ASTR5540: Mathematical Methods (Fall 2018)

MWF 10:00 – 10:50am Duane E126

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Office Hours: MW 1:00 – 2:00pm or readily by appointment

ASTR 5540 (3). Mathematical Methods. Presents an applied mathematics course designed to provide the necessary analytical and numerical background for courses in astrophysics, plasma physics, fluid dynamics, electromagnetism, and radiation transfer. Topics include integration techniques, linear and nonlinear differential equations, WKB and Fourier transform methods, adiabatic invariants, partial differential equations, integral equations, and integrodifferential equations. Draws illustrative examples from the areas of physics listed above. Same as ATOC 5540.

Analytics
Numerics
Statistics – ASTR5550: Observations, Data Analysis, and Statistics

Typical topics that could be covered in a course of this nature:

Infinite series
Vector calculus
Coordinate systems and coordinate transformation
Differential geometry

Linear Algebra -- Matrices, vector spaces, and eigenvalue problems

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Ordinary differential equations
- Series solutions / special functions
- Sturm-Liouville theory and orthogonal functions
- Fourier analysis
- Green functions
- Nonlinear differential equations
- Dynamical systems, maps, and chaos

Partial differential equations
- Separation of variables
- Integral transforms
- Numerical solutions

Complex variables and functions of complex variables

Calculus of variations
Perturbation theory
Integral equations
Integro-differential equations
Group Theory

Textbooks:

Arfken, G.B., Weber, H.J., & Harris, F.E. 2013, *Mathematical Methods for Physicists* (Elsevier).
A resource to come back to.

Riley, K.F., Hobson, M.P., & Bence, S.J. 2006, *Mathematical Methods for Physics and Engineering* (Cambridge).
Very pedagogical (available as pdf)

Pletcher, R.H., Tannehill, J.C., & Anderson, D.A.2013, Compiutational Fluid Mechanics and Heat Transfer (CRC Press).
Gridded methods (not Galerkin or spectral methods)

Bender, C.M. & Orszag, S.A. 1978, Advanced Mathematical Methods for Scientists and Engineers I: Asymptotic Methods and Perturbation Theory (Springer)
Advanced, but excellent. Approximate analytic solutions.

Morse, P.M. & Feshbach, H. 1953, Methods of Theoretical Physics (McGraw-Hill)
Greatest single two-volume math methods book

Smith, G.D. 2004, *Numerical Solution of Partial Differential Equations* (Clarendon).
Old, limited, but good

Strang, G. 1986, Introduction to Applied Mathematics (Wellesley-Cambridge).
An unusual perspective

Press, W.H. et al. 2007, Numerical Recipes (Cambridge University Press
See website numerical.recipes

Course outline (subject to modification):

1. Linear Algebra, matrices, and vector spaces
    a. Matrix definitions, algebra, and operations
    b. Vector spaces, Gram-Schmidt orthogonalization
    c. Eigenvalues and eigenvectors, diagonalization
    d. Martix inversion: Gauss-Jordan and Gaussian elimination. LU and SV decomposition

2. Ordinary differential equations (ODEs):
    a. Classification, existence and uniqueness
    b. Basic methods: exact differentials. integrating factors
    c. Series solutions, singular points, method of Frobenius, second solution
    d. Green functions

3. Numerical Solution of ODEs:
    a. Introduction to computational approaches, finite-difference approximations, order, error and stability
    b. Initial value problems: explicit vs. implicit methods, multi-step methods
    c. Boundary value problems: shooting methods, relaxation
    d. Linear and nonlinear oscillators

4. Integral transforms
    a. Sturm-Liouville problems, eigenfunction expansion
    b. Fourier series and integrals
    c. Fourier and Laplace transforms
    d. Applications, convolution, FFTs

5. Partial differential equations (PDEs):
    a. Classification and boundary conditions: well-posedness
    b. Characteristics
    c. Separation of variables, integral transform methods
    d. Green functions: construction by eigenfunction expansion, integral transform, and delta slope methods

6. Special functions and their application to PDEs:
    a. Bessel functions
    b. Legendre polynomials and spherical harmonics
    c. Application to helioseismology

7. Numerical solution of PDEs:
    a. Introduction to issues
    b. Parabolic equations: FTCS, von Neuman stability, Crank-Nicolson
    c. Hyperbolic equations: failure of FTCS, CFL condition, leapfrog, Lax-Wendroff
    d. Elliptic equations: finite-difference 5-point stencil
    e. Solvers: direct, relaxation (Jacobi, Gauss-Seidel, SOR, multigrid)
    f. Spectral methods, shock capturing schemes

Grading:

Homework (approx one every week or two, collaboration encouraged) 50%
Midterm (take home, no collaboration) 25%
Final (take home, no collaboration) 25%

Travel dates:

9/12, 9/14, 12/7