# DAILY FLOW ROUTING WITH THE MUSKINGUM-CUNGE METHOD IN THE PECOS RIVER RIVERWARE MODEL

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**Abstract:** One of the major efforts for development of a daily timestep water operations model for the Pecos River in New Mexico was to implement a routing methodology that would appropriately represent flood wave travel times (translation) and reduction in peak discharge (attenuation) of flood waves. The model is to be used to evaluate the impacts of modified dam operations on flow conditions in critical habitat for a federally "threatened" fish species. It is important for travel times of flood waves to be represented appropriately. Due to the morphology of the Pecos River and shape of typical inflow hydrographs, flood waves during the summer monsoon season significantly attenuate as these waves propagate down the Pecos River. The Muskingum-Cunge method was selected as a routing method to add to the water operations model, but it was coded in a different manner than it is conventionally coded in other models. The water operations model was developed with the RiverWare software application that is a general river basin modeling tool that runs in an object-oriented modeling environment. While this modeling environment provides flexibility for developing models, it provides a restriction to simulate the entire river system one model timestep at a time. Due to this simulation style, the routing method for each river reach must also run one model timestep at a time. The resulting routing method in RiverWare requires the user to input an incremental routing timestep that will be used to route flood waves within each model timestep. The model then uses other input parameters to determine the best incremental routing spatial step to minimize numerical dispersion. In addition, the water operations model simulates with daily average flows, so assumptions were made to implement the Muskingum-Cunge method that routes instantaneous flows.

## INTRODUCTION

The Pecos River system discussed here is located in eastern New Mexico. In 1987, the Pecos bluntnose shiner (*Notropis simus pecosensis*) was listed as federally threatened under the Endangered Species Act (ESA) of 1973. The Bureau of Reclamation (Reclamation) began consultation with the Fish and Wildlife Service (Service) to determine potential impacts of Pecos River operations on the Pecos bluntnose shiner and its habitat. A biological opinion was issued by the Service which concluded that historical river operations were likely to jeopardize the continued existence of the Pecos bluntnose shiner. One of the Reasonable and Prudent Alternatives from the Biological Opinion directed Reclamation to develop a daily timestep water operations computer model of the Pecos River system. The model would be used to analyze the effects of different operational scenarios on Pecos bluntnose shiner habitat, overall water delivery efficiency, and state-line deliveries. The software selected by Reclamation to simulate the Pecos River surface water resources from Santa Rosa Lake to Avalon Dam is RiverWare (Zagona, *et al*, 2001) developed by the Center for Advanced Decision Support for Water and Environmental Systems (CADSWES) at the University of Colorado at Boulder.

A routing methodology is required for the Pecos River water operations model to simulate flood wave travel time (translation) and reduction in peak discharge (attenuation) as river flows propagate downstream. To determine an appropriate methodology for routing flows in the Pecos River RiverWare model, channel geometry information were used to evaluate whether a kinematic or diffusive wave approximation to the full dynamic wave equation could be used. As a result of this evaluation, the Muskingum-Cunge routing methodology was selected. The Muskingum-Cunge method mimics diffusion with parameters that are a function of the channel geometry. An example of flood wave attenuation evident from average daily Pecos River streamflow data at three model nodes is presented in Figure 1. The routing algorithm implemented in the RiverWare modeling tool was developed as a joint effort on the part of CADSWES, Reclamation, and Tetra Tech.

Several details of the routing method had to be defined before adding the routing method to the RiverWare software. The first issue was that the incremental routing timestep needed by the routing algorithm's finite difference scheme is different than the RiverWare timestep used for the complete Pecos River RiverWare model. An approach was established for setting the grid size for the routing method. RiverWare simulates each designated object for each RiverWare timestep before moving to the next RiverWare timestep. This requires that a separate routing scheme be completed for each daily timestep of RiverWare simulation. An approach was also established for computing a reference discharge to use for computing the Muskingum-Cunge routing parameters. Finally, since the Muskingum-Cunge method simulates with instantaneous discharges, average daily flows used in the Pecos River RiverWare model are converted to instantaneous flows for routing. The resulting instantaneous flows following routing are then converted back to average daily flows.

