MATHEMATICAL METHODS

This model syllabus defines the core material for Mathematical Methods. Instructors should use their discretion in deciding the ordering of topics, the depth to which each is covered, and additional material to include (example: complex analysis would be a logical additional topic that could be covered with a strong class). It is anticipated that instructors will draw upon a range of examples from astrophysics and planetary science to illustrate the core material.

LINEAR ALGEBRA

Brief review of linear algebra, as necessary

Matrix definition, algebra
Vector spaces, Gram-Schmidt orthogonalization
Eigenvalues and eigenvectors, diagonalization
Matrix inversion
Numerical methods for linear algebra

ORDINARY DIFFERENTIAL EQUATIONS

Review of ODEs, as necessary

Classification
Basic solution methods: exact differentials, integrating factors
Series solutions, singular points, Frobenius method, second solution
Green functions
Nonlinear ODEs, perturbation analysis

NUMERICAL METHODS FOR ODEs

Goal: to be able to understand the appropriate techniques to use to solve different ODEs numerically, and their limitations

Introduction to approaches, finite difference methods, concepts of order, accuracy, stability
Techniques for initial value problems
Techniques for boundary value problems

INTEGRAL TRANSFORMS

Understanding the basis of techniques such as convolution, power spectrum estimation, etc

Sturm-Liouville problems, expansion in eigenfunctions

Fourier series and integrals
Fourier and Laplace transforms
Applications e.g. to convolution, Fast Fourier Transforms
Wavelets

PARTIAL DIFFERENTIAL EQUATIONS

Classification of PDEs, boundary conditions
Characteristics
Separation of variables
Solution using integral transforms
Green functions

SPECIAL FUNCTIONS

Bessel functions
Legendre polynomials
Spherical harmonics

NUMERICAL METHODS FOR PDEs

Goal: an understanding of some of the techniques for solving PDEs, and the considerations involved in choosing methods (stability, efficiency, obeying conservation laws, etc).

Introduction to different approaches: finite difference, spectral methods.
Von Neumann stability analysis, the CFL condition
Examples of numerical schemes for hyperbolic and parabolic equations
Relation of the numerical system to the physical PDE
Elliptic equations: Solution via direct methods and via relaxation