Midterm 1, vers. A - Physics 1120 – Spring 2019

NAME ____________________________

Student ID ____________________________

Circle TA’s Name: Beremkulov, Timur Burau, Justin
Corsiglia, Giaco Lucas Feng, Lili Gebre, Yonas McMaken, Tyler
Ni, Yifei Prabhune, Ameya Sumner, Bjorn Wilczak, Matteo

Circle recitation START TIME: 8am 9am 10am 11am Noon 1pm 2pm 3pm 4pm

Please do not open the exam until you are told to.

Your exam should have 14 pages, numbered 1 through 14. The last sheet is for scratch paper. This exam consists of 25 questions, weighted equally. Fill in the bubble sheet with a #2 pencil.

Please fill out your bubble sheet according to the following instructions or you may be penalized. Check each box as you complete the instructions.

- Please circle your TA's name and start time of your recitation section.
- Print and bubble in your name on the bubble sheet.
- Print and bubble in your 9-digit student Identification Number.
- Bubble in your exam version, A or B.
- On your bubble sheet, erase mistakes thoroughly, and make no extraneous marks

As you take the exam, show all your work by the question on the exam and circle the correct answers on your exam. Your circled exam answers and bubbled answers must agree.

I have read and followed the instructions above. I give my word that I have neither given nor received unauthorized assistance on this exam.

Signature ___________________________________________________________________

Try to relax. Budget your time. All your neighbors also think this is a long, hard exam.
Possibly useful information:

c = 1.6 \times 10^{-19} \text{ C},

\varepsilon_0 = 8.85 \times 10^{-12} \text{ (SI units)},

k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9 \text{ (SI units)},

\mu_0 = 4\pi \times 10^{-7} \text{ (SI units)}

Start of exam

1. Which are the units of electric flux \( \Phi = \int \vec{E} \cdot d\vec{a} \)?

   A) \( \frac{N}{C \ m^2} \)
   B) \( C \ m^2 \)
   C) \( \frac{N \ m^2}{C} \)
   D) \( \frac{C}{m^2} \)
   E) \( \frac{N \ m^2}{C^2} \)

\[ d\Phi = \vec{E} \cdot d\vec{a} = E \ dA \cos \theta \]

\[ \frac{N}{C} \ \text{m}^2 \ \text{UNITLESS} \]

So total units are \( \frac{N \ m^2}{C} \)

(INTEGRATION DOES NOT CHANGE UNITS)

2. An electron is given an initial velocity to the left in a region filled with a constant uniform electric field pointing to the right. If the electron interacts with nothing other than this uniform field, its speed will...

   A) steadily increase
   B) steadily decrease
   C) increase, then decrease
   D) decrease, then increase
   E) remain constant

At all points, the electron feels a force

\[ F = q \vec{E} = -e \vec{E} \] (Force points opposite field)

Constant left force creates constant acceleration in the direction of motion
3. Consider the three point charges arranged as shown along the x-axis. What is the magnitude and direction of the net electric force on the +Q charge in the middle?

\[ F_A = k \frac{Q^2}{d^2} \quad F_B = k \frac{3Q^2}{4d^2} \]

\[ \overrightarrow{F_{NET}} = \overrightarrow{F_A} + \overrightarrow{F_B} \]

\[ |F_{NET}| = F_A - F_B \quad \text{in this case} \]

\[ = k \frac{2Q^2}{d^2} - k \frac{3Q^2}{4d^2} - k \frac{Q^2}{d^2} \left( \frac{2}{4} \right) \]

\[ = k \frac{Q^2}{4d^2} \left( \frac{5}{4} \right) \]

4. Which of the following electric fields would hold an electron (mass \( m_e \), charge \( -e \)) stationary at a height \( h \) near the surface of the Earth? Take the positive y direction to be upward and the magnitude of the acceleration due to gravity to be \( g \).

