AC transport measurement in flat band systems
Boulder Summer School Lecture 2

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Mass

crystal lattice. (a) Show that for a hexagonal-close-packed crystal structure the Fourier component $U(\mathbf{G}_c)$ of the crystal potential $U(\mathbf{r})$ is zero. (b) Is $U(2\mathbf{G}_c)$ also zero? (c) Why is it possible in principle to obtain an insulator made up of divalent atoms at the lattice points of a simple hexagonal lattice? (d) Why is it not possible to obtain an insulator made up of monovalent atoms in a hexagonal-close-packed structure?

4. ***Brillouin zones of two-dimensional divalent metal.*** A two-dimensional metal in the form of a square lattice has two conduction electrons per atom. In the almost free electron approximation, sketch carefully the electron and hole energy surfaces. For the electrons choose a zone scheme such that the Fermi surface is shown as closed.

Renormalization of mass by remote bands

Charles Kittel, Introduction to Solid State Physics, 8^{th} edition, p 253

Quantum Geometry in Solids

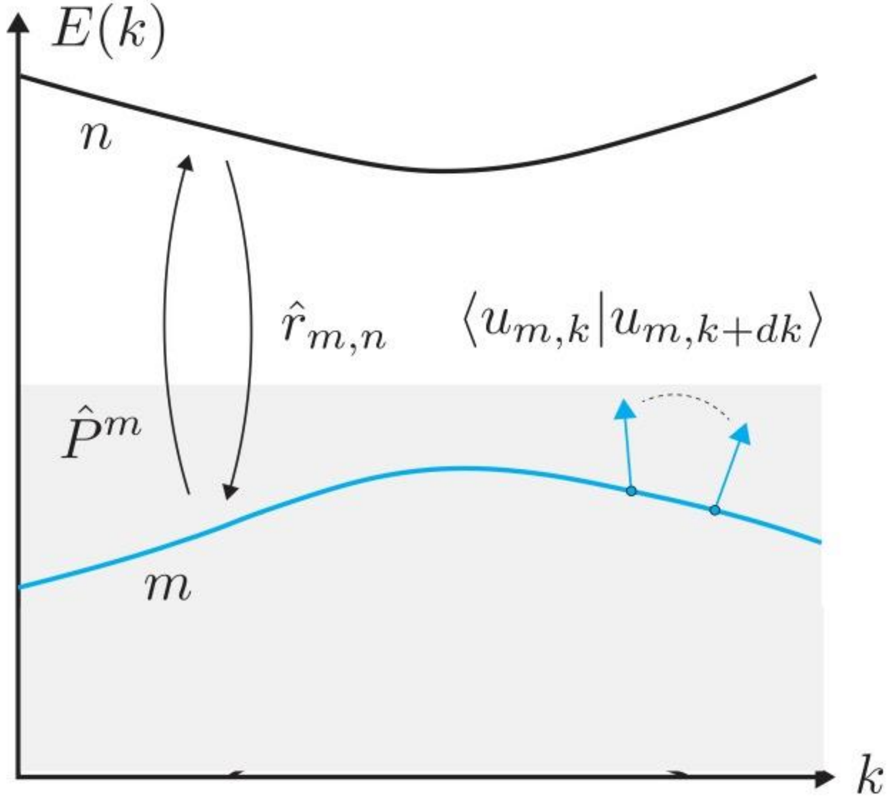
Quantum Geometric Tensor: measure of dipolar fluctuation in Bloch electrons

$$\mathcal{Q}_{\mu\nu} = \mathrm{tr}\left[P\hat{r}_\mu(1-P)\hat{r}_\nu\right] \sim \langle \hat{r}_\mu \hat{r}_\nu \rangle - \langle \hat{r}_\mu \rangle \langle \hat{r}_\nu \rangle$$

$$\mathcal{Q}_{\mu\nu} = g_{\mu\nu} - \tfrac{i}{2}\Omega_{\mu\nu}$$

quantum metric (symmetric)

Berry curvature (antisymmetric)



→ Insulators, metric gives localization of electrons:

$$\ell_g^2 = \mathrm{tr}\ g_{\mu\nu}$$

Resta and Sorella 1997
Souza, Willkens, Martin, 2000

→ Distinguishes insulators from metals: **g diverges in metals** Kohn 1964

Slide from: Raquel Queiroz

Experimentally Observable Quantum Geometric Effect

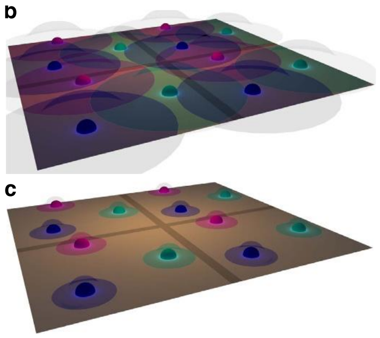
nature COMMUNICATIONS

ARTICLE

Received 17 Sep 2015 | Accepted 19 Oct 2015 | Published 20 Nov 2015 DOI: 10.1038/ncomms9944 OPEN

Superfluidity in topologically nontrivial flat bands

Sebastiano Peotta[1] & Päivi Törmä[1,2]



$$\mathcal{M}_{ij} = \mathcal{M}^{R}_{ij} + i\epsilon_{ij}\, C \geq 0$$

$$\downarrow \qquad\qquad \downarrow$$

$$D_s \geq |C|$$

PHYSICAL REVIEW B **95**, 024515 (2017)

Band geometry, Berry curvature, and superfluid weight

Long Liang, Tuomas I. Vanhala, Sebastiano Peotta, Topi Siro, Ari Harju,* and Päivi Törmä†
COMP Centre of Excellence, Department of Applied Physics, Aalto University, Helsinki, Finland
(Received 6 October 2016; revised manuscript received 10 January 2017; published 27 January 2017)

nature communications

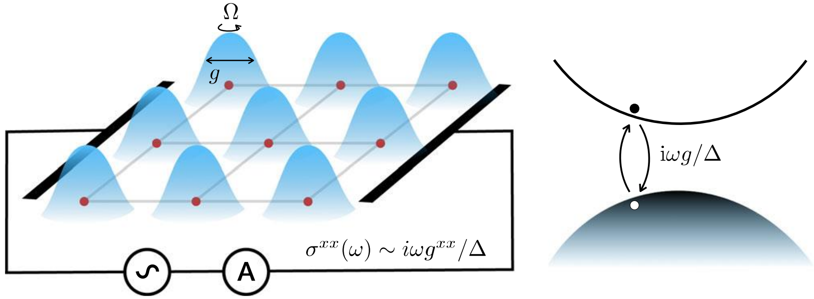
Article https://doi.org/10.1038/s41467-024-48808-x

The quantum geometric origin of capacitance in insulators

Received: 31 August 2023

Accepted: 15 May 2024

Ilia Komissarov[1], Tobias Holder[2,3] & Raquel Queiroz[1,4]



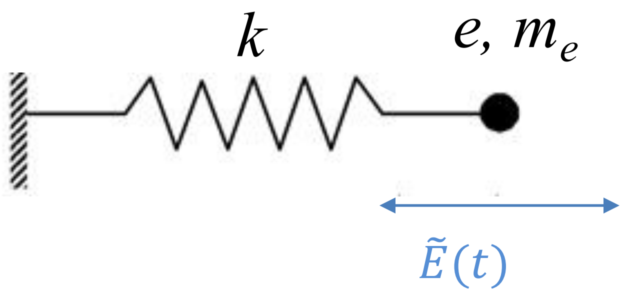
$$\sigma^{\mu\nu}(\omega) = -\frac{e^2}{h} C\epsilon^{\mu\nu} + i\omega c\,\delta^{\mu\nu} + \ldots,$$

For Landau levels for parabolic bands

$$c = \frac{e^2}{h\omega_c} C.$$

AC Conductivity: Measure of Quantum Geometry

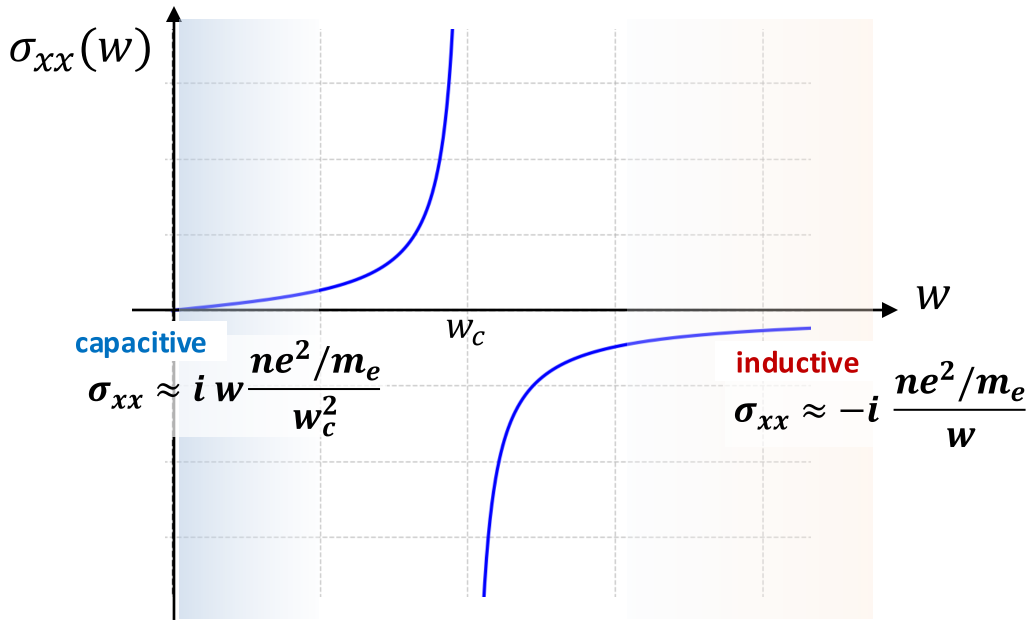
The Lorentz Model for Dielectric Matter



$$\ddot{x}(t) + \omega_c^2 x(t) = -\frac{e}{m_e} E e^{i\omega t}$$

AC conductivity without damping

$$\sigma_{xx}(w) = i\, w \frac{ne^2/m_e}{w_c^2 - w^2}$$



Part I: Superconductors $w \gg w_c \approx 0$

Kinetic Inductance measurement for superfluid stiffness

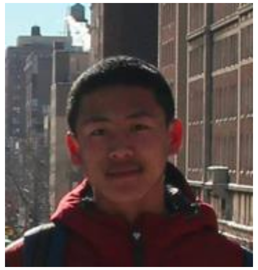
Part II: QH Insulators $w \ll w_c$

Capacitance measurement for quantized dielectric response

Part I:
Superfluid Stiffness Measurement of Twisted Graphene Superconductors

Experiments

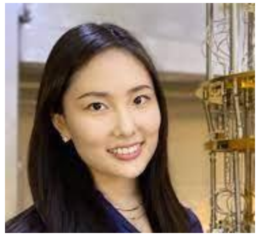

Abhishek Banerjee


Zeyu Hao


Mary Kreidel


Andrew Zimmerman


Isabelle Phinney


Jeong Min Park


Pablo Jarillo-Herrero


Kin Chung Fong

hBN

T. Taniguchi, K. Watanabe

Theory


Pavel Volkov

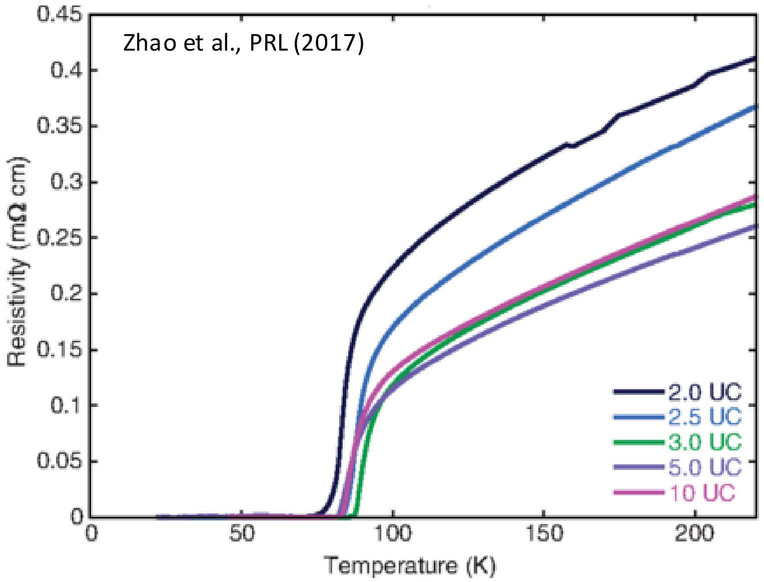

Patrick Ledwith


Ashvin Vishwanath

Funding:

B. Abhishek et al., Nature (2025)

Superfluid Stiffness of Superconductor



Perfect conducting

London Equation

$$\mathbf{J}_s = -\frac{n_s e^2}{m}\mathbf{A}$$

$$\rho_s = \frac{\hbar^2}{4}\frac{n_s}{m}$$

"superfluid stiffness"

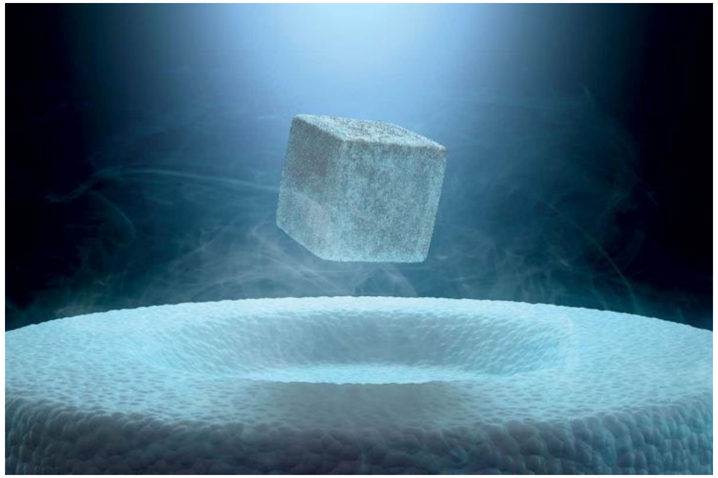


Image from: Kiyoshi Takahase Segundo / Alamy Stock Photo

Perfect Diamagnetic

G-L free energy

$$\mathcal{F} = \frac{1}{2}\int d^d\mathbf{r}\,\rho_s\,|\nabla\phi(\mathbf{r})|^2$$

Superconducting order parameter phase change

$$\Delta_0 e^{i\phi(\mathbf{r})}$$

Superfluid Stiffness Measurement

Superfluid stiffness can be measured by diamagnetic shielding response of superconductors

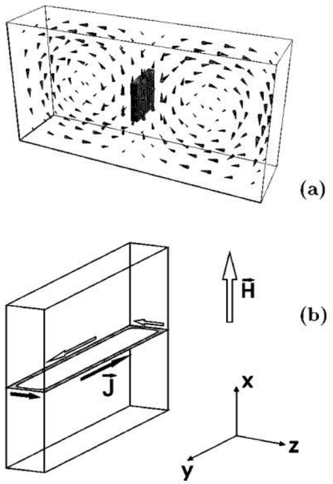
$$\mathbf{J}_s = -\frac{n_s e^2}{m}\mathbf{A} = -\frac{4e^2}{\hbar^2}\rho_s \mathbf{A}$$

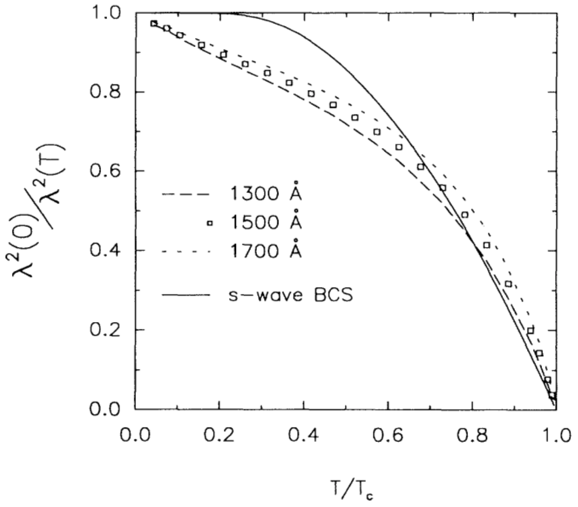
Magnetic Penetration length:

$$\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{\frac{1}{2}} = \left(\frac{\hbar^2}{4\mu_0 e^2 \rho_s}\right)^{\frac{1}{2}}$$

