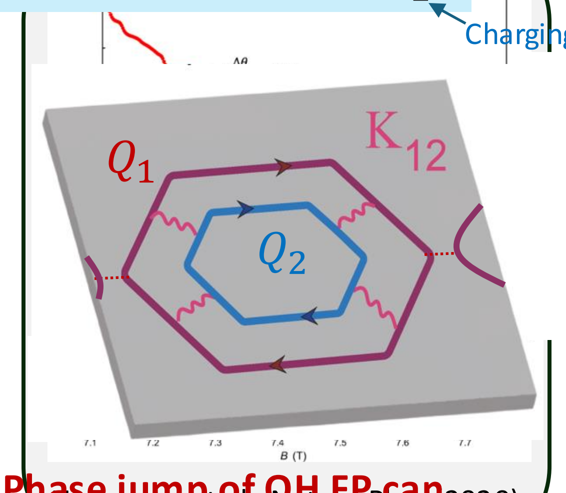
$$= 2\pi(\delta N_{out} + \delta N_{in})$$

Phase jumps

$$\frac{\Delta\theta}{2\pi} = \Delta N_{out} = -\frac{K_{12}}{K_1}$$

Capacitive coupling

Charging energy

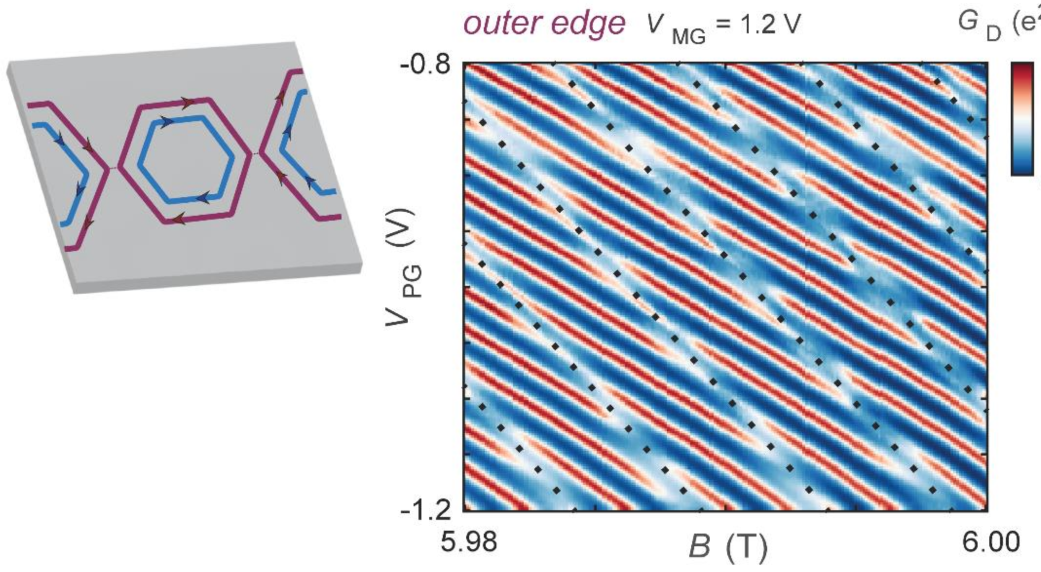


Nakamura et al., Nature Phys (2020)

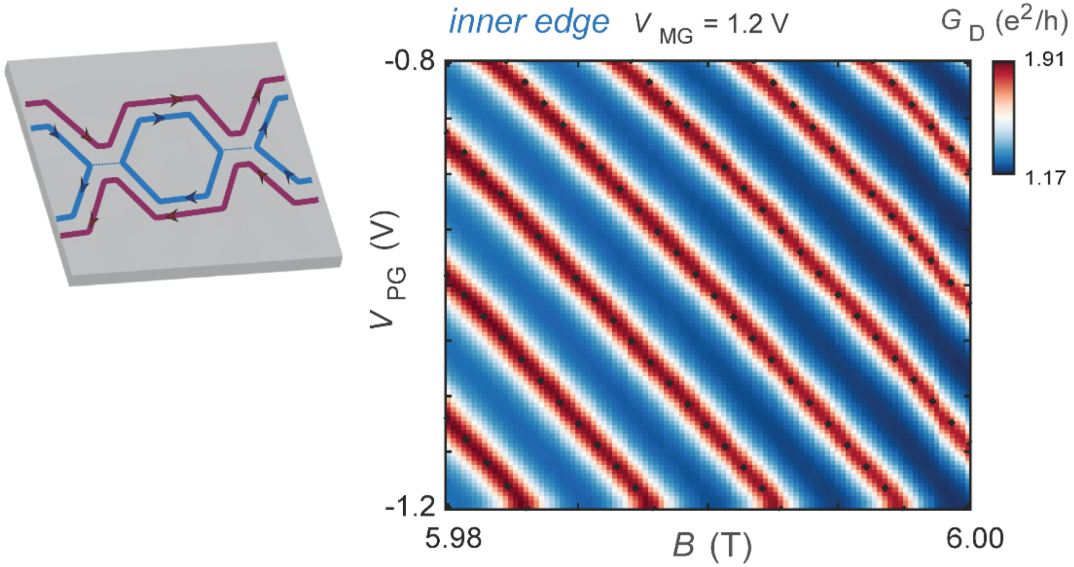
Phase jump of QH FP can detect the charge change of inner QH edge!

Inner Edge versus Outer Edge AB Oscillations

Partition and interference of outer edge

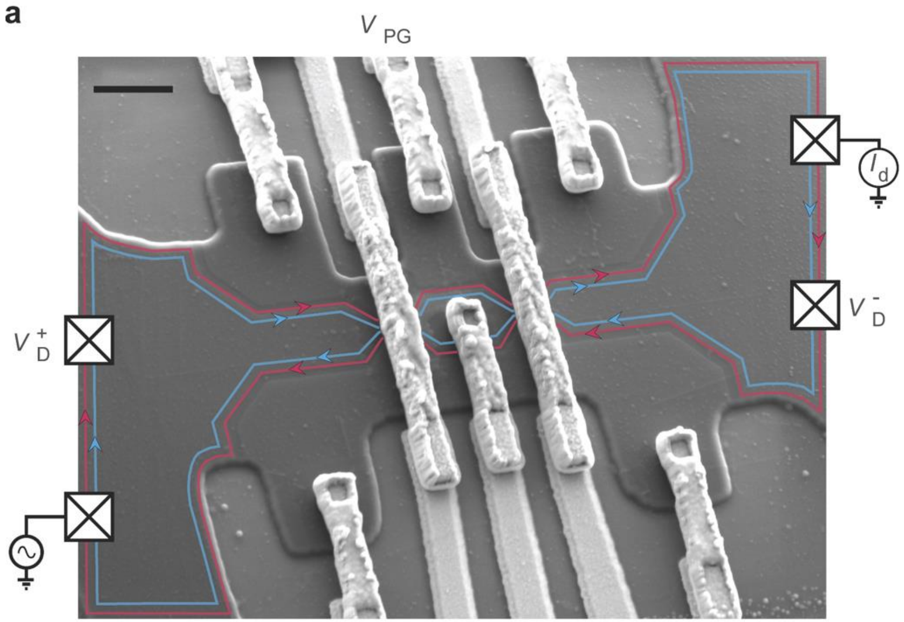


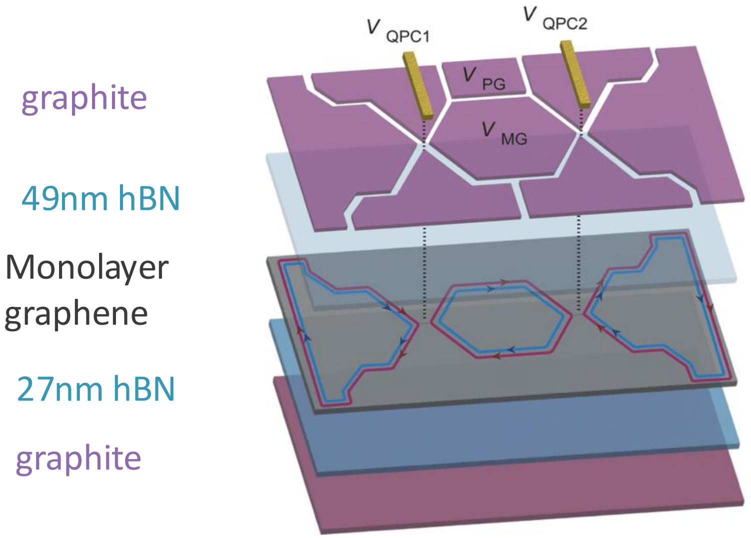
Partition and interference of Inner edge

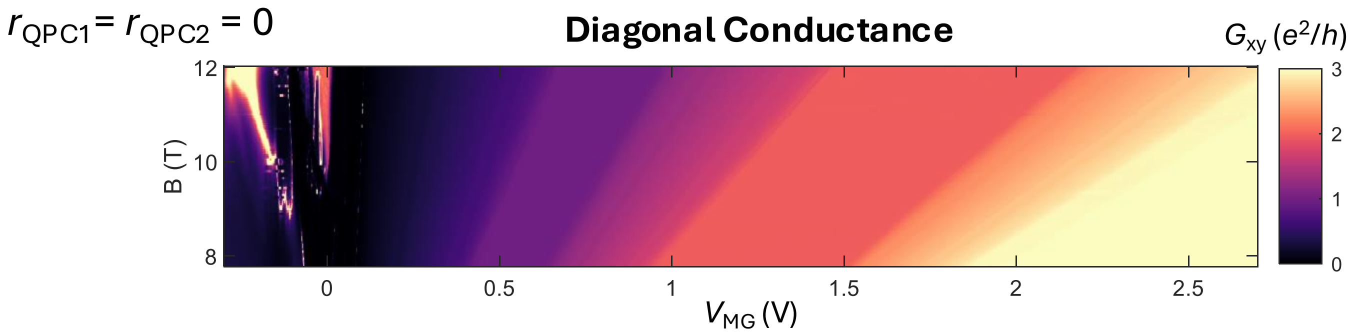


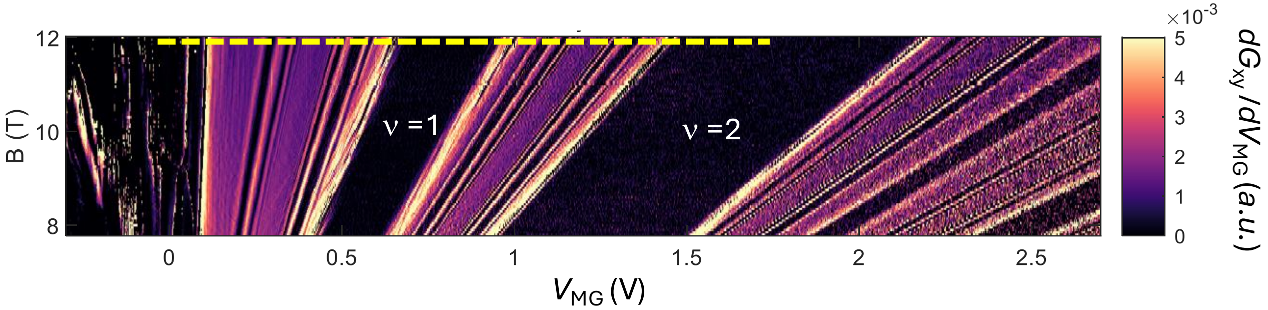
Phase slip lines of the outer edge interference is directly connected to the inner edge AB oscillation!

