ASTR 5540 Mathematical Methods Fall 2019 Toomre
(http://zeus.colorado.edu/astr5540-toomre)

MWF 10:00-10:50am Duane E126

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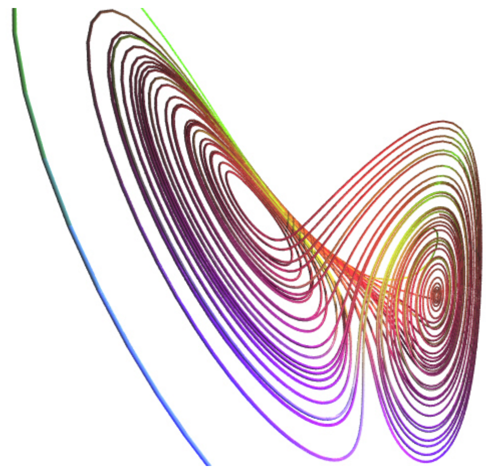

This course is intended to help refine your perspectives about a variety of mathematical methods essential to many areas of research in astrophysical and planetary sciences. Central to these broad disciplines is understanding the properties of differential equations, for these are the building blocks for most models of the underlying physics and dynamics. We turn to combinations of analytical and numerical methods for seeking solutions to both ordinary and partial differential equations.

Part of the material involves brief reviews, followed by discussion of modern methods, including the use of numerical experiments. Topics to be covered encompass ordinary differential equations, complex functions, integral transform techniques, partial differential equations, special functions and asymptotic methods, and the richness of dynamical systems that admit chaos. The lectures are supplemented by problem sets, some of which require use of numerical solutions and experimenting, typically using workstations or laptops and IDL (Interactive Data Language), Mathematica or Matlab as appropriate.

Course textbooks:

RILEY, KF, HOBSON, MP & BENCE SJ, *Mathematical Methods for Physics and Engineering,* 2006, Third Edition, Cambridge, ISBN 978-0-521-67971-8.

(optional) RILEY, KF & Hobson, MP, *Student Solution Manual for Mathematical Methods for Physics and Engineering*, 2006, Third Edition, Cambridge, ISBN 978-0-521-67973-2.

(optional) PRESS, WH, Teukolsky, SA, Vetterling, WT & Flannery, BP, *Numerical Recipes: The Art of Scientific Computing*, 2007, Third Edition, Cambridge, ISBN 978-0-521-88068-8.

Useful reference books:

Acton, *Numerical Methods That (Usually) Work*, 1970
Arfken & Weber, *Mathematical Methods for Physicists, Sixth Edition,* 2005
Bender & Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, 1978
Carrier & Pearson, *Partial Differential Equations*, 1991
Mathews & Walker, *Mathematical Methods of Physics*, 1970
Strang, *Introduction to Applied Mathematics*, 1986

General topics to be discussed (ordering may be adjusted/tuned):

1. Ordinary differential equations:
    a. Review basic methods for seeking solutions in closed form
    b. Greens functions and superposition
    c. Harmonic oscillator applications
    d. Systems of ODEs, eigenvectors and eigenvalues
2. Numerical solutions of ODEs
    a. Introduction to computational approaches
    b. Initial value problems: explicit, implicit methods; multi-step, compound methods
    c. Boundary value problems: shooting, relaxation
    d. Dynamical systems with chaos
3. Asymptotic methods
    a. Regular perturbation theory
    b. Singular perturbations
    c. WKB approximations and turning points
4. Complex functions
    a. Review of analyticity and analytic continuation
    b. Integration in complex plane, integral and residue theorems
    c. Conformal transformations
5. Integral transform methods
    a. Fourier series and integrals
    b. Fourier and Laplace transforms
    c. Applications, signal/noise analysis
6. Partial differential equations
    a. Classification and boundary conditions; well-posedness
    b. Characteristics
    c. Separation of variables and variety of tractable examples
7. Special functions
    a. Legendre polynomials and spherical harmonics
    b. Bessel functions
    c. Sturm-Liouville theory, self-adjoint differential equations
    d. Applications to various PDEs
8. Numerical solutions of PDEs
    a. Introduction to issues
    b. Parabolic equations: FTCS, von Neuman stability, BTCS, Crank-Nicolson
    c. Elliptic equations: finite-difference 5 point formula
    d. Solvers: direct (Gauss elim), relaxation (Jacobi, Gauss-Seidel, SOR, multigrid, ADI), rapid solvers (spectral)