Microwave cavity Penetration Length Measurement

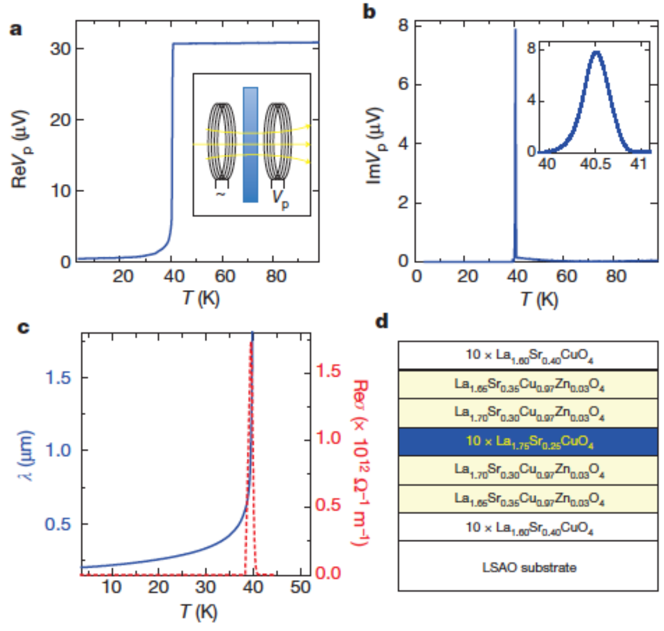
YBCO cuprate single crystal





W. N. Hardy et. al., PRL 70 (1993)

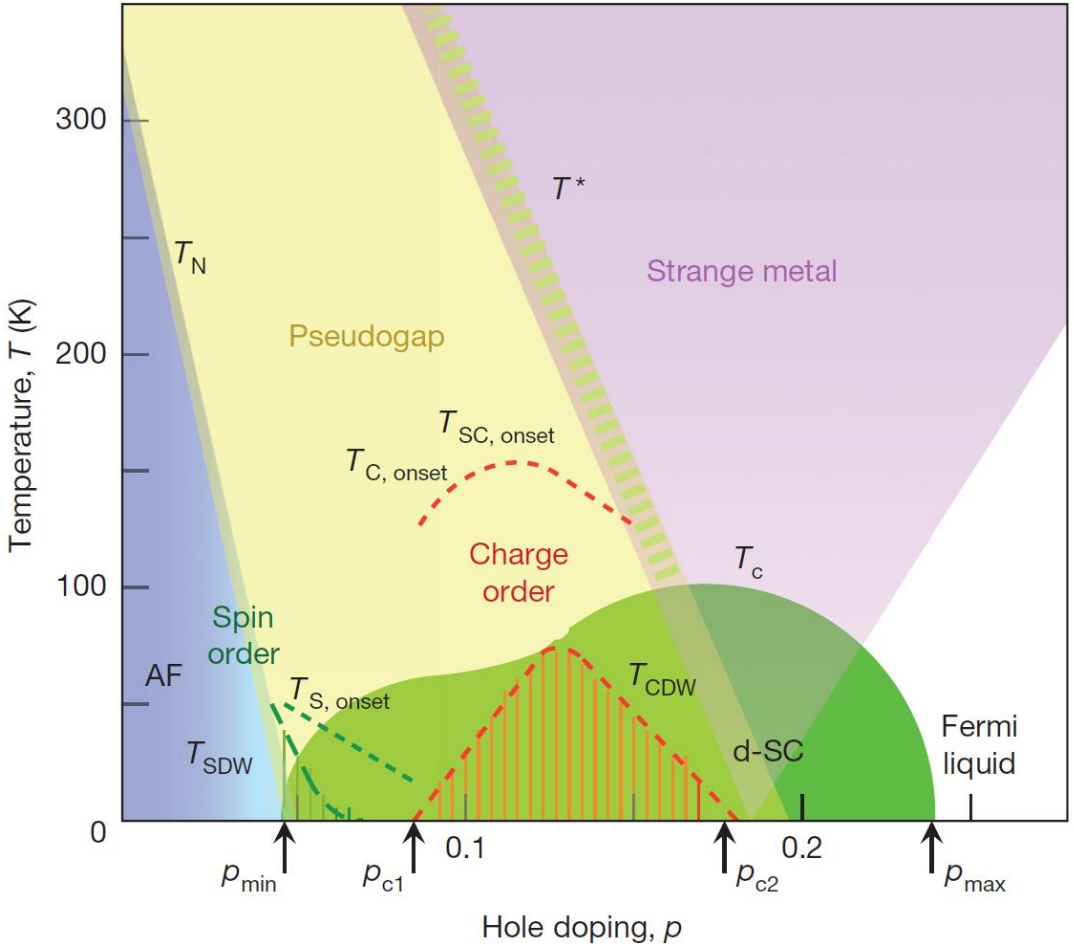
Mutual inductance Measurement



I. Bozovic et. al., Nature 536, 309 (2016)

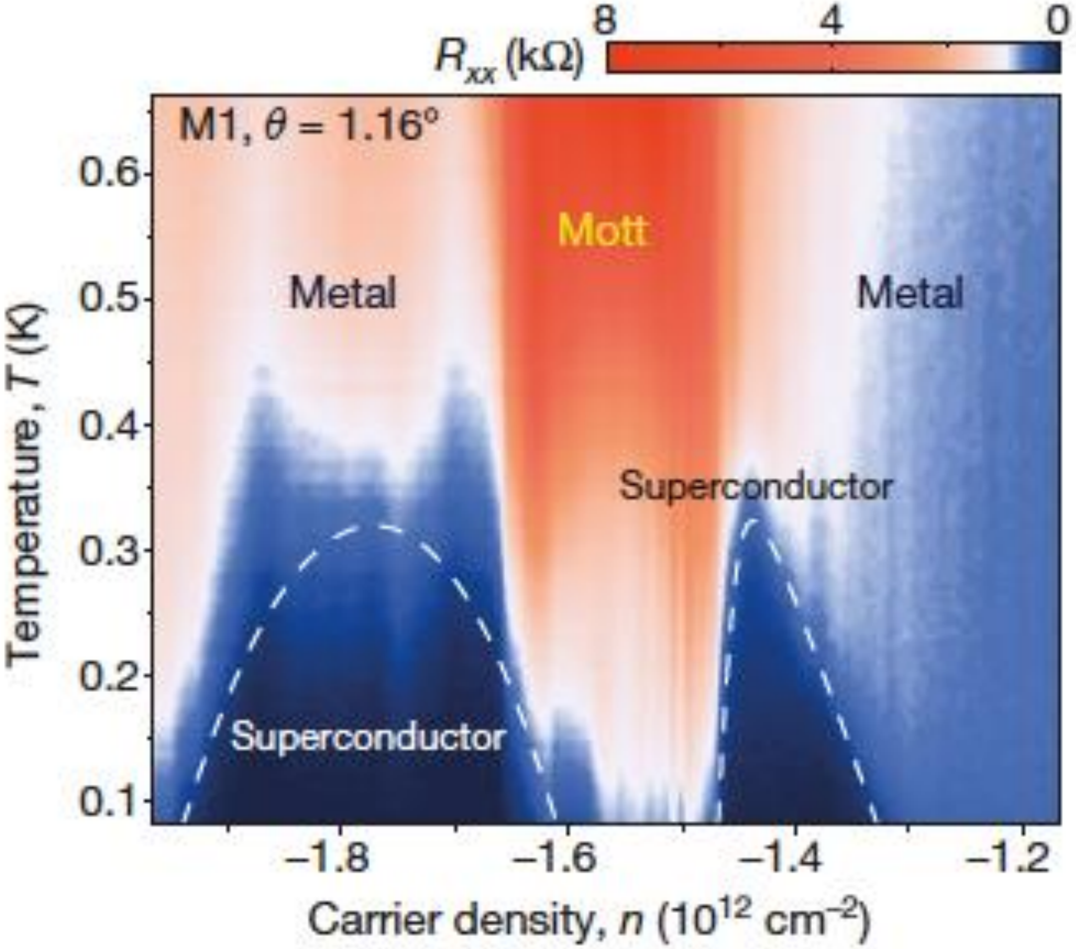
Unusual Superconducting Phase Diagrams

Cuprate superconductivity:
dopped anti-ferromagnetic insulator



B. Keimer *et al.* Nature (2015)

Magic Angle Twist Bilayer Graphene Superconductivity:
dopped correlated insulator

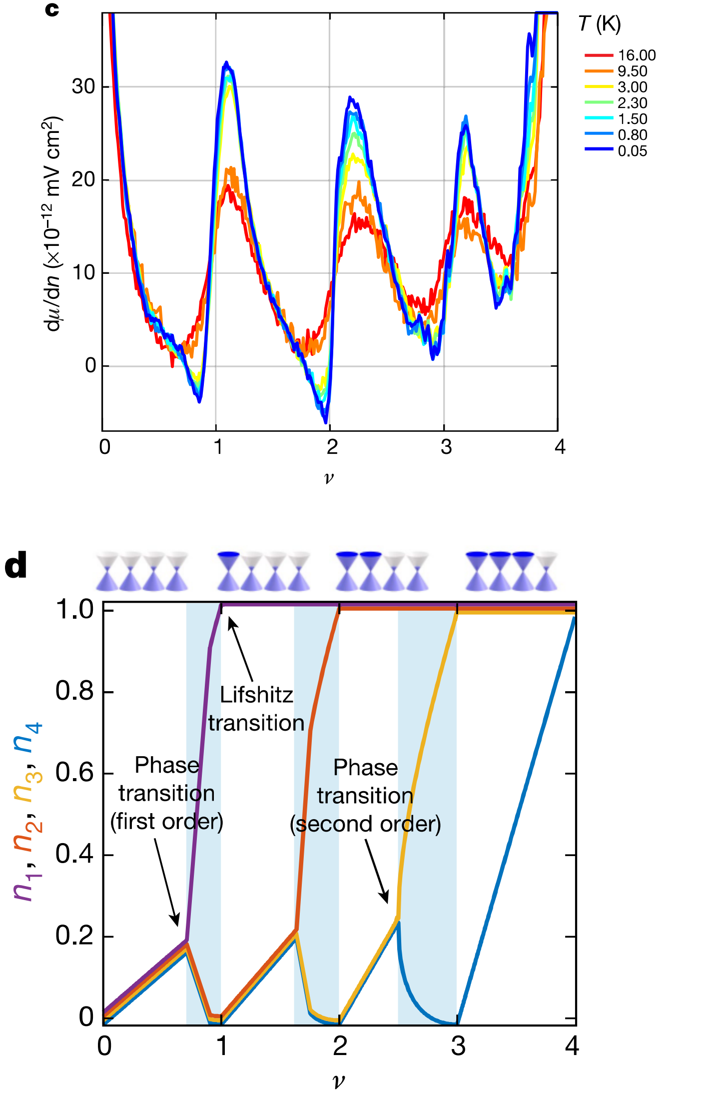


Y. Cao *et al.* Nature (2018)

Graphene Moire Correlated Insulators: Broken Symmetry

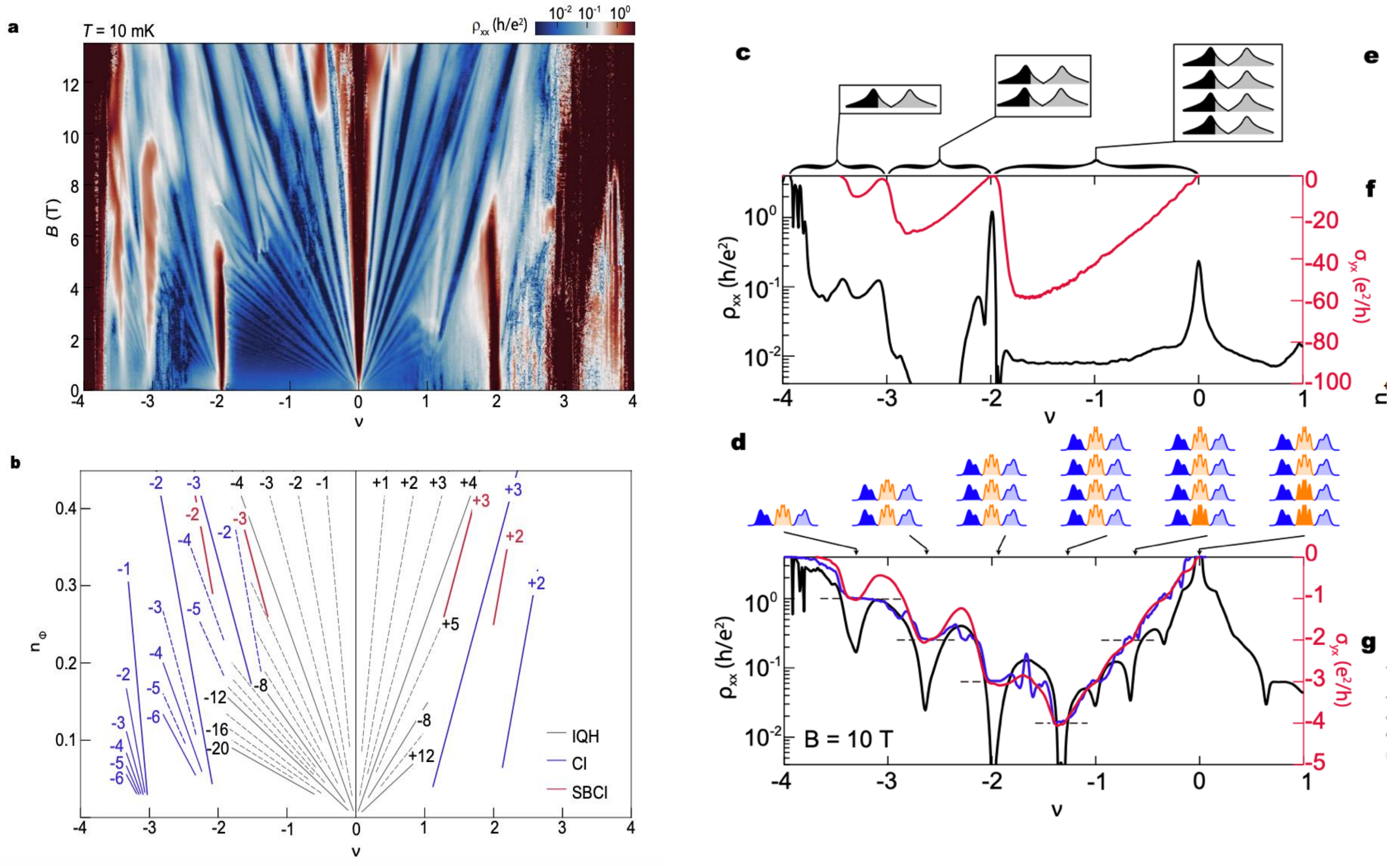
Scanning SET

U. Zondiner *et al*, Nature 582, 203 (2020)



Cascade Transition: broken flavour symmetry

Y. Saito et al, Nature Physics 17, 478 (2021)

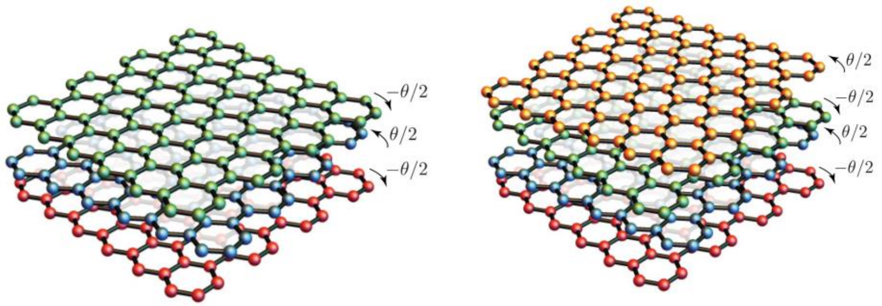


Unusual Fan Diagram: magnetic field induced Chern bands

Multi-layer Graphene Moire

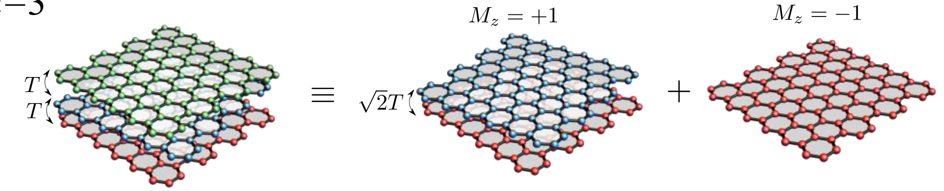
E. Khalaf, A. Kruchkov, G. Tarnopolsky, and A. Vishwanath, PRB 100, 085109 (2020)

Mulitlayer twisted stacked graphene with alternative angles

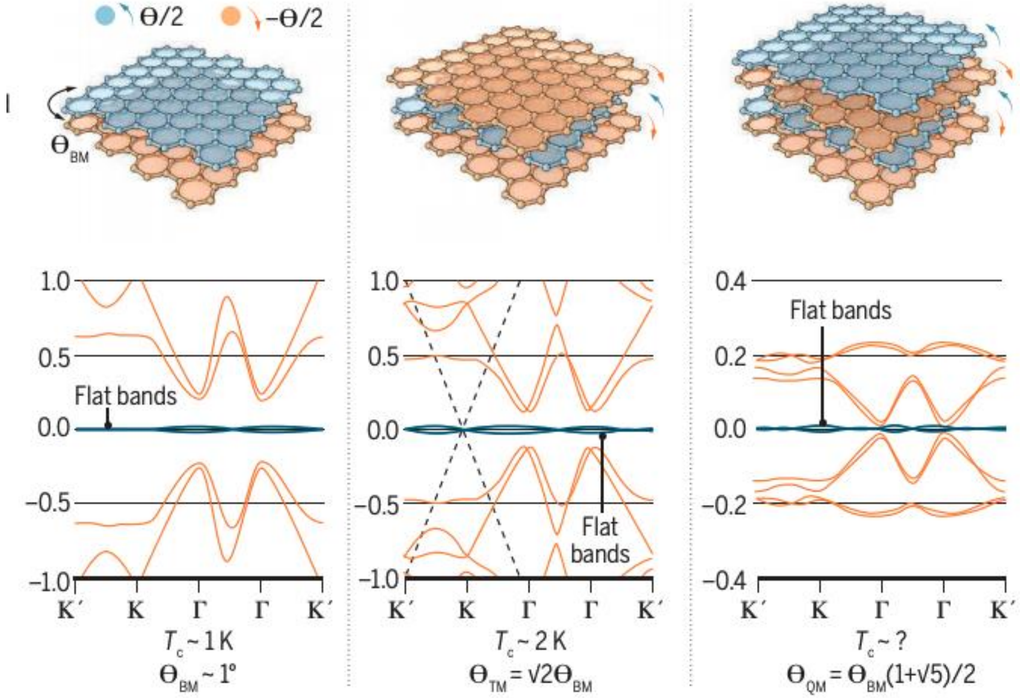


$$H(\boldsymbol{r}) = \begin{pmatrix} -iv\boldsymbol{\sigma}_+ \cdot \boldsymbol{\nabla} & T(\boldsymbol{r}) & 0 & \cdots \\ T^\dagger(\boldsymbol{r}) & -iv\boldsymbol{\sigma}_- \cdot \boldsymbol{\nabla} & T^\dagger(\boldsymbol{r}) & \cdots \\ 0 & T(\boldsymbol{r}) & -iv\boldsymbol{\sigma}_+ \cdot \nabla & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

n=3



Twisted bilayer | Twisted trilayer | Twisted quadruple layer



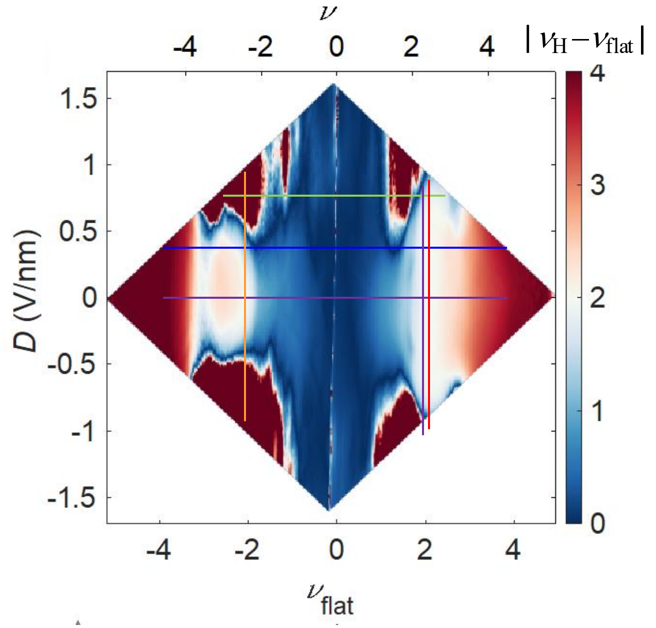
- $2n$+1 layers: single Dirac cone + n copies of TGB at different interlayer coupling
- $2n$ layers: n copies of TGB at different interlayer coupling

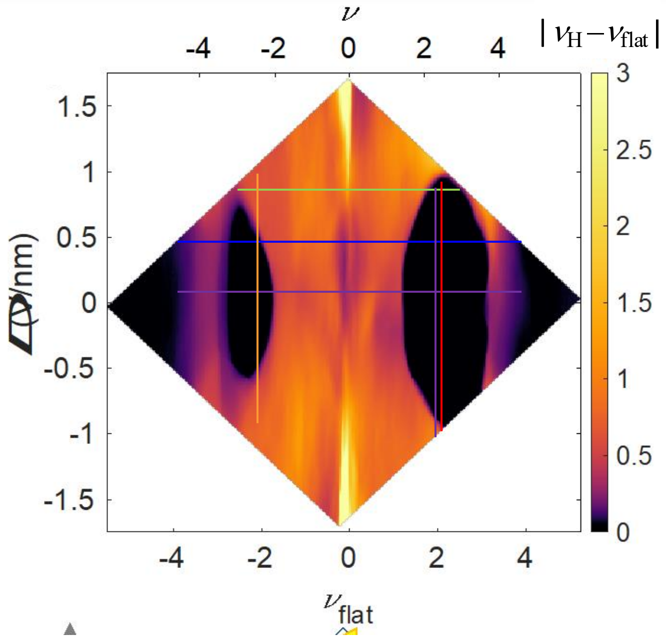
Twisted Quadruple Layer Graphene Normal State and Superconducting State