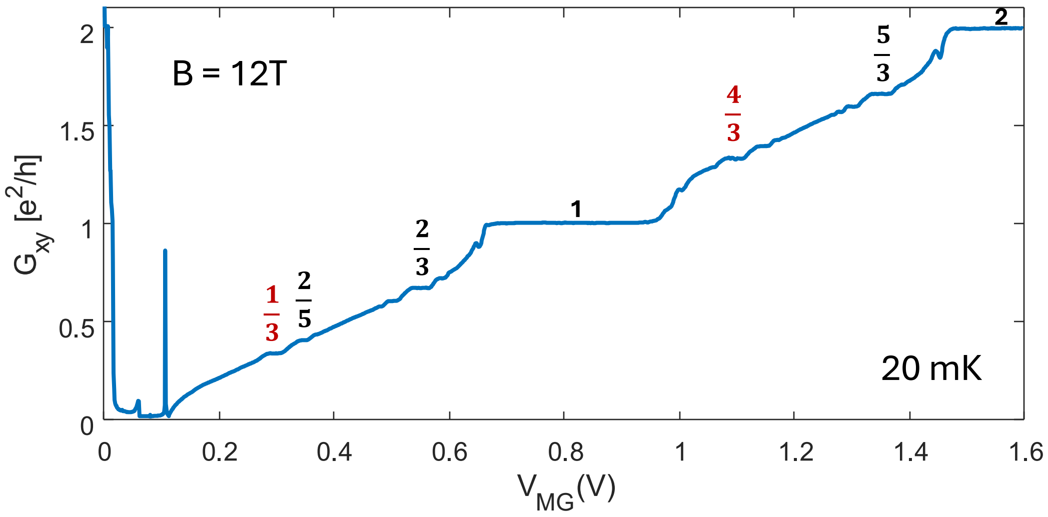
Fractional Quantum Hall Effect Fabry-Perot Interferometer



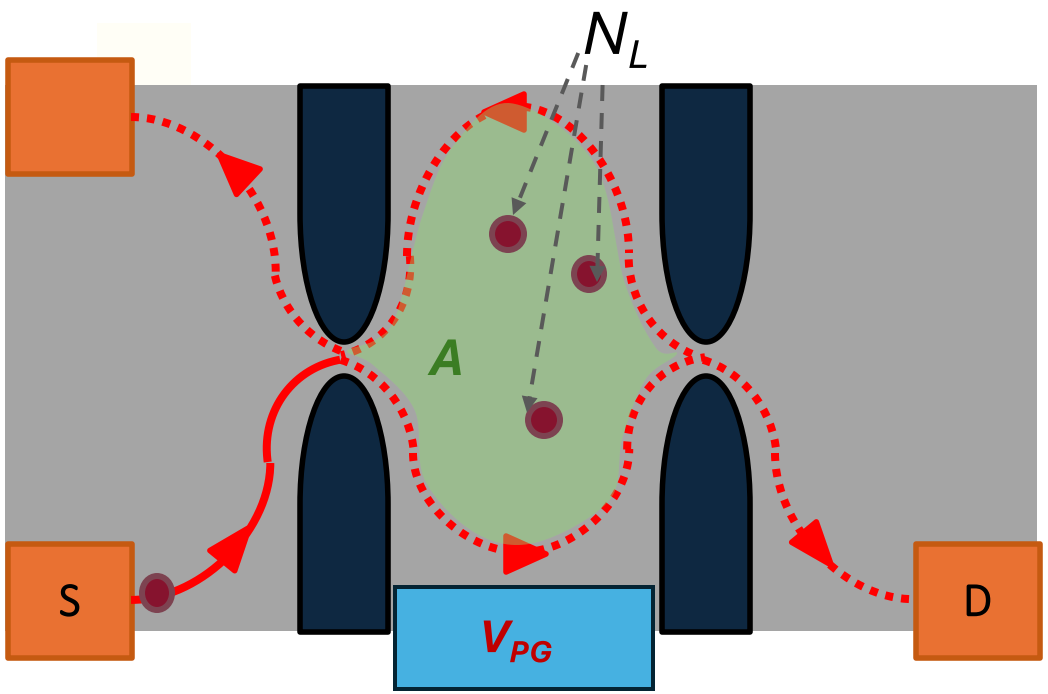








Fabry-Pérot FQH interferometer: abelian anyons

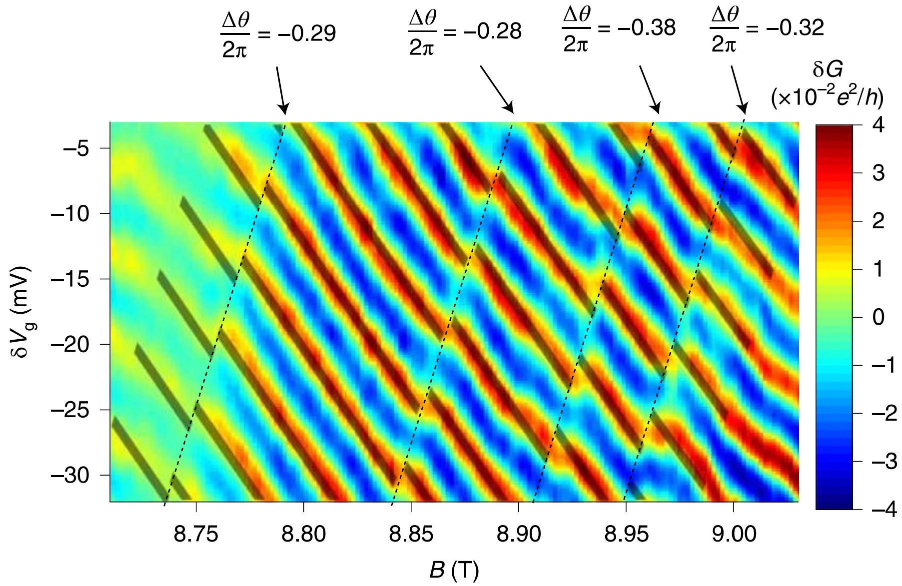


Interference Phase

fractional charge   fractional statistics

$$\phi = 2\pi \frac{e^*}{e}\frac{AB}{\varphi_0} + N_L 2\theta$$

1/3 FQH Interference in GaAs heterostructures



Nakamura et al., Nature Phys. (2020)

Fractionalized Charge

$\frac{e^*}{e} = 1$: electrons, ν=integer

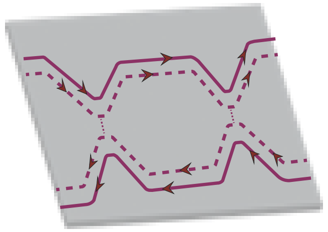
$\frac{e^*}{e} = \frac{1}{3}$: Laughlin state, $\nu=1/3$