A) \( + \frac{m_e g}{e} \hat{y} \)

B) \( - \frac{m_e g}{e} \hat{y} \)

C) \( + \frac{ke}{h^2} \hat{y} \)

D) \( - \frac{ke}{h^2} \hat{y} \)

E) None of these

\[ \vec{F}_E = \vec{E} \]

\[ \vec{F}_e = -m_g \hat{y} \]

\[ \text{NEED} \quad \vec{F}_E = -\vec{F}_e \]

\[ -c\vec{E} = -(-m_g \hat{y}) \]

\[ \vec{E} = -\frac{m_g}{e} \hat{y} \]

**Electrons Feel Upward Forces in Downward Fields!**
5. A positive sodium ion is placed at a particular location in an electric field and released. It immediately experiences an acceleration to the right of magnitude $a$. The sodium ion is removed and a different, unknown ion is brought to the same location and released. The unknown ion experiences a rightward acceleration of magnitude $a/2$. Which of the following MUST be true? (In this problem, assume no gravity.)

A) The unknown ion had half as much charge as the sodium ion.  
B) The unknown ion had twice as much mass as the sodium ion.  
C) The unknown ion had an initial velocity and the sodium did not.  
D) The unknown ion was negatively charged.  
E) None of the above MUST be true.

NONE OF THEM HAVE TO BE TRUE  
(COULD BE OTHERS)

6. A box of mass $m$ and charge $+q$ rests on a horizontal surface. There is a constant, uniform electric field $E$ at an angle $\theta$ below the horizontal, as shown, and there is friction between the box and the surface. If the box slides to the right at a constant velocity $v$, which of the following is a correct expression for the magnitude of the friction force?

A) $mg \sin \theta$  
B) $qvE$  
C) $qE \cos \theta$  
D) $\frac{qE}{\sin \theta}$  
E) None of these

$$F_{F_R} = F_{E_x} = F_E \cos \theta = 8E \cos \theta$$
7. A particle with charge \(-Q\) and mass \(m\) is fired along the y-axis, as shown. A triangle of additional charges are fixed in place along the x- and y-axes, as shown. What is the magnitude of the acceleration of the \(-Q\) charge at the instant when it reaches the origin? Ignore the effects of gravity.

A) zero  
B) \(\frac{kQ^2}{\sqrt{2mr^2}}\)  
C) \(\frac{2kQ^2}{mr^2}\)  
D) \(\frac{4kQ^2}{m^2r^2}\)  
E) Cannot answer without further information

\[ a = \frac{F}{m} = \frac{QE}{m} \]

WE WILL ACCEPT THIS IF YOU WERE WORRIED ABOUT WHETHER COULOMB'S LAW APPLIES SINCE THE CHARGE IS MOVING.

8. Two particles are released from rest with initial distance \(R\) between them. Their masses and charges are given below. If there are no forces other than those the particles exert on one another, which statement correctly describes the magnitudes of the forces and accelerations the particles will experience immediately after release?

\[ \text{particle A} \quad \text{particle B} \]
\[ 3m, +3Q \quad 2m, +2Q \]

A) Both feel the same magnitude force and experience the same magnitude acceleration.  
B) Both experience the same magnitude acceleration, but particle B feels a greater force.  
C) Particle B feels a greater magnitude force and experiences a greater magnitude acceleration.  
D) Both feel the same magnitude force but particle B experiences a greater magnitude acceleration.  
E) Particle A feels a greater magnitude force and experiences a greater magnitude acceleration.

\[ \text{Forces are the same by Newton's 3rd} \]
\[ F = ma \Rightarrow a = \frac{F}{m} \text{ so the more massive charge will accelerate less.} \]
9. Two charges are fixed in place on the x and y-axes as shown. The direction of the electric field at the origin is found to be 45 degrees below the x-axis. Which of the following sets of values for \( Q_1 \) and \( Q_2 \) would be consistent with that observation?