Subtracted Normalized Hall Data

$\nu_H = B/e\rho_{xy}n_s$

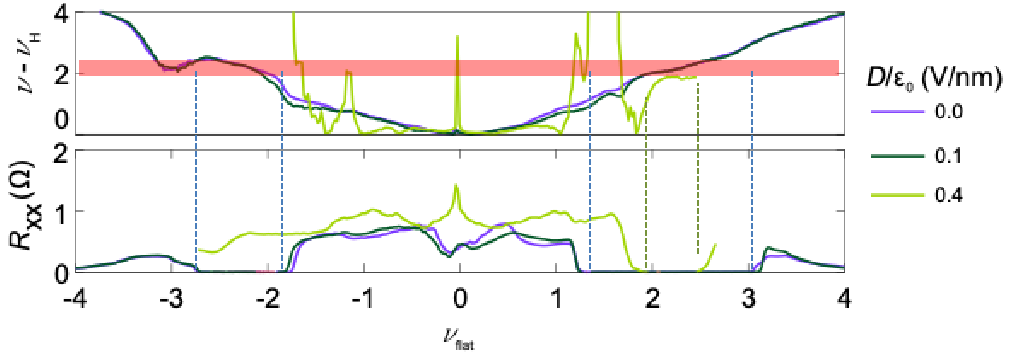
$f = |\nu_H - \nu_{flat}|$

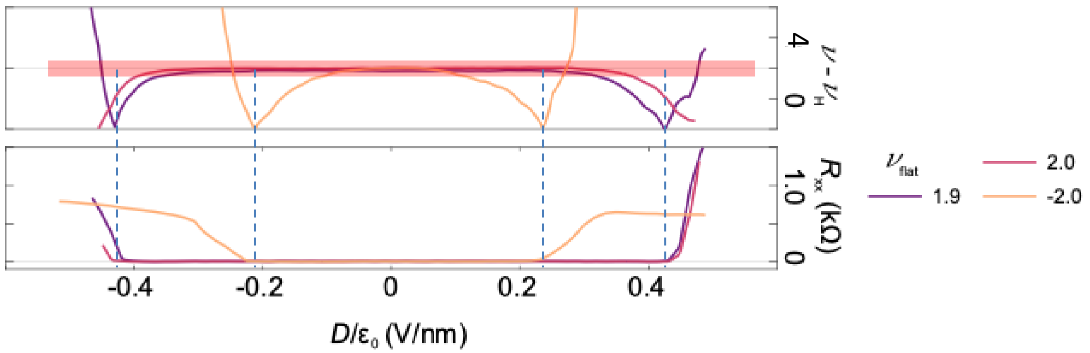




Resistance

Phinney et al., in preparation





- Symmetry broken state of $\nu=\pm2$ exactly coincides with the superconducting regime.
- Connection between SU(4) magnetism and potentially unconventional superconductivity!

Unusual Superconductivity in Twisted Graphene?

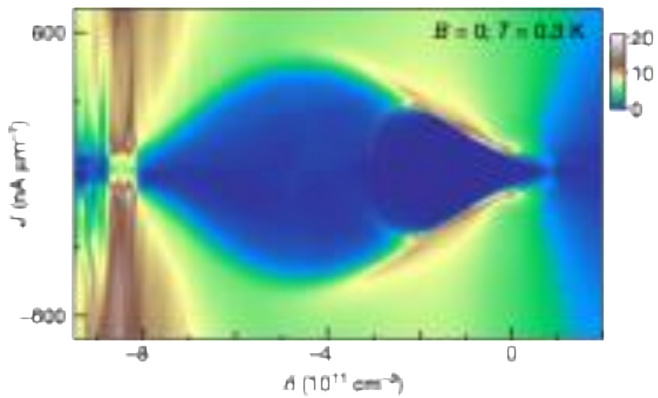
- **Correlation with symmetry breaking in the superconducting state**
- **Critical current nematicity**

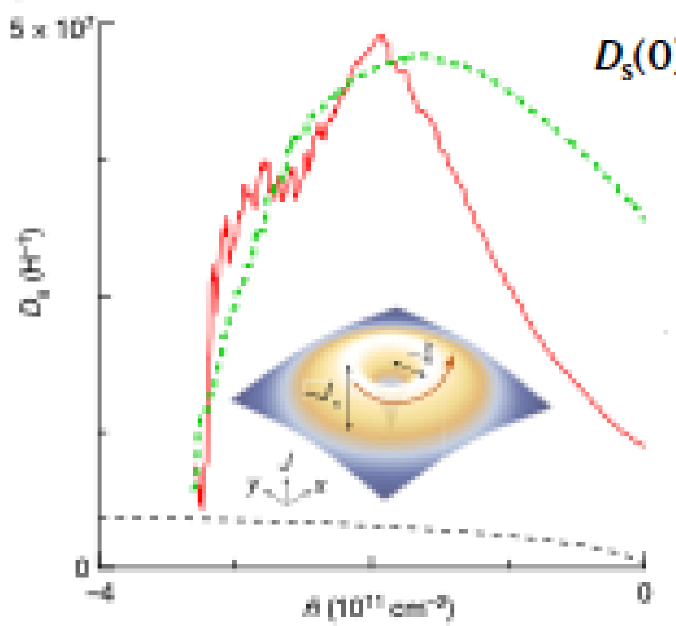
  Cao et al., Science 372(6539), 264–271 (2021)
- **Pauli limit violation**

  Cao et al., Nature 595(7868), 526–531 (2021)
- **STS gap measurement**

  Oh et al., Nature 600(7888), 240–245 (2021);
  Kim et al., Nature 606(7914), 494–500 (2022)
- **Unusual behavior of superfluid stiffness**

Indirect estimation of superfluid stiffness





$$D_s(0) = \frac{2\pi J_{cs}\xi}{\Phi_0}$$

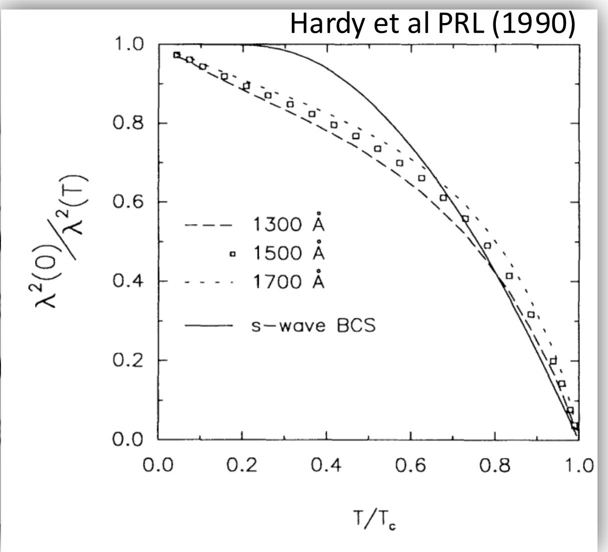
H. Tian et al., Nature (2023); Lau & Bockrath group

Unconventional Superconductivity

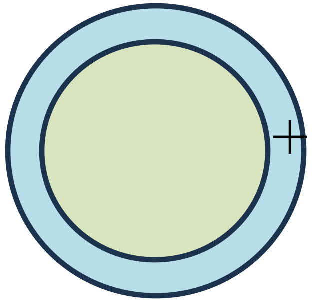
- Unconventional superconductivity : Gap Δ_k breaks symmetry of underlying lattice or goes to zero at some parts of Fermi surface (nodes)

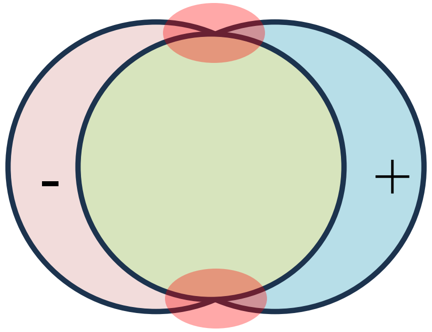
Superconducting order parameter symmetry

l of $\varphi_{\mathbf{k}}, \Delta_{\mathbf{k}}$		Symmetry of $\varphi_{\mathbf{k}}, \Delta_{\mathbf{k}}$		spin fun
0	s-wave	+1	odd	singl
1	p-wave	-1	even	tripl
2	d-wave	+1	odd	singl
3	f-wave	-1	even	tripl
...		...		...



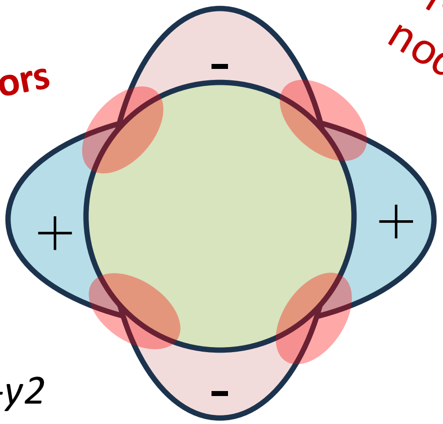
- S-wave: original BCS
- p-wave: superfluid ^{3}He
- d-wave: cuprates





p_x

Nodal superconductors



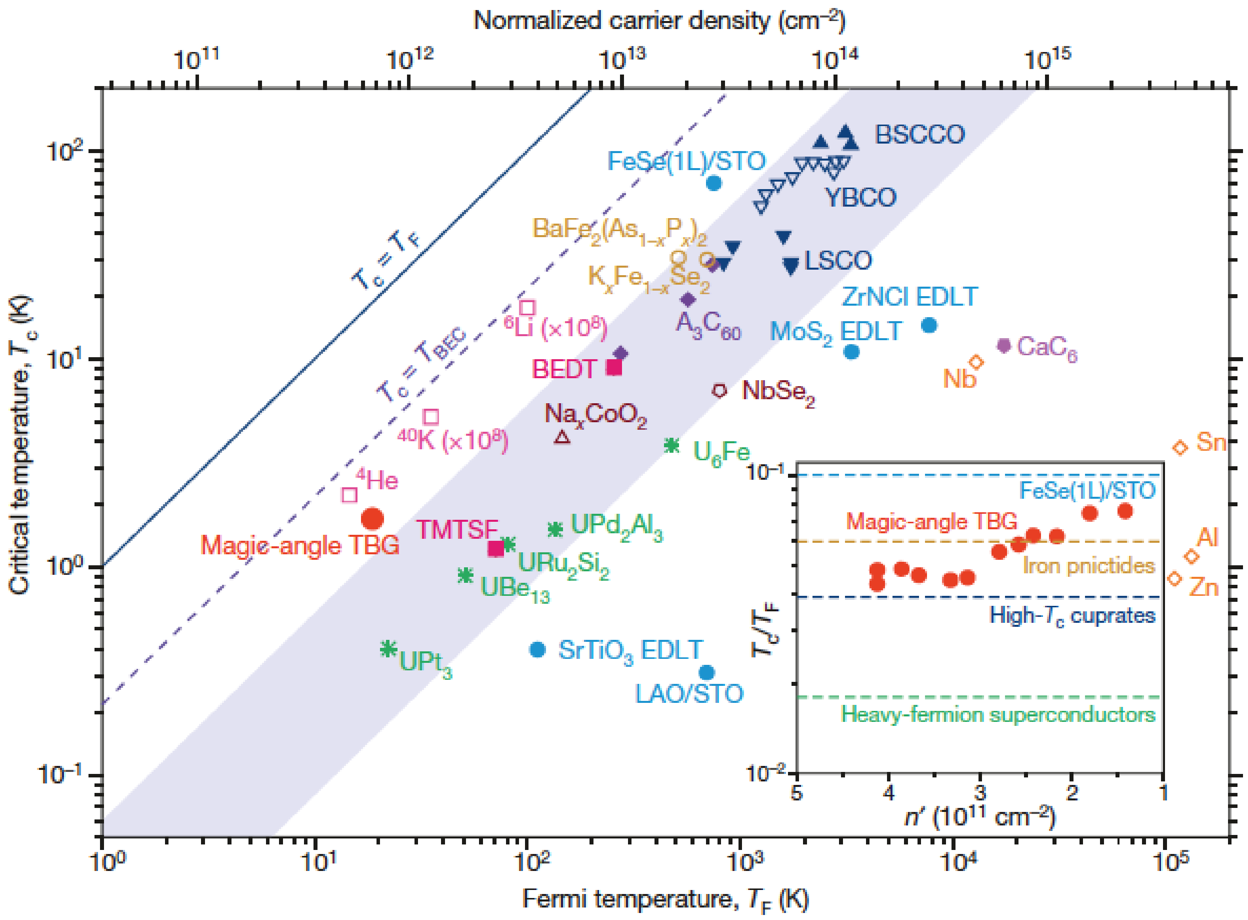
d_{x2-y2}

Thermal excitation of quasiparticles across the nodal gaps

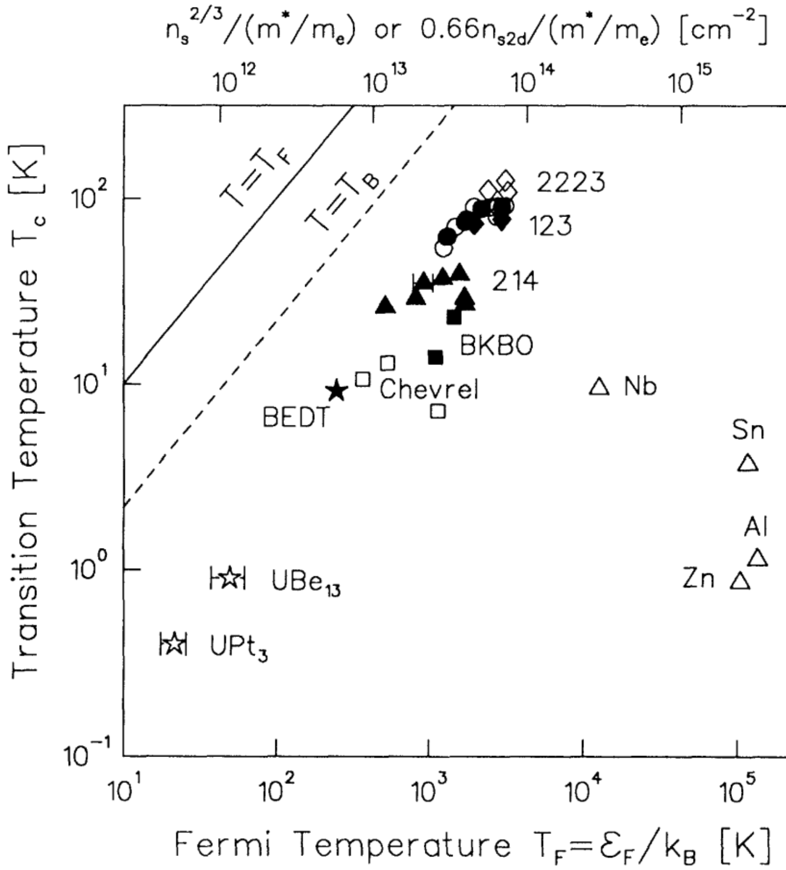
Superconductivity in the strong-coupling limit

$$\frac{\Delta}{E_F} \sim \frac{T_c}{T_F}$$

The Uemura plot

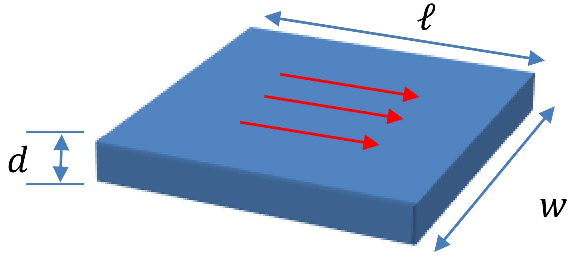


Cao et al., Nature 556, 43 (2018)



Uemura et al., PRL 68, 2712 (1992)

Kinetic Inductance of Superconductors



$$I = (en_e v)(wd)$$

Kinetic energy of charge carrier

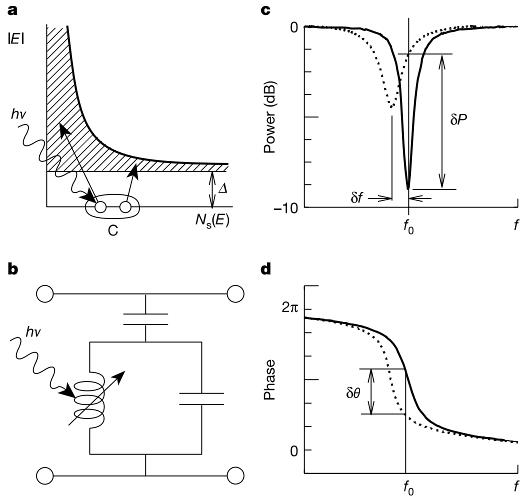
$$E_{kinetic} = \frac{1}{2} m_e v^2 (n_e\, \ell wd) \;\; = \frac{1}{2} L_k I^2$$

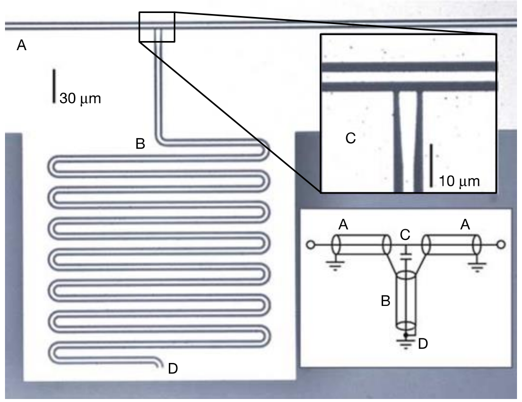
Kinetic Inductance:

$$L_k = \left(\frac{m_e}{n_s e^2}\right)\left(\frac{\ell}{w}\right)$$

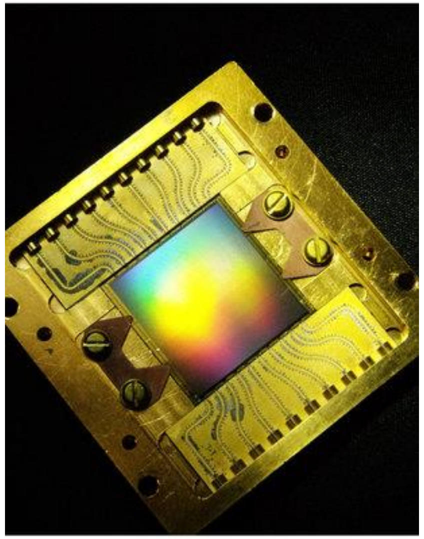
can be detected when there is small dissipation…