θ: exchange phase

$\theta = \pi$: electrons, ν=integer

$\theta = \frac{\pi}{3}$: Laughlin state, $\nu=1/3$

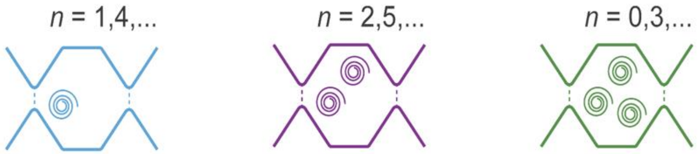
Telegraph Noise in FQH Interference



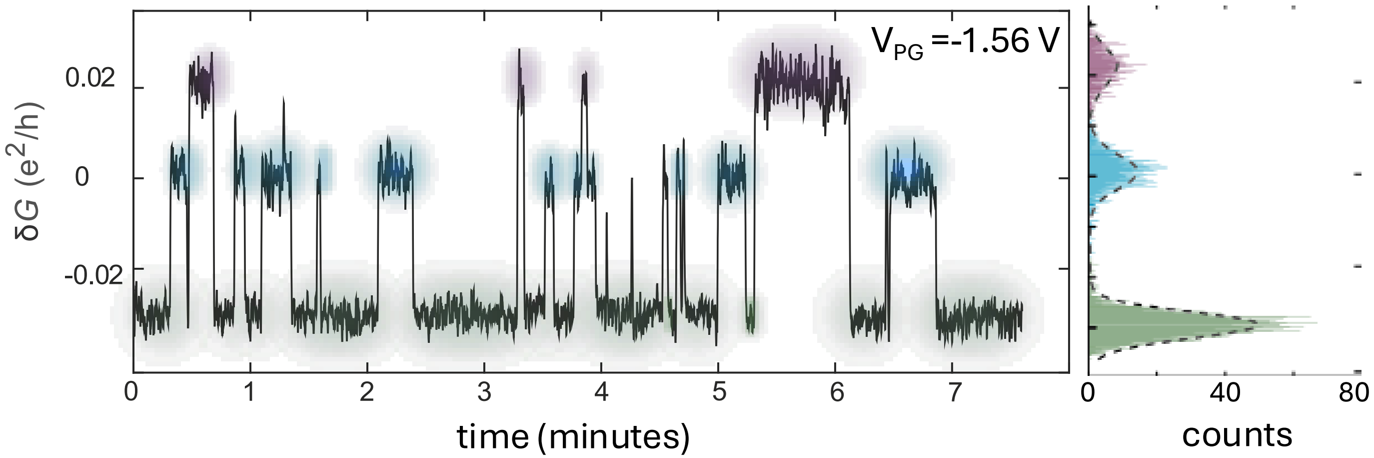
ν=4/3

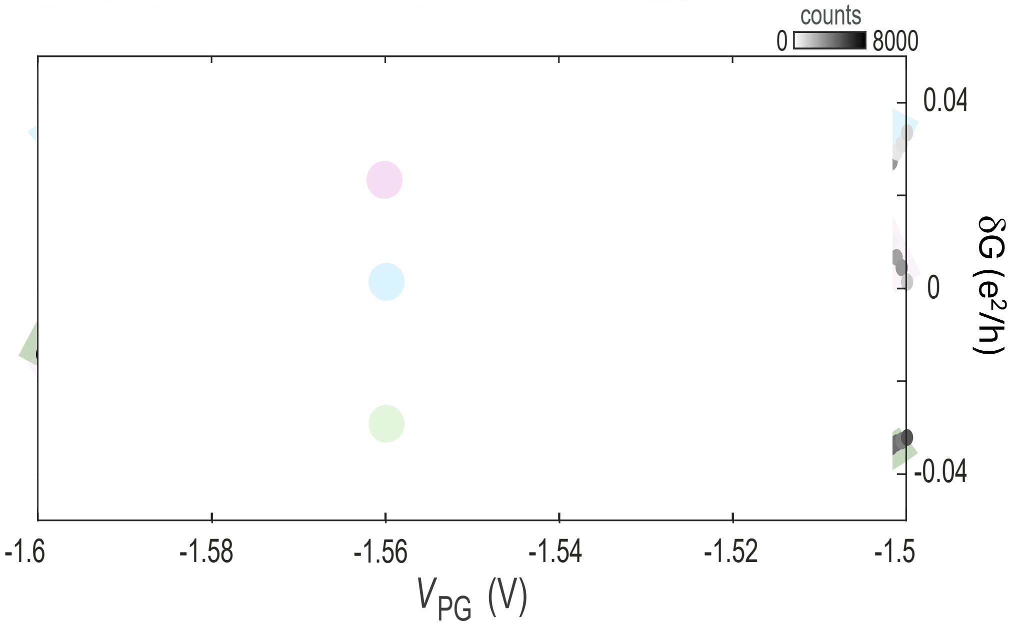
QPCs partition inner FQH edge

12 T @ 20 mK



Interference Signal at Constant Gate Voltages and Magnetic Field





- **Random telegraph noise with switching time ~ 10s sec**
- **Three different switching states: caused by fluctuations in localized anyon number n**
- **conductance can only take 3 discrete values depending on n mod 3**

$$\delta G(n) \approx \beta \cos\left(2\pi\theta + (n \bmod 3)\frac{2\pi}{3}\right)$$

Telegraph Noise in FQH Interference

VOLUME 90, NUMBER 22 PHYSICAL REVIEW LETTERS week ending 6 JUNE 2003

Telegraph Noise and Fractional Statistics in the Quantum Hall Effect

C. L. Kane

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

(Received 28 October 2002; published 2 June 2003)

We study theoretically nonequilibrium noise in the fractional quantum Hall regime for an Aharonov-Bohm ring with a third contact in the middle of the ring. Because of their fractional statistics the tunneling of Laughlin quasiparticles between the inner and outer edges of the ring changes the effective Aharonov-Bohm flux experienced by quasiparticles going around the ring, leading to a change in the conductance across the ring. A small current in the middle contact, therefore, gives rise to fluctuations in the current flowing across the ring which resemble random telegraph noise. We analyze this noise using the chiral Luttinger liquid model. At low frequencies the telegraph noise varies inversely with the tunneling current and can be much larger than the shot noise. We propose that combining the Aharonov-Bohm effect with a noise measurement provides a direct method for observing fractional statistics.

PRL **96,** 226803 (2006) PHYSICAL REVIEW LETTERS week ending 9 JUNE 2006

Switching Noise as a Probe of Statistics in the Fractional Quantum Hall Effect

Eytan Grosfeld,[1] Steven H. Simon,[2] and Ady Stern[1]

[1]*Department of Condensed Matter, Weizmann Institute of Science, Rehovot 76100, Israel*

[2]*Bell Laboratories, Lucent Technologies, 600 Mountain Avenue, Murray Hill, New Jersey 07974, USA*

(Received 27 February 2006; published 8 June 2006)

$$\theta = 2\pi \frac{e^*}{e}\frac{A_{\rm I} B}{\Phi_0} + N_{\rm qp}\theta_{\rm anyon}$$

PHYSICAL REVIEW B **85**, 201302(R) (2012)

Telegraph noise and the Fabry-Perot quantum Hall interferometer

B. Rosenow[1] and Steven H. Simon[2]

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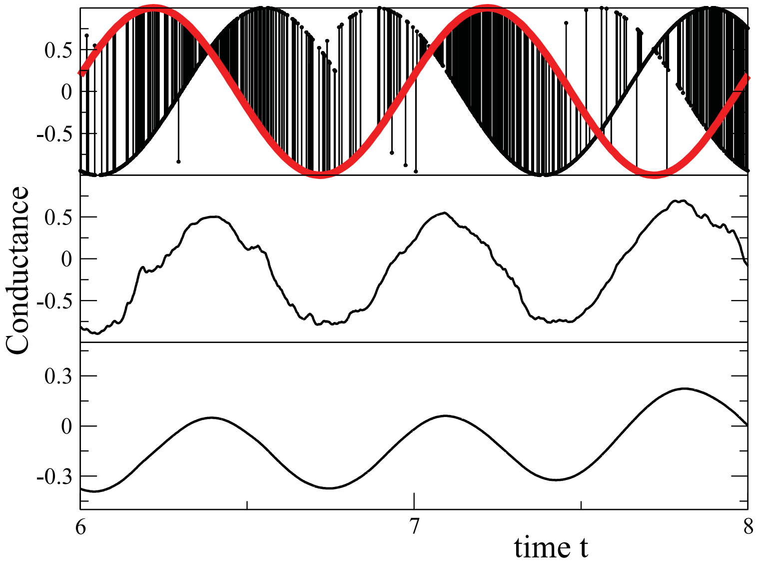
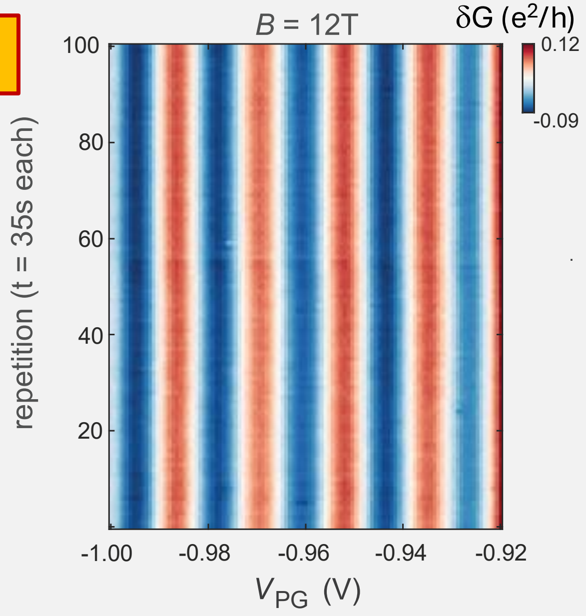
[2]*Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxfordshire OX1 3NP, United Kingdom*

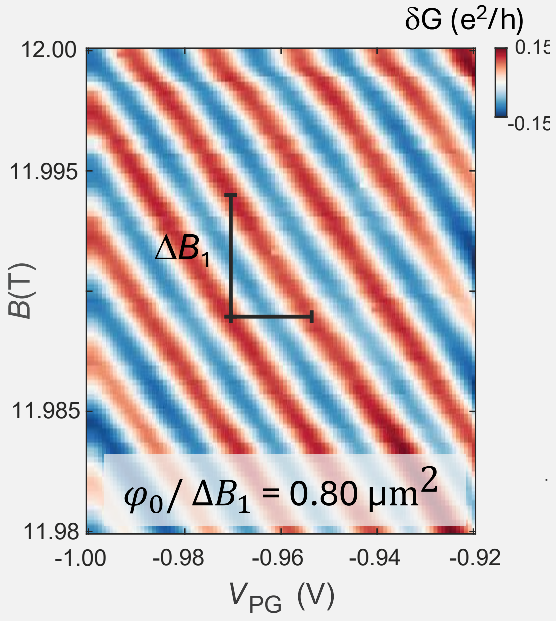
(Received 22 December 2011; published 14 May 2012)



