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Evidence of “Summer Learning Loss” on the i-Ready Diagnostic Assessment

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About CADRE

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Executive Summary

In this report we provide a descriptive look at patterns of "summer learning loss" in reading and mathematics for elementary and middle school students who use the *i-Ready Diagnostic* assessment system produced by Curriculum Associates. We also compare the magnitude of summer learning loss found using *i-Ready* data to that which has been reported for students in similar grade ranges on a similar interim assessment. We find evidence of a significant amount of summer learning loss for the *i-Ready* student population in mathematics for grades K through 7. The magnitudes are practically significant, amounting to between about 30 to 40% of school year growth depending on the grade in question. In reading, summer learning loss appears to be considerably smaller, ranging from about 10 to 20% of school year growth. When compared to modeled estimates of summer learning loss reported by Kuhfeld, Condron, & Downey (2021), estimates based on *i-Ready* data tend to be somewhat larger in math, but smaller in reading.

Overview

When school-age children are given standardized achievement tests at the end of one school year (i.e., in the spring) and the beginning of the next (i.e., in the fall), the observation that a student's performance in the fall tends to be worse than it was in the spring has been described as the phenomenon of "summer learning loss." It is an observation that serves to quantify what many teachers describe anecdotally—the sense that incoming students are not showing the kind of mastery of subject matter content that would be anticipated, even when considering prior year grades and test performance.

Indeed, every summer there are numerous programs and products that are marketed to parents as tools for preventing "summer slide." However, as a phenomenon amenable to research, learning loss can be a thorny concept (see, for example, Alexander, Pitcock & Boulay, 2016). To begin with, there may not be shared perspectives on what it means for learning to have occurred in the first place. One might argue that if learning can be "lost" over the span of a few months, its apparent occurrence in the first place may have been something of an illusion. At the same time, to the extent that learning is a developmental and cyclical process, one that hinges upon the opportunities to have interactions within a structured environment, it stands to reason that when such opportunities are removed (or at least lessened) that learning will proceed more slowly. As such, it may be more appropriate to refer to what happens for many children during the summer months as the effect of a "curricular learning break" as opposed to learning loss more generically. In this sense, when repeated administrations of large-scale assessments on multiple occasions during two successive academic school years are available, there are actually two distinct quantities of interest: (1) the effect of a curricular learning break during the summer on student test performance in the fall, and (2) the average difference in test performance before and after a summer break. The latter, somewhat unfortunately, is what has come to be referred to in the literature as summer learning loss, and for the sake of consistency, we will use the same term in this report.

The most recent and comprehensive quantitative estimates of summer learning loss among elementary and middle-school aged students in the United States come from an analysis of longitudinal patterns using data from cohorts of students between the 2015-16 school year and the 2017-18 school year who took NWEA's Measures of Academic Progress (MAP) Assessments (Kuhfeld, Condron, & Downey, 2021). Our aim in this report is to conduct a similar descriptive analysis on the basis of the population of students who take Curriculum Associates' *i-Ready* Diagnostic assessment. Specifically, we ask: what are the estimates of summer learning loss in math and reading by grade found for students taking *i-Ready* assessments? How should their magnitudes be interpreted, and how do they compare to estimates of summer learning loss reported by Kuhfeld, Condron, & Downey (2021)?

Data

We use *i-Ready* data from the full population of elementary and middle school students across the United States who took assessments in either math or reading in grade K-7 during the 2018-19 school year, and then in grades 1 through 8 during the fall of the 2019-20 school year. We only include students who had scores for both the fall and spring testing occasions during the 2018-19 school year, and who also had a fall score available in the following year (fall 2019). We further restrict our analytic *i-Ready* sample to those students who also had the opportunity to make use of online lessons in between taking the assessments[1]. The resulting samples, shown in Table 1, range from a low of 183,042 students in grade 7 to a high of 382,592 in grade 4.

Table 1. i-Ready Student Populations, Fall 2018- Fall 2019

Grade	Reading	Math
K-1	196,573	191,772
1-2	277,298	295,552
2-3	310,234	340,858
3-4	326,711	356,720
4-5	317,385	357,120
5-6	214,904	256,889
6-7	167,202	204,438
7-8	138,716	166,227

[1] Students in each grade who used the lessons at any point during the school year tended to have slightly lower fall scores than did students who did not use the lessons at all. The effect sizes range from -0.11 to -0.38 in reading, and -0.15 to -0.39 in math. The effect sizes are largest in middle-school grades and therefore suggest that the lessons may be more likely to be used for remediation purposes in middle school than in elementary school.