Broadband Superconducting KI Detector for Arrays





P. K. Day et. al., Nature 425, 817 (2003)



Microwave Kinetic Inductance Detector Array for exoplanet camera

https://www.naoj.org/Projects/SCEXAO/scexaoWEB/

Kinetic Inductance and Superfluid Stiffness of 2D Superconductors

Sheet Kinetic Inductance:

$$L_k^{\square} \equiv L_k\left(\frac{w}{\ell}\right) = \left(\frac{m_e}{n_s e^2}\right)$$

Superfluid stiffness of 2D Superconductors

$$\rho_s = \frac{\hbar^2}{4}\frac{n_s}{m} = \frac{\hbar^2}{4e^2}\frac{1}{L_K^{\square}}$$

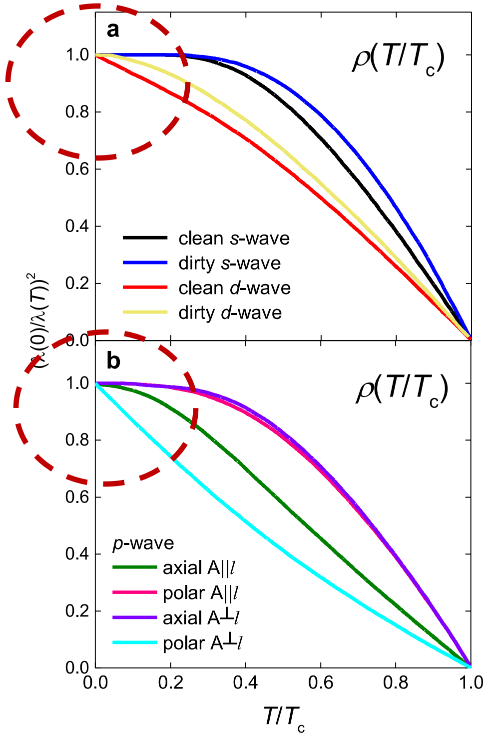
units

$\rho_s/\mathrm{k_B}$ (Kelvin)

$\frac{4e^2}{\hbar^2}\rho_s$ (Henry^{-1})

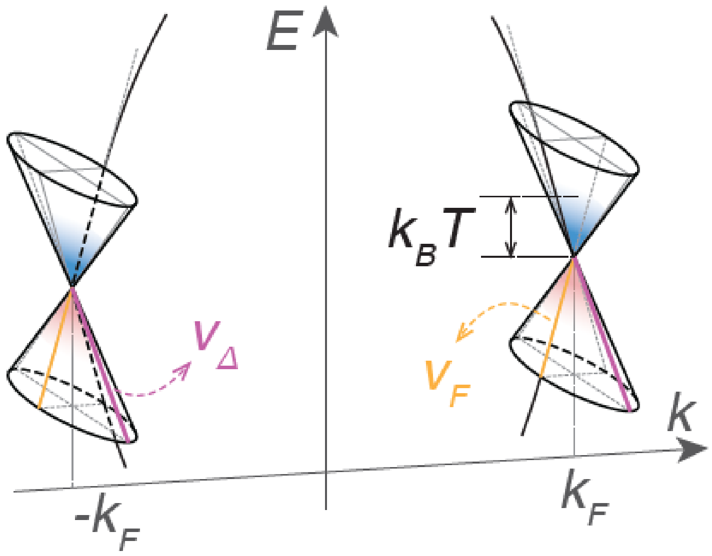
Nodal versus
Nodeless
Superconductors

Nodal versus Nodeless SOP determines T -> 0 behaviors



E. F. Talantsev et. al., Scientific Research 9, 14245 (2019)

Thermal Excitation Across Nodal Superconducting Gaps

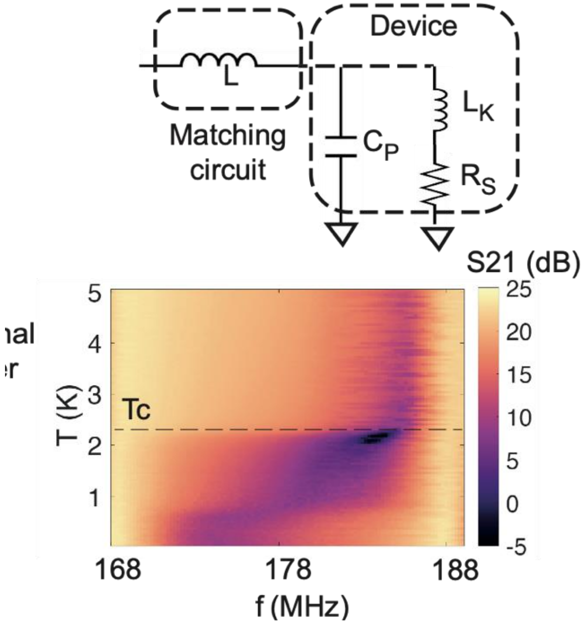


$$\rho_s(T) \propto n_s(T) = n_s(0) - n_{qp}(T)$$

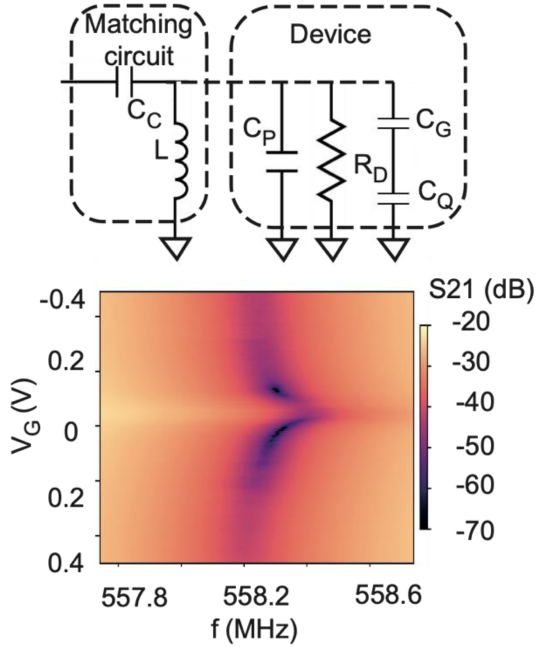
RF Reflectometry for thermodynamic measurement of correlated systems

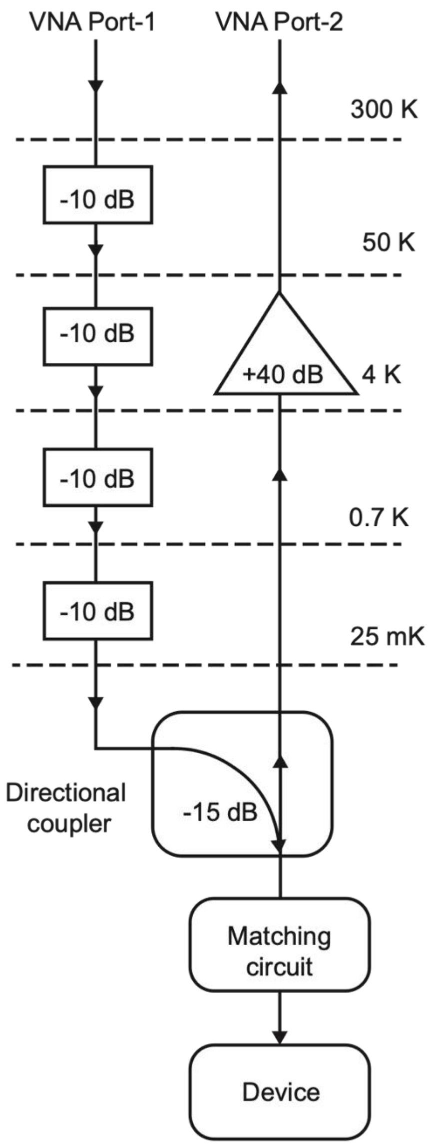
- Radio frequency (100 MHz- 1 GHz) reflectometry measurement for kinetic inductance and quantum capacitance measurement with high energy resolution (< meV)
- Superfluid stiffness (n_s/m), vibrational mode strength, and electron compressibility can be measured

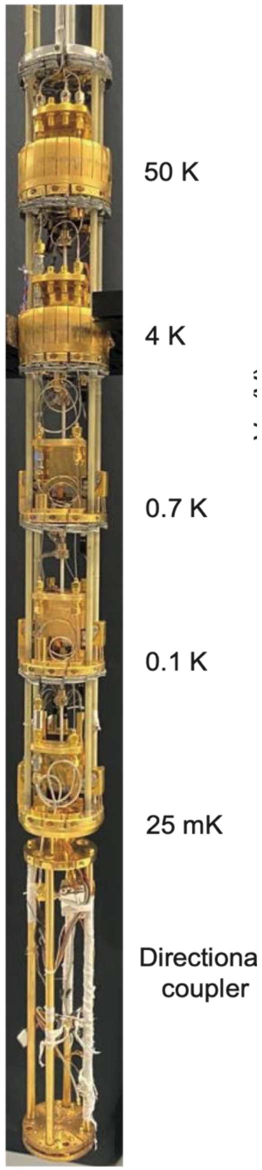
Inductive Load



Capacitive Load



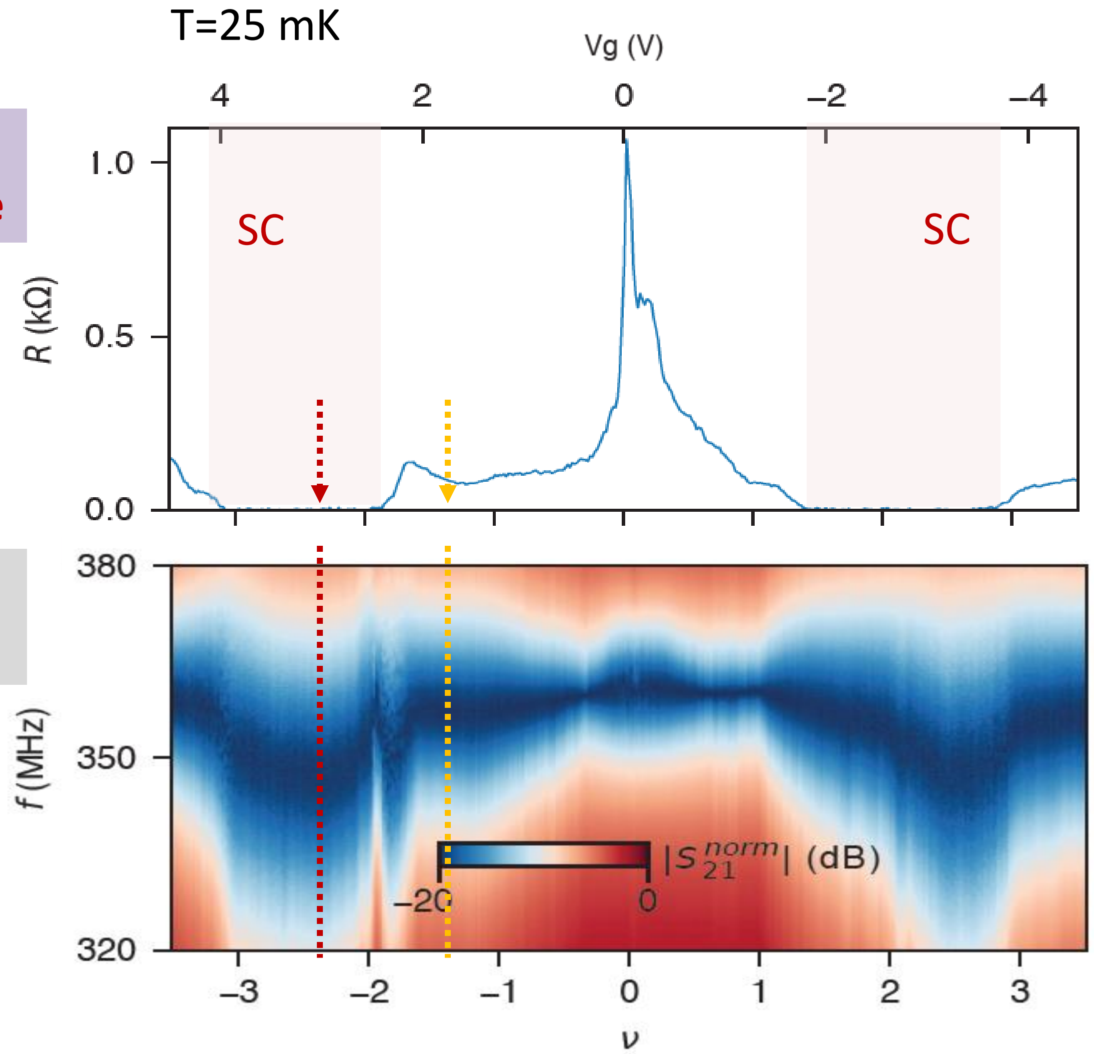


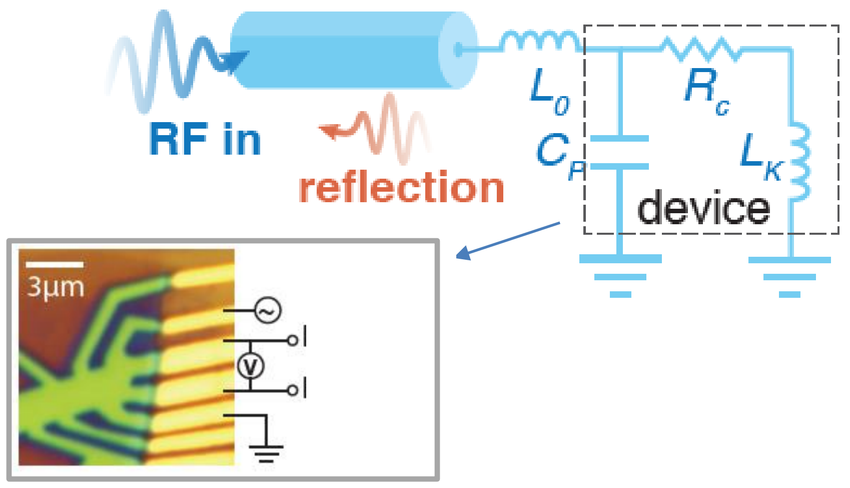


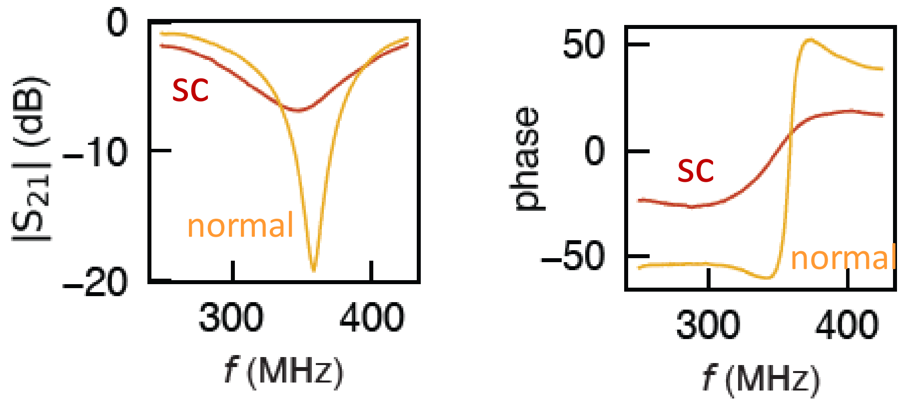
Kinetic Inductance of TTG Superconductors

DC Resistance

RF Reflectance

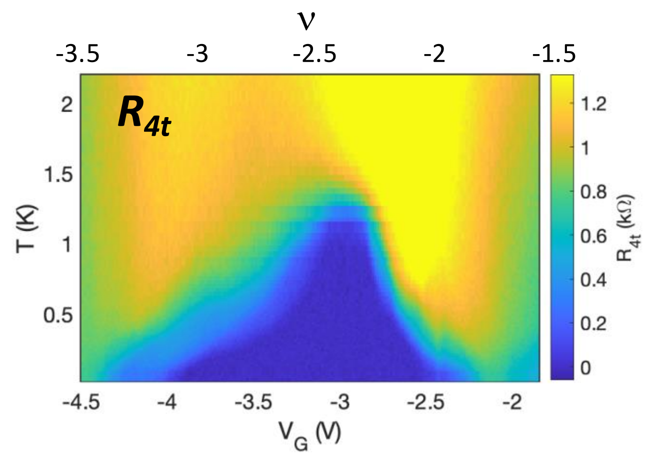


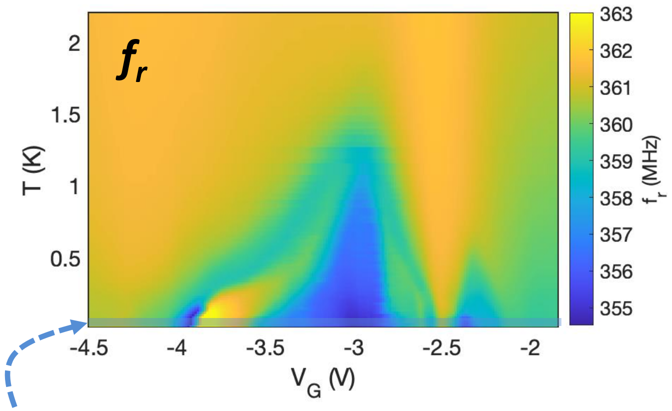


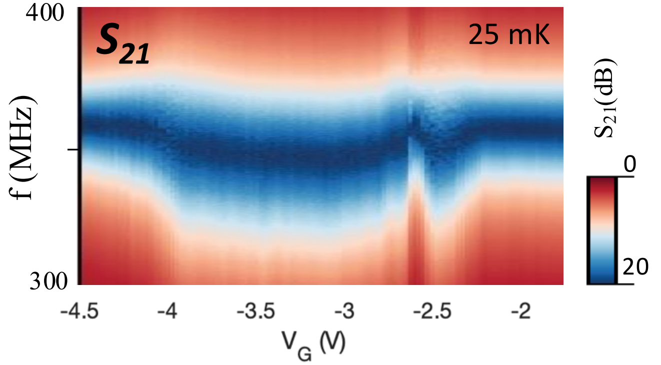


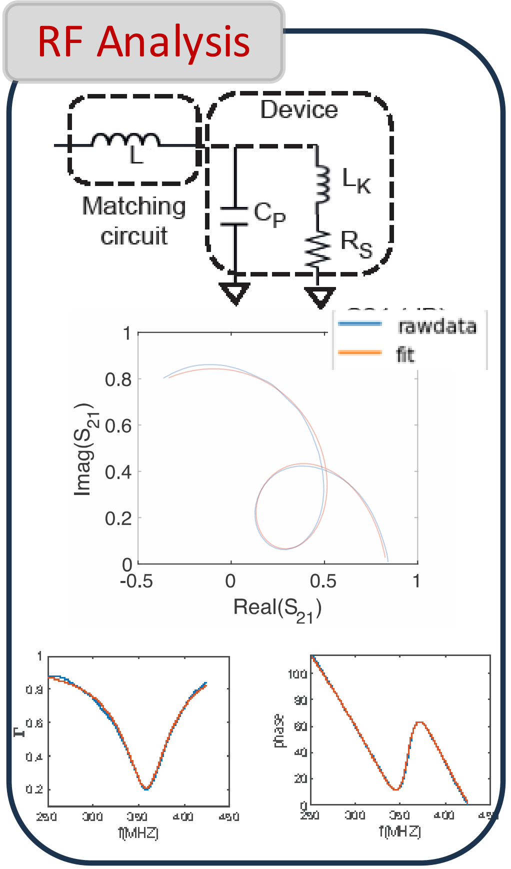
Resonance frequency: $f = f_0\left(1 + \frac{L_K}{2R_s^2C_p}\right)$ $f_0 = \frac{1}{\sqrt{LC_P}}$