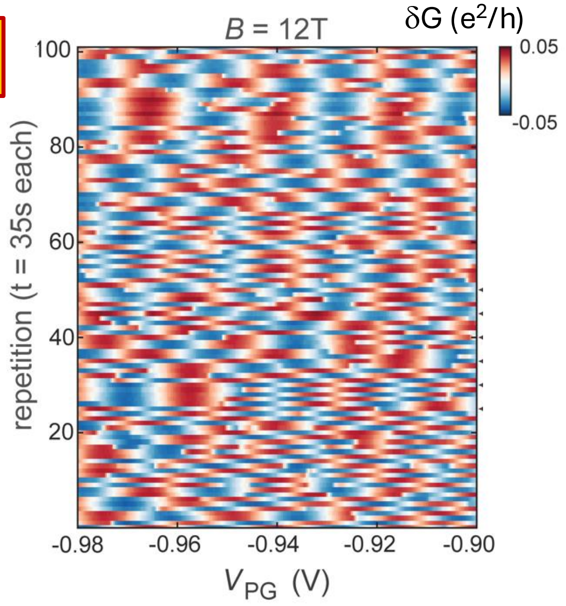
AB Oscillation of FQHE and Phase Shift

ν=1

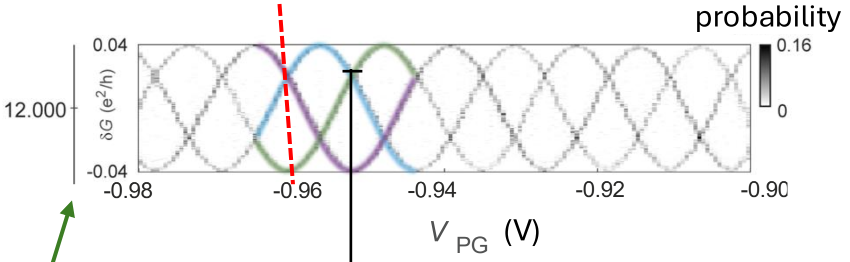




ν=1/3



$\pm\frac{2\pi}{3}$ phase slips



B = 12 T

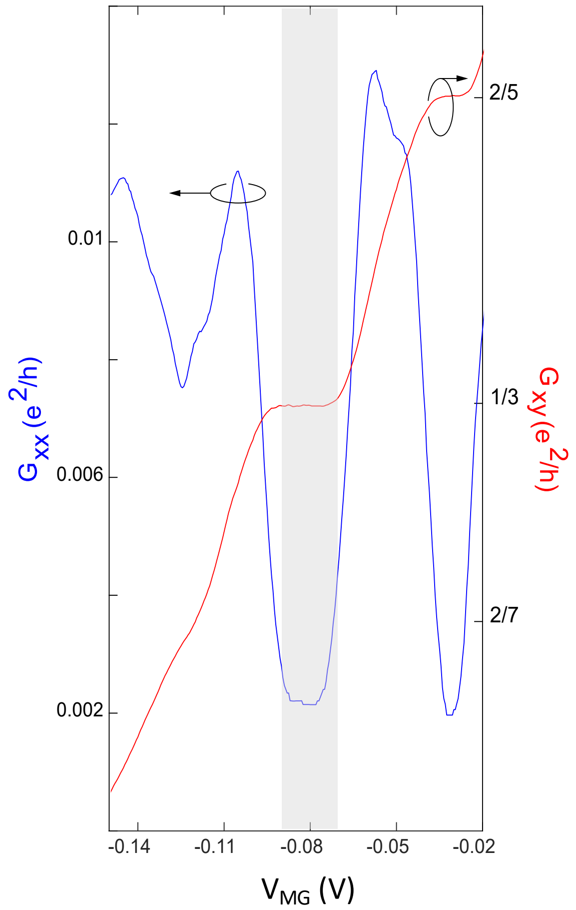
$\Delta B_{1/3}/3$

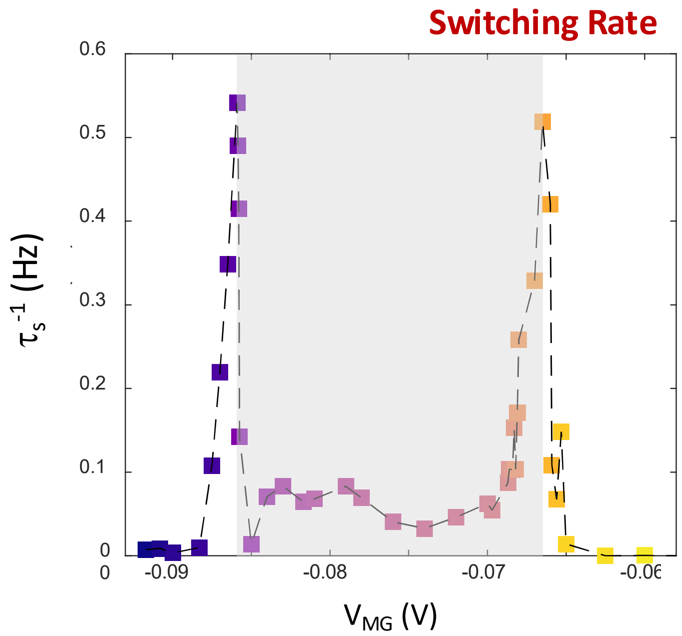
$3\varphi_0/\ \Delta B_{1/3}$ = 0.83 μm^2

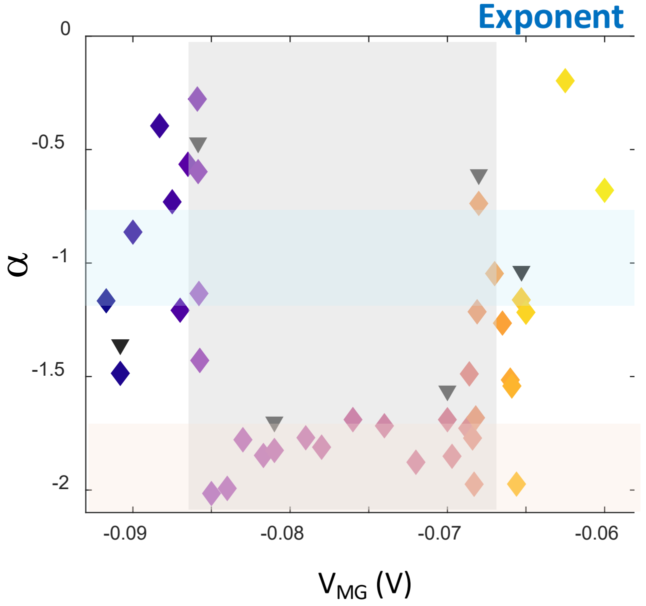
Switching Rate: Density Dependence

ν=1/3

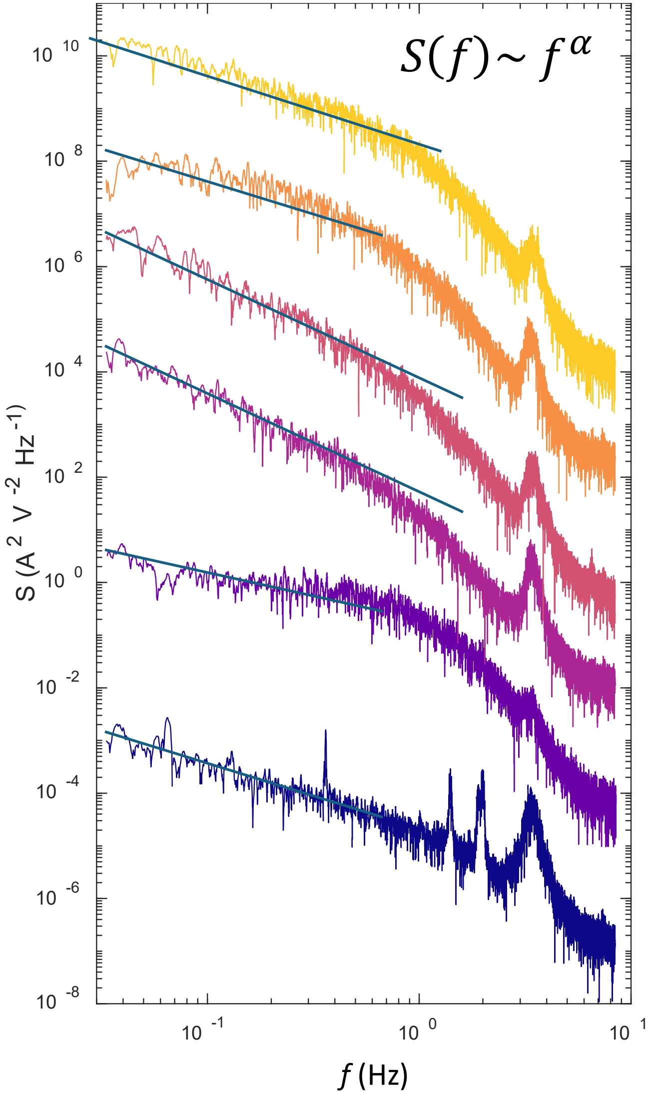
12 T
@ 20 mK







Fourier Transform of $\delta G(t)$: $S(f)$



Random Telegraph Noise

$S \sim 1/f^2$: single time scale

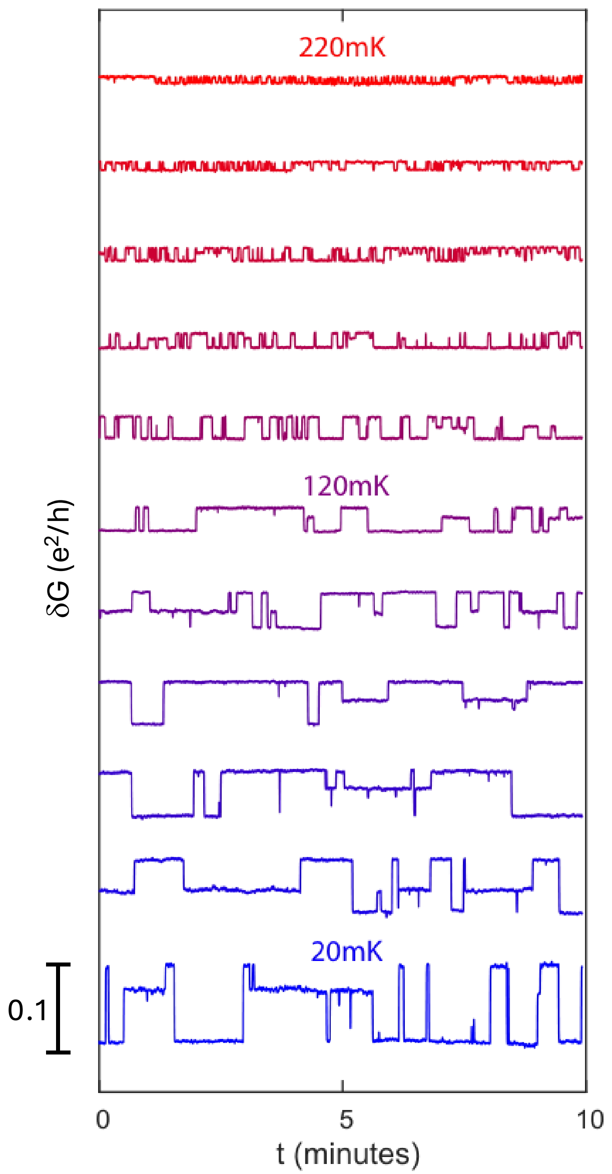
$S \sim 1/f$: multiple time scale

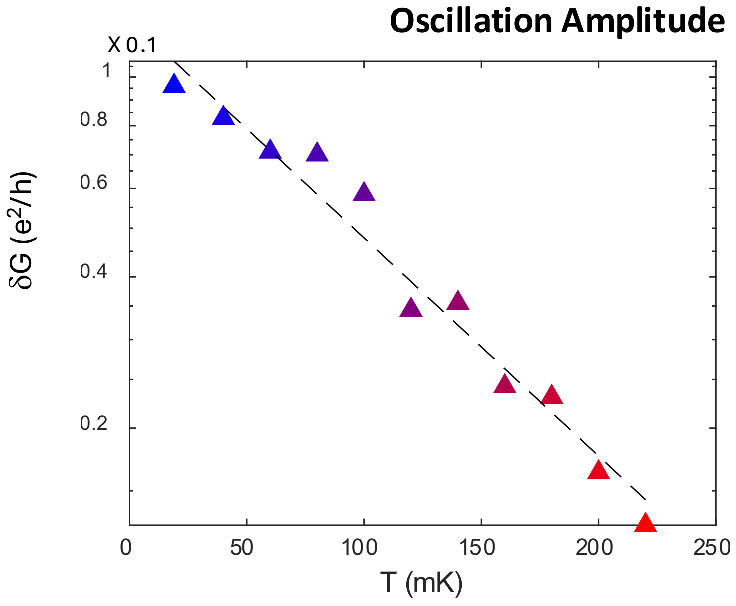
M. B. Weissman Rev. Mod. Phys. 60, 537 (1988)