A) \( Q_1 = +3\mu C, \ Q_2 = +3\mu C \)

B) \( Q_1 = +3\mu C, \ Q_2 = -3\mu C \)

C) \( Q_1 = -3\mu C, \ Q_2 = +3\mu C \)

D) \( Q_1 = -3\mu C, \ Q_2 = -3\mu C \)

E) None of these.

10. In the figure below, two equal and opposite charges \( +Q \) and \( -Q \) are fixed in place on the y-axis. Point A is a point on the x-axis. The distances between the two charges and between the charges and point A are all equal, so that the three form an equilateral triangle.

What is the magnitude of the electric field at point A?

\[
E_{101} = E_{1y} + E_{2y}
\]

A) \( \frac{2\frac{kQ}{r^2}}{r^2} \)

B) \( \frac{2\frac{kQ}{r^2} \cos 60^\circ}{r^2} \)

C) \( \frac{2\frac{kQ}{r^2} \cos 30^\circ}{r^2} \)

D) \( \frac{kQ}{(r \cos 30^\circ)^2} \)

E) Zero
11. A positively charged rod is brought near a small, electrically neutral sphere. The sphere is observed to be attracted to the rod (but does not touch it). Next, a negatively charged rod is brought towards the sphere. What do we expect to observe?

A) The sphere will be attracted to the rod.
B) The sphere will be repelled from the rod.
C) The sphere will not react to the rod.

12. A neutral metal object is placed in the non-uniform electric field shown at right. What is the direction of the net electrostatic force on the object?

A) To the right →
B) To the left ←
C) Upward ↑
D) Downward ↓
E) The net force is zero

13. A charge +Q is spread out uniformly along a semicircular rod, as shown. If we wish to evaluate the total electric field \( \vec{E} \) at point A by integrating over the charge distribution, which expression will be sufficient? Use the given convention for the unit vectors \( \hat{x}, \hat{y}, \hat{z} \).

A) \( \vec{E} = \int dE_x \hat{x} \)
B) \( \vec{E} = \int dE_y \hat{y} \)
C) \( \vec{E} = \int dE_z \hat{z} \)
D) None of these

**Polarization Forces Always Attract**

**Total charge: +Q**

**WEAKER FIELD**

**STRONGER FIELD**

(\( \text{FIELD LINES MORE DENSE} \))

\( F_{\text{top}} = b E_{\text{top}} \)

\( E_{\text{bot}} > E_{\text{top}} \)
14. Consider a uniformly-charged horizontal rod of length $L$ with a charge density $\lambda$. Which of the following is an expression for the magnitude of the total electric field at a point $P$, located along the axis of the rod at a distance $d$ from the left end?

A) $\int_0^L \lambda \, dx$

B) $\int_0^d \lambda \, dx$

C) $\int_0^L \frac{k \lambda dx}{x^2}$

D) $\int_0^d \frac{k \lambda dx}{x^2}$

E) $\int_0^{d+L} \frac{k \lambda dx}{x^2}$

\[ dE = k \frac{dQ}{x^2} = k \frac{\lambda \, dx}{x^2} \]

\[ \vec{E} = \int d\vec{E} = \int_0^{d+L} \frac{k \lambda \, dx}{x^2} \]

15. Consider the following statements:

I) If $\int_S \vec{E} \cdot d\vec{a} = 0$ for a closed surface $S$, then $E = 0$ everywhere on surface $S$.

II) If $E = 0$ everywhere on a closed surface $S$, then $\int_S \vec{E} \cdot d\vec{a} = 0$ for the surface $S$.

Which of these statements are true?

A) Both are true  
B) Neither are true  
C) Only I is true  
D) Only II is true

**Counter-example to (I):**

$E$ is **never** zero on surface, but $\Phi = 0$

(All lines in later go out)
The next two questions refer to this situation: A infinite plastic slab of thickness $t$ has a uniform positive charge density $\rho$ (charge per volume). A student wishes to compute the magnitude $E$ of the electric field at a distance $d$ from the center of the slab ($d > t/2$). She writes down Gauss’s Law \[ \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} \] and sketches the centered, cylindrical Gaussian surface $S$ shown. The cylinder has length $2d$ and end caps each of area $A = \pi r^2$.

