Possibly useful information:

\[ g = 9.8 \text{ m/s}^2 \]

Moments of inertia of circular or spherical objects with mass \( M \) and radius \( R \) about an axis coinciding with the axis of symmetry:

- **Hoop:** \( I = MR^2 \)
- **Solid disk or cylinder:** \( I = \frac{1}{2} MR^2 \)
- **Sphere:** \( I = \frac{2}{5} MR^2 \)

Uniform rod or bar of length \( L \) and mass \( M \) with the axis perpendicular to its length:

- \( I = \frac{1}{3} ML^2 \) when the axis is at one end of the bar.
- \( I = \frac{1}{12} ML^2 \) when the axis is through the middle of the bar.

Parallel Axis Theorem: \( I_{\text{axis}} = I_{\text{CM}} + Md^2 \)

- 30-60-90 triangle
- 45-45-90 triangle
1. A pendulum of length $L = 3.00$ m swings through an angle $\theta = 15.0^\circ$. To 3-place precision, what is the length $d$ of the path traveled by the pendulum mass, in centimeters?

(A) 78.5 cm    B) 26.2 cm    C) 450 cm    D) 262 cm    E) 6.94 cm

$$\Theta = \frac{s}{r} = \frac{d}{L}$$

$$d = L \cdot \Theta = 3 \text{ m} \cdot \frac{15^\circ \cdot \pi \text{ rad}}{180^\circ} = 0.785 \text{ m}$$

2. A toy gun, held stationary, is pointed straight down and contains a dart of mass $m$ on an ideal massless spring with spring constant $k$, which is compressed by a distance $d$. NOTE that $d$ is a positive quantity. What is the **magnitude of the initial acceleration** of the dart immediately after the catch is released? Note that *immediately* after the catch is released, the spring is still compressed by a distance $d$, because it has not yet had time to uncompress. (HINT: Draw a FBD.)

A) $\frac{kd}{mg}$    B) $kd - mg$    C) $\frac{kd}{m} - g$    D) $\frac{kd}{m} + g$    E) None of these

$$Hooke's \ Law: \quad F(x) = -kx = -kd$$

$$\sum F_y = ma_y \implies mg + kd = ma$$

$$a = g + \frac{k}{m} \cdot d$$

3. A small steel ball of mass $m$ is thrown straight down with an initial speed $v_0$ from an initial height $h_0$ above the floor. The ball strikes the floor in a *perfectly elastic* collision and rebounds straight up. Ignore air resistance. What is the **maximum height** of the ball above the floor after it rebounds?

A) $\sqrt{h_0^2 + \frac{v_0^2}{2g}}$    B) $\sqrt{2gh_0 + \frac{v_0^2}{2g}}$    C) $h_0 + \frac{v_0^2}{2g}$    D) $\sqrt{h_0^2 - \frac{v_0^2}{2g}}$

E) None of these.

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2} m(v_0^2) + mg h_0 = 0 + mg h_f$$

$$h_f = h_0 + \frac{v_0^2}{2g}$$

Vers. A
4. A mass m is suspended by two strings as shown. T₁ and T₂ are the (unknown) tensions in the strings. The string with tension T₁ is horizontal and the other string, with tension T₂, makes an angle θ with the horizontal, as shown. What is |T₂y|, the magnitude of y-component of the tension in the right string?

A) mg cosθ  B) \( \frac{mg}{\sin θ} \)  C) mg  D) \( \frac{mg}{\cos θ} \)  E) \( \frac{mg}{\tan θ} \)

\[ \sum F_y = 0 \Rightarrow |T₂y| = mg \]

The next two questions refer to this situation: A pair of objects with masses 4m and m are connected by a hook and have an ideal massless, compressed spring between them. Initially, the objects are at rest. Then the hook unlatches, the spring expands to its relaxed length, and the smaller mass m is then found to be moving right with speed v. Assume no friction and no thermal energy generated anywhere, and no outside forces acting on the masses.

BEFORE

\[ 4m \quad m \]

(rest)

v' = ?

AFTER

\[ 4m \quad m \]

6. After the spring is released, what is the speed of the larger mass 4m?

A) v  B) v/4  C) zero  D) v/3  E) 4v

\[ p_{tot} = 0 = mv + 4mv' \Rightarrow 4v' = v, \quad v' = -\frac{v}{4}, \quad \text{speed } |v'| = \frac{v}{4} \]
7. Two sleds are each pushed across a frictionless ice rink with the same force for the same time. The mass of sled 1 is larger, M1 > M2. The sleds begin at rest, and each is pushed with an identical, constant force, F, for 10 seconds. After each sled has been pushed for 10 seconds, which of the following statements are true?