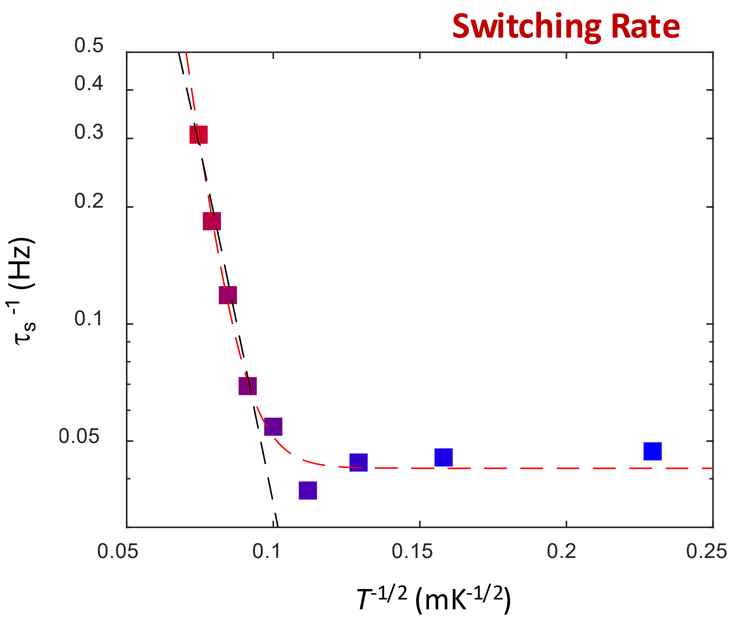
Switching Rate: Temperature Dependence

ν=1/3

12 T







Visibility $\sim e^{-T/T_0}$

Electron temperature can reach ~ 20 mK

Variable-range hopping

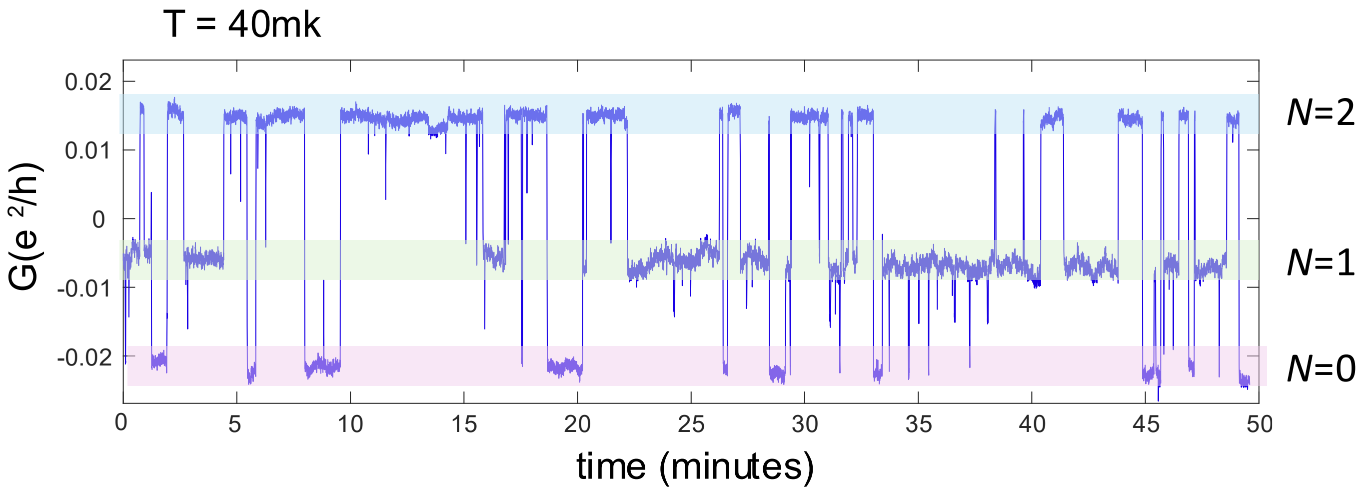
$$\tau_S^{-1} = \tau_{ES}^{-1}\, e^{-\left(\frac{T_{ES}}{T}\right)^{1/2}} + \tau_0^{-1}$$

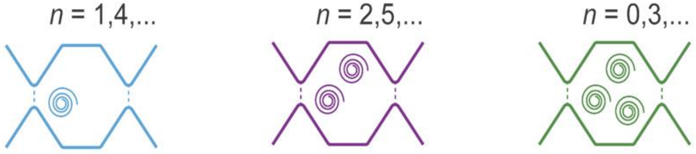
$T_{ES} = 25$ K

Efros and Shklovskii (1975)

Two-time joint probability: Correlation

ν=1/3   12 T

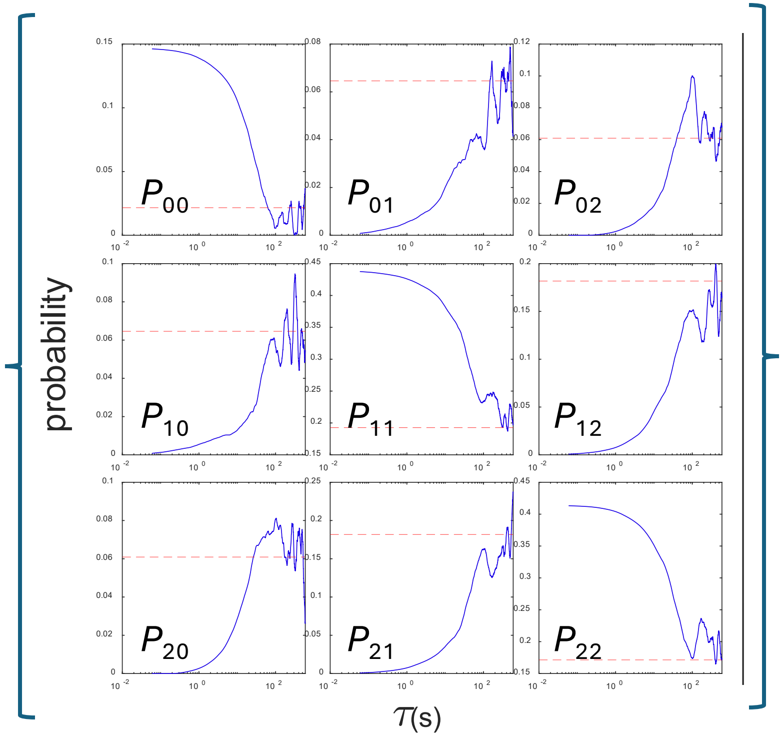




Nee a model to describe $P_{N,N'}(\tau)$

$P_{N,N'}(t_1\text{-}t_2)$:
The joint probability to find the system in state N at time t_1 and state N' at time t_2.

$P_{N,N'}(\tau)=$



arXiv:2402.12432

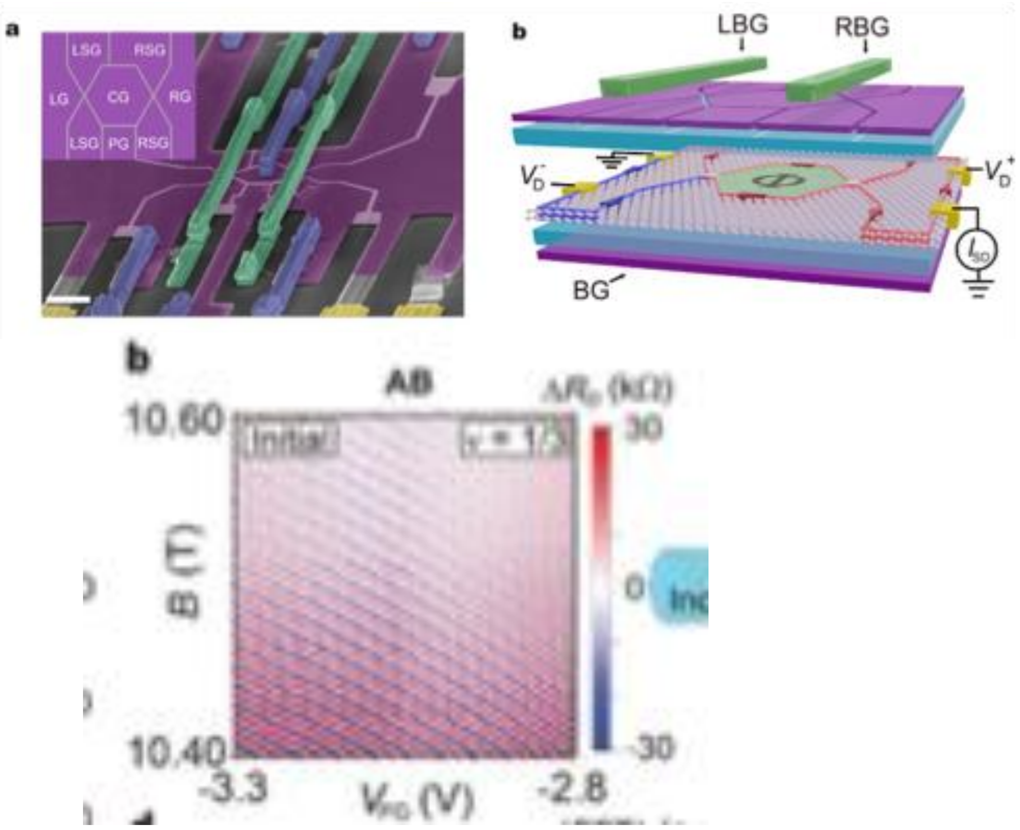
Aharonov-Bohm interference and the evolution of phase jumps in fractional quantum Hall Fabry-Perot interferometers based on bi-layer graphene

Jehyun Kim[1], Himanshu Dev[1], Ravi Kumar[1], Alexey Ilin[1], André Haug[1], Vishal Bhardwaj[1], Changki Hong[1], Kenji Watanabe[2], Takashi Taniguchi[3], Ady Stern[1], and Yuval Ronen[1]*

[1] *Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, Israel*

[2] *Research Center for Functional Materials, National Institute for Materials Science, Tsukuba, Japan*

[3] *International Center for Materials Nano architectonics, National Institute for Materials Science, Tsukuba, Japan*