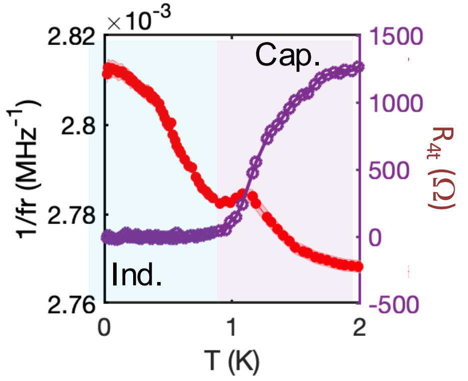
Estimation of Kinetic Inductance from Resonance Frequency Shift

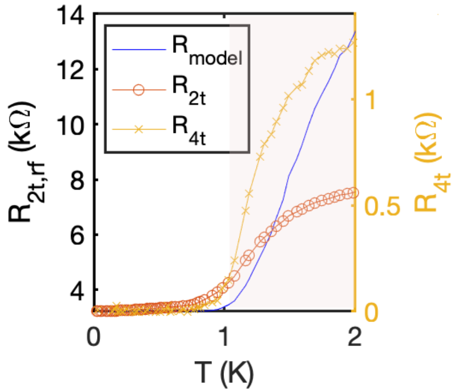


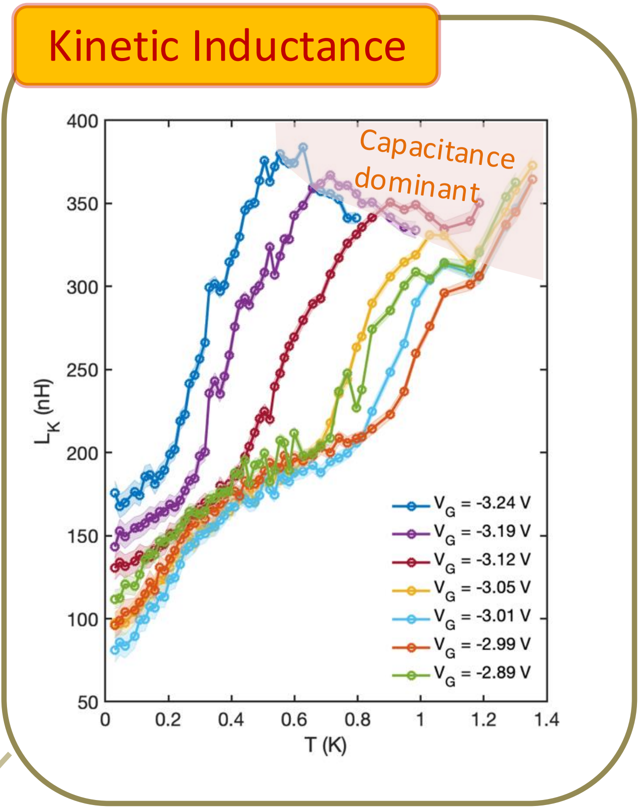










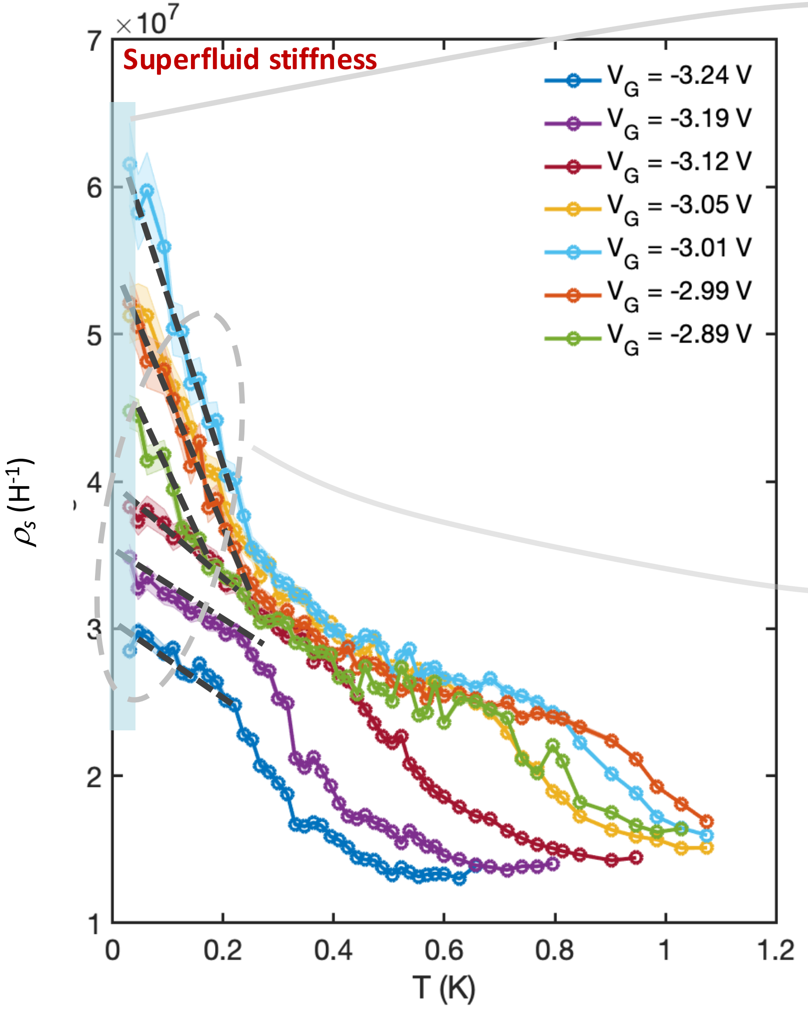


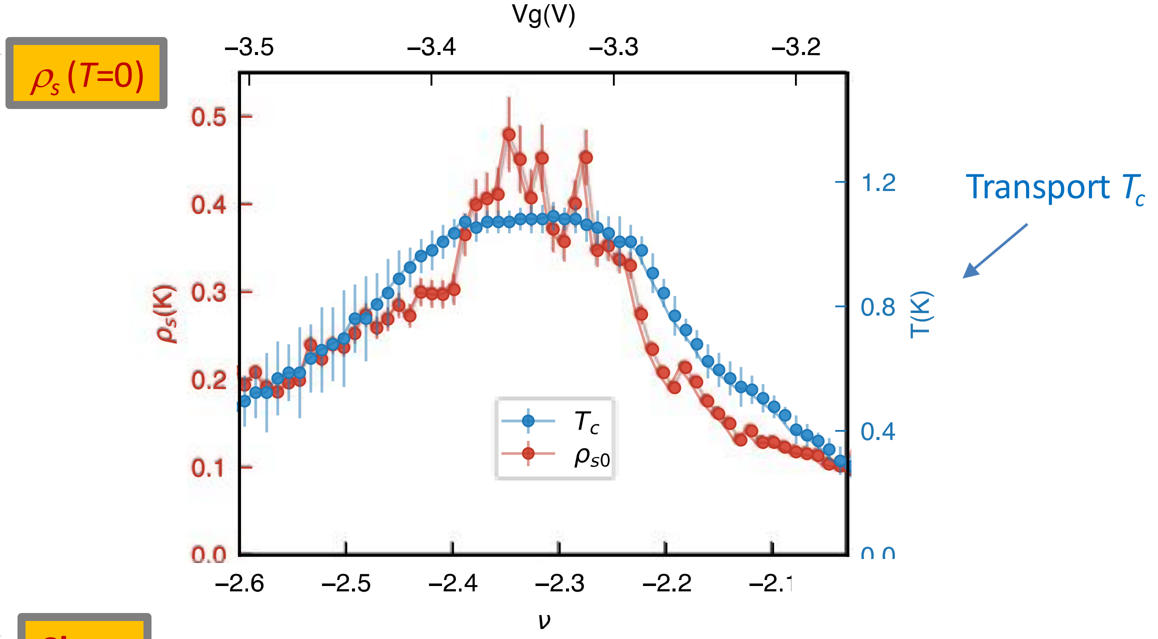
- Both amplitude and phase fit for all frequency
- L_K and R_s are fitting parameter
- $R_s = R_{contact}$ ~ 2 kΩ in superconducting state

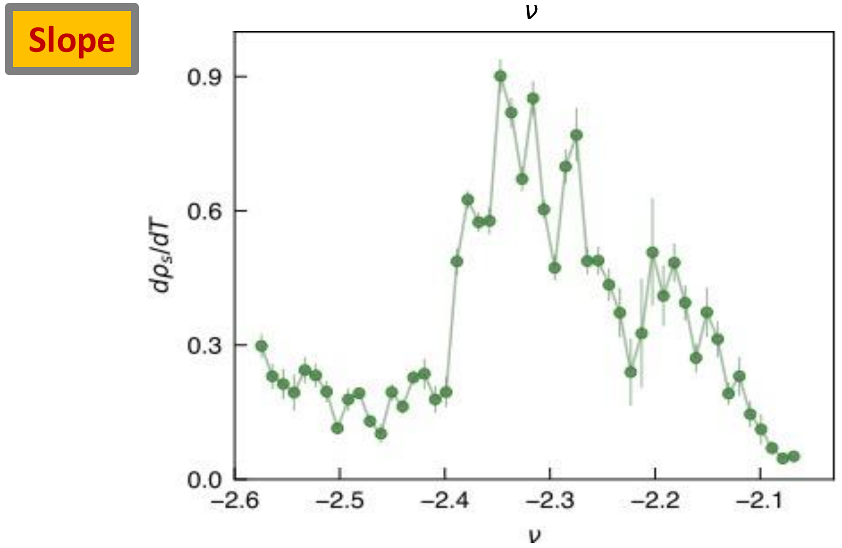
Superfluid stiffness

$$\rho_s = \frac{\hbar^2}{4}\frac{n_s}{m} = \frac{\hbar^2}{4e^2}\frac{1}{L_K}$$

Gate and Temperature Dependent Superfluid Stiffness



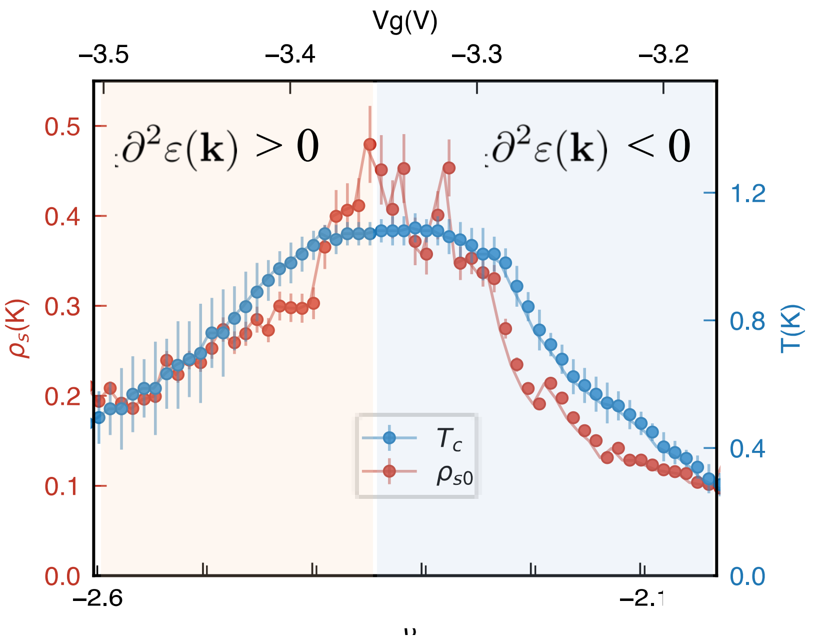




$\delta\rho_s(T) \sim T$ behaviors suggest **nodal** superconductivity!

$\rho_s(0)$ and $d\rho_s/dT$ are strongly correlated to T_c.

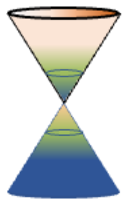
Gate and Temperature Dependent Superfluid Stiffness



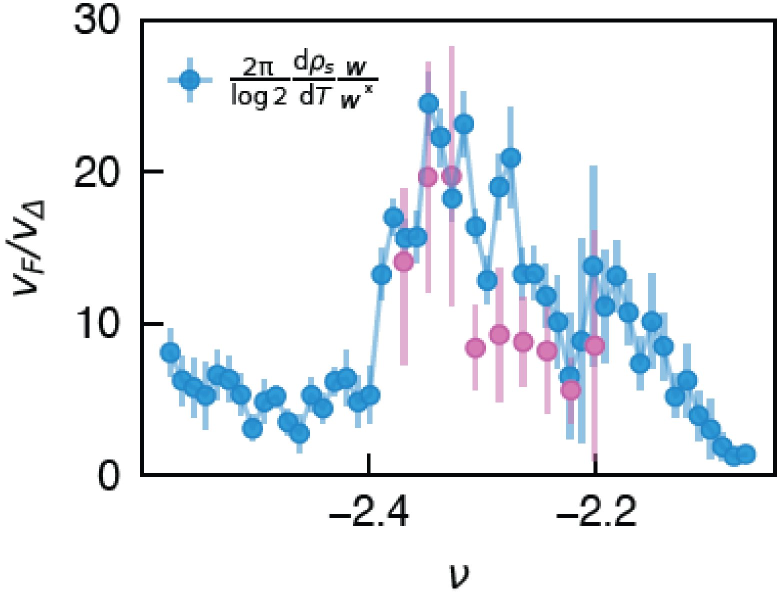
P. A. Lee and X.-G. Wen, Phys. Rev. Lett. 78, 4111 (1997)

Superfluid stiffness for nodal superconductors at a finite temperature

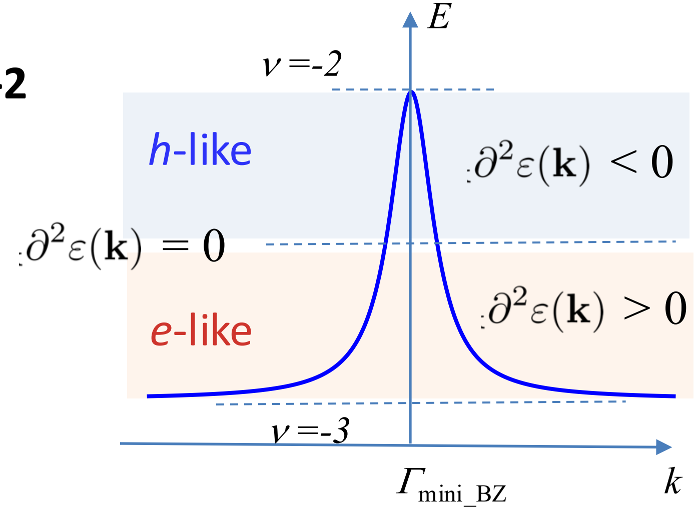
$$\rho_s(T) = \frac{1}{4}\sum_{\mathbf{k}} n_{\mathbf{k}}\partial^2\varepsilon(\mathbf{k}) - \frac{v_F^2}{16T}\sum_{\mathbf{k}} \frac{1}{\cosh^2\frac{\sqrt{\varepsilon_{\mathbf{k}}^2+\Delta^2(\mathbf{k})}}{2T}}$$



$$\rho_s(T=0) = \frac{1}{4}\sum_{\mathbf{k}} n_{\mathbf{k}}\partial^2\varepsilon(\mathbf{k}) \qquad \delta\rho_s(T) = -\frac{v_F}{4\pi v_\Delta} T\log[2]$$



TBG
Hartree-Fock Band for ν = -2

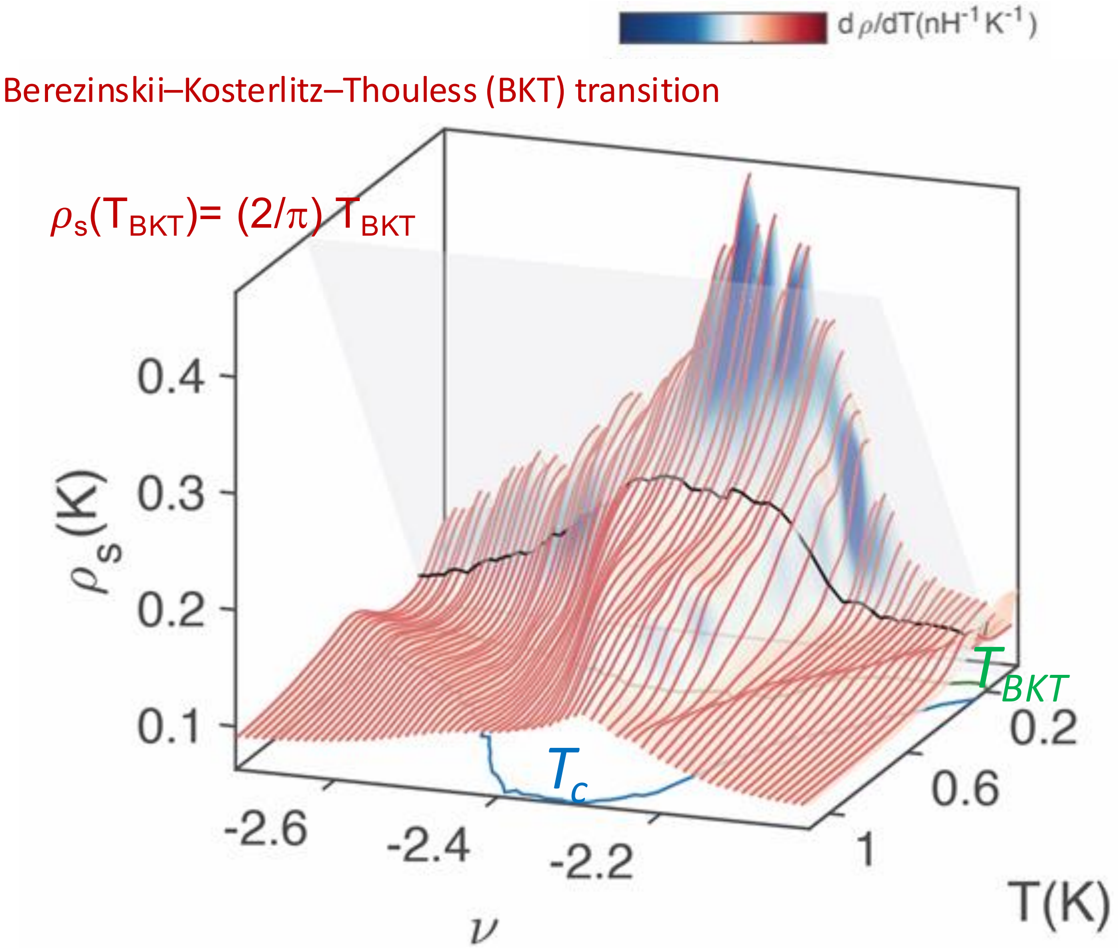


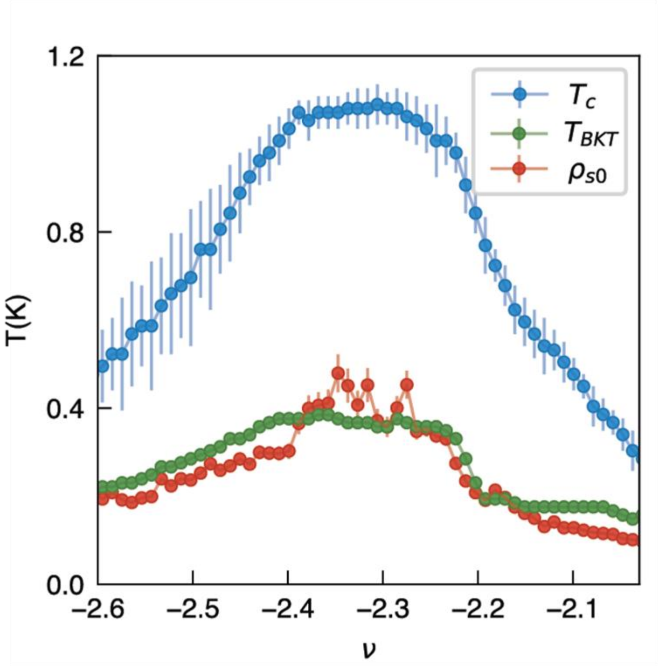
Bultinck et al. , Phys. Rev. X 10, 031034 (2020)