I. After 10 s, they have the same momentum. True
II. After 10 s, they have the same kinetic energy. False
III. After 10 s, they have the same speed. False

\[ \Delta P = F \cdot \Delta t \quad \text{(from } \frac{F}{m_{net}} = \frac{d\vec{v}}{dt}) \]

\[ \text{SAME} \quad \Delta \text{KE} = W_{net} = F \cdot d \quad \text{Not Same} \]

A) Only I is true.
B) Only II is true.
C) Only III is true.
D) Exactly two of the three statements is true.
E) All three of the statements are true.

8. Ball 1 strikes Ball 2 in a collision. The figure shows the initial momenta of both Ball 1 and Ball 2 before the collision and the final momentum of Ball 1 after the collision. There are no forces on the balls except the collision forces, and the balls do not change mass as a result of the collision. As a result of the collision, did Ball 2 speed up, slow down, or keep the same speed? Ball 2.

A) speeds up. B) slows down.
C) has the same speed after the collision.
D) impossible to tell without knowing the masses of the balls
E) impossible to tell without knowing whether the collision was elastic or inelastic.

\[ |\vec{P}_{2i}| = 2, \quad |\vec{P}_{2f}| = \sqrt{1^2 + 3^2} \]

I've used \( \vec{P}_{tot} = \vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f} \)

9. A bike wheel mounted on the front desk of the lecture hall is spinning counter-clockwise, with an initial angular velocity \( \omega_0 = 20.0 \text{ rad/s} \). The wheel has moment of inertia \( I = 20 \text{ kg m}^2 \) about its center. The wheel slows with constant acceleration and, after 10.0 seconds, the wheel is at rest. Through what angle, in rads, did the wheel turn while it was slowing?

A) 50.0 rad \hspace{1cm} B) 100.0 rad \hspace{1cm} C) 150.0 rad \hspace{1cm} D) 200.0 rad

E) There is not enough information to answer the question.

\[ \Delta \omega = \omega_2 - \omega_1 = 0 - \omega \]

\[ \Delta \Theta = \omega_0 t + \frac{1}{2} \alpha t^2, \quad \alpha = \frac{\Delta \omega}{\Delta t} = -\frac{\omega_0}{t} = -20 \frac{\text{rad}}{\text{s}^2} \]

\[ \Delta \Theta = 20(10) + \frac{1}{2}(-2)(10^2) = 200 - 100 = 100 \text{ rad} \]

Vers. A
10. An object consists of 5 point masses, each of mass $m$, connected by massless rods as shown. The four short rods have length $L$ and the long rod has length $2L$. Although the diagram shows the masses as small spheres, assume they are points. What is the moment of inertia about an axis perpendicular to the diagram and through the top mass on the left?

A) $6mL^2$  
B) $8mL^2$  
C) $11mL^2$  
D) $12mL^2$  
E) $13mL^2$

$$I = \sum m_i r_i^2$$
$$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 + m_5 r_5^2$$
$$= m \cdot 0 + m \cdot L^2 + m \cdot L^2 + m \cdot (\sqrt{2}L)^2 + m \cdot (3L)^2$$
$$= m L^2 (1 + 1 + 2 + 9) = 13mL^2$$

11. A very light (massless) rod of length $L$ has a mass $m$ at one end and a frictionless axle at the other end. The rod is released from rest from the horizontal position. What is the magnitude of the initial angular acceleration $\alpha$, just after the rod is released?

A) $g$  
B) $0$  
C) $\frac{2g}{3L}$  
D) $\frac{g}{\sqrt{2}L}$  
E) $\frac{g}{L}$

$$\tau = I \alpha, \ \alpha = \frac{\tau}{I} = \frac{k \cdot \frac{mg}{r} \cdot g}{\frac{mg \cdot L^2}{r^2}} = \frac{g}{L}$$

12. Four uniform identical square tiles, each with edge length $L$ and mass $m$, are arranged as shown. The origin is at the center of the lower left tile, as shown. What is the $y$-component of the center-of-mass of this 4-tile system? $Y_{CM} =$

A) $L/2$  
B) $L$  
C) $5L/4$  
D) $3L/4$

E) None of these

$$Y_{CM} = \frac{1}{M} \sum m_i y_i$$
$$= \frac{1}{4m} \left( m \cdot 0 + m \cdot 0 + m \cdot L + m \cdot 3L \right)$$
$$= \frac{2 \cdot m \cdot L}{4 \cdot m} = \frac{L}{2}$$
The following 2 questions refer to the same ladder leaning on a wall:

13. A uniform ladder of length L and mass M is leaning stationary against a wall at an angle $\theta$ to the horizontal, as shown. The top of the ladder has a massless frictionless roller, so wall exerts only a normal force of magnitude $N_W$ on the top of the ladder. The coefficient of static friction between the floor and the ladder is $\mu_s$. The normal force from the floor has magnitude $N_F$. An extended free-body diagram is shown. Which of these is certainly a correct expression for the magnitude $f$ of the force of friction on the bottom of the ladder?