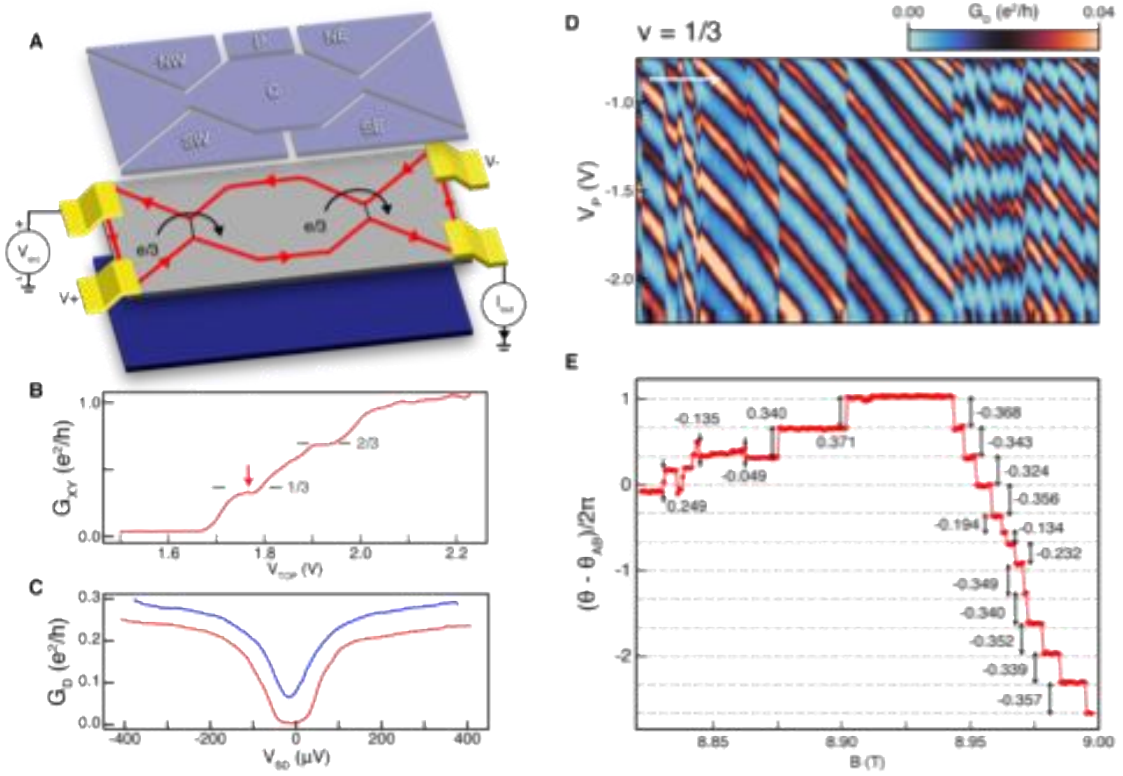
arXiv:2403.19628

Anyonic statistics and slow quasiparticle dynamics in a graphene fractional quantum Hall interferometer

Noah L. Samuelson,[1,*] Liam A. Cohen,[1,*] Will Wang,[1] Simon Blanch,[1] Takashi Taniguchi,[2] Kenji Watanabe,[3] Michael P. Zaletel,[4,5] and Andrea F. Young[1,†]

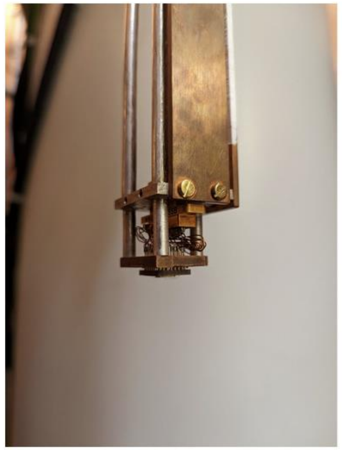
[1] *Department of Physics, University of California at Santa Barbara, Santa Barbara CA 93106, USA*
[2] *International Center for Materials Nanoarchitectonics, National Institute for Materials Science, 1-1 Namiki, Tsukuba 305-0044, Japan*
[3] *Research Center for Functional Materials, National Institute for Materials Science, 1-1 Namiki, Tsukuba 305-0044, Japan*
[4] *Department of Physics, University of California, Berkeley, California 94720, USA*
[5] *Material Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

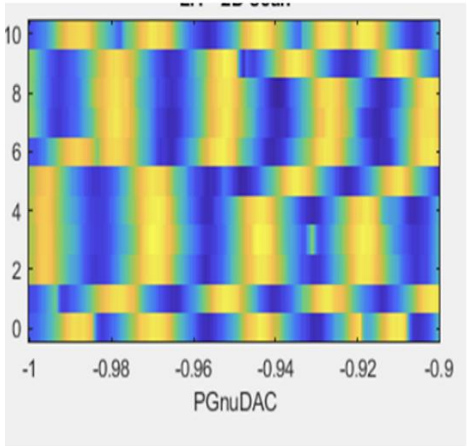
(Dated: March 29, 2024)



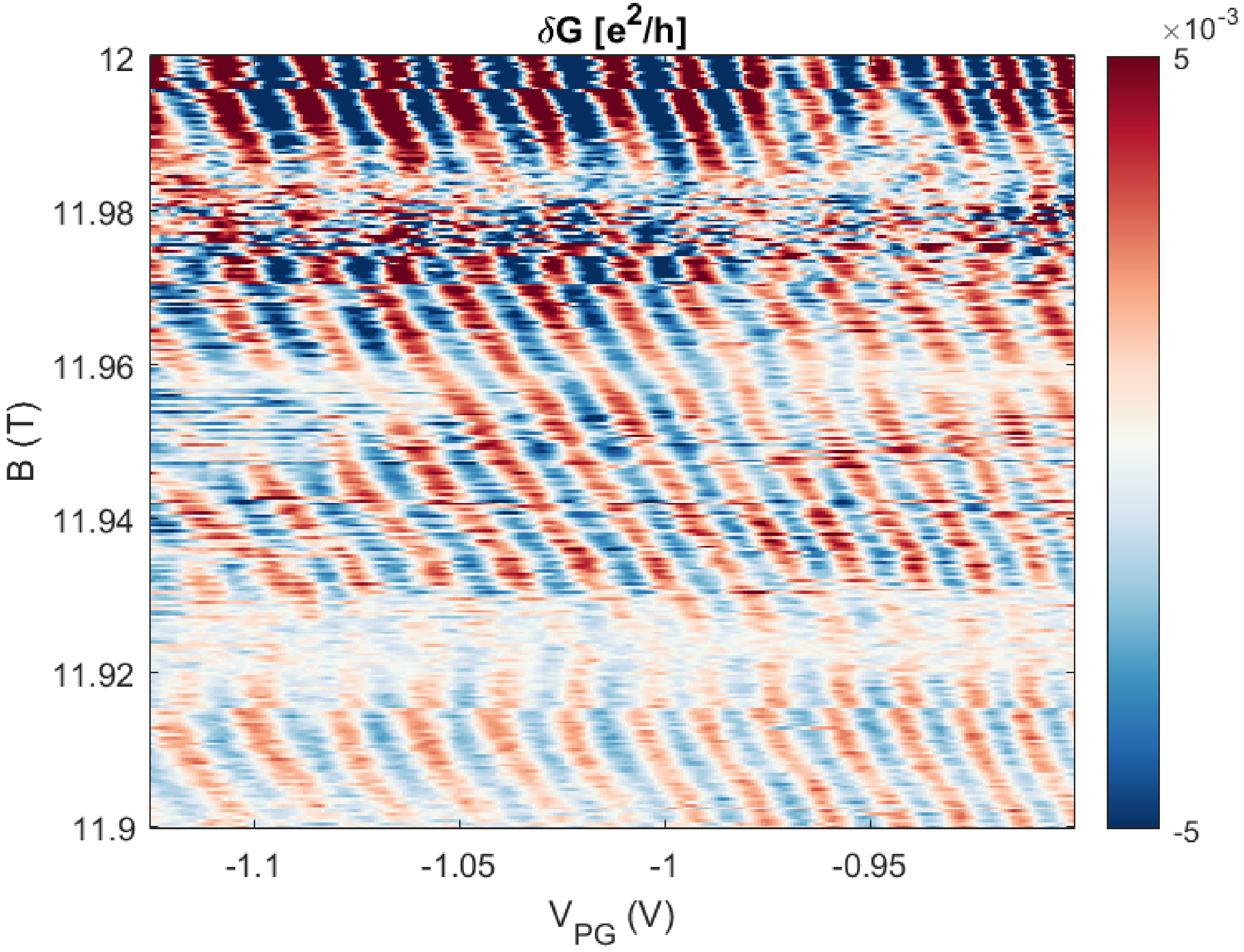
Why Does Quasiparticles Hopping Occur?

Config. 1: bare sample- exposed to ~700mK radiation





switching @ ~0.1 Hz



Pajama plot with static phase shift

Exchange versus Braiding

arXiv:2308.12986

A perspective on anyonic braiding statistics

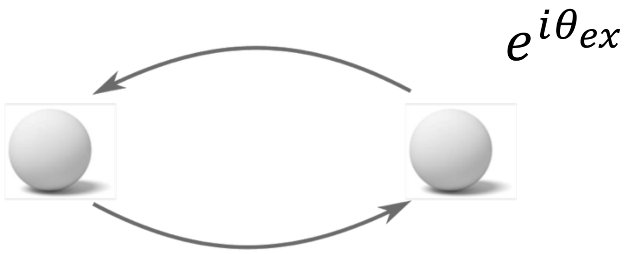
Nicholas Read [1,2] *and Sankar Das Sarma* [3]

This point, that the anyon statistics or statistical phase is not fully determined by a measurement of θ_a, has been somewhat ignored in numerous discussions and commentaries on the experimental result, including in the News and Views piece accompanying the publication [10]. We want to emphasize that, not only is a repeated elementary exchange a braid, but an elementary exchange is also a braid, according to the definition of braids (Artin's work dates originally from the 1920s [5]), and that the fundamental theoretical quantity of interest is θ (modulo 2π), the statistical phase for an elementary exchange, or equivalently $e^{i\theta}$ [3,4,6]; this determines the braiding statistics, while $e^{2i\theta}$ determines $e^{i\theta}$ only up to a factor ± 1.

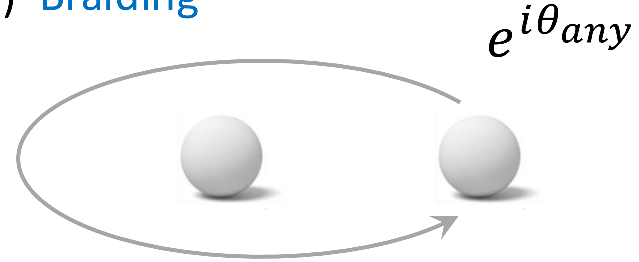
In conclusion, there is no suggestion that the experiment [1] did not correctly measure $e^{i\theta_a} = e^{2i\theta}$, demonstrating that the quasiparticles are anyons, but we do want to point out that it would be of great interest to design and perform an experiment to determine $e^{i\theta}$ uniquely, not only $e^{2i\theta}$.

[5] E. Artin, "Theory of Braids", Ann. Math. **48**, 101 (1947).