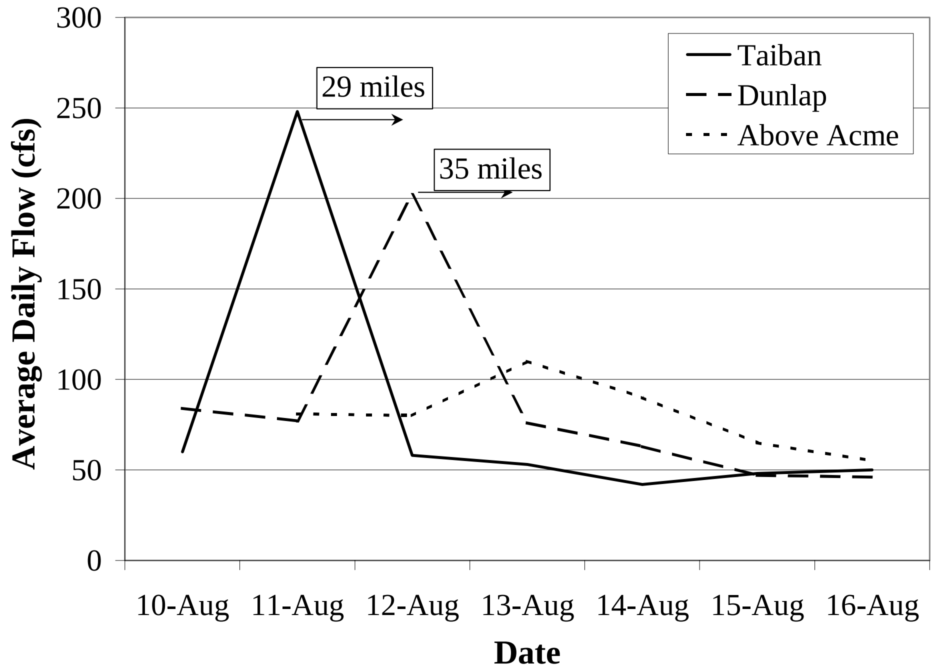


**Figure 1. Example of Flood Wave Attenuation Evident from Pecos River Streamflow Data**

## DISCUSSION

**Routing Method Selection:** Designated criteria were checked to determine an appropriate flood wave routing methodology to use for the Pecos River water operations model. The criteria are used to determine whether terms in the full dynamic wave equation can be neglected to simplify routing. The one-dimensional equation of motion for routing open channel flow is shown below (Chow *et al*, 1988):

$$S_f = S_0 - \frac{\partial y}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \qquad \text{Equation 1}$$

This equation is also known as the St. Venant equation and is derived from the principle of conservation of momentum. If all of the terms in the equation are neglected except for the friction slope ($S_f$) and bed slope ($S_0$), the kinematic wave equation is derived:

$$S_f = S_0 \qquad \text{Equation 2}$$

The kinematic wave equation is sufficient for modeling flood waves on steep sloped rivers. When the pressure gradient term, $\frac{\partial y}{\partial x}$, is considered, the diffusive wave equation is represented:

$$S_f = S_0 - \frac{\partial y}{\partial x} \qquad \text{Equation 3}$$

This term is very important for modeling wave propagation and storage effects within the channel for mild slopes and steeply rising/falling hydrographs as experienced in the Pecos River. Usually, very little accuracy is lost if the convective and local acceleration terms are neglected, $\frac{V}{g}\frac{\partial V}{\partial x}$ and $\frac{1}{g}\frac{\partial V}{\partial t}$, respectively; thus, the diffusive wave equation is typically sufficient to simulate the downstream propagation of a hydrograph. The full dynamic wave equation is usually necessary only for abruptly changing hydrographs (high Froude numbers) such as during a dam breach.

Propagation of a flood wave can be accurately simulated as a kinematic wave if there is no floodwave attenuation. The kinematic wave equation does not predict channel storage, and any computed attenuation is induced by approximations in the numerical solution procedures. Criterion to verify the applicability of the kinematic wave approximation to the full dynamic wave momentum equation is defined below (Ponce, 1989):

$$\frac{t_r S_0 V_0}{d_0} \geq N = 85 \qquad \text{Equation 4}$$

where $t_r$ is time to peak
$S_0$ is bed slope
$V_0$ is average velocity
$d_0$ is average flow depth