Methods

We take two different strategies to arrive at estimates of summer learning loss for the population of students taking *i-Ready* assessments. In the first strategy, we simply compare the average score of students in the spring of one grade (grade "g") to the average score of the same students in the fall of the next grade (grade "g + 1"). This strategy is likely to produce a biased estimate. To see why, let X represent the *i-Ready* score of a student we would observe if the student could be tested on the very last day of the school year grade g, and let Y represent the score we would observe if the student could be tested on the very first day in grade $g + 1$. We could then compute a quantity we will call *SLL* as

$$SLL = Y - X \tag{1}$$

Note that, somewhat confusingly, a positive value of *SLL* should be taken as evidence (putting to the side the role of measurement error) that no learning loss has occurred. In fact, if it is large and positive, it may be taken as evidence of summer learning. In contrast, a negative value for *SLL* is taken as evidence of learning loss. Unfortunately, instead of X and Y, what we actually observe is the score from an *i-Ready* Diagnostic assessment administered some number of weeks prior to the end of the school year, X^*, and the score from an *i-Ready* assessment administered some number of weeks after the beginning of the next school year, Y^*. It follows that

$$X = X^* + \Delta_g \tag{2}$$

$$Y = Y^* - \Delta_{g+1} \tag{3}$$

Where Δ_g represents the additional amount of learning in grade g that would have been captured had the test been given on the last day of the school year in grade g, and Δ_{g+1} represents the new learning that has taken place at the outset of grade $g + 1$ before the fall test has been taken. From this we can compute

$$SLL^* = Y^* - X^* \tag{4}$$

and if we substitute equations 2 and 3 into 4,

$$SLL^* = \left(Y + \Delta_{g+1}\right) - \left(X - \Delta_g\right) = Y - X + \Delta_g + \Delta_{g+1}. \tag{5}$$

It follows that

$$SLL^* = SLL + \{\Delta_g + \Delta_{g+1}\} \tag{6}$$

The upshot is that what a teacher observes as summer learning loss for any individual student when they compare fall test performance to spring test performance (i.e., SLL^*) will only provide an accurate estimate of learning loss when $\Delta_g + \Delta_{g+1} = 0$. To the extent that these terms are positive, SLL^* will be a biased estimate of *SLL*. Of course, both Δ_g and Δ_{g+1} will vary by student (because students vary in when and how they learn, and how they reflect this in their test performance) and by school (because schools will vary in the timing of when they administer spring and fall tests, in their enacted curriculum, in the time between school years, etc.)[2].

[2] For the *i-Ready* data, the mean and SD for the number of days between fall and spring test occasion in each grade was about 241 and 21 respectively; the corresponding number of days between a spring test in grade g and fall test in grade g+1 was about 120 and 20.

Strategy 1 of estimating the mean of SLL^* across students for a given grade and test subject implicitly assumes that the average of $\Delta_g + \Delta_{g+1} = 0$, or at least that it is close enough to 0 to be neglible. For schools in which the spring test is given just a few weeks before the end of the school year and for which the fall test is given just a few weeks after the start of the school year, such an assumption may be plausible. For other schools where the time in which students are exposed to additional instruction is longer, the values for Δ_g and Δ_{g+1} may well be significant.

A second strategy is to attempt a statistical adjustment that controls for differences in Δ_g and Δ_{g+1} across students. To do this we adapt the approach described in Kuhfeld et al. (2021) by specifying a two-level hierarchical linear model. In this model, the subscript t indexes a test occasion (of which there are three), and the subscript i indexes a unique student who has taken a test on each of these three occasions.

$$\text{Level 1: } score_{ti} = \pi_{0i} + \pi_{1i}SYg_i + \pi_{2i}Summer_i + \pi_{3i}SYg1_i + e_{ti} \quad (7)$$

$$\text{Level 2: } \pi_{0i} = \beta_0 + r_{0i} \quad (8)$$

$$\text{Combined Model: } score_{ti} = \beta_0 + \pi_{1i}SYg_i + \pi_{2i}Summer_i + \pi_{3i}SYg1_i + r_{0i} + e_{ti} \quad (9)$$