Exchange



(double) Braiding



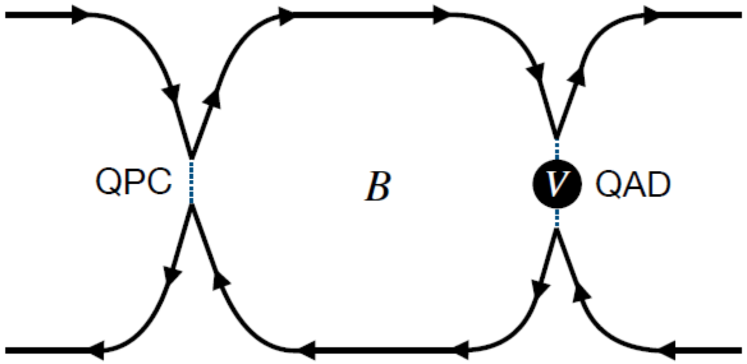
$$\theta_{any} = 2\theta_{ex}$$

Proposal for Probing Exchange Phase

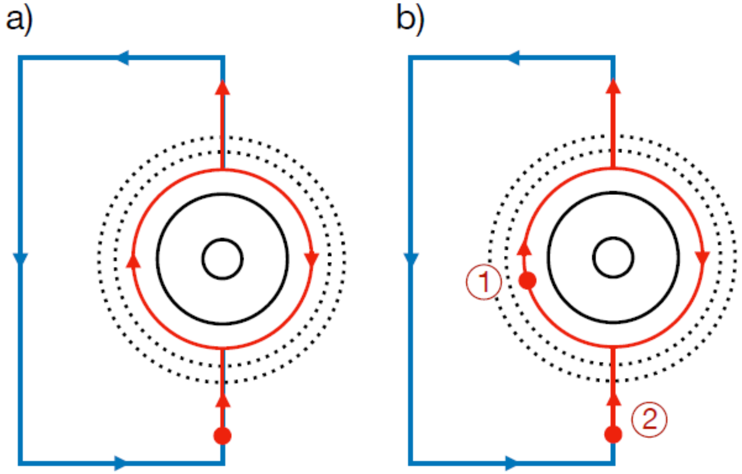
arXiv:2403.12139

A modified interferometer to measure anyonic braiding statistics

Steven A. Kivelson and Chaitanya Murthy[1]
[1]Department of Physics, Stanford University, Stanford, CA 94305, USA



$$\phi = 2\pi \frac{e^*}{e}\frac{AB}{\Phi_0} + 2\theta_{ex}N_L + \eta\theta_{ex}$$

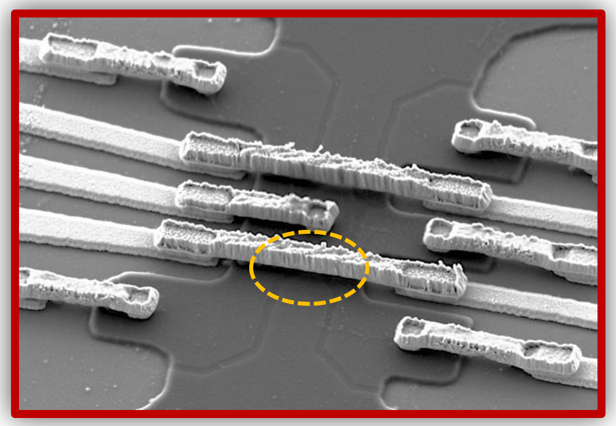


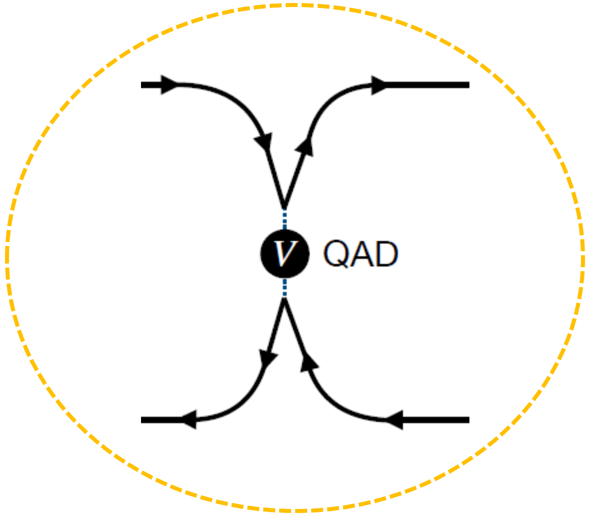
"Direct" tunneling

"Cooperative" tunneling, edge QP exchanges with existing one already on dot

- In standard FP interferometer, only can measure $\theta_{braid} = 2\theta_{ex}$
- Additional dot provides a mechanism for a single exchange within the device, enabling direct observation of θ_{ex}