Most flood waves traveling in mild slope river channels have some physical diffusion and are better simulated by a diffusive wave approximation to the full dynamic wave momentum equation. To determine if a flood wave is appropriately modeled as a diffusive wave, the following criterion is checked where g is gravitational acceleration (Ponce, 1989):

$$t_r S_0 \sqrt{\frac{g}{d_0}} \geq M = 15 \qquad \text{Equation 5}$$

The required parameters to check these criteria were developed for each reach of the Pecos River represented in the Pecos River RiverWare model. Power functions involving cross section geometry, flow, and average velocity were developed for the study reaches. Time to peak versus discharge relationships were determined from wave celerity calculations completed with the power function relationships. The value for $t_r$ was assumed to be the travel time through the specific reach for the flowrate being checked. The results from the checks indicate that a diffusive wave routing methodology is needed to effectively route low flows in the Pecos River. The diffusive wave criteria is exceeded for the lower Pecos reaches for flows greater than 2000 cfs, but most discharges in the Pecos River are less than 2000 cfs due to discharge restrictions through the gates at Sumner Dam. The results indicate that the full dynamic equation would be needed to simulate a release greater than 2000 cfs.

**Muskingum-Cunge Method:** Before discussing the issues for adding the Muskingum-Cunge method to RiverWare, a succinct derivation of the Muskingum-Cunge method is presented. The Muskingum-Cunge method involves use of a finite difference scheme to solve the Muskingum equation where the parameters in the Muskingum equation are determined based on the grid spacing for the finite difference scheme and channel geometry characteristics. The Muskingum equation represents the relationship between reach storage and discharge as a flood wave propagates through a reach. The hysteresis effect in the relationship between reach storage and discharge is represented in Figure 2. This concept is also depicted in Figure 3 where the first case represents the storage in the reach during the rising limb of a hydrograph, the second case represents uniform flow, and the third case represents the storage during the falling limb of the hydrograph. This hysteresis effect is due to the different flood wave speeds during the rising and falling limb of the hydrograph. For the same river stage, the flood wave moves faster during the rising limb of the hydrograph. The effect from this variable reach storage-discharge relationship is mimicked by the Muskingum equation for reach storage, $S$:

