MATHEMATICAL METHODS: ASTR-5540

This document presents topical guidelines for instructors of one of the five APS core graduate courses. It is provided as a reference to support instructors in their syllabus preparation, and to assist the APS Examinations Committee in their review of those syllabi. Following each set of primary/recommended topics (in black), we list suggested optional topics (in *violet*) and example applications to APS research fields (in green) suitable for student projects, scientific coding, or homework exercises. It is anticipated that instructors focus at least two-thirds of class time on the primary course topics, with the remaining time spent on optional topics or other related topics of the instructor's choosing. Instructors are encouraged to draw upon a range of examples from astrophysics, planetary science, and solar/space physics to illustrate the core material. The current version of these guidelines was adopted by the AY20-21 and AY21-22 Graduate Curriculum and Concerns Committees (GCCC). Future changes/updates will be made regularly; alternately, changes can be proposed to the GCCC.

Linear Algebra

Brief review of matrices, inversion, and linear systems (as necessary)
Eigenvalues and eigenvectors; diagonalization
Numerical methods for matrices
Root finding (e.g., Newton-Raphson); interpolation
Vector spaces; Gram-Schmidt orthogonalization; techniques for sparse matrices
Application: linear least-squares; fitting data to models (e.g., galaxy M-σ relation)
Application: rotation matrices for transforming coordinate systems

Ordinary Differential Equations: Analytic Methods

Classification of ODEs
Basic solution methods (separable equations; integrating factors)
Series solutions; singular points; *Frobenius method*
Nonlinear ODEs; perturbation analysis
Application: coupled ODEs for nuclear/chemical reaction chains; predator-prey equations

Ordinary Differential Equations: Numerical Methods

Introduction to approaches; concepts of order, accuracy, stability
Finite-difference techniques for initial & boundary value problems
Application: solution of the Friedmann equations in cosmology
Application: leapfrog & symplectic techniques for N-body dynamical systems

Special Functions

ODE-based functions: Bessel, Legendre, Chebyshev, Laguerre; spherical harmonics
Integral-based functions: gamma, beta, error functions; exponential, Fresnel integrals
Application: spherical harmonics: asteroseismology, magnetic fields, or gravitational waves

Integral Transforms

Sturm-Liouville problems; expansions in eigenfunctions
Fourier series and integrals
Fourier and Laplace transforms; convolution & power spectra; FFTs
Wavelet transforms
Application: time-series problems in stellar variability, exoplanet radial velocities, transits

Partial Differential Equations: Analytic Methods

Classification of PDEs, boundary conditions; characteristics
Separation of variable techniques
Solutions using integral transforms
Green's functions

Partial Differential Equations: Numerical Methods

Introduction to approaches: finite-difference, finite-element, spectral
Von Neumann stability analysis; CFL condition
Comparison of schemes for hyperbolic, parabolic, & elliptic equations
Application: boundary-value problems for interiors of planets, stars, & compact objects