In the combined model shown in Equation 9, the variables SYg_i, $Summer_i$, and $SYg1_i$ represent the cumulative amount of time (in months)[3] that a given student has experienced the school year in grade g, the summer following grade g, and the school year in grade g+1 at the time of a test given on occasion t. The parameters π_{1i} and π_{3i} represent the average linear monthly growth rate in school year g and g + 1, respectively. The parameter π_{2i} represents the monthly growth rate in the summer months between grades g and g+1 (expected to be negative if summer learning loss exists). The parameter π_{0i} represents the score that would be predicted for a student at the outset of grade g, and it is specified as a random effect (i.e., $\pi_{0i} = \beta_0 + r_{0i}$). The terms e_{ti} and r_{0i} are assumed to be independent and normally distributed with means of 0 and variance terms σ_e^2 and σ_r^2. A statistically adjusted value for SLL, which we denote as "SLL^a" is derived from this model by multiplying the estimate for π_{2i} by 2.5 months (a typical duration for the summer break between school years). We estimate the model in Equation 9 using the `lme4` package (Bates et al., 2015) in the R Computing Environment.

In what follows, for a given test subject (reading or math), and base grade (Kindergarten through grade 7), we report two different values for the average summer learning loss across the analytic sample of *i-Ready* test-takers that correspond to each of the two strategies described above.

$$\overline{SLL}^* = \frac{\sum SLL_i^*}{N} \quad (10)$$

and

$$SLL^a = \hat{\pi}_{2i} * 2.5 \quad (11)$$

where in equation 11 the "a" superscript stands for "adjusted" and N represents the size of the grade and subject specific sample (see Table 1).

To evaluate the practical significance of $\overline{SLL}^*$ and SLL^a, we express each in effect size units after dividing by the SD of the base year spring test score. We also do this by expressing $\overline{SLL}^*$ as a

[3] For details on the approach to the coding of these time variables by student, see Appendix E and Table E2 from the online supplementary materials[3] for Kuhfeld et. al (2021). Also available upon request of this report's first author.

proportion of the average fall to spring test score gain during the base grade school year, and by expressing SLL^a as a proportion of the *modeled* gain during the prior school year. This latter denominator is computed by multiplying the estimate for π_{1i} by 9.5 (the typical length of a school year in months). Note that summer learning loss estimates for *i-Ready* in reading and math can only by compared to those previously reported for in Kuhfeld et al (2021) on the basis of SLL^a for the base grades of K, 1, 3, 4, 6 and 7.

Results

Table 2 provides key descriptive statistics relevant to *i-Ready* test scores by grade along with lesson usage patterns. The rows labelled SLL^* represent the $\overline{SLL}^*$ quantity introduced in the previous section (i.e., Equation 11). On the basis of this statistic, summer learning loss is really only evident in math, where score declines on the *i-Ready* scale average about 7 points. In contrast, average scores in reading stay about the same from spring to fall. The variability in student growth across any two test occasions—whether from fall to spring, or spring to fall—is substantial. The SDs across grades range from about 25 to 30 points on the *i-Ready* scale for reading, and about 15 and 20 points on the *i-Ready* scale for math.

Tables 3 and 4 summarize and compare estimates for summer learning loss based on *i-Ready* reading and math assessments using the two different strategies discussed in the previous section. The first pair of main columns in each table compares the magnitudes in *i-Ready* scale score units. The next two pairs of columns re-express these values as proportions of the spring test SD of the base grade (i.e., in "effect size" units), and the magnitude of the fall to spring gain during the base grade school year. Focusing first on SLL^*—the quantity that teachers and school administrators would notice if they compared fall test scores to prior grade spring test scores—we can see that the magnitudes for math are much larger than those for reading. In fact, summer learning loss in reading is only noticeable for grade 2, where it amounts to 3 scale score units. When expressed in effect size units or as a proportion of the average base grade school year gain, none of the SLL^* values appear practically significant. In contrast, the magnitude of observed *SLL* for math is between about -.20 and -.30 effect size units between grades K through 5, and between grade 4 and 7 the effect sizes decrease to -.10. When expressed as a proportion of the average base grade school year gain, the SLL^* values in grades 4 through 7 are larger than those from K through 3. This is because school year growth in math shows a general pattern of decline (also known as deceleration) across grades[4]. Hence the same value of SLL^* will be a larger proprtion of school year growth in later grades relative to earlier grades.