$$S = k\left[XI + (1 - X)O\right] \qquad \text{Equation 6}$$

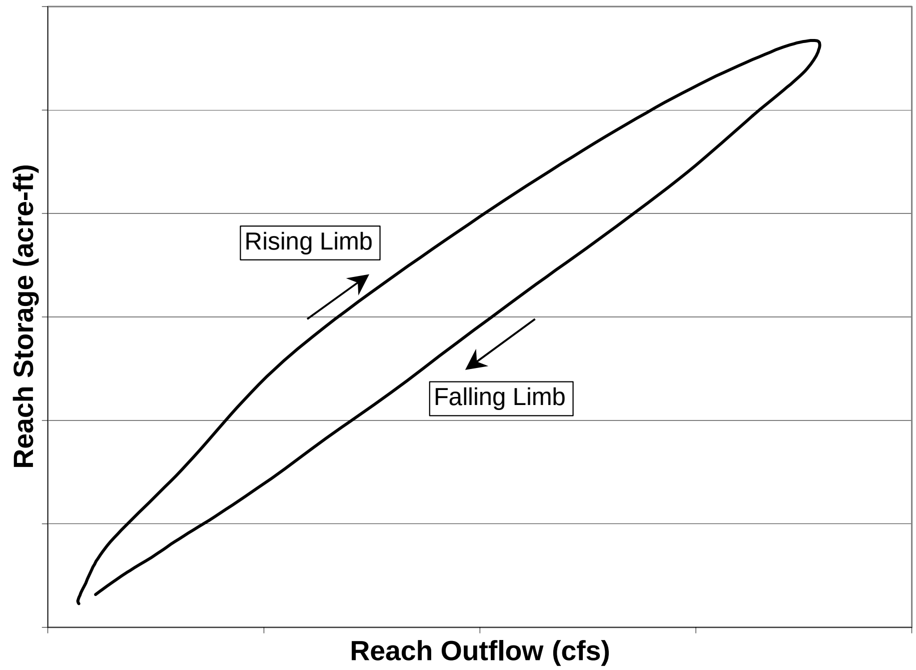


**Figure 2. Storage in a River Reach versus Reach Outflow**

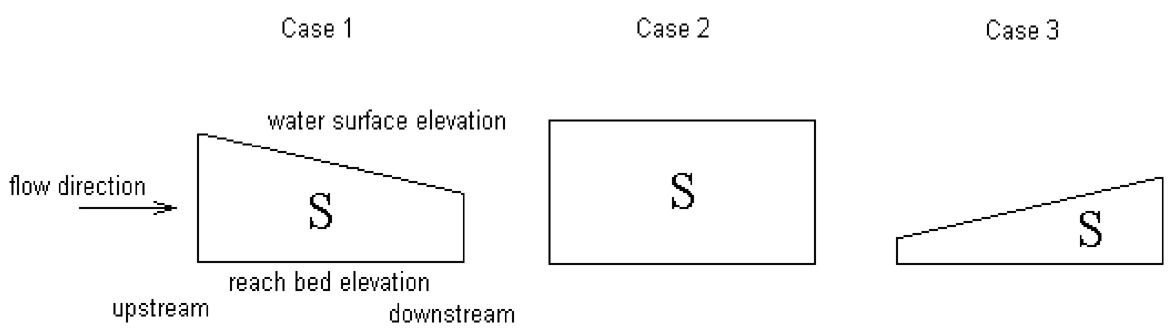


**Figure 3. Depiction of Reach Storage as a Flood Wave Propagates Downstream**

The inflow and outflow to the reach are represented by *I* and *O*, and *k* and *X* are the Muskingum travel time and diffusion parameters, respectively. The equation for continuity (conservation of mass) for the reach is defined below:

$$\frac{dS}{dt} = I - O \qquad \text{Equation 7}$$

Integrating this equation over an incremental timestep yields the following equation where the volume of the inflow and outflow over the timestep are represented by trapezoidal approximations:

$$S_{t+\Delta t} = S_t + \frac{I_t + I_{t+1}}{2}\Delta t - \frac{O_t + O_{t+\Delta t}}{2}\Delta t \qquad \text{Equation 8}$$

Combining this equation with the solutions for $S_{t+\Delta t}$ and $S_t$ from Equation 6 yields the following:

$$O_{t+\Delta t} = C_0 I_{t+\Delta t} + C_1 I_t + C_2 O_t \qquad \text{Equation 9}$$

where

$$C_0 = \frac{\left(\Delta t / k\right) - 2X}{2(1 - X) + \left(\Delta t / k\right)} \qquad \text{Equation 10}$$

$$C_1 = \frac{\left(\Delta t / k\right) + 2X}{2(1 - X) + \left(\Delta t / k\right)} \qquad \text{Equation 11}$$

$$C_2 = \frac{2(1 - X) - \left(\Delta t / k\right)}{2(1 - X) + \left(\Delta t / k\right)} \qquad \text{Equation 12}$$

The values of the Muskingum $k$ and $X$ parameters can be calibrated from streamflow data or the values can be determined from the finite difference grid spacing and channel geometry information. The latter is referred to as the Muskingum-Cunge method. After completing a Taylor series expansion of the outflow in the continuity equation (Equation 7) and differentiating the Muskingum equation (Equation 6), the resulting two equations can be compared to define equations for the Muskingum $k$ and $X$ parameters. The hydraulic diffusivity in the physical diffusive wave equation is set to the numerical diffusion coefficient from the Muskingum method. Hydraulic diffusivity is the coefficient of the second order term in the physical diffusive wave equation. This second order term accounts for wave diffusion. This relation allows for diffusion to be incorporated into the Muskingum scheme as a function of the channel cross section geometry. Refer to Appendix B in Engineering Hydrology by Ponce for documentation of this derivation (Ponce, 1989). The results from this derivation are Equations 13 and 14 for the Muskingum $k$ and $X$ parameters:

$$k = \frac{\Delta x}{c} \qquad \text{Equation 13}$$

where $\Delta x$ is the incremental spatial step for the finite difference scheme and *c* is the wave celerity.

$$X = \frac{1}{2}\left(1 - \frac{Q_{reference}}{ReachSlope * c * TopWidth * \Delta x}\right) \qquad \text{Equation 14}$$

For development of the Muskingum-Cunge method, the Courant number, C, and the cell Reynold's number, D, can be computed as defined and then used to compute $C_0$, $C_1$, and $C_2$.

$$C = c\frac{\Delta t}{\Delta x} \qquad \text{Equation 15}$$

$$D = \frac{Q_{reference}}{ReachSlope * c * TopWidth * \Delta x} \qquad \text{Equation 16}$$