16. What is the correct expression for the enclosed charge $Q_{\text{enc}}$?

A) $2EA$  B) $\rho At$  C) $4\pi r t$  D) $2d A \rho$  E) None of these

\[ Q = \rho \cdot V = \rho At \]

\[ \text{CH}_b = \frac{\text{CH}_e \cdot V_{OL}}{V_{OL}} \]

17. How does the magnitude of the E-field outside this slab, vary with distance $d$ from the center of the slab ($d > t/2$)?

A) $E \propto d$  B) $E \propto 1/d$  C) $E \propto d^2$

D) $E \propto 1/d^2$  E) $E$ is constant, independent of $d$

From Gauss: \[ \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} = \frac{CA t}{\varepsilon_0} \]

\[ \int_{\text{ends}} E dA = \rho At/\varepsilon_0 \]

\[ \int_S dA = 2EA = \frac{\rho At}{\varepsilon_0} \]

\[ S \delta E = \frac{\rho t}{2\varepsilon_0} \]

$E_{\text{const at ends}}$
18. Consider two imaginary closed spherical surfaces S1 and S2. S1 encloses a positive charge +Q, off-center as shown, and there are no other charges nearby. S2 also encloses a positive charge +Q, off-center, and there is a negative charge −Q, just outside the surface.

Which surface has the greater magnitude electric flux?
A) S1  B) S2  C) Both surfaces have the same magnitude electric flux, which is non-zero.  D) Both surfaces have zero total flux.

\[ \Phi = \frac{Q_{\text{in}}}{\varepsilon_0} \]

\[ \text{Gauss: Same } Q_{\text{in}}, \text{ so same flux. } Q+Q \text{ so } \Phi \neq 0 \]

19. Consider the closed surface consisting of a sphere of radius R centered on the origin, as shown. This closed surface consists of two open surfaces: S1, the upper hemisphere above the xy-plane, z > 0, and S2, the lower hemisphere below the xy plane, z < 0. There is a constant uniform electric field \( \vec{E} = E \hat{k} \) filling all space. What is the magnitude of the electric flux through the upper, open hemisphere S1?

\[ |\Phi| = \left| \int_{S_1} \vec{E} \cdot d\vec{a} \right| = \]

A) \( \pi R^2 E \)  B) \( 2\pi R^2 E \)  C) \( \frac{2}{3} \pi R^3 E \)  D) \( 2\pi R E \)  E) zero

\[ \Phi_{\text{circle}} = EA = \pi R^2 E \]

(\( E \) \text{ const, } \| \to \hat{A} \) on circle)

\[ \text{Flux through } S_1 = \text{flux through shaded circle} \]
The next two questions refer to this situation: A cubical box (with edge length L=2 meters) is placed so that its edges are parallel to the coordinate axes, as shown. The space in and around the box is filled with a constant, uniform electric field given by \( \vec{E} = A\hat{x} + B\hat{y} + C\hat{z} \), where \( A = 3 \) N/C (newtons per coulomb), \( B = 2 \) N/C, and \( C = 1 \) N/C.

**Surface is flat and \( \vec{E} \) is const, so**

\[
\Phi = \vec{E} \cdot \hat{A} = (A\hat{x} + B\hat{y} + C\hat{z}) \cdot 4m^2 \hat{z} = -4C \frac{z^2}{2}
\]

**20.** In SI units, what is the magnitude of the electric flux through the bottom shaded surface (the surface at \( z = 0 \))?  
A) zero  
B) 36  
C) 8  
D) 4  
E) None of these

\[
(\Phi) = 4C \quad \text{SEE ABOVE}
\]

\[
= 4
\]

**21.** The total charge enclosed by the cubical box is...

A) zero  
B) positive  
C) negative  
D) Not enough information to answer the question