A) $\mu_s N_F$  
B) $\frac{2Mg}{\cos \theta}$  
C) $Mg \sin \theta$  
D) $N_F - N_W$

E) $N_W$  
$\sum F_x = 0 \Rightarrow N_W = f$

14. A person of mass $m$ now starts climbing the ladder, and the ladder does not slip. It remains stationary. What happens to the magnitude $\tau_{net}$ of the net torque on the ladder about an axis at the bottom end of the ladder as the person ascends the stationary ladder? The magnitude $\tau_{net}$ ...

A) increases  
B) decreases  
C) remains constant.

$\tau_{net} = 0$ since ladder is in static equilibrium

15. A clay block of mass $m$ moving to the right with speed $3v_o$ strikes a second block of the same mass $m$ moving to the left with speed $v_o$. The two clay blocks **stick together** after the collision. There are no outside forces acting on the blocks. What is the total kinetic energy of the system after the collision?

A) $(1/4) m v_o^2$  
B) $(5/2) m v_o^2$  
C) $(2/3) m v_o^2$  
D) $m v_o^2$

E) 0, all KE is lost

$P_{tot, before} = P_{tot, after}$

$\frac{1}{2} (mass) (speed)^2 = \frac{1}{2} (2m) v'_o^2 = m v_o^2$
16. A structure consists of a uniform rod of mass $m_0$ and length $L$ with an axis of rotation at its center, and two masses $m_1$ and $m_2$ attached at its ends (where $m_2 > m_1$). So the total mass of the structure is $(m_0 + m_1 + m_2)$. The rod is initially held horizontally and then released from rest. Consider the following two statements:

I. The net torque on the structure, immediately after release, depends on the mass $m_0$ of the rod. **False**

II. The moment-of-inertia of the structure depends on the mass $m_0$ of the rod. **True**

Which statements are true?

A) Both are true.  
B) Neither are true.  
C) Only I is true.  
D) Only II is true.

\[ I_{rod} = \sum_i m_i r_i^2 \neq 0 \]

\[ \tau \text{ due to mass of rod} = 0 \text{ since weight of rod acts at c.m.} \]

17. A robotic vehicle travels across flat ground, with a position vector given by

\[ \vec{r} = (2 \cdot t^2 - 4 \cdot t + 12) \hat{i} + 3 \cdot t \hat{j} \]

where $t$ is the time in seconds and units on the numbers are such that the position components are in meters. To 3-place precision, what is the speed of the vehicle at $t = 1.00 \text{ s}$?

A) $0.708 \text{ m/s}$  
B) $6.71 \text{ m/s}$  
C) $3.00 \text{ m/s}$  
D) $5.00 \text{ m/s}$  
E) None of these

\[ \dot{\vec{r}} = \frac{d\vec{r}}{dt} = (4 \cdot t - 4) \hat{i} + 3 \hat{j} \]

\[ \Rightarrow |\dot{\vec{r}}| = 3 \]

18. As the mass swings down freely from its initial horizontal position to the vertical position, what happens to the magnitude of the angular acceleration $\alpha$ and the magnitude of the angular velocity $\omega$?

A) $|\alpha|$ increases and $|\omega|$ increases  
B) $|\alpha|$ stays constant and $|\omega|$ increases  
C) $|\alpha|$ decreases and $|\omega|$ increases  
D) $|\alpha|$ decreases and $|\omega|$ decreases  
E) $|\alpha|$ stays constant and $|\omega|$ stays constant

\[ \alpha = \frac{\tau}{I} = \frac{mgL \sin \theta}{I} \rightarrow 0 \]

$\omega$ increases since $\alpha \neq 0$
19. Three objects, all with the same mass, all start from rest at the same height at the top of a ramp and then slide or roll down the ramp. At the bottom of the ramp, they take off horizontally, and then hit the ground at a horizontal distance \( x \) from the take-off point, as shown. The 3 objects are 1) a frictionless sliding ice cube, 2) a hoop, and 3) a disk. The hoop and disk have the same radius and roll without slipping. Which object hits the ground at the \textbf{largest} distance \( x \)?

\[ E_{\text{tot}} = Mgh_i = \text{SAME} \]
\[ = KE_f = KE_{\text{trans}} + KE_{\text{rot}} \]
\[ \text{largest for ice} \quad \text{0 for ice} \]

A) the hoop   B) the disk   C) the ice cube   D) they all hit at the same distance \( x \)
E) the hoop and the disk hit at the same distance, larger than the distance of the ice cube.