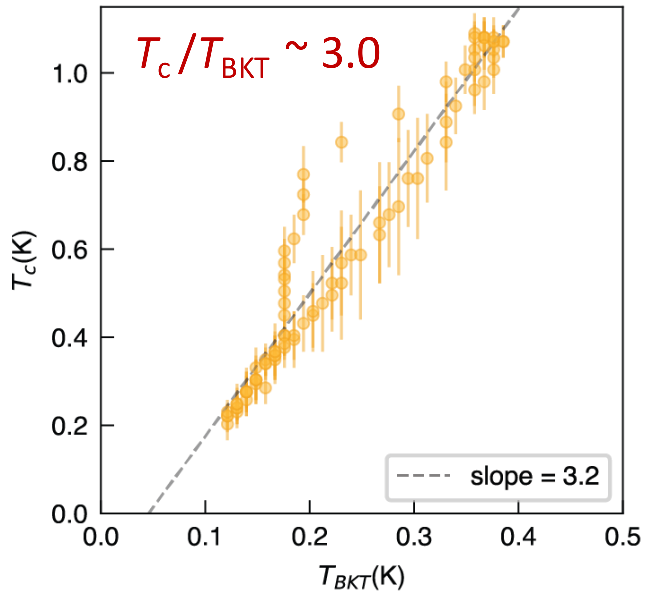
High Temperature Dependent Superfluid Stiffness

Berezinskii–Kosterlitz–Thouless (BKT) transition

$\rho_s(T_{BKT}) = (2/\pi)\, T_{BKT}$







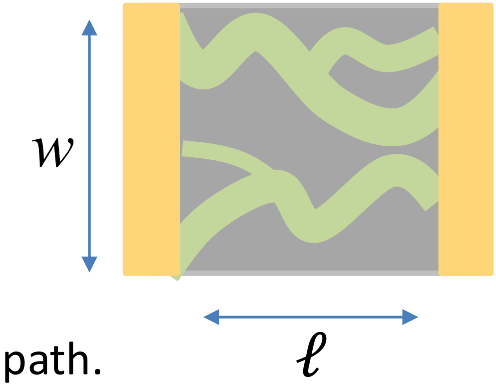
$T_{BKT} << T_c$??

Experimental ρ_s

$$\rho_s = \frac{\hbar^2}{4}\frac{n_s}{m} = \frac{\hbar^2}{4e^2}\frac{1}{L_K}\left(\frac{\ell}{w}\right)$$

$$w^* = w/3$$

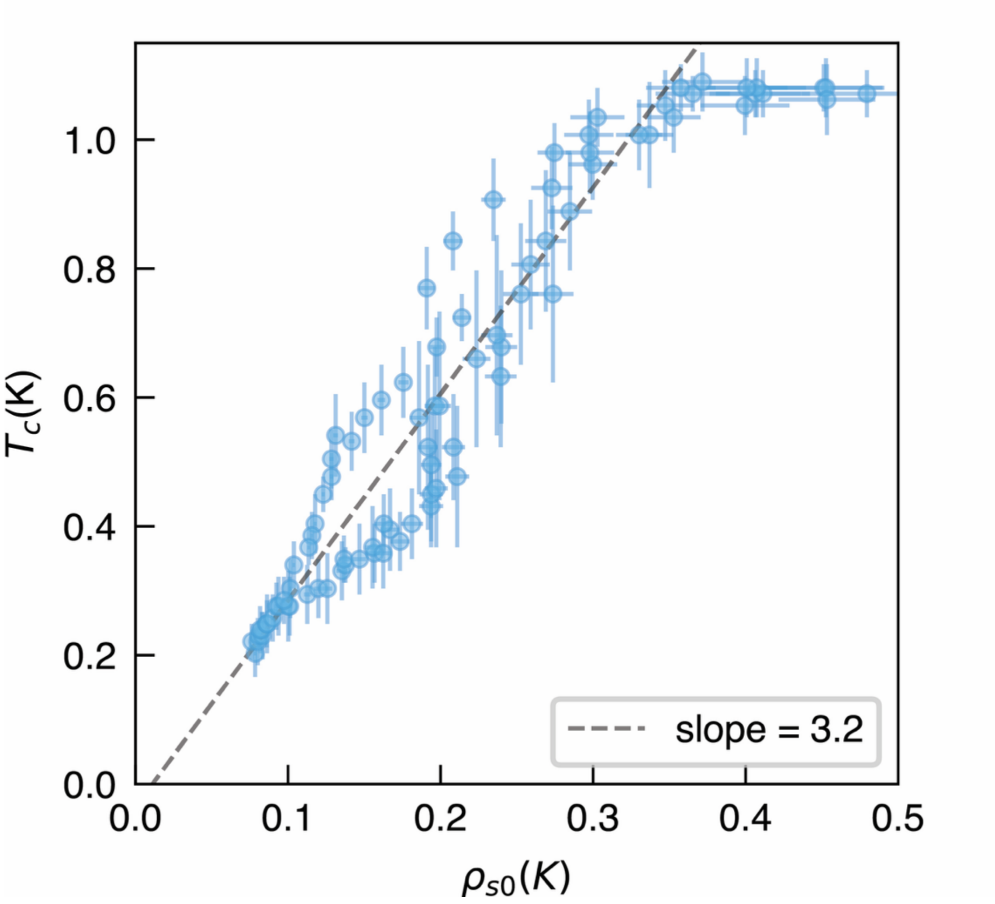
Percolation paths



Inhomogeneity across the sample created percolation superconducting path.

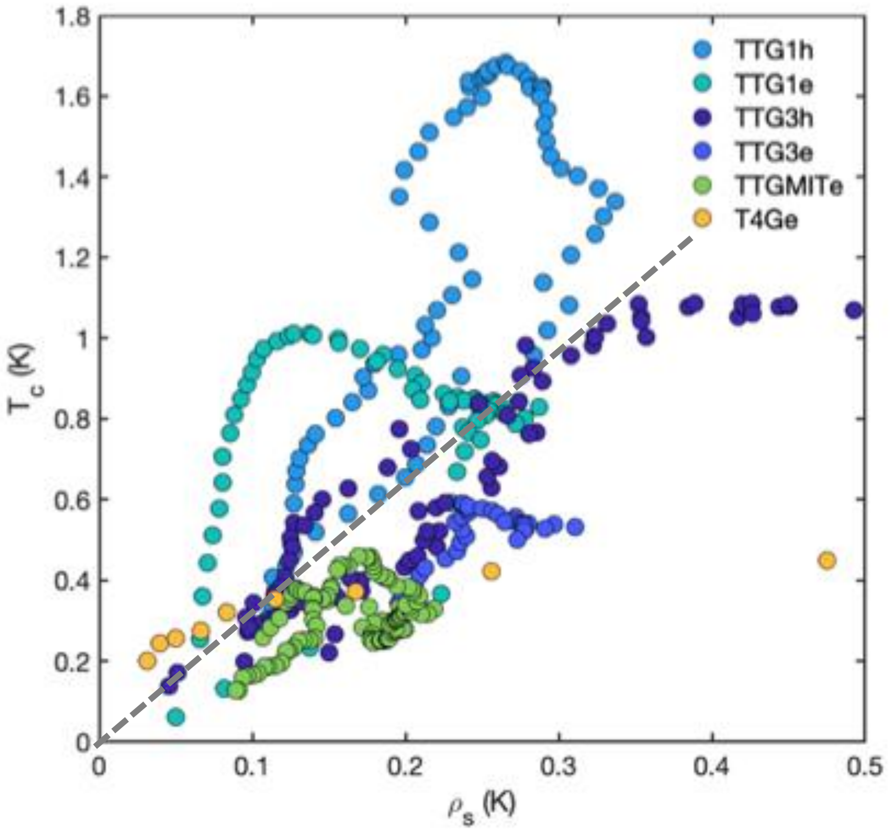
The Uemura Plot for Twisted Superconductors

Following Uemura, Y. J. et al., *Phys. Rev. Lett.* **62**, 2317–2320 (1989).



$$\frac{T_C}{E_F} = \frac{T_C}{4\pi\rho_s} = \left(\frac{w^*}{w}\right)\frac{T_C}{4\pi\rho_s^{ex}} \approx 0.08$$

Many samples both *e-* and *h-* SC domes



Note that inhomogeneity percolation may underestimate ρ_s in the disordered samples.

Superfluid Stiffness and Quantum Geometry

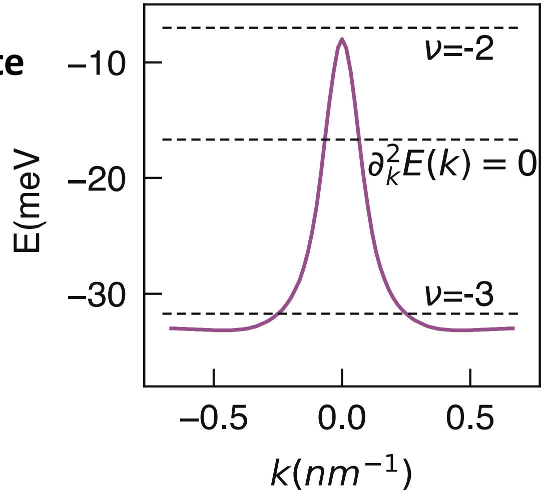
Superfluid stiffness in a parabolic band

$$\rho_s = \frac{\hbar^2}{4}\frac{n_s}{m^*} = \frac{1}{4}\sum_{\mathbf{k}} n_{\mathbf{k}} \underbrace{\partial_{\mathbf{k}}^2 \varepsilon(\mathbf{k})}_{\approx\, 0}$$

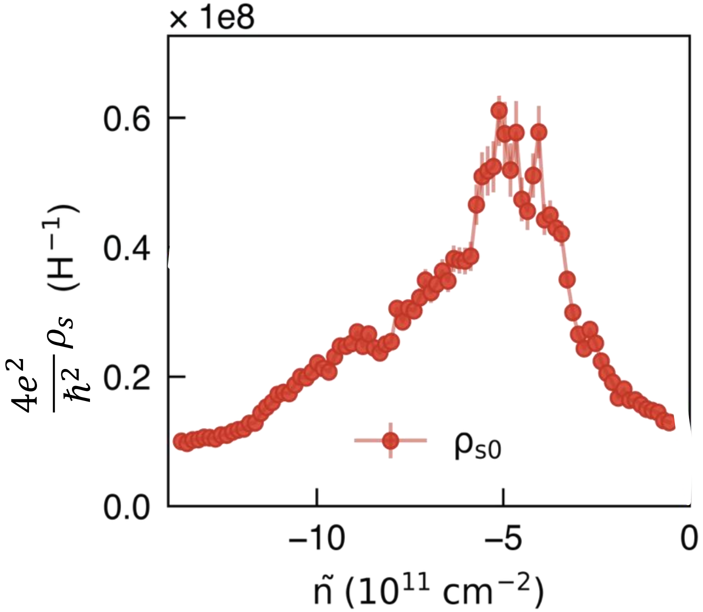
For a flat band

Quantum geometry consideration is needed!

Hartree-Fock Band for Symmetry Broken State -3<ν<-2

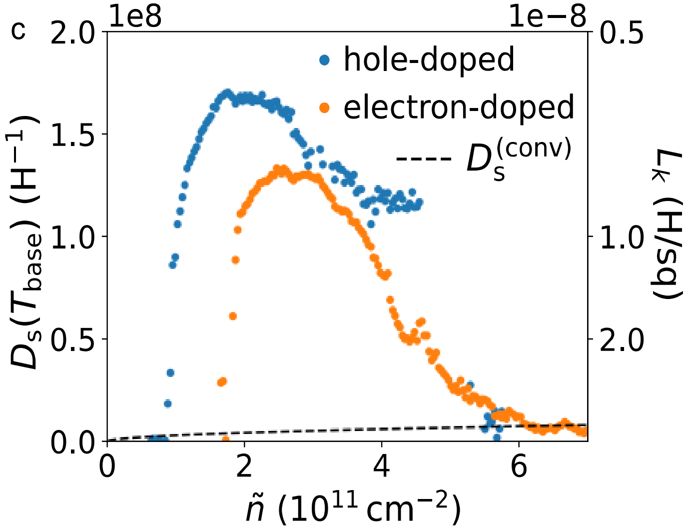


MATTG: Our data/theory



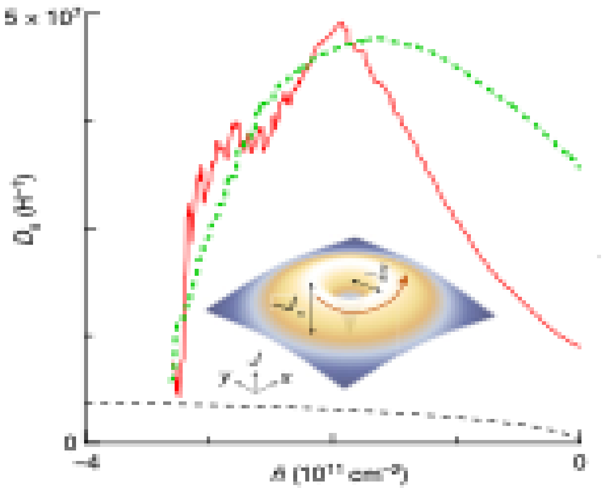
B. Abhishek et al., Nature (2025)

MATBG: MIT



M. Tanaka et al., Nature (2025)

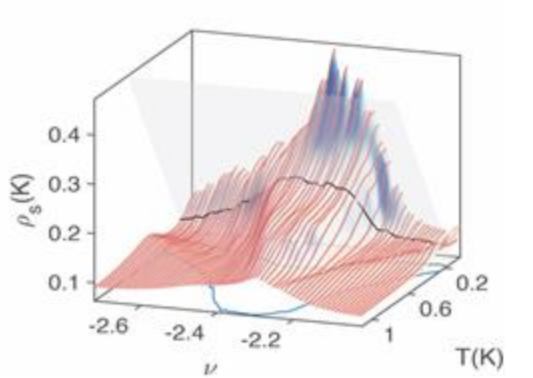
MATBG: OSU



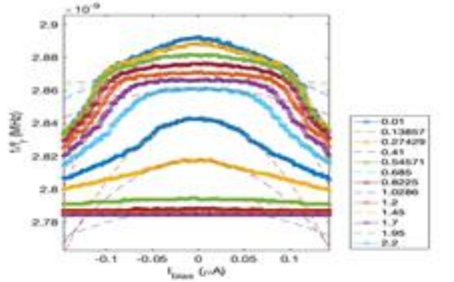
H. Tian et al., Nature (2023)

Summary I: Superfluid Stiffness of TTG

- The kinetic inductance measurement in 2D superconductor probes temperature dependent superfluid stiffness

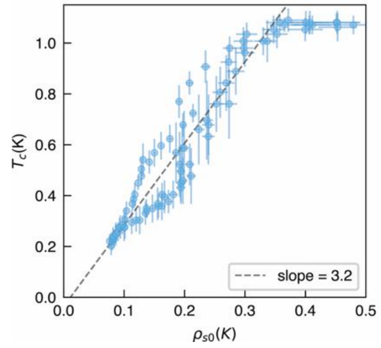


- Hartree-Fock flavor symmetry broken band can explain the low temperature behaviors of superfluid stiffness qualitatively



- Strong nonlinear Meissner effect suggested nodal superconductivty.

- Strongly coupled superconducting limit, Tc/EF ~ 10%, was estimated.



- Quantum metric consideration for superfluid stiffness with flavor polarization is required.