[2] This feature is not unique to the *i-Ready Diagnostic* vertical scale. A trend of growth deceleration as students enter the middle school grades tends to be evident on interm and state assessments alike. For a summary of these trends on state assessments, see Dadey & Briggs, 2012.

Table 2. Descriptive Statistics for i-Ready Test Scores, Fall 2018-Fall 2019

	Reading		Math	
Variable	Mean	SD	Mean	SD
Grade K				
Fall 18 Test	346.7	29.6	344.2	21.6
Winter 19 Test	380.4	34.2	365.6	22.1
Spring 19 Test	404.8	38.7	381.3	23.7
Fall 19 Test	405.7	41.9	376.0	24.6
SLL*	0.9	25.3	-5.2	18.0
SY Gain	58.2	31.0	37.1	19.6
Grade 1				
Fall 18 Test	403.3	40.6	376.7	23.2
Winter 19 Test	434.7	44.6	396.3	23.8
Spring 19 Test	457.6	47.4	409.7	25.7
Fall 19 Test	458.5	50.4	402.2	25.3
SLL*	0.9	25.2	-7.5	16.1
SY Gain	54.3	30.7	33.0	17.8
Grade 2				
Fall 18 Test	457.6	50.0	402.4	24.4
Winter 19 Test	484.5	49.6	419.2	24.9
Spring 19 Test	499.8	49.5	432.0	26.7
Fall 19 Test	496.8	51.7	425.9	25.4
SLL*	-3.1	25.9	-6.0	14.9
SY Gain	42.2	28.9	29.6	16.9
Grade 3				
Fall 18 Test	494.9	51.6	425.2	25.9
Winter 19 Test	515.9	50.2	441.8	25.4
Spring 19 Test	527.7	51.6	455.4	29.2
Fall 19 Test	527.0	52.7	447.2	27.3
SLL*	-0.7	25.9	-8.2	15.0
SY Gain	32.9	28.4	30.1	17.1

	Reading		Math	
Variable	Mean	SD	Mean	SD
Grade 4				
Fall 18 Test	525.6	52.5	448.6	28.6
Winter 19 Test	541.6	51.1	461.9	28.3
Spring 19 Test	550.6	53.0	474.3	31.7
Fall 19 Test	551.0	53.3	464.5	29.1
SLL*	0.4	26.4	-9.8	15.3
SY Gain	25.0	28.1	25.7	17.2
Grade 5				
Fall 18 Test	545.4	53.6	464.3	30.1
Winter 19 Test	559.8	52.6	475.1	29.9
Spring 19 Test	567.3	55.2	483.8	32.8
Fall 19 Test	566.5	55.8	476.7	31.7
SLL*	-0.8	28.4	-7.2	15.8
SY Gain	21.9	29.3	19.5	17.1
Grade 6				
Fall 18 Test	561.3	56.6	475.4	32.0
Winter 19 Test	571.0	56.6	484.1	32.9
Spring 19 Test	577.6	58.8	490.8	35.9
Fall 19 Test	577.6	58.5	484.3	35.1
SLL*	0.0	30.9	-6.5	18.1
SY Gain	16.3	32.1	15.4	18.5
Grade 7				
Fall 18 Test	572.3	59.1	482.0	33.8
Winter 19 Test	580.4	59.2	488.9	35.2
Spring 19 Test	585.7	61.3	493.8	38.3
Fall 19 Test	587.3	59.5	489.9	37.7
SLL*	1.6	33.0	-4.0	19.3
SY Gain	13.4	34.5	11.9	20.1

Note: SY Gain = Spring 19 Test – Fall 18 Test; *SLL** = Fall 19 Test – Spring 19 Test

All the estimates of average summer learning loss tend to increase in magnitude when attempting to control for differences in the time periods before the fall test period and after the spring test period. (This suggests that $\Delta_g + \Delta_{g+1} > 0$ and that *SLL** may underestimate the magnitude of *SLL*.) In reading, we can now see evidence of small but arguably practically significant score declines in each of the elementary school grades of K through 5. In math, estimates expressed in effect size units increase in grades K through 4 from a range of -.20 to -.30 to a range of -.40 to -.53. The difference in *SLL** and SLL^a when expressed as a proportion of the average school year gain are less extreme (and in some cases, such as grades 6 and 7, do not really change at all) because in the case of SLL^a, the modeled school year gain (i.e., the denominator) also tends to be adjusted upward. A summary conclusion about the amount of summer learning loss evident from *i-Ready* assessment data is that, on average, over the summer months students “lose” about 10% of the growth in test scores observed over the course of the previous school year in reading, and about 30% in math.