20. Three identical blocks, labeled I, II, and III, are on a horizontal surface, initially at rest with identical orientations, as shown. Blocks I and II have no friction between them and the table, but there is friction between Block III and the table. All three blocks are pulled with forces of equal magnitudes and direction, as shown. The forces on blocks I and III are exerted at the centers of the blocks; the force on block II is exerted at a point on the far right of the block as shown. The centers-of-mass of the blocks are marked with an \( \times \). What is the correct ranking of the accelerations of the center-of-masses of the three blocks?

\[ F_{\text{net}} = F \quad \text{NO friction} \]
\[ F_{\text{net}} = F - f \quad \text{Friction} \]

A) \( a_1 > a_{II} > a_{III} \)   B) \( a_1 = a_{II} = a_{III} \)   C) \( a_1 > a_{II} = a_{III} \)   D) \( a_1 > a_{II} > a_{III} \)
E) Impossible to answer without more information

\[ F_{\text{net}} = ma, \quad a = \frac{F_{\text{net}}}{m} \]

Vers. A
21. A uniform horizontal rod of length $L$ and mass $M$ has a frictionless hinge at the left end and is held in place by a wire at the right end as shown. The wire makes an angle $\theta$ with the rod. Which expression is equal to the magnitude of the horizontal component of the force on the left end of the rod by the hinge?

A) zero  B) $Mg \frac{L}{2}$  C) $T \cos \theta$
C) $T \sin \theta - Mg$  E) None of these

\[ \sum F_x = 0 \]

22. In the diagram below, the final momentum $\vec{p}_2$ of an object after a collision and its change in momentum $\Delta \vec{p}$ are shown. Which choice gives the correct vector for the initial momentum $\vec{p}_1$?

\[ \vec{p}_1 + \Delta \vec{p} = \vec{p}_2 \]

23. Two gliders, labeled A and B, are on a frictionless air track, initially heading toward each other. The mass of A is greater than the mass of B. After they collide, glider A is moving left and glider B is moving right, with equal speeds, as shown. (The arrows in the diagram only show direction of motion; their lengths do not indicate speed or magnitude of momentum.)

Before the collision, how does the magnitude of the momentum of A compare to the magnitude of the momentum of B?

A) $|p_A| > |p_B|$  B) $|p_A| < |p_B|$  C) $|p_A| = |p_B|$

D) Not enough information is given to answer the question.

Since $m_A > m_B$, you can see that $\vec{p}_{\text{tot}}$ is LEFT, after.

$\Rightarrow$ So $\vec{p}_{\text{tot}} = \vec{p}_A + \vec{p}_B$ must be LEFT before collision.

Vers. A $\Rightarrow |p_B| > |p_A|$ before collision
24. Two objects labeled A and B are attached to a merry-go-round which is spinning at a constant rate of 6 rpm (revolutions per minute). There is a motor on the merry-go-round which keeps the rotation rate constant. Object A is near the center and object B is near the rim. Consider the following 4 quantities of each object:

1) the speed \( v \),
2) the magnitude of the angular velocity \( \omega \), SAME \( \omega \) (\( \gamma = 0 \))
3) the magnitude \( \alpha \) of the angular acceleration, SAME \( \alpha \)
4) the magnitude \( a \) of the linear acceleration.

How many of these 4 quantities are larger for object B than for object A?

A) None of them is larger for B  B) Only 1 of them is larger for B.
C) Only 2 are larger for B  D) 3 of them are larger for B
E) All 4 (object B has larger \( v \), \( \omega \), \( \alpha \), and \( a \))

\( v = \omega \cdot r \) larger for B
\( \omega \) same for both
\( \alpha = 0 \) same for both
\( a = \frac{v^2}{r} = \omega^2 \cdot r \) larger for B

25. A uniform disk of mass 3M and radius R is rotating on a frictionless axis about its center with initial angular velocity of magnitude \( \omega_0 \). A non-rotating uniform disk of mass M and the same radius R is dropped directly on top the spinning mass 3M, as shown. Because of friction, the two masses quickly start spinning together and achieve the same final magnitude angular velocity \( \omega_f \). What is \( \omega_f \)?

A) \( \omega_0 \)  B) \( \frac{2}{3} \omega_0 \)  C) \( \frac{\omega_0}{3} \)  D) \( \frac{3}{2} \omega_0 \)  E) \( \frac{3}{4} \omega_0 \)

\[ I_B = \frac{1}{2} (3M) R^2 \quad \text{(bottom)} \]
\[ I_T = \frac{1}{2} MR^2 \quad \text{(top)} \]
\[ I_{tot} = I_B \omega_0 = (I_T + I_B) \omega_f \]

\[ (\frac{1}{2} 3M R^2) \omega_0 = \frac{1}{2} (1M + 3M) R^2 \omega_f \]
\[ 3 \omega_0 = 4 \omega_f \quad \omega_f = \frac{3}{4} \omega_0 \]

Vers. A