Part II: AC Conductivity in Quantum Hall States in Graphene

Experiments

Abhishek Banerjee

Terry Phang

Zhongying Yan

Tom Werkmeister

Theory

Ilia Komissarov

Tobias Holder

Raquel Queiroz

hBN

T. Taniguchi, K. Watanabe

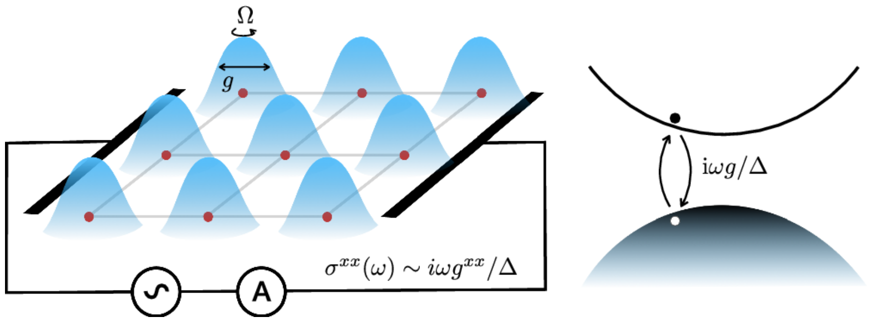
Funding:

Dielectric Response of Quantum Hall Insulator

The quantum geometric origin of capacitance in insulators

Ilia Komissarov, Tobias Holder & Raquel Queiroz

Nature Communications 15, Article number: 4621 (2024) | Cite this article



capacitive response

$$\sigma^{\mu\nu}(\omega) = -\frac{e^2}{h} C \varepsilon^{\mu\nu} + i\omega \overbrace{c\,\delta^{\mu\nu}} + \ldots$$

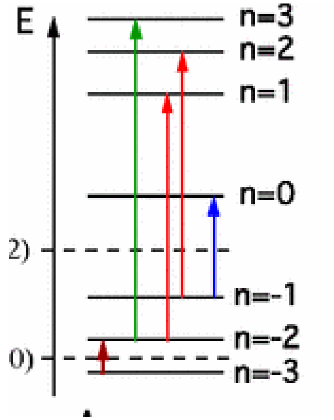
For quantum Hall states of massive electrons:

$$c = \frac{e^2}{h\omega_c} C$$

Chern number

For graphene LLs:

Inter Landau level transitions

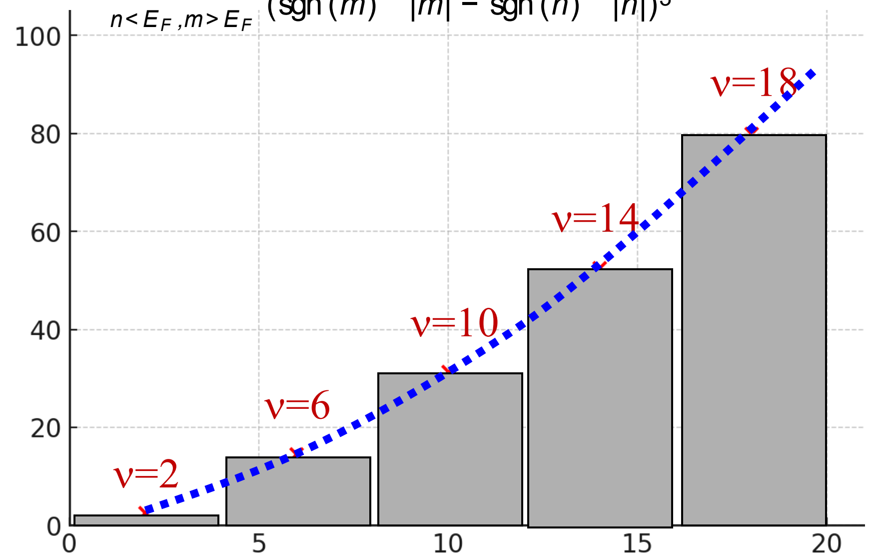


$$c_g = \frac{e^2}{2\pi v_F \sqrt{2\hbar e B}} C_{E_F}$$

Effective Chern number

X $n<E_F, m>E_F$ (sgn (m) p |m| − sgn (n) p |n|)^3 $\delta_{||m|-|n||=1}$

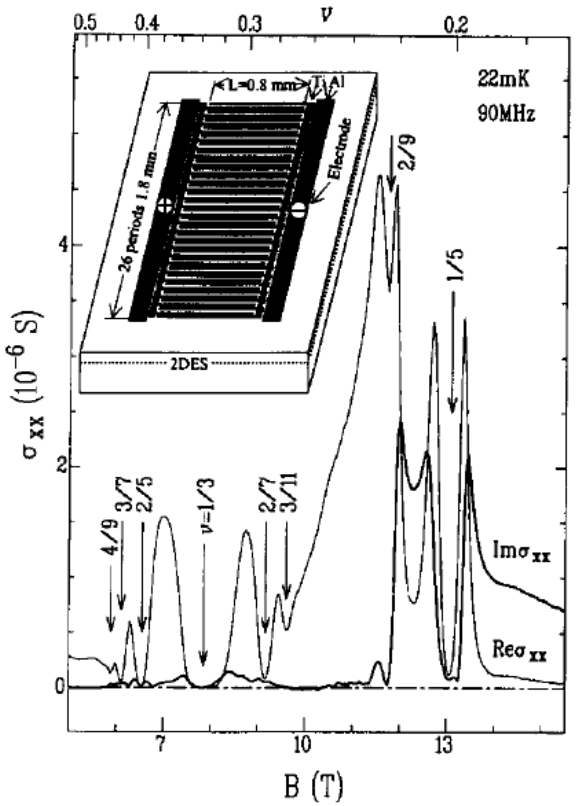
C_{E_F}



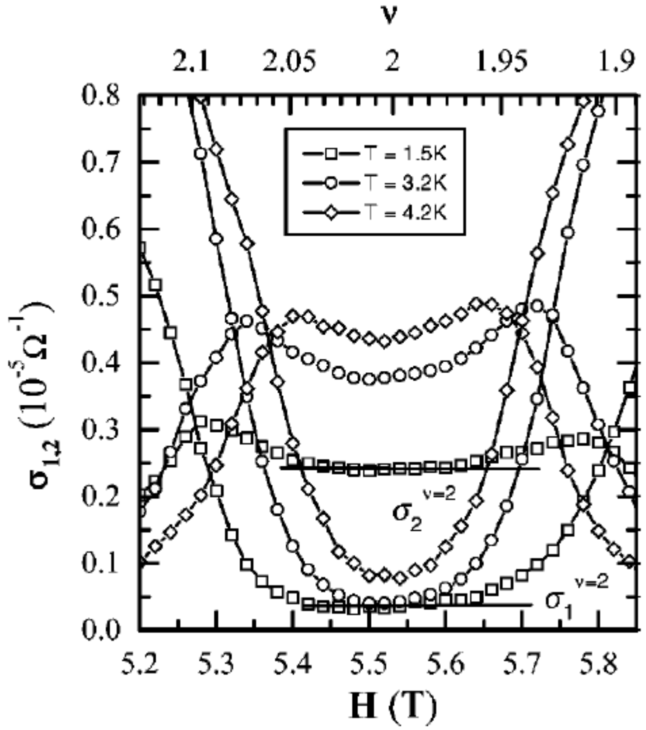
Komisarrov et al.

Microwave conductance in Quantum Hall Insulators

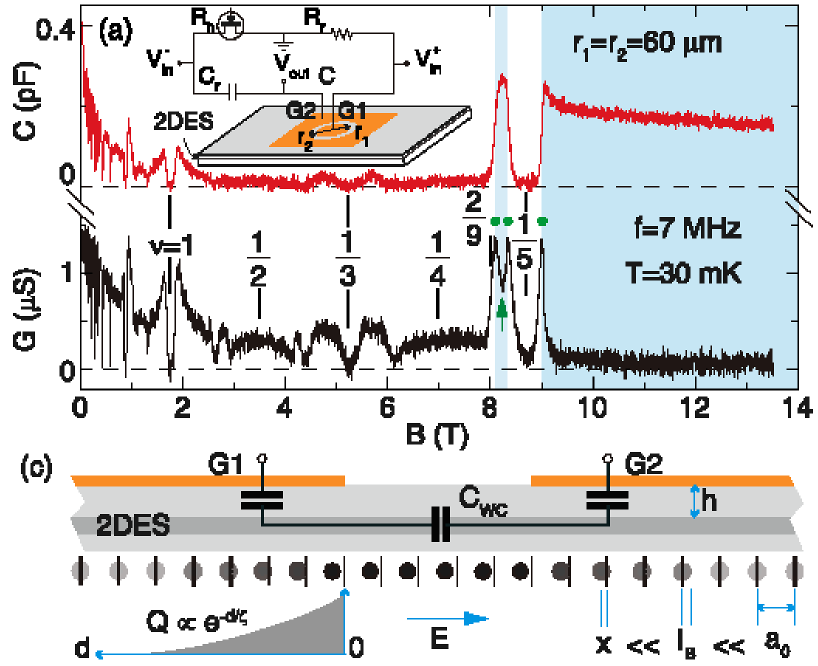
Mainly focus on quasiparticle Wigner crystal



Y.P. Li , D.C. Tsui,.., M. Shayegan,
Solid State Communications, 95, 9(1995)

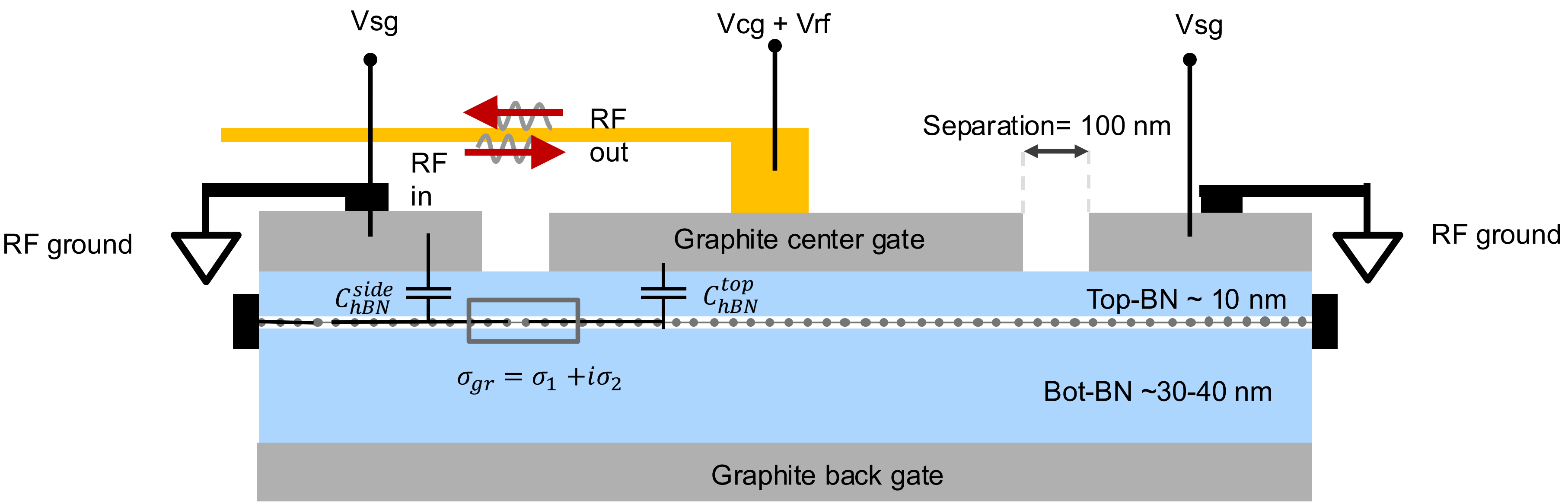


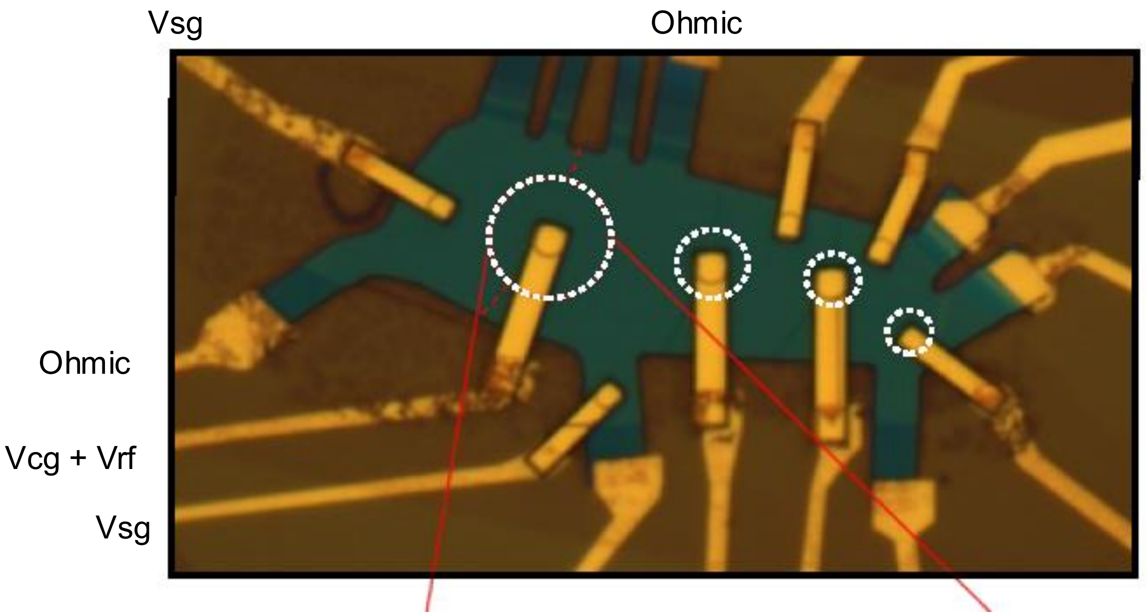
Drichko,… Galperin" *PRB* 92.20 (2015)



Zhao, Lili, et al, PRL, 130.24 (2023): 246401.

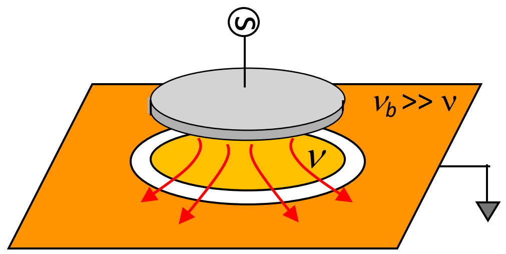
Microwave conductance in Graphene Quantum Hall Insulators





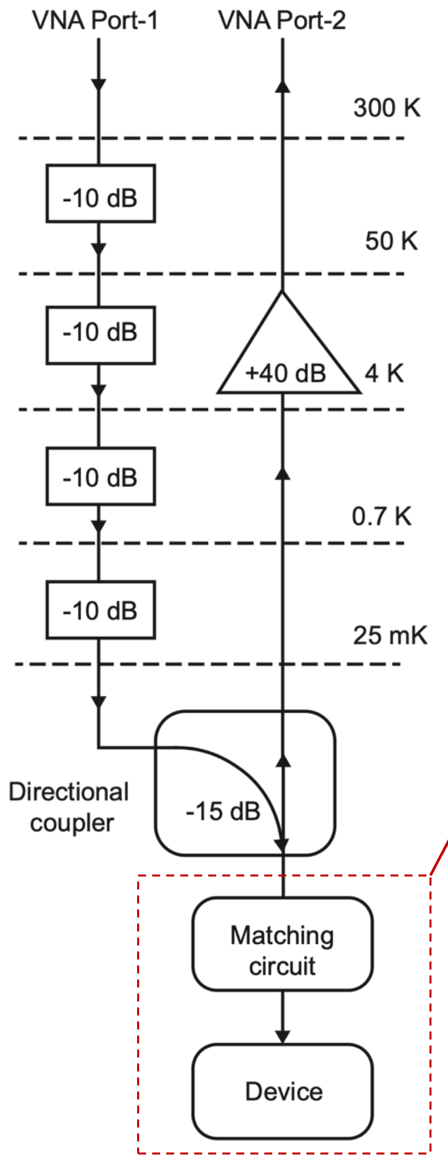
Lollipop device geometry, Diameter = 8 μm

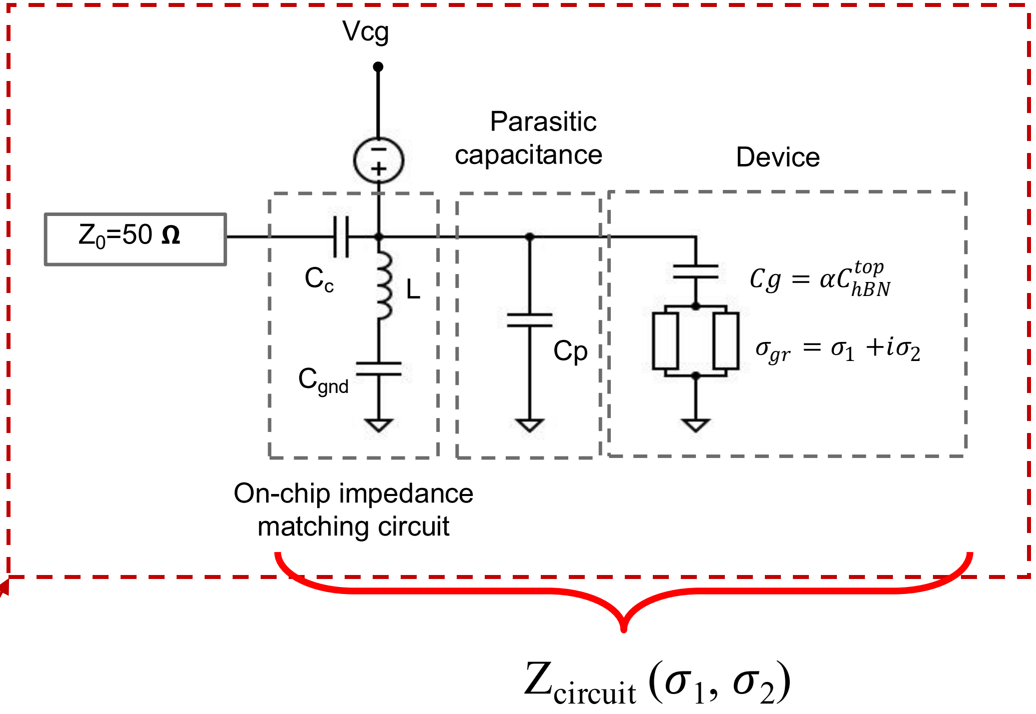
Capacitively coupled
Corbino device Geometry



Measuring $\tilde{\sigma}(w)$

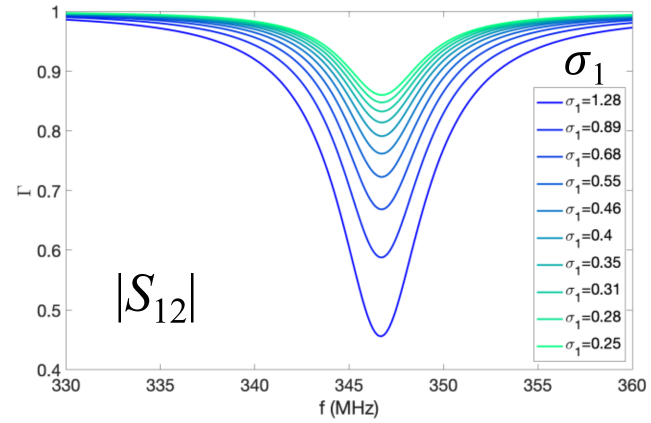
Matching Circuit and Microwave Reflectometry for AC Conductivity Measurement

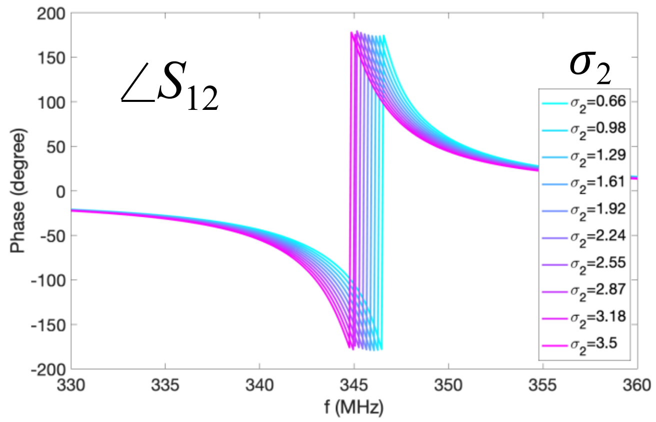


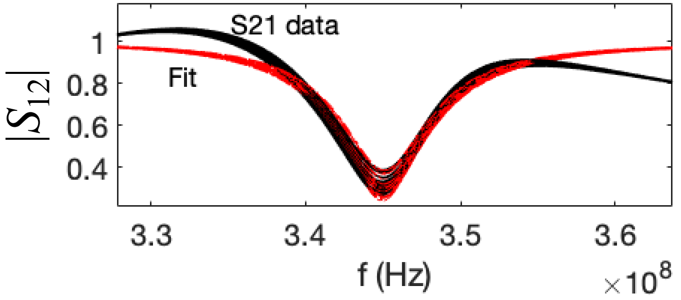


$$S_{12}(\omega) = \frac{Z_{\text{circuit}}(\omega) - Z_0}{Z_{\text{circuit}}(\omega) + Z_0}$$