For a closed surface in a const field,  
\( \Phi = 0 \) (all lines going in also go out)

\[\text{Gauss: } \Phi = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

\[0 = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

So \( Q_{\text{enc}} = 0 \)
22. A device called a **spherical capacitor** consists of an inner metal sphere and an outer spherical metal shell. The inner sphere has radius \( R_1 \) with total net positive charge \( +Q \). The hollow metal outer spherical shell has total negative charge \( -Q \) and with inner and outer radii \( R_2 \) and \( R_3 \). **There are no other charges nearby.** What can you say about the magnitude \( E \) of the electric field in the space between inner sphere and the outer shell at distance \( r \) from the center \( (R_1 < r < R_2) \)?

A) \( E = \frac{kQ}{r^2} \)  
B) \( E < \frac{kQ}{r^2} \), but not zero  
C) \( E > \frac{kQ}{r^2} \)  
D) \( E = 0 \)

E) \( E \) can be equal, less than, or greater than \( \frac{kQ}{r^2} \), depending on the exact value of \( r \) in the interval \( (R_1 < r < R_2) \).

\[
\oint_{\text{on surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} = \frac{+Q}{\varepsilon_0}
\]

\( E \) const. \( \Rightarrow \) \( E A = \frac{Q_{\text{enc}}}{\varepsilon_0} \)

\[
E \frac{4\pi r^2}{\varepsilon_0} = \frac{Q_{\text{enc}}}{\varepsilon_0} \Rightarrow E = \frac{Q_{\text{enc}}}{4\pi \varepsilon_0 r^2} = \frac{kQ}{r^2}
\]

23. A solid metal cube with a negative net charge of \( -Q \) is in **electrostatic equilibrium**. Consider a small, imaginary cubical surface \( S \) entirely contained within the metal cube and off-center as shown. What can you say about the total flux \( \Phi = \oint \vec{E} \cdot d\vec{A} \) through the closed surface \( S \)?

A) \( \Phi > 0 \)  
B) \( \Phi = 0 \)  
C) \( \Phi < 0 \)  
D) Not enough information given to answer the question

\[
\oint \vec{E} \cdot d\vec{A} = 0 \quad \text{because} \quad \vec{E} = 0
\]

**In a metal at equilibrium**

Alternatively: \( Q_{\text{enc}} = 0 \) inside a metal so \( \Phi = \frac{Q_{\text{enc}}}{\varepsilon_0} = 0 \) (at \( \Phi \).)
24. Three charged particles labeled 1, 2, and 3 are shown in the figure below. Particle 1 has a charge of +3 mC. What is the charge of particle 2 (the one in the middle)?

Particle 2 has charge
A) +2 mC  B) –2 mC  C) –2.5 mC  D) –3.6 mC  E) None of these

12 LINES FROM CHARGE 3 REPRESENT 3 mC
SO EACH LINE REPRESENTS 3/12 = 0.25 mC

CHARGE 2 HAS 10 LINES = 10 * 0.25 mC = 2.5 mC

25. Three parallel, infinite planes (shown edge-on in the diagram) are all charged with the same magnitude charge per area \( \sigma \). The two planes on the right have a positive charge density \( +\sigma \) and the plane on the left has a negative charge density \( –\sigma \). The three planes divide space up into four regions labeled I, II, III, and IV, as shown. Note: these are planes of charge, not charged metal plates.

How does the magnitude of the field in region III (\( E_{III} \)), compare to the magnitude of the field in region II (\( E_{II} \))? 

A) \( E_{II} = 2 E_{III} \)  B) \( E_{II} = 3 E_{III} \)  C) \( E_{II} = (1/2) E_{III} \)

D) \( E_{III} = 0, E_{II} \neq 0 \)  E) None of these

\( \vec{E}_0 = -3 E_0 \hat{x} \)
\( \vec{E}_{IV} = -2 E_0 \hat{x} + E_0 \hat{x} = -E_0 \hat{x} \)
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Except for those words at the top. And these.