Table 3. Summer Learning Loss in Reading Following Each Grade based on i-Ready Assessments

	i-Ready Scale Units		Effect Size Units		As Prop of SY Gain	
	SLL*	SLL[a]	SLL*	SLL[a]	SLL*	SLL[a]
Grade K	0.9	-8.5	0.02	-0.22	--	0.12
Grade 1	0.9	-8.5	0.02	-0.18	--	0.13
Grade 2	-3.1	-10.2	-0.06	-0.21	0.07	0.20
Grade 3	-0.7	-5.5	-0.01	-0.11	0.02	0.14
Grade 4	0.4	-2.7	0.01	-0.05	--	0.09
Grade 5	-0.8	-2.9	-0.01	-0.05	0.04	0.11
Grade 6	0.0	-1.1	0.00	-0.02	--	0.06
Grade 7	1.6	0.5	0.03	0.01	--	0.03

Table 4. Summer Learning Loss in Math Following Each Grade based on i-Ready Assessments

	i-Ready Scale Units		Effect Size Units		As Prop of SY Gain	
	SLL*	SLL[a]	SLL*	SLL[a]	SLL*	SLL[a]
Grade K	-5.2	-23.8	-0.22	-0.53	0.14	0.28
Grade 1	-7.5	-25.8	-0.29	-0.52	0.23	0.34
Grade 2	-6.0	-26.8	-0.22	-0.42	0.20	0.32
Grade 3	-8.1	-29.3	-0.28	-0.42	0.27	0.34
Grade 4	-9.7	-31.7	-0.31	-0.40	0.38	0.42
Grade 5	-7.1	-32.8	-0.21	-0.28	0.37	0.41
Grade 6	-6.5	-35.7	-0.18	-0.21	0.42	0.42
Grade 7	-3.9	-38.1	-0.10	-0.12	0.34	0.35

As the results shown in Tables 3 and 4 focus on averages, it can be easy to lose sight of the variability in summer learning loss evident in Table 2. Figure 1 presents boxplot summaries of the distribution of *SLL** values by subject and grade. The height of each box represents the interquartile range; the lines (or "whiskers") extending from the boxes indicate the minimum and maximum values. The interquartile range of *SLL** in reading and math are about 25 and 20 *i-Ready* scale score units. For the *i-Ready* reading assessment, increases from spring to fall testing of about 10 scale score units are observed just about as frequently as decreases. In math, gains from spring to fall are only observed for the top 25% *i-Ready* test-takers.

Figure 1. Distribution of SLL* on i-Ready Assessments by Subject and Grade.