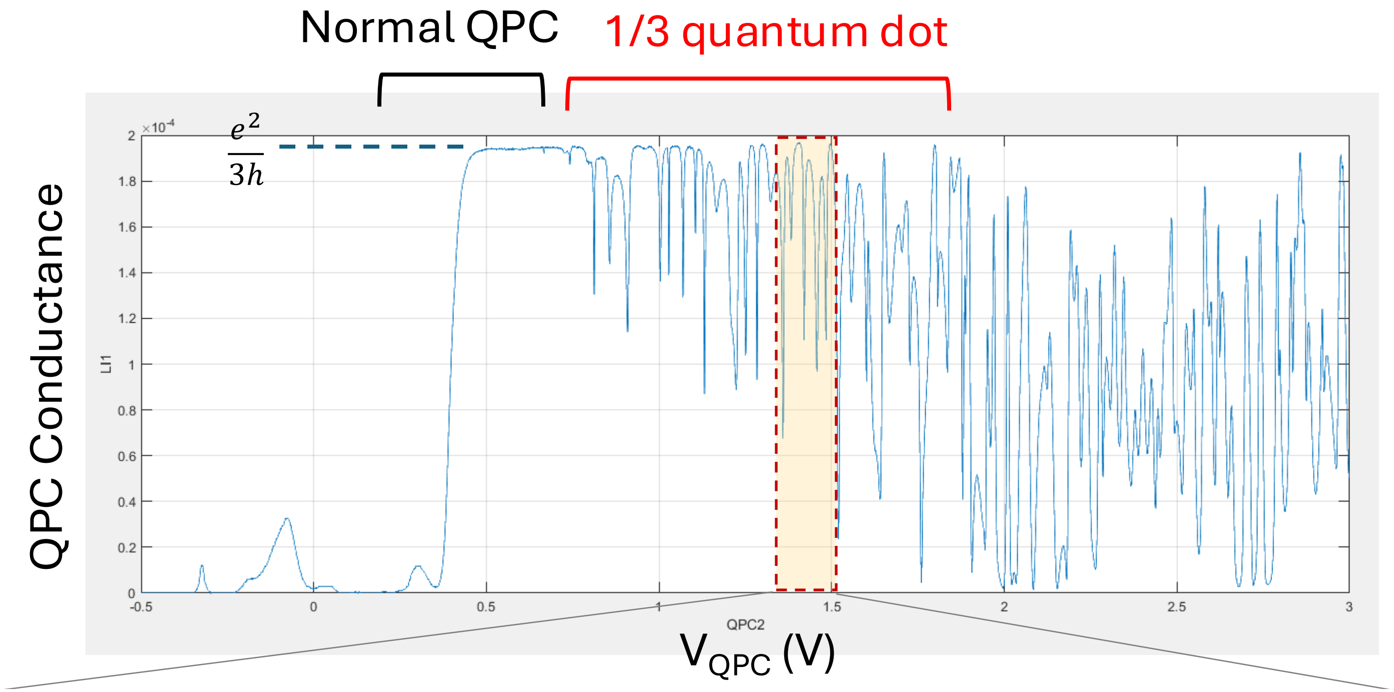
Very Preliminary Experimental Data

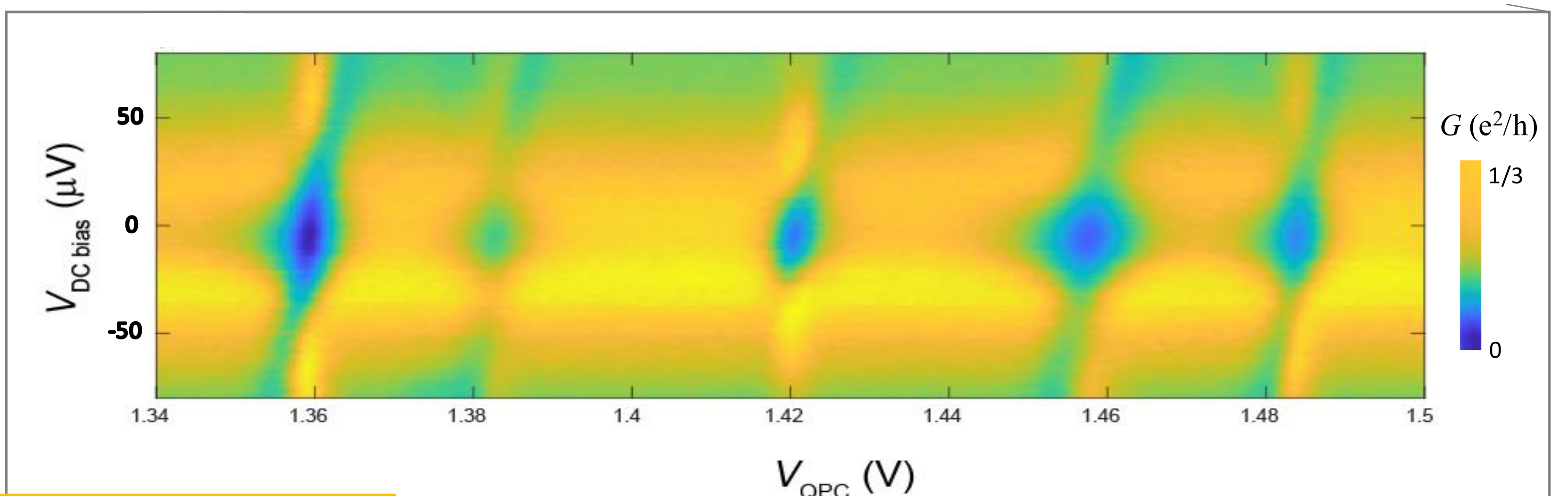




Accidental quantum dot formation underneath of QPC

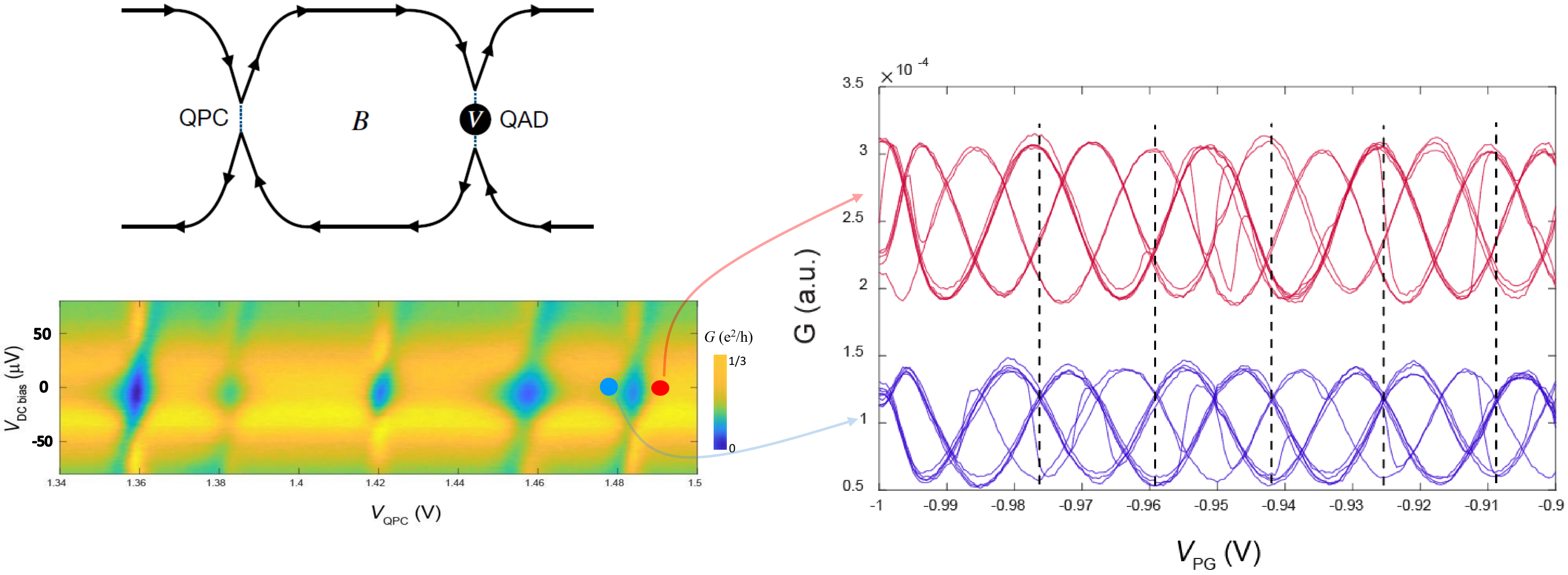
ν=1/3





Coulomb blockade Resonant

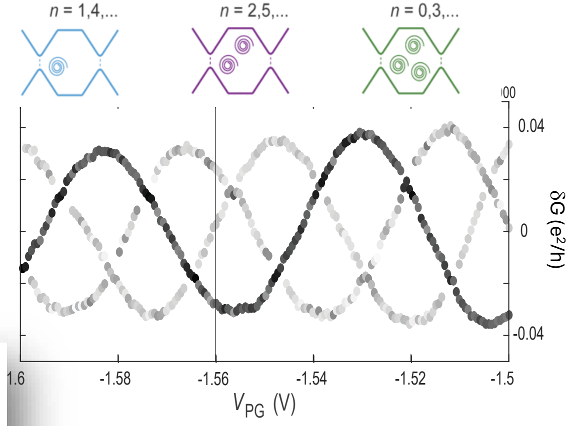
Phase Flipping Across the QAD Resonance



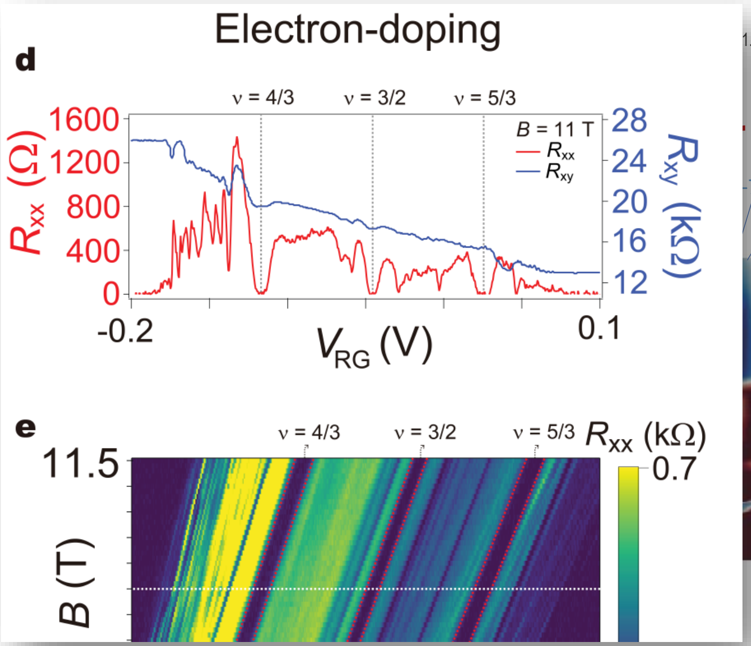
Experimentally, for RTN interferometer should see **60 deg phase shift in triple helix across quantum dot resonance** (shift from direct to cooperative tunneling)

Summary and Outlook

- **Fabry-Perot Interference for Integer and Fractional Quantum Hall States.**
- **Phase slips related to the quasi-particle occupancy in the interferometers.**
- **Abelian anyon braiding phase has been identified.**

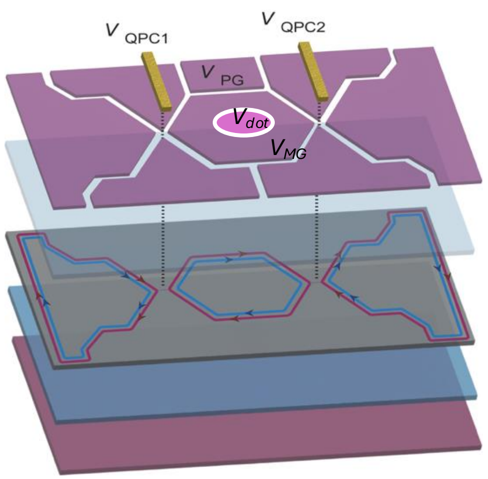


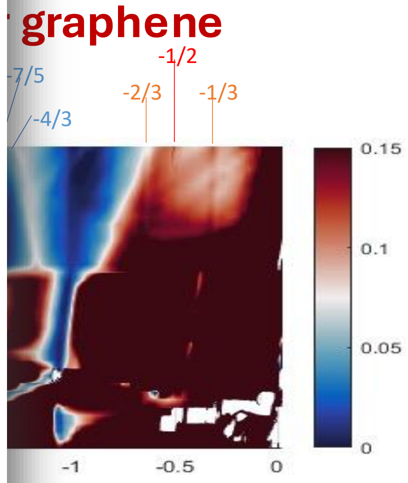
J. Kim *et al.*, arXiv:2412.19886



Outlook

Engineered quasi-particle occupancy: Anti-dot





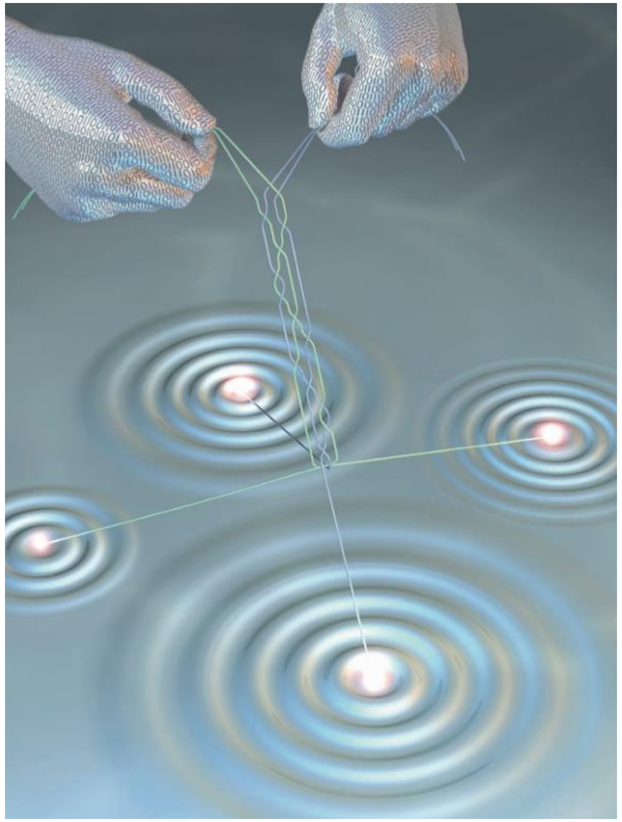
@ 16 T, 250 mK

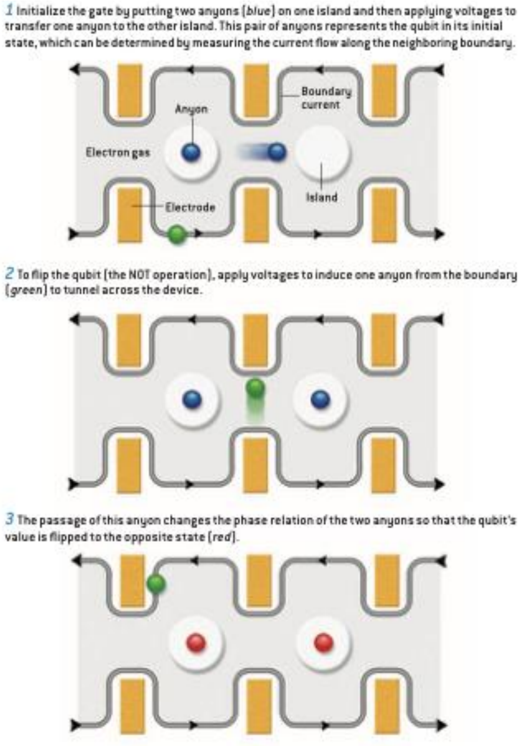
Nonabelian Anyon for Topologically Protected Qubit

PRL 94, 166802 (2005) PHYSICAL REVIEW LETTERS week ending 29 APRIL 2005

Topologically Protected Qubits from a Possible Non-Abelian Fractional Quantum Hall State

Sankar Das Sarma,[1] Michael Freedman,[2] and Chetan Nayak[2,3]





Graham P. Collins, SCIENTIFIC AMERICAN April 2006

PHYSICAL REVIEW X 13, 011028 (2023)

Interference Measurements of Non-Abelian $e/4$ & Abelian $e/2$ Quasiparticle Braiding

R. L. Willett,[1,*,†] K. Shtengel,[2] C. Nayak,[3,4] L. N. Pfeiffer,[5] Y. J. Chung,[5] M. L. Peabody,[1] K. W. Baldwin,[5] and K. W. West[5]