After manipulating these equations with Equations 10 through 14, the following equations for $C_0$, $C_1$, and $C_2$ are derived.

$$C_0 = \frac{-1 + C + D}{1 + C + D} \qquad \text{Equation 17}$$

$$C_1 = \frac{1 + C - D}{1 + C + D} \qquad \text{Equation 18}$$

$$C_2 = \frac{1 - C + D}{1 + C + D} \qquad \text{Equation 19}$$

**<u>Computation of Wave Celerity</u>:** J.A. Seddon (1900) studied the computation of wave celerity for unsteady flow in rivers. He concluded that the celerity is equal to $\frac{\partial Q}{\partial A}$ (the partial derivative of flow with respect to flow area). Celerity is the speed of a monoclinal rising wave and is not equivalent to the average velocity of a floodwave. Wave celerity depends on channel geometry, slope, and roughness. When applying Manning's equation for triangular, wide rectangular, and wide parabolic shaped cross sections, the ratio of the celerity to the average velocity is 1.33, 1.67, and 1.44, respectively (Kohler, et al, 1975). For the Pecos River, the following power relationships were developed for reaches represented in the Pecos River RiverWare model, and these relationships are used to determine the celerity for the Muskingum-Cunge routing computations.

$$A = \alpha_1 Q^{\beta_1} \qquad \text{Equation 20}$$

$$V = \alpha_2 Q^{\beta_2} \qquad \text{Equation 21}$$

where $A$ is the cross section area ($ft^2$)
$Q$ is the discharge (cfs)
$V$ is the average velocity (ft/s)
$\alpha$ and $\beta$ are regression power coefficients and exponents

Equation 20 needs to be manipulated to solve for flow as a function of area:

$$Q = \delta A^{\varepsilon} \qquad \text{Equation 22}$$

where

$$\varepsilon = \frac{1}{\beta_1} \qquad \delta = \left(\frac{1}{\alpha_1}\right)^{\frac{1}{\beta_1}} \qquad \text{Equation 23}$$

$$c = \frac{\partial Q}{\partial A} = \varepsilon \delta A^{\varepsilon - 1} = \frac{1}{\beta_1} V \qquad \text{Equation 24}$$

The information for the regression power functions is input into the Pecos River RiverWare model.

**Muskingum-Cunge Method in RiverWare:** Before reviewing the issues with adding the routing method to RiverWare, it's important to recognize the difference between the routing timestep and the RiverWare model timestep. The RiverWare model timestep is one day for the Pecos River RiverWare model. Since RiverWare runs for the entire river system for each model timestep before progressing to the next model timestep, a separate routing scheme simulates for each model timestep. A smaller incremental routing timestep is used for the routing scheme. An appropriate routing timestep is entered by the user for each reach object within the RiverWare model. An appropriate corresponding spatial step for the routing scheme is determined by the model as discussed below. The total reach length is also entered by the RiverWare user.

**Reference Discharge:** A reference discharge is used to determine the cell Reynolds number within the Muskingum-Cunge finite difference scheme. Within RiverWare, this reference flow is set to the average of three known flow values: the flow at the previous incremental routing timestep and current incremental spatial step, the flow at the previous incremental routing timestep and previous incremental spatial step, and the flow at the previous incremental routing timestep and the previous incremental spatial step. The top width is determined for the reference discharge based on the regression power function between top width and flow entered by the RiverWare user, and the slope for the reach is also entered by the user.

For the routing method within RiverWare, the user inputs an incremental timestep for the finite difference scheme, and an appropriate corresponding incremental spatial step is determined by the model. The incremental spatial step is determined such that the Courant number, C, will be close to one to reduce the effects of numerical dispersion. Since the discharge will vary for a simulation, the Courant number will also vary. To pick a value for the incremental spatial step that minimizes the effects of numerical dispersion, the user inputs maximum and minimum discharges expected for a simulation, and the incremental spatial step is determined using the average of these two discharges:

$$Q_{to-calc-\Delta x} = \frac{1}{2}\left(Q_{\max} + Q_{\min}\right) \qquad \text{Equation 25}$$

The wave celerity computed with this reference discharge, the input power functions, and Equation 24 are used in Equation 15 with the input Δt to compute the corresponding Δx such that the Courant number will be 1.0. This Δx is used with the input Δt for the entire simulation for that reach within the Pecos River RiverWare model. Generally, the maximum release through the gates at Sumner Dam is a good value to enter for a maximum flow, and a base flow of ten cubic feet per second could be used for the minimum flow. The maximum release from the gates at Sumner Dam is approximately 1400 cfs. If storm inflows result in discharges greater than 1400 cfs, the flood peak is recommended for the estimated peak flow.