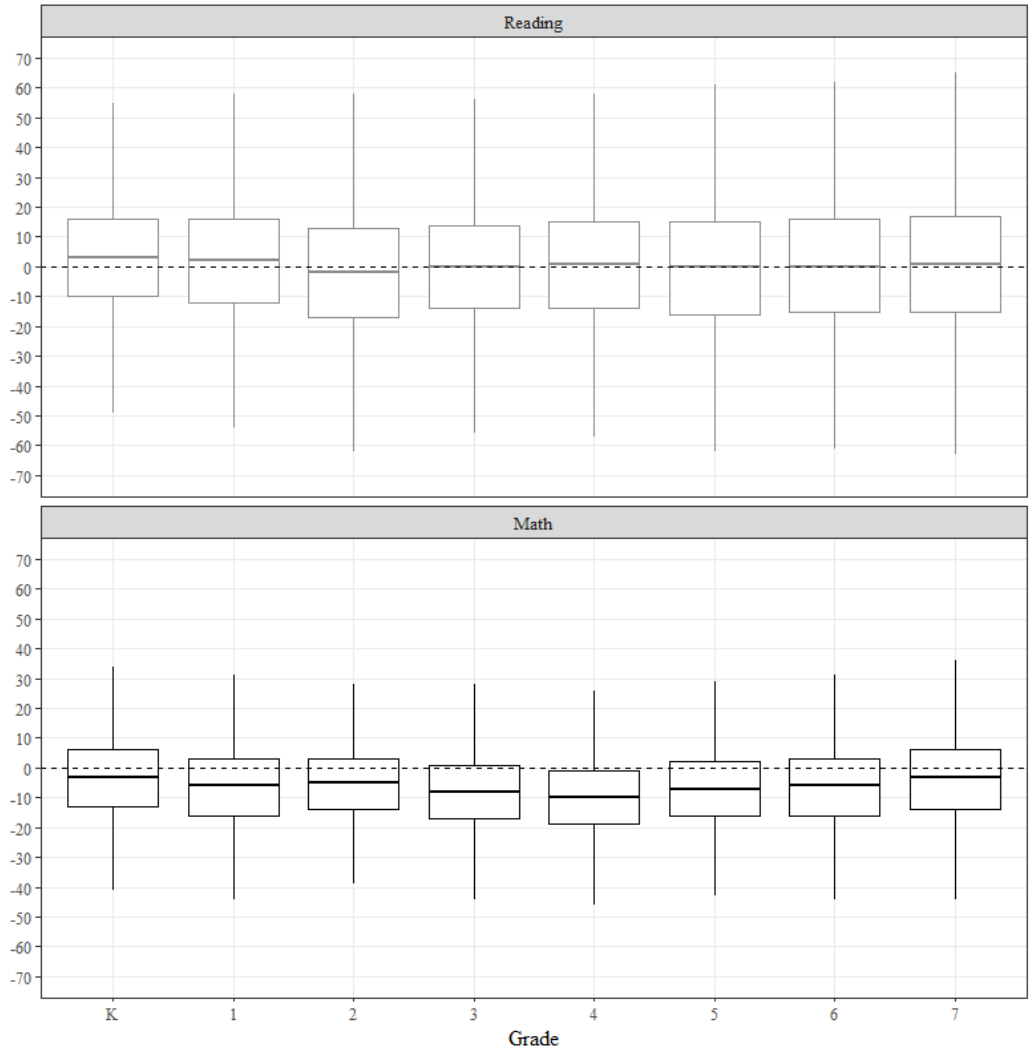

Table 5 compares average model-based estimates summer learning loss in reading and math on *i-Ready* to those that have been previously reported using MAP data. The two sources of data paint a similar picture about learning loss in reading in the summers following Kindergarten and grade 1: whether using *i-Ready* or MAP, score decreases over the summer represent about 10% of gains that were made during the prior school year. From here the patterns tend to diverge. Although SLL^a for MAP data also tends to be higher in math than in reading, the differences are much smaller than evident on *i-Ready*. On *i-Ready*, SLL^a in reading remains small enough to be mostly negligible to non-existent by the middle school grades of 6 and 7; on MAP, SLL^a in reading increases in both the upper elementary and middle school grades to as much as 28% of a prior school year gain. In math, SLL^a on *i-Ready* tends to be larger than that evident for MAP. It tends to be about 36% of a prior school year gain on *i-Ready*, and 25% on MAP, a difference of about 11%. As we will discuss in the concluding section of this report, there are many possible explanations for these differences.

Table 5. Modeled Summer Learning Loss by Interim Assessment Product

Subject/ Grade	SLL as Proportion of Spring SD		SLL as Proportion of School Year Gain	
	i-Ready	MAP	i-Ready	MAP
Reading				
K-1	-0.22	-0.18	0.12	0.12
1-2	-0.18	-0.17	0.13	0.13
2-3	-0.21		0.20	
3-4	-0.11	-0.17	0.14	0.21
4-5	-0.05	-0.14	0.09	0.22
5-6	-0.05		0.11	
6-7	-0.02	-0.11	0.06	0.28
7-8	0.01	-0.06	--	0.20
Math				
K-1	-0.53	-0.19	0.28	0.14
1-2	-0.52	-0.29	0.34	0.23
2-3	-0.42		0.32	
3-4	-0.42	-0.30	0.34	0.26
4-5	-0.40	-0.25	0.42	0.28
5-6	-0.28		0.41	
6-7	-0.21	-0.20	0.42	0.34
7-8	-0.12	-0.11	0.35	0.26

Source for MAP values: Calculations based on Tables 1 and 2 from Kuhfeld, Condron & Downey, 2021.