Microwave reflectometry to measure the reflection co-efficient S_{21}

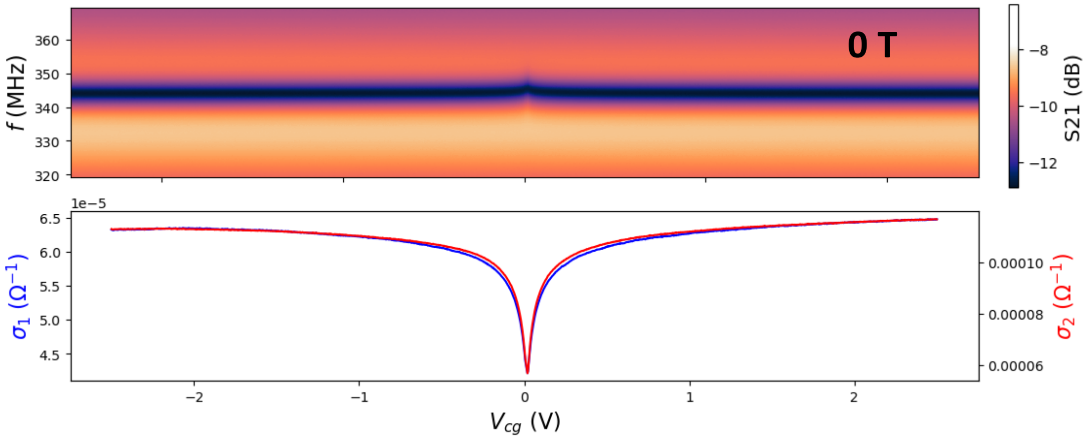


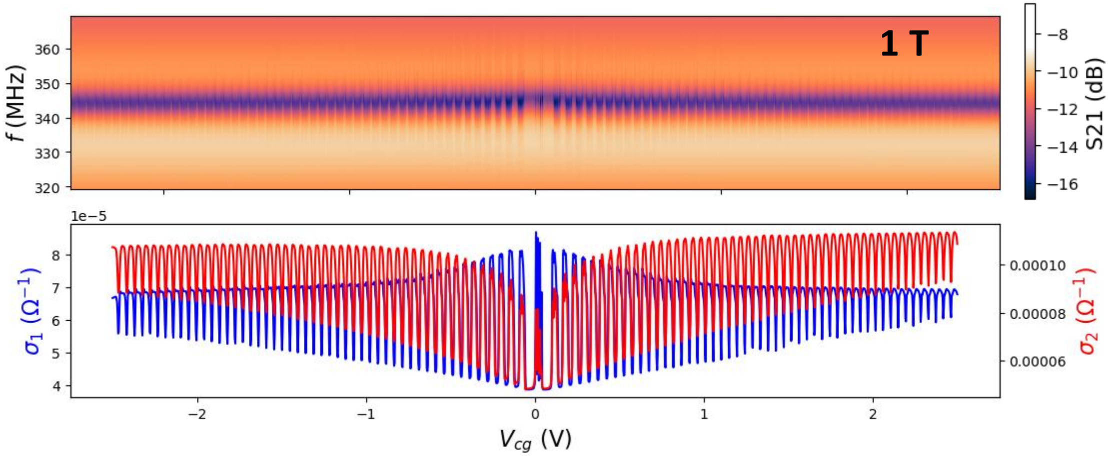


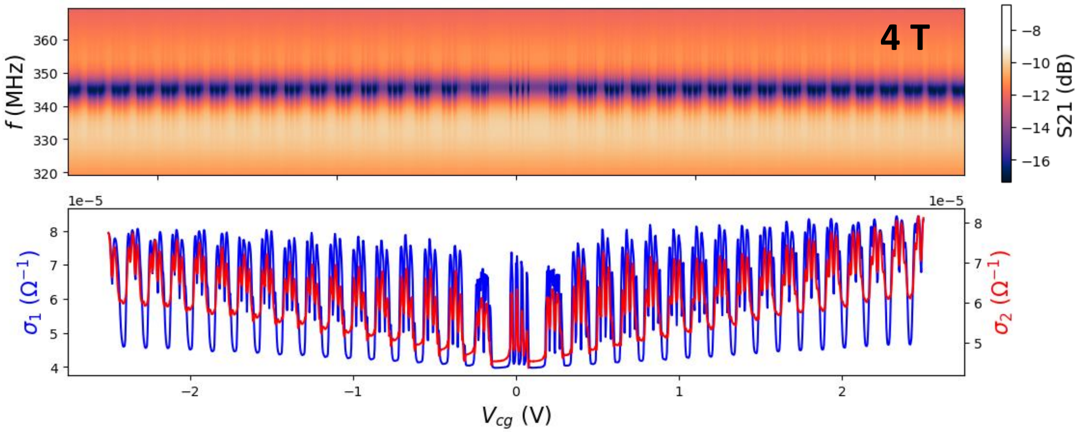


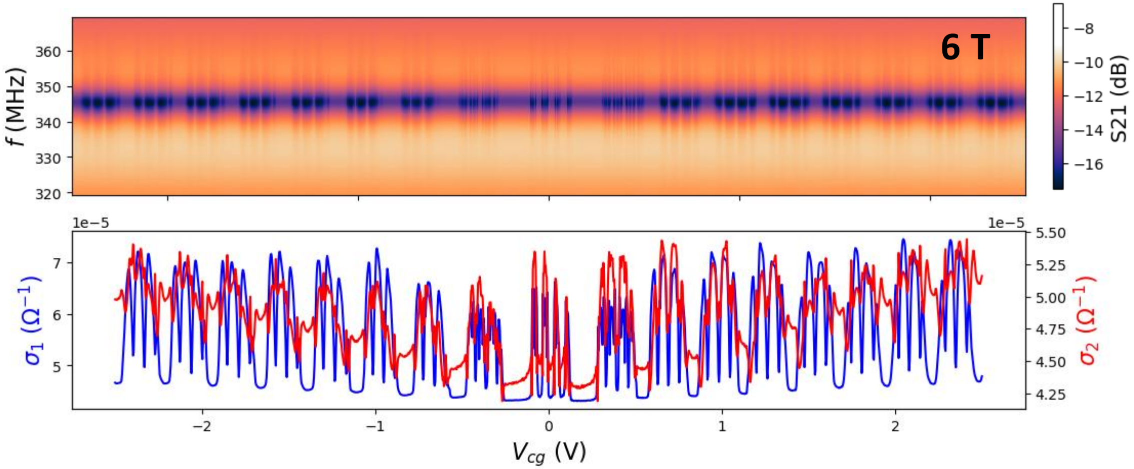
AC Microwave Conductance versus Gate Voltage

$$\tilde{\sigma}(w) = \sigma_1 + i\sigma_2$$

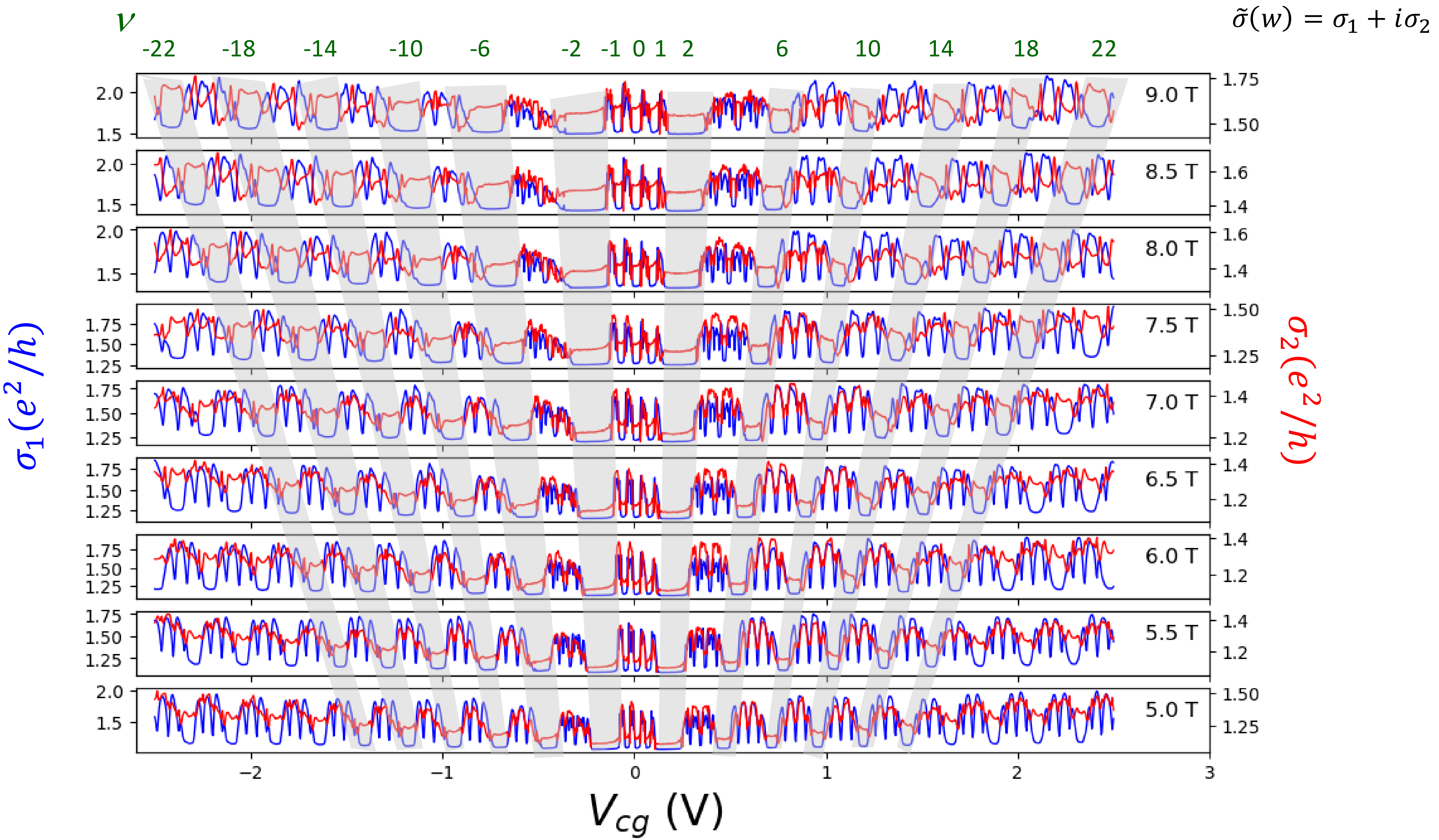




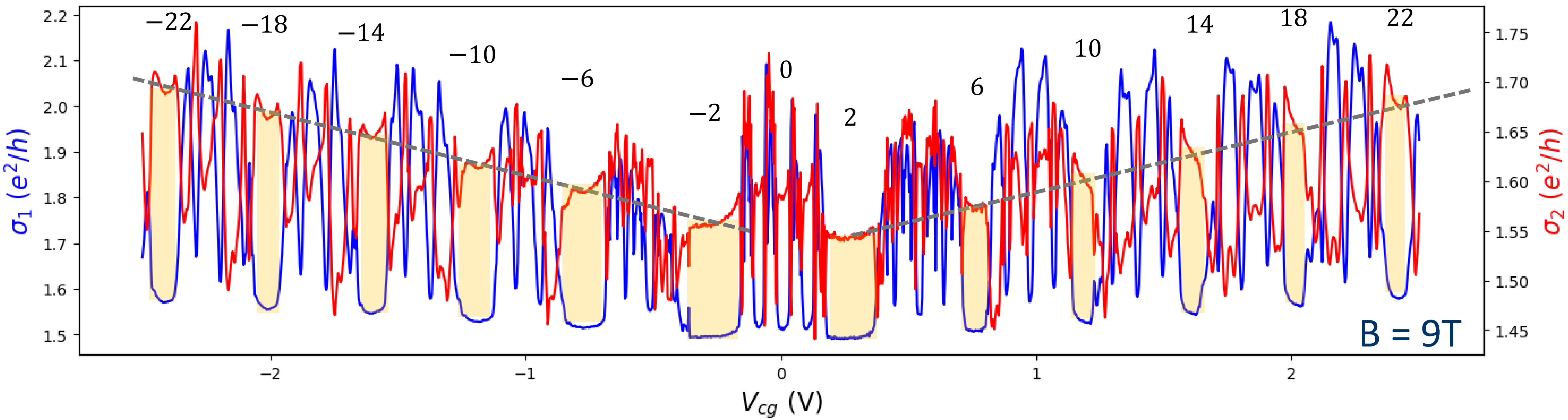




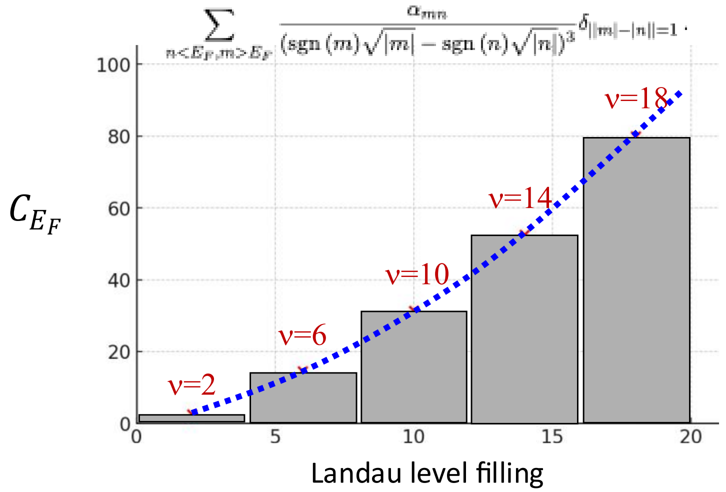
Emergence of Plateau features in σ_2



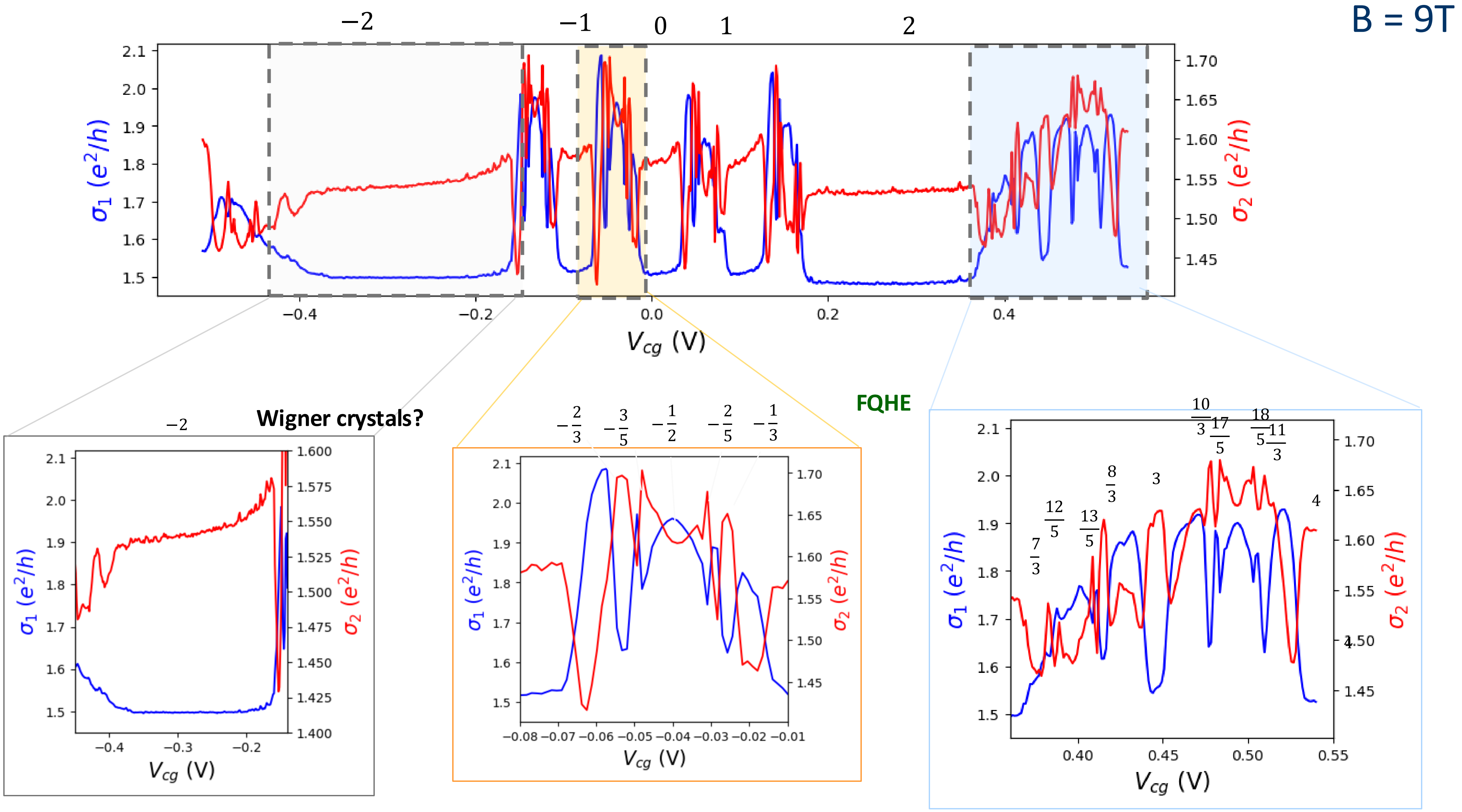
Dielectric Response of Quantum Hall Insulator and Quantum Metric



- Increasing σ_2 with increasing ν.
- Consistent with quantum metric calculation at a fixed magnetic fields qualitatively .
- Further quantitative analysis is required.

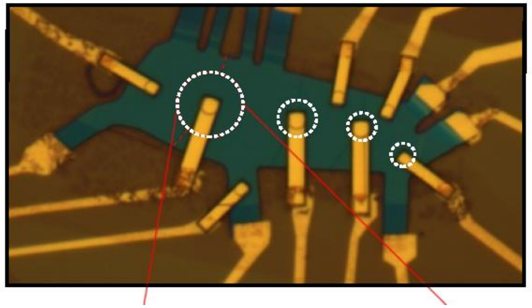


Dielectric Response of Wigner Crystals and Fractional Quantum Hall States

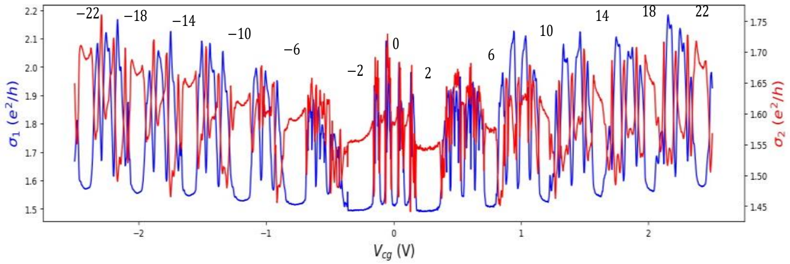


Summary II: Quantized Dielectric Response in Graphene QH Insulators

- AC conductivity of quantum Hall insulator can be probed by microwave reflectometry in graphene capacitively coupled Corbino device geometry



- Graphene quantum Hall insulator exhibits quantized response of dielectric response following quantum geometric calculation



- Rich features found in capacitive response in the edge of QHE plateaus and fractional quantum Hall regime.