The value of the Muskingum *X* parameter cannot be less than zero or greater than 0.5. If the Muskingum *X* parameter is greater than 0.5, the wave will amplify, and a value less than 0.0 represents reach storage moving upstream. This translates to mean that the cell Reynolds number cannot be less than zero or greater than 1.0. If the resulting value of Δx is too small, the cell Reynolds number could be greater than 1.0. RiverWare will abort if the value of the cell Reynolds number is not within these boundaries. This occurs when the user inputs a very small value for Δt. For each reach in the Pecos River RiverWare model, appropriate Δt values were determined such that the Courant number would be close to one for typical discharges to be simulated. The selected Δt for each reach is presented in Table 1.

**Table 1. Selected Δt Values used in the Muskingum-Cunge Routing Method within RiverWare (hours)**

| | |
|---|---|
| Santa Rosa to Puerto de Luna | 1 |
| Sumner to Taiban | 1 |
| Taiban to Dunlap | 2 |
| Dunlap to Above Acme | 2 |
| Above Acme to Acme | 2 |
| Acme to Hagerman (Dexter) | 4 |
| Hagerman (Dexter) to Lake Arthur | 4 |
| Lake Arthur to Artesia | 4 |
| Artesia to Kaiser | 2 |
| Brantley to Damsite 3 | 1 |

**Flow Conversion:** The Muskingum-Cunge routing method requires instantaneous flows for the inflow hydrograph, but the flows used in the Pecos River RiverWare model are daily average flows. The daily average flows must be converted to instantaneous inflows for routing, and the routing results must be converted back to daily average flows. Assumptions are made to estimate the instantaneous flows at each initial incremental routing timestep to provide the necessary initial conditions for the finite difference scheme. The inflow at each initial incremental timestep is determined by interpolating between the inflow for the previous day and the inflow for the current day. The instantaneous routed outflow is converted back to a daily average outflow by using the final instantaneous outflow. This methodology was tested against another methodology for converting between average daily flows and instantaneous flows. For the other configuration, the instantaneous inflows are determined by using the inflow for the current day as the inflow at each initial incremental timestep, and the final instantaneous routed outflows at each incremental timestep are averaged to get the average daily outflow. Both methods yield the same results and conserve volume 100%. The interpolation method is used in the current RiverWare code.

The initial flows at each incremental spatial step must also be known to provide the boundary conditions for the finite difference scheme. The inflows to each reach are input to the RiverWare model by the user, and this flow is used for the initial flow at each spatial step. Essentially, steady flow is assumed along the entire reach for the initial timestep. The flows at each spatial step at the end of the routing scheme for each model timestep are saved and used as the initial flows at each spatial step for the next model timestep. As a result, even though a separate routing scheme simulates for each model timestep, the incremental spatial step cannot be changed during a model simulation; thus, the same incremental routing timestep and spatial step are used for the entire RiverWare simulation.

## FUTURE ENHANCEMENTS

Within RiverWare, the routing parameters, $C_0$, $C_1$, and $C_2$, are adjusted at each day of simulation based on the inflow for that day. The reference flow in Equation 16 is recomputed for each day of simulation. This reference flow along with corresponding values for the wave celerity and top width are used to compute new values for the cell Reynolds number, D, and the Courant number, C. This adjustment is made to assure the routing parameters are appropriately computed for the given inflow; however, changing the C and D values in the middle of a simulation results in a volume conservation error. This error can be significant for sharply rising and falling hydrographs, but the volume conservation error for a typical annual Pecos River hydrograph is generally less than 1%. This error is monitored as part of a simulation. This issue will be reviewed to evaluate alternatives for reducing this volume conservation error. Another discrepancy with the current method pertains to the determined incremental spatial step used during routing. This spatial step is not necessarily a perfect integer factor of the input reach length, so a small fraction of the reach length is neglected during routing.

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