Summary and Discussion

In this report we have presented descriptive evidence in regard to the amount of summer learning loss experienced by the population of students in the 2018-19 school year who were enrolled in schools that administered the *i-Ready* assessments in reading and math. We applied two strategies to estimate this quantity. The first strategy, which produced quantities that we labelled *SLL**, is based on taking the simple difference in the *i-Ready* test score for students in the spring of a base grade and then subtracting this from the *i-Ready* test score for the same students in the fall of the next grade. A negative value would be taken as evidence of "learning loss." The second strategy, which produces quantities that we labelled SLL^a, is based on a statistical adjustment to account for differences in the timing of spring and fall test occasions, as well as differences in the length of summers across school districts.

We find evidence of average summer learning loss in math at all grade levels, whether this is estimated using *SLL** or SLL^a. The magnitudes are practically significant, amounting to at least 20% of school year growth, and in some cases as much as 42%. In reading, a smaller amount of average summer learning loss—generally around 10% of school year growth—is only evident when estimated using SLL^a. When estimated using *SLL**, evidence of average summer learning loss in reading—when it exists at all—tends to be practically insignificant.

With the exception of grades K and 1 in reading, the magnitudes of SLL^a by grade and subject based on the *i-Ready* test-taking population differs from that which has been reported for the MAP test-taking population in Kuhfeld et. al., 2021. There may be substantive explanations for these differences. The schools and school districts that choose to purchase *i-Ready* or MAP represent self-selected samples, so there is no way to establish the extent to which they represent comparable populations. It is also possible that some of these differences might be attributable to differences between *i-Ready* and MAP in the choice and overlap of content across test administrations.

Another possible source of confounding comes from the fact that we have implemented a simplified version of the modeling approach described by Kuhfeld et al. Where Kuhfeld et al. used five longitudinal test occasions per student, we are only using three. Furthermore, where Kuhfeld et al. specified a three-level model with all slope coefficients at level 1 as random effects, we have specified a two-level model with only the intercept term as a random effect. We chose this simpler model because we were primarily concerned with the estimation of fixed effects for level 1 slopes, as opposed to modeling and explaining variability in these slopes as a function of student and school variables at levels 2 and 3. The more complex model also makes much stronger assumptions, and can only be estimated with specialized software that takes hours to run by grade and subject.

We examined the possible sensitivity of our results to choice of HLM specification by estimating the more complex three-level model from Kuhfeld et al. on our Kindergarten base grade population of test-takers in mathematics using the specialized software HLM 8 (Raudenbush & Congdon, 2021). The respective base grade school year slopes under the simple and complex HLM specifications were 4.821 and 4.751; the respective summer months slopes were -5.594 and -5.045. The estimates are different, but probably not different enough to significantly change our conclusions about the summer learning loss magnitudes. For example, if the point estimates from the more complex HLM had been used, *SLL*^{a} would go from -12.61 to -13.99. When expressed as a proportion of the modeled school year gain, the quantity shifts from .28 to .31. Nonetheless, future work in this area should be attentive to choices in model specification, as well as implications for model identification.

In this report we used two different strategies to get estimates of average summer learning loss by grade and test subject. Both are important to report. The *SLL** quantity is the value that teachers, parents, and students themselves can actually observe. As shown in the methods section, this quantity almost surely underestimates summer learning loss, because it does not account for learning that may have occurred after the spring testing of a base grade, and prior to the fall testing of a subsequent grade. Although the *SLL*^{a} quantity seems preferable in the sense that it attempts to model and account for these additional increments, the model is still fairly crude. That is, the model assumes that students learn at the same linear rate for every month of an academic school year. If, instead, learning follows more of a nonlinear S-shaped logistic curve—slower during the first and last few weeks of a school year—then *SLL*^{a} will be likely to overestimate average summer learning loss. Unfortunately, there are not enough test occasions to accurately model a nonlinear trajectory, nor is it clear that the *i-Ready* tests (or for that matter, any existing large-scale assessments) would be sufficiently sensitive to pick up small differences in the knowledge and skills students develop over the course of a few weeks (as opposed to a few months). Therefore, it is probably best to think of the *SLL** and *SLL*^{a} estimates presented in this report as lower and upper bounds on the magnitude of summer learning loss by grade and test subject.

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